Wilsonian and Large N approaches to Non-Fermi Liquids

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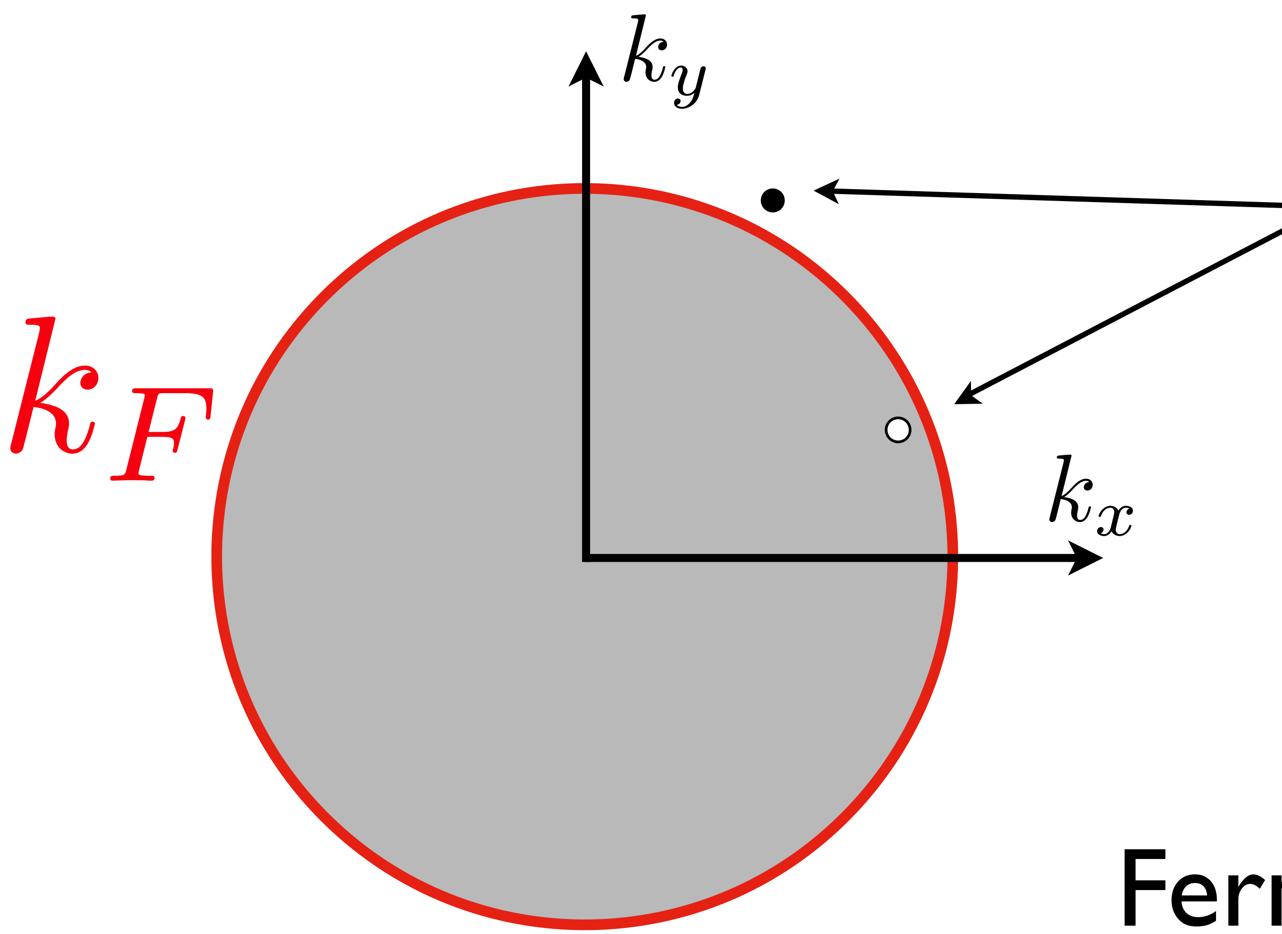
1307.0004, 1312.3321 and work in preparation

Introduction to Fermi Liquids

Fermions at finite density
have a Fermi surface



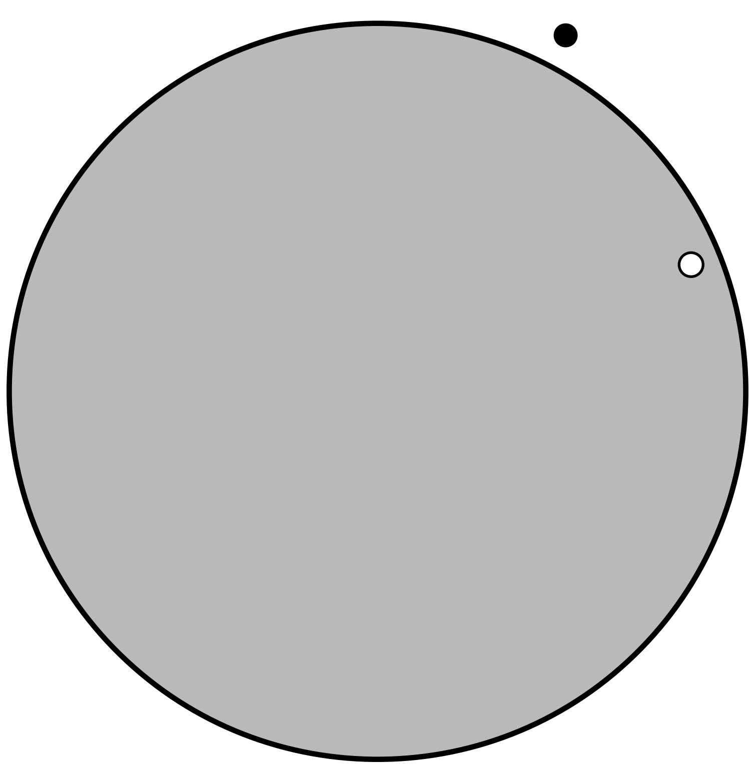
Fermi energy

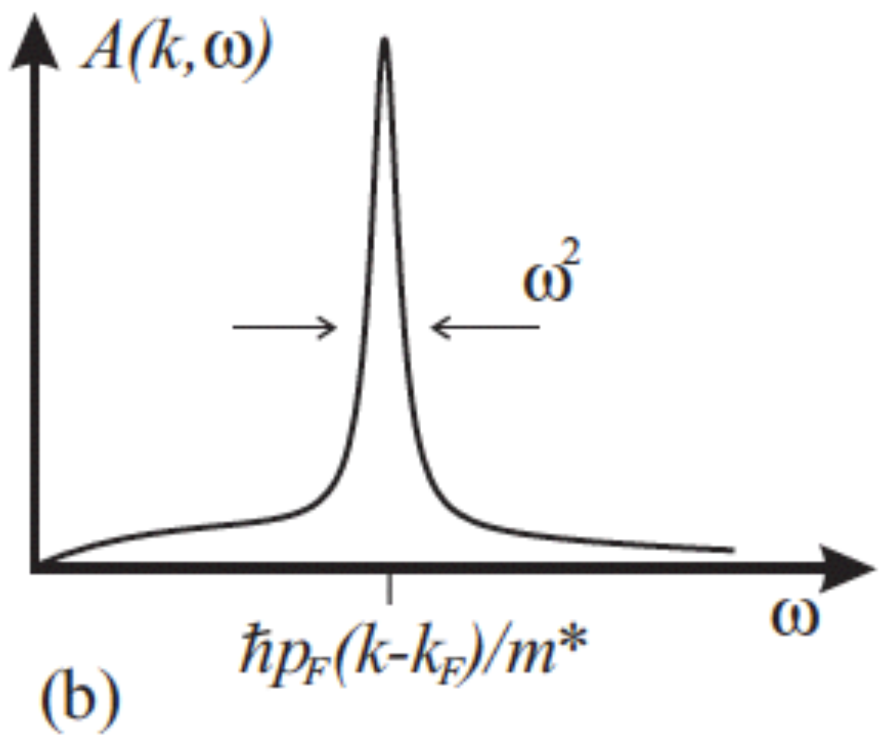
$$E = \frac{k^2}{2m} - \epsilon_F$$

Fermi momentum: $\frac{k_F^2}{2m} = \epsilon_F$

Landau Fermi Liquids

In simple metals, excitations are weakly coupled quasi-particles





$$\frac{1}{\tau} = \mathrm{Im}(\Sigma) \sim \frac{\omega^2}{k_F}$$

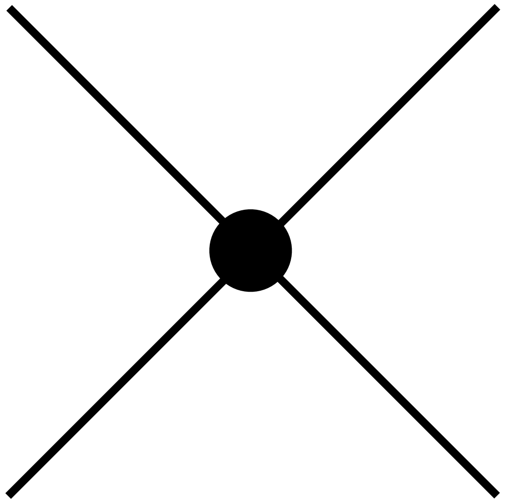
Landau Fermi Liquids

Why are emergent quasiparticles well-described by weak coupling?

Modern EFT description:
(almost) all interactions are irrelevant

Shankar
Polchinski

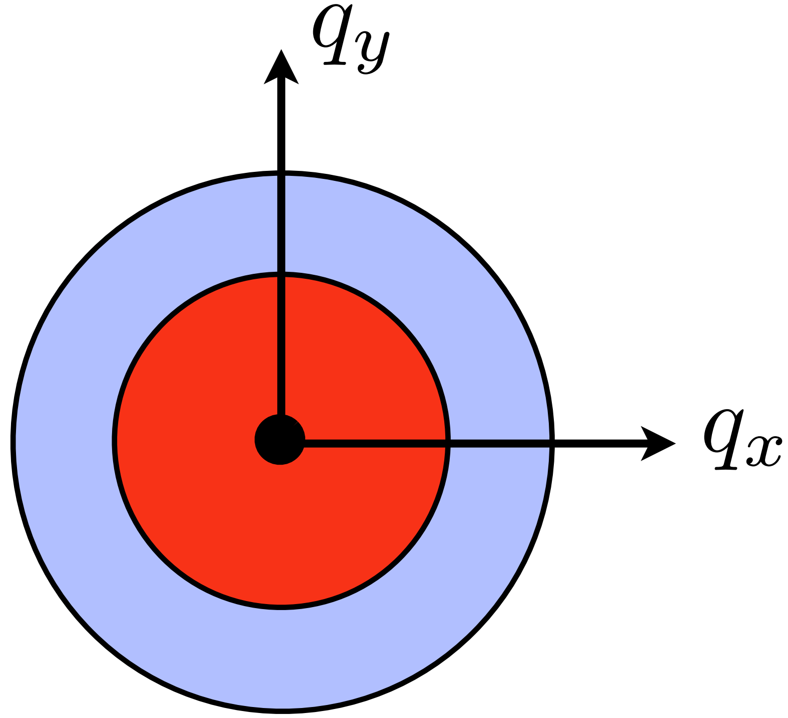
$$\frac{\psi^\dagger \psi \psi^\dagger \psi}{\Lambda}$$



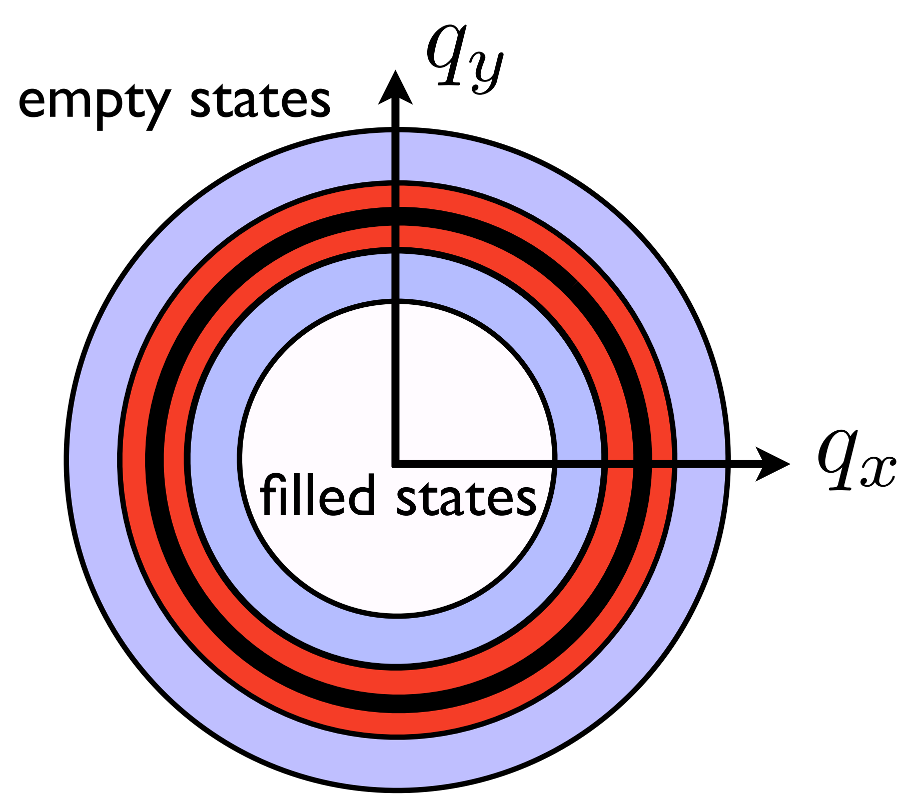
Landau Fermi Liquids

Scaling:

Standard:



Fermi Surface:



Fix angle and scale toward nearest point on Fermi surface:

$$\vec{q} = \hat{\theta}(k_F + \ell)$$

$$\omega \to e^{\lambda}\omega \quad \ell \to e^{\lambda}\ell$$

Landau Fermi Liquids

$$S_2 = \int dS^{d-1} \left[\int d\omega d\ell \psi^\dagger (\omega - v_F \ell) \psi \right]$$

$$\ell \equiv |k| - k_F$$

$$\omega \to e^\lambda \omega$$

$$\ell \to e^\lambda \ell$$

So we see that the fermions should scale as

$$\psi \to e^{-\frac{3}{2}\lambda} \psi$$

Landau Fermi Liquids

First interaction is four-fermion interaction

$$S_4 = \int d^{d-1}S_1 d\omega_1 d\ell_1 \ldots d^{d-1}S_4 d\omega_4 d\ell_4 \, \delta(\omega_1 + \omega_2 + \omega_e + \omega_4)$$

