Aspects of holographic finite density matter

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Introduction

I will make some comments on finite density strongly interacting states.

Most recent paper: arXiv:1312.0463 (with R. Davison and M. Goykhman)

Related work by Karch, Starinets, Son; Faulkner and Iqbal; Nickel and Son; Hartnoll and Shaghoulian, many others.

Introduction

We generally expect physics at low energies to be described by an effective theory. At finite density Fermi liquid theory is an example of such an effective theory.

It would be great to have such a description for theories at finite density that are described by holography. At the moment we don't have a complete picture although we do know quite a bit about the responses of finite density system.

Especially interesting for strongly interacting fermions. One way is to place back-reacting fermions in the bulk ($1/N$ corections). But we really need to understand the starting point: $N = \infty$ limit.

Outline

Fermi liquid

Fermi statistics of quasiparticle excitations. Change in energy due to the change in distribution function δn

$$\frac{1}{V}\delta E = \int \epsilon_0(k)\delta n(\mathbf{k})d\tau + \int f(\mathbf{k},\mathbf{k}')\delta n(\mathbf{k})\delta n(\mathbf{k}')d\tau d\tau'$$

$\epsilon_0(k)$ defines dispersion relation and Fermi velocity $\upsilon_F = \frac{\partial\epsilon_0(k)}{\partial k}\Big|_{k=k_F}$; effective mass $m^* = \frac{k_F}{\upsilon_F}$; and interaction $f(\mathbf{k},\mathbf{k}') = \frac{k_F m^*}{\pi^2\hbar^3}F(\vartheta)$. Also, $d\tau = d^3p/(2\pi\hbar)^3$.

Convenient to expand: $F(\vartheta) = \sum(2l+1)F_l P_l(\vartheta)$
F_l –Landau parameters

Fermi liquid

Note that it is natural to expand to second order in δn:

$$\frac{1}{V}\delta(E-\mu N)=\int(\epsilon_0(k)-\mu)\delta n(\mathbf{k})d\tau+\int f(\mathbf{k},\mathbf{k}')\delta n(\mathbf{k})\delta n(\mathbf{k}')d\tau d\tau'$$

If α is a small parameter characterizing deviation from degenerate state, $(\epsilon-\mu),\delta n\sim\alpha$ and both terms are $\mathcal{O}(\alpha^2)$.

Fermi liquid

$$m^* = \mu \left(1 + \frac{F_1}{3}\right), \qquad u_1^2 = \frac{\upsilon_F^2}{3}(1 + F_0)\left(1 + \frac{F_1}{3}\right)$$

The most important thermodynamical fact about near degenerate Fermi liquid is heat capacity which vanishes linearly with temperature. This is because quasiparticles are excited in the narrow region of momenta $\sim T$ with an average energy $\sim T$.

$$C_\upsilon = T \left(\frac{\partial S}{\partial T}\right)_V = \frac{m_* k_F}{3} T$$

Zero sound

An interesting dynamical property is the existence of a sound (gapless) excitation at small temperature. The speed of zero sound u_0 is determined by

$$(s - \cos\theta)\nu(\theta, \varphi) = \cos\theta \int F(\vartheta')\nu(\theta', \varphi')\frac{d\Omega'}{4\pi}$$

where $s = u_0/\upsilon_F$ and $\nu(\theta, \varphi)$ parametrizes deformations of spherical Fermi surface; θ denotes polar angle with respect to the direction of sound propagation and ϑ' is the angle between (θ, φ) and (θ', φ').

Zero sound

This equation comes from $n = n_0(\vec{p}) + \delta n(\vec{p}, \vec{r}, t)$

$$\frac{\partial \delta n}{\partial t} + \frac{\partial \delta n}{\partial \vec{r}}\frac{\partial \epsilon_0}{\partial \vec{p}} - \frac{\partial n_0}{\partial \vec{p}}\frac{\partial \delta \epsilon}{\partial \vec{r}} = 0$$

Now use $\partial n_0/\partial \vec{p} = -\hat{n}\delta(p - p_F) = -\vec{v}\delta(\epsilon - \epsilon_F)$ and $\delta n = \delta(\epsilon - \epsilon_F)\nu(\hat{n})\exp(i\vec{k}\vec{r} - i\omega t)$ to obtain

$$(\omega - \vec{v}\vec{k})\nu(\hat{n}) = \vec{v}\vec{k}\int f(\vec{p}, \vec{p}')\delta(\epsilon - \epsilon_F)\nu(\hat{n}')d\tau'$$

Zero sound lives in the $\omega\tau \sim w\mu/T^2 \gg 1$ regime, as opposed to the normal sound which corresponds to $\omega\tau \ll 1$ and necessarily has $u_1 = 1/\sqrt{d-1}$.

