Alberto Nicolis
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“Non-relativistic” theories

w/ L. Delacretaz, S. Dubovsky, T. Gregoire, S. Endlich, B. Horn,
L. Hui, W. Irvine, R. Penco, F. Piazza, R. Porto, R. Rattazzi, R. Rosen,
S. Sabharwal, S. Sibiryakov, D. T. Son, J. Wang, X. Xiao

JHEP 0603, JHEP 1104, JHEP 1206, PRD 85 (2012),
PRL 110 (2013), JCAP 1310, PRD 88 (2013), JHEP 1311, PRD (2014)
hep-th 1303.3289, 1307.0517, 1310.2272, 1311.6491, ...

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Relativistic non-relativistic theories

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Usually

condensed matter EFTs = ~~relativity~~

1. v<<1
2. preferred frame

Note: relativity = Lorentz or Galilei

For us instead

condensed matter EFTs = ~~relativity~~ spontaneously

Note: relativity = Lorentz or Galilei

Or equivalently

condensed matter EFTs = relativity
non-linearly
realized

Note: relativity = Lorentz or Galilei

More precisely:

Poincaré $\begin{cases} P^\mu & \text{translations} \\ J^i & \text{rotations} \\ K^i & \text{boosts} \end{cases}$ + internal symmetries ("Q")

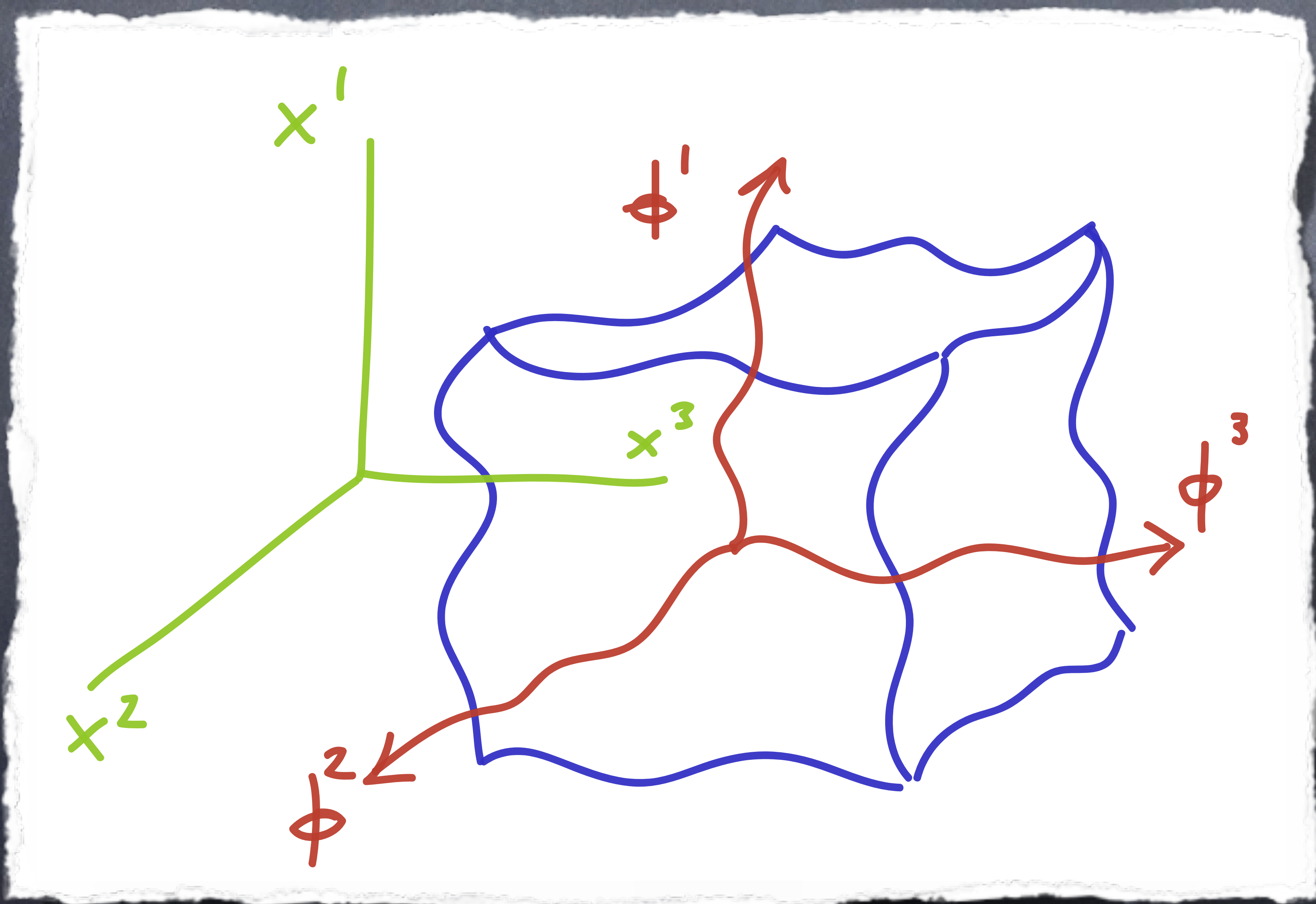
⇓

$\begin{cases} \bar{P}^\mu & \text{(new) translations} \\ \bar{J}^i & \text{(new) rotations (convenience)} \end{cases}$

Example: solids and fluids

Dof: volume elements' positions

$\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$



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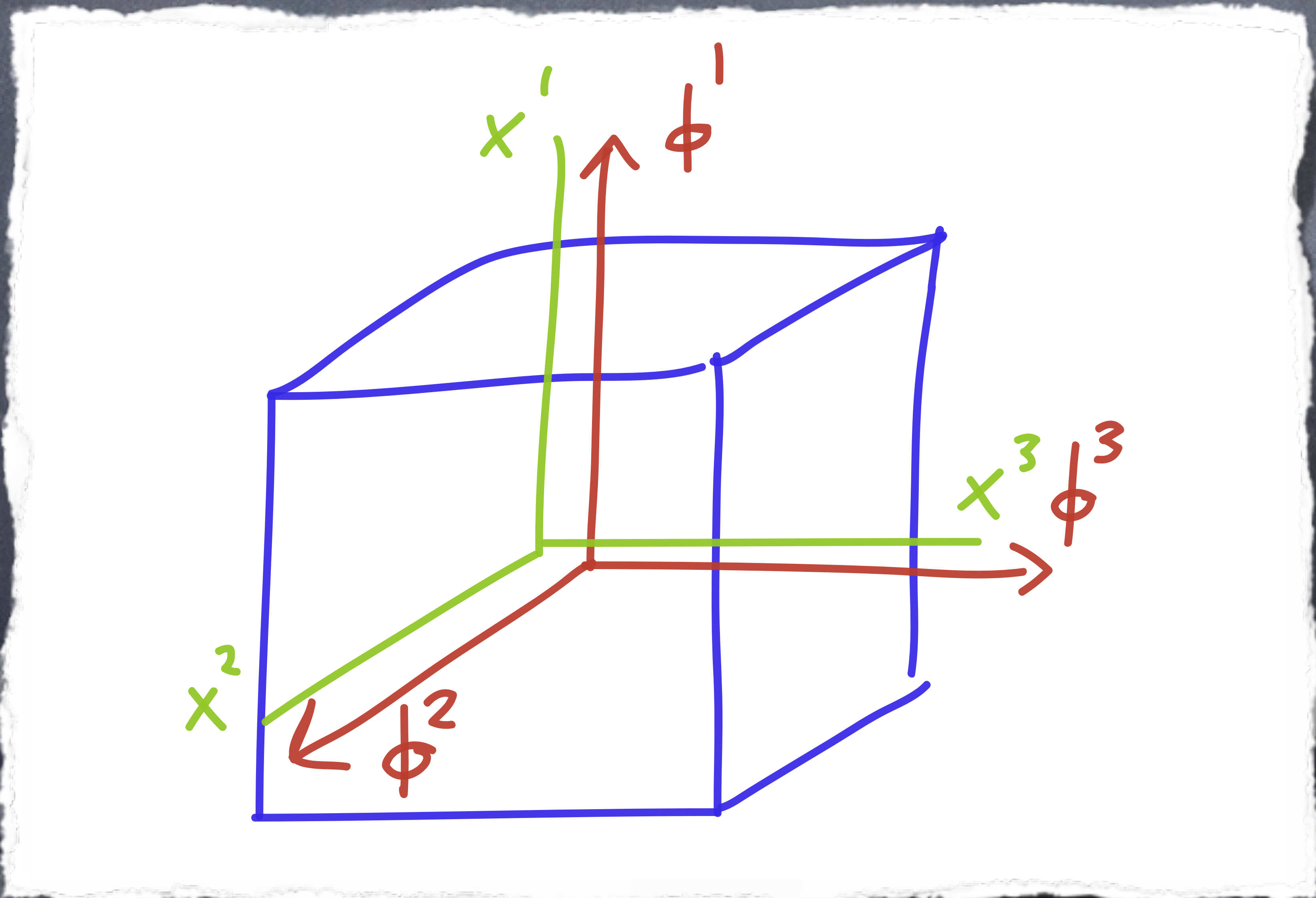


$$\langle \phi^I \rangle_{\text{eq}} = x^I$$

Symmetries: Poincaré + internal

$$\left.\begin{array}{l}\phi^I \to \phi^I + a^I \\ \phi^I \to SO(3)\,\phi^I\end{array}\right\}$$ recover homogeneity/isotropy

($\langle\phi^I\rangle_{\rm eq} = x^I$ preserves diagonal combinations)

$$\phi^I \to \xi^I(\phi) \qquad \det\frac{\partial\xi^I}{\partial\phi^J} = 1$$ fluid vs solid

Action: $S = \int d^4x \, F(b)$ $\qquad b = \sqrt{\det \partial_\mu \phi^I \partial^\mu \phi^J}$

Correct hydrodynamics ($T_{\mu\nu}$ + eom)

with

$$\begin{aligned} \rho &= -F \\ p &= F - F' \, b \\ u^\mu &= \frac{1}{6\, b} \, \epsilon\epsilon \, \partial\phi\partial\phi\partial\phi \end{aligned}$$

Relativistic, non-linear

(Dubovsky, Gregoire, Nicolis, Rattazzi 2005) ground state (at given p): $\phi^I = x^I$

Nambu-Goldstone modes: $\phi^I = x^I + \pi^I$

$$\mathcal{L} \to \left(\dot{\pi}^I\right)^2 - c_s^2\left(\partial_I \pi^I\right)^2 + \text{interactions}$$

longitudinal = sound $\omega = c_s k$

transverse = vortices $\omega = 0$

Applications

- Sound-vortex interactions (Endlich, Nicolis 2013)
- Hall viscosity in 2+1 D (Nicolis, Son 2011) (Haehl, Rangamani 2013) (Geracie, Son soon)
- Fluids with quantum anomalies (Dubovsky, Hui, Nicolis 2013) (Haehl, Loganayagam, Rangamani 2013)
- Finite T relativistic superfluids (Nicolis 2011)
- Quantum hydrodynamics (Endlich, Nicolis, Rattazzi, Wang 2013) (Goldberger, Rothstein soon)
- Dissipative hydrodynamics (Endlich, Nicolis, Porto, Wang 2012) (Grozdanov, Polonyi 2012)
- Alternative inflationary models (Endlich, Nicolis, Wang 2012) (Bartolo, Matarrese, Peloso, Ricciardone 2013)

Sound-vortex interactions

(Endlich, Nicolis 2013)

