Higher Spin Gravity

Per Kraus

- Plan: broad overview of higher spin gravity and its application to AdS/CFT
- Various reviews:
  - Vasiliev: hep-th/9910096
  - Gaberdiel, Gopakumar, arxiv: 1207.6697
  - Giombi, Yin, arxiv: 1208.4036
  - Ammon, Gutperle, PK, Perlmutter, arxiv: 1208.5182
  - Iazeolla, Sundell, arxiv: 1208.4077
  - Bekaert, Cnockaert, Iazeolla, Vasiliev: hep-th/0503128
  - Didenko, Skvortsov, arxiv: 1401.2975
  - Rahman, arxiv: 1307.3199
  - Sagnotti, arxiv: 1112.4285

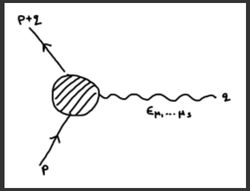
Introduction

- Much of what we know about fundamental physics is captured by the study of massless particles of spins: s = 0, 1/2, 1, 3/2, 2
- Principles of quantum mechanics and relativity tightly constrain possibilities, and *almost* rule out extensions to s>2
- Restrictions refer to theories of *interacting massless particles with a Lorentz invariant S-matrix, coupled to charged particles*
- *Free* massless particles of any spin are of course consistent, and nature provides us with interacting *massive* particles of s>2

s > 2?

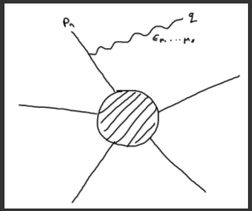
- Weinberg (1964) exposed the primary obstacle to constructing higher spin theories

Vertex for emission of spin-s particle in soft limit:



$$\pm Q_n p^{\mu_1} \dots p^{\mu_s} \epsilon_{\mu_1 \dots \mu_s}$$

S-matrix amplitude with soft emission:



$$\mathcal{M} = \mathcal{M}_0 \sum_n (\pm)_n \frac{Q_n p_n^{\mu_1} \dots p_n^{\mu_s} \epsilon_{\mu_1 \dots \mu_s}}{p_n \cdot q}$$

Lorentz invariance requires invariance under $\delta\epsilon_{\mu_1 \dots \mu_s} = q_{(\mu_1}\lambda_{\mu_2 \dots \mu_s)}$

Implies conservation law $\sum_n (\pm)_n Q_n p_n^{\mu_1} \dots p_n^{\mu_{s-1}} = 0$

For s>2 this renders S-matrix trivial, unless Q=0

Free field theory

- Fronsdal (1978) wrote down actions for free massless spin-s particles in Minkowski space

Double traceless, symmetric tensor: $\phi_{\mu_1\ldots\mu_s}$

$$\eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\phi_{\mu_1\ldots\mu_s}=0$$

Fronsdal tensor:

$$\begin{aligned}\mathcal{F}_{\mu_1\ldots\mu_s}&=\partial^2\phi_{\mu_1\ldots\mu_s}-\partial_{(\mu_1}\partial\cdot\phi_{\mu_2\ldots\mu_s)}\\&+\partial_{(\mu_1}\partial_{\mu_2}\eta^{\nu_1\nu_2}\phi_{(\nu_1\nu_2\mu_3\ldots\mu_s)}\end{aligned}$$

Gauge invariance: $\delta\phi_{\mu_1\ldots\mu_s}=\partial_{(\mu_1}\lambda_{\mu_2\ldots\mu_s)}$ $\qquad\eta^{\mu_1\mu_2}\lambda_{\mu_1\ldots\mu_{s-1}}=0$

Field equation: $\mathcal{F}_{\mu_1\ldots\mu_s}=0$

Gauge invariance does not survive in curved background:

$$\eta_{\mu\nu}\to g_{\mu\nu}\ ,\quad \partial_\mu\to\nabla_\mu$$

obstacle is: $[\nabla_\mu,\nabla_\nu]\neq 0$

Only in (A)dS can one add terms to cancel non-invariance

Higher spins in AdS

- Higher spin particles can propagate consistently in AdS, and can furthermore interact
- Introducing interactions in the Fronsdal metric-type formulation is cumbersome, and so far has led to only partial results for cubic vertices
- Vasiliev formulation is in terms of generalized frame fields
    - infinite tower of HS particles
    - nonlocal interactions

