Consequences of F-theorem for Phase Transitions & Stability of 3D Gauge Theories

Tarun Grover

KITP

arXiv:1211.1392

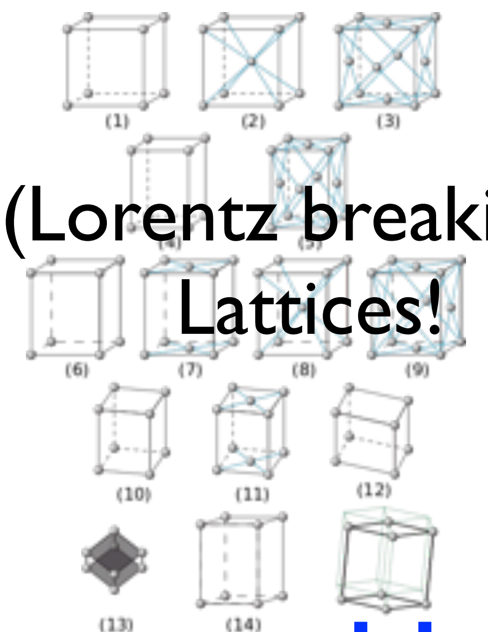
Outline

- Introduction
- Entanglement Entropy & Renormalization.
- Applications of Entanglement monotonicity.

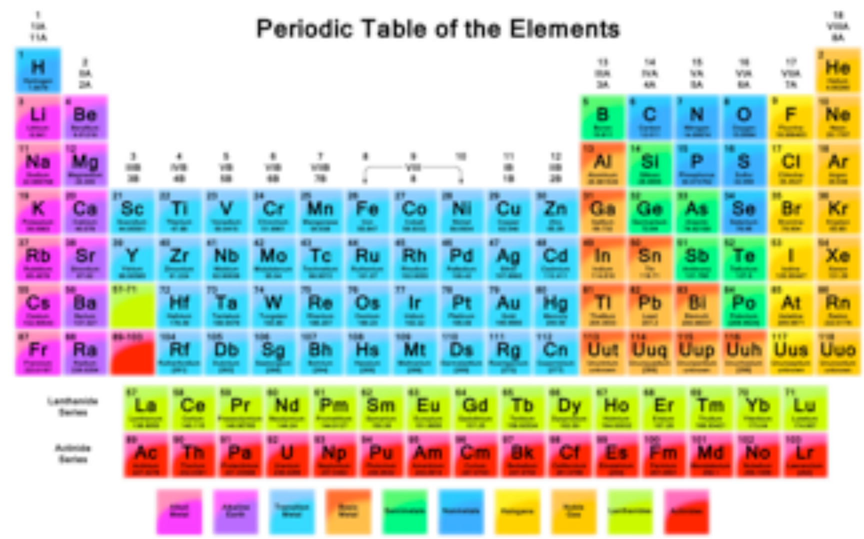
Condensed Matter Physics =

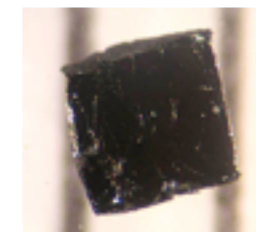
Collective behavior of interacting, many ($\approx$ infinite) particle systems.

Enormously many systems & experiments...



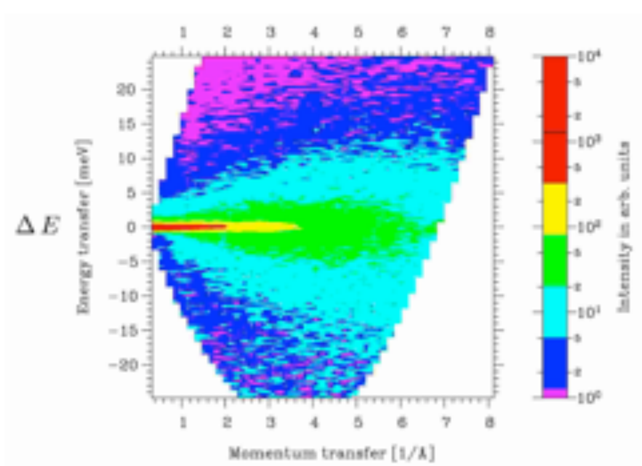
(Lorentz breaking)
Lattices!



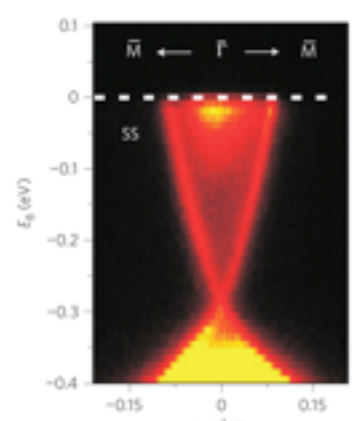


Crystals, potentially with disorder.

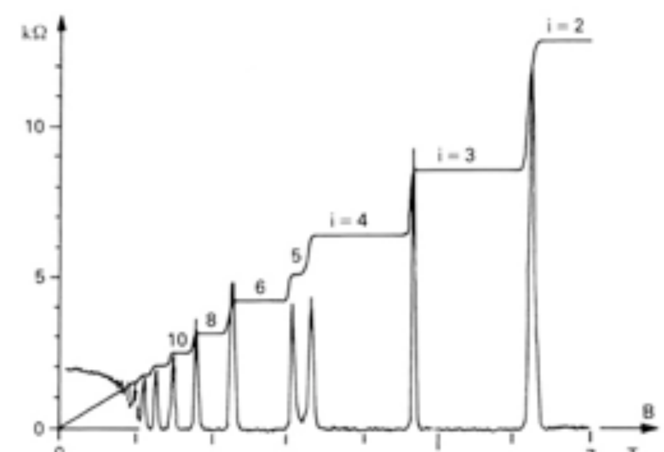
How to make sense of it all?



scattering

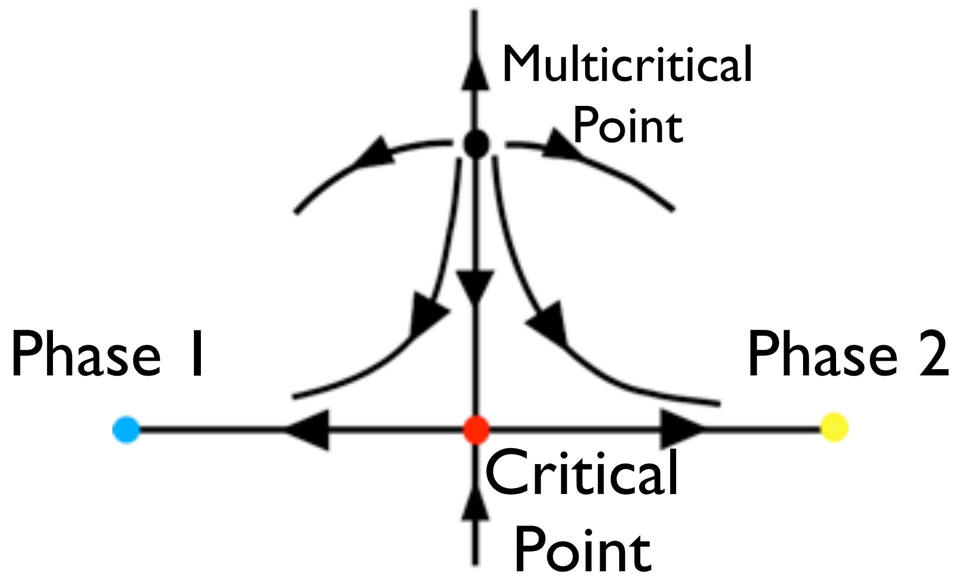


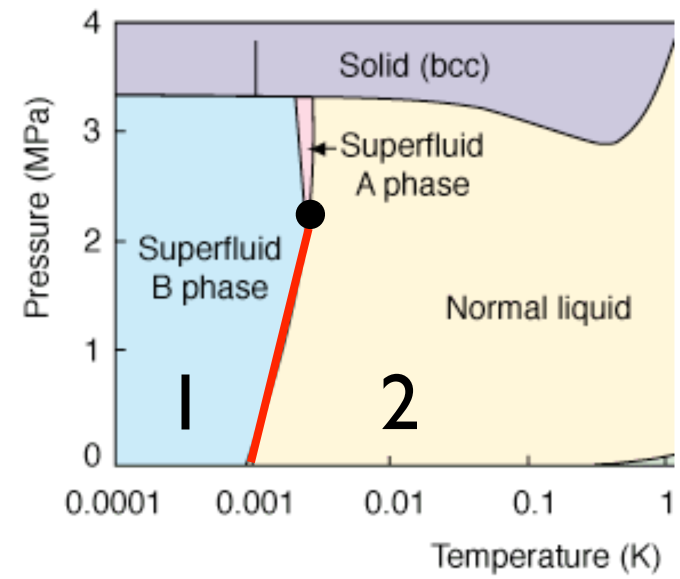
photoemission



resistivity etc.