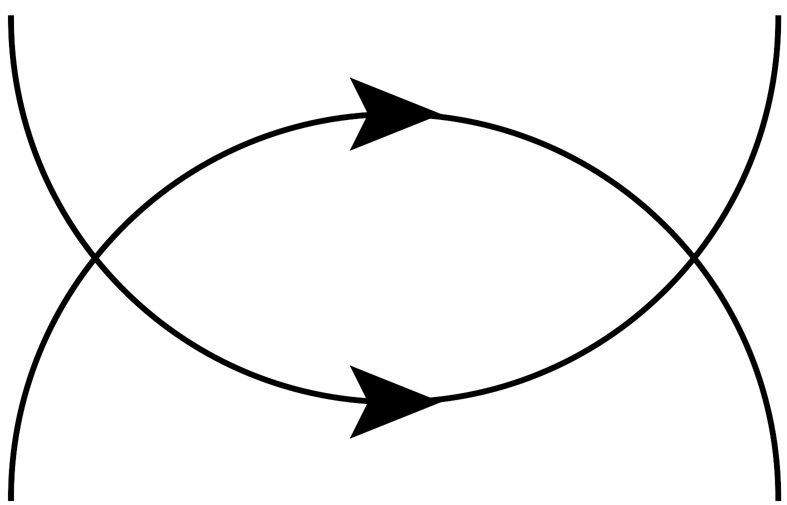
$$V(\theta_i)\psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 \;\; \delta^d(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

It naively scales like e^{λ} and is irrelevant

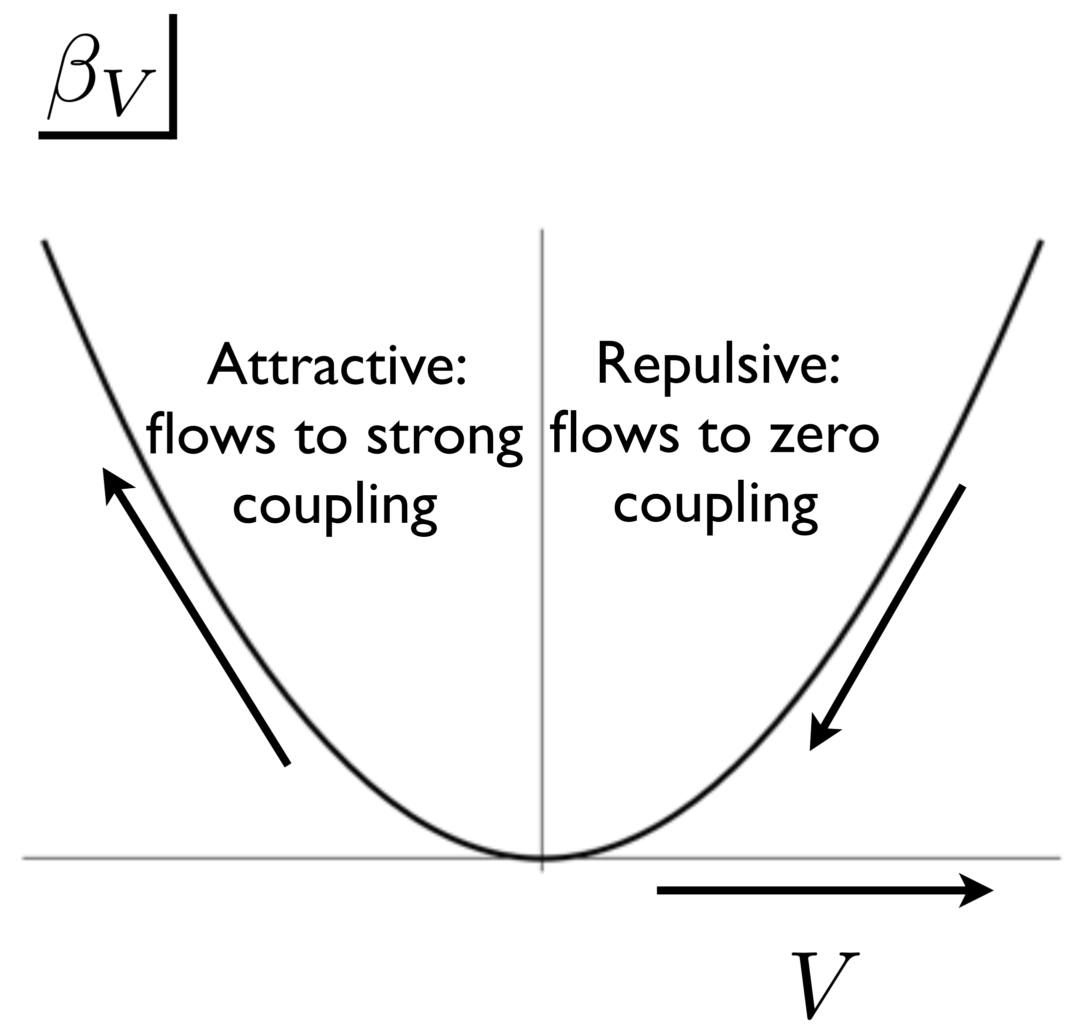
But for certain kinematic configurations, the delta function scales like $e^{-\lambda}$ and the interaction becomes *marginal*

Landau Fermi Liquids

BCS instability:
At one-loop, the interaction between antipodal points runs and becomes marginally relevant/irrelevant



$$\frac{dV}{d\log\mu} = V^2$$

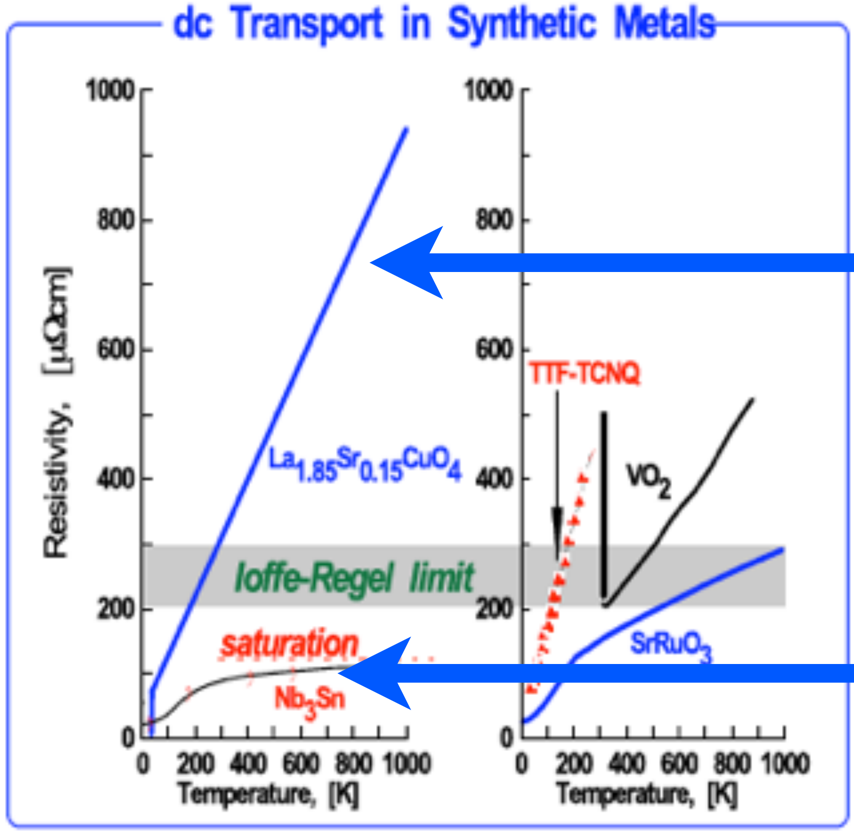


Fermi vs. Non-Fermi

Ioffe-Regel Resistivity Limit on Fermi Liquids

Quasi-particle transport:

resistivity is inversely proportional to mean free path



"Non-Fermi Liquid" metal exceeds bound

"Fermi Liquid" metal doesn't exceed bound

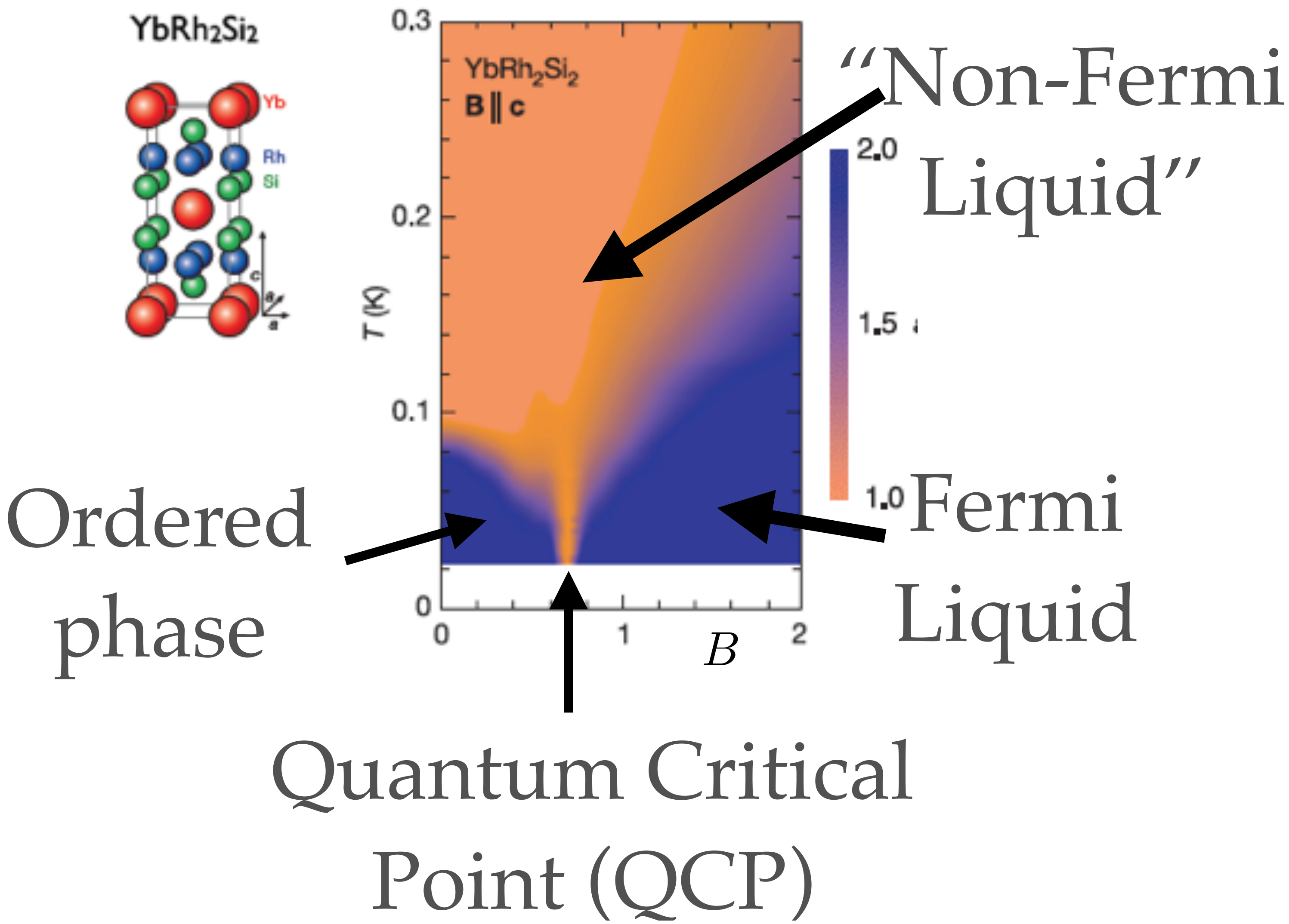
Courtesy of D. Basov (UCSD)

If resistivity is too large, then mean free path is shorter than lattice spacing, and quasi-particle description doesn't make sense

“Non-Fermi” Liquids

Many materials have fascinating new properties that make them fall outside of the Fermi Liquid description

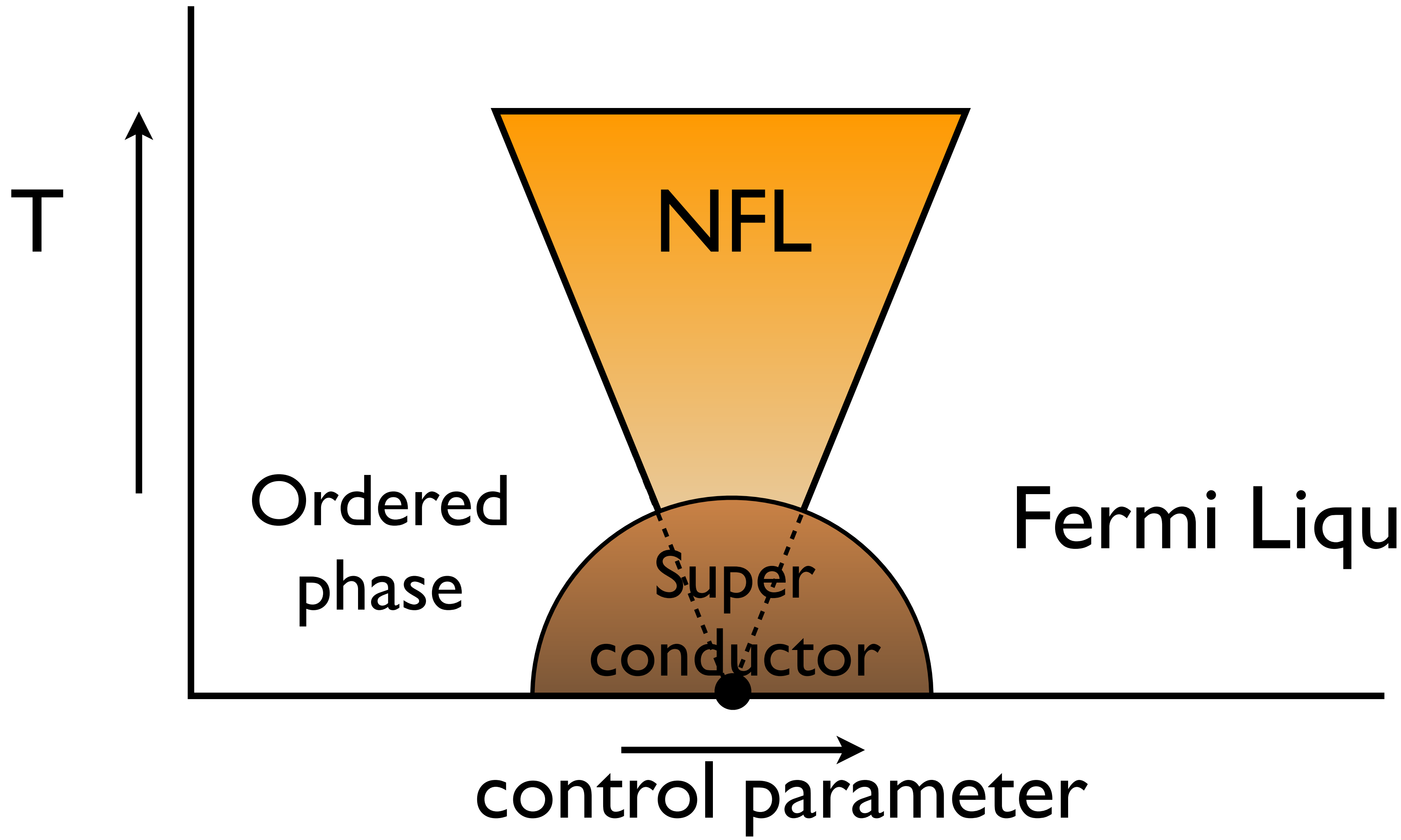
- Resistivity Linear in T
- Violate Ioffe-Regel bound
- Superconductivity often occurs at high temperature
- Often Located near Quantum Critical Points