Higher spin currents

- Massless HS fields in bulk (with suitable boundary conditions) imply conserved HS currents on boundary. Their correlators are highly constrained
- Maldacena/Zhiboedov (2011) studied constraints on such correlators in d=3 (boundary dimension)
- Under mild assumptions they found:
  - a single HS current implies an infinite tower of even spin currents
  - correlators of these currents are the same as in a theory of free bosons or fermions

$$J^{(s)} \sim \phi \partial^s \phi \ , \quad \psi \partial^{s-1} \psi$$

- No analogous theorem in d=2

Vasiliev system

- Construction in all D, but here restrict to D=3,4
- Background independent field equations, but no conventional action

$$dW = W \wedge \star W$$

$$dB = W \star B - B \star W$$

$$dS = W \star S - S \star W$$

$$S \star S = \mathcal{C} + R(B)$$

- W, B and S are master fields, functions of spacetime coords and internal spinor variables
  - W = higher spin gauge fields
  - B = scalar field and higher spin Weyl tensors
  - S = auxiliary fields

$$\boxed{dW = W \wedge \star W}$$

- Recall how AdS can be described as a flat connection

Generators of O(D-1,2): (P_a, M_{ab})

Define O(D-1,2) connection: $A = A^a P_a + \frac{1}{2} A^{ab} M_{ab}$

Field strength: $$\begin{aligned} F &= dA + A \wedge A \\ &= T^a P_a + (R^{ab} + A^a \wedge A^b) M_{ab} \end{aligned}$$

Under identification: $$\begin{aligned} A^a_\mu &= e^a_\mu \quad \text{vielbein} \\ A^{ab}_\mu &= \omega^{ab}_\mu \quad \text{spin connection} \end{aligned}$$

F=0 defines condition for AdS_D with torsionless connection

- D=3 solutions of Einstein gravity with Λ<0 are locally AdS_3. Of course not true for D>3

Higher spin extension

- Natural to embed O(D-1,2) in larger HS algebra. Focus here on D=3: $O(2,2) \cong SL(2) \oplus SL(2) \subset \text{HS algebra}$

Realize SL(2) in terms of spinors:

$$T_{\alpha\beta} = y_{(\alpha} \star y_{\beta)} \quad \alpha, \beta = 1, 2$$

$$[y_\alpha, y_\beta]_\star = \epsilon_{\alpha\beta}$$

$$\star = \text{Moyal star product}$$

Extend to all symmetrized products of spinors:

$$y_{(\alpha_1} \star y_{\alpha_2} \star \cdots \star y_{\alpha_{2n})}$$

Star-commutators define infinite dimensional Lie algebra hs[1/2]

Generalize to hs[λ] by using “deformed oscillators”:

$$[\tilde{y}_\alpha, \tilde{y}_\beta]_\star = \epsilon_{\alpha\beta}(1+\nu k) \;, \quad k^2 = 1 \;, \quad k\tilde{y}_\alpha = -\tilde{y}_\alpha k$$

$$\lambda = \tfrac{1+\nu}{2}$$

hs[λ] is the natural analytic continuation of SL(N) to noninteger N

- A natural D=3 HS algebra is thus $\mathrm{hs}[\lambda] \oplus \mathrm{hs}[\lambda]$

  Vasiliev equation $dW = W \wedge \star W$ is the flatness conditions for the corresponding connection

  Einstein gravity in D=3 has no propagating degrees of freedom, but still has rich physics:

  - BTZ black holes (quotients of AdS)
  - boundary gravitons and asymptotic Virasoro algebra

  These are generalized in interesting ways in HS context:

  - Generalizations of BTZ carrying HS charges
  - asymptotic Virasoro → asymptotic W-algebras

- D=4 version of this story is less rich

$$\boxed{dB = W \star B - B \star W}$$

- covariant constancy condition yields the scalar wave equation and equations for HS Weyl tensors

Constant solution $B = \nu$ fixes the HS algebra

Expanded around AdS_3 one finds the KG equation for a scalar field of mass $m^2 = \lambda^2 - 1, \quad \lambda = (1+\nu)/2$

$$B = \nu + C_0(x) + C^{\alpha\beta}(x)\tilde{y}_\alpha \tilde{y}_\beta + C^{\alpha\beta\gamma\delta}(x)\tilde{y}_\alpha \tilde{y}_\beta \tilde{y}_\gamma \tilde{y}_\delta + \dots$$

Star-multiplication against W relates different components

System is solved via $C^{\alpha_1 \dots \alpha_s} \sim \partial^s C_0$ and $(\nabla^2 + m^2)C_0 = 0$

- Interesting to ask how wave equation is modified in presence of HS gauge fields

Wave equation in HS background

Decomposing: $W = A \oplus \overline{A}$ and using notation:

$$V^s_m \sim (y_1)^{n_1}(y_2)^{n_2} \ , \quad n_1 + n_2 = 2s - 2 \ , \quad n_1 - n_2 = 2m$$

