AdS Cluster Decomposition and Deficit Angles from the CFT Bootstrap

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Outline

I. Use AdS Kinematics to **motivate and define** Cluster Decomposition, and long-range corrections to it, **purely as a statement in the CFT**.

II. We will **Prove** results using the CFT bootstrap, in generality in $d > 2$, and in a special large central charge limit for $d=2$.

Cluster Decomposition: Very Coarse Locality

For some sufficiently large separation, perhaps truly gargantuan, well-separated processes decouple.

We will also address the rate of decoupling, relating it to the bulk gravity and other forces.

Formal Definition of Cluster Decomposition?

$\psi_c =$ 

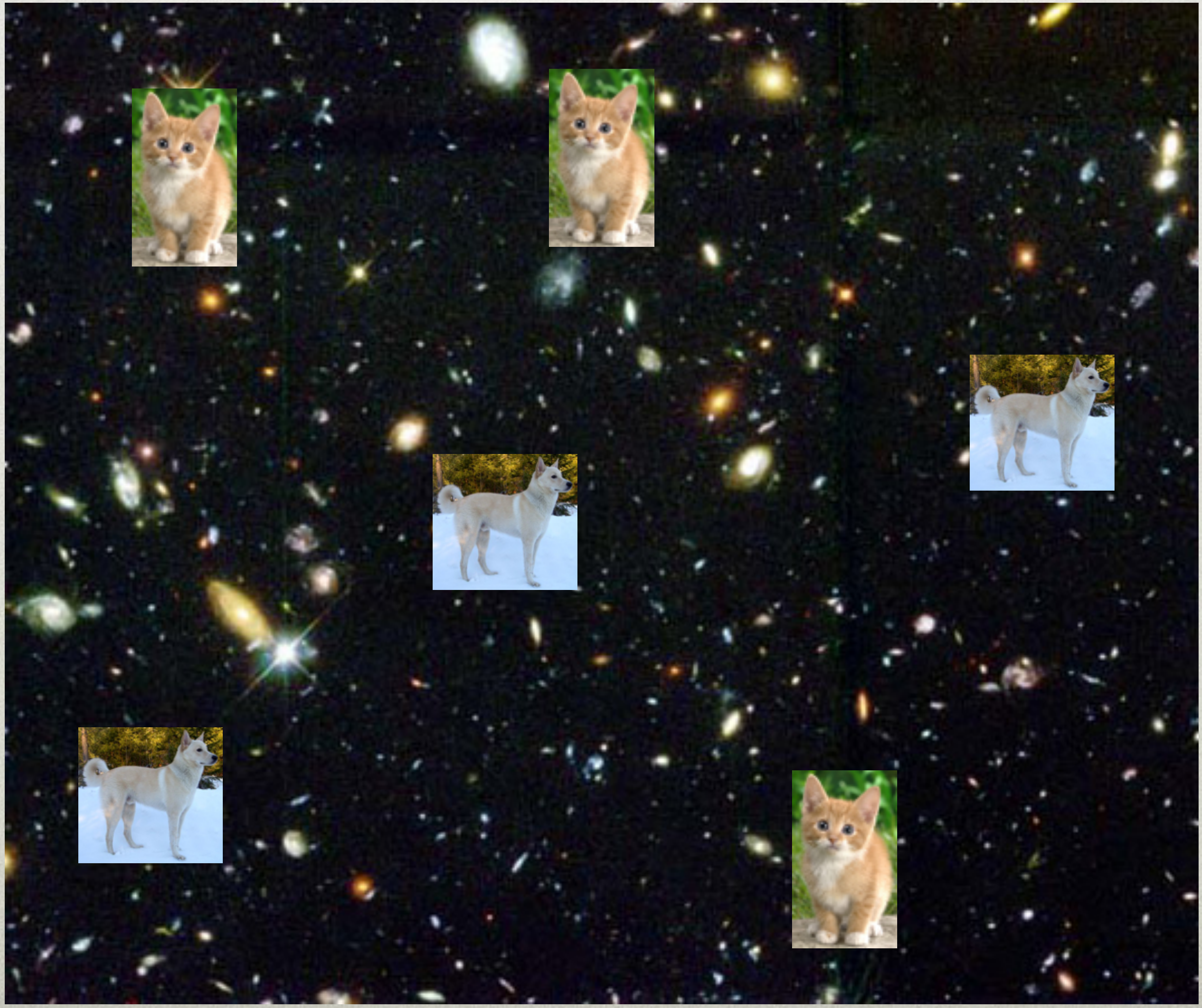
$\psi_d =$ 

$\Longrightarrow \; \exists \; \psi_{cd} =$ 

Statement about **structure of the Hilbert Space**:

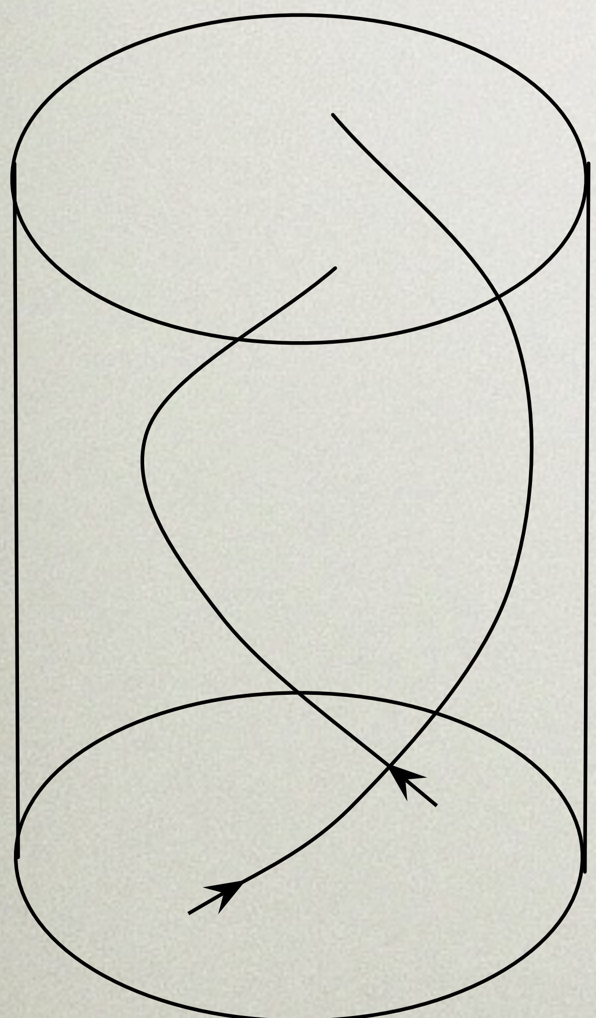
$$\mathcal{H}_{AdS} = \mathcal{H}_{CFT} \approx \text{Fock Space}$$

A Fock Space at Large Separation



How to Define Distant Furballs?