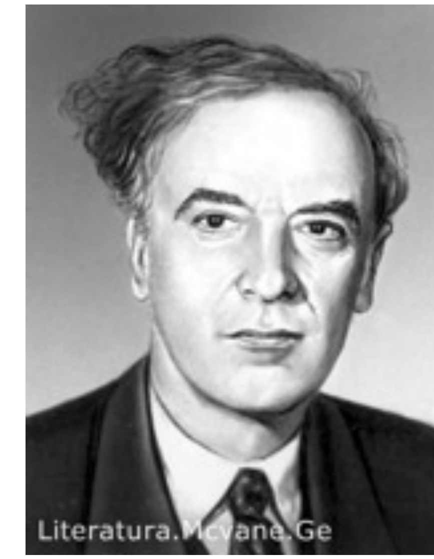
Quantum Critical Points

A Recurring theme: NFLs arise near Quantum Phase Transitions

(Phase transition at zero temp)



Landau-Ginzburg-Wilson

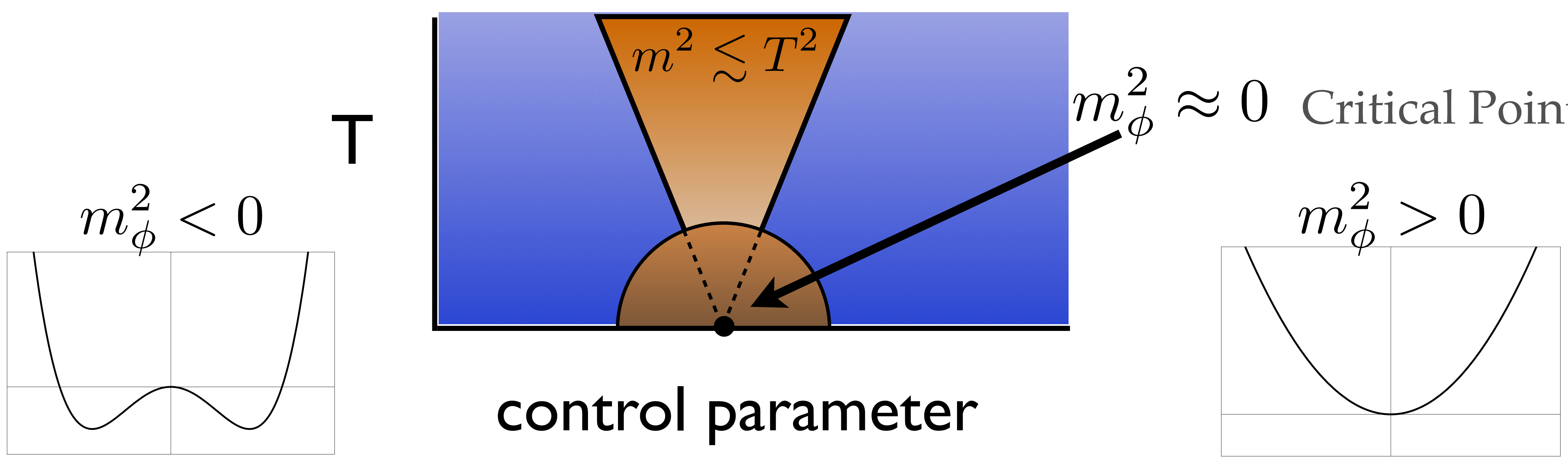


Write down Lagrangian for the order parameter of the phase transition

$$\mathcal{L} \sim \dot{\phi}^2 - (\nabla\phi)^2 - m^2\phi^2 - \lambda\phi^4 + \ldots$$

(ϕ should transform according to the symmetries it breaks)

Near critical point: ϕ is a nearly massless fluctuating boson



EFTs of Non-Fermi Liquids

As a high energy physicist, I will take some lessons from the study of QCD:

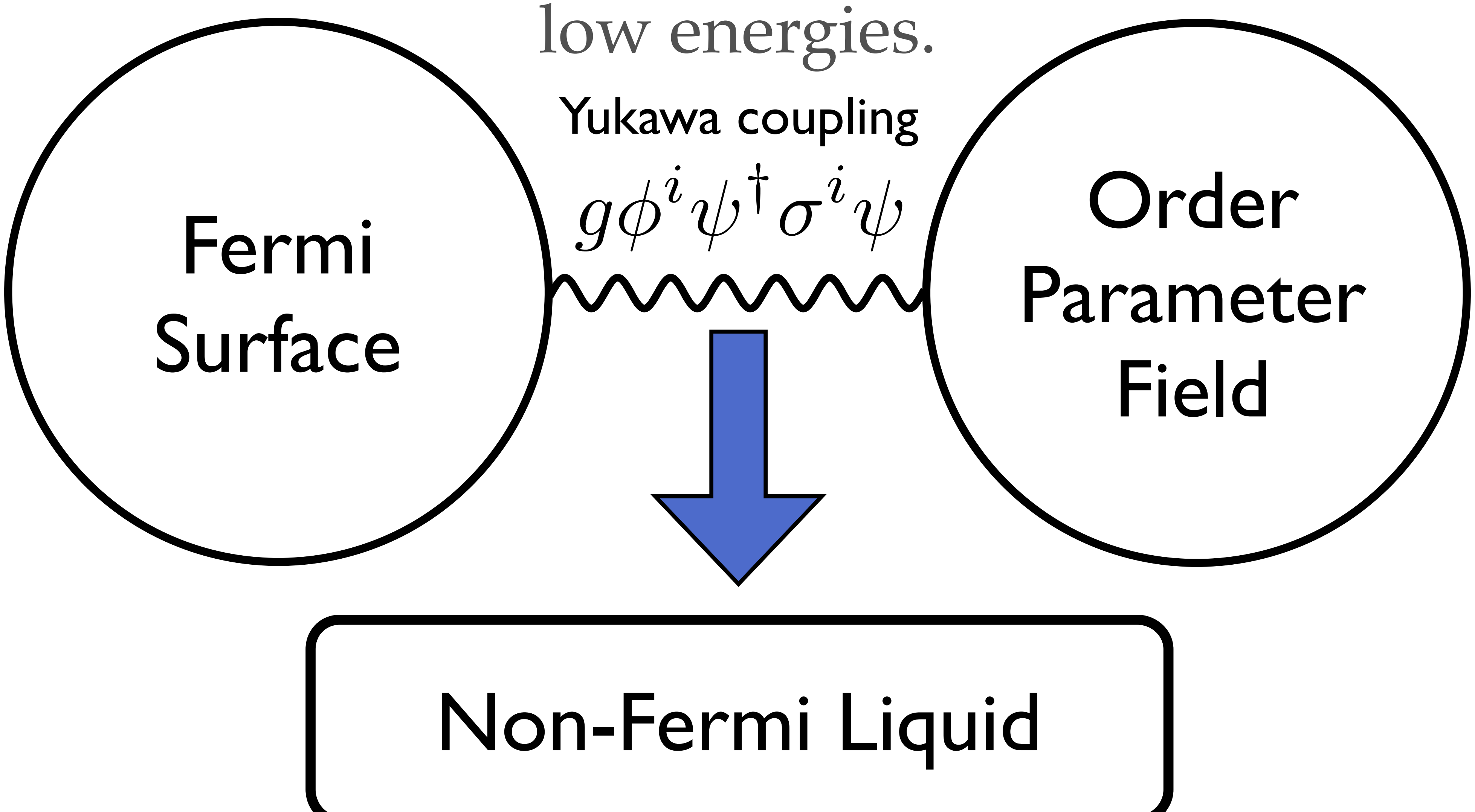
1) It was hard to see *a priori* what QFTs (if any!) could explain deep inelastic scattering

The classification and study of local QFTs was wildly successful

2) Confinement especially was hard to tackle directly, and simplifying special cases (2d, large N, SUSY) played a crucial role in our qualitative understanding

EFTs and Non-Fermi Liquids

Now we have a great EFT problem:
Choose our light degrees of freedom and add interactions.
Integrate out high energy modes and see what happens at low energies.



EFTs of Non-Fermi Liquids

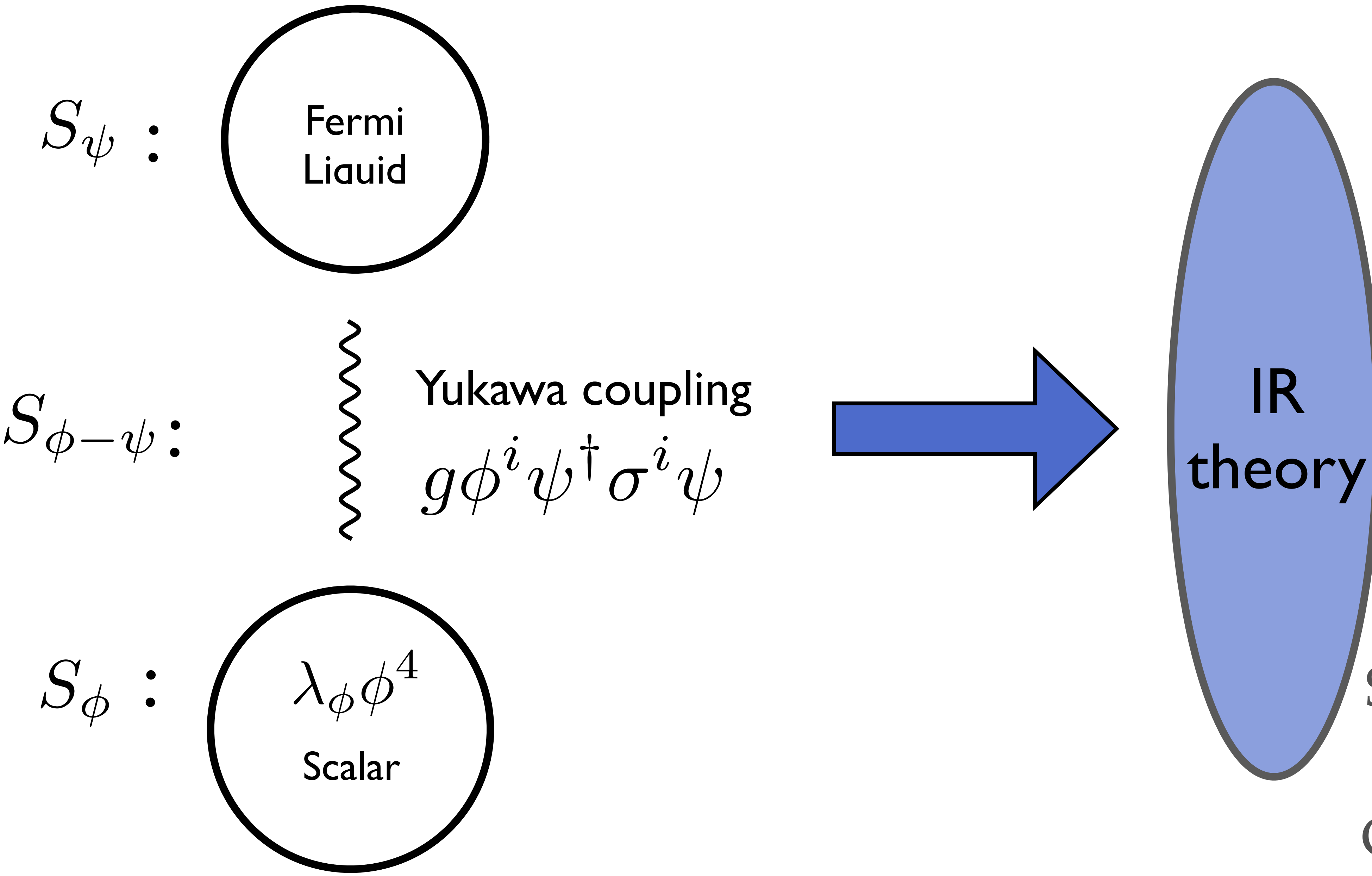
Wilsonian approach: start with *local* action in UV and integrate out high energy modes

We will not add *by hand* any terms like

$$\phi \frac{1}{\nabla^2} \phi$$

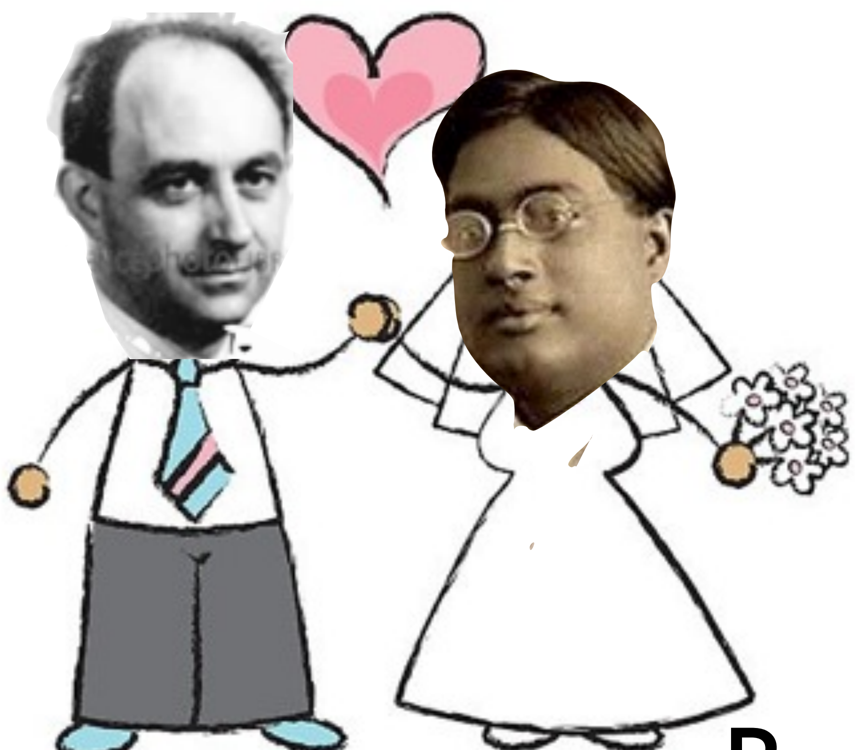
Quantum Critical Points

$$S = S_\psi + S_\phi + S_{\phi-\psi}$$

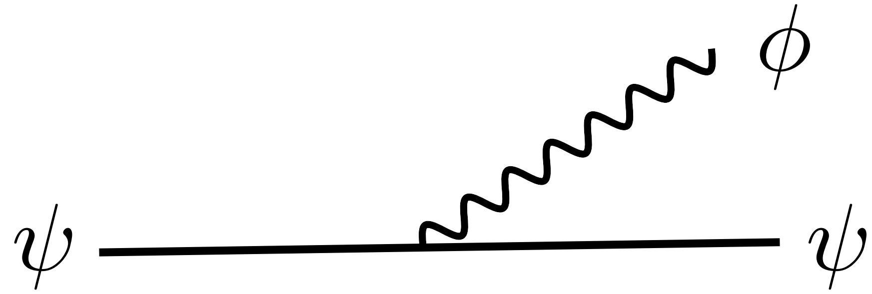


Hertz, Millis, Sachdev, Metlitski, Belitz, Todadri Chubukov, Lee, Xu

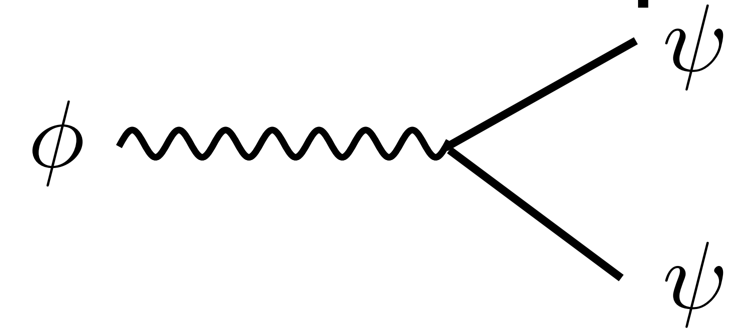
Marriage of Landau's Two Great Frameworks



Fermions can decay:
Non-Fermi Liquid



Bosons can decay to
particle/hole pairs:
"Landau damping"



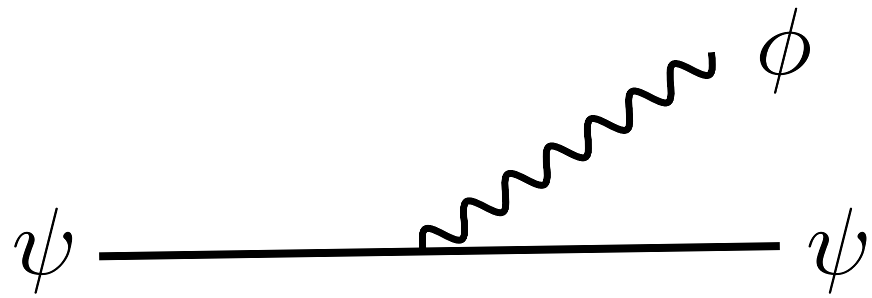
Titanic Struggle

Fermions renormalize bosons and vice versa

Who wins?