Phases, Phase Transitions & Universality



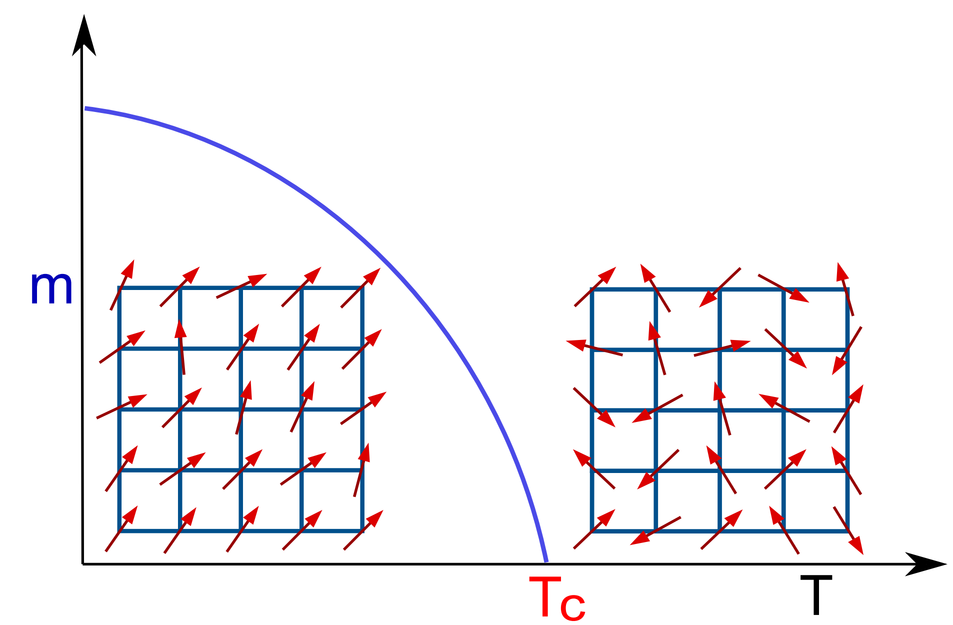


Low-energy, long-distance physics "Universal": determined solely by the RG fixed point.

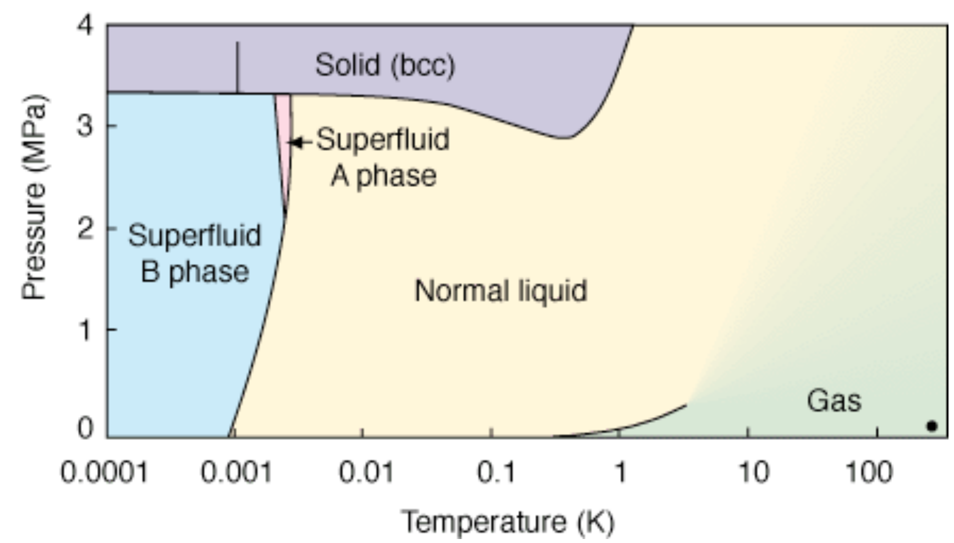
Often, Emergent Symmetries at a fixed point.

Most phases break *some* global symmetry at T = 0 in D>2

Ferromagnet



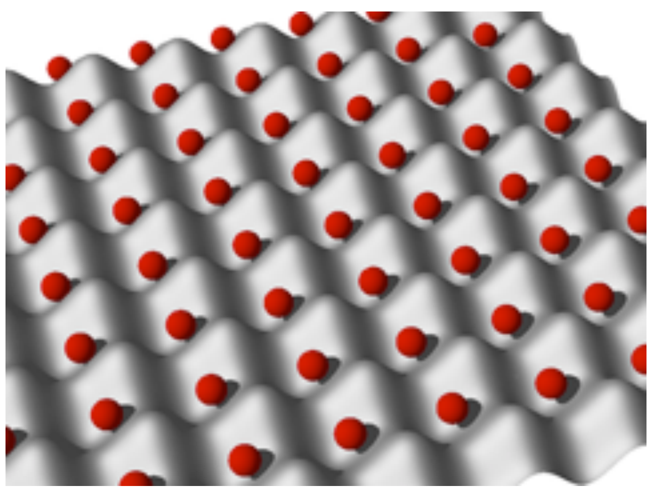
Superfluid He-3

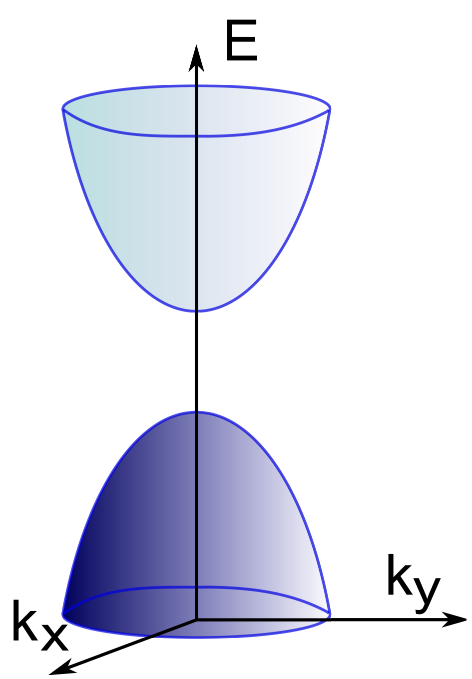


- Local order parameter "protects" a symmetry broken phase.
- Also makes it more classical.

Phases without Order Parameter?

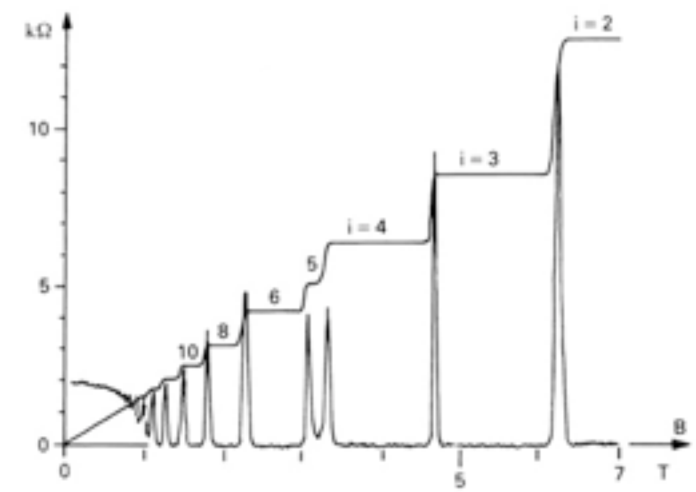
"Trivial"



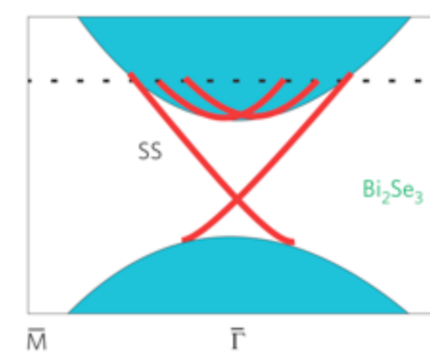


Integer filling Mott Insulator, Band insulator...

