Chern-Simons Vector Models and Bose-Fermi Dualities

Ran Yacoby

Princeton University

Quantum Fields Beyond Perturbation Theory, KITP
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Chern-Simons Vector Models

$U(N)_k$ Chern-Simons + matter

$$\mathcal{L} = \frac{ik}{4\pi} \int \mathrm{Tr} \left(A \wedge dA + \frac{2}{3} A^3 \right) + S_{\mathrm{matter}}$$

Chern-Simons Vector Models

- Matter only in fundamental representation
- Focus on non-supersymmetric theories

- Naively, cannot do much beyond perturbation theory
- However, amazing cancelations appear in planar perturbation theory ($N, k \to \infty$ $\lambda = N/k$)

Chern-Simons Vector Models: Large N

Holography

- CS vector models $\overset{AdS/\mathrm{CFT}}{\longleftrightarrow}$ high-spin gravity
- High-spin symmetry at large N

Planar limit is remarkably simple

- Lines of fixed points
- Compute planar 2-point, 3-point functions, thermal partition function
- Non-SUSic dualities (``3d bosonization''),

$$\mathrm{CS} + \phi \equiv \mathrm{CS} + \psi$$

Vector Models Holography: Bosons

$k \to \infty$, $(\lambda \to 0) \implies U(N)$ Vector models in singlet sector

[Klebanov,Polyakov 2002] , [Sezgin,Sundell 2005]

$$S = \int d^3x \partial^\mu \phi_i^\dagger \partial_\mu \phi^i , \quad i = 1, \ldots, N .$$

CFT

Single-trace primaries:

$$J_s = \phi_i^\dagger \partial^s \phi^i \quad , \Delta_s = s + 1$$
$$J_0 = \phi_i^\dagger \phi^i \quad , \Delta_0 = 1$$

3-point functions:

$$\langle J_s J_{s'} J_{s''} \rangle \sim \frac{1}{\sqrt{N}}$$

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Vasiliev Type A

Single-particle states:

A_μ , $g_{\mu\nu}$, HS fields

Scalar : $m^2 = -2/R_{AdS}^2$

Bulk interactions:

$$G_N \sim \frac{1}{N}$$

Vector Models Holography: Bosons

Critical Bosonic Vector Models: $\delta\mathcal{L} = (\phi_i^\dagger \phi^i)^2$

Free Boson (UV)

↓

Wilson-Fischer (IR)

- Currents ($s \geq 1$): $\Delta_s^{IR} = \Delta_s^{UV} + O(1/N)$
- Scalar: $\Delta^{IR} = 2 + O(1/N)$
- Different b.c. for bulk scalar
- In planar limit, related to free theory by Legendre transform w.r.t. $J_0 = \phi^\dagger \phi$

Vector Models Holography: Fermions

$$S = \int d^3x \, \psi_i^\dagger \gamma^\mu \partial_\mu \psi^i$$

- Single-trace primaries:

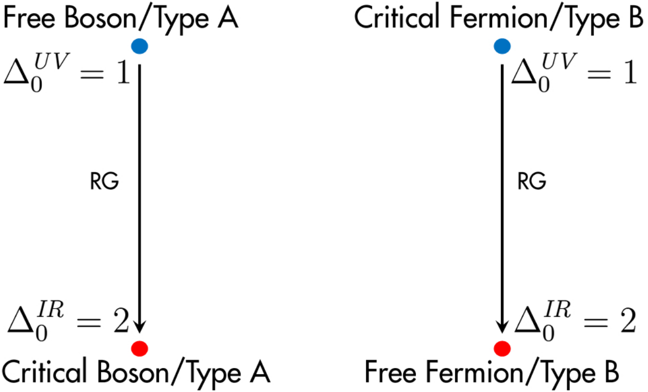
$$J_s = \psi_i^\dagger \gamma \partial^{s-1} \psi^i \, , \; \Delta_s = s+1$$
$$J_0 = \psi_i^\dagger \psi^i \qquad , \; \Delta_0 = 2$$

- J_0 is parity odd
- Dual to type B Vasiliev [Sezgin,Sundell 2005]