Fermions can decay:

Non-Fermi Liquid



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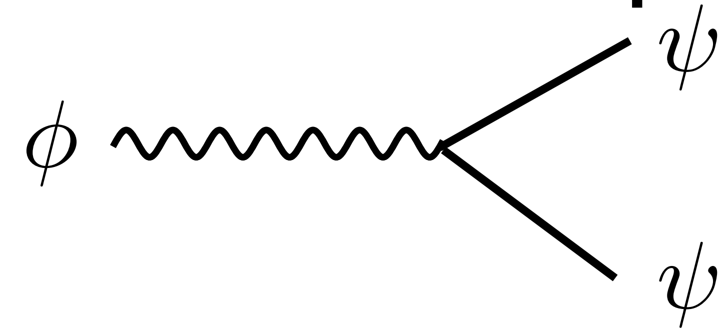

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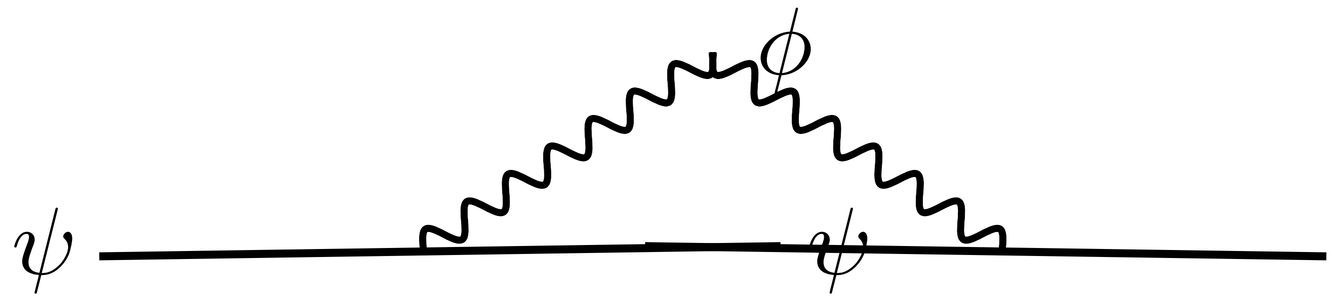

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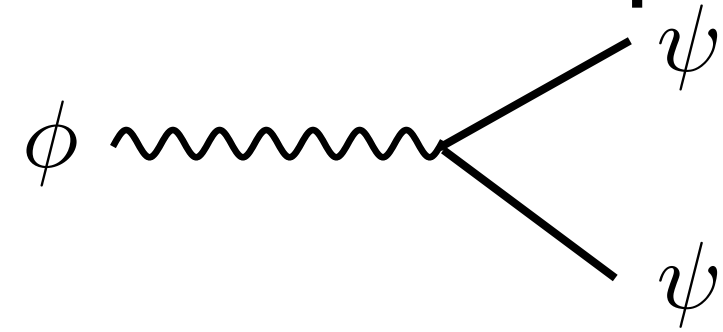

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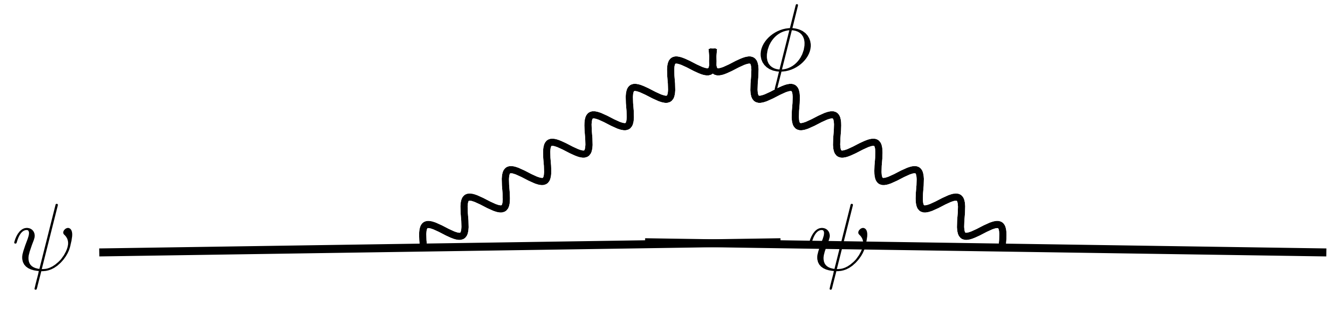

Bosons can decay to particle/hole pairs:
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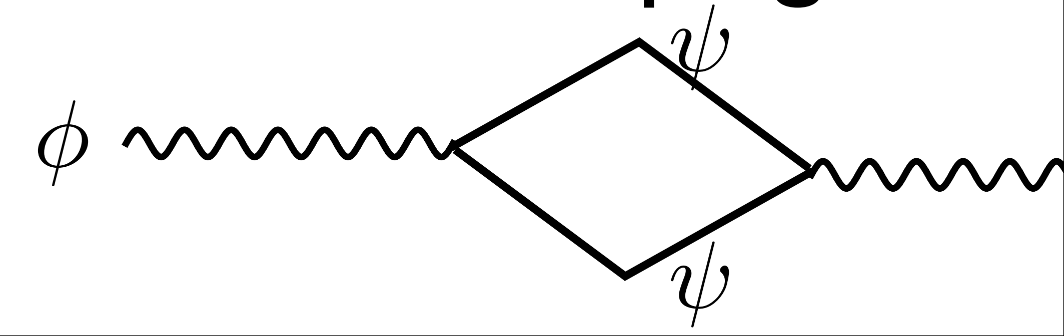
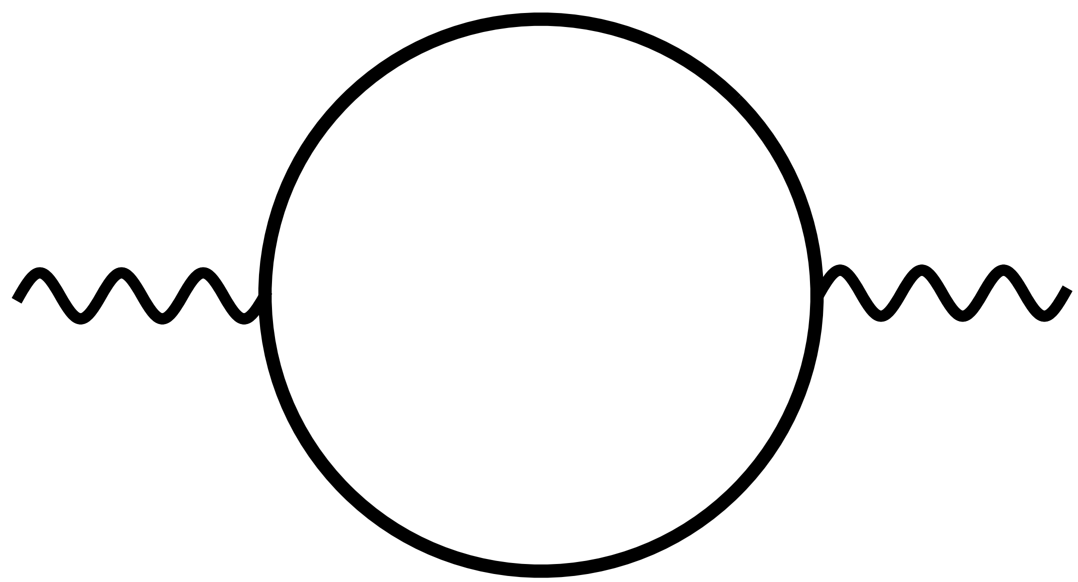
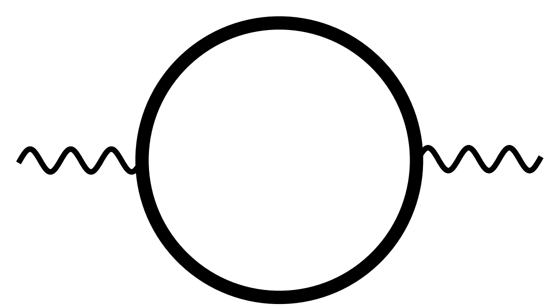


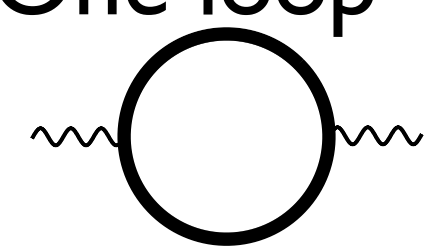
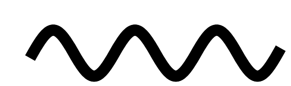
Landau Damping



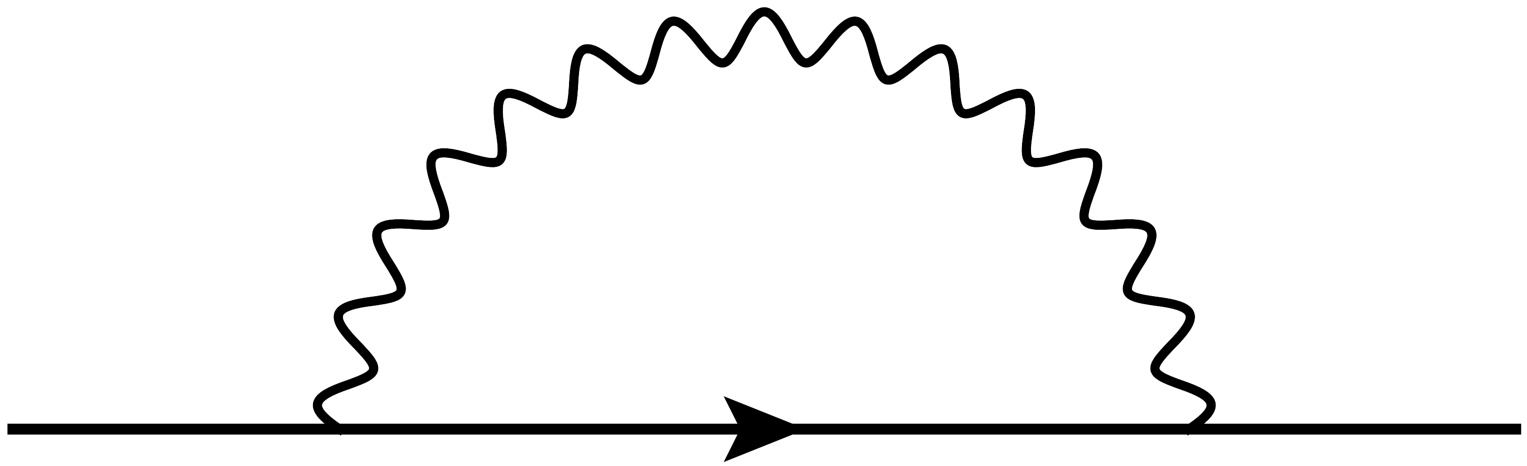
One-loop boson self-energy
has non-analytic term

 $\sim M^2 F(q_0/q)$

One loop vs tree-level

Strong coupling at IR scale:  > 

Anomalous Dimension



Wavefunction
renormalization

This is a more familiar effect from a particle physicist's point of view:

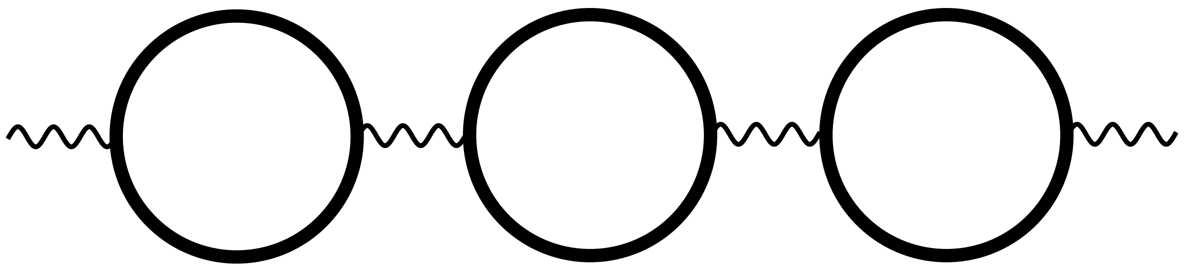
The log divergent piece changes the scaling dimension of the fermion field

Landau Damping

Mainstream philosophy
Hertz (1976):

"Fermions Win"

"Keep 1PI diagrams but drop all others, resum to get new kinetic term"



"Then feed this back into corrections to fermion"

Landau Damping

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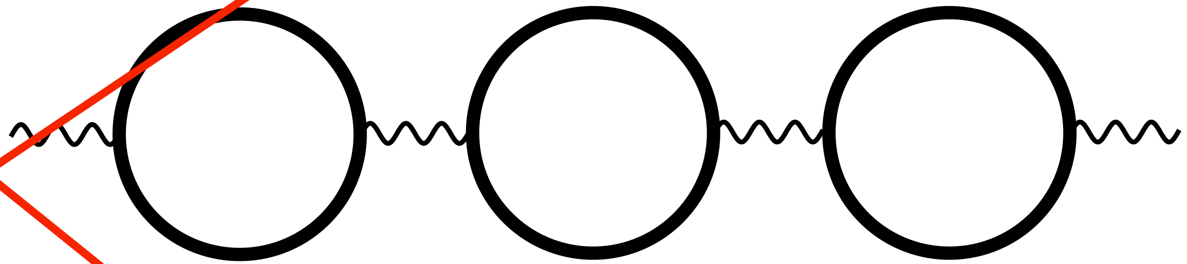