Seemingly Non-trivial



Klitzing et al (1980)

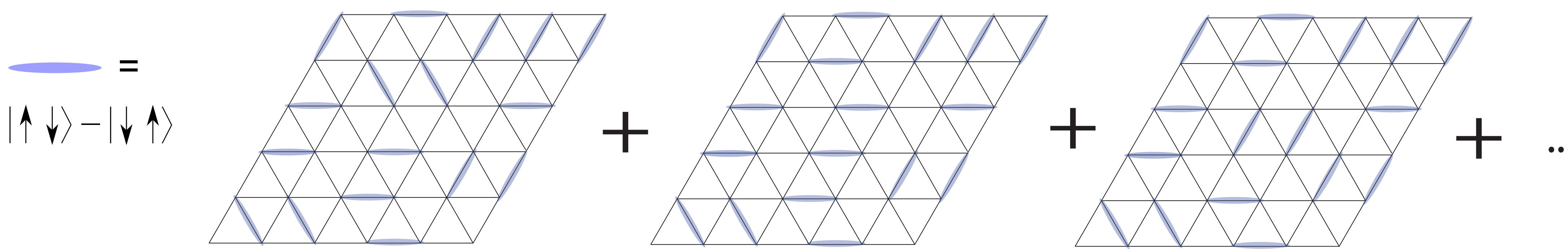


Hasan et al (2009)

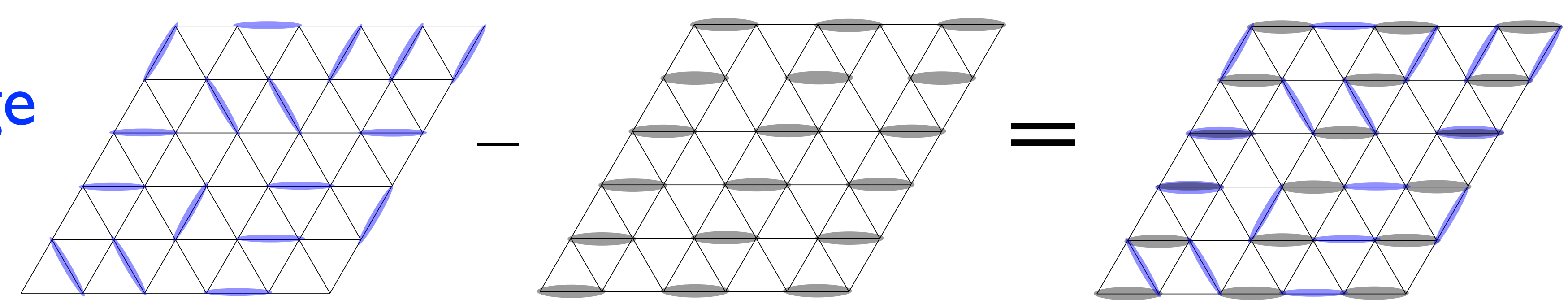
Quantum Hall, Topological Insulators, Gapless spin-liquids...

Heuristics of Phases without Order Parameter

Example: Spin-1/2 SU(2) spins on lattice. Ground state =



Emergent Gauge structure!

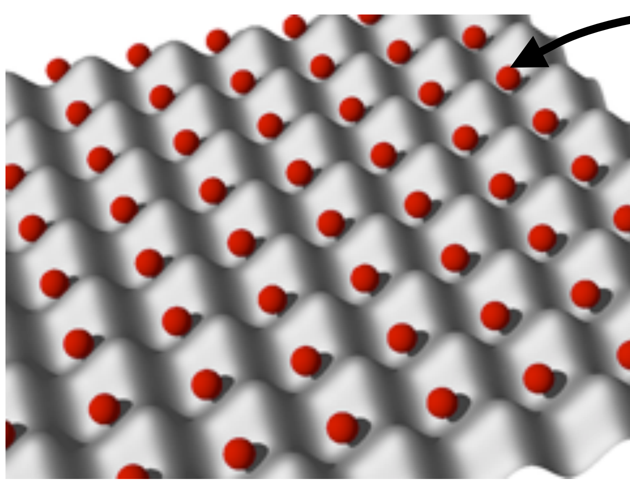


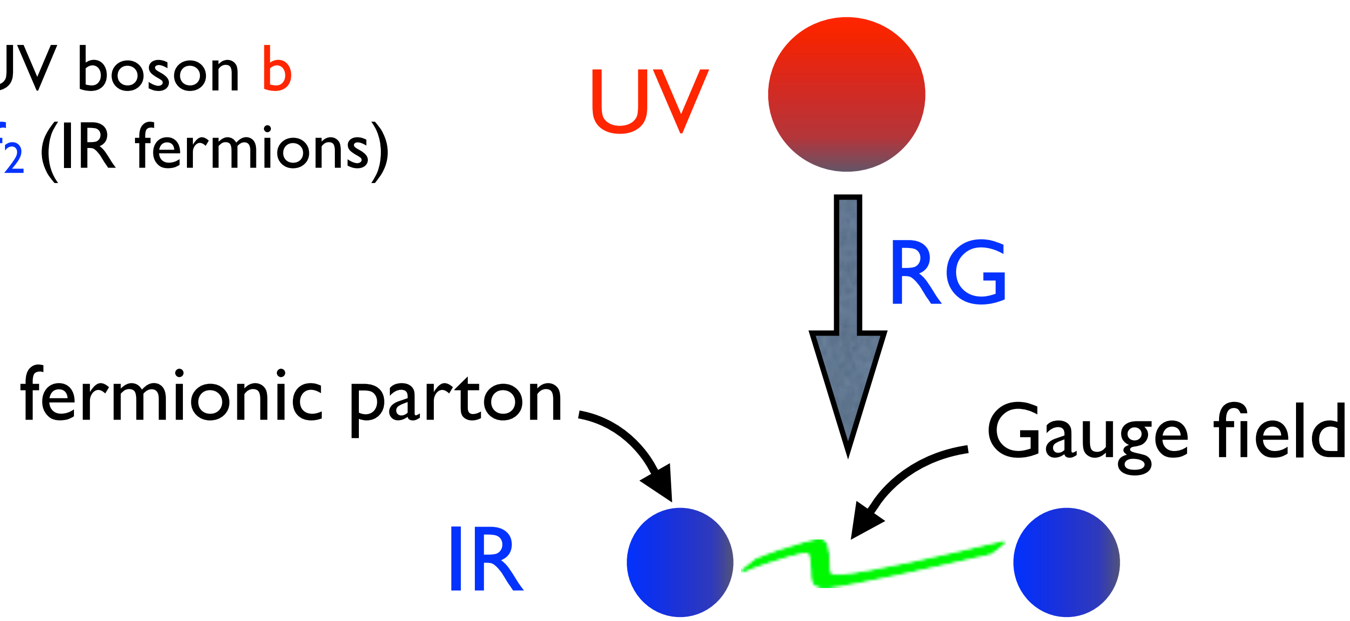
"Quantum Spin-liquid" Anderson 1973

Kivelson, Rokhsar, Wen, Kitaev, Sondhi, Moessner, Fisher, Senthil, and many others.

Mechanics of Emergent Gauge Structure.