Then AdS is described by the connection:

$$\begin{aligned} A &= e^\rho V^2_1 dz + V^2_0 d\rho \\ \overline{A} &= e^\rho V^2_{-1} d\bar{z} - V^2_0 d\rho \end{aligned} \quad \Longrightarrow \quad ds^2 = d\rho^2 + e^{2\rho} dz d\bar{z}$$

Add in a spin-s perturbation via:

$$A \to A + \eta e^{(s-1)\rho} V^s_{s-1} d\bar{z} \quad \Longrightarrow \quad \phi_{\bar{z}\bar{z}\ldots\bar{z}} \sim \eta e^{2(s-1)\rho}$$

Scalar wave equation becomes:

$$\left(\nabla^2 + m^2 + 4\eta e^{-2\rho}(-\partial)^s\right) C_0 = 0$$

Illustrates appearance of higher order derivatives, and makes it clear that in a general background have nonlocal wave equation

- D>3: expanded around AdS get Fronsdal equations for higher spin gauge fields

Higher orders

$$dW = W \wedge \star W$$

$$dB = W \star B - B \star W$$

$$dS = W \star S - S \star W$$

$$S \star S = \mathcal{C} + R(B)$$

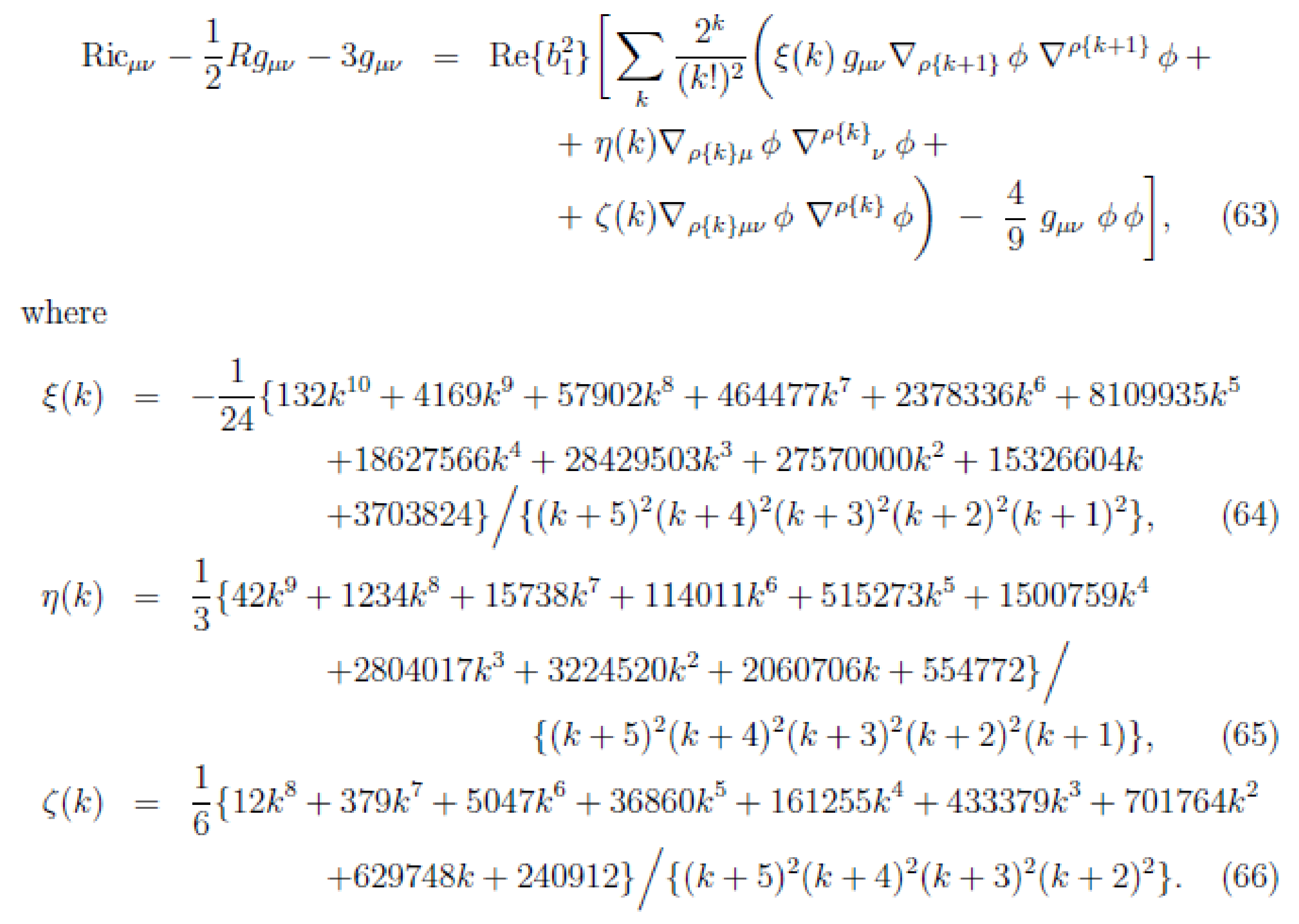
Master fields are functions of two sets of spinors: (y_α, z_α)

