Non-equilibrium dynamics of ultracold bosons in optical lattices.

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Overview

1. *Introduction to ultracold bosons*

2. *Ramp dynamics of the Bose-Hubbard model*

3. *Periodic dynamics: freezing*

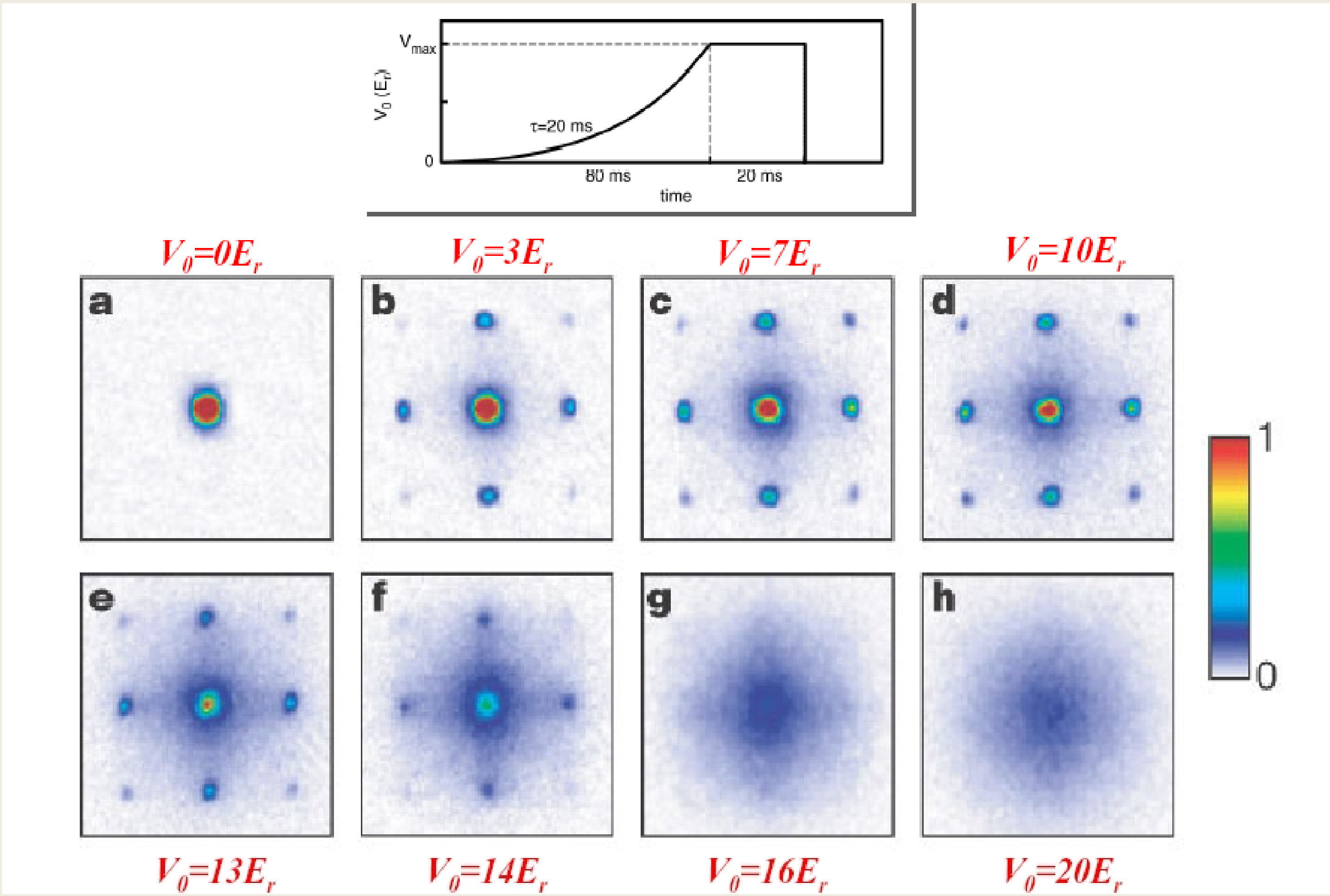
4. *Bosons in an electric field: Theory and experiments*

5. *Dynamics of bosons in an electric field*

6. *Conclusion*

Introduction to ultracold bosons

State of cold bosons in a lattice: experiment **Bloch 2001**



From BEC to the Mott state

Apply counter propagating laser: standing wave of light.

The atoms feel a potential **V = -α |E|²**

Energy Scales

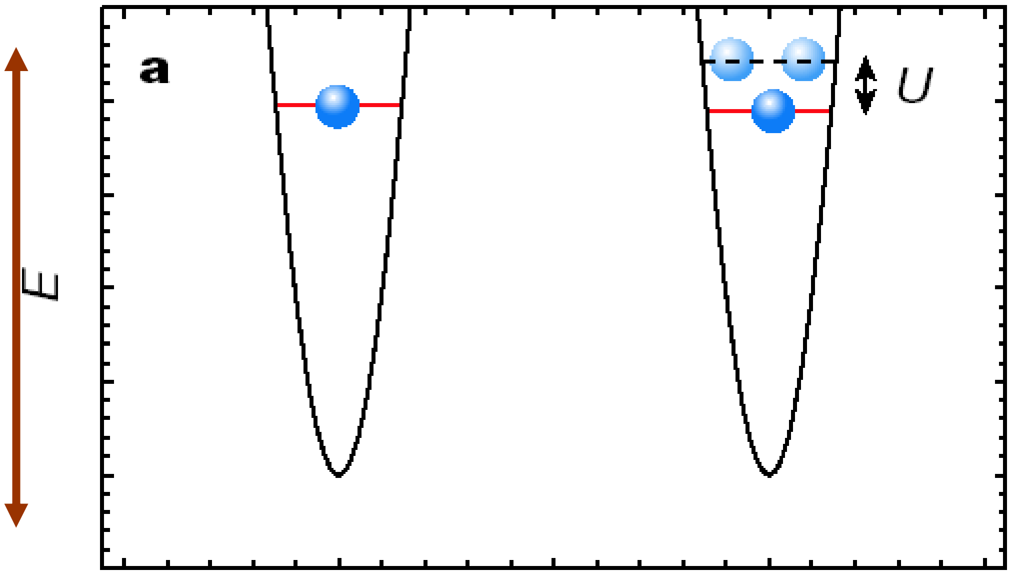
$\delta E_n = 5E_r \sim 20\ U$
$U \sim 10 - 300\ t$

For a deep enough potential, the atoms are localized : Mott insulator described by single band Bose-Hubbard model.

Model Hamiltonian

$$\mathcal{H} = -t \sum_{<i,j>} \left(b_i^\dagger b_j + \text{h.c.}\right) + \frac{U}{2}\sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

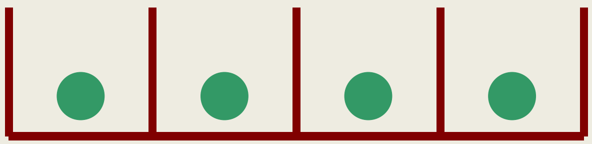
$20E_r$



Ignore higher bands

Mott-Superfluid transition: preliminary analysis

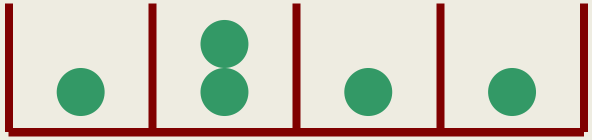
Mott state with 1 boson per site



$$\mathcal{H}_{\text{on}-\text{site}} = \frac{U}{2}\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

Stable ground state for $0 < \mu < U$

Adding a particle to the Mott state

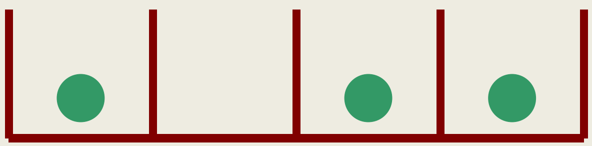


Mott state is destabilized when the excitation energy touches 0.

$$\delta E_p = (-\mu + U) - 2zt$$

$$t_p^c = (-\mu + U)/2z$$

Removing a particle from the Mott state



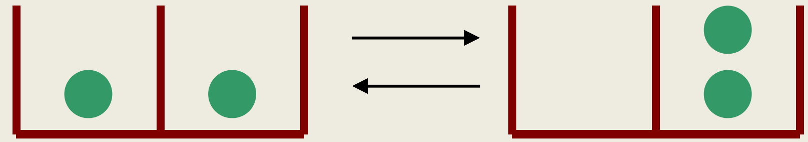
$$\delta E_p = \mu - \_zt$$

$$t_c^h = \mu/{}^{-}z$$

Destabilization of the Mott state via addition of particles/hole: onset of superfluidity

Beyond this simple picture

Higher order energy calculation by Freericks and Monien: Inclusion of up to $O(t^3/U^3)$ virtual processes.