Spectral function

Consider summing "bubble" diagrams in the current-current correlator

$$\langle J^0(\omega, q) J^0(-\omega, -q)\rangle = \frac{\chi_0(q,\omega)}{1 - F_0 \chi_0(q,\omega)} \,,$$

where $\chi_0(q,\omega) = \chi_0^r(q,\omega) + i\,\chi_0^{(i)}(q,\omega)$ The spectral function

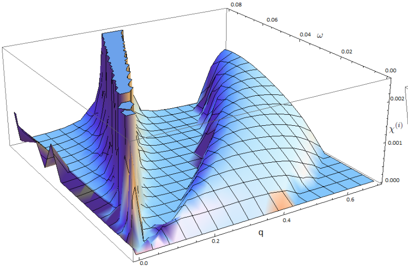
$$Im\langle J^0(\omega, q) J^0(-\omega, -q)\rangle = \frac{\chi_0^{(i)}(q,\omega)}{(1 - F_0 \chi_0^{(r)}(q,\omega))^2 + (\chi_0^{(i)}(q,\omega))^2} \,.$$

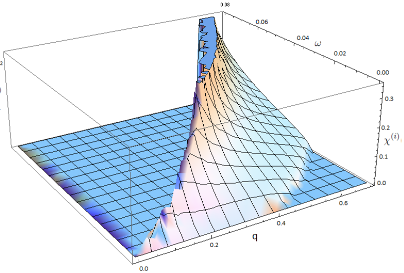
Zero sound and particle-hole continuum

Delta function peak, representing zero sound, appears as the solution of

$$1 - F_0 \chi_0^{(r)}(q, \omega) = 0$$

Upper edge of the particle-hole continuum is at $\omega = v_F q$.





Zero sound and particle-hole continuum

The location of zero sound pole is determined by $\chi_0^{(r)}(q,\omega) = 1/F_0$. The equation is equivalent to the integral equation for $s = u_0/\upsilon_F$ with $F = F_0$

$$\frac{s}{2}\log\frac{s+1}{s-1} - 1 = \frac{1}{F_0},$$

In the limit of small interactions, $F_0 \ll 1$, $s \simeq 1$, $u_0/u_1 \simeq \sqrt{d-1}$.

In the opposite limit $F_0 \gg 1$, $s \simeq \sqrt{F_0/3}$, $u_0/u_1 \to 1$.

The upper edge of the particle-hole continuum is defined by $\omega \simeq \upsilon_F k$ and is separated from the zero sound when $\upsilon_F \ll u_0$.

Summary

It the regime where Fermi liquid is a good description $\omega, T \ll \mu$ the following things happen:

- Sound mode interpolates between regular sound $\omega\mu/T^2 \ll 1$ and zero sound $\omega\mu/T^2 \gg 1$
- When F_0 or F_1 are large, $v_F/u_0 \ll 1$ and spectral density is entirely given by the sound pole.

AdS/CFT for probe branes

$d = 3 + 1$ dimensional: N D3 branes stretched along 0123 directions and a Dp brane stretched along 0123 $4, \ldots, p$. Matter interacting with $\mathcal{N} = 4$ SYM.

$d = 2 + 1$ dimensional: N D3 branes stretched along 0123 directions and a Dp brane stretched along 012 $4, \ldots, p + 1$. This describe defect matter + $\mathcal{N} = 4$ SYM.

Examples: (d,p)=(3,5) susy defect; (3,7) - defect fermions; (4,7) - susy hyper

AdS/CFT for probe branes

At strong 't Hooft coupling the dynamics of the fundamental matter living on the intersection is described by the DBI action in $AdS_5 \times S^5$ background.

Finite temperature = asymptotically AdS black hole.

Finite density = Gauge field flux on the probe brane world volume. At sufficiently large chemical potential the symmetric state represents the vacuum of the theory.