Vector Models Holography: Evidence

- Spectrum matches [Klebanov,Polyakov 2002] ,[Sezgin,Sundell 2001]
- All 3-point functions agree [Giombi, Yin 2010]
- CFT + High-spin symmetry $\Rightarrow$ free bosons/fermions

  [Maldacena, Zhiboedov 2011]



Parity Violating Vasiliev Theories

- $\theta_0 =$ continuous ~~parity~~ parameter of bulk equations
- Classicaly spectrum is unchanged
- Interactions are modified

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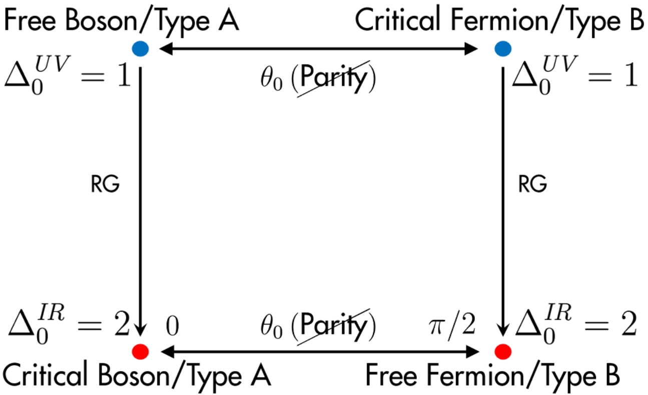


CS Vector Models ↔ ~~Parity~~ Vasiliev

$$\mathcal{L} = \frac{ik}{4\pi}\int \mathrm{Tr}\left(A\wedge dA + \frac{2}{3}A^3\right) + S_{\mathrm{matter}}$$

$$N, k \to \infty\,, \qquad \lambda = N/k$$

Conjecture [Giombi, Yin 2010]

$\theta_0 \overset{?}{\sim} \lambda \Longrightarrow$ Far reaching implications on CFT side.

- Lines of fixed points $\forall\lambda$ [Aharony, Gur-Ari, RY 2011] , [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 2011]
- $\gamma_{J_s} = O(1/N)$
- $U(N_b)_{k_b} + \mathrm{critical}\,\phi^i \equiv U(N_f)_{k_f} + \psi^i$

Spectrum Non-Renormalization

$$\underbrace{\partial \cdot J_s}_{(\Delta,\text{spin})=(s+2,s-1)} = \underbrace{0}_{\text{single}-\text{trace}} + \frac{\text{f}(\lambda)}{N} :J_{s'}J_{s''}: + \cdots .$$

- $\langle \partial \cdot J_s J_s \rangle = O(1/N) \Rightarrow \Delta_s = s + 1 + O(1/N)$
- $\langle \partial \cdot J_s J_{s'} J_{s''} \rangle \neq 0$ to leading order $\Rightarrow$ ~~high-spin sym.~~ in planar limit
- However,

$$\langle \partial \cdot J_s J_{s'} J_{s''} \rangle = O(1/N)$$

in ∞-ly many 3-point functions $\Rightarrow$ ``Slightly broken high-spin symmetry''

[Aharony, Gur-Ari, RY 2011] , [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 2011]

3-point Functions: From HS-Symmetry

- Assume ∃ 3d CFT with slightly broken HS-symmetry.
- Define $\tilde{N}, \tilde{\lambda}$ by

$$\langle J_2 J_2 \rangle \equiv \tilde{N} \langle J_2 J_2 \rangle_{\text{bos.}} \,,$$

$$\partial \cdot J_4 = \frac{\tilde{\lambda}}{\tilde{N}} : J_2 J_0 : + \cdots$$

- All 3-point functions are determined by $\tilde{N}, \tilde{\lambda}$
  [Maldacena,Zhiboedov 2012]

3-point Functions: From HS-Symmetry

$$\langle J_s J_s \rangle_{s\geq 1} \equiv \tilde{N} \langle J_s J_s \rangle_{\text{bos.}} \,,$$