- Strong quantum fluctuations may "fractionalize" UV degrees into "partons".
- Partons interact via gauge field and *may* deconfine in IR.

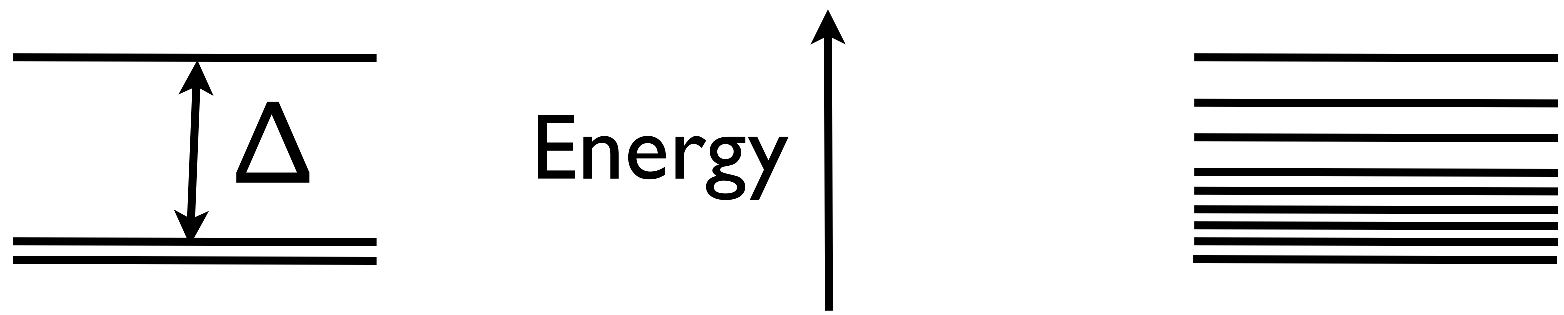




Wen 1990

Effective field theory of Phases without Order Parameter?

Gapped Spectrum | **Gapless Spectrum**



Topological Quantum Field Theories

e.g. $\mathcal{L} = \frac{1}{4\pi} A \wedge dA$

Always Stable in the IR!

CFTs, Fermi surface coupled to order parameters, etc.

e.g. $$\mathcal{L} = \sum_{a=1}^{N_f} \overline{\psi}_a \left[-i\gamma_\mu \left(\partial_\mu + i a_\mu \right) \right] \psi_a + \frac{1}{2g^2} F_{\mu\nu} F_{\mu\nu}$$

Stability?

Outline

- Introduction
- Entanglement Entropy & Renormalization.
- Applications of Entanglement monotonicity.

- Quantum Entanglement can often detect universal properties of a phase, given only the ground state wavefunction.

"Which phase is it?"

- This talk:

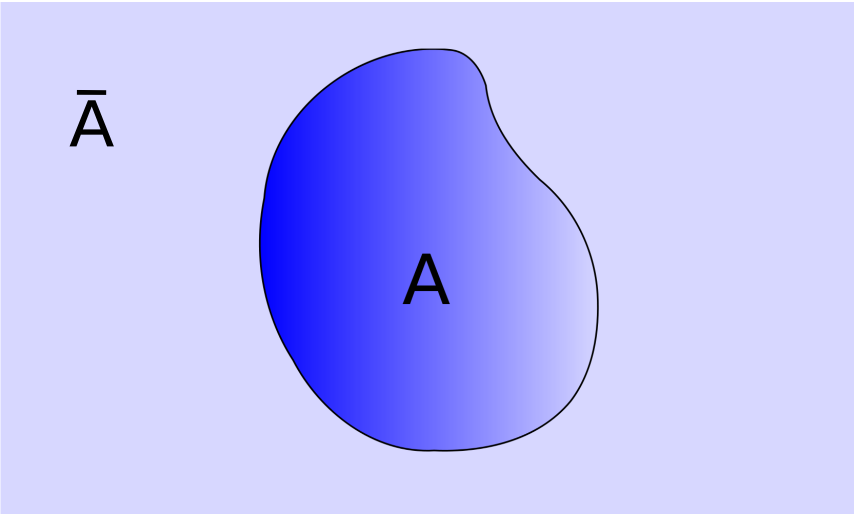
RG Flows from Quantum Entanglement

"Is the phase stable?"

"If not, what are its instabilities?"

Entanglement Entropy

- Divide system into two parts...



Reduced density-matrix for A:

$$\rho_A = \text{Trace}_{\bar{A}} |\psi\rangle\langle\psi|$$

- von-Neumann entropy: $S = -\text{Trace}(\rho_A \log \rho_A)$

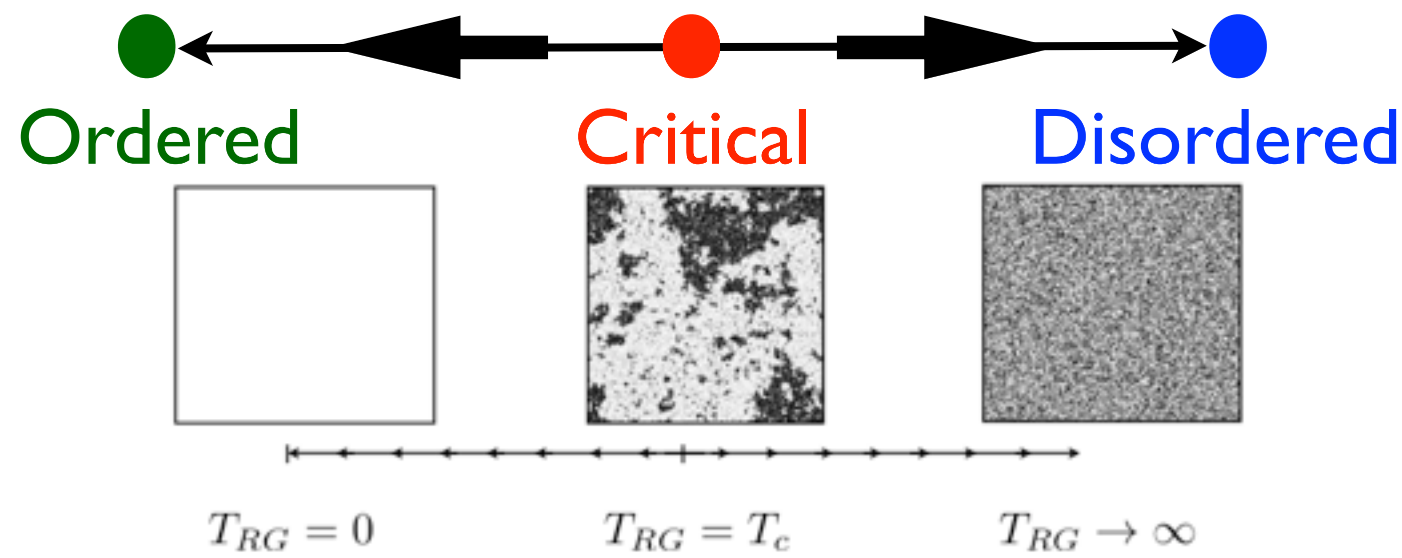
- Renyi entropies: $S_n = -\frac{1}{n-1} \log(\text{Trace}\rho_A^n)$

- Zero if and only if product state: $\psi = \phi_A \otimes \phi_{\overline{A}}$

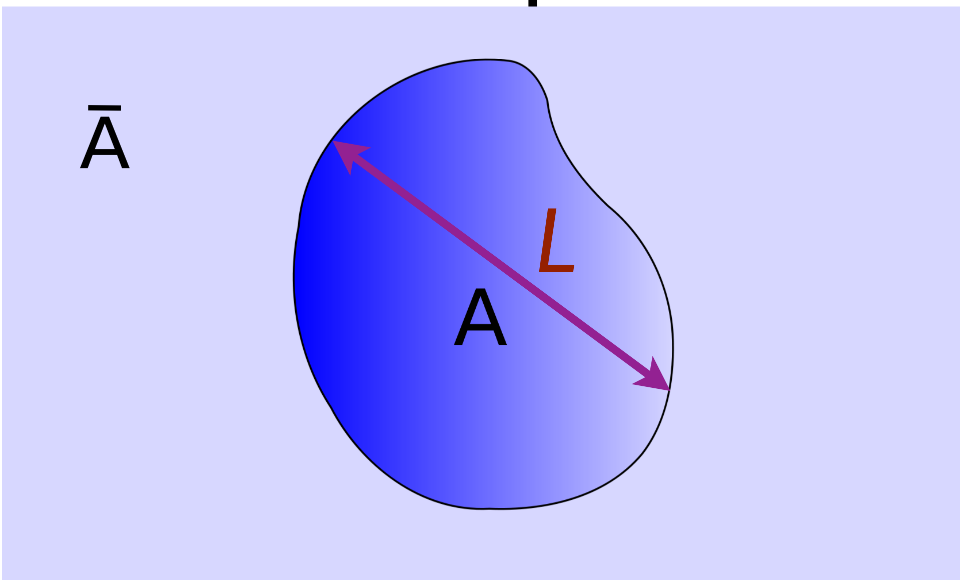
$S = \log(2)$ for EPR singlet $|\psi\rangle = |\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle$

Entanglement & Renormalization Group

- RG Probes system at different length scales.



