Wilson Lines in Higher Spin Gravity

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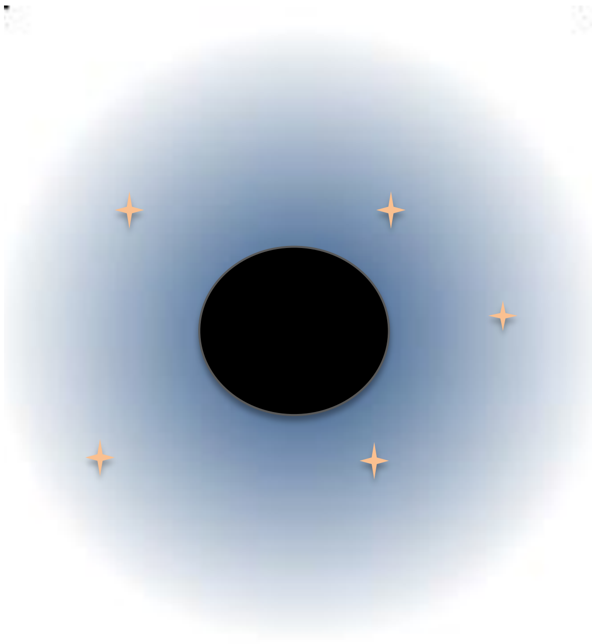
Introduction

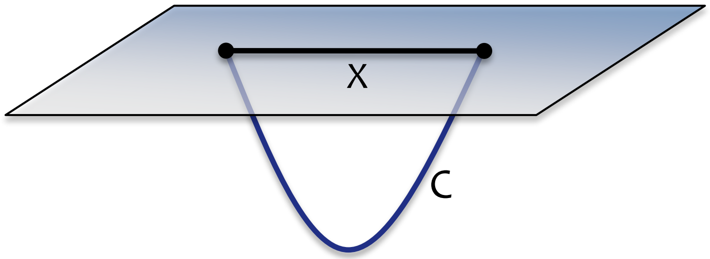
Universal features of gravity

If Gravity equals Geometry …



Laws of thermodynamics

[Bekenstein, Hawking]



Quantum information
(Entanglement Entropy)

[Ryu & Takayanagi]

… Not every gravitational theory admits a geometrical description (or at least not an obvious one)…

Vasiliev's Higher Spin Gravity

- Infinite number of coupled massless spin fields.
- Gauge group acts non trivially on all fields.

Tractable example of AdS/CFT:
analytic tractability and intrinsic complexity

[Sezgin & Sundell; Klebanov & Polyakov; Gaberdiel & Gopakumar]

Challenge: Does higher spin gravity know about entanglement?

Goal: design a massive probe which will generalize the notion of geodesic for non-local theories.

Our proposal: Wilson line captures the correct dynamics

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}\left(\mathcal{P}\exp\int_C \mathcal{A}\right) = \int \mathcal{D}U \exp[-S(U;\mathcal{A})_C]$$

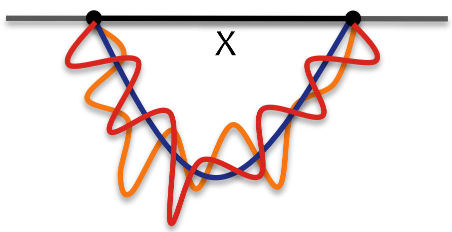
Encodes quantum numbers of the probe. It is an infinite dimensional representation.

Auxiliary field.
Captures dynamics of the probe.

Introduction

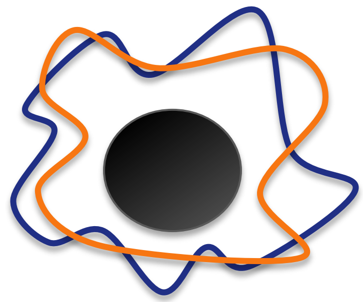
Open Paths

$$S_{\mathrm{EE}} = -\mathrm{Tr}(\rho_X \log \rho_X) = -\log\left(W_{\mathcal{R}}(C)\right)$$