AdS



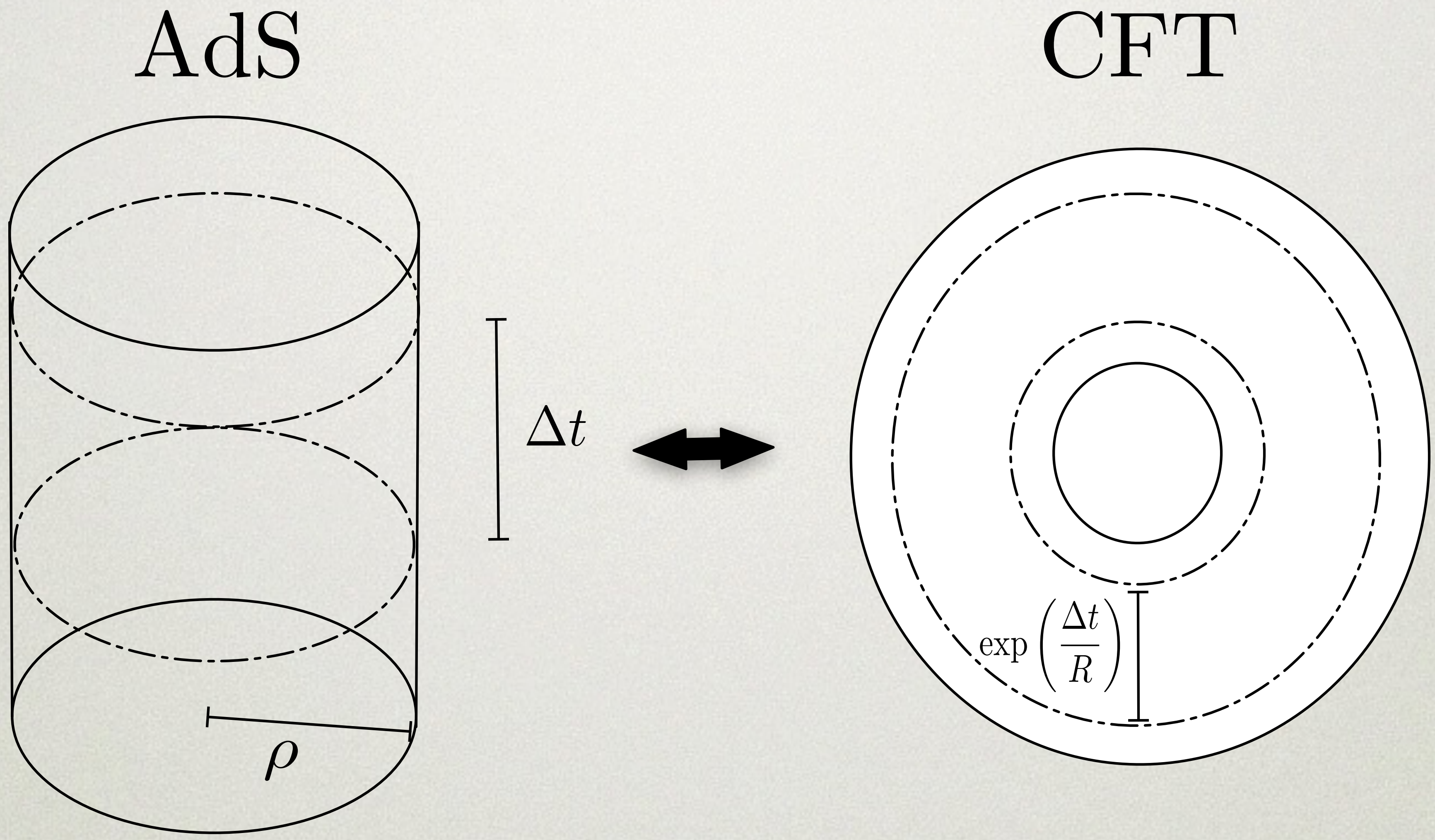
Geodesic separation between cat & dog:

$$\kappa \sim R_{AdS} \log \ell$$

Are there two furball states at **large angular momentum**???

Review of AdS/CFT and Expectations

Energies and Dimensions in AdS/CFT



$$H_{AdS} = D_{CFT}$$

Representations of Conformal Symmetry

The momentum and special conformal generators act as raising and lowering operators wrt Dimension

$$[D, P_\mu] = P_\mu \quad [D, K_\mu] = -K_\mu$$

Irreducible reps built from primaries:

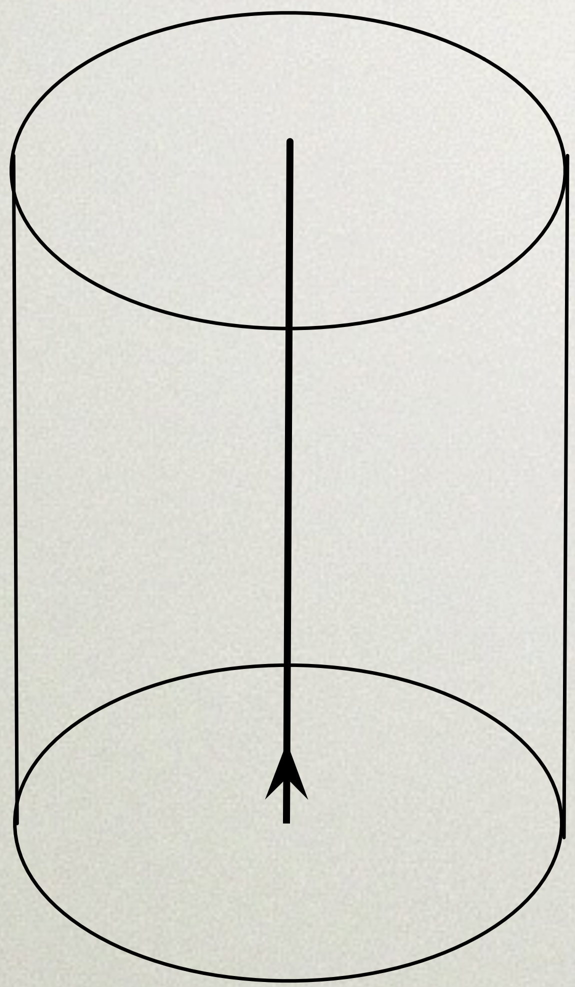
$$[K_\mu, \mathcal{O}(0)] = 0 \;\text{ or }\; K_\mu|\psi_{\mathcal{O}}\rangle = 0$$