Subsonic regime: $v \ll c_s$

Nearly incompressible

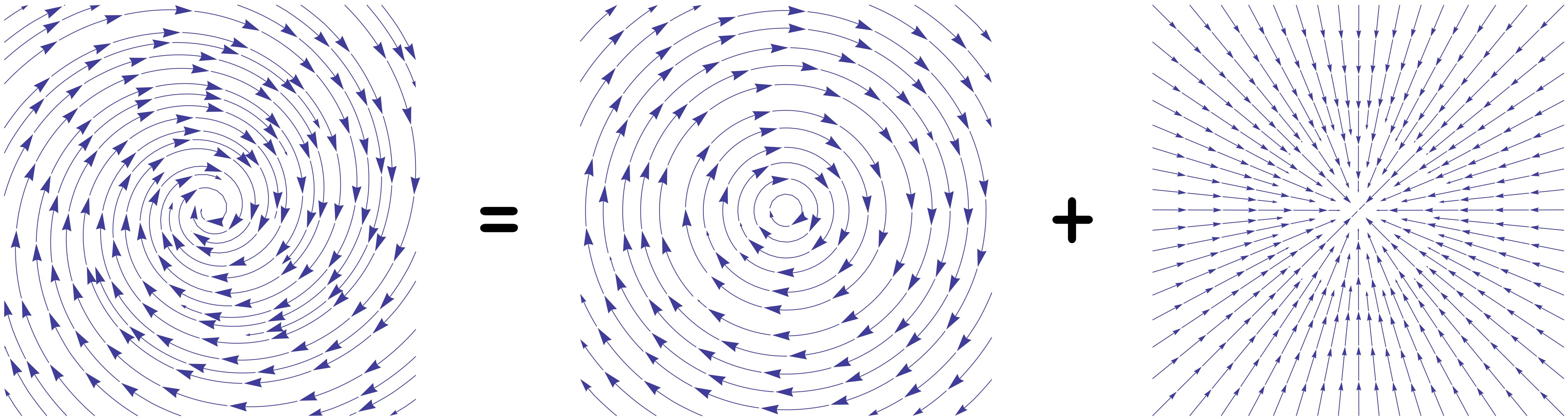
sound waves difficult to excite

treat vortices non-linearly

treat sound perturbatively

integrate it out

Vortex-sound decomposition



$$\phi^I(\vec{x},t) = \phi_0^I(\vec{x},t) + \underbrace{\delta\phi^I}_{\text{compression}}(\vec{x},t)$$

$$\det \frac{\partial \phi_0^I}{\partial x^j} = 1$$

Expand the action in powers of $\delta\phi$ and v_0/c_s

Wednesday, February 12, 14

The action, expanded

$$S = S_{x_0} + S_\psi + S_{\rm int}$$

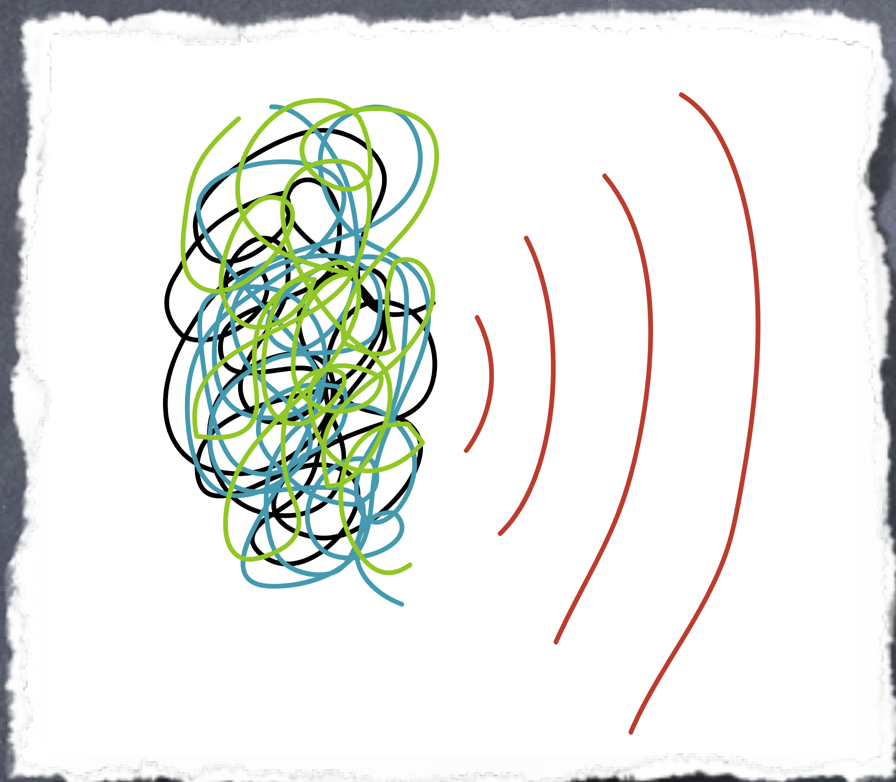
$$S_{x_0} = (\rho + p)\int d^3\phi dt\left[\tfrac{1}{2}v_0^2 + \tfrac{1}{8}v_0^4\left(1/c^2 - c_s^2/c^4\right) + \ldots\right]$$

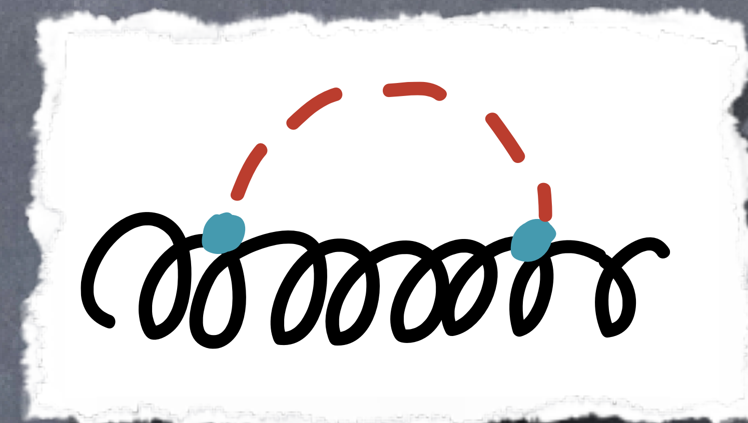
$$S_\psi = (\rho + p)\int d^3x_0 dt\left[\tfrac{1}{2}\left(\nabla_0\dot\psi\right)^2 - \tfrac{1}{2}c_s^2\left(\nabla_0^2\psi\right)^2 + \ldots\right]$$

$$S_{\rm int} = (\rho + p)\int d^3x_0 dt\left[-\tfrac{1}{2}c_s^2/c^2\left(\nabla_0^2\psi\right)v_0^2 - \vec\nabla_0\psi\cdot\left(\vec v_0\cdot\vec\nabla_0\right)\vec v_0 + \ldots\right]$$

$$v_0 \equiv \partial_t x_0(\vec\phi, t)$$

The sound of turbulence



= Im ()

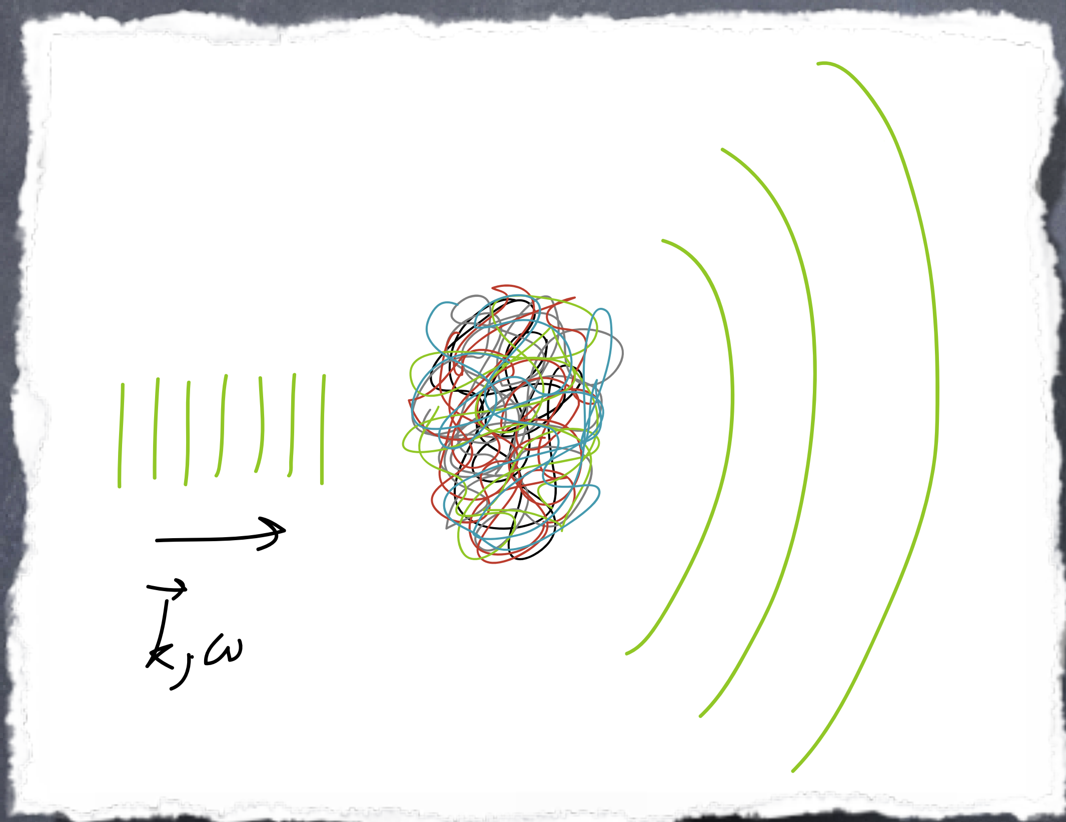
$$P = \frac{\rho + p}{c_s^5} \langle \dddot{Q} \dddot{Q} \rangle$$

$$Q_{ij} \equiv \int d^3x \left(v_i v_j - \frac{c_s^2}{c^2} v^2 \, \delta_{ij} \right)$$

(Lighthill 1954 + relativistic correction)

(similar to Goldberger, Rothstein 2004)

Probing turbulence with sound waves



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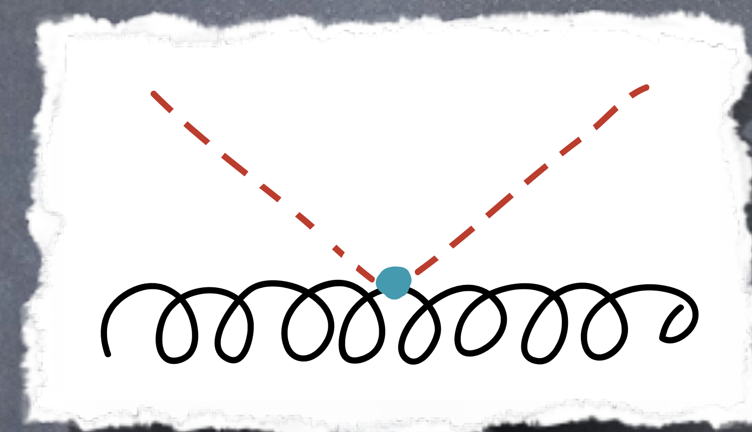
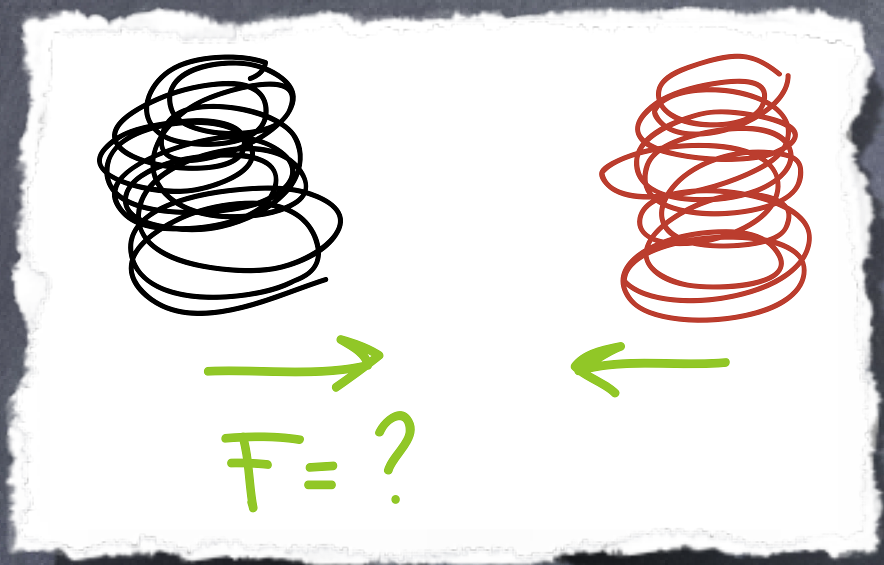