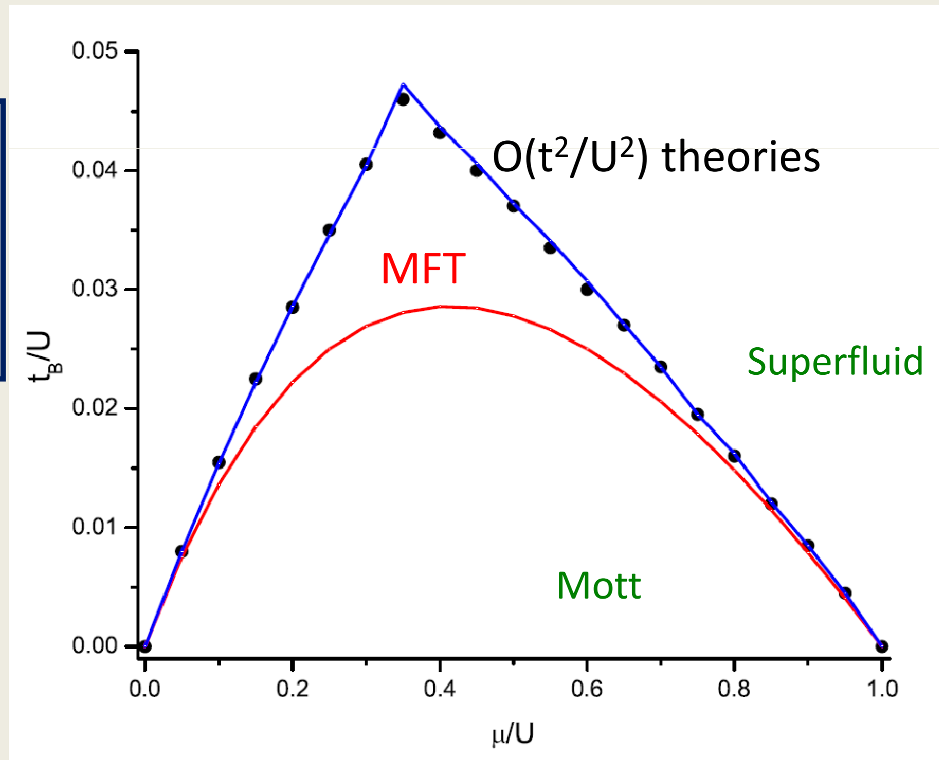


Mean-field theory (Fisher 89, Seshadri 93)

Quantum Monte Carlo studies for 2D & 3D systems: Trivedi and Krauth, B. Sansone-Capponegro

Predicts a quantum phase transition with z=2 (except at the tip of the Mott lobe where z=1).

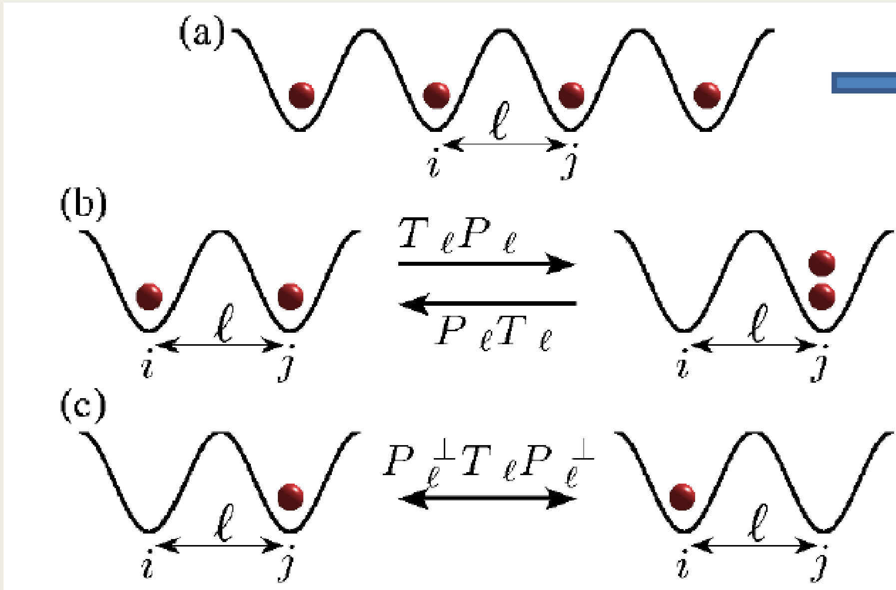
Phase diagram for n=1 and d=3



No method for studying dynamics beyond mean-field theory

Projection Operator Method

Distinguishing between hopping processes



Mott state

Distinguish between two types of hopping processes using a projection operator technique

Define a projection operator

$$P_\ell = |\bar{n}\rangle\langle\bar{n}|_{\mathbf{r}} \times |\bar{n}\rangle\langle\bar{n}|_{\mathbf{r}'}$$

Divide the hopping to classes (b) and (c)

$$T = \sum_{\langle \mathbf{r}\mathbf{r}'\rangle} -J b^\dagger_{\mathbf{r}} b_{\mathbf{r}'} = \sum_\ell T_\ell = \sum_\ell [(P_\ell T_\ell + T_\ell P_\ell) + P_\ell^\perp T_\ell P_\ell^\perp]$$

Building fluctuations over MFT

Design a transformation which eliminate hopping processes of class (b) perturbatively in J/U.

$$S \equiv S[J] = \sum_\ell i[P_\ell, T_\ell]/U$$

Obtain the effective Hamiltonian

$$H^* = \exp(iS)\mathcal{H}\exp(-iS)$$

$$\begin{aligned} H^* &= H_0 + \sum_\ell P_\ell^\perp T_\ell P_\ell^\perp - \frac{1}{U}\sum_\ell \Big[P_\ell T_\ell^2 + T_\ell^2 P_\ell \\ &- P_\ell T_\ell^2 P_\ell - T_\ell P_\ell T_\ell\Big] - \frac{1}{U}\sum_{\langle \ell\ell'\rangle}\Big[P_\ell T_\ell T_{\ell'} - T_\ell P_\ell T_{\ell'} \\ &+ \frac{1}{2}\Big(T_\ell P_\ell P_{\ell'} T_{\ell'} - P_\ell T_\ell T_{\ell'} P_{\ell'}\Big) + \text{h.c.}\Big] \qquad (2) \end{aligned}$$

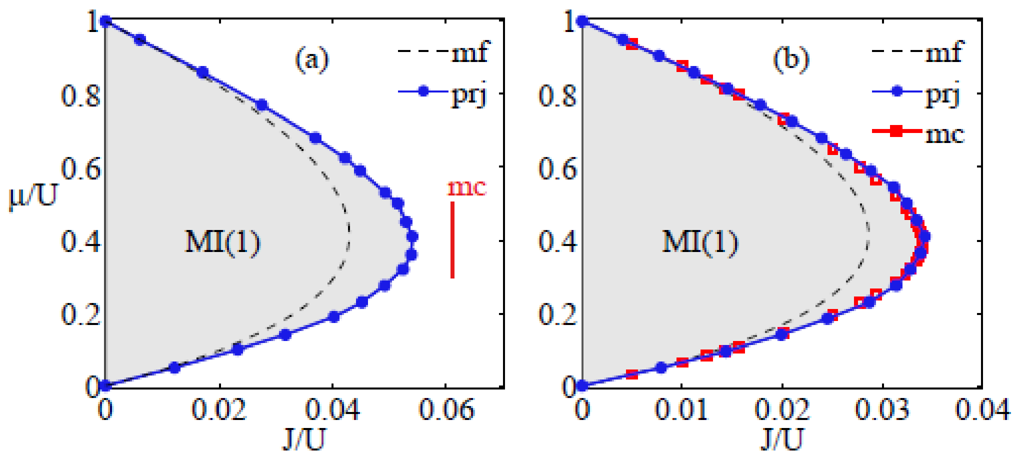
Use the effective Hamiltonian to compute the ground state energy and hence the phase diagram

$$E = \langle\psi|\mathcal{H}|\psi\rangle = \langle\psi'|H^*|\psi'\rangle + O(z^3J^3/U^2)$$

$$|\psi'\rangle = \exp(iS)|\psi\rangle$$

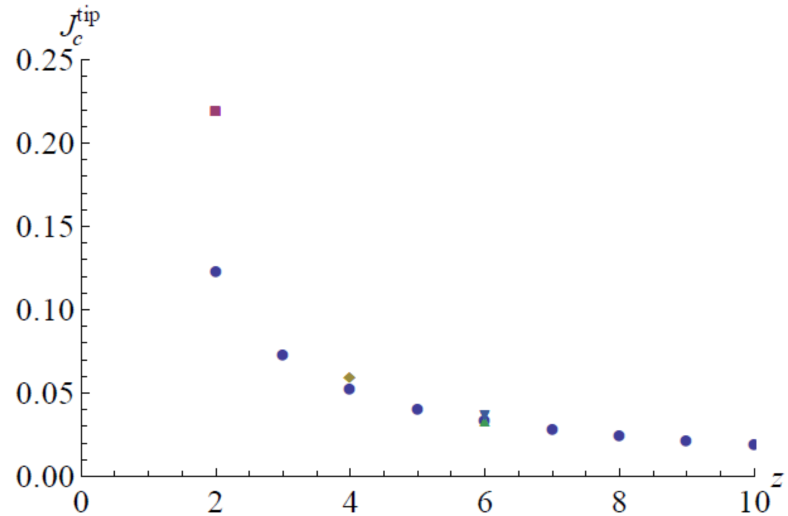
$$|\psi'\rangle = \prod_{\mathbf{r}} \sum f_n^{(\mathbf{r})}|n\rangle$$

Equilibrium phase diagram



Accurate reproduction of the phase diagram in d=3 for square lattice.

Does better than standard mean-field or strong coupling expansion in d=2.