Can derive a unitarity relation for $\tau = \Delta - \ell$

$$\Delta_s \geq \frac{d}{2} - 1 \;\text{ and }\; \tau_\ell \geq d - 2$$

Conformal Symmetry and Primary States

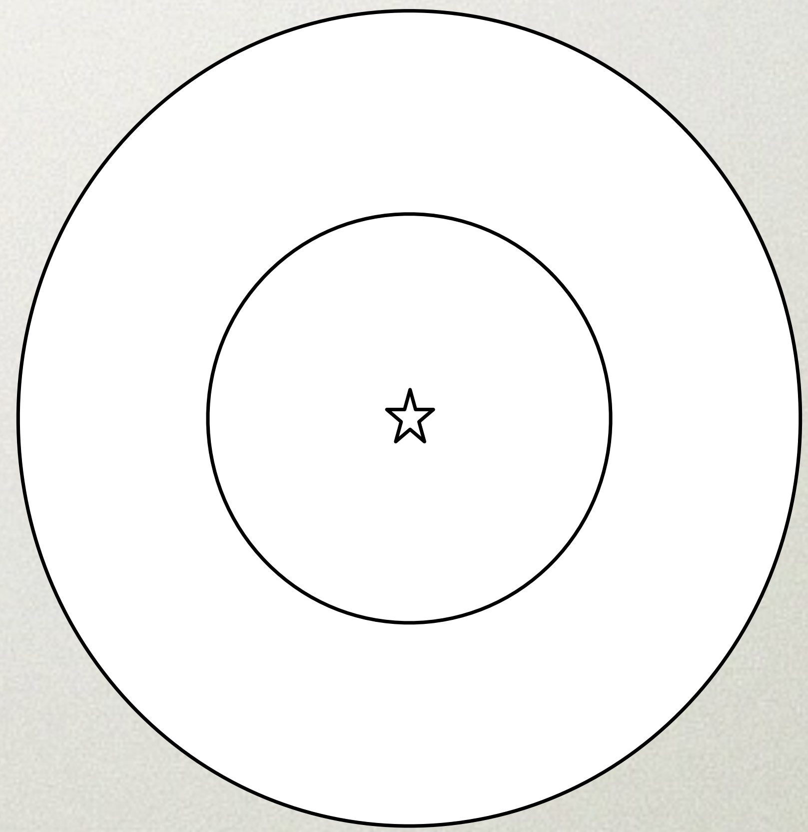
AdS



$$\psi_0(t,\rho) = e^{i\Delta t}\cos^{\Delta}(\rho)$$

CoM of Ground State

CFT



$$\mathcal{O}|0\rangle$$

CFT Primary State

Excited/Descendant States

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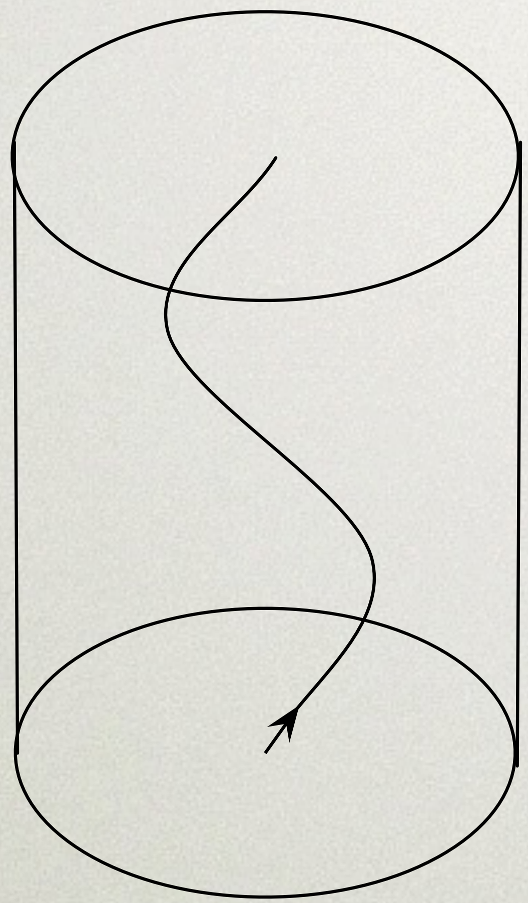
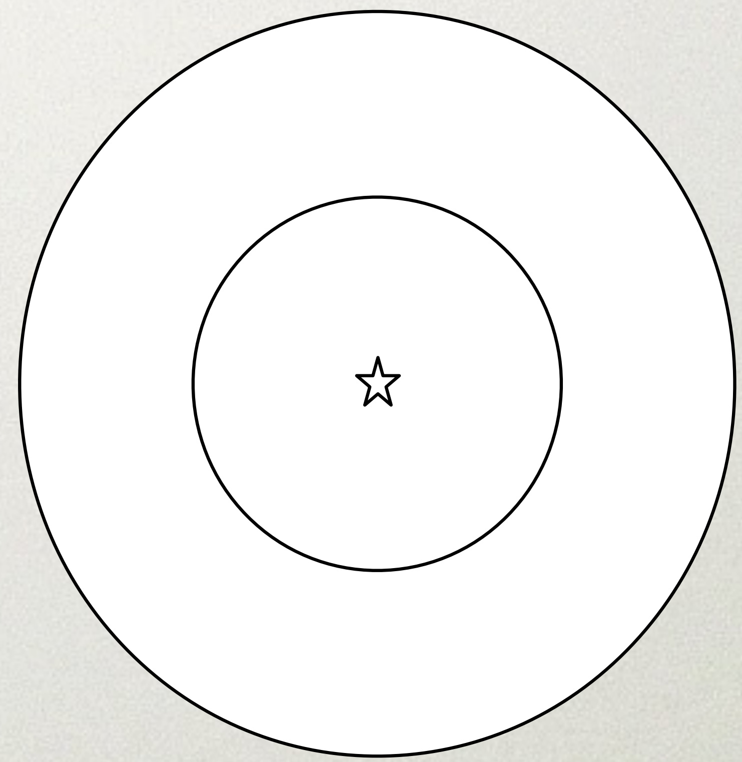


CFT



$\psi_{n,\ell}(t,\rho,\Omega)$

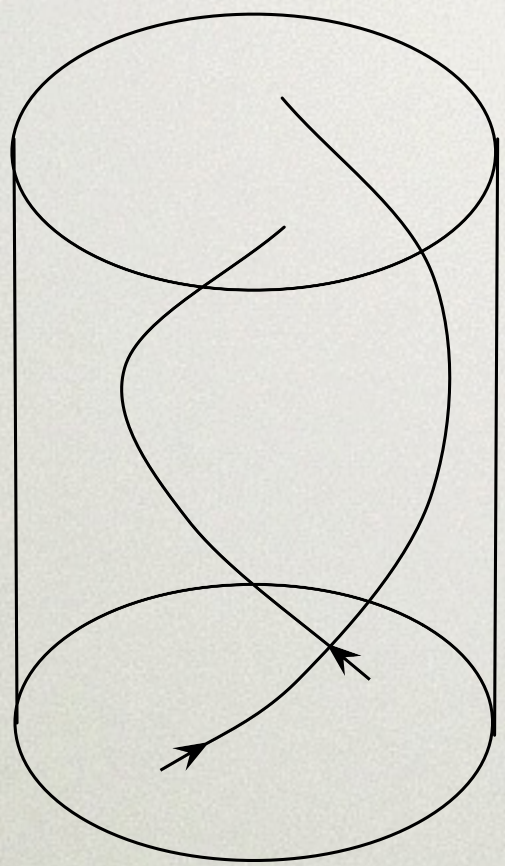
$\left(\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O}\right)|0\rangle$