- Entanglement as a tool to probe different scales?



Entanglement Scaling at RG Fixed Points

Entanglement = Non-local "Order parameter" for phases and phase transitions

Phase Transitions (Conformal Field Theories)

1D: $S \sim c \log(L) + O(1/L)$

2D: $S \sim L - \gamma + O(1/L)$

3D: $S \sim L^2 + a \log(L) + O(1/L)$

Phases

"Topological Entanglement Entropy" → γ

Topologically Ordered Phase: $S \sim L - \gamma$

Fermi Surface: $S \sim k L^{D-1} \log(L)$

Holzhey, Wilczek, Larsen; Cardy, Calabrese; Casini, Huerta; Ryu, Takayanagi; Kitaev, Preskill; Wen, Levin; Gioev, Klich, and many others.

Entanglement *along* RG flow

Universal part of quantum entanglement for CFTs decreases between RG fixed points!

1D: $S_{line\ segement} \sim c \log(L)$

Zamolodchikov 1986

c-theorem (Zamolodchikov): "c" descreases under RG.

3D: $S_{sphere} \sim L^2 + a \log(L)$

Cardy 1988; Jack, Osborn 1990; Komargodski, Schwimmer 2011.

a-theorem (Cardy): "a" decreases under RG.

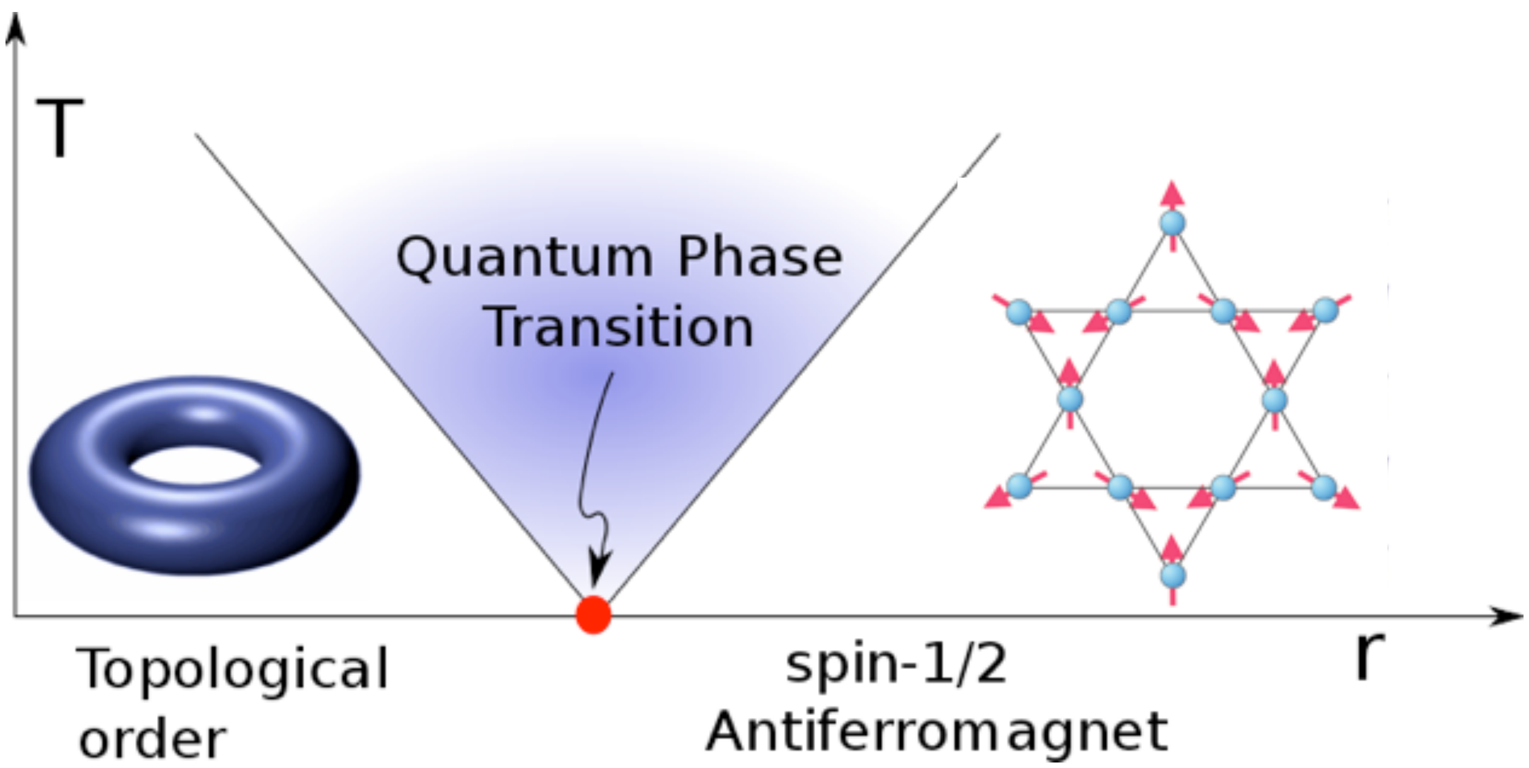
2D: $S_{circle} \sim L - \gamma$

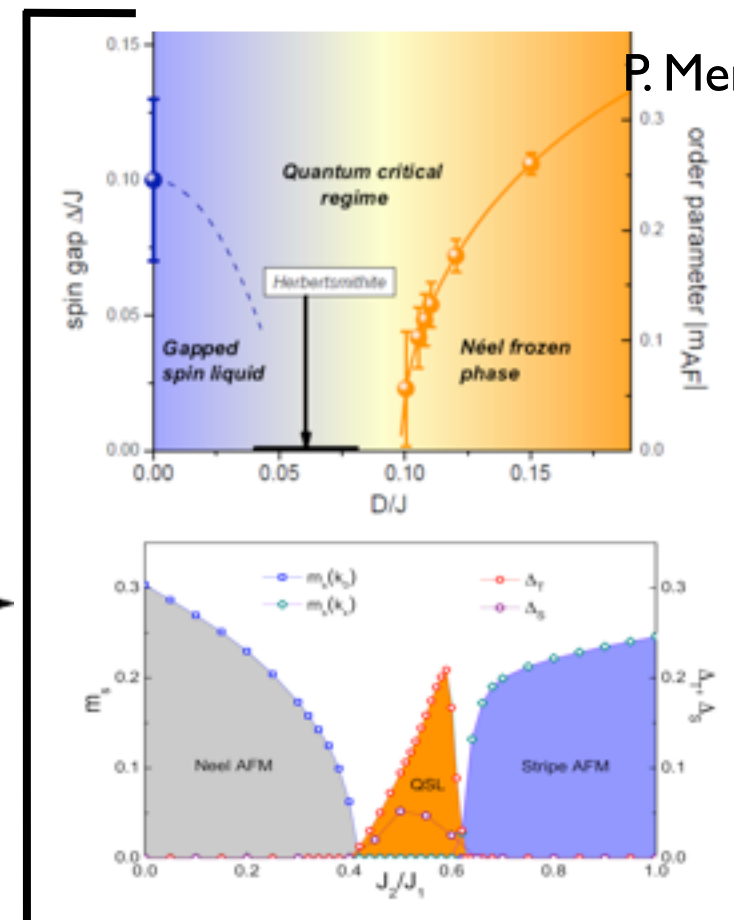
γ/F-theorem (Klebanov et al, Casini, Huerta, Myers): "γ" decreases under RG

Outline

- Introduction
- Entanglement Entropy & Renormalization.
- Applications of Entanglement monotonicity.