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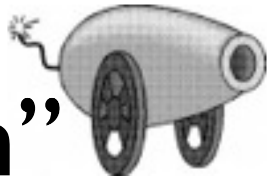
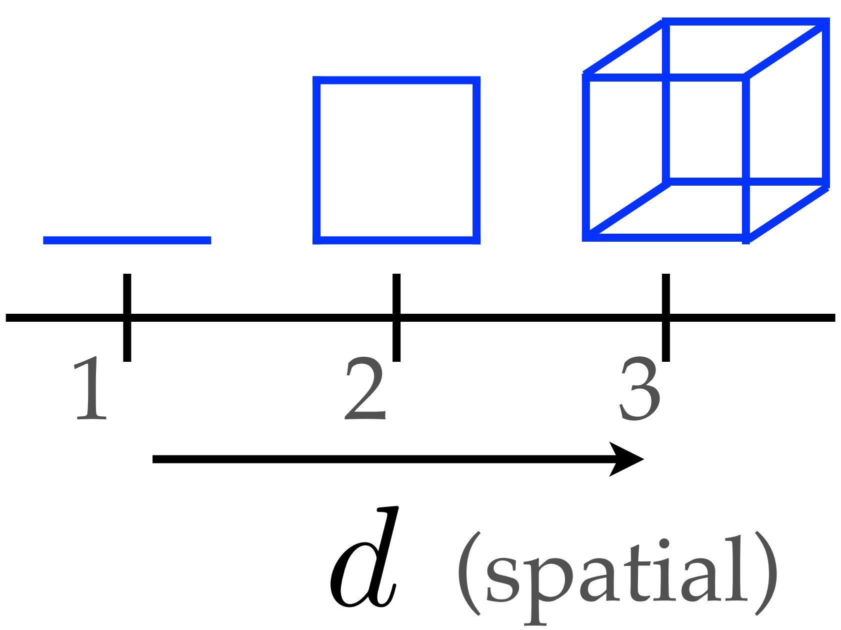


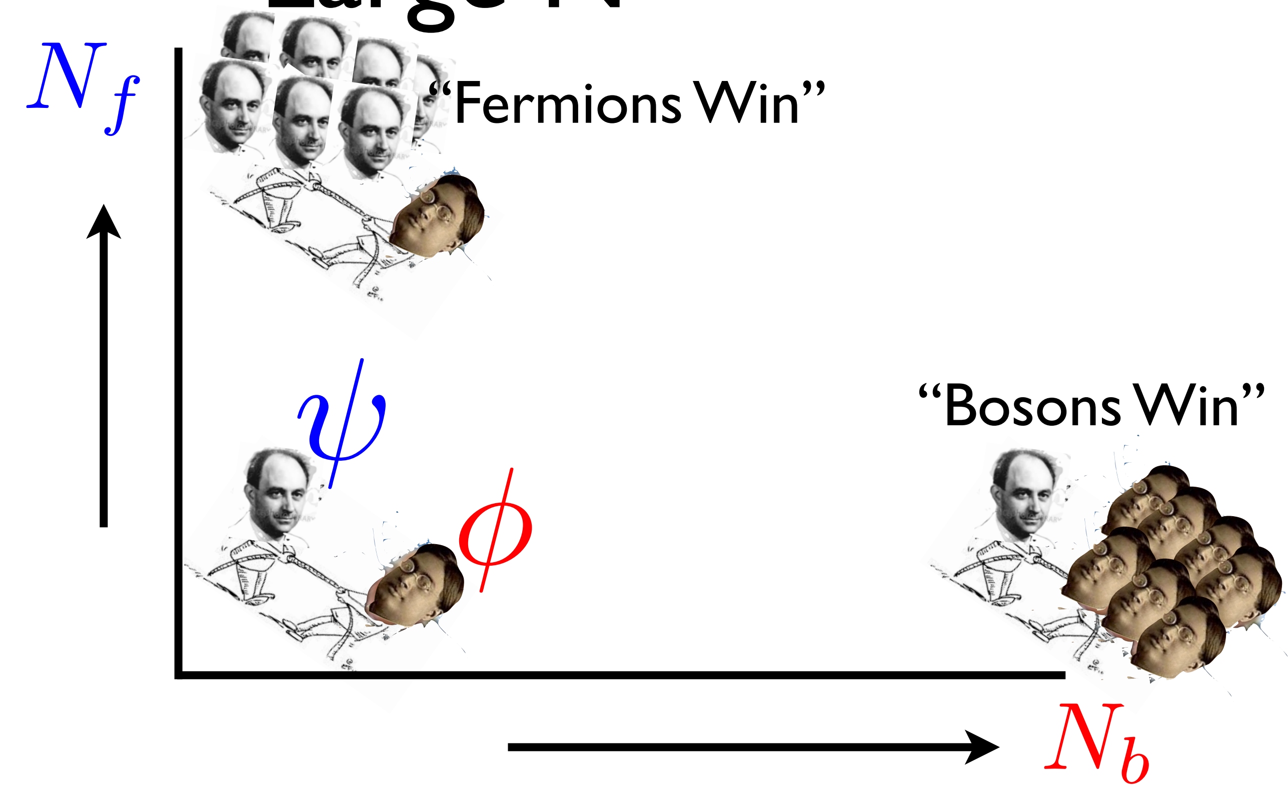
Looking for Controlled Limits

Speed of fermions vs bosons

Dimension: small $\epsilon = 3 - d$



Large N



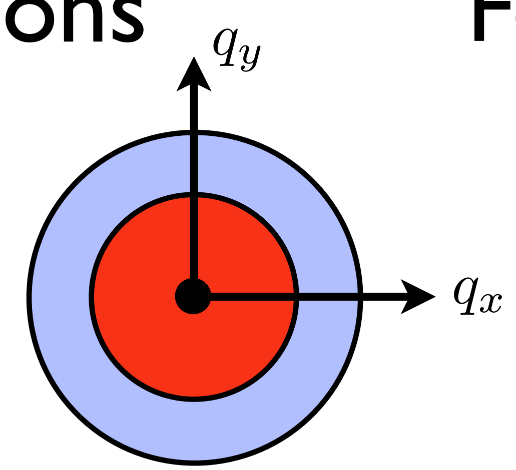
Epsilon Expansion

Work near upper critical dimension to find a scale-invariant fixed point at weak coupling

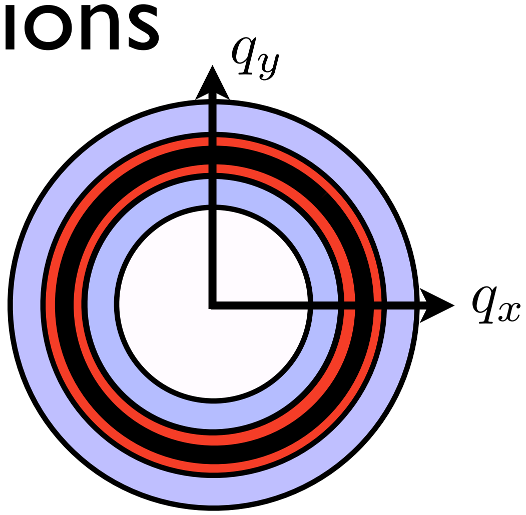
All three couplings are classically marginal in $d = 3$

$$g\phi\psi^\dagger\psi \qquad \lambda_\phi \phi^4$$
$$\lambda_\psi (\psi^\dagger \psi)^2$$

Bosons



Fermions



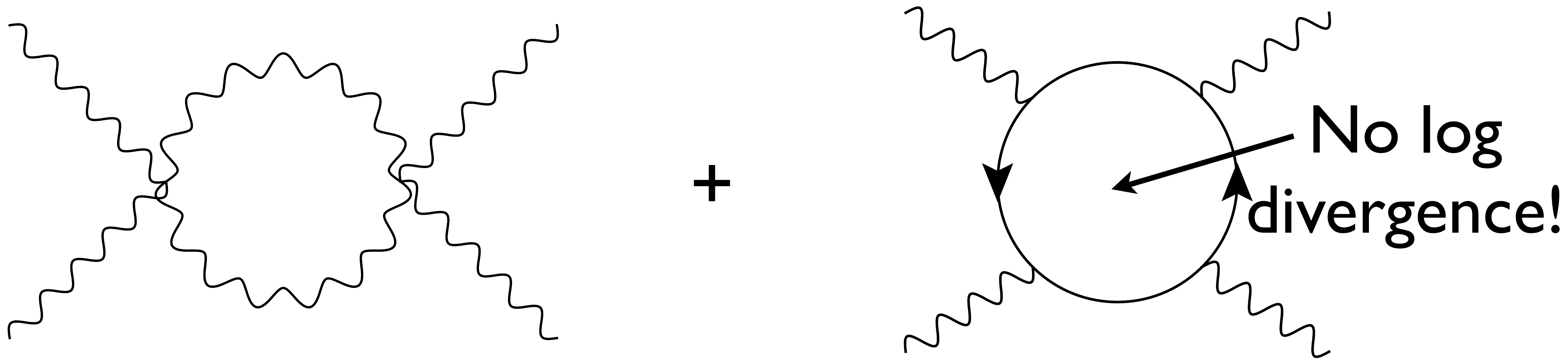
$$g \to e^{\frac{3-d}{2}\lambda} g$$

$$\lambda_\phi \to e^{(3-d)\lambda} \lambda_\phi$$

$$\lambda_\psi \to \lambda_\psi$$

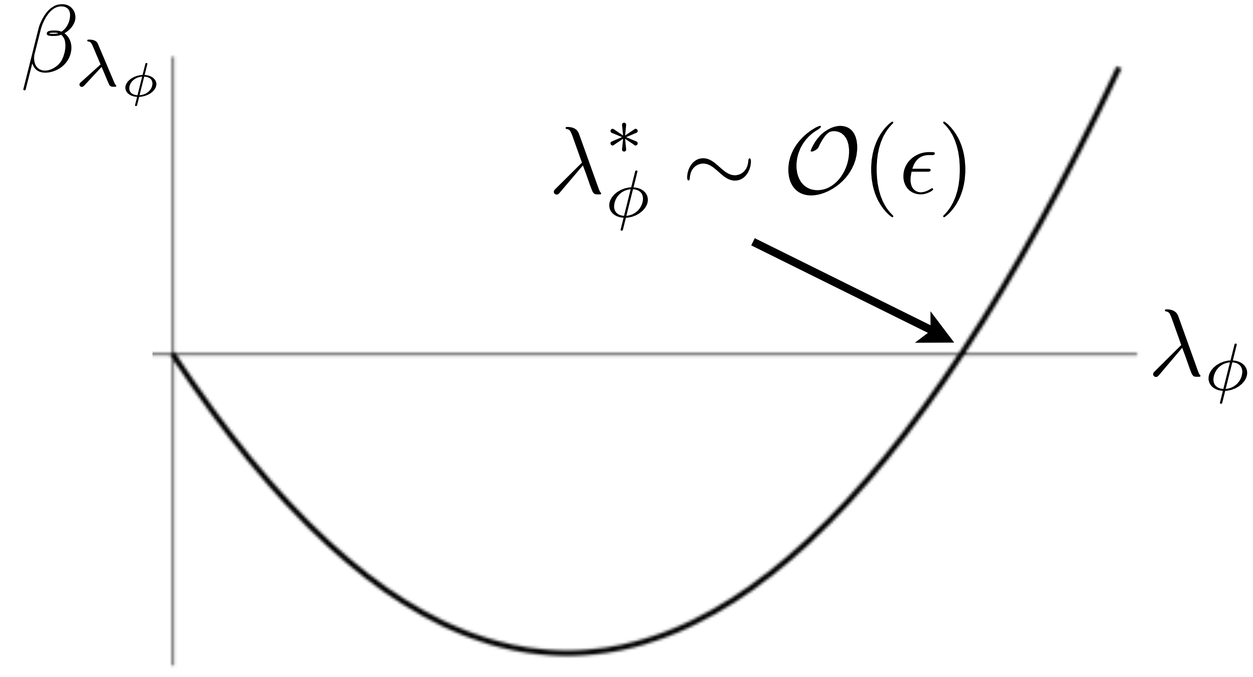
Epsilon Expansion

$$d = 3 - \epsilon$$



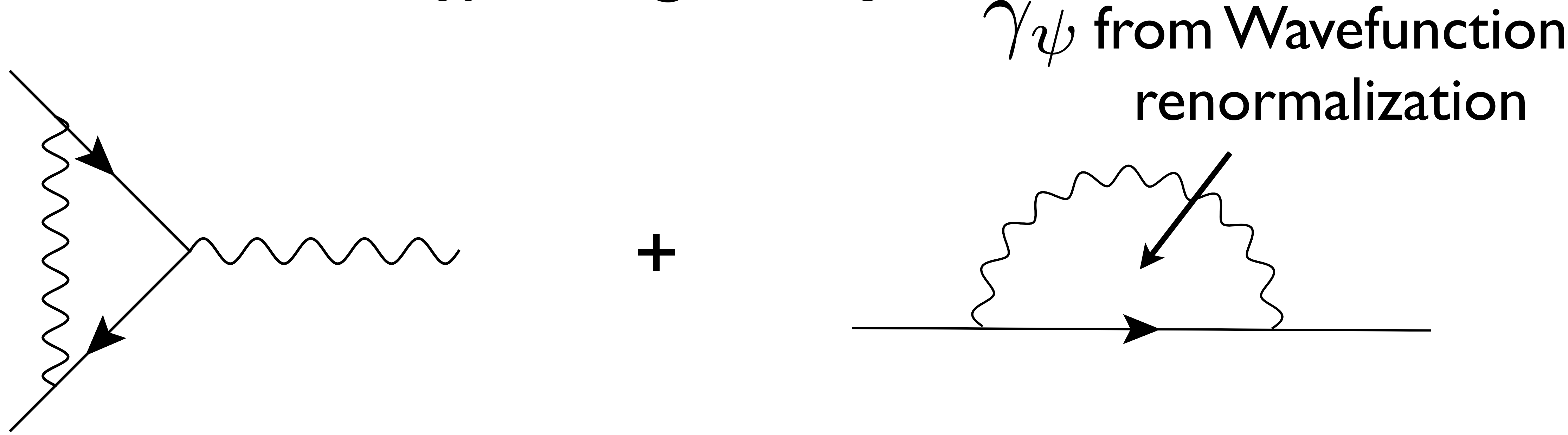
Scalar quartic running is the same as in Wilson Fisher

$$\frac{d}{d \log \mu} \lambda_\phi = -\epsilon \lambda_\phi + a_{\lambda_\phi} \lambda_\phi^2$$