Center of Mass for Excited State

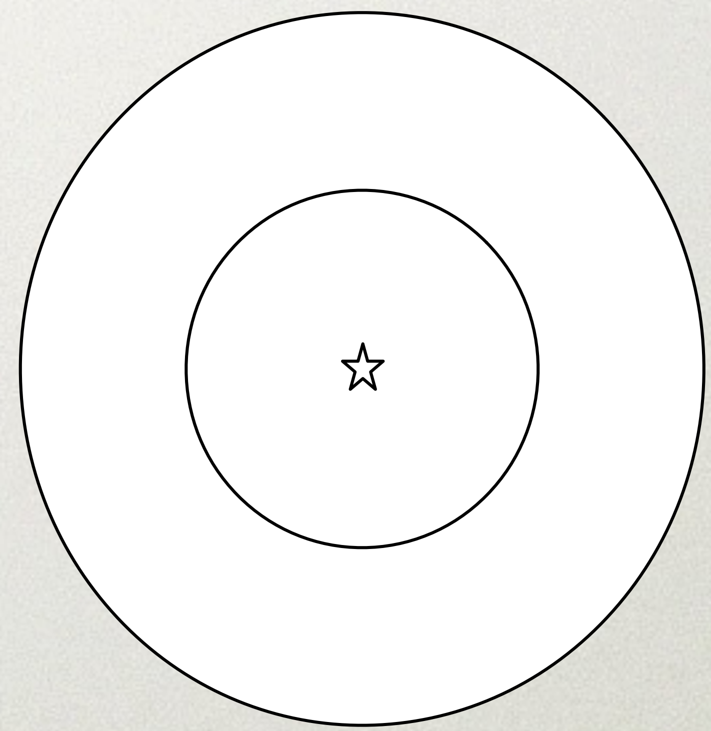
Descendant of a Primary

Now Specialize to Two Particle States?

AdS



CFT



$\psi_{n,\ell}(t_i, \rho_i, \Omega_i)$

Two Particle State, CoM at Origin

$\left(\mathcal{O}\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O}\right)|0\rangle$

`Double-Trace' Primary

Two Particle Kinematics

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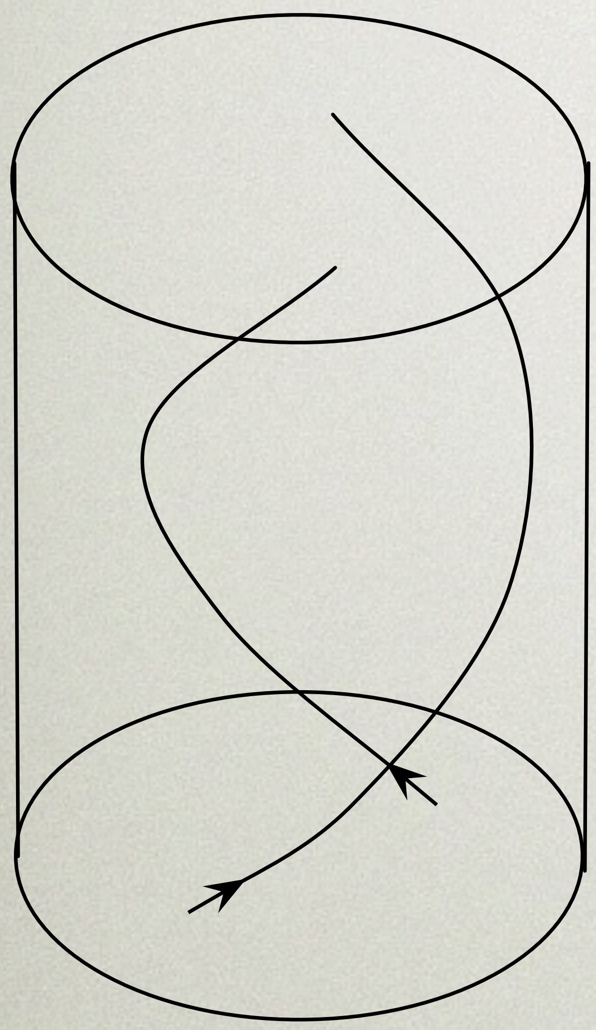


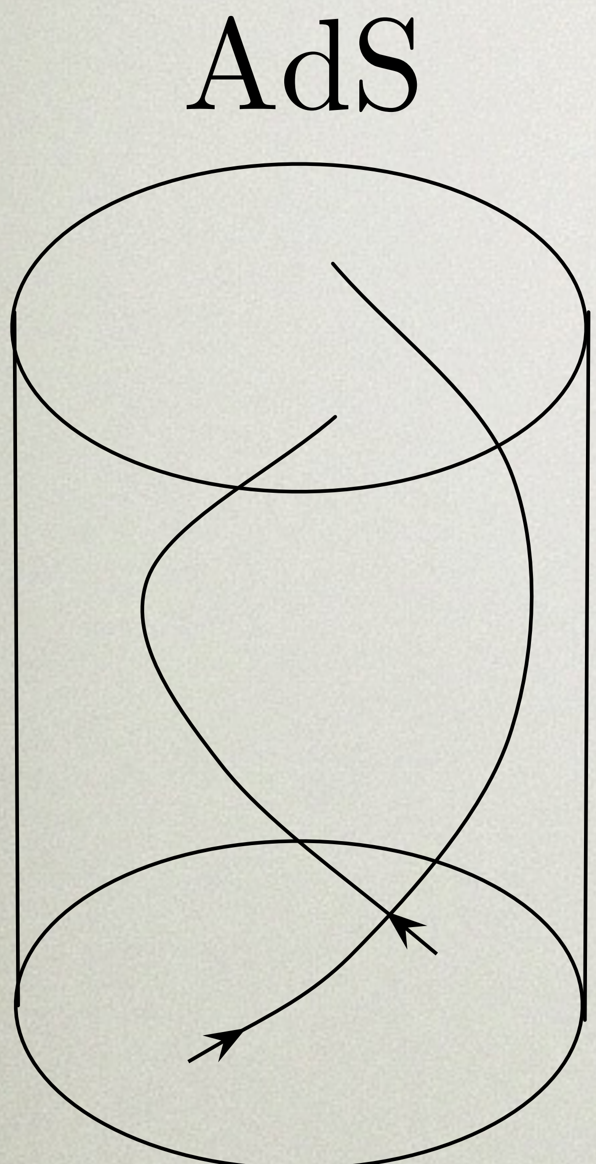
$$\left(\mathcal{O}\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O}\right)|0\rangle$$

Double-Trace Primary

Geodesic distance between objects:

$$\kappa \approx R_{AdS} \log\left[\frac{\ell}{\Delta_{\mathcal{O}}}\right]$$

Two Particle Physics



$$\left(\mathcal{O}\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O}\right)|0\rangle$$