At linearized order can ignore the z’s, but not beyond:

backreaction: $\delta B \to \delta S \to \delta W$

Physical fields are identified with those at z=0. So the flatness condition $dW = W \wedge \star W$ in fact describes nontrivial dynamics. Nonlocality of star product in z-space implies complicated dynamics among physical fields.

Direct attempt to find stress tensor on RHS of D=4 Einstein equations yields …



$$\begin{aligned}\mathrm{Ric}_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - 3g_{\mu\nu} \;=\; & \mathrm{Re}\{b_1^2\}\Bigg[\sum_k \frac{2^k}{(k!)^2}\Bigg(\xi(k)\, g_{\mu\nu} \nabla_{\rho\{k+1\}}\, \phi\ \nabla^{\rho\{k+1\}}\, \phi + \\ & + \eta(k)\nabla_{\rho\{k\}\mu}\, \phi\ \nabla^{\rho\{k\}}{}_{\nu}\, \phi + \\ & + \zeta(k)\nabla_{\rho\{k\}\mu\nu}\, \phi\ \nabla^{\rho\{k\}}\, \phi\Bigg) \;-\; \frac{4}{9}\, g_{\mu\nu}\ \phi\, \phi\Bigg], \qquad (63)\end{aligned}$$

where

$$\begin{aligned}\xi(k) \;=\; & -\frac{1}{24}\{132k^{10} + 4169k^9 + 57902k^8 + 464477k^7 + 2378336k^6 + 8109935k^5 \\ & +18627566k^4 + 28429503k^3 + 27570000k^2 + 15326604k \\ & +3703824\}\Big/\{(k+5)^2(k+4)^2(k+3)^2(k+2)^2(k+1)^2\}, \qquad (64)\\ \eta(k) \;=\; & \frac{1}{3}\{42k^9 + 1234k^8 + 15738k^7 + 114011k^6 + 515273k^5 + 1500759k^4 \\ & +2804017k^3 + 3224520k^2 + 2060706k + 554772\}\Big/ \\ & \{(k+5)^2(k+4)^2(k+3)^2(k+2)^2(k+1)\}, \qquad (65)\\ \zeta(k) \;=\; & \frac{1}{6}\{12k^8 + 379k^7 + 5047k^6 + 36860k^5 + 161255k^4 + 433379k^3 + 701764k^2 \\ & +629748k + 240912\}\Big/\{(k+5)^2(k+4)^2(k+3)^2(k+2)^2\}. \qquad (66)\end{aligned}$$

(Kristianson/Rajan)

- Note that for D>3 metric perturbations source HS fields, so no truncation to metric sector

AdS/CFT Dualities

$\mathrm{AdS}_4/\mathrm{CFT}_3$ duality

Spectrum of minimal Vasiliev theory is an infinite tower of HS gauge fields and a scalar with mass:

$$m^2 = -2 \quad \Rightarrow \quad \Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2} = 1,\ 2$$

Simplest versions preserve parity, with scalar being parity odd or even. Together with scalar boundary conditions gives four possibilities. Klebanov/Polyakov and Sezgin/Sundell (2002) proposed that these correspond to large N boson or fermion vector models in the singlet sector:

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a$$

$$J^{(0)} = \phi^a \phi^a \ , \quad \Delta = 1\ , \quad P = +$$

$$J^{(s)} \sim \phi^a \partial^s_\mu \phi^a \ , \quad \Delta = s+1$$

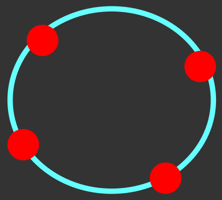
$$\mathcal{L} = \overline{\psi}^a i \partial\!\!\!/ \psi^a$$

$$J^{(0)} = \overline{\psi}^a \psi^a \ , \quad \Delta = 2\ , \quad P = -$$

$$J^{(s)} \sim \overline{\psi}^a \partial^{s-1}_\mu \psi^a \ , \quad \Delta = s+1$$

These free theories obviously have exact HS symmetry

Current correlators computed from Wick contractions still look "nontrivial"



Boundary conditions in the bulk also preserve HS symmetry. As long as computations of bulk correlators respect basic CFT properties, they are then guaranteed to match those of the CFT according to the result of Maldacena/Zhiboedov.