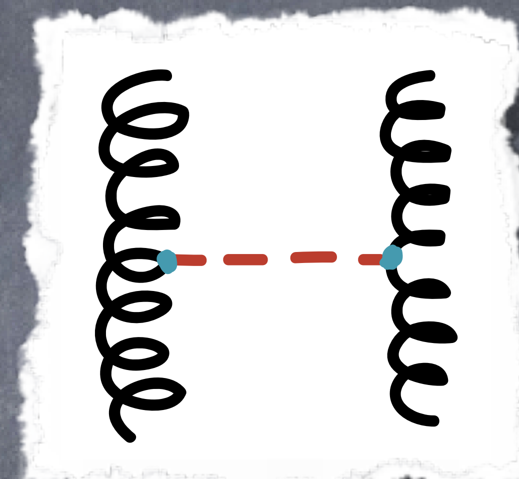
$$\frac{d\sigma}{d\Omega} = \frac{\omega^4}{c_s^6}\left[1 - \frac{c_s^2}{c^2} + \frac{c_s^4}{c^4}\right]\left|\tilde{v}(\Delta\vec{k})\right|^2$$

(Lund, Rojas 1989 + relativistic correction)

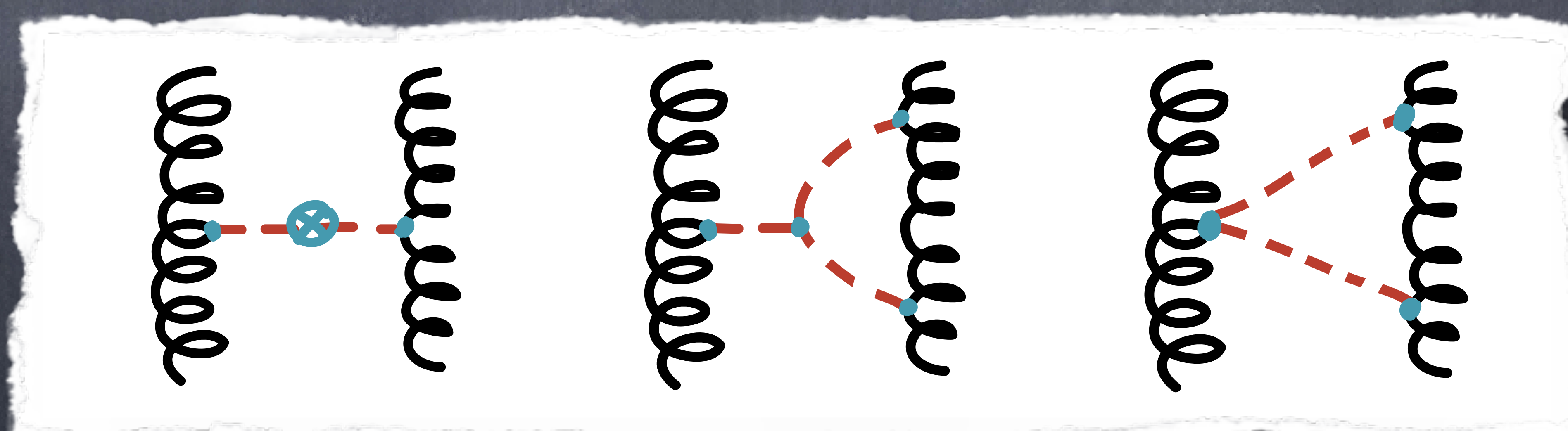
Sound mediated vortex-vortex potential



Leading order



Next to leading order



Long range potential:

$$V \sim \frac{(\rho + p)}{c_s^2} \cdot \frac{q_1 q_2}{r^3} \sim E_{\rm kin} \big(v/c_s\big)^2 (\ell/r)^3$$

$$q \equiv \int_{\rm vortex} d^3x \; v^2$$

Useful? Detectable? Known?

Long range potential:

$$V \sim \frac{(\rho + p)}{c_s^2} \cdot \frac{q_1 q_2}{r^3} \sim E_{\rm kin} \big(v/c_s\big)^2 (\ell/r)^3$$

$$q \equiv \int_{\rm vortex} d^3x \; v^2$$

Useful? Detectable? Known?

? ? No

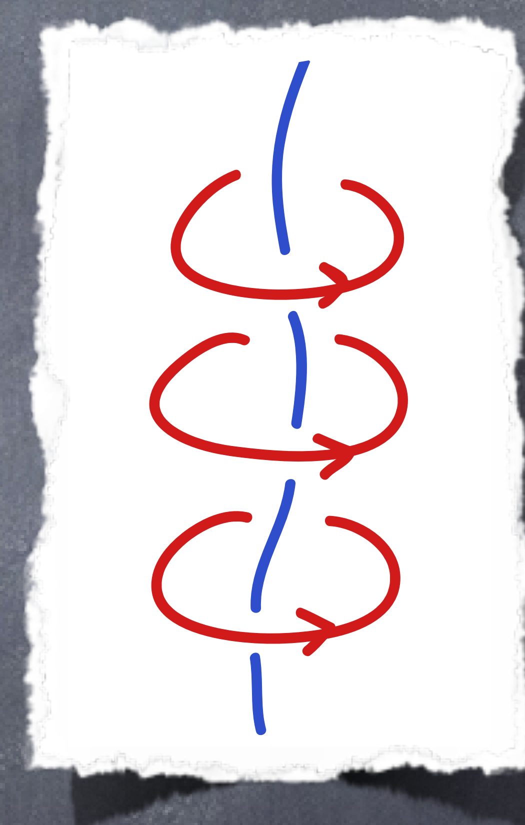
(William Irvine, U. of Chicago)

potential ⇒ force $F = -\partial_r V$, right?

Not really.

For vortex lines

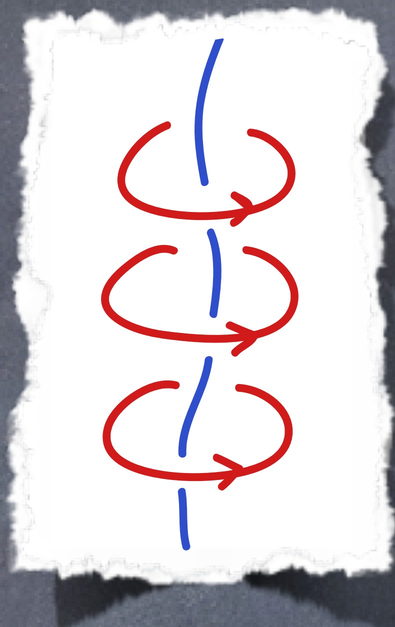
$$\vec{v}(\vec{x}) = -\frac{\Gamma}{4\pi}\int \frac{(\vec{x}-\vec{x}\,')}{|\vec{x}-\vec{x}\,'|^3}\times d\vec{x}\,'$$

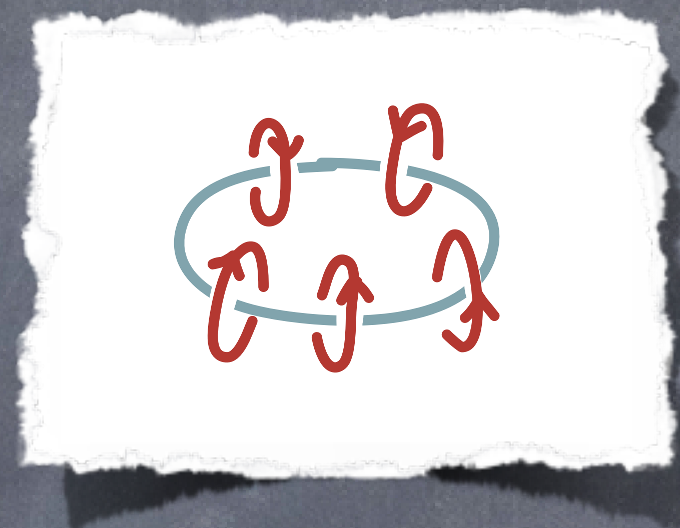


1st order EOM!

Unlike $m\vec{a} = \vec{F}_{\rm ext}$ ⇒ No room for “forces”