Closed Paths

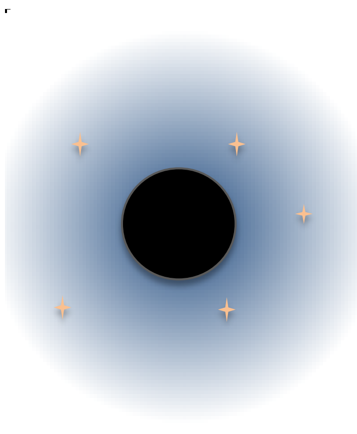
$$S_{\mathrm{thermal}} = -\log\left(W_{\mathcal{R}}(C)\right)$$

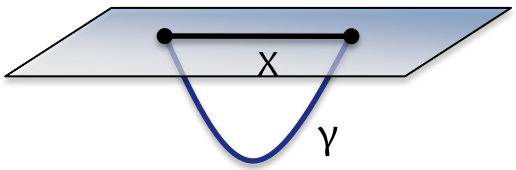


AdS3 Gravity à la Chern-Simons

Warm up: 3D gravity

Reminder: 3D gravity has no local degrees of freedom…





As a topological theory,
how does 3d gravity knows about thermodynamics and entanglement?

AdS3 Gravity à la Chern-Simons

$$\mathcal{A} \in so(2,2) = SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$$

$$S_{\text{EH}} = S_{CS}[A] - S_{CS}[\bar{A}]$$

Dictionary to gravity

$$A = \omega + \frac{e}{\ell}$$

$$\bar{A} = \omega - \frac{e}{\ell}$$

Gauge transformations

$$A_\mu \to L(x)\,(A_\mu + \partial_\mu)\,L^{-1}(x)$$

$$\bar{A}_\mu \to R^{-1}(x)\,(\bar{A}_\mu + \partial_\mu)\,R(x)$$

$$L, R \in SL(2,\mathbb{R})$$

Explicit construction of the Wilson line

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}\left(\mathcal{P}\exp\int_C \mathcal{A}\right) = \int \mathcal{D}U \exp[-S(U;\mathcal{A})_C]$$

[Witten; Carlip]

Effective acting along the curve

$$\begin{aligned} S(U,P;\mathcal{A})_C &= S(U)_{C,\mathrm{free}} + S(U;\mathcal{A})_{C,\mathrm{int}} \\ &= \int_C ds\,\left(\mathrm{Tr}\left(PU^{-1}D_sU\right) + \lambda(s)\left(\mathrm{Tr}(P^2) - c_2\right)\right) \end{aligned}$$

Lagrange multiplier **Casimir representation**

Auxiliary field and its conjugate momenta

$$U(s) \to LU(s)R \qquad P(s) \to R^{-1}P(s)R\ , \qquad L, R \in SL(2,\mathbb{R})$$

Explicit construction of the Wilson line

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}} \left(\mathcal{P} \exp \int_C \mathcal{A} \right) = \int \mathcal{D}U \exp[-S(U;\mathcal{A})_C]$$

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Covariant derivative couples probe to background

$$D_sU = \frac{d}{ds}U + A_sU - U\bar{A}_s \ , \quad A_s \equiv A_\mu \frac{dx^\mu}{ds}$$

Recovering the geodesic equation: Set $U(s) = 1$

$$\frac{d}{ds}\left((A-\bar{A})_\mu \frac{dx^\mu}{ds}\right) + [\bar{A}_\mu, A_\nu]\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0$$

eom reduces to geodesic eqn!

$$\begin{aligned} S_{C,\text{on-shell}} &= \sqrt{c_2}\int_C ds \sqrt{\text{Tr}\left((A-\bar{A})_\mu (A-\bar{A})_\nu \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\right)} \\ &= \sqrt{2c_2}\int_C ds \sqrt{g_{\mu\nu}(x)\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}} \end{aligned}$$

Action is the proper distance!

$$W_{\mathcal{R}}(x_i, x_f) \sim \exp\left(-\sqrt{2c_2}L(x_i, x_f)\right)$$

But please remember: the path is not important! This is a little miracle of SL(2,R)