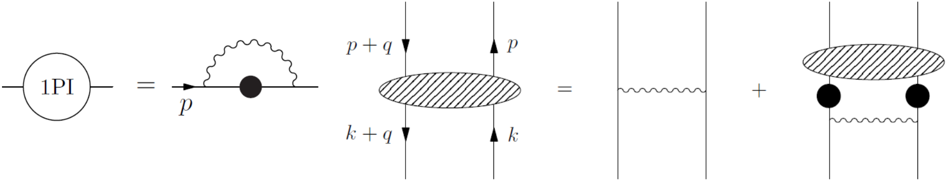
$$\langle J_s J_{s'} J_{s''} \rangle_{s\geq 1} = \tilde{N} \left[\frac{1}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\text{bos.}} + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\text{fer.}} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\text{odd}} \right] \,.$$

- $\tilde{N}$ is the coefficient of all 2-point functions
- Only three conformal structures appear in 3-point
- Coefficients of structures are independent of J_s
- Interpolation between bosons and fermions
- Relation to bulk $\tilde{\lambda} = \tan(\theta_0)$ [Chang, Minwalla, Sharma, Yin 2012]

$$\tilde{N}, \tilde{\lambda} \xleftrightarrow{?} N, \lambda$$

3-Point Function: Direct Computation

- Light-cone gauge $\Rightarrow A \wedge A \wedge A = 0$
- Perturbation theory efficiently organized in terms of ladder diagrams

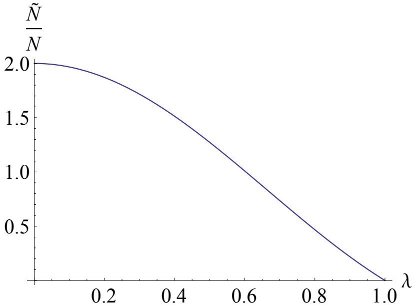


- Solvable when $q^{\pm} = 0\,,\ q^3 \neq 0$

[Aharony, Gur-Ari, RY 2012] ,[Gur-Ari,RY 2012]

3-Point Function: Results

$$\langle TT\rangle = \underbrace{2N\frac{\sin(\pi\lambda)}{\pi\lambda}}_{\tilde{N}}\langle TT\rangle_{\text{free}}$$



- $\tilde{N} =$ monotonically decreasing
- $|k| = |k_0| + N \Rightarrow |\lambda| \leq 1$
- $\tilde{\lambda} = \tan\left(\frac{\pi\lambda}{2}\right)$

All planar 3-point functions

$$\langle J_s J_{s'} J_{s''}\rangle_{s\geq 1} =$$

$$2N\frac{\sin(\pi\lambda)}{\pi\lambda}\left[\cos^2\left(\frac{\pi\lambda}{2}\right)\langle\cdot\rangle_{\text{bos.}} + \sin^2\left(\frac{\pi\lambda}{2}\right)\langle\cdot\rangle_{\text{fer.}} + \frac{\sin(\pi\lambda)}{2}\langle\cdot\rangle_{\text{odd}}\right]$$

Bose-Fermi Duality

- By comparing correlators in bosonic and fermionic theories, one finds the duality map

$$U(N_b)_{k_b} + \text{critical}\, \phi^i \longleftrightarrow U(N_f)_{k_f} + \psi^i$$
$$k_f = -k_b\,, \quad N_f = |k_b| - N_b$$

- Level-rank duality (in terms of the ``renormalized level'')
- Weak-strong: $|\lambda_b| = 1 - |\lambda_f|$

Duality: Generalizations

- Can repeat computation for any number of matter fields

Example: [Aharony, Giombi, Gur-Ari, Maldacena, RY 2012]

$$\mathcal{L} = \frac{ik}{4\pi} CS(A) + i\bar{\psi}\not{D}\psi + |D_\mu\phi|^2 + \frac{\lambda_4}{N}(\bar{\psi}\psi)(\phi^\dagger\phi) + \frac{\lambda_6}{3!N^2}(\phi^\dagger\phi)^3$$

$$x_4 \equiv \frac{\lambda_4}{4\pi\lambda}\ , \qquad x_6 \equiv \frac{1}{4}\left(1 + \frac{\lambda_6}{8\pi^2\lambda}\right)$$

$$x_4 \leftrightarrow \frac{1}{x_4}\ , \qquad x_6 \leftrightarrow \frac{1}{x_4^3}\left(1 + x_4^3 - x_6\right)$$

- Self-dual point, $x_4 = x_6 = 1$, corresponds to $\mathcal{N} = 2$ $U(N)_k$ with 1 chiral

[Giveon,Kutasov 2008] ,[Benini,Closset,Cremonesi 2011]

Free Energy

- Equality of 3-point functions follows from symmetry
- Can compute thermal free energy
- This gives a highly non-trivial test of dualities

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On $\mathbb{R}^2 \times S^1$ can turn on $a = \oint_{S^1} A$

What is the large N saddle-point of the holonomy ?