Vortex lines and vortex rings are fascinating objects





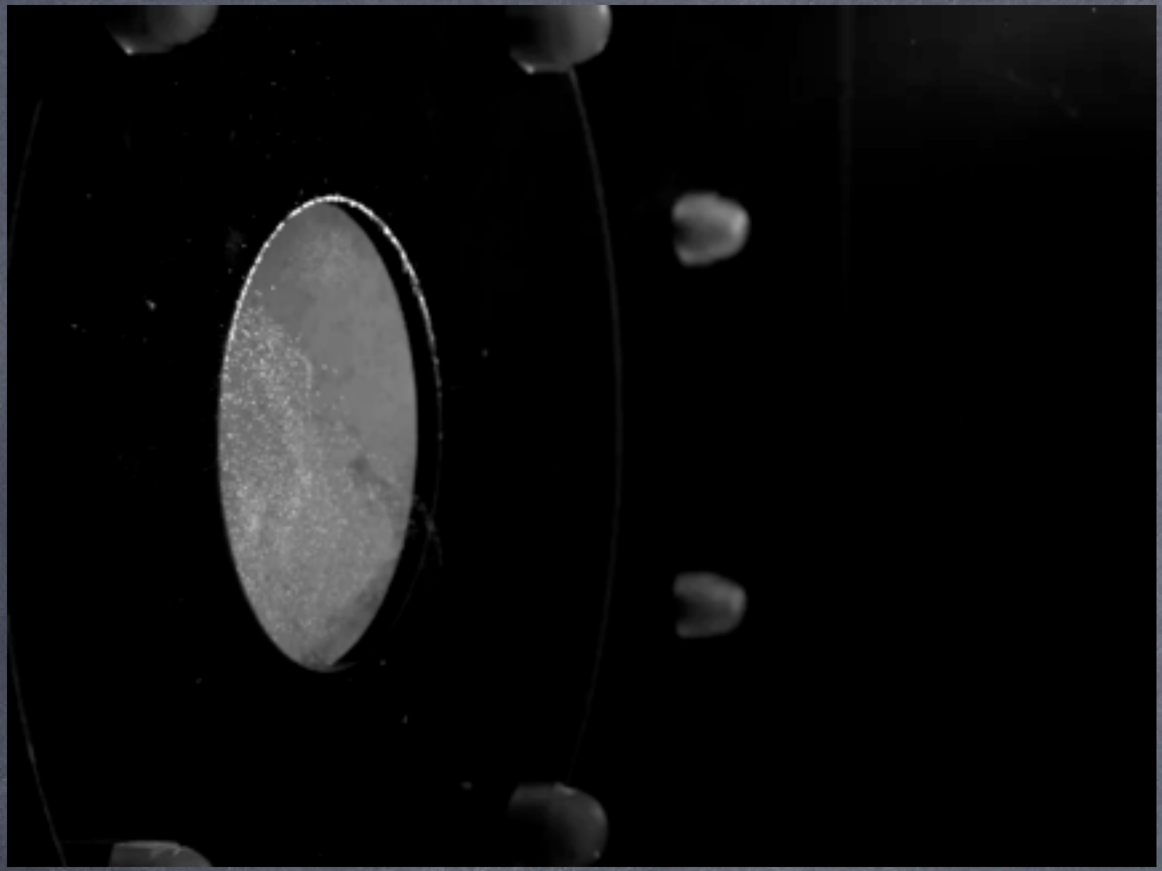
Irvine Lab

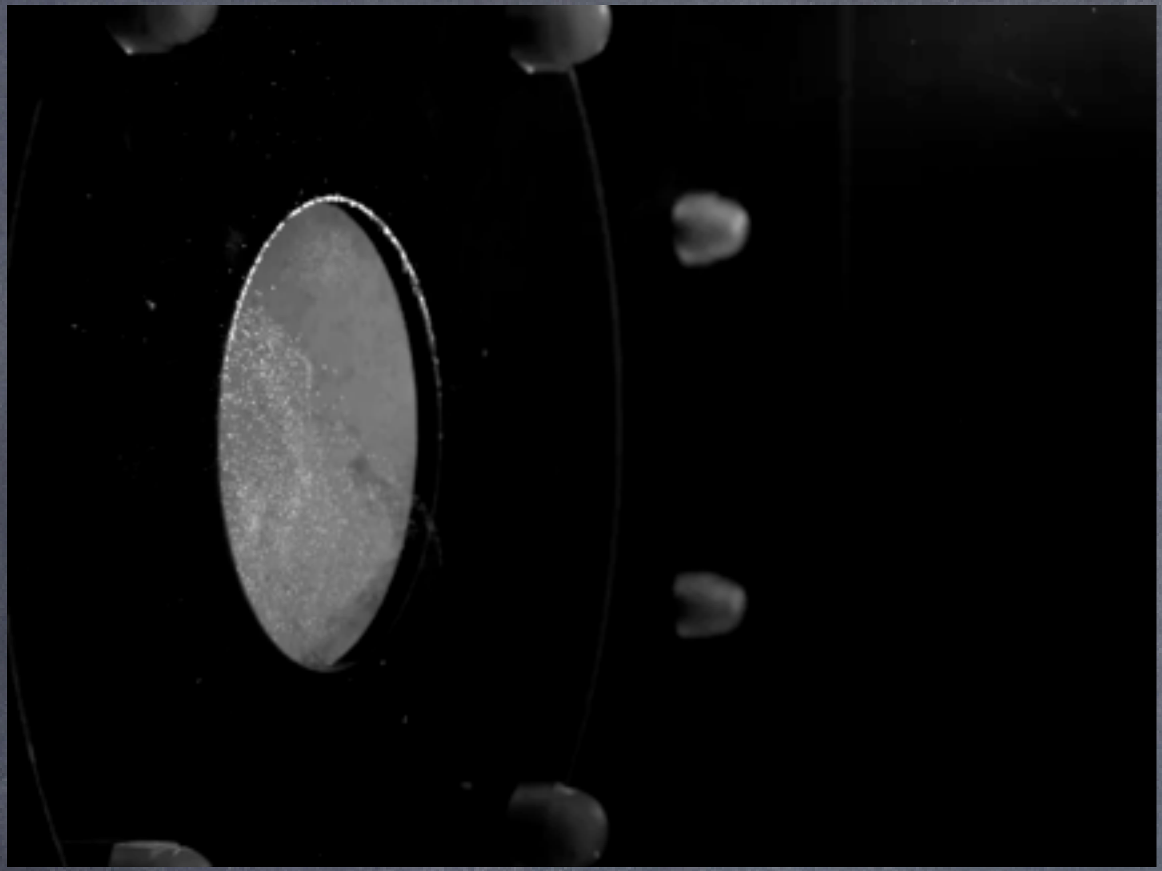
Other groups

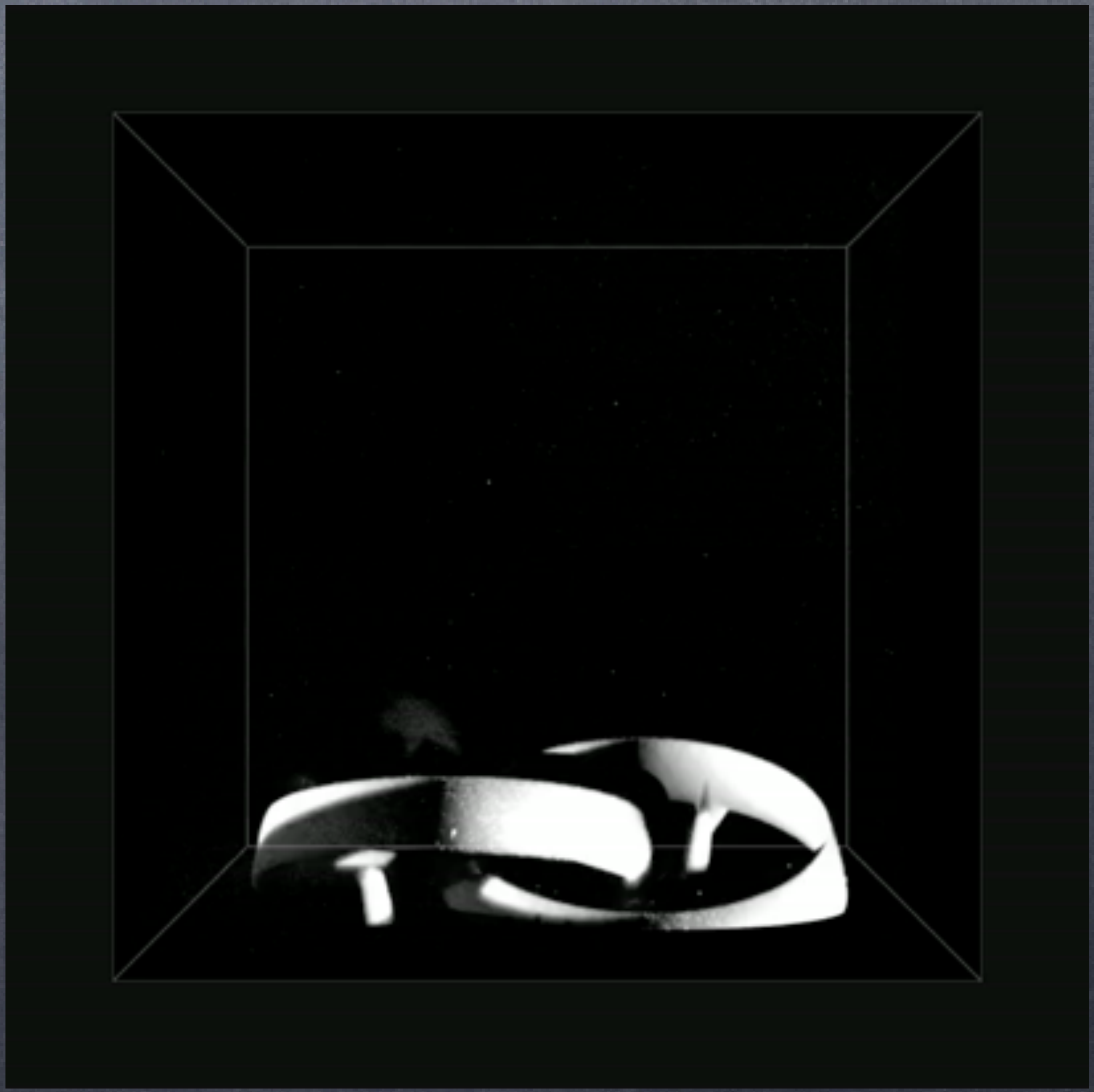
Superfluid turbulence

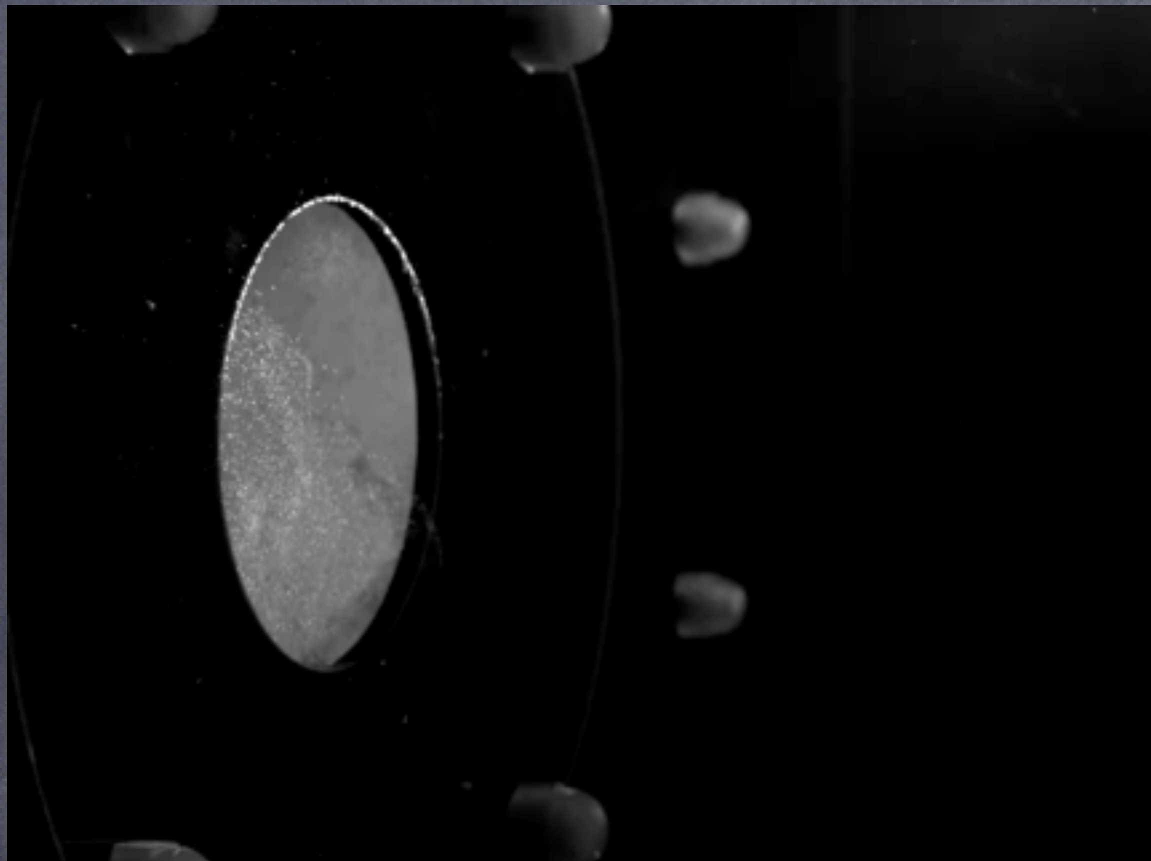
Pulsars

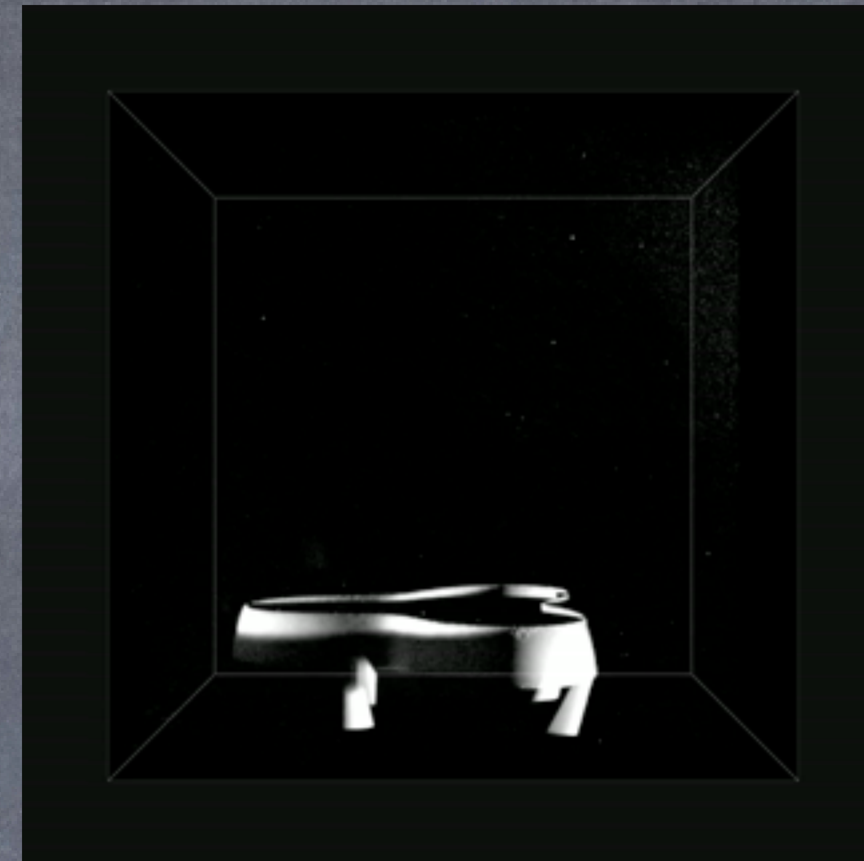
How to makes sense of their dynamics?