Double-Trace Primary

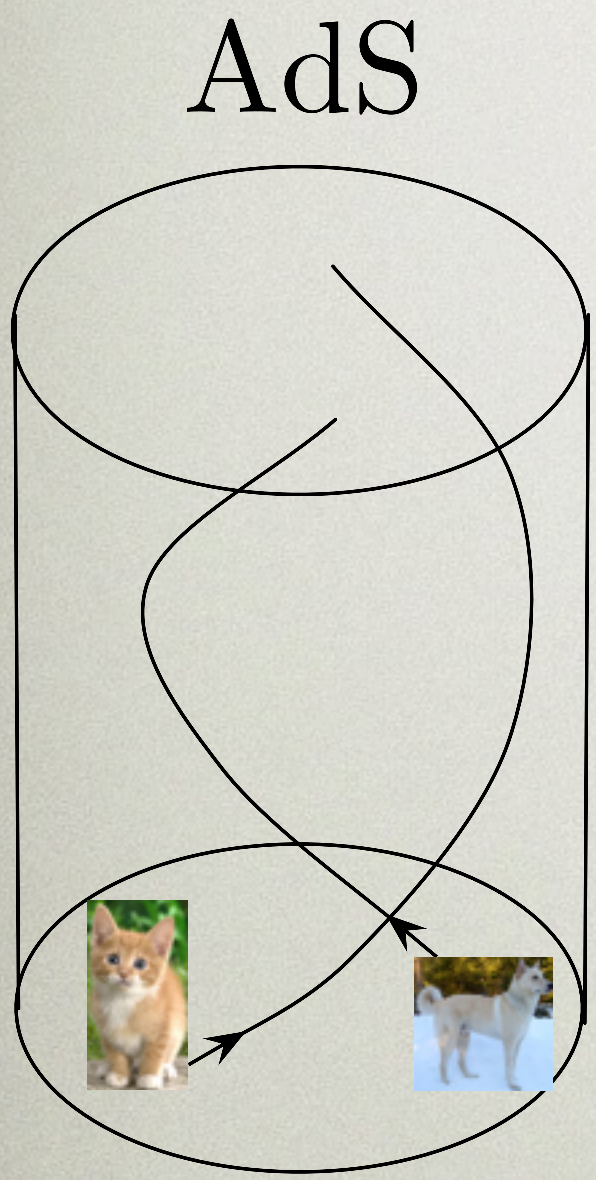
Bulk Energy = CFT Dimension:

$$\Delta_{n,\ell} = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,\ell)$$

View anomalous dimension as

$$\gamma(n,\ell) \to \gamma\left(n, e^{\kappa/R_{AdS}}\right)$$

Two Furball Physics?



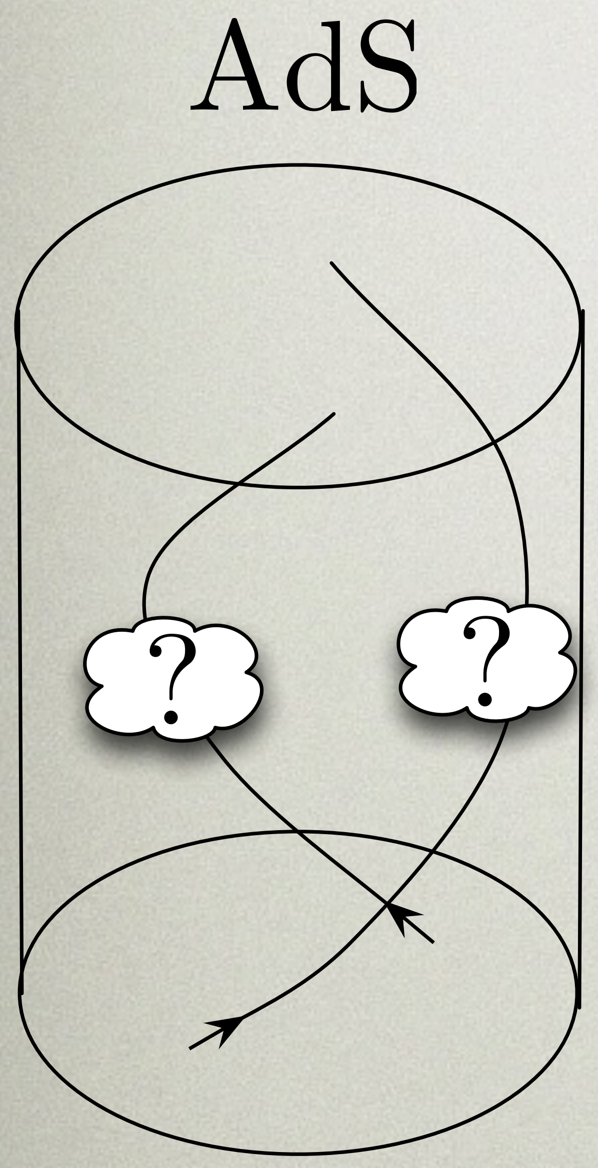
AdS Energy = **CFT Dimension**:

$$E_{n\ell} = E_c + E_d + 2n + \ell + \gamma(n, \ell)$$

Anomalous dimension, $\gamma(n, \ell)$, is a kind of `**binding energy**'.

Existence of states as $\ell \to \infty$ with vanishing $\gamma(n, \ell)$ implies **AdS Cluster Decomposion**.

Two Anything Physics?



In any CFT whatsoever in d > 2,
and in some d=2 limits, we will **prove** that
two **anything** states exist at large spin.

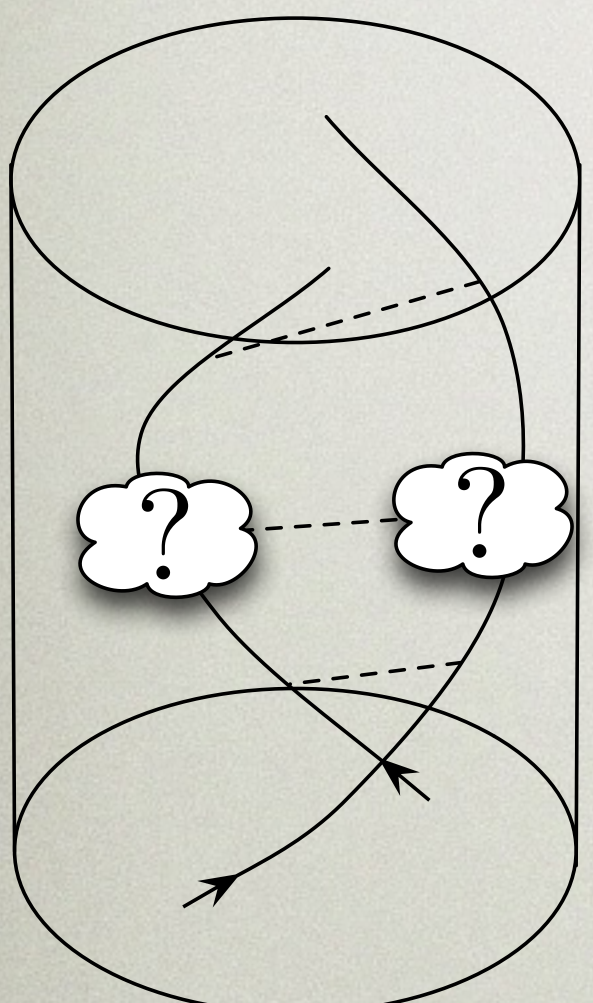
$$|\psi_{cd,\ell}\rangle$$

Generally, proves existence
of Fock space at large separation.

Expectations for Anomalous Dimensions from Distant Objects in AdS?