Wilson line and Entanglement Entropy

$$W_{\mathcal{R}}(x_i, x_f) \sim \exp\left(-\sqrt{2c_2}L(x_i, x_f)\right)$$

Casimir = mass^2 probe = (dimension twist field in CFT)^2

$$m = h + \bar{h}$$

$$h = \bar{h} = \frac{c}{24}\left(n - \frac{1}{n}\right)$$

[Cardy & Calabrese]

$$c = \frac{3\ell}{2G_3}$$

[Brown & Henneaux]

$$2c_2 = 4h(h-1) \underset{n\to 1}{\longrightarrow} \left(\frac{c}{6}\right)^2$$

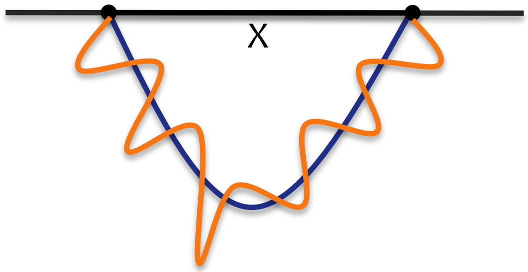
$$S_{\mathrm{EE}} = -\log\left(W_{\mathcal{R}}(C)\right)$$

Follows from Lewkowycz & Maldacena

Warm up: 3D gravity

Entanglement Entropy

(open path; end points at boundary)



$$S_{\mathrm{EE}} = -\log\left(W_{\mathcal{R}}(C)\right) = \frac{c}{3}\log\left(\frac{\Delta X}{\epsilon}\right)$$

Infinite system, zero temperature

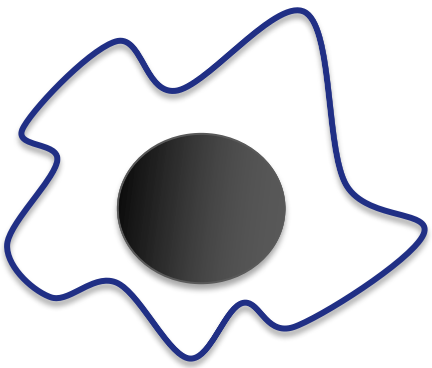
$$S_{\mathrm{EE}} = -\log\left(W_{\mathcal{R}}(C)\right) = \frac{c}{3}\log\left(\frac{\beta}{\pi\epsilon}\sinh\left(\frac{\pi\Delta X}{\beta}\right)\right)$$

Infinite system, finite temperature

Thermal entropy

(closed spatial path)

$$S_{\mathrm{thermal}} = -\log\left(W_{\mathcal{R}}(C)\right) = \frac{A_H}{4G_3}$$



SL(3) Higher Spin Gravity

$$S_{\rm HS} = S_{CS}[A] - S_{CS}[\bar{A}] \qquad A, \bar{A} \in SL(3, \mathbb{R})$$

[Bergshoeff, Blencowe & Stelle; Campoleoni, Fredenhagen, Pfenninger & S. Theisen]

Gravitational interpretation: Interacting theory of a graviton with a massless spin-3 field

Interesting solutions:

- AdS3 and BTZ black hole
  [Bañados,Teitelboim & Zanelli]
- Higher Spin Black holes: non-zero spin-3 potential (β, μ)
  [Gutperle & Kraus]
- Domain walls, Janus solutions, conical defects, …
  [Ammon, Gutperle, Kraus & Perlmutter; Gutperle; AC, Gopakumar, Gutperle & Raeymaekers]

Constructing a massive probe

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}\left(\mathcal{P}\exp\int_C \mathcal{A}\right) = \int \mathcal{D}U \exp[-S(U;\mathcal{A})_C]$$

Effective acting along the curve

$$S(U,P;A,\bar{A})_C \;=\; \int_C ds\Big(\mathrm{Tr}(PU^{-1}D_sU) + \lambda_2(\mathrm{Tr}(P^2) - c_2) + \lambda_3(\mathrm{Tr}(P^3) - c_3)\Big)$$

Cubic and quadratic casimir

Covariant derivative couples probe to background

$$D_sU = \frac{d}{ds}U + A_sU - U\bar{A}_s\ , \quad A_s \equiv A_\mu \frac{dx^\mu}{ds}$$

Auxiliary field and its conjugate momenta

$$U(s) \to LU(s)R\ , \quad P \to R^{-1}P(s)R\ , \qquad L, R \in SL(3,\mathbb{R})$$

Wilson line and Entanglement Entropy

Follow the same logic… Wilson line is a massive probe that induces a conical deficit on the background.