Accurate for large z as can be checked by comparing with QMC data for 2D triangular (z=6) and 3D cubic lattice

Allows for straightforward generalization for treatment of dynamics

Non-equilibrium dynamics: Linear ramp

Consider a linear ramp of $J(t)=J_i+(J_f-J_i)\,t/\tau$.
For dynamics, one needs to solve the Sch. Eq.

$$i\hbar\partial_t|\psi\rangle = \mathcal{H}[J(t)]|\psi\rangle$$

Make a time dependent transformation to address the dynamics by projecting on the instantaneous low-energy sector.

$$|\psi'\rangle = \exp(iS[J(t)])|\psi\rangle$$

$$(i\hbar\partial_t + \partial S/\partial t)|\psi'\rangle = H^*[J(t)]|\psi'\rangle$$

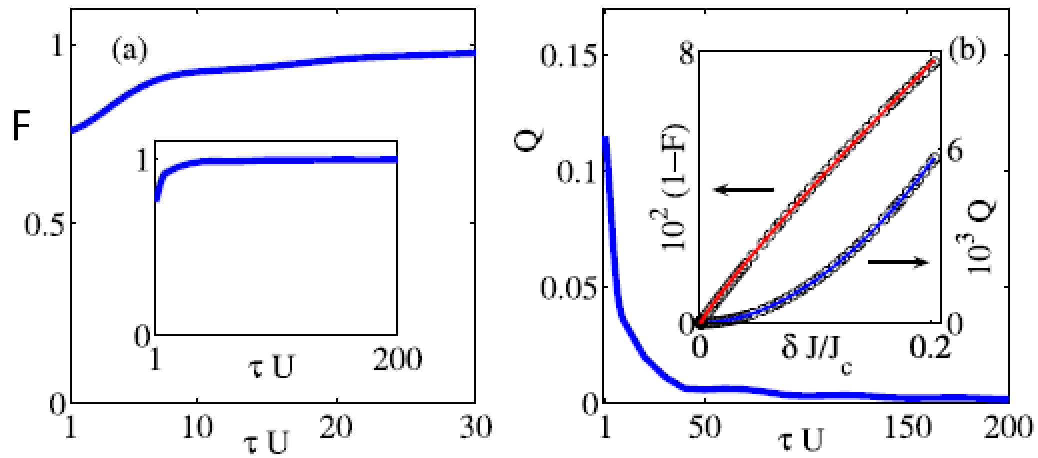
The method provides an accurate description of the ramp if J(t)/U <<1 and hence can treat slow and fast ramps at equal footing.

Takes care of particle/hole production due to finite ramp rate beyond mft.

$$\begin{aligned} i\hbar\partial_t f_n^{(\mathbf{r})} &= \delta E[\{f_n(t)\};J(t)]/\delta f_n^{*(\mathbf{r})} + i\hbar\frac{(J_f-J_i)}{U\tau} \\ &\times \sum_{\langle \mathbf{r}'\rangle_{\mathbf{r}}} \sqrt{n} f_{n-1}^{(\mathbf{r})}\left[\delta_{n\bar{n}}\varphi_{\mathbf{r}'\bar{n}} - \delta_{n,\bar{n}+1}\varphi_{\mathbf{r}',\bar{n}-1}\right] \\ &+\sqrt{n+1} f_{n+1}^{(\mathbf{r})}\left[\delta_{n\bar{n}}\varphi^*_{\mathbf{r}',\bar{n}-1} - \delta_{n,\bar{n}-1}\varphi^*_{\mathbf{r}'\bar{n}}\right] \end{aligned}$$

$$\varphi_{\mathbf{r}}[\Phi_{\mathbf{r}}] = \langle\psi'|b_{\mathbf{r}}|\psi'\rangle[\langle\psi'|b_{\mathbf{r}}^2|\psi'\rangle]$$

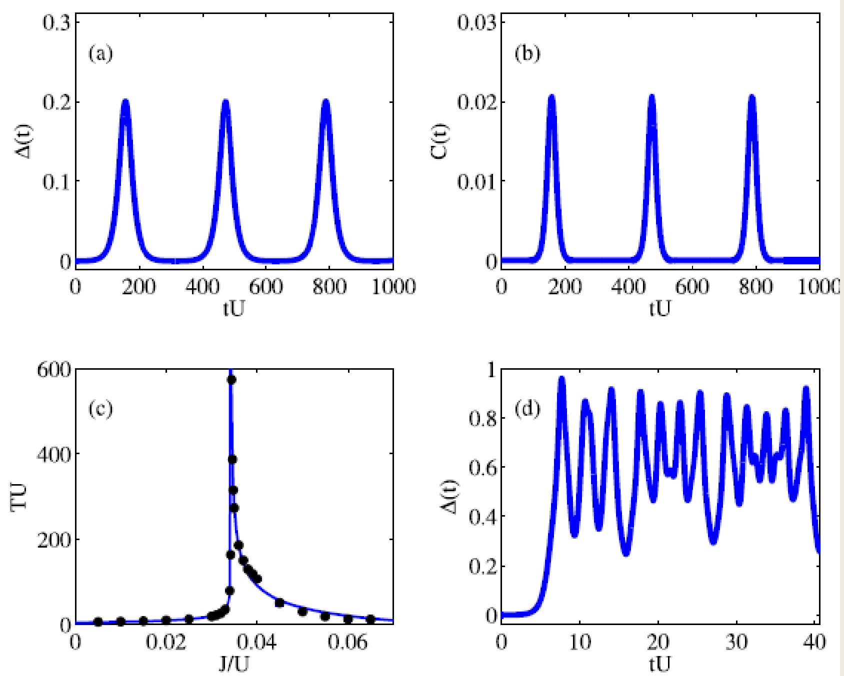
$$E = \langle\psi|\mathcal{H}|\psi\rangle$$



$$P = 1 - |\langle \psi_G | \psi(t_f) \rangle|^2$$

$$Q = \langle \psi_c | \mathcal{H}[J_f] | \psi_c \rangle - E_G[J_f]$$

Absence of critical scaling: may be understood as the inability of the system to access the critical (k=0) modes.



$$\Delta_{\mathbf{r}}(t) = \langle \psi(t) | b_{\mathbf{r}} | \psi(t) \rangle = \langle \psi'(t) | b'_{\mathbf{r}} | \psi'(t) \rangle$$

$$C_{\mathbf{r}}(t) = \langle \psi'(t) | b'_{\mathbf{r}} b'_{\mathbf{r}} | \psi'(t) \rangle - \Delta^2_{\mathbf{r}}(t)$$

Fast quench from the Mott to the SF phase; study of superfluid dynamics.

Single frequency pattern near the critical Point; more complicated deeper in the SF phase.

Strong quantum fluctuations near the QCP; justification of going beyond mft.

Experiments with ultracold bosons on a lattice: finite rate dynamics

2D BEC confined in a trap and in the presence of an optical lattice.

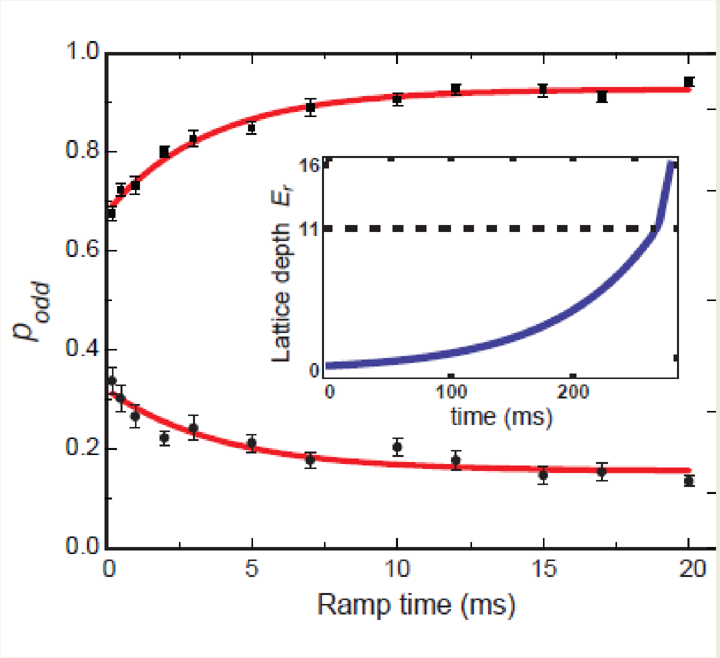
Single site imaging done by light-assisted collision which can reliably detect even/odd occupation of a site. In the present experiment they detect sites with n=1.

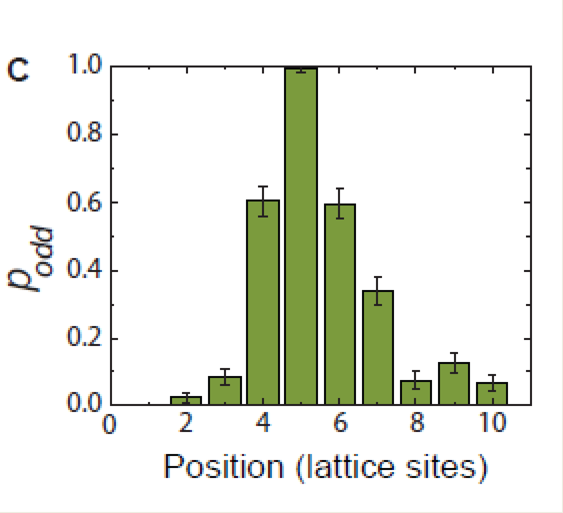
Ramp from the SF side near the QCP to deep inside the Mott phase in a linear ramp with different ramp rates.

The no. of sites with odd n displays plateau like behavior and approaches the adiabatic limit when the ramp time is increased asymptotically.