Borne out by earlier 2009 computation of 3-point functions by Giombi/Yin

These theories can be perturbed by “double trace” operators

$$\frac{\lambda}{N}(\phi^a\phi^a)^2\ , \quad \Delta = 2 \qquad\qquad \frac{\lambda}{N}(\overline{\psi}^a\psi^a)^2\ , \quad \Delta = 4$$

Easily studied using standard large N methods

Scalar theory flows to IR fixed point with $\Delta[\phi^a\phi^a] = 2 + O(\frac{1}{N})$

Fermion theory flows to UV fixed point with $\Delta[\overline{\psi}^a\psi^a] = 1 + O(\frac{1}{N})$

At finite N higher spin symmetry is not exact. However, breaking is due to “multi-trace” operators and so suppressed by 1/N:

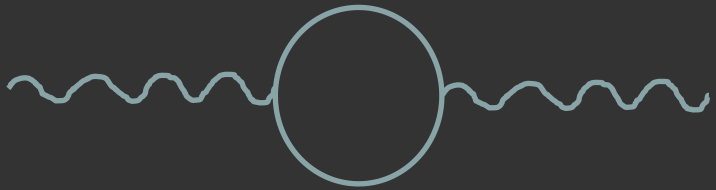
$$\partial \cdot J^{(s)} = \text{twist 3 operator}$$

if no single trace twist 3 operator: $\partial \cdot J^{(s)} = \frac{1}{N} J J' + \dots$

“slightly broken HS symmetry” (Maldacena/Zhiboedov)

Corresponding statement in the bulk is that choice of scalar boundary conditions can break HS symmetry due to quantum effects.

HS gauge fields get mass. Novelty here is that the massless gauge field pairs up with a multi-particle state to gain mass

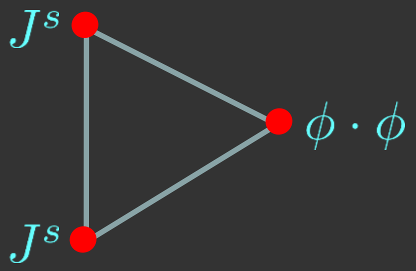


Only for one choice of boundary condition does two-particle state sit in correct representation to pair up with gauge field

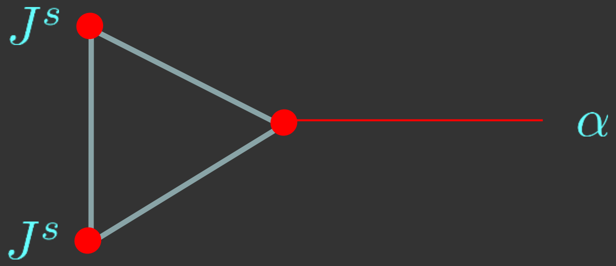
(Girardello, Poratti, Zafforoni)

- A simple argument shows that bulk-CFT agreement in free case implies agreement in interacting case (Giombi/Yin)

e.g. current-current-scalar 3-point function in free theory:



In interacting theory we introduce auxiliary field α coupling via αφφ, and correlator becomes



In IR α acquire dimension 2, and propagator |p|. Extra factor of |p| matches modification of scalar propagator in bulk upon change of boundary conditions

More generally, slightly broken HS symmetry still allows one to say a lot about correlators (Maldacena/Zhiboedov)

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle = \tilde{N} \left[\frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{bos}} + \frac{1}{1+\tilde{\lambda}^2} \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{fer}} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{odd}} \right]$$

- $\tilde{N}$ is some large (integer) free parameter
- $\tilde{\lambda}$ is another parameter

An important realization is in terms of Chern-Simons vector models

$\tilde{\lambda} = \lambda = \frac{N}{k}$ k = CS level

At finite λ the CS gauge field does more than just impose the singlet constraint. Theory is parity violating. (Giombi et al, Aharony et al)

Structure has direct analog in Vasiliev theory via parity violating phase in interaction function

$$f(X) = \frac{!}{4} + X e^{i\theta} \qquad \theta = \theta(\lambda)$$

Recover parity invariant theories for $e^{i\theta} = 1, i$ (Chang et. al.)