Application I: Entanglement monotonicity & Quantum Phase Transitions





P. Mendels

Balents et al 2011;
Wen et al 2011

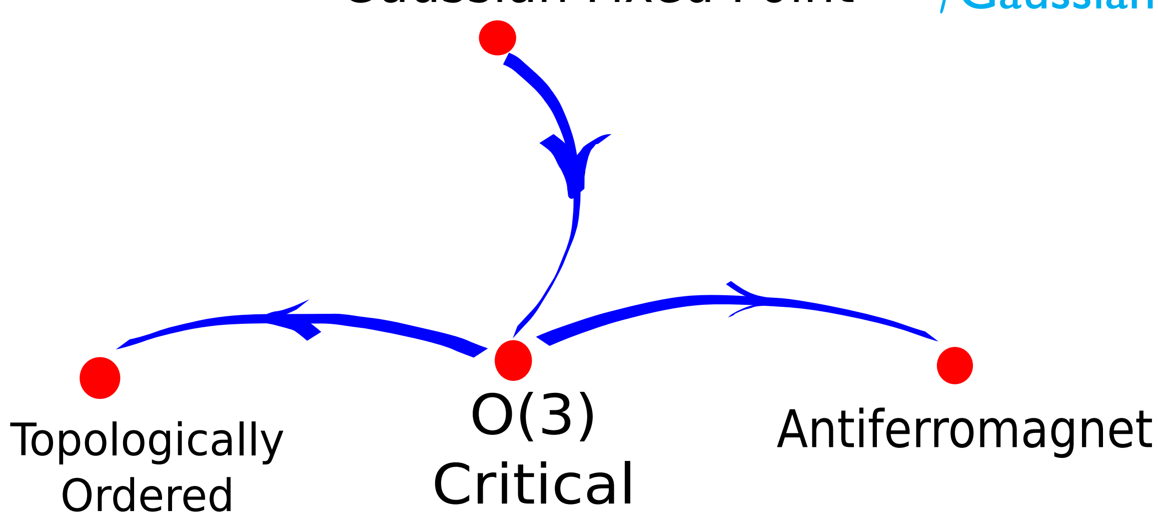
Question: Nature of transition?

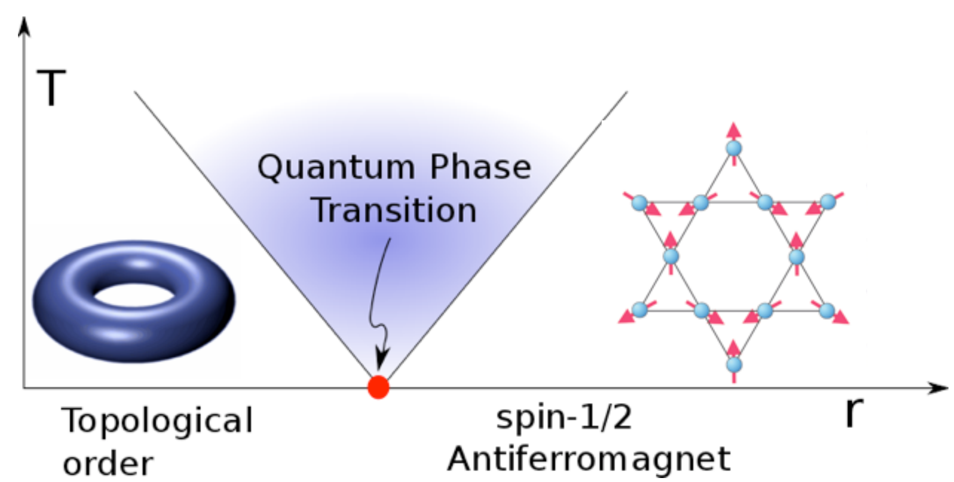
naive Landau-Ginzburg reasoning:
O(3) Wilson-Fisher.

A No-Go Theorem for Quantum Phase Transitions

RG flow <u>assuming</u> O(3) transition

$\gamma_{\text{Gaussian}} = 3 \times \gamma_{\text{Scalar}} = 0.18$





$$\gamma_{\text{Topological}} = \log(D) \geq \log(\sqrt{2}) \approx 0.35$$

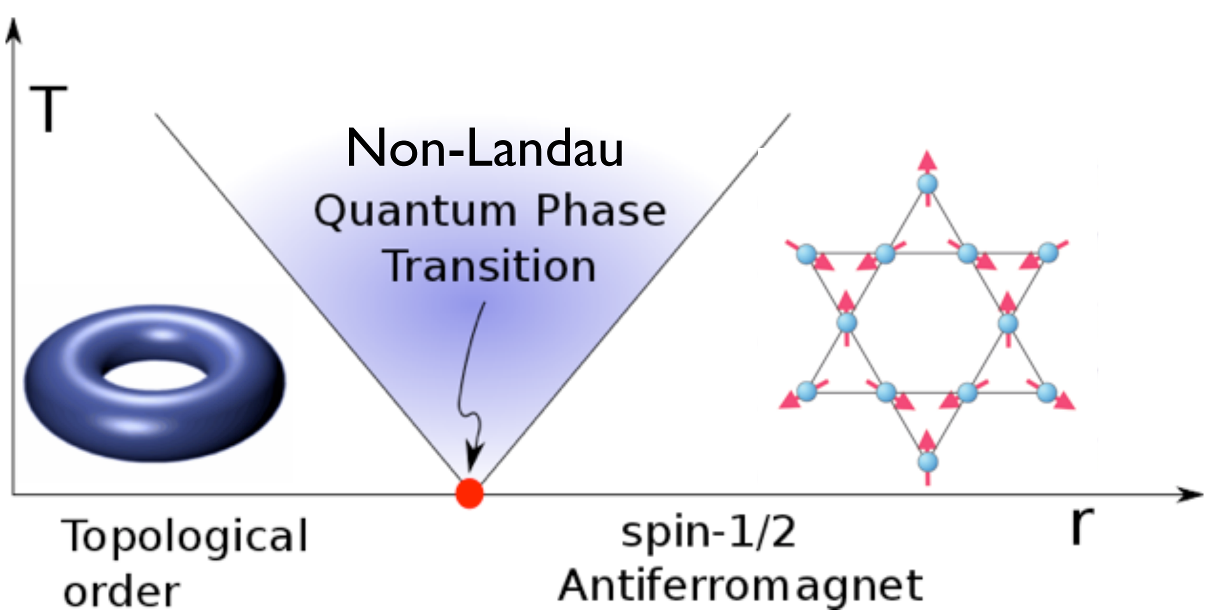
Contradiction with entanglement monotonicity!

$\Rightarrow$ **O(3) Transition impossible.** TG 2012

Obvious generalizations (SF$\longleftrightarrow$FQH, Nematic $\longleftrightarrow$ $\mathbb{Z}_2$ Spin liquid ...)