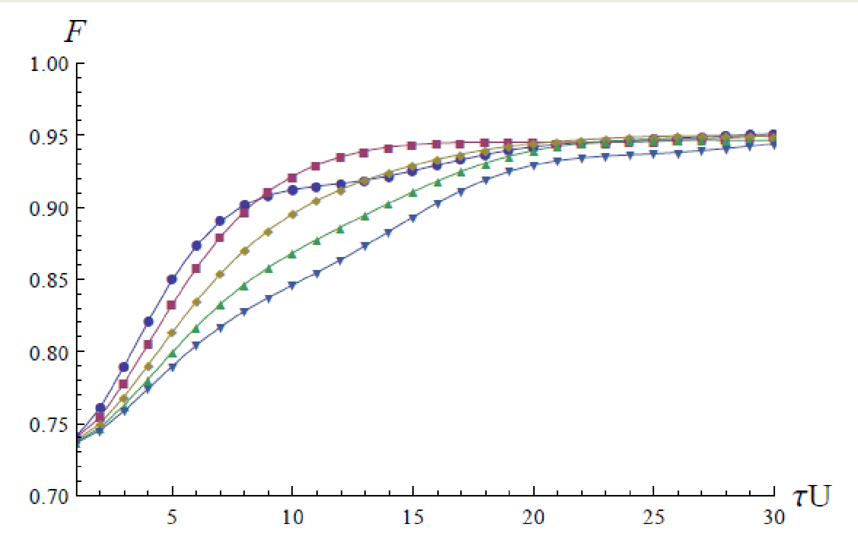
No signature of scaling behavior. Interesting spatial patterns.

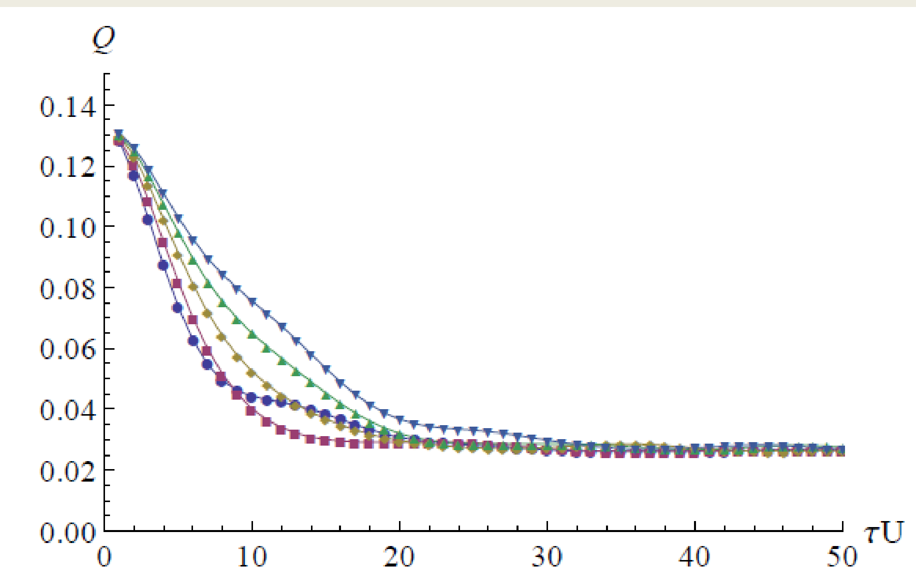
W. Bakr et al. Science 2010





Power law ramp





Use a power-law ramp protocol

$$J(t) = J_i + (J_f - J_i)(t/\tau)^{\alpha}$$

Slope of both F and Q depends on α; however, the plateau-like behavior at large τ is independent of α.

↓

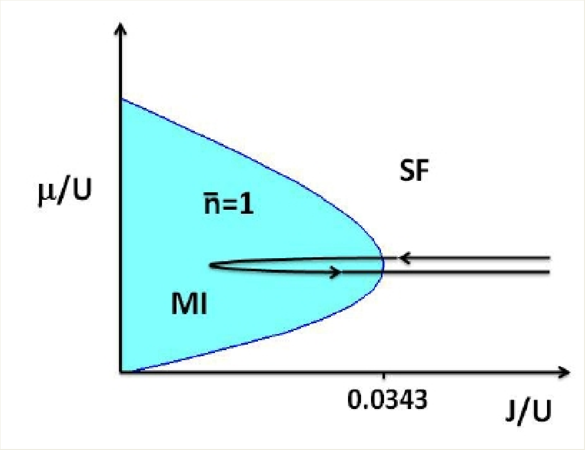
Absence of Kibble-Zureck scaling for any α due to lack of contribution of small k (critical) modes in the dynamics.

Periodic protocol: dynamics induced freezing

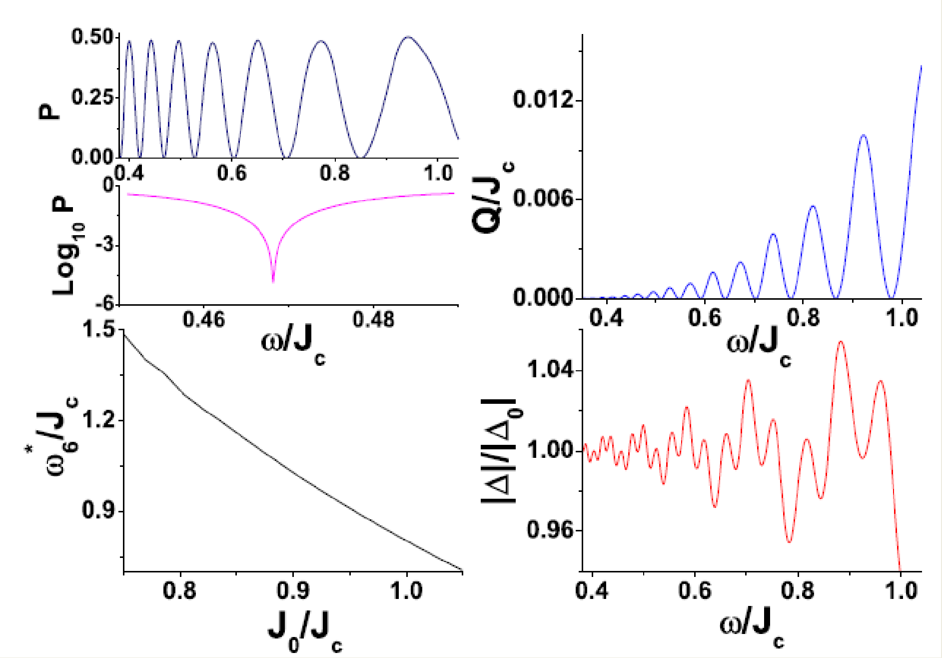
Dynamics induced freezing

Tune J from superfluid phase to Mott
and back through the tip of the Mott lobe.

$$J(t) = J_0 + \delta J \cos(\omega t)$$



Key result



There are specific frequencies at which the wavefunction of the system comes back to itself after a cycle of the drive leading to P=1-F -> 0.

Dynamics induced freezing

Mean-field analysis: A qualitative picture

1. ***Choose a gutzwiller wavefunction:*** $|\psi(\mathbf{r},t)\rangle_{\rm mf} = \prod_{\mathbf{r}} \sum_n f_n(t)|n\rangle$

2. ***The mean-field equations for f_n***
   E_n is the on-site energy of the state |n>

$$(i\partial_t - E_n)f_n = \tilde{\Delta}(t)\sqrt{n}f_{n-1} + \tilde{\Delta}^*(t)\sqrt{n+1}f_{n+1},$$

$$\tilde{\Delta}(t) = -zJ(t)\sum_n \sqrt{n} f^*_{n-1} f_n,$$

3. ***Numerical solution of this equation indicates that f_n vanishes for n>2 for all ranges of drive frequencies studied.***

$$\begin{aligned} i\partial_t f_0 &= -zJ(t)[|f_1|^2 f_0 + \sqrt{2} f_2^* f_1^2] \\ i\partial_t f_2 &= E_2 f_2 - zJ(t)[2|f_1|^2 f_2 + \sqrt{2} f_0^* f_1^2] \\ i\partial_t f_1 &= E_1 f_1 - zJ(t)[(2|f_2|^2 + |f_0|^2) f_1 + 2\sqrt{2} f_1^* f_2 f_0]. \end{aligned}$$

4. ***Analysis of these equations leads to the relation involving***
   $f_n = r_n(t)\exp[i\phi_n(t)]$.

$$\partial_t |f_0|^2 = \partial_t |f_2|^2 = -\partial_t |f_1|^2/2.$$

$$r^2_{2[0]}(t) = -(r_1^2(t) - 1)/2 + [-]\eta,$$

5. Thus one can describe the system in terms of three real variables : amplitude of state |1> and the sum and difference of the relative phases.