$\mathrm{AdS}_3/\mathrm{CFT}_2$ duality

Big clue comes from asymptotic symmetry group.
Recall that for Einstein gravity Brown and Henneaux established existence of asymptotic Virasoro algebra with c = 3L/2G.
Easily seen in the SL(2) x SL(2) CS formulation of 3D gravity:

$$A = \left(e^{\rho} L_1 - \tfrac{2\pi}{k} e^{-\rho} \mathcal{L}(z,\overline{z}) L_{{}_1} \right) dz + L_0 d\rho$$

asymptotically AdS connection

Look for gauge transformations preserving this form. Find free functions worth $\epsilon(z)$ with:

$$\delta\mathcal{L} = -\tfrac{k}{4\pi}\partial^3\epsilon(z) - 2\partial\epsilon(z)\mathcal{L} - \epsilon(z)\partial\mathcal{L} \qquad\qquad c = 6k = \tfrac{3L}{2G}$$

Now extend to the HS algebra:

$$A = \left(e^{\rho} L_1 - \tfrac{2\pi}{k} e^{-\rho} \mathcal{L}(z,\overline{z}) L_{{}_1} + \tfrac{1}{k} e^{-2\rho} V^{3}_{{}_2} J_3(z,\overline{z}) + e^{-3\rho} V^{4}_{-3} J_4(z,\overline{z}) + \ldots \right) dz$$

asymptotic Virasoro → asymptotic W-algebra $W_\infty(\lambda)$

(Campoleoni et. al., Henneaux/Rey, Hartman/Gaberdiel)

In general, gauge transformation parameters are field dependent, and this leads to quadratic nonlinearities in the algebra. Classical analysis yields classical algebra. Need 1/k corrections to convert to quantum commutator algebra obeying Jacobi identities.

Since Virasoro minimal models have coset representation

$$\frac{SU(2)_k \oplus SU(2)_1}{SU(2)_{k+1}}$$

rather natural to suppose that HS bulk theory is dual to W-minimal model

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}$$

(Gaberdiel/Gopakumar)

- nontrivial interacting theories with HS currents
- everything is computable *in principle* (and a lot in practice)

$$\boxed{\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}}$$

central charge: $c = (N-1)\left(1 - \frac{N(N+1)}{(N+k)(N+k+1)}\right)$

primaries: (Λ_+, Λ_-)

$$h = \frac{1}{2(N+k)(N+k+1)}\left(|(N+k+1)(\Lambda_+ + \rho) - (N+k)(\Lambda_- + \rho)|^2 - \rho^2\right)$$

$\rho = \mathrm{SU(N)\ Weyl\ Vector}$

Need a large c limit to connect with classical theory in bulk

- ‘t Hooft limit: $N, k \to \infty\ ,\quad \lambda = \frac{N}{N+k}\ \mathrm{fixed}$ (Gaberdiel/Gopakumar)
- “semi-classical limit”: $k \to -N-1, \quad N\ \mathrm{fixed}$ (Perlmutter, Prochazka, Raeymakers)

nonunitary, but leads to remarkably direct comparison between bulk and boundary

Much information contained in SL(N) x SL(N) CS coupled to C:

$$dA + A \wedge A = 0 \ , \quad d\overline{A} + \overline{A} \wedge \overline{A} = 0$$

$$dC + AC - C\overline{A} = 0$$ C in some rep of SL(N) x SL(N)

Can recover full spectrum of CFT primaries

Behavior of coset primaries (Λ_+, Λ_-) depends qualitatively on whether or not $\Lambda_- = 0$

$$h_{(0,\Lambda_-)} \approx -\frac{C_2(\Lambda_-)}{N(N^2-1)}c$$ heavy

$$h_{(\Lambda_+,0)} \approx -\Lambda \cdot \rho = -\sum_{j=1}^{N} \frac{j(N-j)}{2} \Lambda_j$$ light

Since c ~ 1/G_N, expect heavy primaries to be described by solitonic solutions in the bulk, while light primaries are field quanta. General primary is then a collection of quanta bound to soliton.

Solitons (Castro et. al.)

Flat connections of the Euclidean SL(N,C) theory

Demand that holonomy around angular AdS direction lies in the center of the gauge group: $H = Pe^{\oint A} \in Z_N$

Writing $H = e^{\omega}$, holonomy conditions restricts eigenvalues of ω as

$$\omega_j = 2\pi i(\underset{\substack{\uparrow \\ \text{nonnegative integer}}}{r_j} + \tfrac{N+1}{2} - \tfrac{B}{N} - j)\ , \quad B = \textstyle\sum_j r_j$$

Energy and higher spin charges encoded in r_j

By equating r_j to the number of boxes in the jth row of a Young tableau, the soliton defines a representation of SU(N), and its energy expressed in terms of the weight vector reads

$$h_\Lambda = -\tfrac{C_2(\Lambda_-)}{N(N^2-1)}c$$

matches dimension of CFT primary (0,Λ)