Anomalous Dimensions with Distance in D > 2

AdS



At large spin in d > 2, we will show that the interaction energies are universal:

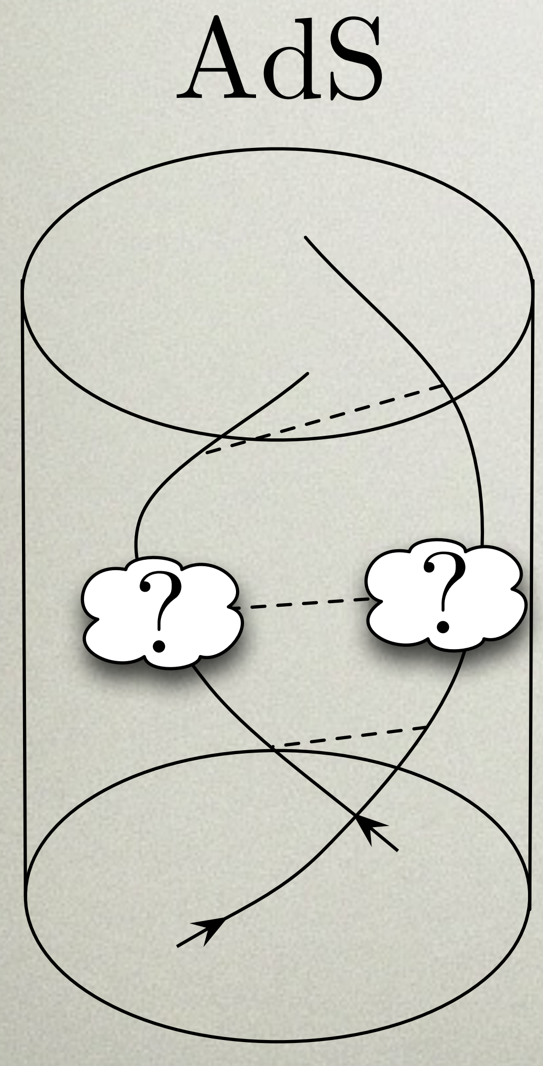
$$\gamma(n,\ell) \sim \frac{\gamma_n}{\ell^{\tau_{exch}}} \sim \gamma_n \exp[-\tau_{exch}\kappa]$$

Can be computed and matched in the `Newtonian' limit of AdS-Schwarzschild if $\tau_{exch} = d-2$.

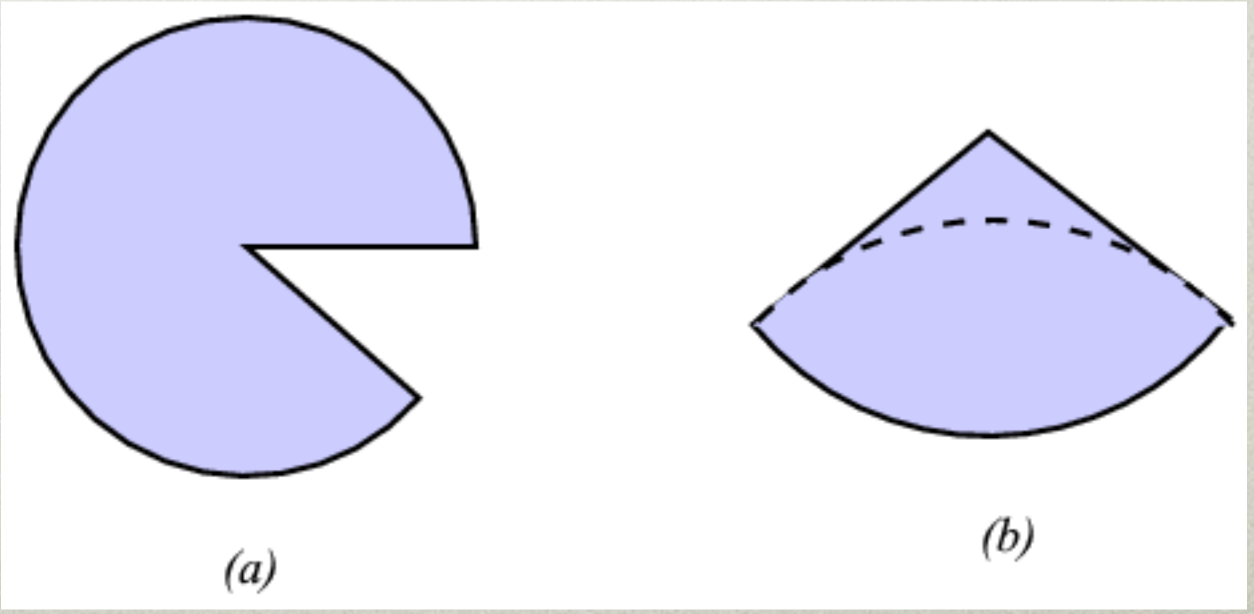
Deficit Angles in D=2

AdS Solution for a Sub-Planckian Object:

$$ds^2 = \cosh^2(\kappa)dt^2 - d\kappa^2 - (1 - 8GM)\sinh^2(\kappa)d\phi^2$$

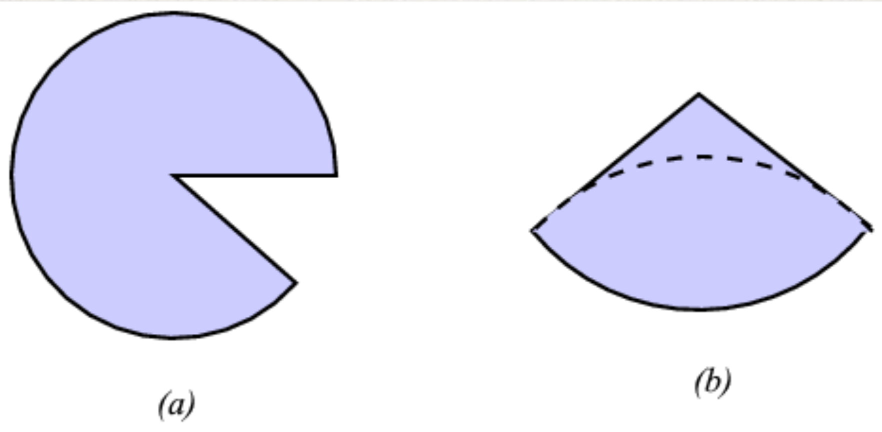


In 2+1 dimensional AdS, expect deficit angles, detectable near infinity.



D=2 Energy Shifts

Deficit angle implies time for orbits, corresponding to constant phase shift.



Leads to a **constant** energy shift at large separations.

$$\gamma \approx -\frac{6\Delta_1\Delta_2}{c}$$

Let's Prove that EVERY CFT in D>2 Dimensions Has a Fock Space

Theorem to Prove (for Any CFT in d>2)

Consider OPE of **any** two scalar primary operators:

$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) = \sum_{\Delta,\ell} c^{12}_{\Delta,\ell}\mathcal{O}_{\Delta,\ell}(x)$$