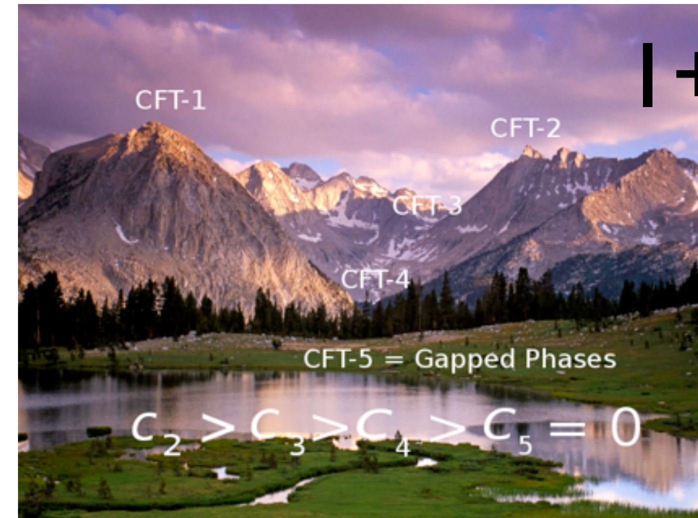
Lesson

Phase transitions out of topologically ordered phases *necessarily* lie beyond Landau-Ginzburg paradigm



Contrast: 1+1-D Vs 2+1-D

1+1-D

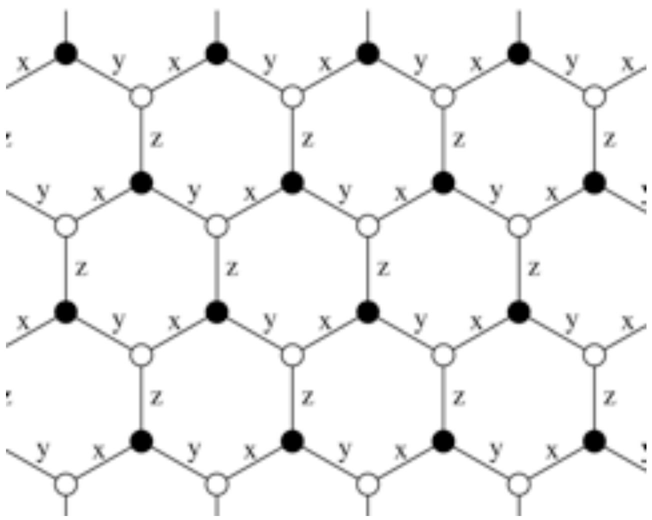


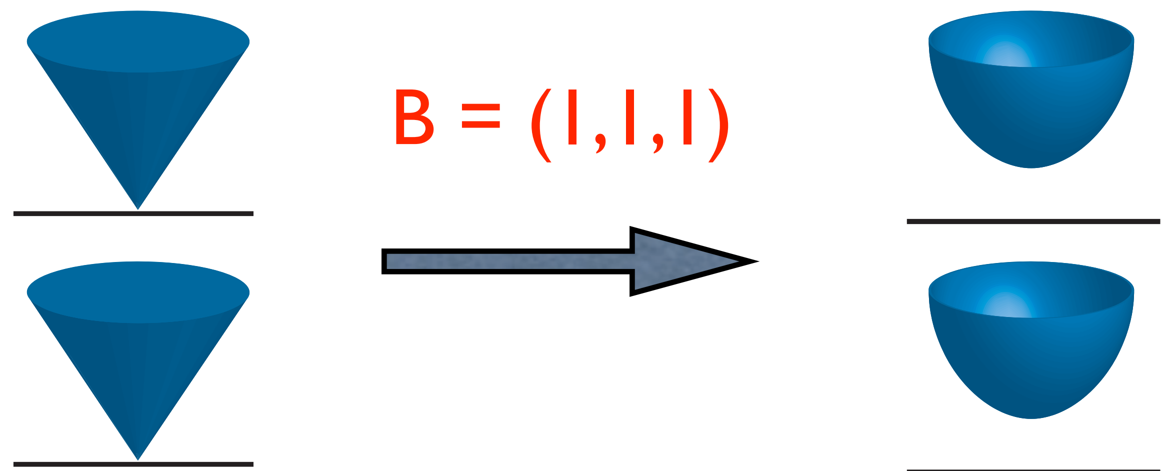
2+1-D

- In 1+1-D, with no symmetries, unique gapped CFT ($c = 0$)
- All c > 0 CFTs can be taken to this unique massive CFT.
- In 2+1-d, more than one gapped CFT. Distinct "Topological Ordered Phases".

F-theorem ⇒ A gapless theory may not be deformable to a given massive theory in 2+1-d.

Aside: F-theorem & Kitaev's Honeycomb Model





$$\gamma = \underbrace{\gamma_{\text{Dirac}}}_{0.22} + \underbrace{\gamma_{\mathbb{Z}_2 \text{ Topological}}}_{\log(2)} \longrightarrow \underbrace{\gamma_{\text{Ising Topological}}}_{\log(2)}$$

(Yao, Qi 2010)

Lesson: When QFT = CFT + TQFT, important to keep track of total F.

Application II: Stability of Gapless Spin-liquids

Gapless spin-liquids = Phases without order parameter or well-defined quasiparticles.

Low-energy theory = Interacting gauge-matter theories.

However, many instabilities in 2+1-d!

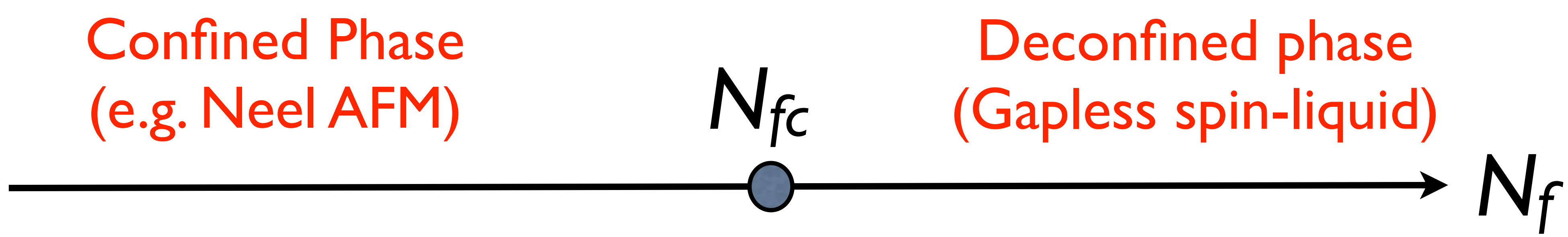
Classic problem from 1970's:

Stability of gauge theories against confinement and/or symmetry breaking?

F-theorem &
Stability of of QED-3

$$\mathcal{L}_{QED-3} = \sum_{a=1}^{N_f} \overline{\psi}_a \left[-i\gamma_\mu \left(\partial_\mu + i a_\mu \right) \right] \psi_a + \frac{1}{2g^2} F_{\mu\nu} F_{\mu\nu}$$

N_f determined by parton band-structure.



Hermele et al 2005

Critical value of N_f above which spin-liquid stable?

Instabilities of QED-3

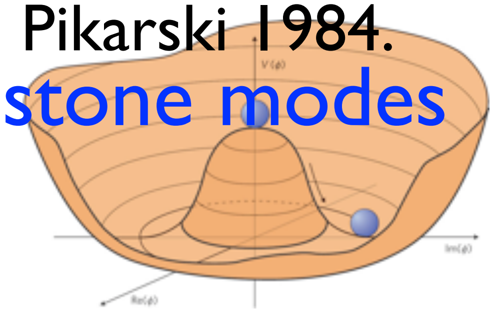
Non-compact QED-3

Chiral symmetry breaking (CSB):

$U(N_f) \to U(N_f/2)\times U(N_f/2)$

Vafa, Witten 1984; Pikarski 1984.

Low-energy theory? $N^2_f/2$ Goldstone modes + Photon



Compact QED-3

Confinement: Polyakov 1977

Proliferation of monopoles.

Low-energy theory? Depends on monopole quantum numbers. May be accompanied by chiral symmetry breaking.

Entanglement Monotonicity & Stability of Non-compact QED-3

CSB generates $N^2_f/2$ Goldstone modes.

But, these are too many to satisfy entanglement monotonicity when N_f large...

$\gamma_{QED-3} \propto N_f$ while $\gamma_{\text{Goldstone}} \propto N_f^2$

Rough estimate: $N_{fc} \simeq 2 \times \frac{\gamma_{\text{Free Dirac Fermion}}}{\gamma_{\text{Free Real Scalar}}} \simeq 8$

TG 2012

cf. Appelquist et al 1999

Entanglement Monotonicity & Stability of Non-compact QED-3

Two approaches to make further progress...