Free Energy: Holonomy

- Integrate out all fields treating $a = \oint_{S^1} A$ as background

$$F(a)/T = \underbrace{N^2(\cdots)}_{\text{pure CS}} + \underbrace{V_2 T^2 \mathrm{tr}(V(a))}_{\text{fundamental matter}\sim N V_2 T^2} \quad .$$

- Gauge invariance $\Rightarrow$ F only depends on eigenvalues of a, which are compact
- For F on flat space need $V_2 T^2 \gg N$
- $V(a) =$ attracts eigenvalues to zero

Contribution of matter $\gg$ pure CS $\overset{?}{\Longrightarrow}$ $a = 0$

Free Energy: Holonomy

- Consider pure CS on $T^2 \times \mathbb{R}$

$$S_{CS} \to \frac{k}{2\pi} \int dt\, \mathrm{tr}\left(a\dot{b}\right) , \qquad a\,,b = \oint_{\sigma_{a,b}} A$$

- Can diagonalize $a\,,b \to a^i\,,b^i\,, \quad i = 1,\ldots,N$

$$b^i \sim b^i + 2\pi\,, \quad \pi_b^i = \frac{k}{2\pi} a^i \Longrightarrow a^i = \frac{2\pi}{k}\mathbb{Z}$$

- Eigenvalues are free fermions [Brezin, Itzykson, Parisi, Zuber 1977] , [Douglas 1994]

$$a^i \sim a^i + 2\pi\,,\ a^i = \frac{2\pi}{k}\mathbb{Z} \Rightarrow \#\mathrm{g.s.} = \binom{|k|}{N}$$

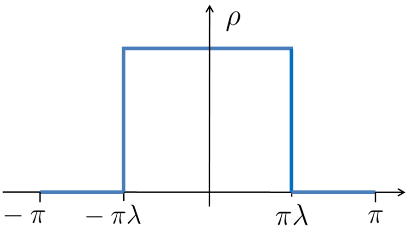
Free Energy: Holonomy

Summary: Pure CS on $T^2 \times \mathbb{R}$

- $a^i = \frac{2\pi}{k}\mathbb{Z}$
- a^i are fermions

CS + Matter

- $V(a)$ attracts a_i towards zero
- a_i spread uniformly around zero
- Width: $\frac{2\pi}{k} \times N = 2\pi\lambda$



[Aharony, Giombi, Gur-Ari, Maldacena, RY 2012]

Free Energy: Results

- Example: critical-boson

$$F/T = \frac{NV_2T^2}{2\pi^2 i\lambda}\left[\frac{\mu^2}{3}\mathrm{Li}_2(e^{-\mu-\pi i\lambda}) + \int_\mu^\infty dy\, y\, \mathrm{Li}_2(e^{-y-\pi i\lambda}) - \mathrm{c.c.}\right]$$

$$-\lambda\mu = \frac{1}{\pi i}\left[\mathrm{Li}_2(e^{-\mu-\pi i\lambda}) - \mathrm{c.c.}\right]$$

- Free energy of most general renormalizable theory with one scalar and fermion [Aharony, Giombi, Gur-Ari, Maldacena, RY 2012] ,[Jain, Minwalla, Yokoyama 2013]
- Phase structure on $S^2 \times S^1$ [Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama 2013]

Finite N Evidence ?

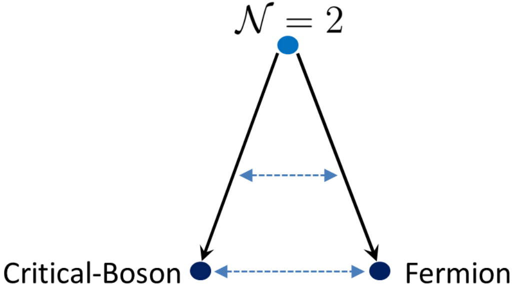
So far have only indirect evidence...