Holographic zero sound

What about zero sound? (massless excitation in the regime, $\mu \gg \omega, q \gg T^2/\mu$) Consider gauge field fluctuations on the $D7$ branes at finite μ.

Technically one needs to expand the DBI action for the probe brane to quadratic order in fluctuations. Need to solve the second order differential equation for fluctuations with incoming boundary conditions at the horizon. Looking for quasinormal mode with $\omega, q \to 0$.

D3/Dp; d=3; massless

EOM for $E = \omega A_2 + qA_0$, $\vec{q} = q\hat{x}_2$.

$$E'' + \frac{2}{z}\left(\frac{1}{1+z^{-4}} + 2(1 - \frac{\omega^2 - q^2(1+z^4)^{-2}}{\omega^2 - q^2(1+z^4)^{-1}})\right)E' + (\omega^2 - \frac{q^2}{1+z^4})E = 0$$

can be solved separately in the $\omega z \ll 1$ regime and $z \gg 1$ regime ($e^{i\omega z}/z$). The solutions than can be matched and the quasinormal mode can be found.

$$\omega = \pm\frac{q}{\sqrt{2}} - \frac{iq^2\sqrt{\lambda}}{4K(1/2)\mu}$$

[true for $\omega \gg T$, otherwise $q^2/\mu \to (q^2 + T^2)/\mu$]

D3/D7; massive

This exercise can be repeated in the case of massless fermions. Complication: gauge field fluctuations couple to the fluctuations of the probe brane profile. One needs to solve the system of 2 coupled ODEs, but this can be done and the zero sound mode is uncovered with the speed of sound

$$s = \frac{\mu^2 - m^2}{3\mu^2 - m^2}$$

The speed of zero sound is equal to the speed of normal sound. This is true in ALL holographic finite matter examples studied so far. Also, no structure in spectral density other than zero sound.

Gravity+gauge field+...

We considered a couple of cases (RN-AdS and $S \sim T$ BH) and studied low energy excitations of finite density holographic matter in the $T^2/\mu^2 \ll \omega/\mu \ll 1$ regime.

Naively this is where the zero sound lives in the Landau Fermi liquid setting (and it violates hydro). However, we observed that the sound mode is always described by hydro (as long as $\omega, T \ll \mu$).

It was hard to observe this in the probe brane case for technical reasons.

$S \sim T$ background

Consider the bulk Lagrangian

$$S_2 \simeq \int d^5x\sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{8}{L^2}e^{\phi/\sqrt{6}} - \frac{4}{L^2}e^{-2\phi/\sqrt{6}} + 2e^{2\phi/\sqrt{6}}F_{ab}F^{ab}\right)$$

with the black hole solution which has

$$T = \frac{r_H}{\pi L^2}, \qquad \mu = \frac{\sqrt{2}Q}{L^2}, \qquad s = \frac{r_H}{4GL^3}(r_H^2 + Q^2)$$

$$\hat{\rho} = \frac{\sqrt{2}Qs}{2\pi r_H}, \qquad \varepsilon = 3P = \frac{3\left(r_H^2 + Q^2\right)^2}{16\pi GL^5}$$

$S \sim T$ background

Low temperature heat capacity is

$$c_V = \frac{\pi L^3}{8G}\mu^2 T = \frac{1}{3x^3}\mu^2 T, \qquad x \ll 1$$

This should be compared with $c_V = k_F m^*/3T$. This, and Luttinger theorem

$$\rho = \frac{1}{6\pi^2}\frac{\mu^3}{x^3} = \frac{k_F^3}{6\pi^2}$$

fix

$$k_F = \frac{\mu}{x}, \qquad m^* = \frac{\mu}{x^2}, \qquad \upsilon_F = x$$

$S \sim T$ background

Comparing the expression for m^* with FL prediction we get

$$\frac{F_1}{3} = x^{-2}, \qquad x \ll 1$$

On the other hand, the expression for the 1st sound gives $F_0 = 0$. Now go back to the integral equation for $s = u_0/\upsilon_F$. Assume azimuthal independence and write

$$\nu(\theta, \varphi) = \sum_l P_l(\cos\theta)\nu_l(2l+1)$$

which gives

$$\nu_l + \sum_l \Omega_{ll'}(s) F_{l'} \nu_{l'} = 0$$

$S \sim T$ background

This implies $\det[\delta_{ll'} + \Omega_{ll'}(s) F_{l'}] = 0$
Assuming $F_{n>1} = \mathcal{O}(1)$ the determinant is

$$1 - \frac{F_0 \tilde{s}^2}{3} - \frac{F_1 \tilde{s}^2}{5} - \frac{F_0 F_1 \tilde{s}^2}{9} - \frac{4}{225} F_1 F_2 \tilde{s}^2 + \mathcal{O}(\tilde{s}^2) = 0$$

where $\tilde{s} = 1/s = v_F/u_0 \sim x$.