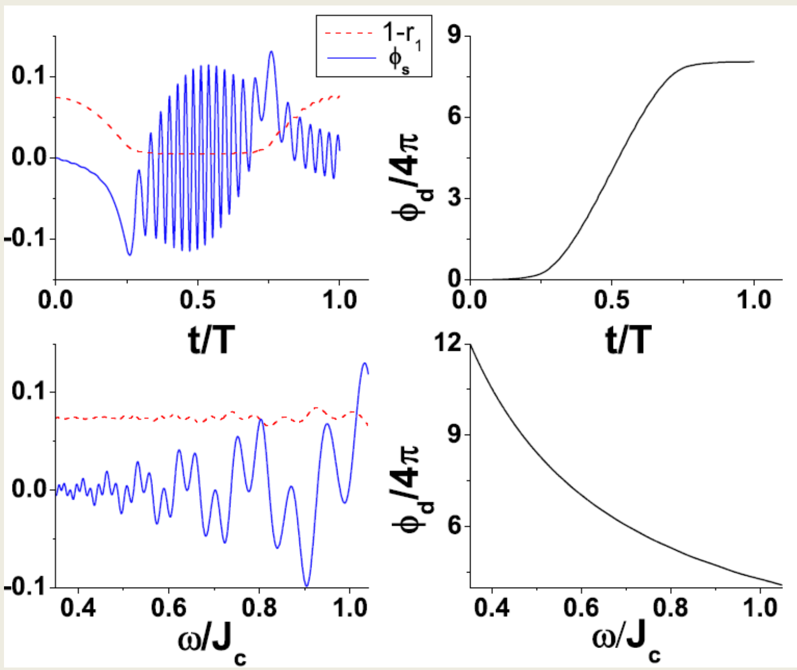
$$\phi_s=\phi_0+\phi_2-2\phi_1 \quad \phi_d=\phi_2-\phi_0$$

$$\begin{aligned}
\partial_t r_1 &= -\sqrt{2} z J(t) \sin(\phi_s) r_1 g_0(r_1), \\
\partial_t \phi_s &= -U + zJ(t)\left[g_1(r_1) - g_2(r_1)\cos(\phi_s)\right], \qquad (6) \\
\partial_t \phi_d &= -U + 2\mu + zJ(t) r_1^2 \left[1 - 4\sqrt{2}\eta \cos(\phi_s)/g_0(r_1)\right],
\end{aligned}$$

$$\begin{aligned}
g_0(r_1) &= \sqrt{(1-r_1^2)^2 - 4\eta^2}, \quad g_1(r_1) = 6r_1^2 - 3 - 2\eta, \\
g_2(r_1) &= 2\sqrt{2}\left[r_1^2(r_1^2-1)/g_0(r_1) + g_0(r_1)\right]. \qquad (7)
\end{aligned}$$

6. One can construct a frequency-independent relation between r_1 and ϕ_s

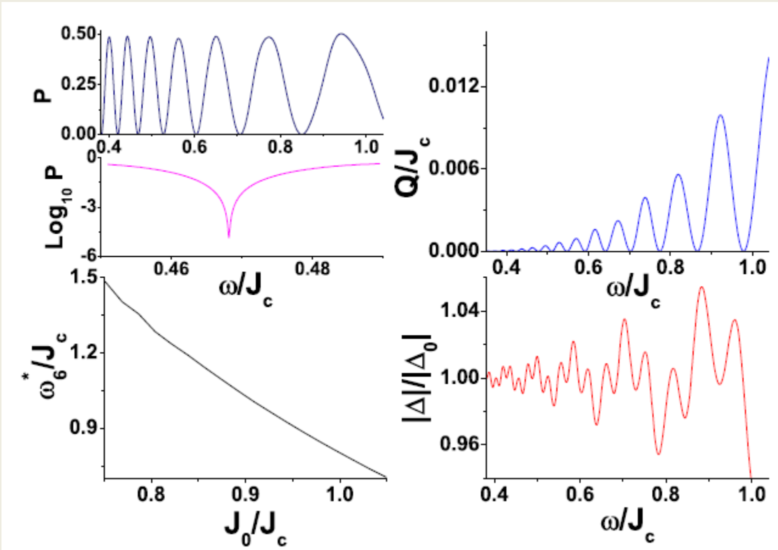
$$dr_1/d\phi_s = \frac{-\sqrt{2}\sin(\phi_s) r_1 g_0(r_1)}{[g_1(r_1) - g_2(r_1)\cos(\phi_s)] - U/zJ(t')}.$$

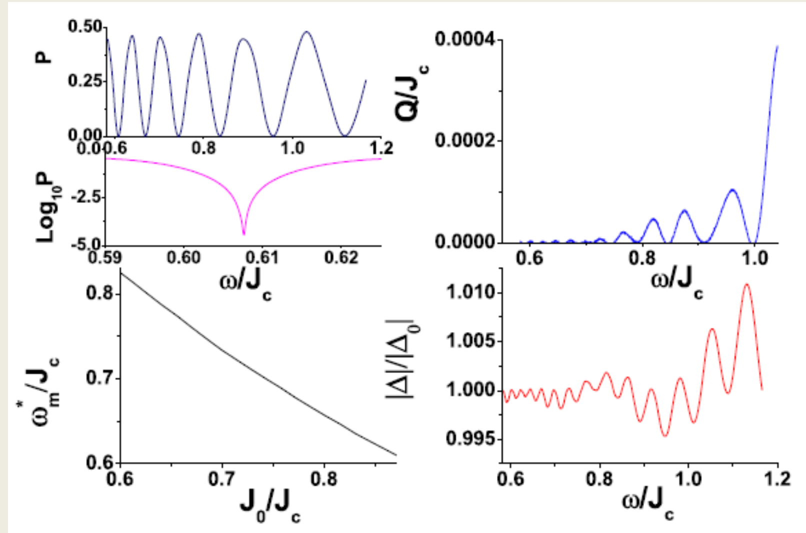


← ***Numerical solution of (6)***

There is a range of frequency for which r_1 and ϕ_s remain close to their original values; dynamics induced freezing occurs when $\phi_d/4\pi$ = n within this range.

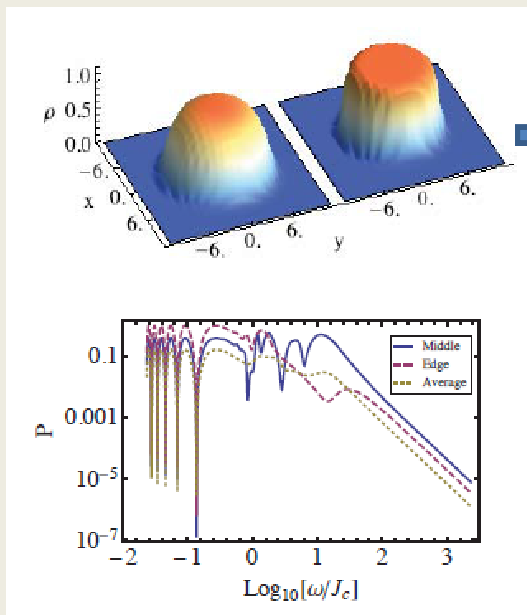
Robustness against quantum fluctuations and presence of a trap





Mean-field theory

Projection operator formalism



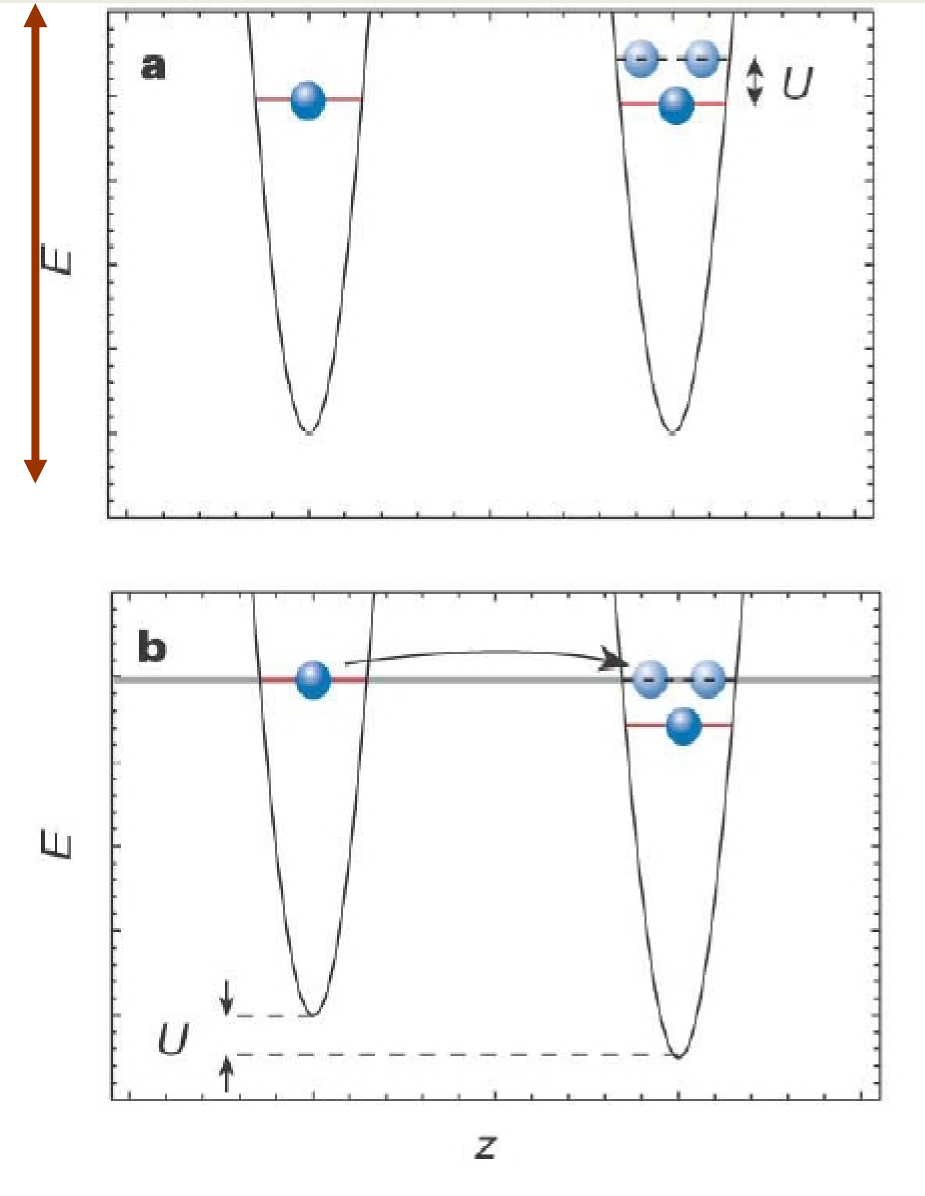
Density distribution of the bosons inside a trap

