Thermally isolated Luttinger liquids with noisy Hamiltonians

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L. D'Alessio (BU) and A. Rahmani (LANL), arXiv:1207.1441v1

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Outline

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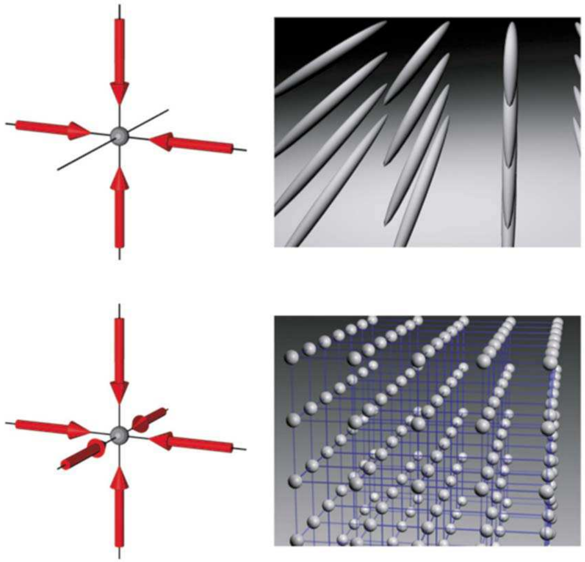
1. Introduction and motivation
2. Generic setup
3. Two types of energy moments
4. Energy fluctuations in noisy Luttinger liquids
5. Correlation functions
6. Fokker-Planck approach
7. Summary

Introduction and motivation

Cold atoms in optical lattices provide clean realizations of many-body Hamiltonians in isolation from a thermal environment.



Bloch, Dalibard, and Zwerger, RMP (2008).

Motivated extensive studies of nonequilibrium quantum dynamics of thermally isolated systems:

Polkovnikov, Sengupta, Silva, and Vengalattore, RMP (2011).

Unitary evolution with Schrödinger equation, time-dependent Hamiltonian.

Fully deterministic evolution:

$$i\partial_t \psi(t) = H(g(t))\psi(t)$$

Introduction and motivation

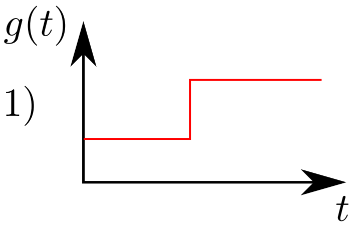
Given $H(t) \equiv H(g(t))$ and $\psi(0)$, what happens at a later time t?

Examples:

1. Quantum quench: $g(t) = g_1$ for $t < 0$, and $g(t) = g_2$ for $t > 0$.

2. Linear ramping: $g(t) = g_0$ for $t < 0$, and $g(t) = g_0 + \gamma t$ for $t > 0$.

1)



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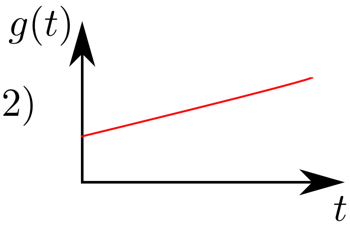

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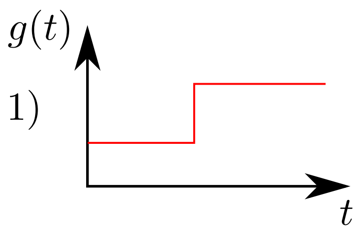
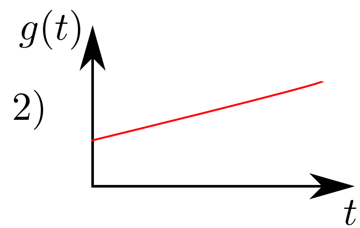
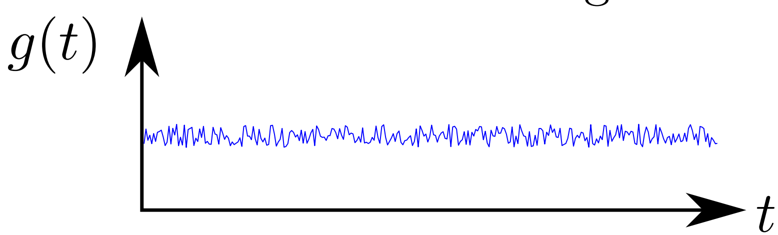




What about stochastic driving of the system?