Imposing $u_0 = u_1$ leads to

$$\frac{4}{25} \frac{F_1(5 + F_2)}{(1 + F_0)(3 + F_1)} = \mathcal{O}(\tilde{s}^2)$$

which means $F_2 = -5 + \mathcal{O}(\tilde{s}^2)$. Critical point for Pomeranchuk instability.

Observables

What does this mean? Let's go over observables:

$k_F \simeq \mu/x$, which means we can't see Friedel oscillations from a tree-level calculation.

$v_F \simeq x$, which means we don't see particle-hole continuum. Only zero sound pole appears in the spectral function.

log violation in EE is $k_F^2 L^2 \log L \simeq \frac{L^2}{x^2} \log L$, can't be seen at tree level (which gives $1/x^3$)

Observables

Zero sound with azimuthal dependence exists when the following equation is satisfied:

$$1 - \frac{3F_1(5+F_2)S_1^2}{25(1+F_0)(3+F_1)} = \mathcal{O}(\tilde{s}^2)$$

where $S_1 = u_0^{(1)}/u_0^{(1)}$. Precisely when $F_2 = -5$ it disappears.

Observables

Another useful quantity is quasiparticle lifetime

$$\tau = \frac{8\pi}{{m^*}^3 \langle W \rangle T^2}$$

where

$$\langle W \rangle = \frac{2\pi^5}{{m^*}^2 k_F^2} \int \frac{d\Omega}{4\pi} \frac{1}{\cos(\theta/2)} \Big| \sum_l \frac{F_l}{1 + F_l/(2l+1)} P_l(\cos\theta) \Big|^2$$

For generic F_l, $\tau \sim k_F^2/m^* T^2$ and $\eta/s \sim \mu^3/T^3$, while observed value is $\eta/s = 1/4\pi$. But for $F_2 = -5$, τ is parametrically suppressed.

Observables

Another issue is the applicability of hydro: determined by $\omega\tau \ll 1$ or $\omega\mu \ll T^2$. But in our case $\omega\mu x \ll T^2$ which is always true in the limit $x \to 0$. So hydro is supposed to be applicable all the way to $T = 0$, and at that point the sound merges with the zero sound; that's why the speed has conformal value.

This is precisely what we observe:

$$\omega = \pm\sqrt{\frac{dP}{d\varepsilon}}k - i\frac{2\eta}{3(\varepsilon + P)}k^2 + \dots$$

[with $\eta = 4\pi s$ of course]

Higher derivative gravity

Can we engineer a gravity dual with $\eta/s \sim \mu^3/T^3$. The answer is yes. Two-derivative gravity

$$S_2 \simeq \int d^5x\sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{8}{L^2}e^{\phi/\sqrt{6}} - \frac{4}{L^2}e^{-2\phi/\sqrt{6}} + 2e^{2\phi/\sqrt{6}}F_{ab}F^{ab}\right)$$

needs to be supplemented by a dilation-Gauss-Bonnet term

$$S_4 \simeq \int d^5x\sqrt{g}\left(3e^{\frac{7}{2\sqrt{6}}\phi} + \frac{3}{2}e^{-\frac{7}{\sqrt{6}}\phi}\right)\left(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\right)$$

Summary, future directions

- Fermi liquid with tuned parameters exhibits the same collective behavior as holographic matter
- Is there a gravitational model where the speed of zero sound is not equal to conformal value? Then one could see interpolation between hydro and non-hydro regimes.
- Made first steps in this direction by considering certain higher-derivative gravity. Need to analyze QNM dispersion relation.
- Zero sound on the probe brane shows interpolation between the collisionless thermal $T^2/\mu^2 \ll \omega/\mu \ll T/\mu$ and collisionless quantum $T/\mu \ll \omega/\mu$ regimes. We don't see this in gravity+E&M.

THE END

Thank you!