Robust freezing phenomenon

Bosons in an electric field

Applying an electric field to the Mott state

$$20E_r$$



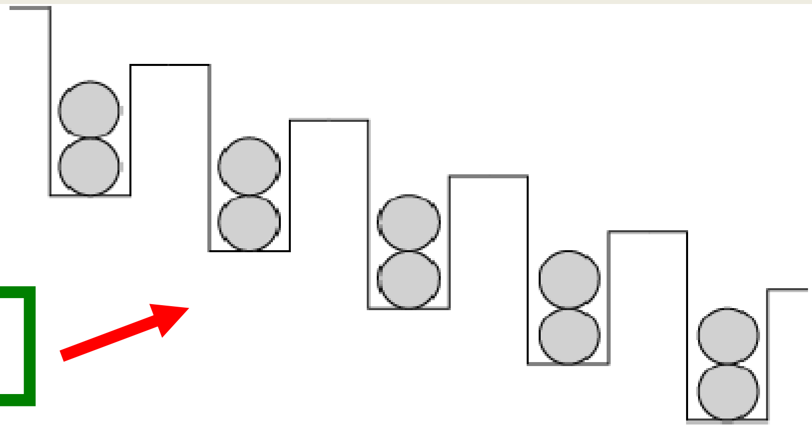
Energy Scales:

$\hbar\omega_v = 5E_r \approx 20\ U$

$U \approx 10\text{–}300\ J$

Construction of an effective model: 1D

Parent Mott state



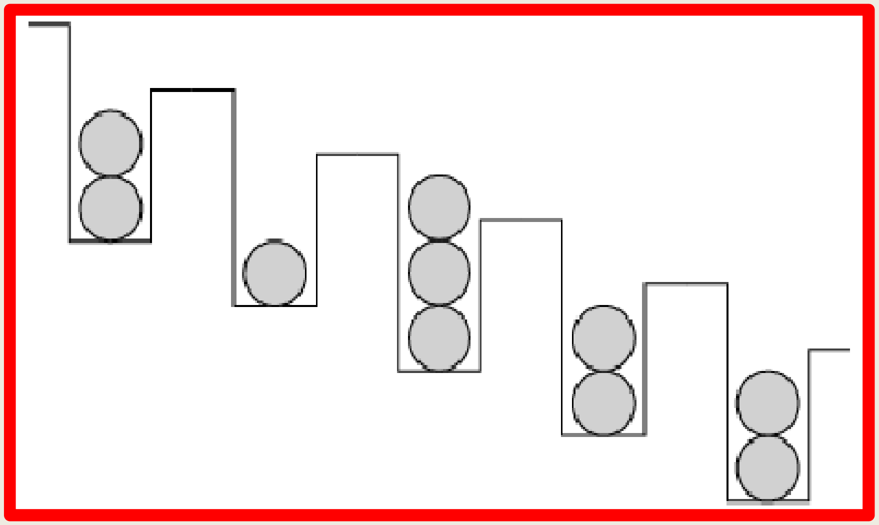
$$H = -w\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

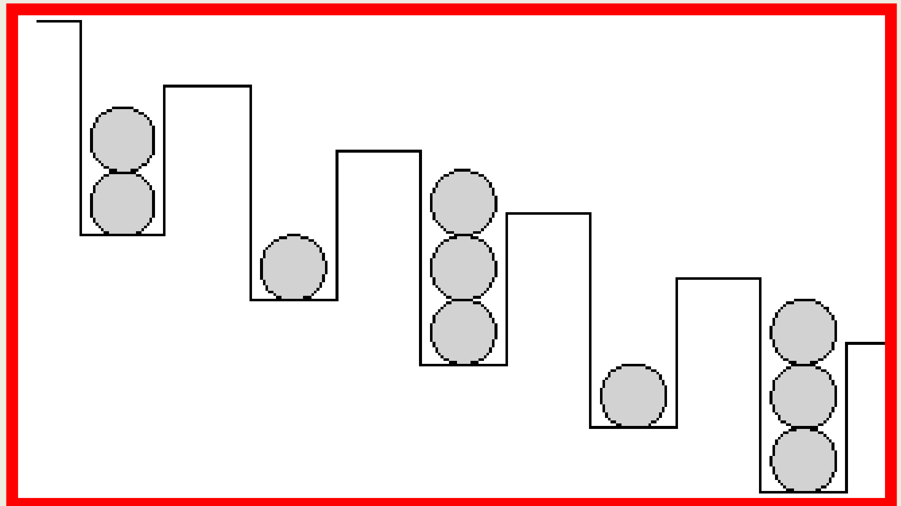
$$n_i = b_i^\dagger b_i$$

$$\left|U - E\right|, w \ll E, U$$

Describe spectrum in subspace of states resonantly coupled to the Mott insulator

Neutral dipoles





Resonantly coupled to the parent Mott state when U=E.

Neutral dipole state with energy U-E.

Two dipoles which are not nearest neighbors with energy 2(U-E).

Effective dipole Hamiltonian: 1D

$d_\ell^\dagger \Rightarrow$ Creates dipole on link ℓ

$$H_d = -\sqrt{6}w\sum_\ell \left(d_\ell^\dagger + d_\ell\right) + (U-E)\sum_\ell d_\ell^\dagger d_\ell$$

Constraints: $d_\ell^\dagger d_\ell \le 1 \quad ; \quad d_{\ell+1}^\dagger d_{\ell+1} d_\ell^\dagger d_\ell = 0$

Determine phase diagram of H_d as a function of $(U\text{-}E)/w$

Note: there is no explicit dipole hopping term.

However, dipole hopping is generated by the interplay of terms in H_d and the constraints.

Weak Electric Field

For weak electric field, the ground state is dipole vaccum and the low-energy excitations are single dipole

- The effective Hamiltonian for the dipoles for weak E:

$$\mathcal{H}_{d,\mathrm{eff}} = (U-E)\sum_l \Big[|l><l| + \frac{w^2 n_0(n_0+1)}{(U-E)^2}\big(|l><l| + |l+1><l| + |l><l+1|\big)\Big]$$

- Lowest energy excitations: Single band of dipole excitations.
- These excitations soften as E approaches U. This is a precursor of the appearance of Ising density wave with period 2.
- Higher excited states consists of multiparticle continuum.

Strong Electric field

- The ground state is a state of maximum dipoles.

- Because of the constraint of not having two dipoles on consecutive sites, we have two degenerate ground states

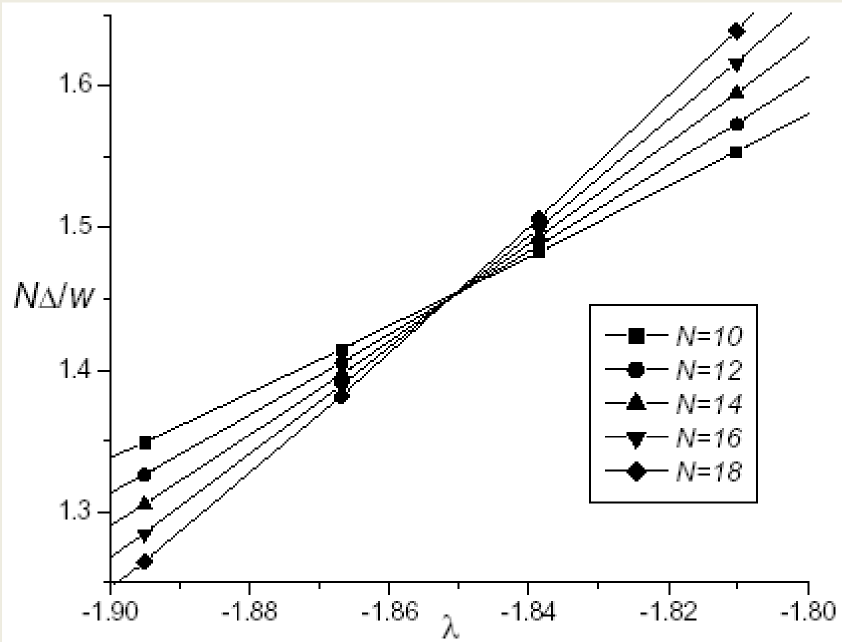
$$\cdots d_1^\dagger d_3^\dagger d_5^\dagger d_7^\dagger d_9^\dagger d_{11}^\dagger \cdots |0\rangle \quad or \quad \cdots d_2^\dagger d_4^\dagger d_6^\dagger d_8^\dagger d_{10}^\dagger d_{12}^\dagger \cdots |0\rangle$$

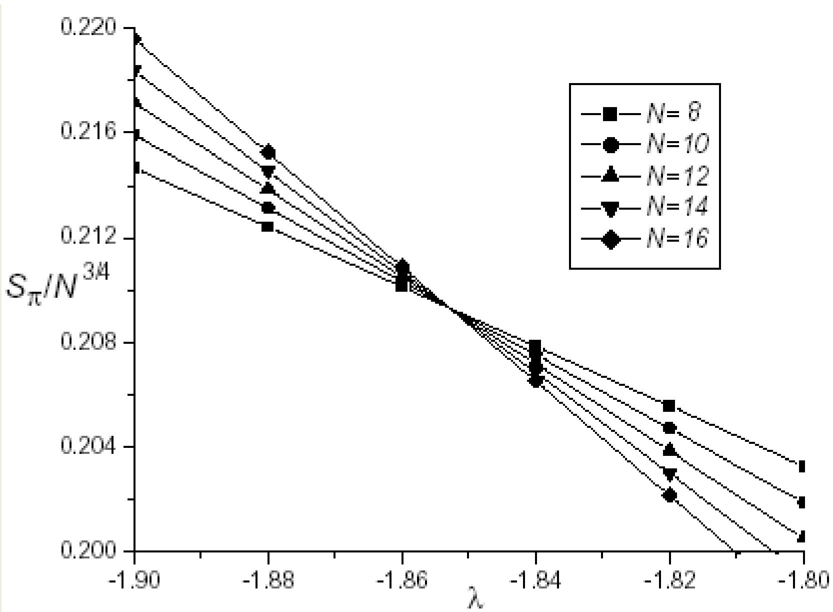
- The ground state breaks Z2 symmetry.