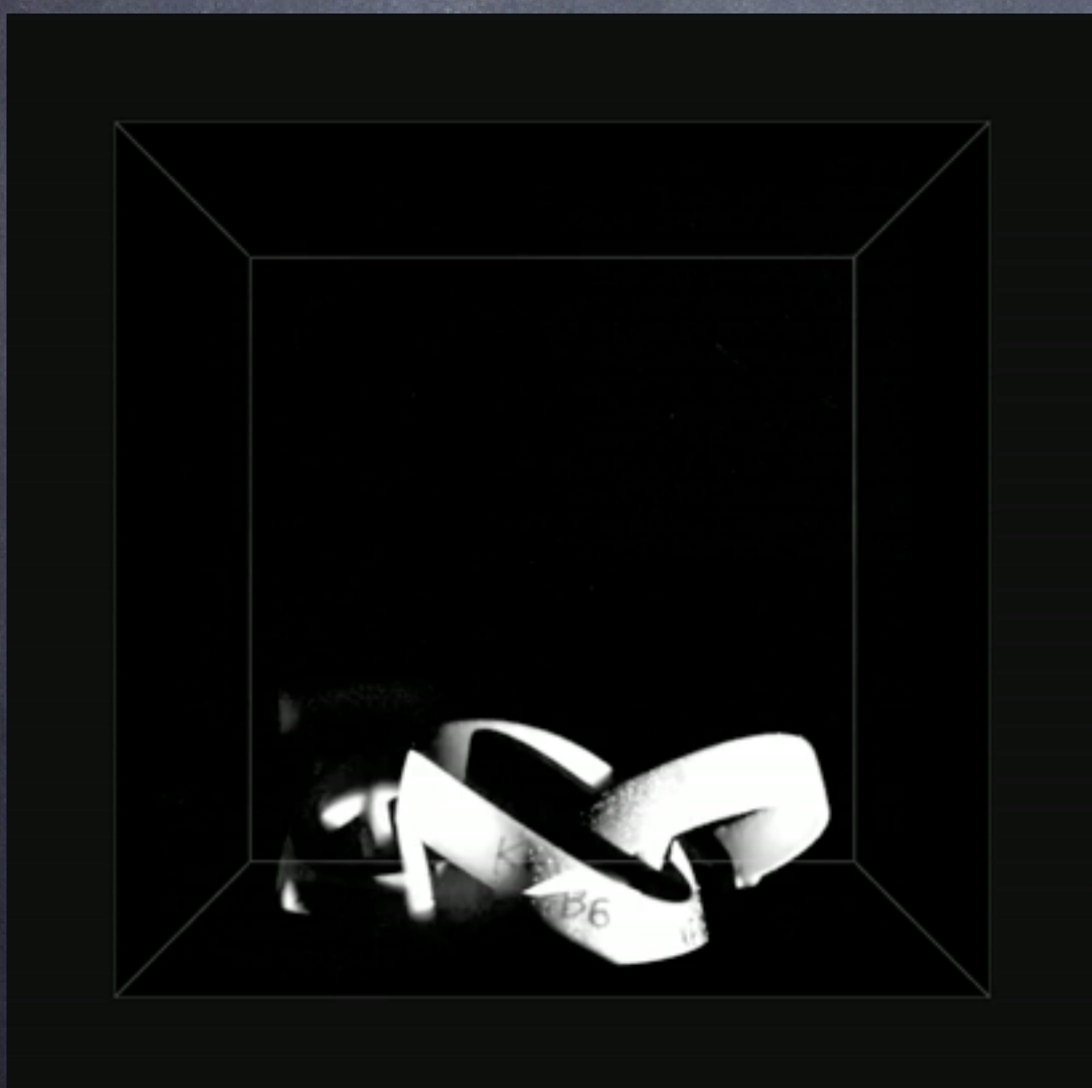


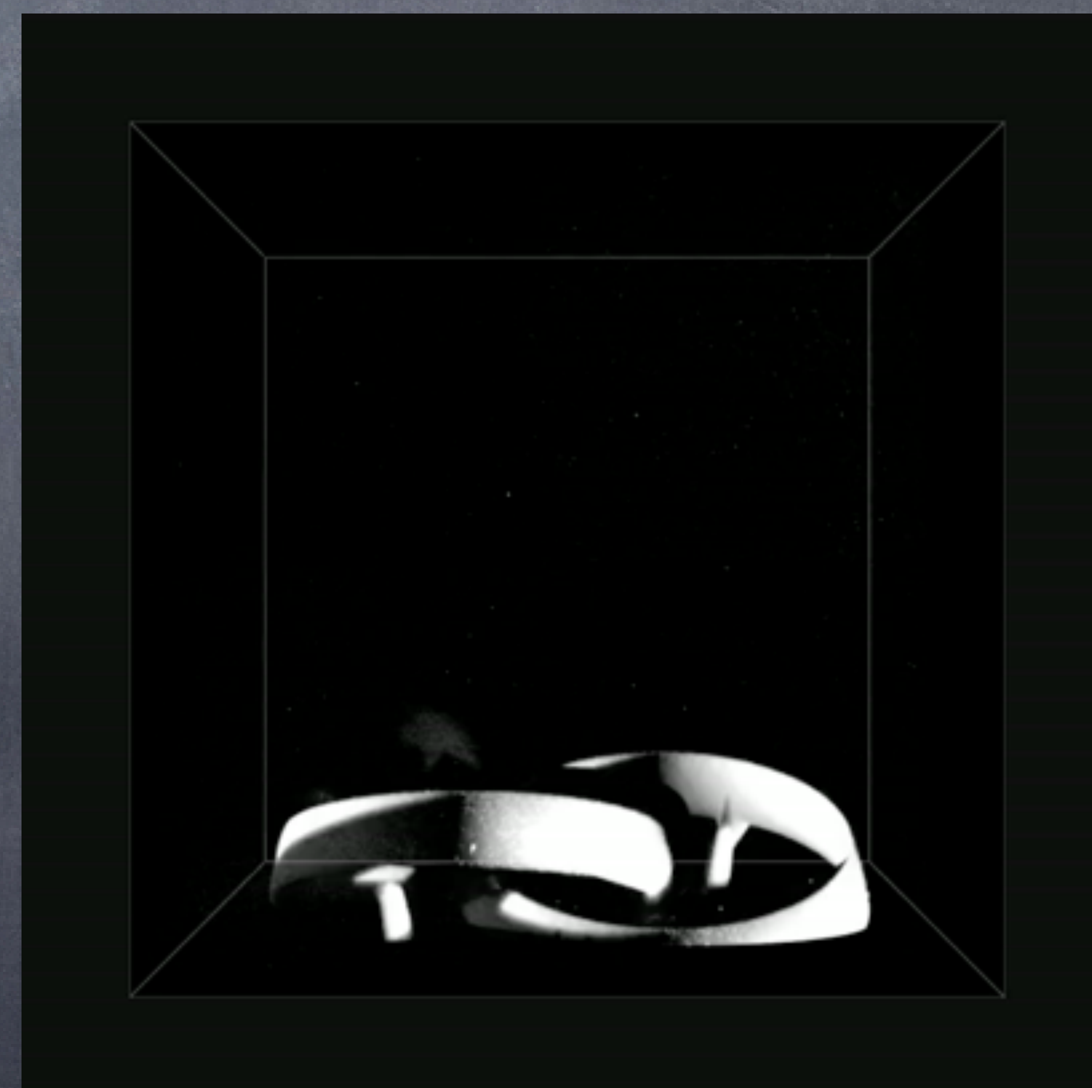


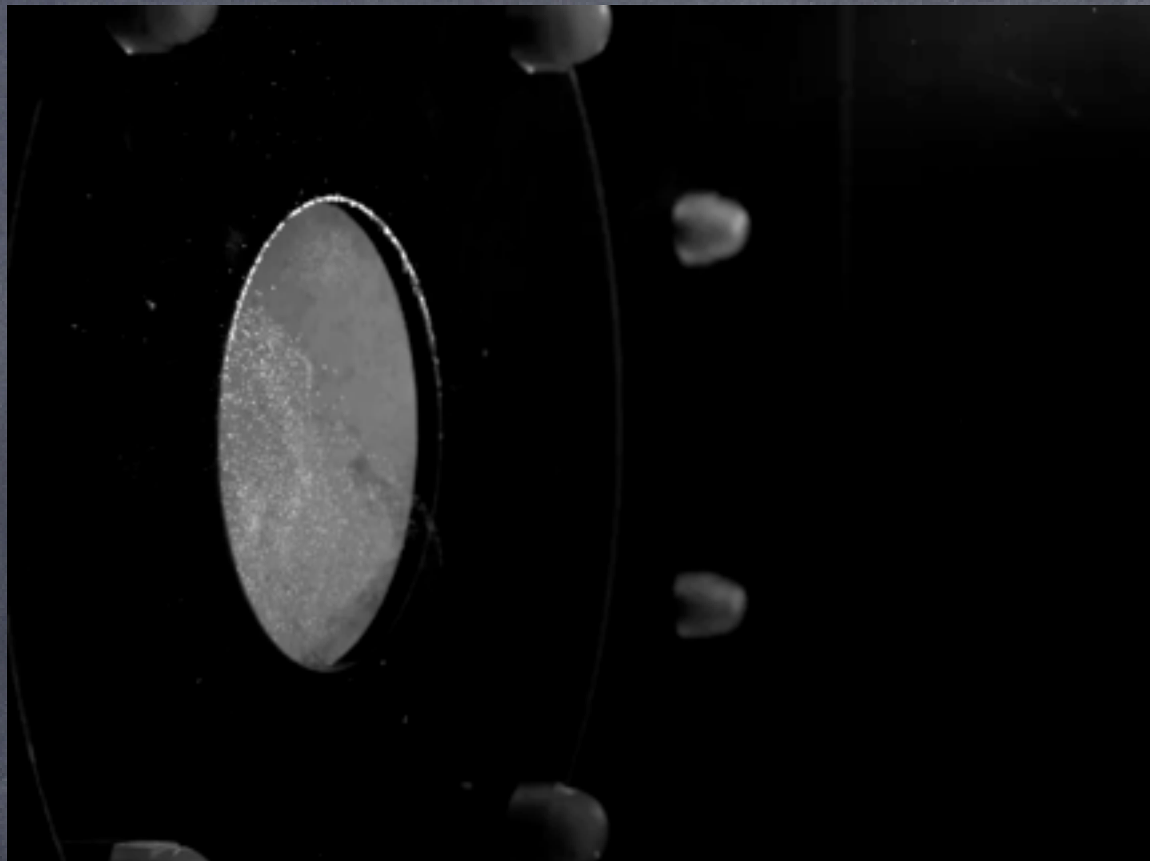


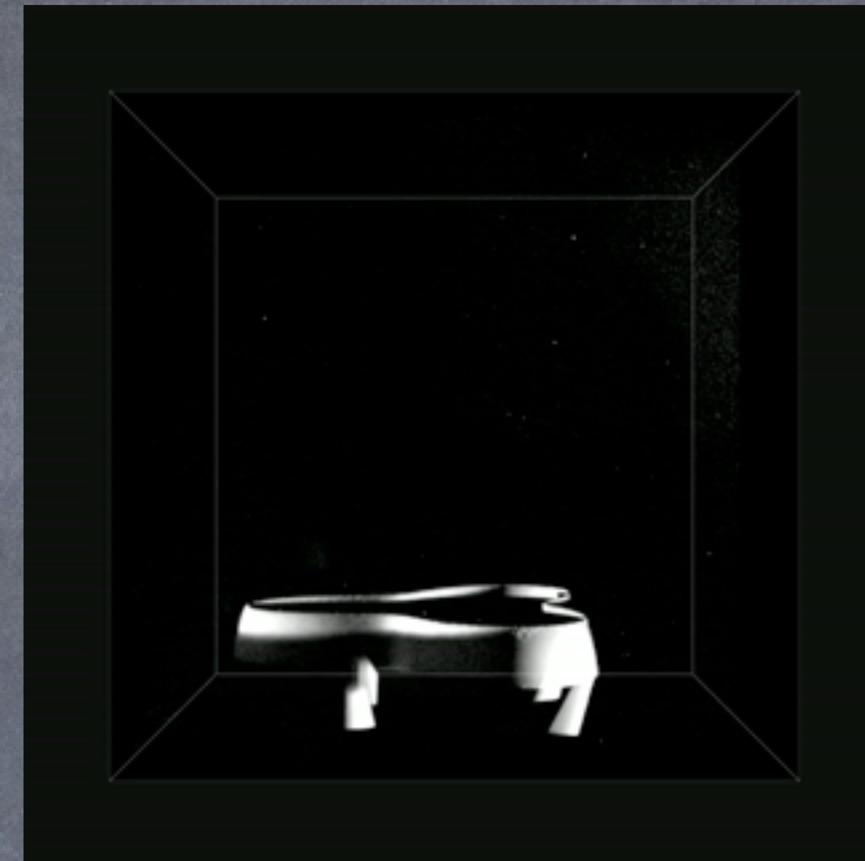


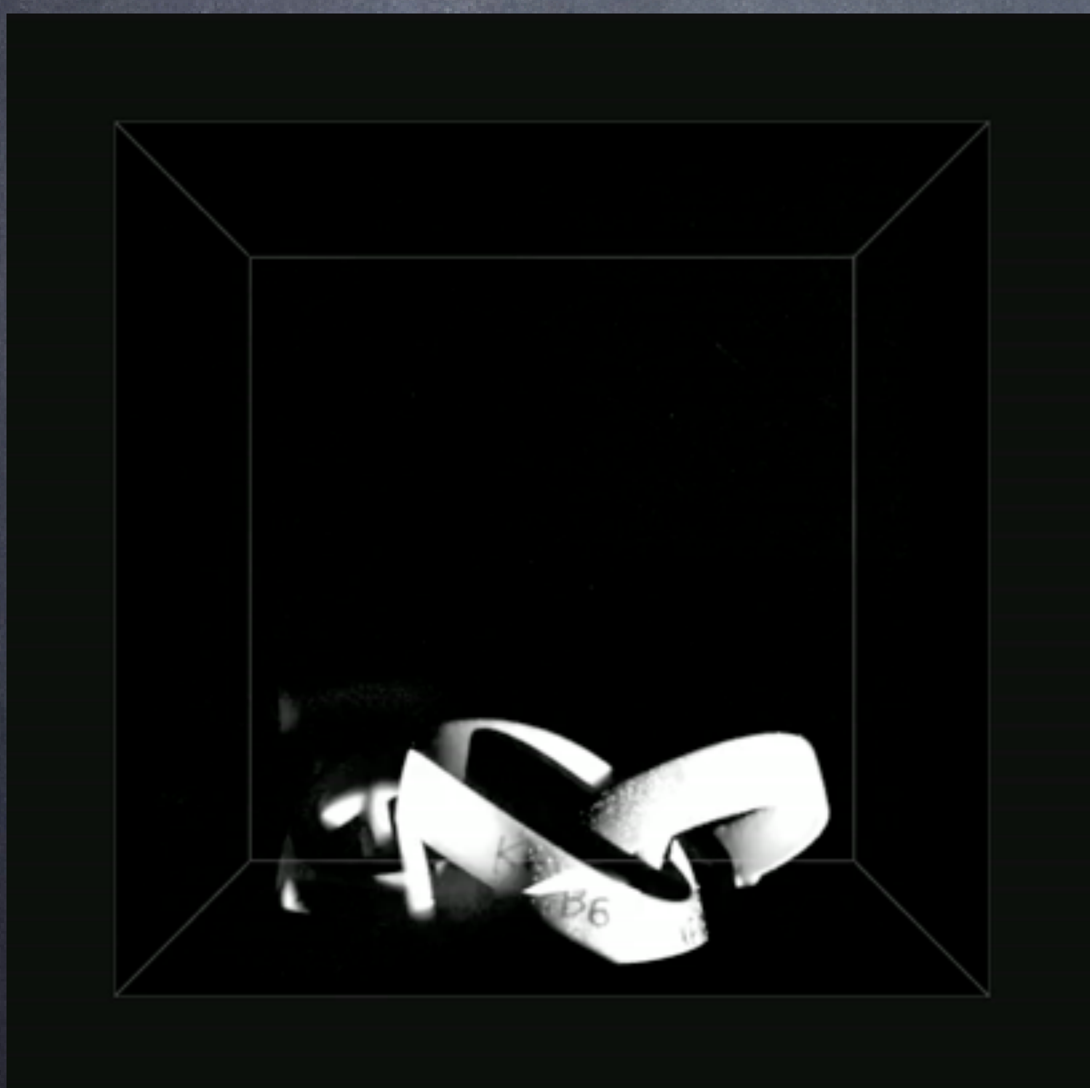


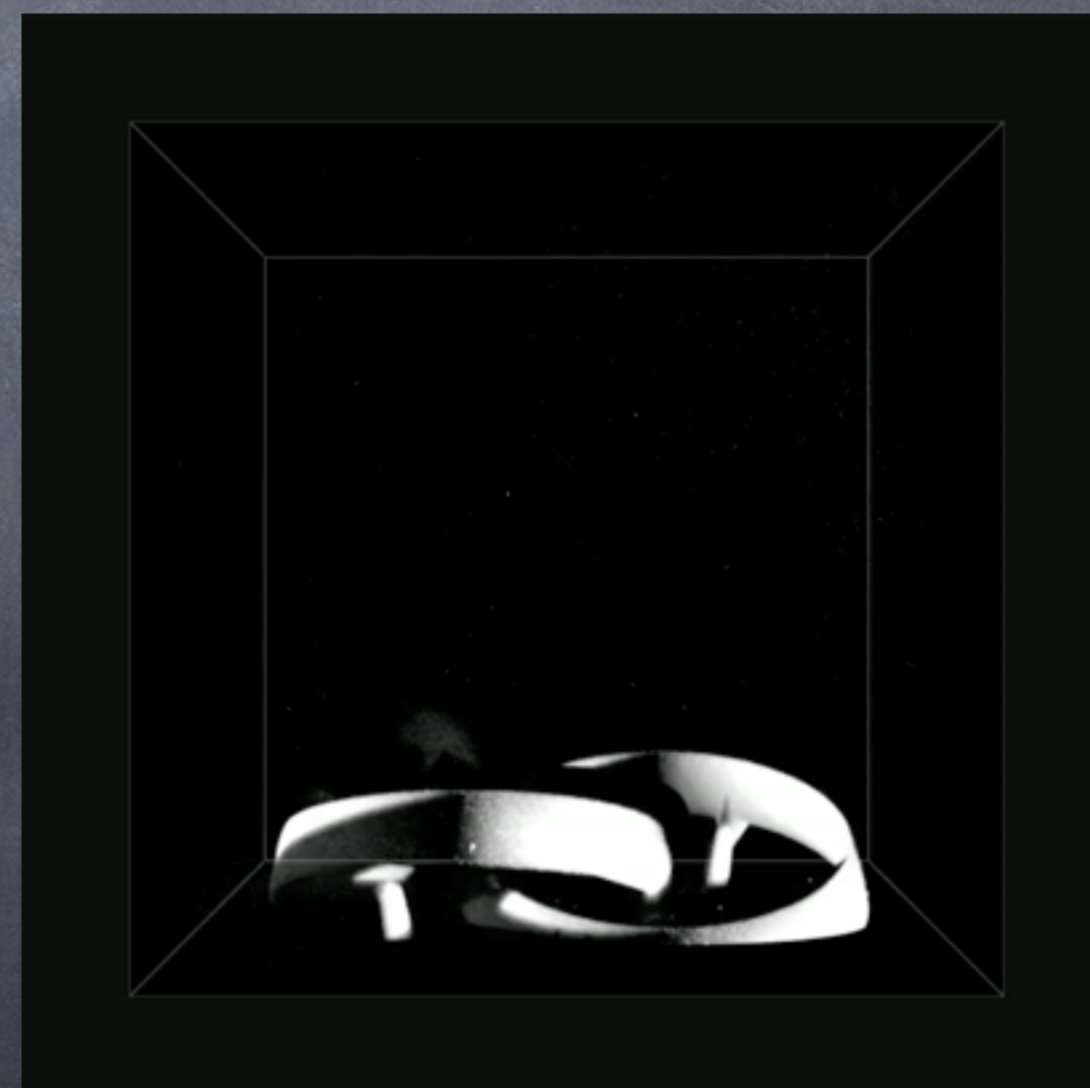












Effective field theory, again

$$\mathcal{L} = -(\rho + p)\left[\Gamma \int d\lambda\, \epsilon^{ijk}\, X^i\, \partial_t\, X^j\, \partial_\lambda X^k + \Gamma^2 \int d\lambda d\lambda' \, \frac{\partial_\lambda \vec{X} \cdot \partial_{\lambda'} \vec{X}'}{|\vec{X} - \vec{X}'|}\right]$$

EOM: $\vec{v}(\vec{x}) = -\frac{\Gamma}{4\pi} \int \frac{(\vec{x} - \vec{x}\,')}{|\vec{x} - \vec{x}\,'|^3} \times d\vec{x}\,'$

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$$\int d^3x\, (\partial_i A_j)^2 - \Gamma \int d\lambda\, \partial_\lambda \vec{X} \cdot \vec{A}(\vec{X}, t)$$

Effective field theory, again

$$\mathcal{L} = -(\rho + p)\left[\Gamma \int d\lambda\, \epsilon^{ijk}\, X^i\, \partial_t\, X^j\, \partial_\lambda X^k + \Gamma^2 \int d\lambda d\lambda'\, \frac{\partial_\lambda \vec{X} \cdot \partial_{\lambda'} \vec{X}'}{|\vec{X} - \vec{X}'|}\right]$$