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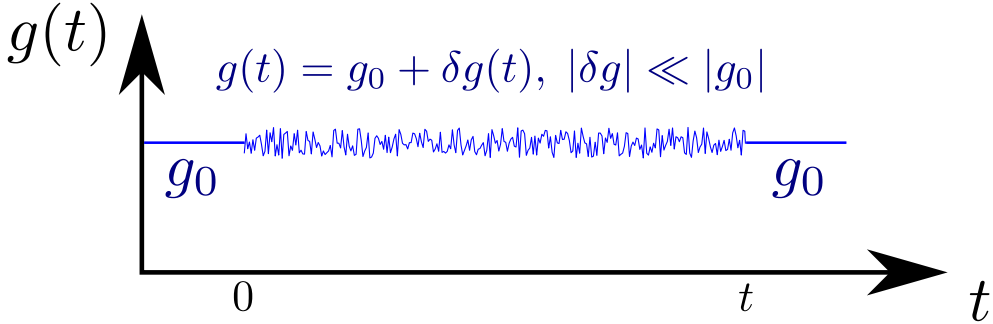
Generic setup



New ingredient: Averaging over noise.

Recent studies:
Bunin, D'Alessio, Kafri, and Polkovnikov (2011).
Marino and Silva (2012).
Pichler, Schachenmayer, Simon, Zoller, and Daley (2012).

Questions:

- How can we characterize the absorbed energy?
- What happens to the correlation functions?

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Two types of energy moments

The absorbed energy is a time-dependent random variable ϵ. We are interested in $\mathrm{E}(\epsilon)$ and $\mathrm{Var}(\epsilon)$.

What do we mean by ϵ? We have a coherent superposition for each realization of noise.

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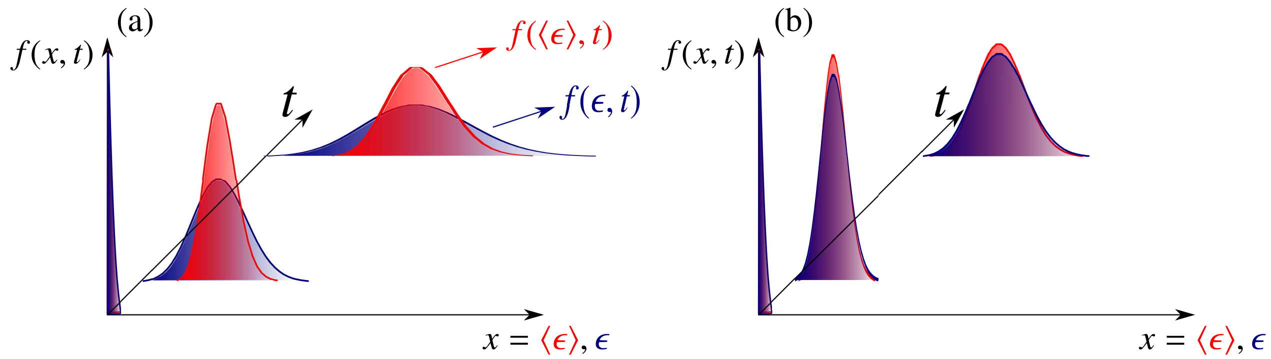
Two sources of fluctuations:

Classical (different wave functions): $f(\psi, t)$

Quantum (internal structure of wave functions): $|\psi\rangle = \sum_n c_n^{\psi} |n\rangle$



Two types of energy moments

This leads to two types of moments:

$$\begin{cases} \overline{\langle \epsilon^m \rangle} = \int d\psi f(\psi,t) \langle \psi | H_0^m | \psi \rangle = \mathrm{tr}\left[H_0^m \rho(t) \right] \\ \overline{\langle \epsilon \rangle^m} = \int d\psi f(\psi,t) \left(\langle \psi | H_0 | \psi \rangle \right)^m \end{cases}$$

One average: $\mathrm{E}(\epsilon) = \overline{\langle \epsilon \rangle}$

Two variances: $\mathrm{Var}_1(\epsilon) = \overline{\langle \epsilon^2 \rangle} - \left(\overline{\langle \epsilon \rangle} \right)^2$

$$\mathrm{Var}_2(\epsilon) = \overline{\langle \epsilon \rangle^2} - \left(\overline{\langle \epsilon \rangle} \right)^2$$

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Energy fluctuations in a noisy Luttinger liquid

$$H(K) = u \sum_{q>0} \left(K\, \Pi_q \Pi_{-q} + \frac{1}{K}\, q^2\, \Phi_q \Phi_{-q}\right)$$

$$H(K(t)) = H(K_0 + \delta K(t)) \text{ for } t > 0$$

Traditional way: integrate out the noise at the outset.

Leads to quartic terms in the bosonic fields.

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An alternative approach: working directly with wave functions.