Quanta

Recall the CFT at k=-(N+1) has primaries

$$(\Lambda,0) \qquad h=-\Lambda\cdot\rho=-\sum_{j=1}^{N}\frac{j(N-j)}{2}\Lambda_j$$

e.g. $h_{(f,0)}=\frac{1-N}{2}$

For the master field equation $dC+AC-C\overline{A}=0$ with connections in the defining representation of SL(N), one finds that Tr(C) obey KG equation with:

$$m^2=-1+N^2 \quad \rightarrow \quad h=\frac{1}{2}-\sqrt{1+m^2}=\frac{1-N}{2}$$

Repeating this for a general representation Λ one finds a multiplet of scalar fields corresponding to the decomposition into SL(2) reps. Highest weight states have masses that reproduce CFT spectrum above. (PK, Perlmutter)

Attaching these to solitons accounts for full CFT spectrum

Correlation functions

Large class of correlators can be computed just by manipulating NxN matrices. Solutions can be generated via gauge transformations

$$A = g^{-1}dg \ , \quad \overline{A} = \overline{g}^{-1}d\overline{g}$$
$$C = g^{-1}c\overline{g} \ , \quad c = \text{constant}$$

$\mathrm{Tr}(C) =$ physical scalar

Scalar bulk-boundary propagator obtained from highest weight c

$$L_{-1}c = cL_1 = 0$$

Use this to compute class of 4-point functions:

$$\langle \underbrace{\mathcal{O}_{\Lambda_+}(z_1)\overline{\mathcal{O}}_{\Lambda_+}(z_2)}_{\text{light}}\underbrace{\mathcal{O}_{\Lambda_-}(z_3)\overline{\mathcal{O}}_{\Lambda_-}(z_4)}_{\text{heavy}}\rangle$$

Compute by solving scalar wave equation in soliton background

example: $\langle \mathcal{O}_f(z)\overline{\mathcal{O}}_f(1)\mathcal{O}_{\Lambda_-}(0)\overline{\mathcal{O}}_{\Lambda_-}(\infty)\rangle = \left| z^{\frac{N-1}{2}} \sum_{j=1}^{N} \frac{z^{\omega_j}}{\prod_{l\neq j}(\omega_l-\omega_j)}\right|^2$

CFT side: Coulomb gas formulation; hypergeometric functions and sum over confomal blocks. Collapses to above upon setting k = -(N+1).

(Hijano, PK, Perlmutter)

‘t Hooft limit

Main open question here is the bulk description of the “light states”

e.g. $h_{(\Lambda,\Lambda)} \sim \frac{\lambda^2}{N^2} C_2(\Lambda) \to 0$ (Gaberdiel/Gopakumar)

Needed to match the 1/N expansion of the CFT, but are not present in the Vasiliev spectrum.

Analogous issue arises in AdS_4/CFT_3 case when we put the CFT on a Riemann surface, e.g T^2 x R.
The CS gauge field enforcing the singlet constraint now gives rise to zero energy states from holonomies around T^2 directions.
Actually, at finite N scalar determinants shift the energy levels, yielding $E \sim \frac{\sqrt{\lambda}}{N} \to 0$ (Banerjee et.al.)

Bulk theory apparently needs an extra “topological” sector, since these states are absent for CFT on S^2 x R

Black holes: D=4

If classical "black hole" solutions exist we would expect their entropy to behave as $S \sim \frac{1}{G_N} \sim N$

Can compare this to the theory of N free scalars in the singlet sector on S^2 x R (Shenker/Yin)

At asymptotically high T we can ignore singlet condition and $S \sim NT^2$
This N dependence turns out to be valid only for

$$T > \sqrt{N} = \frac{1}{\sqrt{G_N}} = T_{Pl}$$

This is to be contrasted with ordinary AdS/CFT, where we have:

$$S \sim N^0 \ , \quad T < O(1)$$ gas of particles

$$S \sim N^2 T^2 \ , \quad T > O(1)$$ black hole

In HS case, abundance of light states enhances the entropy of the thermal gas, pushing up the phase transition temperature

Black holes: D=4

Looking for HS black holes in D=4 is a technically challenging task (recall: AdS-Schwarzschild is not a solution!)

Main candidate (Didenko/Vasiliev) based on HS version of the Kerr-Schild form of the Kerr solution

$$ds^2 = (\eta_{\mu\nu} + H k_\mu k_\nu) dx^\mu dx^\nu$$

$$k^\mu k_\mu = 0 \ , \quad k^\mu \nabla_\mu k^\nu = 0$$

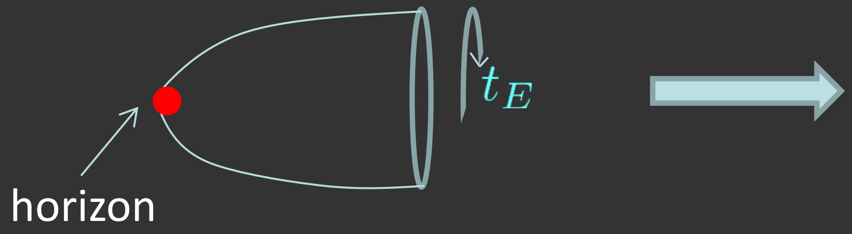
metric perturbation solves the linearized equations