Quadratic Casimir ~ mass^2 probe = (dimension twist field in CFT)^2
Cubic Casimir ~ (spin-3 charge)^3 = (spin-3 of twist field)^3

$$m = h + \bar{h} \qquad h = \bar{h} = \frac{c}{24}\left(n - \frac{1}{n}\right) \qquad c = \frac{3\ell}{2G_3}$$

[Campoleoni, Fredenhagen, Pfenninger & Theisen; Henneaux & Rey]

$$2c_2 = 4h(h-1) \underset{n\to 1}{\longrightarrow} \left(\frac{c}{6}\right)^2$$

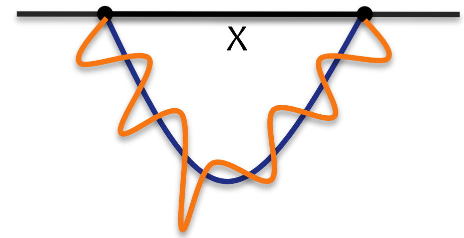
$$c_3 = \frac{3}{8}w\left(h^2 - \frac{1}{4}w^2\right) = 0$$

$$S_{\mathrm{EE}} = -\log\left(W_{\mathcal{R}}(C)\right)$$

Higher spin gravity

Entanglement Entropy

(open path; end points at boundary)



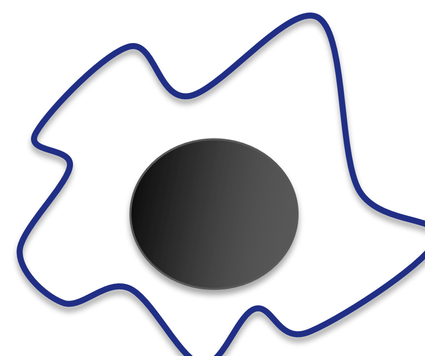
$$S_{\rm EE} = \Delta X \left(2\sqrt{2\pi\mathcal{L}k}\frac{\sqrt{1-\frac{3}{4C}}}{1-\frac{3}{2C}} \right) \qquad (\Delta X \gg \beta;\ \ \mu : \text{fixed};\ \ \beta : \text{fixed})$$

$$S_{\rm EE} = \frac{c}{3}\log\left(\frac{\beta}{\pi\epsilon}\sinh\left(\frac{\pi\Delta X}{\beta}\right)\right) \qquad (\mu \to 0\ ;\ \ \beta : \text{fixed})$$

$$S_{\rm EE} = \frac{c}{3}\log\left(\frac{\Delta X}{\epsilon}\left|1-\frac{16\mu^2}{\Delta X^2}\right|^{1/4}\right) \qquad (\beta \to \infty\ ;\ \ \mu : \text{fixed})$$

Thermal entropy

(closed spatial path)



$$S_{\rm thermal} = 2\pi\sqrt{\frac{c_2}{2}\mathrm{tr}_f((\lambda_\phi - \bar{\lambda}_\phi)L_0)}$$

Comments

- If the background is AdS (or BTZ) we recover all previous formulas
- Setting U=1 is not allowed. Geodesic equation is not recovered (which is good!).
- Good agreement with thermodynamics of higher spin black holes.
- Agreement with J. de Boer and J.I. Jottar [1306.4347[hep-th]]
- Puzzles for domain wall backgrounds. Not every background satisfy monoticity of entanglement entropy.
- Preliminary results for conical defects: non-trivial agreements with CFT!
- Extending results of Cardy-Calabrese for non-vanishing spin-3 potential is needed in the CFT.
- Still more to explore!

Summary

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}} \left(\mathcal{P} \exp \int_C \mathcal{A} \right) = \int \mathcal{D}U \exp[-S(U;\mathcal{A})_C]$$

- A new way to cast RT formula for AdS3 gravity.
- For closed paths, our proposal computes thermal entropy.
- Our formula reproduces non-trivial results of entanglement entropy.

Outlook

- Generalization to Vasiliev's theory with infinite number of higher spin fields.
- Interpretation of quantum corrections to the Wilson line.
- Renyi entropies in Chern-Simons formulation. [w J. de Boer and J. I. Jottar]
- Entanglement entropy in the presence of diff anomalies. [w M. Ammon , S. Detournay, N. Iqbal and E. Perlmutter]
- Puzzles regarding sub-additivity. Better understanding in the CFT is needed.