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$$\int d^3x\, (\partial_i A_j)^2 - \Gamma \int d\lambda\, \partial_\lambda \vec{X} \cdot \vec{A}(\vec{X}, t)$$

Magnetostatics	Incompressible Hydro
current $\vec{J}$	vorticity $\vec{\omega}$
magnetic field $\vec{B}$	velocity field $\vec{v}$
vector potential $\vec{A}$	hydrophoton $\vec{A}$

Point-particle limit

$$\mathcal{L} = \sum_n \left[\vec{\mu}_n \cdot \dot{\vec{x}}_n + \vec{\mu}_n \cdot (\vec{\nabla} \times \vec{A})\right] - \int d^3x \left(\partial_i A_j\right)^2$$

$$\to \sum_n \left(\vec{\mu}_n \cdot \dot{\vec{x}}_n - \mu_n^{3/2} \log \mu_n\right) - \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

Peculiar conservation laws:

$$\vec{P} = \sum_n \vec{\mu}_n$$

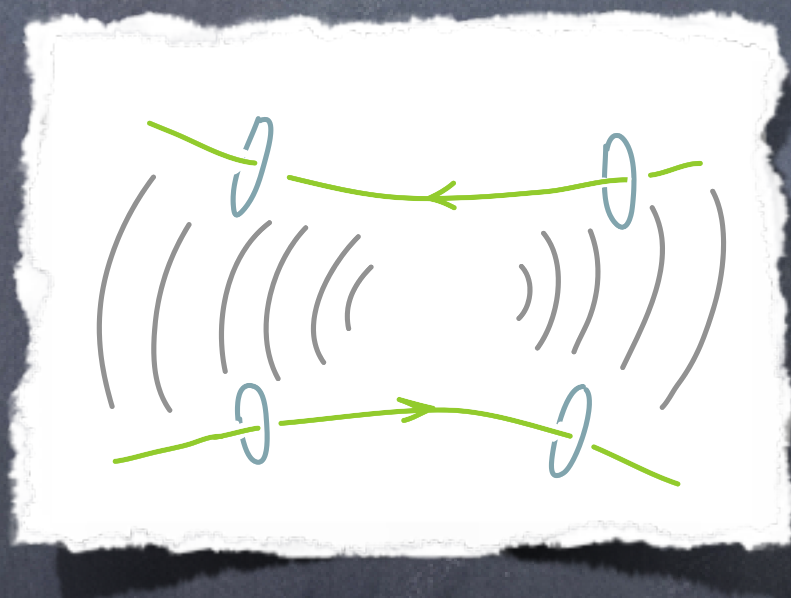
$$\vec{L} = \sum_n \vec{x}_n \times \vec{\mu}_n$$

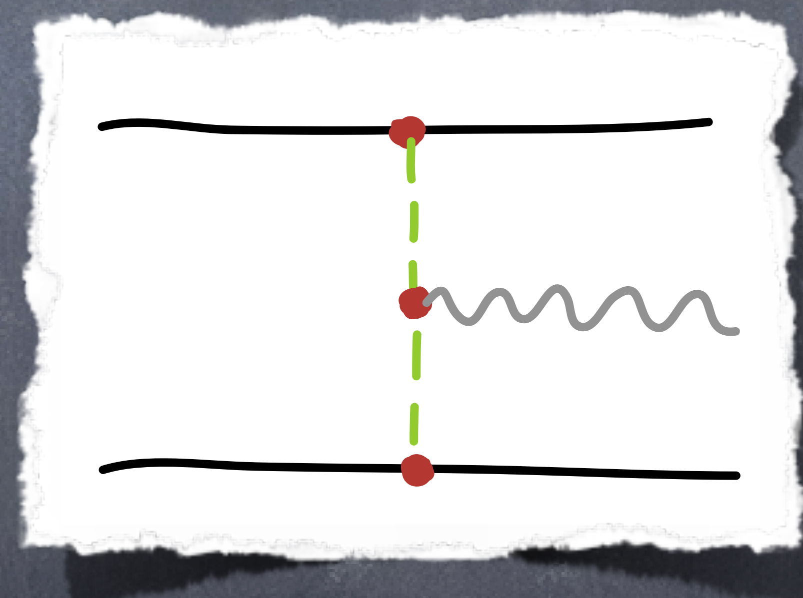
$$E = \sum_n \mu_n^{3/2} \log \mu_n + \sum_{n \neq m} \frac{\vec{\mu}_n \cdot \vec{\mu}_m - 3(\vec{\mu}_m \cdot \hat{r})(\vec{\mu}_n \cdot \hat{r})}{r^3}$$