Two harmonic oscillators per mode q: $H = \sum_{q>0} \left(H_q^{\Re} + H_q^{\Im} \right)$

The wave function for any $K(t)$ can be written as

$$\Psi(\{\Phi_q\}, t) = \prod_{q>0} \left(\frac{2 \, q [\Re \, z_q(t)]}{\pi} \right)^{\frac{1}{2}} \exp \left(-q \, z_q(t) \, |\Phi_q|^2 \right)$$

Energy fluctuations in a noisy Luttinger liquid

The parameters z_q satisfy:

$$i\dot{z}_q(t) = \tfrac{q}{K(t)}\left[(K(t)\,z_q(t))^2 - 1\right], \qquad z_q(0) = K_0^{-1}$$

Langevin equation:

$$\begin{cases} i\dot{z}_q = \tfrac{q}{K_0}\left(K_0^2 z_q^2 - 1\right) - q\left(K_0^2 z_q^2 + 1\right)\delta\alpha \\ \qquad\qquad \delta\alpha(t) = -\delta K(t)/K_0^2 \\ \overline{\delta\alpha(t_1)\delta\alpha(t_2)} = W^2\delta(t_1 - t_2) \end{cases}$$

Energy fluctuations in a noisy Luttinger liquid

Perturbative treatment:

Experimentally, the regime of interest is $|\delta z_q| \ll K_0^{-1}$.

To leading order:

$i\,\delta \dot{z}_q = 2q\ (\delta z_q - \delta\alpha) \Longrightarrow \delta z_q(t) = 2iq \int_0^t dt'\, e^{2iq(t'-t)} \delta\alpha(t')$.

$$\boxed{\overline{\langle \epsilon \rangle} \approx L\pi^3\, K_0^2 W^2 t/4}$$

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$$\langle\epsilon^2\rangle \equiv \langle H^2\rangle, \quad \langle\epsilon\rangle^2 \equiv \langle H\rangle^2, \quad H = \sum_{q>0}\left(H_q^{\Re} + H_q^{\Im}\right)$$

$$\langle H_{q_1} H_{q_2}\rangle \qquad \langle H_{q_1}\rangle\langle H_{q_2}\rangle$$

only different for $q_1 = q_2$ in the same $\Re$ or $\Im$ sector

Energy fluctuations in a noisy Luttinger liquid

$$\overline{\langle \epsilon^2 \rangle} = \overline{\langle \epsilon \rangle}^2 + 2 \sum_q \left(\overline{\langle H_q^2 \rangle} - \overline{\langle H_q \rangle^2} \right)$$

Energy fluctuations in a noisy Luttinger liquid

$$\underbrace{\overline{\langle \epsilon^2 \rangle} - \left(\overline{\langle \epsilon \rangle}\right)^2}_{\mathcal{O}(L^2)} = \underbrace{\overline{\langle \epsilon \rangle^2} - \left(\overline{\langle \epsilon \rangle}\right)^2}_{\mathcal{O}(L^2)} + \underbrace{2 \sum_q \left(\overline{\langle H_q^2 \rangle} - \overline{\langle H_q \rangle^2} \right)}_{\mathcal{O}(L)}$$

In the thermodynamic limit classical fluctuations dominate.

Energy fluctuations in a noisy Luttinger liquid

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In the thermodynamic limit classical fluctuations dominate.

$$\overline{\langle\epsilon\rangle^2} - \left(\overline{\langle\epsilon\rangle}\right)^2 = \frac{1}{16}\pi^6 K_0^4 W^4 t^2 L^2 \mathcal{F}(\pi t)$$

$$\mathrm{Var}(\epsilon) = \mathcal{F}(\pi t)\left[\mathrm{E}(\epsilon)\right]^2$$

independent of K_0 and W

$$\mathcal{F}(x) \sim \begin{cases} 2\left(1 - \frac{4}{9}x^2\right), & x \ll 1 \\ \frac{16\pi}{7x}, & x \gg 1 \end{cases}$$

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Correlation functions

Expressions in terms of z_q:

$$\langle \Phi(x)\Phi(x')\rangle = \frac{1}{L}\sum_{q>0} \frac{\cos\left[q(x-x')\right]}{q\Re z_q}$$

Correlation function	Ground state	Correction due to noise
$\overline{\langle \Phi(x)\Phi(x')\rangle}$	$\ln(x-x')$	$\propto W^2 t^3 (x-x')^{-4}$
$\overline{\langle \partial_x \Phi(x) \partial_{x'} \Phi(x')\rangle}$	$(x-x')^{-2}$	$\propto W^2 t^3 (x-x')^{-6}$
$\overline{\langle e^{i\nu\Phi(x)} e^{-i\nu\Phi(x')}\rangle}$	$(x-x')^{-\nu^2/2}$	$\propto W^2 t^3 (x-x')^{-4-\nu^2/2}$

$$\mathcal{C}^{(0)}(x) \sim x^{-\Delta},\ \mathcal{C}(x) - \mathcal{C}^{(0)}(x) \propto W^2 t^3 x^{-\Delta-4}$$

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Fokker-Planck approach

So far everything has been purterbatve in δz_q (linearized Langevin equation and expanded observables).