For each n there **exists** infinitely many operators

$$\mathcal{O}_{\Delta,\ell} \ \text{ with } \ \Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,\ell)$$

as $\ell \to \infty$, where the anomalous dimensions

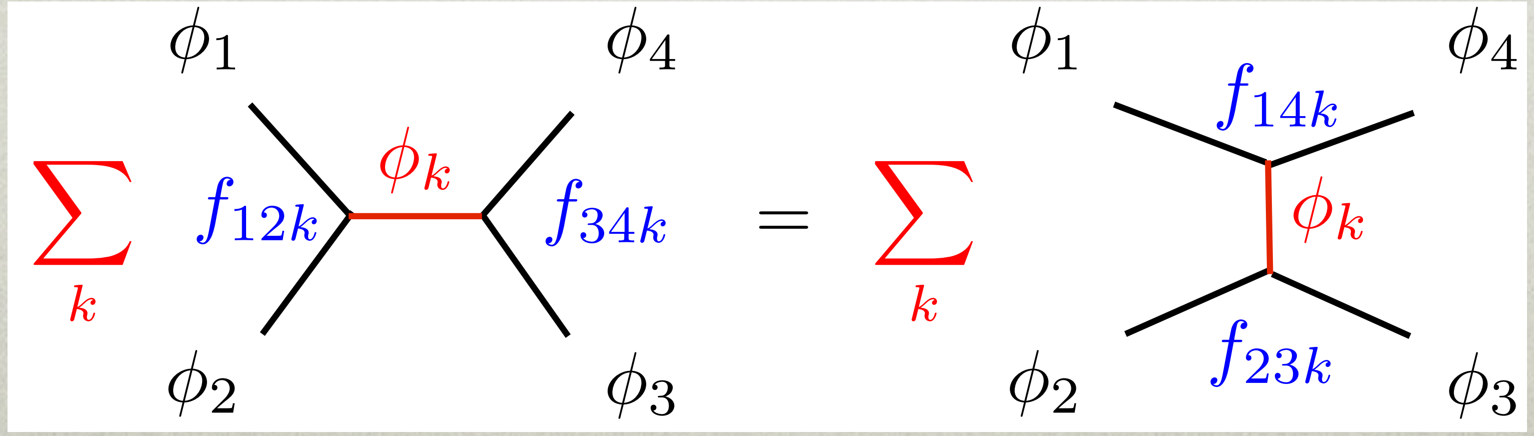
$$\gamma(n,\ell) \to \frac{\gamma_n}{\ell^{\tau_m}} \ \text{ or } \ \gamma_n e^{-\tau_m \kappa}$$

from leading twist, generically $T_{\mu\nu}$

What is the Bootstrap?

- Conformal Symmetry
- Unitarity
- Crossing Symmetry

What can we learn from the fundamental principles?



Consider the 4-pt CFT Correlators

Recall that 4-pt correlators can be written

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{A(u,v)}{(x_{13}^2 x_{24}^2)^{\Delta_\phi}}$$

where the conformal cross-ratios are

$$u = \left(\frac{x_{12}^2 x_{34}^2}{x_{24}^2 x_{13}^2}\right), \qquad v = \left(\frac{x_{14}^2 x_{23}^2}{x_{24}^2 x_{13}^2}\right)$$

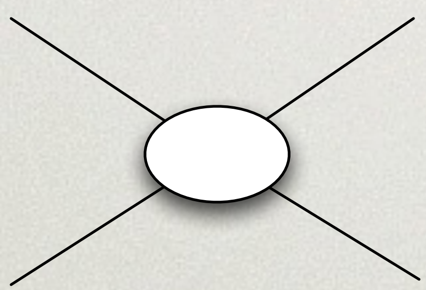
We can use elementary quantum mechanics to rewrite this in a different way...

The Conformal Partial Wave Decomposition

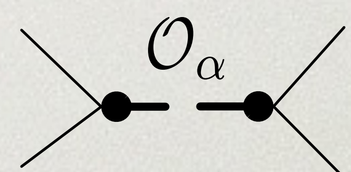
Insert 1, organize according to conformal symmetry:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \left(\sum_\alpha |\alpha\rangle\langle\alpha| \right) \mathcal{O}_3 \mathcal{O}_4 \rangle$$

Since **operators = states** in the CFT, write 4-pt as



$$= \sum_\alpha$$

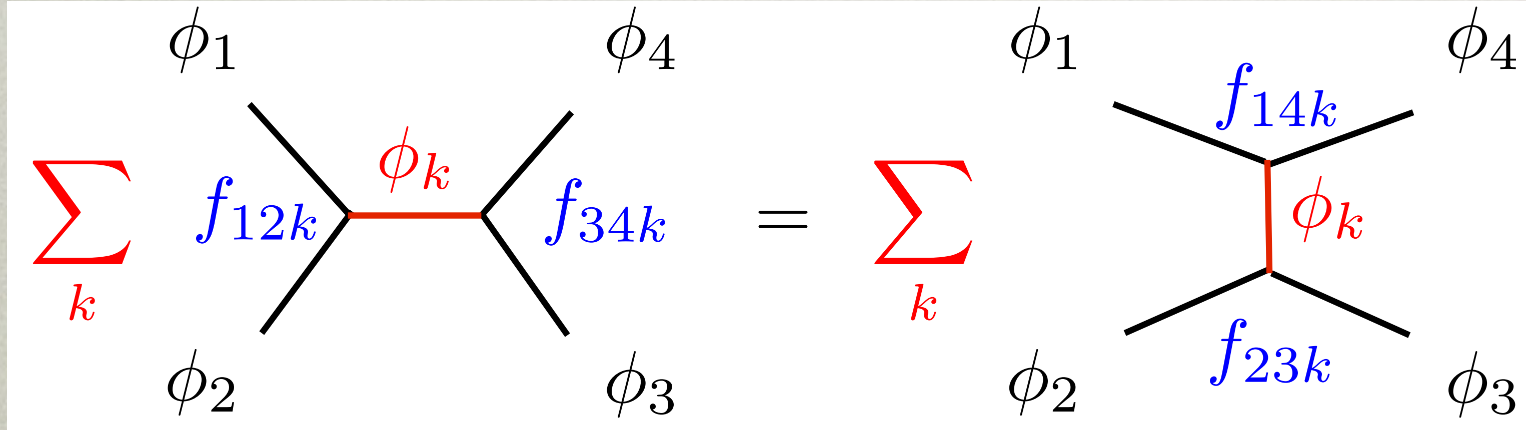


A sum over exchange of all **primary operators**, magnitude given by product of 3-pt correlators.

Formulate CFT Bootstrap

Crossing symmetry gives the **Bootstrap Equation**:

$$\frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_{\mathcal{O}} P_{\mathcal{O}} g_{\tau_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v) = \frac{1}{(x_{14}^2 x_{23}^2)^{\Delta_\phi}} \sum_{\mathcal{O}} P_{\mathcal{O}} g_{\tau_{\mathcal{O}},\ell_{\mathcal{O}}}(v,u)$$



Unitarity:

$$P_{\mathcal{O}} = f^2 > 0$$

The Idea of the Proof: A Scattering Analogy

Conformal Partial Waves ~ Partial Wave Amplitudes

Free propagation and massless exchange require large amplitude at large ℓ, e.g.

$$\frac{1}{1-\cos\theta} = \sum_{\ell} \cos^{\ell}\theta$$

Completely analogous CFT phenomenon implies existence of large ℓ states.

Consider the CFT Bootstrap for Generalized Free Theories...

(Warm Up Example) Generalized Free Theory

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{(x_{12}^2x_{34}^2)^{\Delta_\phi}} + \frac{1}{(x_{13}^2x_{24}^2)^{\Delta_\phi}} + \frac{1}{(x_{14}^2x_{23}^2)^{\Delta_\phi}}$$

$$= \frac{1}{(x_{13}^2x_{24}^2)^{\Delta_\phi}}\left(u^{-\Delta_\phi}+1+v^{-\Delta_\phi}\right).$$

What is the conformal block decomposition?

$$u^{-\Delta_\phi}+1+v^{-\Delta_\phi} = v^{-\Delta_\phi} + v^{-\Delta_\phi}\sum_{n,\ell} P_{2\Delta_\phi+2n,\ell}\, g_{2\Delta_\phi+2n,\ell}(v,u)$$

Focus on the singularity as $u \to 0$

Lightcone OPE limit.