- #1: Use large-N_f result to improve estimate.

$$\begin{cases} \gamma_{QED-3} & \approx \quad N_f\, \gamma_{Dirac} + \frac{1}{2}\log(\pi N_f/8) \\ & \geq \quad \gamma_{Goldstone} = (N_f^2/2+1)\gamma_{scalar} \end{cases}$$

Klebanov et al 2012

$\Rightarrow$Chiral symmetry restored above $N_f \approx 10$

- #2: "Sandwich" QED-3 between two well understood fixed points.

Upper bound on critical flavors by "Sandwiching"

- Deform Chiral SQED-3 by gapping out bosons to obtain QED-3.

- RG flow controlled at large N_f.

- Assumption: No other fixed points besides QED-3 with same matter content and symmetries (Vafa-Witten theorem strongly constrains IR).

$$\gamma_{SQED-3} \geq \gamma_{QED-3} + \gamma_{Dirac} \quad (1)$$

$$\gamma_{QED-3} \geq \gamma_{Goldstone} \quad (2)$$

Upper bound on critical flavors by "Sandwiching"

$$\gamma_{SQED-3} \geq \gamma_{Goldstone} + \gamma_{Dirac}$$

$$\gamma_{SQED-3} = N_f \log(2) + \frac{1}{2}\log\left(\frac{N_f \pi}{2}\right) + \left(\frac{-1}{4} + \frac{10}{3\pi^2}\right)\frac{1}{N_f} + O(N_f^{-2})$$

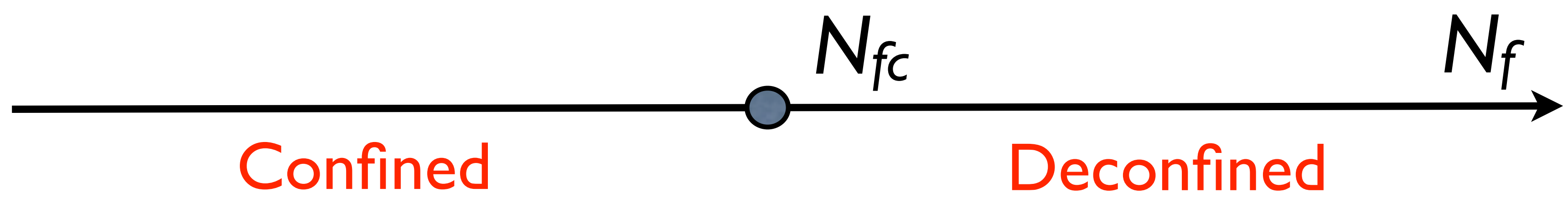
Klebanov, Pufu, Safdi, Sachdev (Localization)

$$\gamma_{\text{Goldstone}} = 2N_f^2 \gamma_{scalar} + \gamma_{scalar}$$

Deconfinement for $N_f > 13$

TG 2013

Entanglement Monotonicity & Stability of Compact QED-3



Vafa-Witten theorem in 2+1-d:

Massless particles in IR when $N_f > 5$.

What are these massless particles?

One guess:
Goldstone modes due to confinement.

Entanglement Monotonicity & Deconfinement in QED-3

Four Possible Scenarios...

- Confinement without massless particles (not possible for $N_f > 5$, Vafa-Witten)
- Confinement with massless Goldstone modes (not possible for $N_f > 13$, Entanglement monotonicity)
- Deconfinement with mass gap (not possible for $N_f > 5$, Vafa-Witten)
- Deconfinement with massless fermions

Deconfinement with massless fermions at least for $N_f > 13$

TG 2012

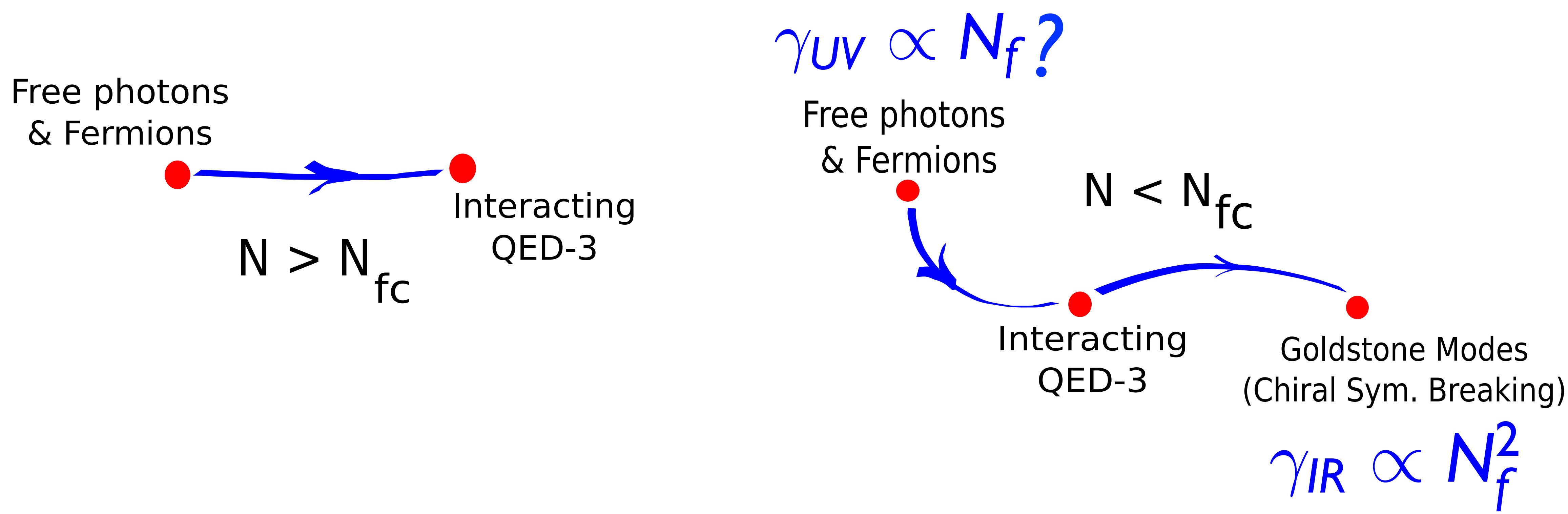
cf. Pufu, Mezei 2013: large-N_f Monopole scaling dimension estimate $N_{fc} \simeq 12$.

Non-abelian Gauge theories?

- Similar arguments imply that one expects deconfinement and chiral symmetry restoration at least when

$$N_f > 2N_c \frac{\gamma_{\text{Dirac}}}{\gamma_{\text{scalar}}} \approx 8\,N_c$$

A Better Bound?



Deconfinement for $N_f > 7$ *under certain assumptions.*

Aside: Application to multicomponent Landau-Ginzburg models

Consider O(m)⊕O(n) vector models.

When m,n = O(1), the IR critical fixed point has O
symmetry.

Aharony 1975
Vicari et al 2002

On the other hand, when m,n >> 1, the most stable fixed point is believed to be decoupled O(m), O(n) model.

$$\gamma_{O(m+n)} \approx (m+n)\gamma_{\text{scalar}} - c$$

$$>$$

$$\gamma_{O(m)} + \gamma_{O(n)} \approx (m+n)\gamma_{\text{scalar}} - 2c$$

Klebanov et al
$c = \zeta(3)/16\pi^2$

(work in progress)

Some Questions

- Field theoretic proof of F-theorem?
- Consequences of strong subadditivity for non-relativistic systems? Instabilities of Fermi and non-Fermi liquids?
- Relation between "F" and degrees of freedom in CFT? (for TQFTs, expression for "F" reminiscent of Cardy's formula for density of states in 1+1-D).
- Why do odd dimensions differ from even with respect to topological contribution?
- Violation of c-theorems (including 1+1-d) in lattice models?

Acknowledgements

Thank you to

Igor Klebanov, Joe Polchniski, Leon Balents, Max Metlitski, Nabil Iqbal & Vijay Kumar for useful and interesting conversations.