Interactions with sound:

$$\mathcal{L} = \int d^3x\, (\vec{\nabla} \times \vec{A})^i ((\vec{\nabla} \times \vec{A}) \cdot \vec{\nabla}) \psi^i + \ldots$$

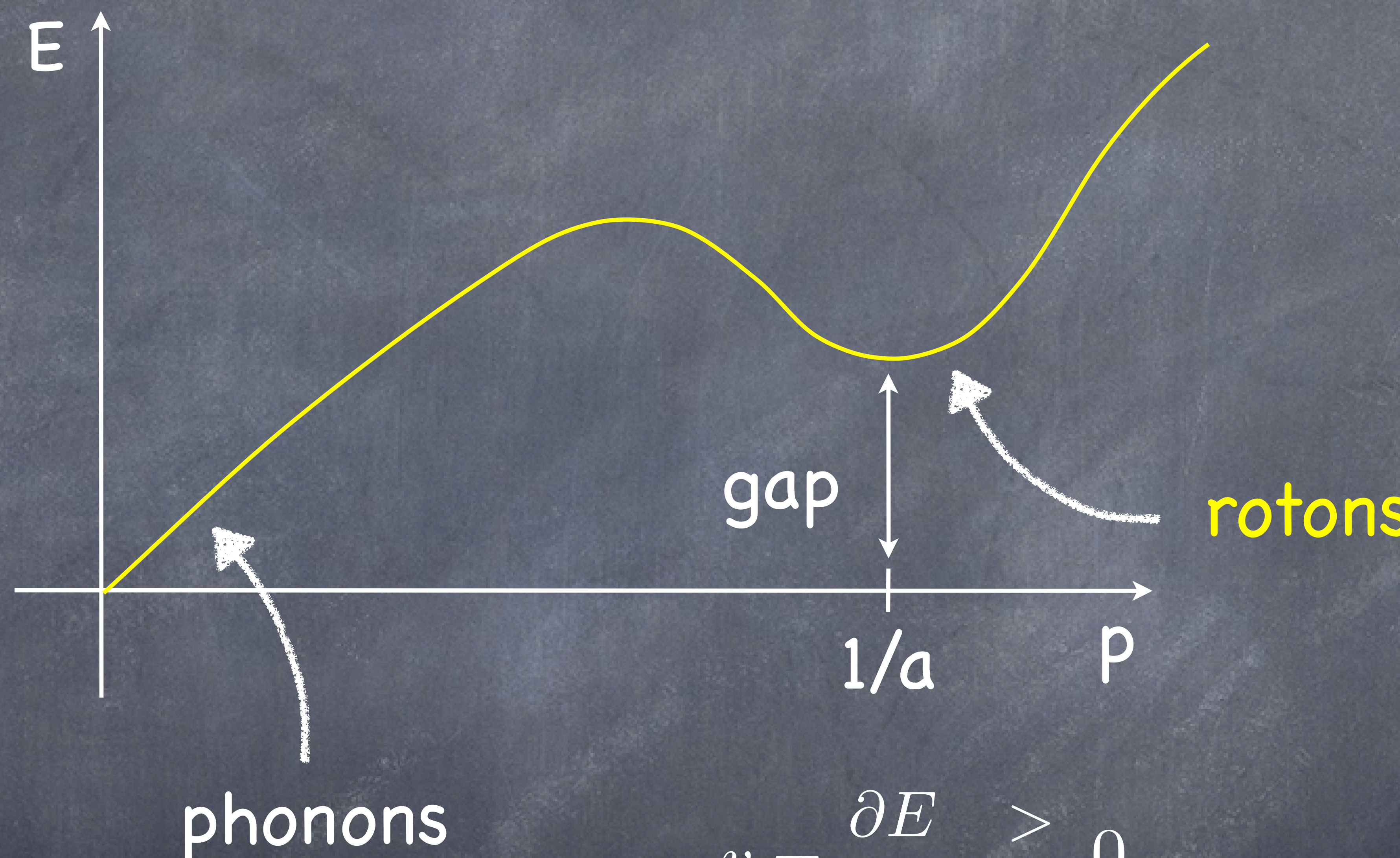
Ex: sound emission in vortex ring collisions





$$P = \frac{21}{2\pi} \frac{w_0 (R_1^2 \Gamma_1)^2 (R_2^2 \Gamma_2)^2 v^4}{c_s^5 r^{10}(t)} \quad \sim E_{\text{kin}} \omega \cdot (R/r)^{10} \cdot (v/c_s)^5$$

Work in progress: Rotons in Helium 4



$$v = \frac{\partial E}{\partial p} \gtreqless 0$$

usually thought of as microscopic vortex rings.
can we check?

Works, but fairly redundant SSB pattern...

$$\text{Poincaré} \begin{cases} P^\mu & \text{translations} \\ J^i & \text{rotations} \\ K^i & \text{boosts} \end{cases} + \text{internal ISO(3)} \begin{cases} Q^I \\ \tilde{Q}^I \end{cases}$$

$$\Downarrow$$

$$\begin{cases} P^t \\ \bar{P}^i = P^i + Q^i \\ \bar{J}^i = J^i + \tilde{Q}^i \end{cases}$$

simpler description?

Just break boosts:

Poincaré $\begin{cases} P^\mu & \text{translations} \\ J^i & \text{rotations} \\ K^i & \text{boosts} \end{cases}$

$$\Downarrow$$

$$\begin{cases} P^\mu \\ J^i \end{cases}$$

e.g.: $\langle V^\mu(x) \rangle = \delta^\mu_0$

(Nicolis, Penco, Piazza, Rattazzi, Rosen, soon)

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$$\text{Poincaré} \begin{cases} P^\mu & \text{translations} \\ J^i & \text{rotations} \\ K^i & \text{boosts} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} P^\mu \\ J^i \end{cases}$$

"framid"

e.g.: $\langle V^\mu(x) \rangle = \delta^\mu_0$

(Nicolis, Penco, Piazza, Rattazzi, Rosen, soon)

3 Goldstones: "boostons"

simple analysis: $V^\mu(x) = \left(e^{i\vec{\eta}(x)\cdot\vec{K}}\right)^\mu{}_\alpha\, \delta_0^\alpha$

$$\mathcal{L}_{\text{eff}} \supset (\partial_\mu V^\mu)^2, (\partial_\mu V_\nu)^2, (V^\mu \partial_\mu V_\nu)^2 + \ldots$$

coset-ology: $\Omega(x) = e^{iP_\mu x^\mu} e^{i\vec{\eta}(x)\cdot\vec{K}}$

$$\Omega^{-1}\partial_\mu \Omega = \ldots \quad \rightarrow \quad \mathcal{D}_t \eta_i,\, \mathcal{D}_i \eta_j \sim \partial\eta + \mathcal{O}(\partial\,\eta^n)$$

$$\mathcal{L}_{\text{eff}} \supset (\mathcal{D}_t \eta_i)^2, (\mathcal{D}_i \eta_i)^2, (\mathcal{D}_i \eta_j)^2 + \ldots$$

same result.

framid = solid (fluid) ?!?

match an observable: $\mathcal{M}_{2\to 2}$

Different naive scaling:

$$\mathcal{L}_{\rm solid} = F(\partial_\mu \phi^I \partial^\mu \phi^J) \sim (\partial\pi)^2 + (\partial\pi)^3 + (\partial\pi)^4 + \ldots$$

$$\mathcal{L}_{\rm framid} \sim (\partial\eta)^2 + \partial^2\eta^3 + \partial^2\eta^4 + \ldots$$

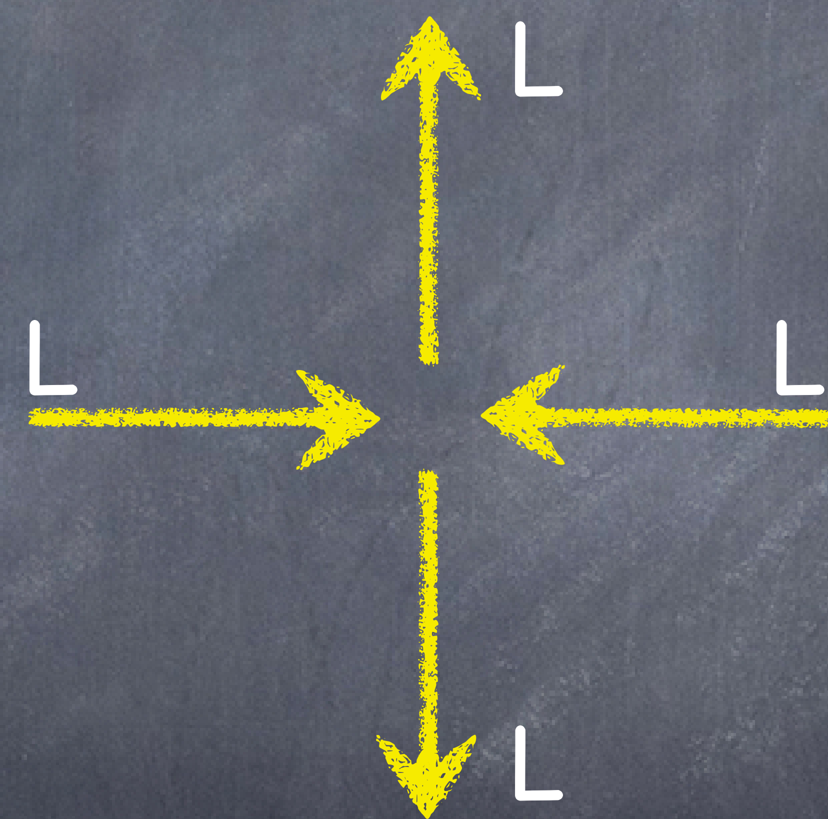
Barring cancellations:

$$\mathcal{M}_{\rm solid} \propto E^4 \quad \text{vs.} \quad \mathcal{M}_{\rm framid} \propto E^2$$

Cancellations ?

NO: $$\mathcal{M}_{\rm framid} = 2\frac{E^2}{f^2}\Big[-6+4c_T^2-2c_L^2-\frac{(1-c_L^2)^2}{c_T^2}\Big]$$

for:



$$\mathcal{L}_{\rm framid} = \tfrac{1}{2}f^2\big[(\partial_t\eta_i)^2 - c_T^2(\partial_i\eta_j)^2 - (c_L^2-c_T^2)(\partial_i\eta_i)^2+\dots\big] \, .$$

So, a framid is NOT a solid in disguise

Yet, much simpler SSB pattern:

$$\left\{ \begin{array}{l} P^\mu \\ J^i \\ K^i \end{array} \right. + \left\{ \begin{array}{l} Q^I \\ \tilde{Q}^I \end{array} \right. \qquad \Downarrow \qquad \left\{ \begin{array}{l} P^t \\ \bar{P}^i = P^i + Q^i \\ \bar{J}^i = J^i + \tilde{Q}^i \end{array} \right.$$