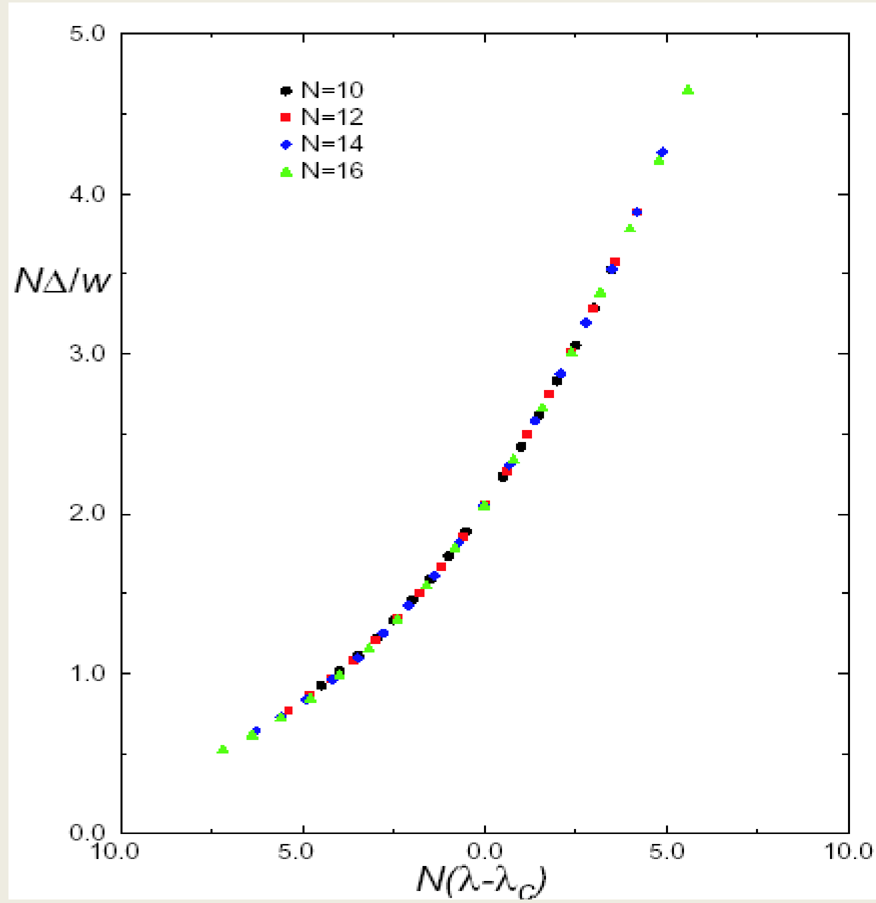
- The first excited state consists of band of domain walls between the two filled dipole states.

- Similar to the behavior of Ising model in a transverse field.

Intermediate electric field: QPT

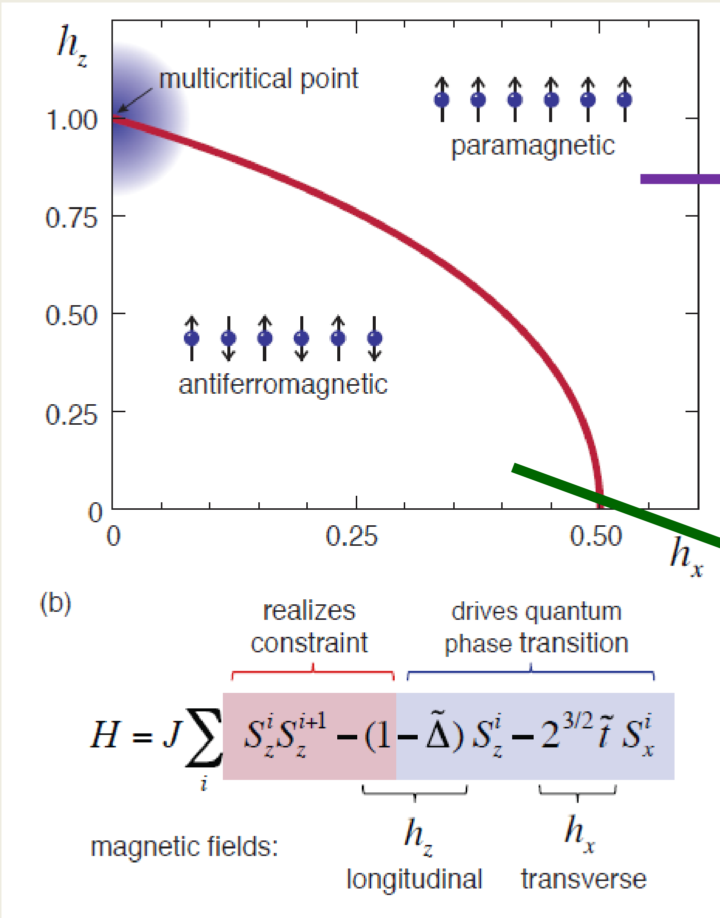


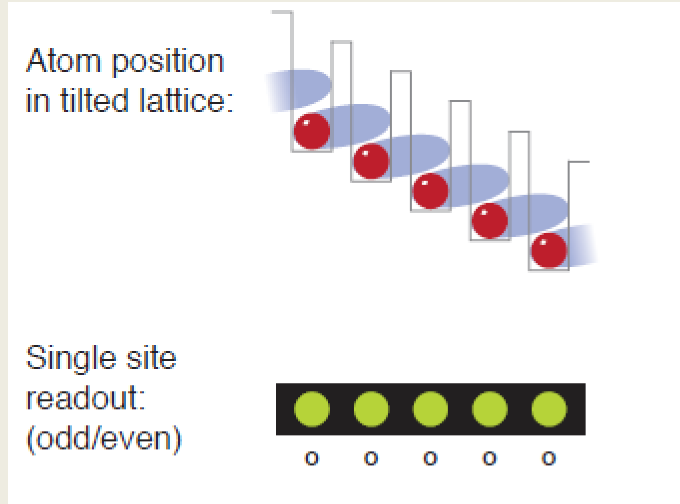


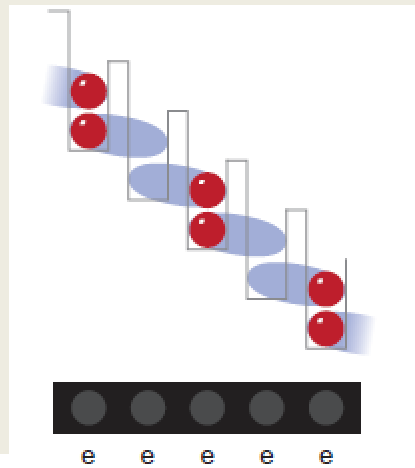


Quantum phase transition at E-U=1.853w. Ising universality.

Recent Experimental observation of Ising order (Bakr et al Nature 2010)







$$S_z^j = \frac{1}{2} - d_j^\dagger d_j, \; S_x^j = \frac{1}{2}\left(d_j^\dagger + d_j\right), \text{ and } S_y^j = \frac{i}{2}\left(d_j^\dagger - d_j\right)$$

First experimental realization of effective Ising model in ultracold atom system

Quench dynamics across the quantum critical point

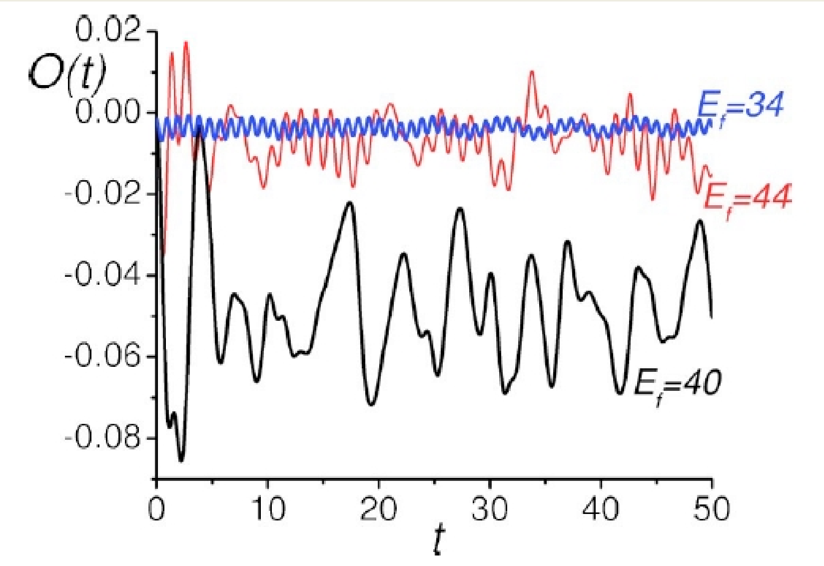
Tune the electric field from E_i to E_f instantaneously