Physical interpretation not clear

Black holes: D=3

Higher spin generalizations of the BTZ black hole exist.
They carry higher spin charge, and their free energy matches up with the high temperature behavior of the generalized CFT partition function $Z(\beta, \mu_3, \ldots) = \mathrm{Tr}\left[e^{\beta(E+\mu_3 Q_3+\ldots)}\right]$

Due to HS gauge transformations, not clear how to define a black hole in terms of metric-like variables
For the Euclidean BH, the condition that the time circle smoothly closes off at the horizon has a HS analog: holonomy around Euclidean time circle should like in center of gauge group (Gutperle, PK)



$$H = Pe^{\oint_t A} \subset \text{center}$$

If rhs is true, there exists a gauge in which metric looks like lhs

(Ammon, Gutperle, PK, Perlmutter)

Black holes: D=3

To proceed, one needs to find flat connections in the hs(λ)xhs(λ) CS theory carrying HS charges and with holonomy in the center of gauge group. This can be done perturbatively in HS chemical potentials.

Next extract thermodynamic quantities and compare to CFT

$$\ln Z(\tau,\alpha) = \frac{i\pi k}{2\tau}\left[1 - \frac{4}{3}\frac{\alpha^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\alpha^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\alpha^6}{\tau^{12}} + \cdots\right]$$

valid for: $\tau \to 0, \quad \alpha \to 0, \quad \frac{\alpha}{\tau^2} \quad \text{fixed}$

bulk and CFT match (PK/Perlmutter; Gaberdiel/Hartman)

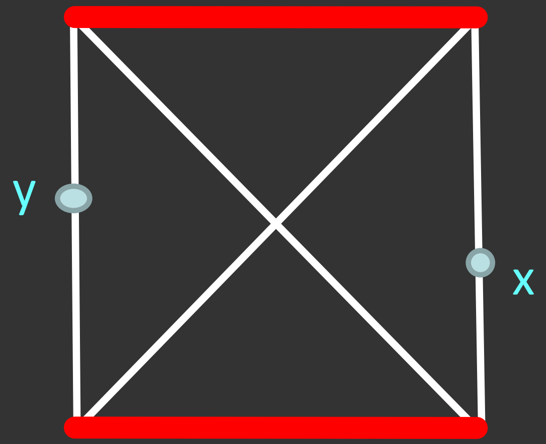
Actually, there are a number of subtleties involved

(Banados et al; Perez et al; de Boer Jottar; Ammon et. al.; Compere et al. ….)

Simplest story: compute CS version of Wald entropy, and express result in terms of charges in an asymptotically AdS gauge

Bottom line: it works!

To map out physical causal structure we can compute AdS/CFT two-point functions of probe scalars, and look for lightcone singularities



- Black hole causal structure: G(x,y) nonsingular

$$\frac{i\alpha e^{\rho/2}}{16\tau^2}\Big[\cosh^2(2\overline{Z})\left(-4(Z+\overline{Z})(\cosh(4Z)-2)-\sinh(4Z)\right)$$
$$+4e^{2\rho}\tau\overline{\tau}\sinh(4\overline{Z})\left(-4(Z+\overline{Z})\sinh(4Z)+2(\cosh(4Z)-1)\right)$$
$$-(4e^{2\rho}\tau\overline{\tau})^2\sinh^2(2\overline{Z})\left(4(Z+\overline{Z})(\cosh(4Z)+2)-3\sinh(4Z)\right)\Big]$$
$$\times\left(\cosh(2Z)\cosh(2\overline{Z})+4e^{2\rho}\tau\overline{\tau}\sinh(2Z)\sinh(2\overline{Z})\right)^{-5/2} \;,$$

$$Z=\frac{iz}{4\tau}$$

(PK, Perlmutter)

No singularities. Consistent with black hole causal structure

Matches CFT correlator (Gaberdiel, Jin, Perlmutter)

Glaring omissions

- HS dS/CFT (Anninos, Hartman, Strominger)
- Connection to string theory:

  HS dual to ABJ theory via susy version of Vasiliev with Chan-Paton factors and alternative boundary conditions. Leads to picture of IIA string as flux tube of strongly coupled Vasiliev particles

  (Chang, Minwalla, Sharma, Yin)
- One-loop corrections (Giombi, Klebanov, Safdi)
- Constructing bulk from CFT (Douglas et. al.; Jevicki et. al.)
- …