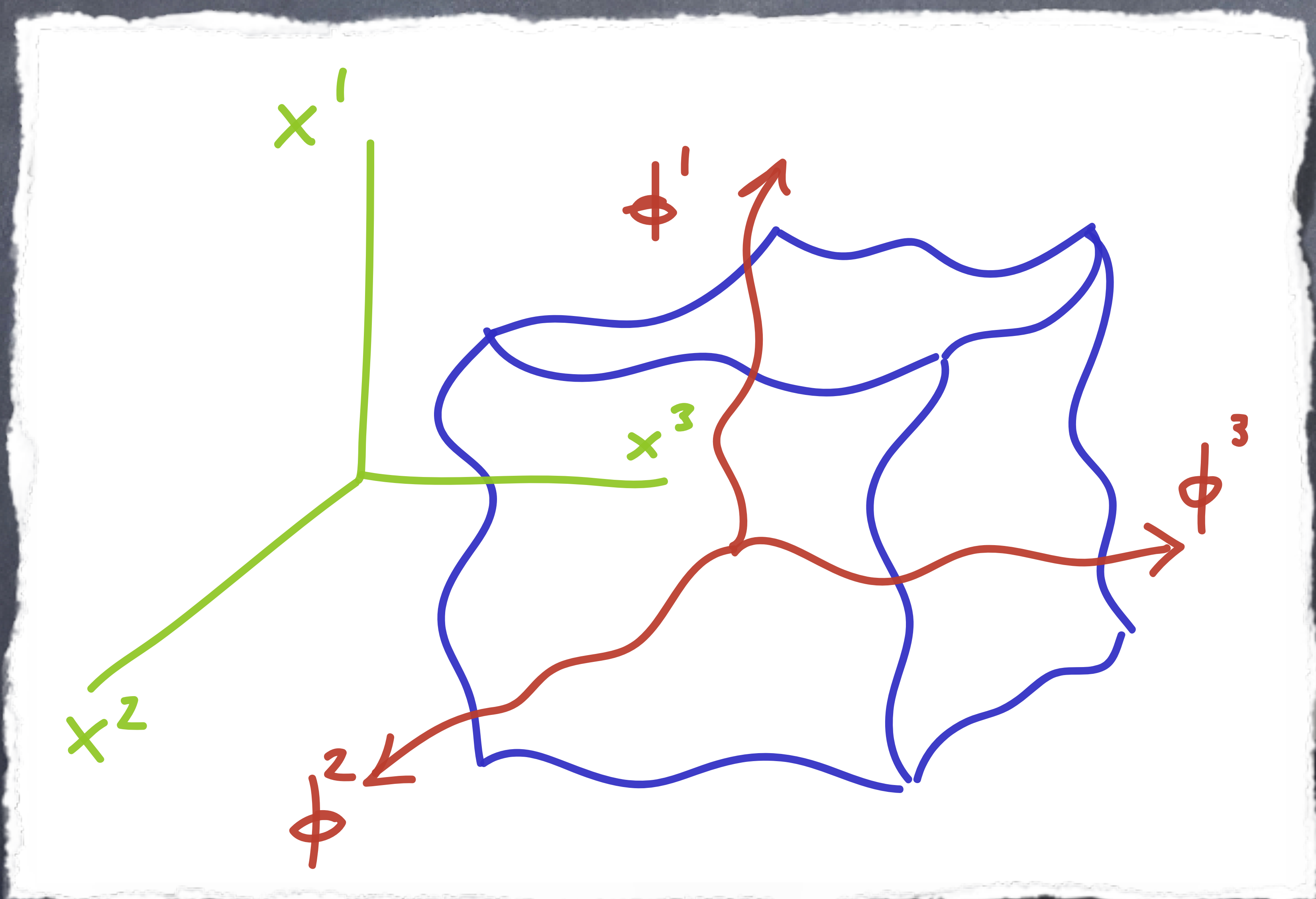
vs.

$$\left\{ \begin{array}{l} P^\mu \\ J^i \\ K^i \end{array} \right. \qquad \Downarrow \qquad \left\{ \begin{array}{l} P^\mu \\ J^i \end{array} \right.$$

Why don't we see framids in the lab?

1. Condensed matter is made up of "stuff".

We need this picture:



$$\phi^I(\vec{x}, t)$$

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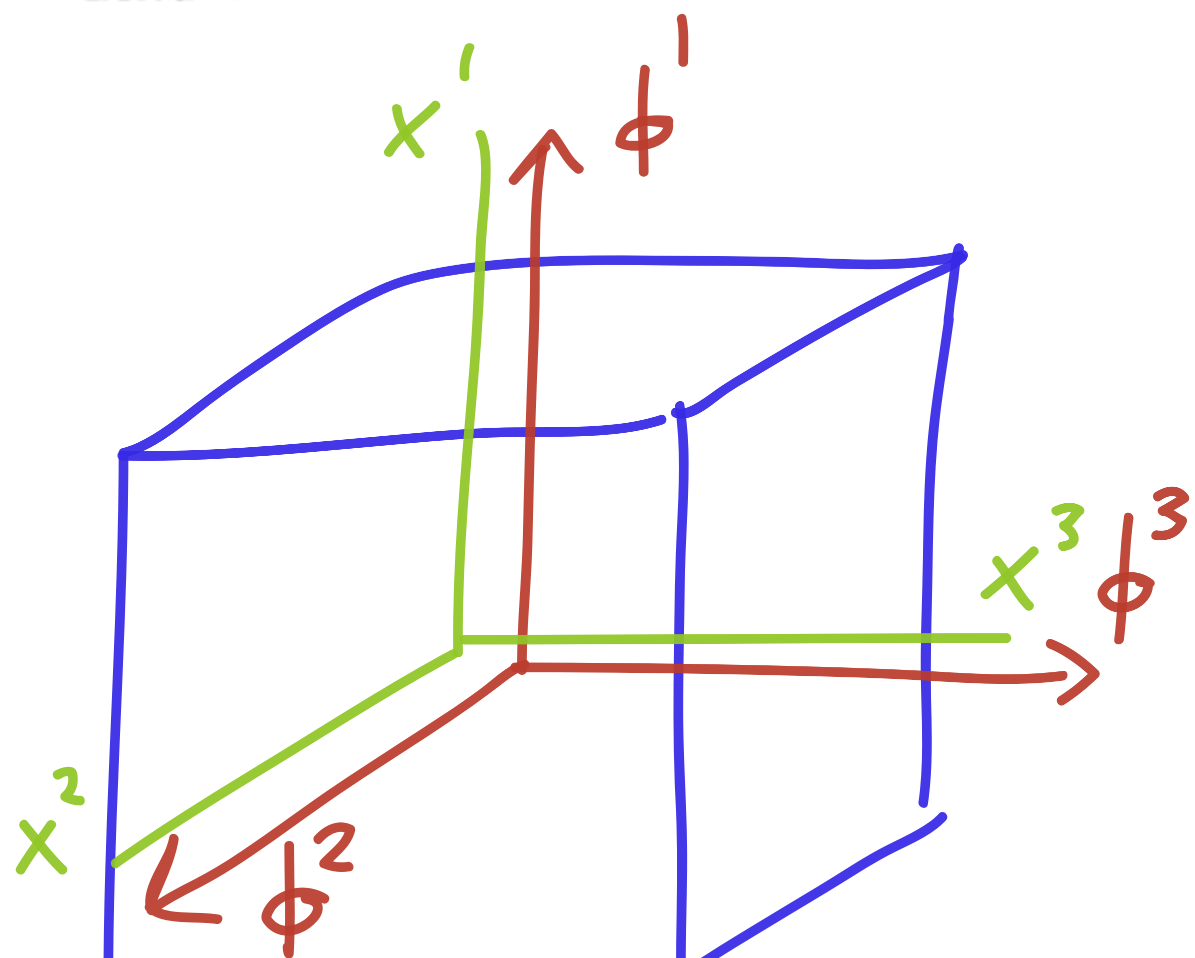


$$\phi^I(\vec{x}, t)$$

$$\langle \phi^I \rangle_{\text{eq}} = x^I$$

Superfluids violate this intuition: $\langle \Phi(x) \rangle = e^{i\mu t}$

... where is the stuff?

2. Maybe it is technically natural to have cs<<1 for solids and fluids, but not framids.

In fact, the radiative stability of cs<<1 is a consequence of ...

Superfluids violate this intuition: $\langle \Phi(x) \rangle = e^{i\mu t}$

... where is the stuff?

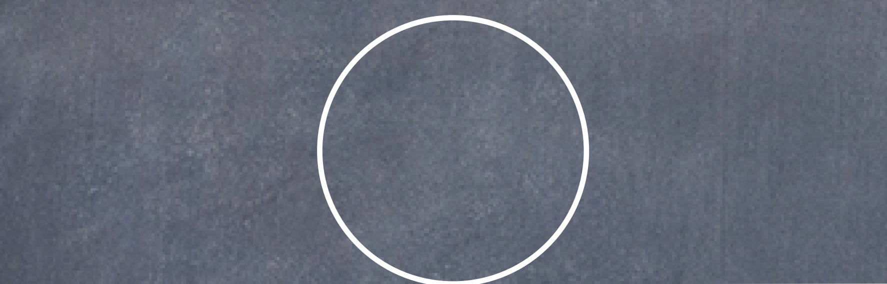
2. Maybe it is technically natural to have cs<<1 for solids and fluids, but not framids.

In fact, the radiative stability of cs<<1 is a consequence of ... nothing

Generic cs<<1 action

$$S = \int d^3x dt\, A(\dot{\pi}^2 - c_s^2(\nabla\pi)^2) + \text{interactions}$$

If interactions $\supset$ "large" $(\nabla\pi)^4$



⇨ large cs, right?

NO: $t \to t'/c_s$

$$S = \int d^3x dt'\, (\pi'^{\,2} - (\nabla\pi)^2) + \text{interactions}$$

Cut off loops at strong coupling scale

⇨ at most O(1) renormalization

3. No standard thermodynamical deformations:

$$V^\mu(x) = \left(e^{i\vec{\eta}(x)\cdot\vec{K}}\right)^\mu{}_\alpha \, \delta_0^\alpha$$

By def., the background can only be boosted

For a solid or fluid: $\langle\phi^I\rangle = \alpha^I{}_J x^J$

is a solution for all $\alpha^I{}_J$

⇨ The medium can be deformed homogeneously

4. Intrinsically relativistic stress-energy tensor

$$\mathcal{L}_{\text{eff}} \supset (\partial_\mu V^\mu)^2, (\partial_\mu V_\nu)^2, (V^\mu \partial_\mu V_\nu)^2 + \dots$$

⇨ $T_{\mu\nu} \sim \partial^2 V^n + \dots \quad \to \quad 0$ for V = const

For a solid or fluid: $T_{\mu\nu} = F(\partial\phi) \neq 0$

for $(\partial\phi)$ = const

More in general: $T_{\mu\nu}^{\text{framid}} \to \Lambda \eta_{\mu\nu}$

⇨ relativistic p

Gapped Goldstones

(Nicolis, Piazza 2012)
(Nicolis, Penco, Piazza, Rosen 2013)
(Brauner, Murayama, Watanabe 2013)
(Kapustin 2012)

Unbroken Poincaré, and broken internal symmetries

⇨ standard Goldstone theorem (#, m=0)

Broken Poincaré, broken internal symmetries

⇨ theorem less powerful (e.g. $\Gamma \sim k^5$)

⇨ more possibilities

New counting rules

For internal symmetries

$$n_1 = \#\text{Goldstones w/ } \omega \sim k$$
$$n_2 = \#\text{Goldstones w/ } \omega \sim k^2$$

$$n_1 + 2 \cdot n_2 = \#\text{broken generators}$$ (Nielsen, Chadha 1976)

For spacetime symmetries

$$\#\text{Goldstones} \leq \#\text{broken generators}$$ (Ivanov, Ogievetsky 1975) (Low, Manohar 2002)

(e.g. point particle)

Exact # depends on the system (Nicolis, Penco, Piazza, Rosen 2013)

Gaps at finite charge density

Finite density for broken Q (superfluid):

$$\bar{H}|\mu\rangle \equiv (H - \mu Q)|\mu\rangle = 0$$

⇨ H broken

⇨ excitations: eigenstates of $\bar{H}$

If other broken Q_a's don't commute w/ Q

⇨ pseudo-Goldstones

No explicit breaking ⇨ gap can be computed exactly

(Nicolis, Piazza 2012)

Choose basis such that

$$[Q, Q_\alpha] = 0$$
$$[Q, Q_a^\pm] = \pm i q_a \, Q_a^\mp$$

Broken Q_α's ⟹ gapless Goldstones ($n_1 + 2 \cdot n_2$)

Broken $Q_a^\pm$'s ⟹ gapped Goldstones

$E_a = q_a \mu$ for $k \to 0$

exact non-perturbative result

(Nicolis, Piazza 2012)

More gapped Goldstones

From a coset construction of the Goldstone EFT:

gapless:
$$n_1 = \#\text{Goldstones w/ } \omega \sim k$$
$$n_2 = \#\text{Goldstones w/ } \omega \sim k^2$$

gapped:
$$n_3 = \#\text{Goldstones w/ } \omega_a = q_a \mu$$
$$n_4 = \#\text{Goldstones w/ } \omega \sim \mu$$

Type 4: partners of type 2 and 3

$$n_2 \leq n_4 \leq n_2 + n_3$$

(Nicolis, Penco, Piazza, Rosen 2013)
(cf. Kapustin 2012)

Conclusions

For certain questions in CM, a lot of mileage from taking into account spacetime symmetries.