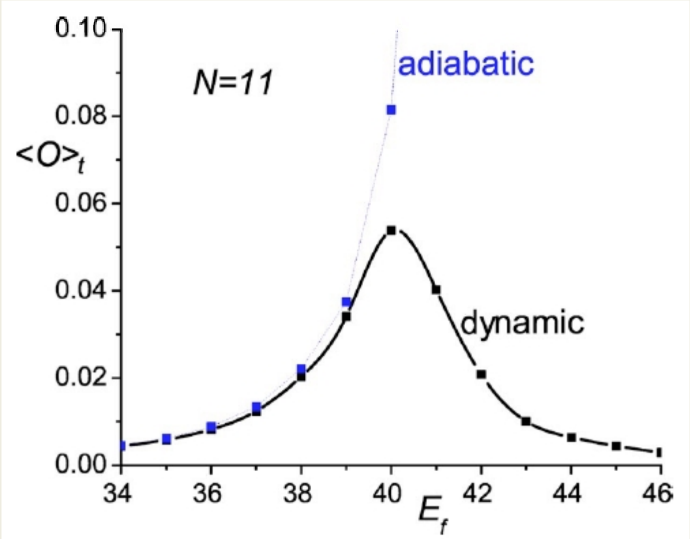
$$|\Psi(t)\rangle = \sum_n c_n \exp(-i\epsilon_n t/\hbar)|n\rangle$$

Compute the dipole order Parameter as a function of time

$$O = \frac{1}{N}\langle\Psi|\sum_\ell (-1)^\ell d_\ell^\dagger d_\ell|\Psi\rangle$$

$$O(t) = \frac{1}{N}\sum_{m,n} c_m c_n \cos[(E_m - E_n)t/\hbar]\langle m|\sum_\ell (-1)^\ell d_\ell^\dagger d_\ell|n\rangle.$$





The time averaged value of the order parameter is maximal near the QCP

Dynamics with a finite rate: Kibble-Zureck scaling

Change the electric field linearly in time with a finite rate v

$$H_d(t) \;=\; [U - \mathcal{E}(t)] \sum_{\ell} d_\ell^\dagger d_\ell - J \sum_{\ell} (d_\ell^\dagger + d_\ell),$$

Quantities of interest

Theory

Experiment

$$Q(t) = \langle \psi(t) | H(t) | \psi(t) \rangle - E_G(t),$$
$$F(t) = \log[|\langle \psi(t) | \psi_G(t) \rangle|^2],$$
$$n_d(t) = \langle \psi(t) | \textstyle\sum_\ell (1 + 2S_\ell^z) | \psi(t) \rangle,$$
$$D(t) = n_d(t) - \langle \psi_G(t) | \textstyle\sum_\ell (1 + 2S_\ell^z) | \psi_G(t) \rangle,$$
$$C_{ij}(t) = \langle \psi(t) | S_i^z S_j^z | \psi(t) \rangle,$$

Scaling laws for finite –size systems

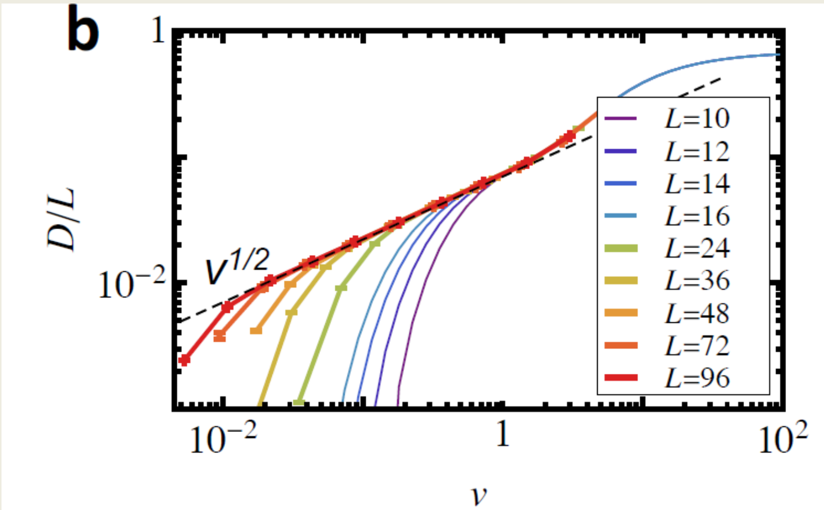
$$Q \sim L^d v^{(d+z)\nu/(z\nu+1)} g_r(vL^{1/\nu+z}),$$
$$F \sim L^d v^{d\nu/(z\nu+1)} f_r(vL^{1/\nu+z}),$$

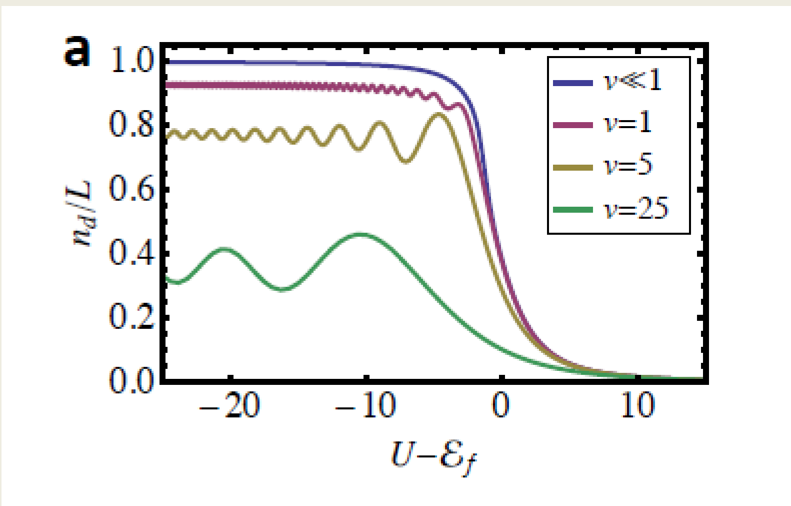
$$g_r(x \ll 1) \sim x^{2-(d+z)\nu/(z\nu+1)}$$
$$f_r(x \ll 1) \sim x^{2-d\nu/(z\nu+1)}$$

Q ~ v^2 (v) for slow (intermediate) quench. These are termed as LZ(KZ) regimes for finite-size systems.

Kibble-Zureck scaling for finite-sized system

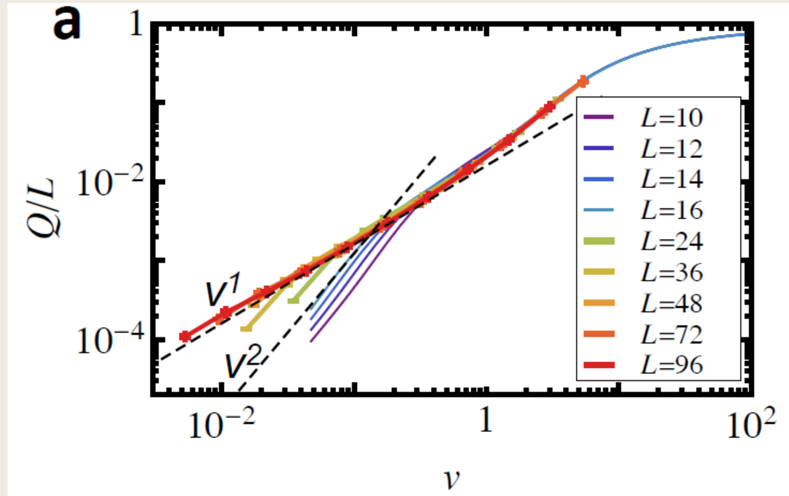
Expected scaling laws for Q and F

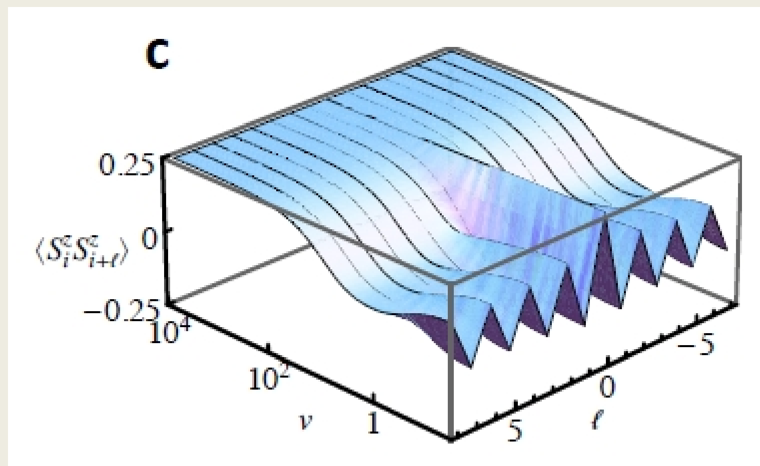




Dipole dynamics

Observation of Kibble-Zurek law for intermediate v with Ising exponents.





Correlation function