Generalized Free Theory

In the limit that $u \to 0$

$$u^{-\Delta_\phi} \approx v^{-\Delta_\phi} \sum_{\tau,\ell} P_{\tau,\ell}\, g_{\tau,\ell}(v,u)$$

The conformal blocks each behave as

$$g_{\tau,\ell}(v,u) = a v^{\frac{\tau}{2}} \log u + b v^{\frac{\tau}{2}} + \cdots$$

We cannot recover the LHS without an infinite sum.

Must be infinite sum over spins, ℓ.

Generalized Free Theory Operators

So we discovered the operators

$$\phi\partial_{\mu_1}\cdots\partial_{\mu_\ell}\phi$$

Looking at the sub-leading v dependence of

$$u^{-\Delta_\phi} = v^{-\Delta_\phi}\sum_{\tau,\ell} P_{\tau,\ell}\, g_{\tau,\ell}(v,u)$$

In the $u \to 0$ limit, also find operators

$$\phi(\partial^2)^n\partial_{\mu_1}\cdots\partial_{\mu_\ell}\phi$$

Trivial result here. What about general CFTs???

Any D>2 CFT

General Bootstrap

Separating out the disconnected piece, have

$$u^{-\Delta_\phi} + \sum_{\tau,\ell} P_{\tau,\ell} u^{\frac{\tau}{2}-\Delta_\phi} f_{\tau,\ell}(u,v) = v^{-\Delta_\phi} + \sum_{\tau,\ell} P_{\tau,\ell} v^{\frac{\tau}{2}-\Delta_\phi} f_{\tau,\ell}(v,u)$$

Have singularity in a general CFT! Recall unitarity:

$$\Delta_s \geq \frac{d}{2} - 1 \quad \text{and} \quad \tau_\ell \geq d-2$$

Disconnected (identity) piece **cleanly separated** for

$$d \geq 3$$

Need Virasoro in d=2… later.

Cluster Decomposition

This is used to prove existence of the operators

$$\mathcal{O}_{\Delta,\ell} \quad \text{with} \quad \Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,\ell)$$

in the limit of large spins.

Physical interpretation? By analogy with scattering...

$$g_{\tau,\ell}(u,v) \quad \text{like} \quad \mathcal{A}_\ell(s) P_\ell(\cos\theta)$$

Need divergence at large spin to capture
disconnected piece ~ free propagation

Anomalous Dimensions?

Anomalous dimensions from sub-leading corrections:

$$u^{-\Delta_\phi} + P_m u^{\frac{\tau_m}{2}-\Delta_\phi}\log v + \cdots = \sum_{\tau,\ell} P_{\tau,\ell} v^{\frac{\tau}{2}-\Delta_\phi} f_{\tau,\ell}(v,u)$$

Note that by unitarity we always have:

$$-\Delta_\phi < \frac{\tau_m}{2} - \Delta_\phi < 0$$

Sub-leading correction can only come from a sum over infinitely many spins on RHS:

$$v^{\frac{\tau}{2}-\Delta_\phi} = v^n \left(1 + \gamma(n,\ell)\log v + \cdots\right)$$

Matching LHS and RHS gives desired result.

The D=2 Case with Virasoro Symmetry at large central charge

What makes D=2 Different?

The identity is not the only **twist zero** state!

$$u^{-\Delta_\phi} + \sum_{\tau,\ell} P_{\tau,\ell} u^{\frac{\tau}{2}-\Delta_\phi} f_{\tau,\ell}(u,v) = v^{-\Delta_\phi} + \sum_{\tau,\ell} P_{\tau,\ell} v^{\frac{\tau}{2}-\Delta_\phi} f_{\tau,\ell}(v,u)$$

Sum over Virasoro descendants changes singularity structure in lightcone OPE limit on the **left-hand side**.

Interpret as AdS gravitational effects that **do not vanish** at large separation.

Need Virasoro Blocks in Lightcone OPE Limit

Not obvious from the literature.

Most techniques expand in OPE limit, without clear generalization to our case.

We will study the very simple limit of

$$h_1, h_2, c \to \infty$$

where we take

$$h_i/c \to 0, \quad h_1 h_2/c \text{ fixed}$$

Compute by Brute Force

$$V_k = \sum_{m_1, m_2, \cdots m_k} \frac{1}{k!} \frac{\langle \phi\phi L_{-m_1} \cdots L_{-m_k} \rangle \langle L_{m_k} \cdots L_{m_1} \phi\phi \rangle}{\langle L_{m_k} \cdots L_{m_1} L_{-m_1} \cdots L_{-m_k} \rangle}$$

Obtain a simple exponentiation in our limit:

$$V_{cl} \equiv e^{V_1} = \exp\left[2 \frac{h_1 h_2}{c} z^2 {}_2F_1(2, 2, 4; z) \right]$$

Agrees with other computations of semi-classical blocks expanded in z.

Lightcone OPE limit

In the lightcone OPE limit $u \to 0$ get power

$$V_{cl} \approx u^{-\Delta_\phi + \frac{3\Delta_\phi^2}{c}}$$

This corresponds to a constant shift in the dimensions of 2-phi states:

$$\Delta_{2\phi} = 2\Delta_\phi + \ell - \frac{6\Delta_\phi^2}{c} + \cdots$$

Reproducing effect of the AdS Deficit angle, directly from a limit of the bootstrap.

Philosophy - Spacetime in Quantum Theories

Proved theorems about the CFT spectrum with striking AdS **interpretation**.

Spacetime is `merely' a set of coordinate labels associated with the states and operators of a quantum mechanical system.

It's a **useful** idea when the Hamiltonian of the system is **approximately local** in these coordinates.

Conclusions & Future Directions

- All d > 2 CFTs have states that evolve via Dilatations like objects in global AdS satisfying cluster decomposition
- corrections can be computed purely from the bootstrap, giving long-range forces, e.g. Gravity
- D=2 CFTs in a large central charge limit satisfy a modified theorem, but to generalize further need lightcone OPE limit of Virasoro blocks!?
- Use BTZ to make a prediction for these blocks?

Large Spin Operators

Conformal Block Coeffs and Blocks are known:

$$P_{2\Delta_\phi,\ell} \sim \ell^{2\Delta_\phi - \frac{3}{2}}$$

$$g_{\tau,\ell}(v,u) \sim v^{\frac{\tau}{2}-\Delta_\phi} \frac{e^{-\ell\sqrt{u}}}{\sqrt[4]{u}}$$

Thus the sum over coeffs times blocks gives:

$$u^{-\Delta_\phi} \propto \int^\infty d\ell \, v^{\frac{\tau}{2}-\Delta_\phi} \ell^{2\Delta_\phi - \frac{3}{2}} \frac{e^{-\ell\sqrt{u}}}{\sqrt[4]{u}}$$

Singularity reproduced by large spin operators with

$$\tau = 2\Delta_\phi \quad \text{as} \quad \ell \to \infty$$