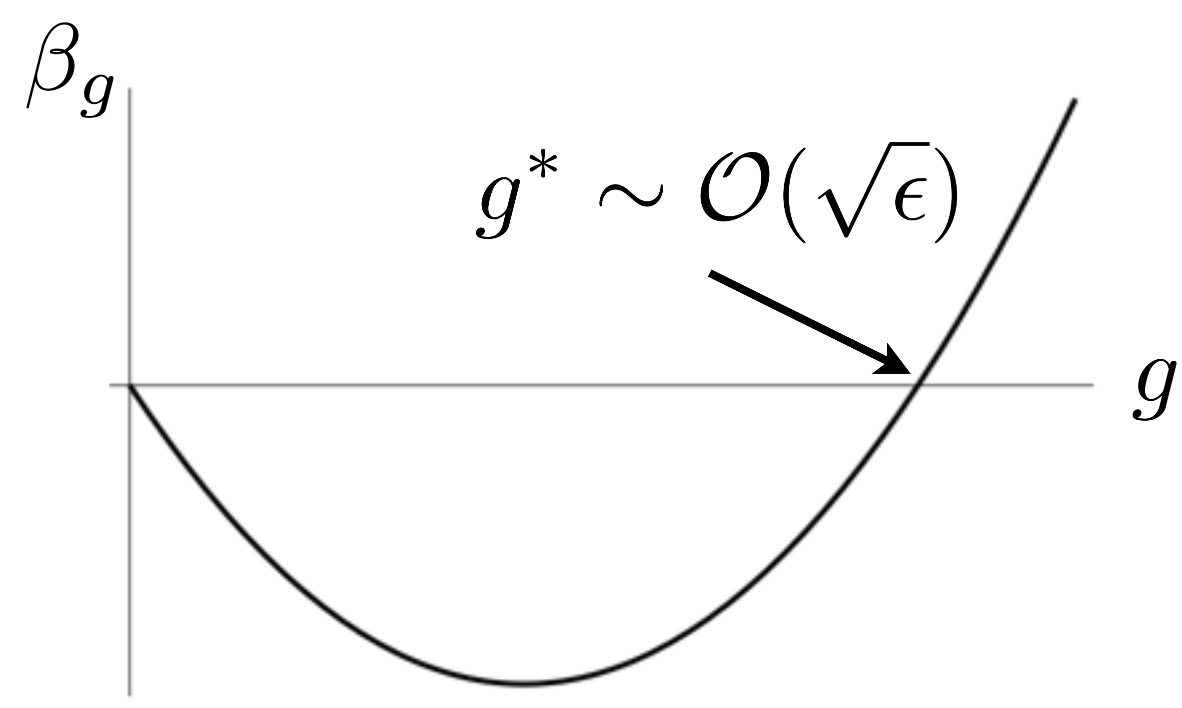
Epsilon Expansion

$$d = 3 - \epsilon$$



Yukawa runs to IR fixed point

$$\frac{d}{d\log\mu} g = -g\left(\frac{\epsilon}{2} - a_g g^2\right)$$
$$+\mathcal{O}(g^2\epsilon)$$

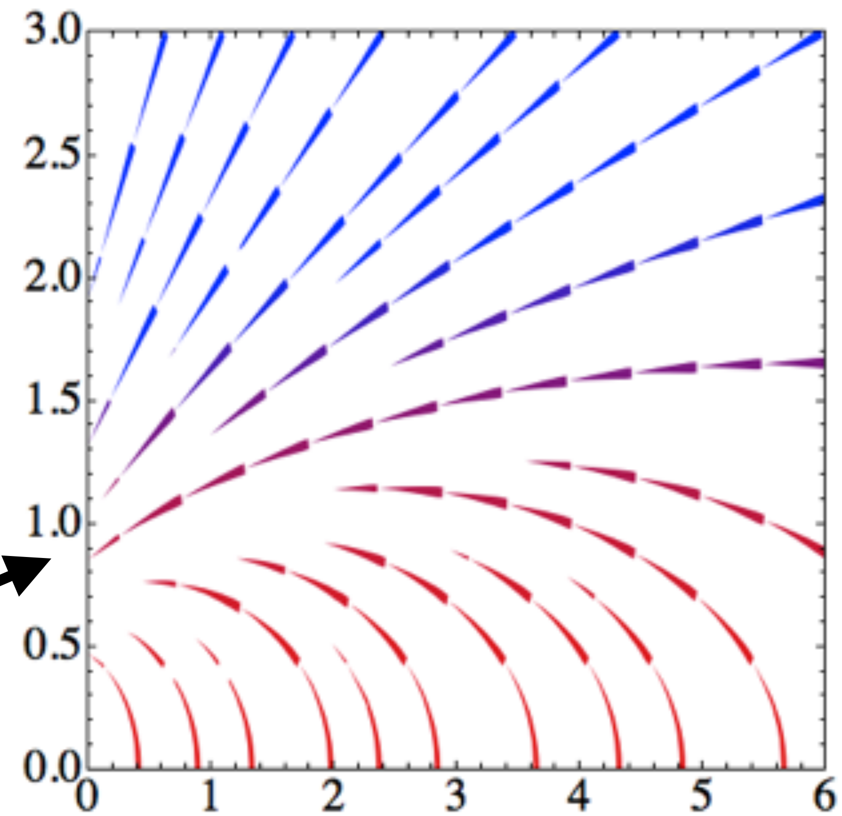


Results from RG Flow

ALF, Kachru, Kaplan, Raghu

Non-trivial scale-invariant fixed point in d<3

$d = 2.98$



Coupling g

fixed point

v

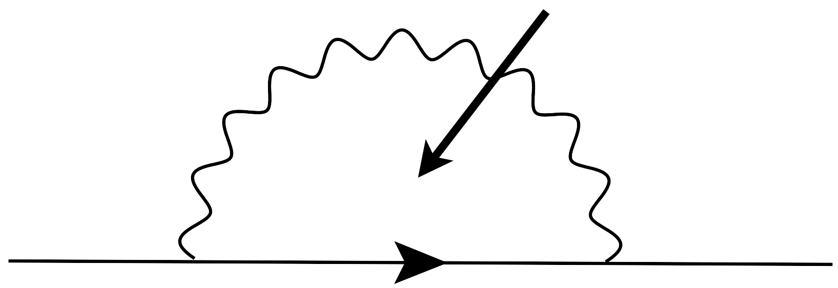
fermion velocity $v = \frac{k_F}{m}$

Epsilon Expansion

$$d = 3 - \epsilon$$

γ_ψ from Wavefunction renormalization

$$2\gamma_\psi \sim \frac{\epsilon}{2}$$



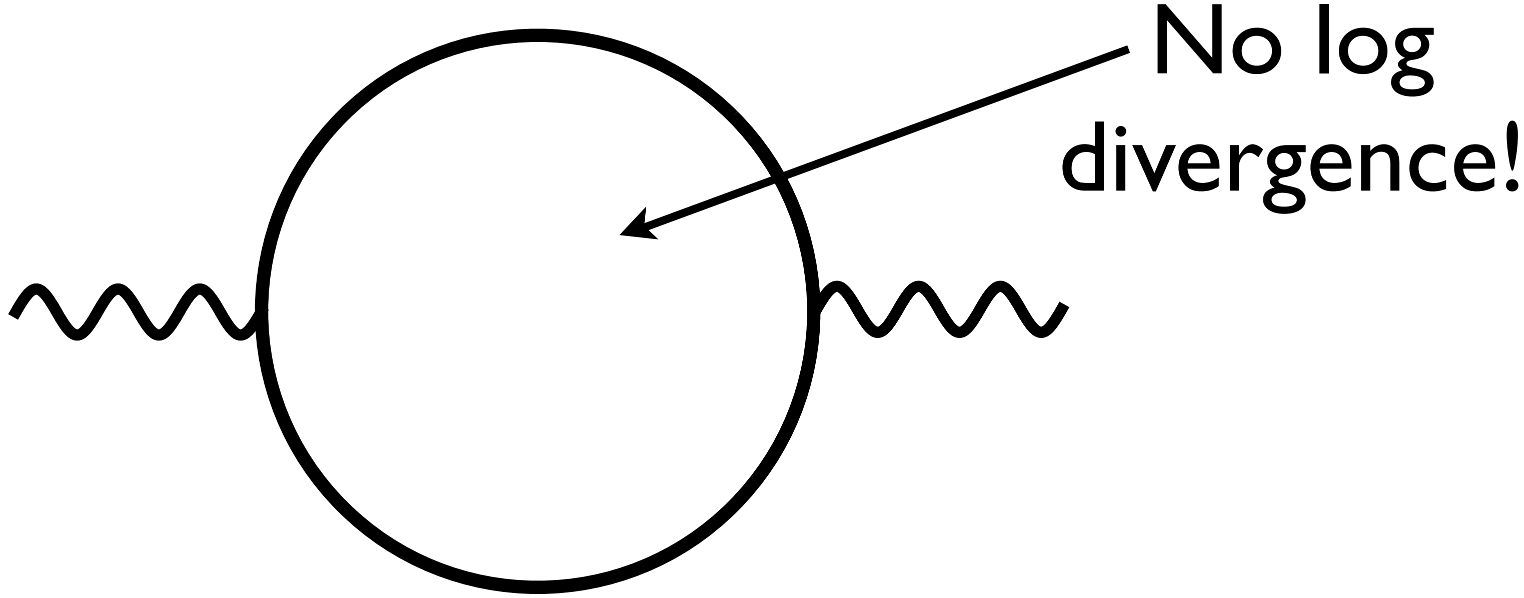
Scale-invariant fixed point with non-vanishing anomalous dimension

Fermion Green's function at fixed point must take the form

$$G(\omega, \ell) = \frac{1}{\omega^{1-2\gamma_\psi}} f(\frac{\omega}{\ell})$$

Epsilon Expansion

Landau damping has no effect on RG

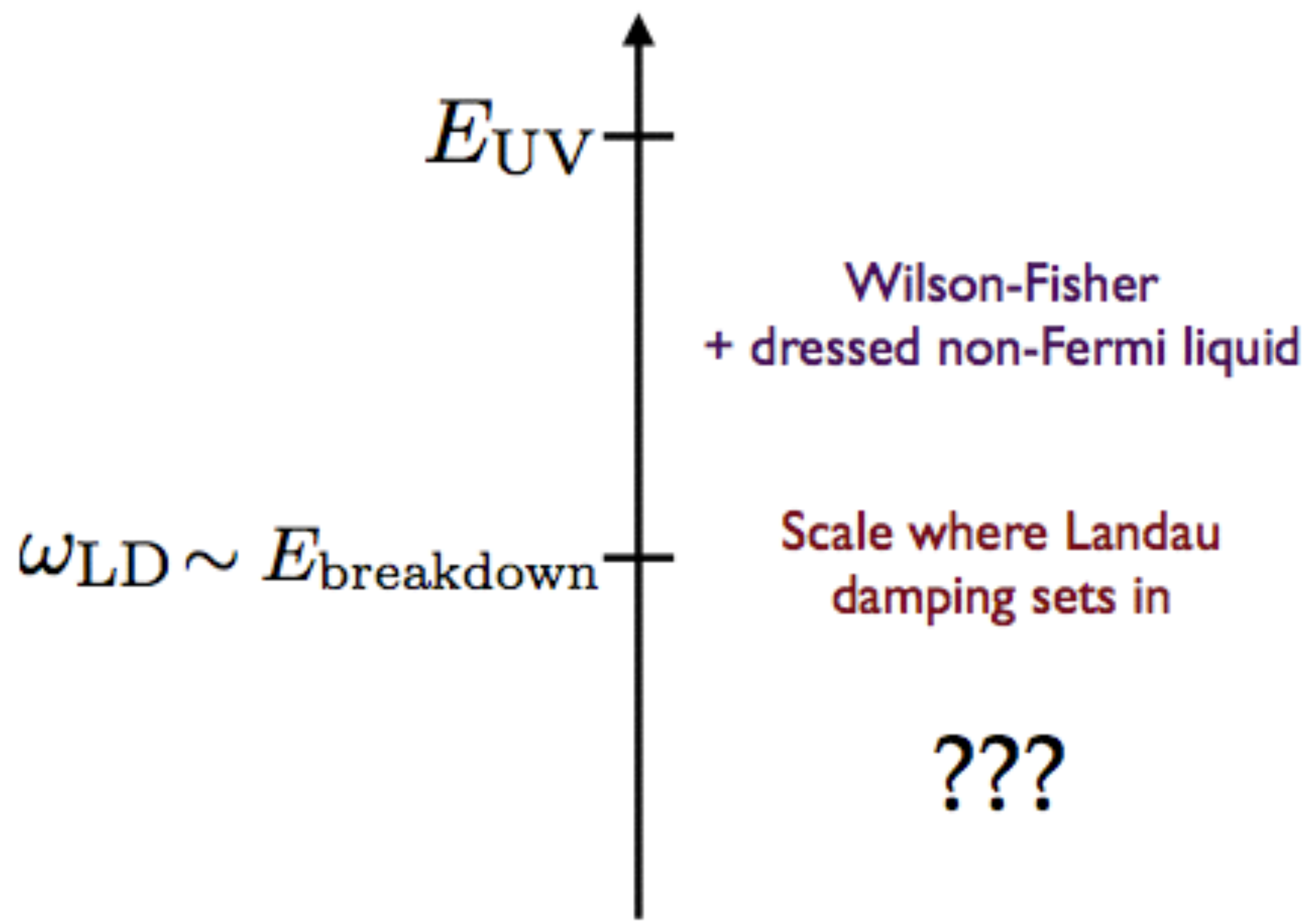


Furthermore, Landau damping pushed to very low scale

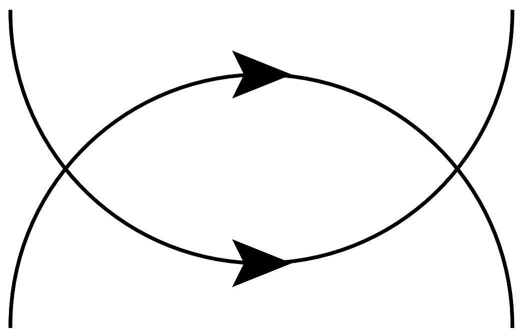
$$\Pi(q_0, q) \sim g^2 M^2 F(q_0/q)$$

Epsilon Expansion

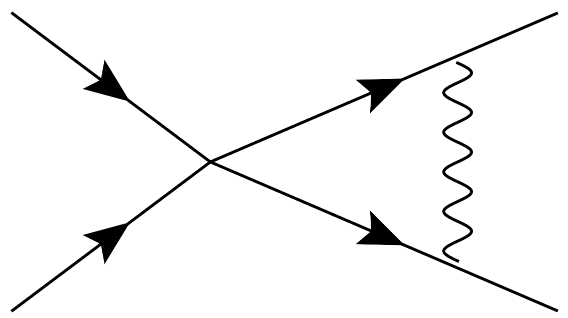
Landau damping pushed to very low scale



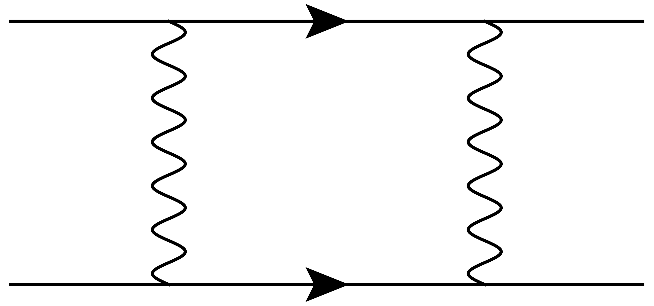
BCS Instability



λ_ψ^2



$\lambda_\psi g^2 = \lambda_\psi \mathcal{O}(\epsilon)$



$\mathcal{O}(g^4) = \mathcal{O}(\epsilon^2)$

BCS instability is a higher order effect and happens only at exponentially lower scales (if at all)