Can we go beyond this approximation?

$$i\dot{z}_q = \frac{q}{K_0}\left(K_0^2 z_q^2 - 1\right) - q\left(K_0^2 z_q^2 + 1\right)\delta\alpha$$

Systematic approach based on Fokker-Plank equation:

General formalism:

$\vec{a}$: a vector of real stochastic variables

$$\partial_t a_i = h_i(\vec{a}) + g_i(\vec{a})\gamma(t), \quad \overline{\gamma(t)\gamma(t')} = 2\delta(t - t')$$

Fokker-Planck approach

$$\partial_t f(\vec{a},t) = \mathcal{D} f(\vec{a},t) \qquad \mathcal{D} = -\frac{\partial}{\partial a_i} h_i - \frac{\partial}{\partial a_i}\frac{\partial g_i}{\partial a_j} g_j + \frac{\partial}{\partial a_i}\frac{\partial}{\partial a_j} g_i g_j$$

$$\overline{G(\vec{a})}\Big|_t = \int \prod_i da_i \, G(\vec{a}) \, e^{\mathcal{D}t} f(\vec{a},0)$$

$$\overline{G(\vec{a})}\Big|_t = \int \prod_i da_i \, f(\vec{a},0) \, e^{\mathcal{D}^\dagger t} G(\vec{a})$$

$$\mathcal{D}^\dagger = \left(h_i + \frac{\partial g_i}{\partial a_j} g_j \right) \frac{\partial}{\partial a_i} + g_i g_j \frac{\partial}{\partial a_i}\frac{\partial}{\partial a_j}$$

$$f(\vec{a},0) = \prod_i \delta(a_i - a_i(0))$$

A systematic way to obtain a Taylor expansion to a finite order, and an exact solution if we can resum.

Fokker-Planck approach

Noise-averaged energy

Perturbative

$\langle H_q(K_0)\rangle \approx \frac{q}{4}\, K_0^2 |\delta z_q|^2$ $\quad i\,\delta\dot{z}_q = 2q\;(\delta z_q - \delta\alpha)$ $\quad \overline{\langle H_q\rangle} \approx q^3 K_0^2 W^2 t$

Fokker-Planck approach

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Nonperturbative

$\mathscr{R}_q \equiv \Re z_q, \quad \mathscr{I}_q \equiv \Im z_q$

$$\langle H_q(K_0)\rangle = \frac{q}{2}\left\{\frac{1}{2K_0\mathscr{R}_q}\left[1 + K_0^2(\mathscr{R}_q^2 + \mathscr{I}_q^2)\right] - 1\right\}$$

$$\dot{\mathscr{R}}_q = 2K_0 q\, \mathscr{R}_i \mathscr{I}_q 2K_0^2 q_i\, \mathscr{R}_q \mathscr{I}_q\, \delta\alpha,$$

$$\dot{\mathscr{I}}_q = K_0 q\ \left(\mathscr{I}_q^2 - \mathscr{R}_q^2 + K_0^{-2}\right) - K_0^2 q\ \left(\mathscr{I}_q^2 - \mathscr{R}_q^2 - K_0^{-2}\right)\ \delta\alpha.$$

$$\overline{\langle H_q \rangle} = \frac{q}{2}\left[\exp\left(2q^2 W^2 K_0^2 t\right) - 1\right]$$

$$\overline{\langle \epsilon \rangle} = \frac{L}{8\pi K_0^2 W^2 t}\left(e^{2K_0^2\pi^2 W^2 t} - 2\pi^2 K_0^2 W^2 t - 1\right)$$

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Summary

- We studied a thermally isolated Luttinger liquid driven with a noisy Luttinger parameter by a mapping to classical statistical mechanics.

- We characterized energy fluctuations by two types of energy moments, and argued that classical fluctuations dominate in the thermodynamic limit.

- We found two relationships that exhibited some universal features:

$$\mathrm{Var}(\epsilon) = \mathcal{F}(\pi t)\left[\mathrm{E}(\epsilon)\right]^2$$

$$\mathcal{C}(x) - \mathcal{C}^{(0)}(x) \propto W^2 t^3 x^{-\Delta-4}, \quad \mathcal{C}^{(0)}(x) \sim x^{-\Delta}$$