- Consistency with level-rank duality
- $\not{SUSY}$ deformations of $\mathcal{N} = 2$ with one chiral multiplet [Jain, Minwalla, Yokoyama 2013]
- Matching mass deformations of fermion and critical-boson CS-vector models [Aharony, Gur-Ari, RY 2011]

Finite N Evidence ?

$\cancel{SUSY}$ deformations of $\mathcal{N}=2$ with one chiral multiplet

- $\mathcal{N}=2$ duality is believed to hold also at finite N, k [Giveon,Kutasov 2008] , [Benini, Closset, Cremonesi 2011]
- Implies duality on space of deformations
- Free energy for most general renormalizable theory with one boson and femrion[Jain, Minwalla, Yokoyama 2013]

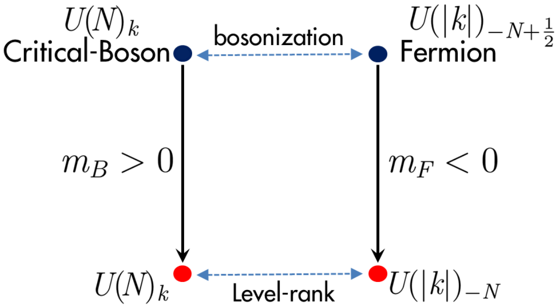


Finite N Evidence ?

Mass deformations

- Flow to pure CS and compare with level-rank duality
- Parity anomaly $\Rightarrow k_F^{IR} = k_F^{UV} + \frac{1}{2}\mathrm{sign}(m_F)$

How can this fermionic property map to the bosonic side ?



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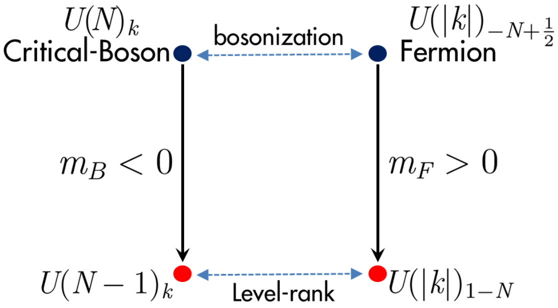
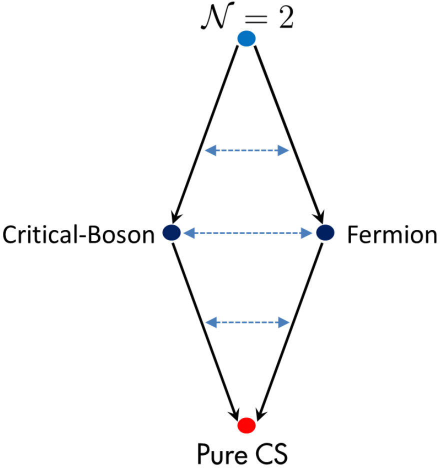


Finite N Evidence: Summary



Open Questions

Holography

- Put theory on different manifolds [Banerjee, Hellerman, Maltz, Shenker 2012]
- Higher-point correlators:
  - Have other ~~parity~~ parameters in the bulk: $\theta_0, \theta_2, \theta_4, \ldots$
  - θ_2 naively contributes to 5-point functions and higher
  - From bulk p.o.v. θ_i are arbitrary, but there is no boundary analog

Open Questions

Bosonization

- Applications, e.g. $U(1)$-model

$$U(1)_{k_0}\ \text{critical}-\text{boson} \overset{??}{\longleftrightarrow} U(k_0)_{-\frac{1}{2}}\ \text{fermion}$$

- Fermi-surface $\leftrightarrow$ Bose-Einstein
- Anyons, non-local operators
- Finite N

Conclusions

- Chern-Simons vector models $\xleftrightarrow{AdS/CFT}$ Vasiliev + ~~Parity~~
- Big simplifications for $N \to \infty$
  - High-spin symmetry
  - Dimensions of operators
  - 3-point functions
  - Thermal free energy
- $U(N)_k + \phi^i + \phi^4 \longleftrightarrow U(|k| - N)_{-N+\frac{1}{2}} + \psi^i$

Thank You !!!