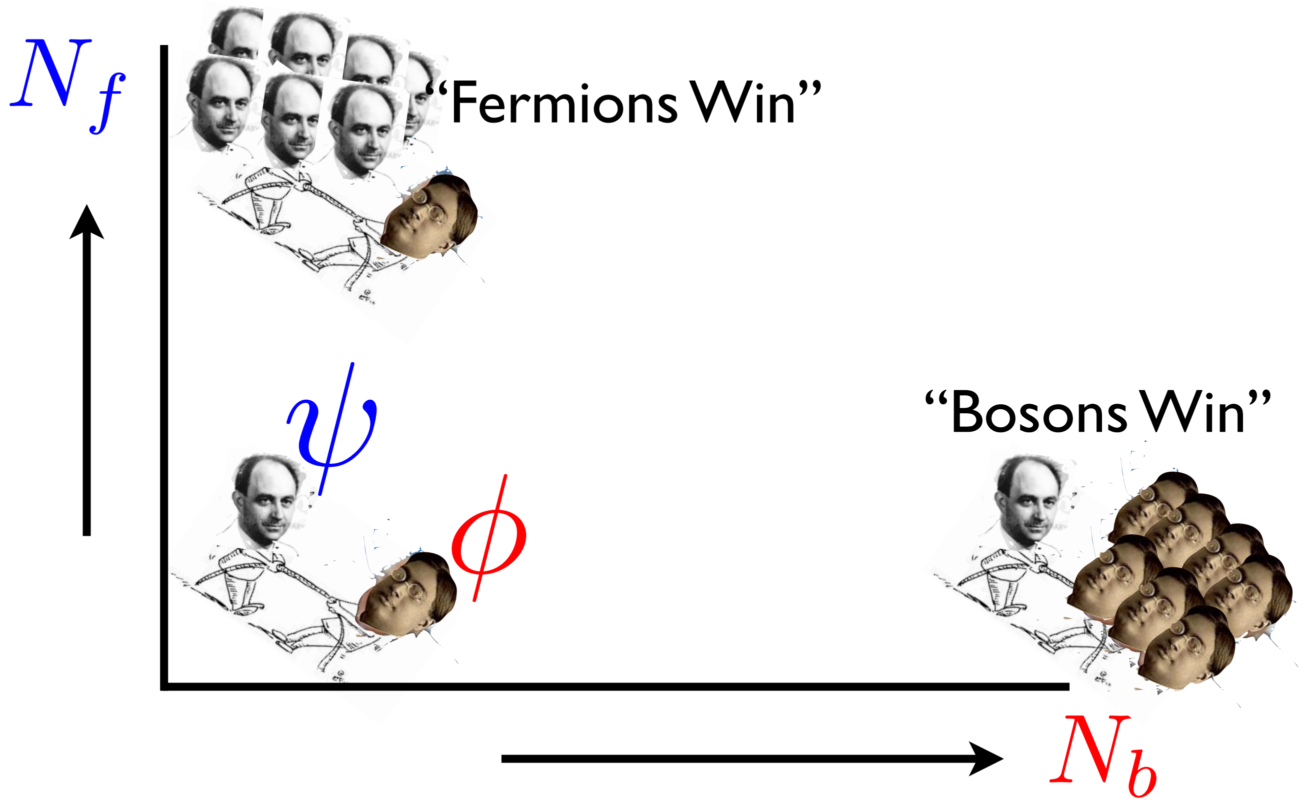
Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\Box$	$\bar{\Box}$

Now we will look at simplifications in large N limits

We will find qualitatively different dependence at large N_b as compared with large N_f

This indicates a rich phase diagram of such theories

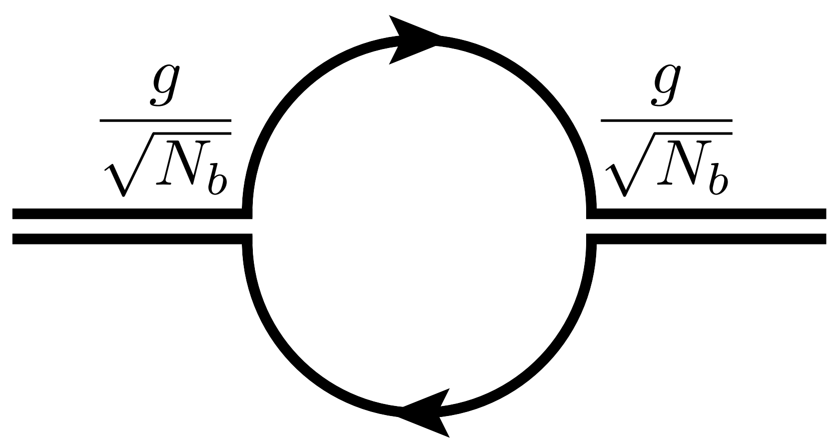


Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

"Bosons Win"



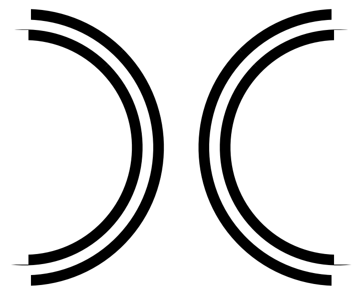
Landau Damping is a non-planar diagram and has no effect at infinite N_b

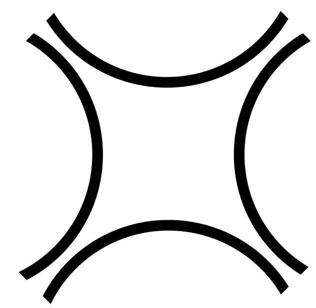
Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

$$\frac{\lambda_\phi^{(2)}}{8N_b^2}(\mathrm{tr}[\phi^2])^2 \qquad \frac{\lambda_\phi^{(1)}}{8N_b}\mathrm{tr}[\phi^4]$$



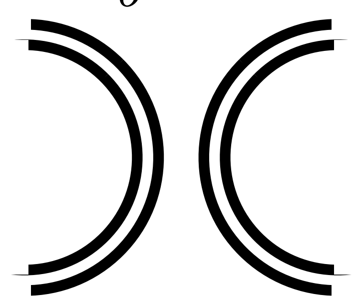


Large N Dials

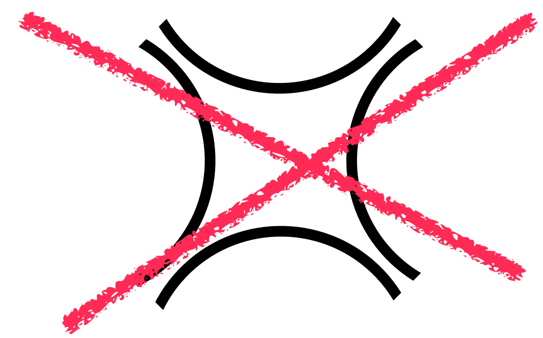
	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

$\frac{\lambda_\phi^{(2)}}{8N_b^2}(\mathrm{tr}[\phi^2])^2$



$\frac{\lambda_\phi^{(1)}}{8N_b}\mathrm{tr}[\phi^4]$



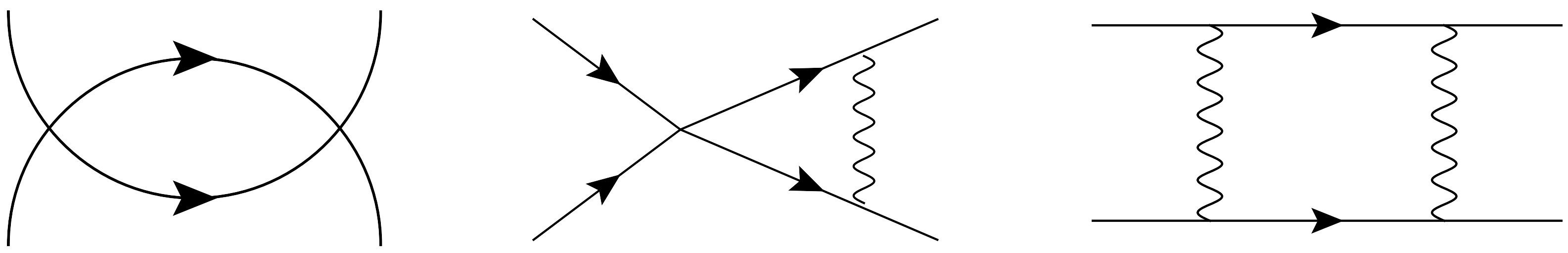
One can set $\lambda_\phi^{(1)} = 0$ naturally (in the 't Hooft sense)

Then the ϕ sector is isomorphic to the SO(N_b^2) Wilson-Fisher fixed point

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed



are all $\mathcal{O}(\frac{1}{N_b})$

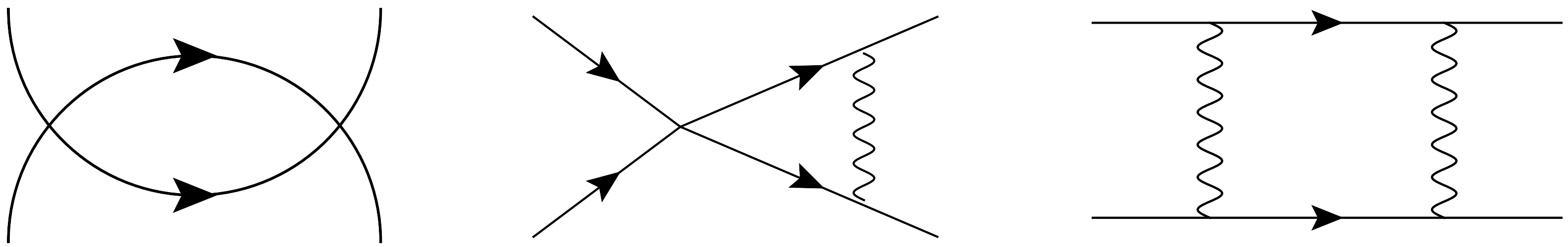
The only contribution to four-fermi running is wavefunction renormalization

$$\frac{d\lambda_\psi}{d\log\mu} = 4\gamma_\psi\lambda_\psi$$

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

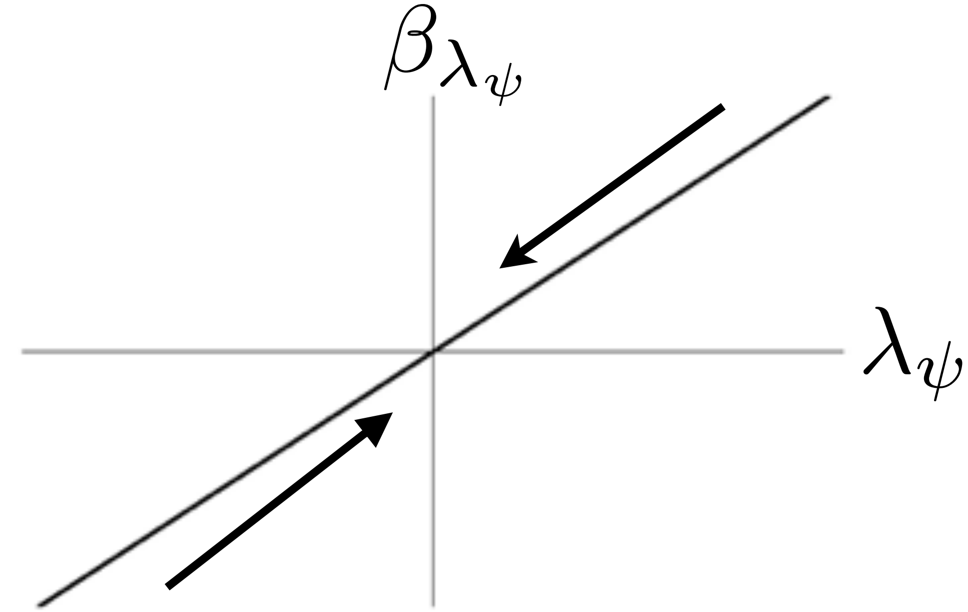


are all $\mathcal{O}(\frac{1}{N_b})$

The only contribution to four-fermi running is wavefunction renormalization

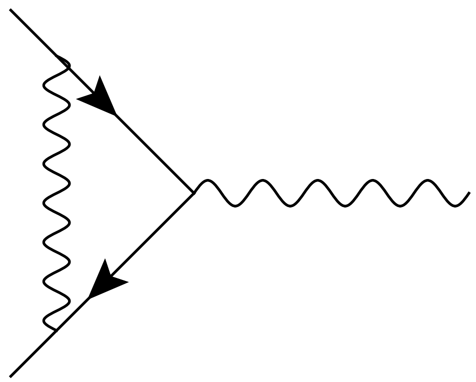
$$\frac{d\lambda_\psi}{d\log\mu} = 4\gamma_\psi \lambda_\psi$$

Stable against superconductivity



Large N Dials

At $N_b \to \infty$ N_f fixed



is $\mathcal{O}(\frac{1}{\sqrt{N_b}})$

So all running of g is through wavefunction renormalization:

$$\frac{d}{d\log\mu}g = -g\left(\frac{\epsilon}{2} - 2\gamma_\psi(g)\right)$$

Scale-invariant fixed point even for $\epsilon \sim \mathcal{O}(1)$

$$2\gamma_\psi = \frac{\epsilon}{2}$$

The fermion Green's function therefore takes the form

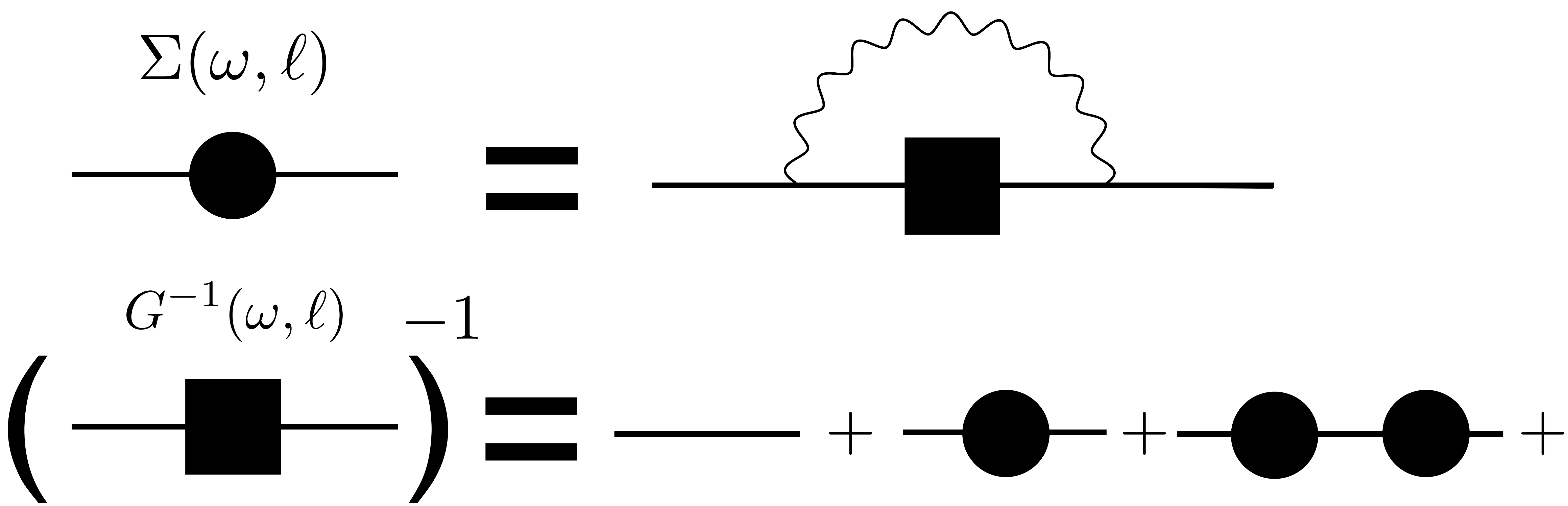
$$G(\omega,\ell) = \frac{1}{\omega^{1-2\gamma_\psi}} f(\frac{\omega}{\ell})$$

Large N Dials

At $N_b \to \infty$ N_f fixed

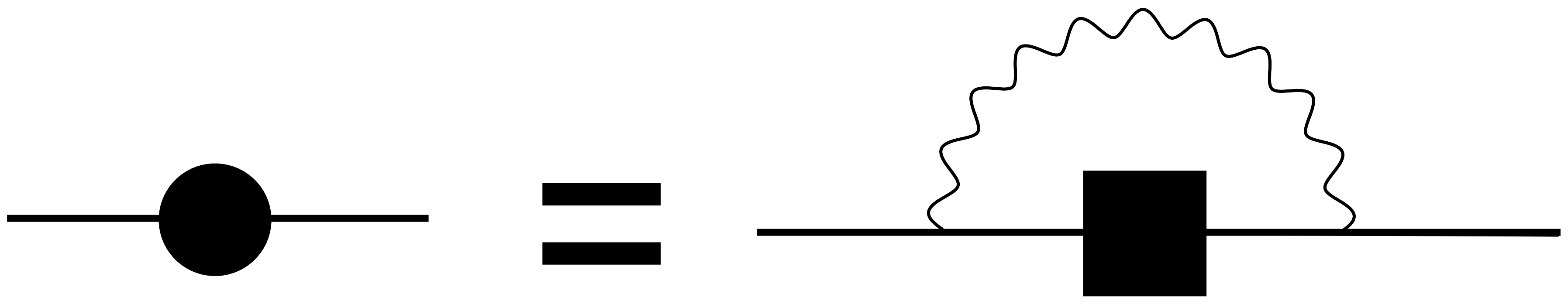
Actually, we can even calculate the scaling function $f\left(\frac{\omega}{\ell}\right)$

Gap equation for fermion Green's function



Large N Dials

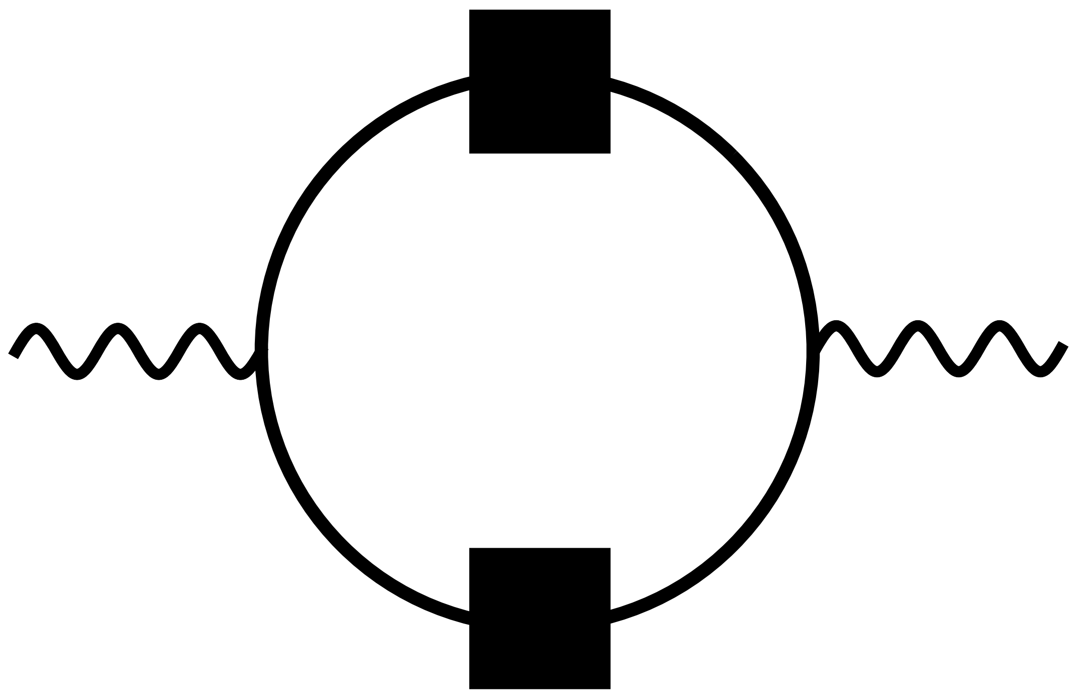
At $N_b \to \infty \quad N_f$ fixed



Solution: $G(\omega, \ell) = \frac{1}{\omega^{1-\frac{\epsilon}{2}}}$ $\qquad f\left(\frac{\omega}{\ell}\right) = 1$

Large N Landau Damping

Now we can look at 1/N correction to boson



$$d = 2 : \qquad \Pi(q_0, q) \sim \frac{g^2}{N_b} q_0 k_F \log(q_0/\Lambda)$$

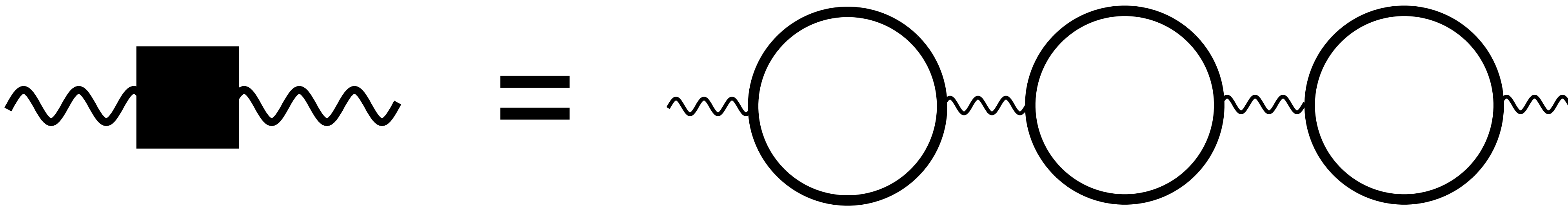
Very different from the boson self-energy in the original "Hertz" treatment!

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_f \to \infty$ N_b fixed

"Fermions Win"



Hertz's theory is exact: $G_\phi(q_0, q) = \dfrac{1}{q_0^2 + c_s^2 q^2 + \Pi(q_0, q)}$

1/N Issues

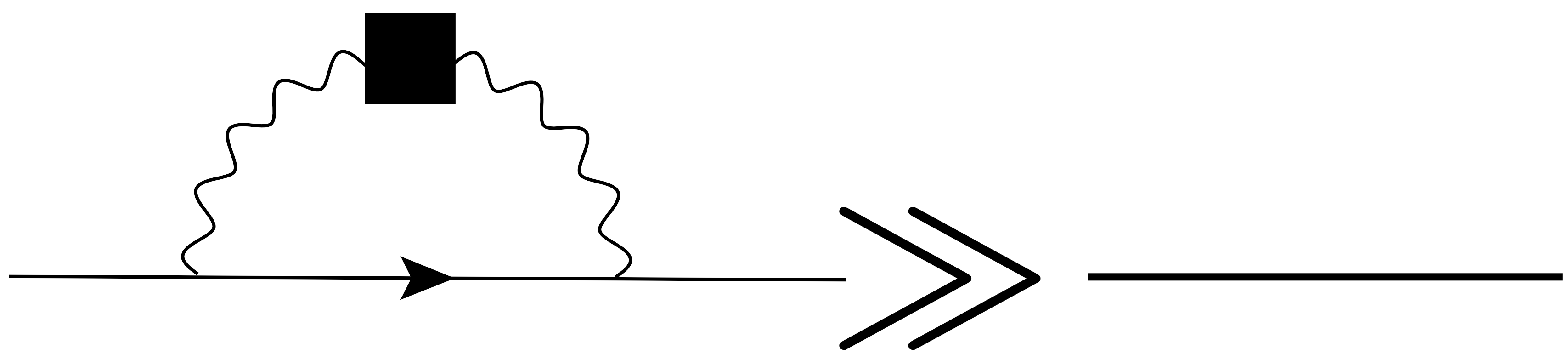
	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

If we look at subleading orders in 1/N, non-planar diagrams dominate deep in the IR

$d = 2$

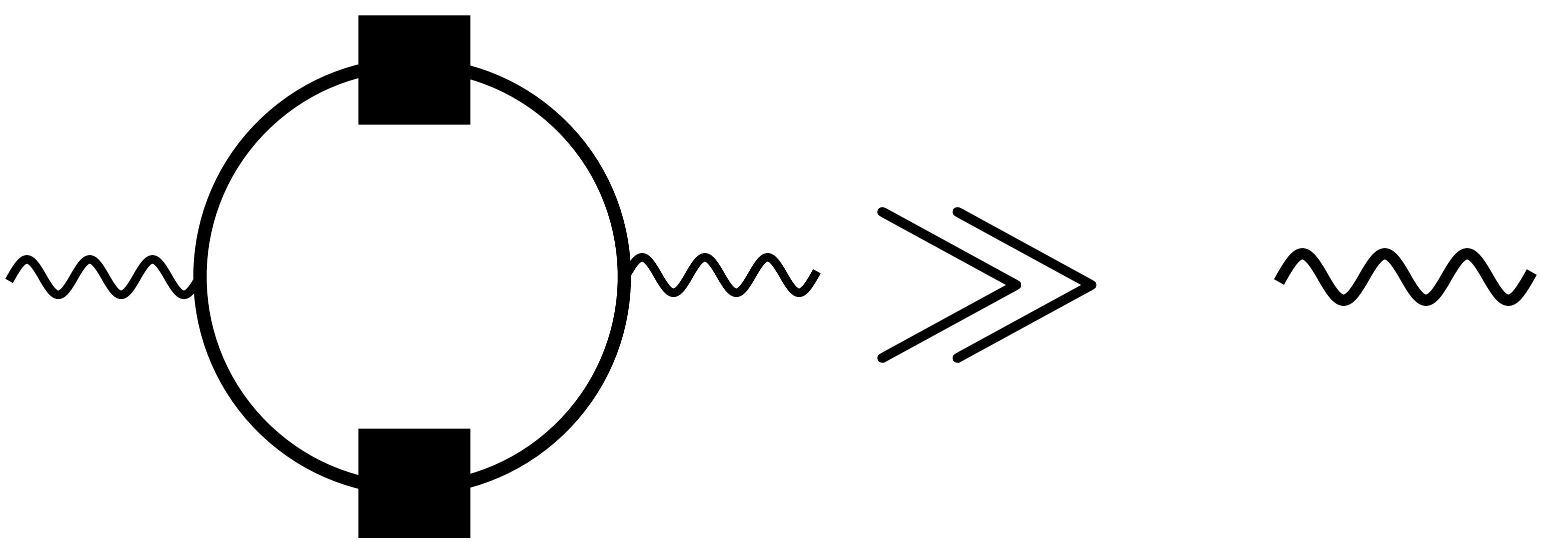
S.S. Lee

$\frac{1}{N_f}$



at $\omega \lesssim \frac{g^4}{m N_f^3}$

$\frac{1}{N_b}$



at $\omega \lesssim \frac{g^2 m}{N_b}$

1/N Issues

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

If we look at subleading orders in 1/N, non-planar diagrams dominate deep in the IR

$d = 2$

S.S. Lee

$$\frac{1}{N_f}$$

at $\omega \lesssim \frac{g^4}{m N_f^3}$

Complicated effects arise as we leave the regime of small parameters

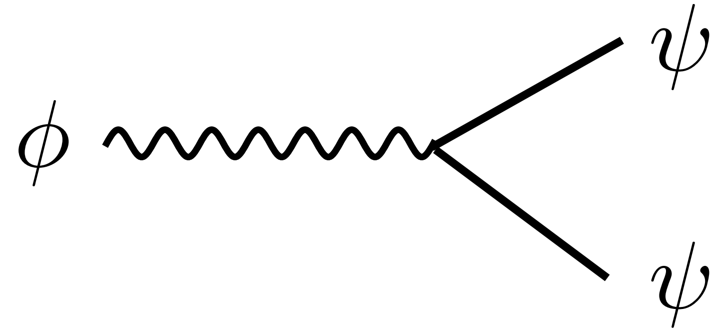
$$\frac{1}{N_b}$$

at $\omega \lesssim \frac{g^2 m}{N_b}$

Slow Fermions

Small fermion velocity is similar to large N:
no Landau damping as velocity goes to zero

Essentially a kinematic effect:
Bosons cannot decay to very slow fermions

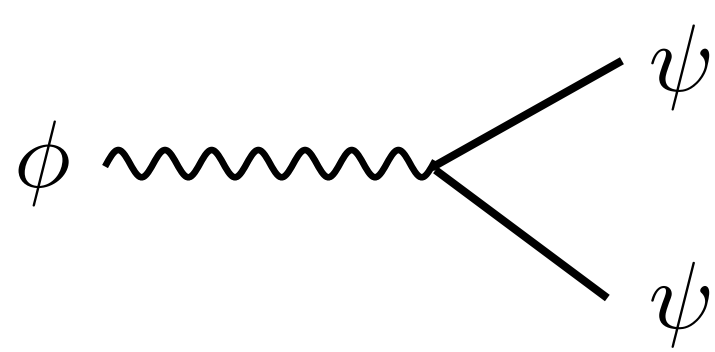


Slow Fermions

Advantages over large N:

a) velocity is a more generic physical parameter

b) velocity runs to zero under RG flow, so there is a "basin of attraction" for zero velocity



Conclusion

Non-Fermi liquids have new dynamics in need of a theoretical description

We are looking for local EFTs of the Fermi surface (plus light states) that exhibit similar dynamics

A rich structure of such theories exists depending on various parameters of the theory

In some limits (large N, small ϵ, small v) the theory can be solved and leads to new fixed points

The End