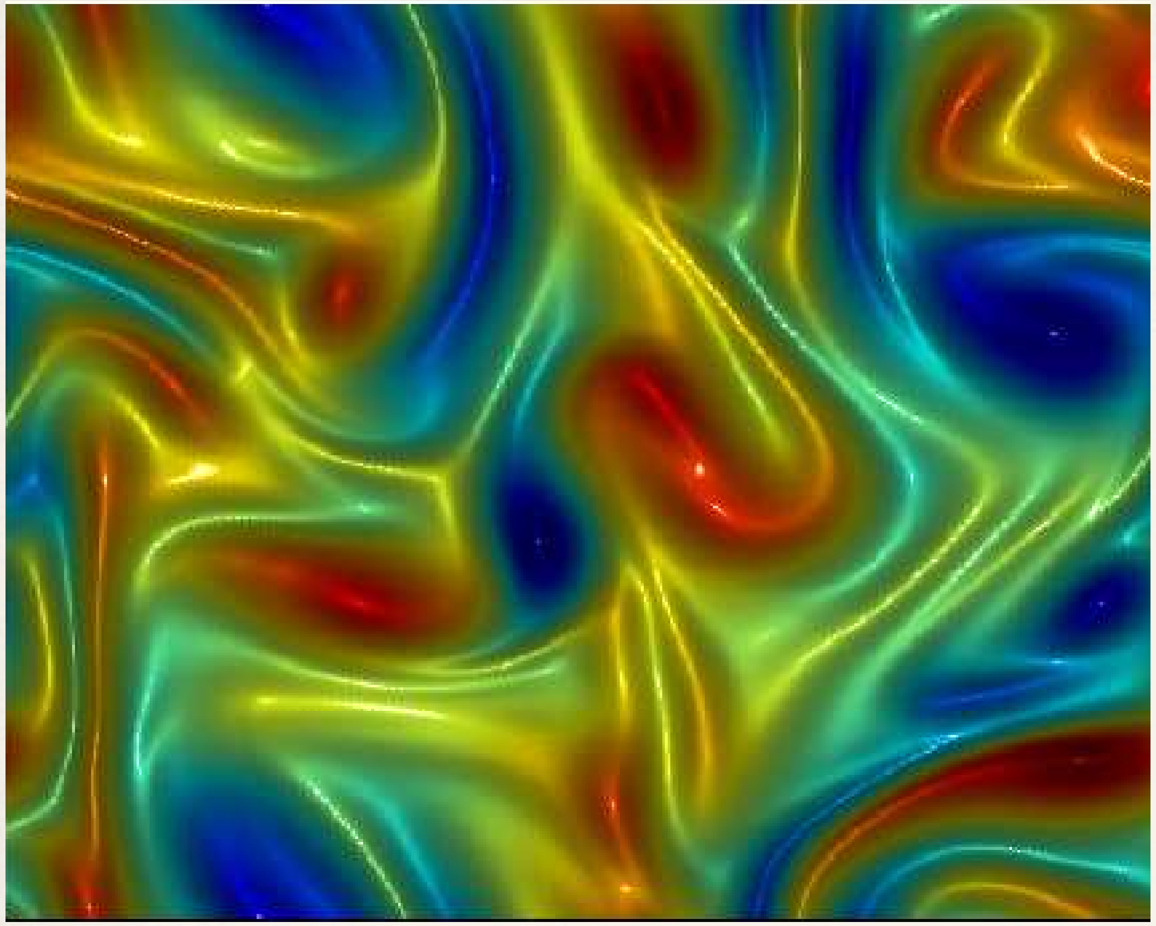
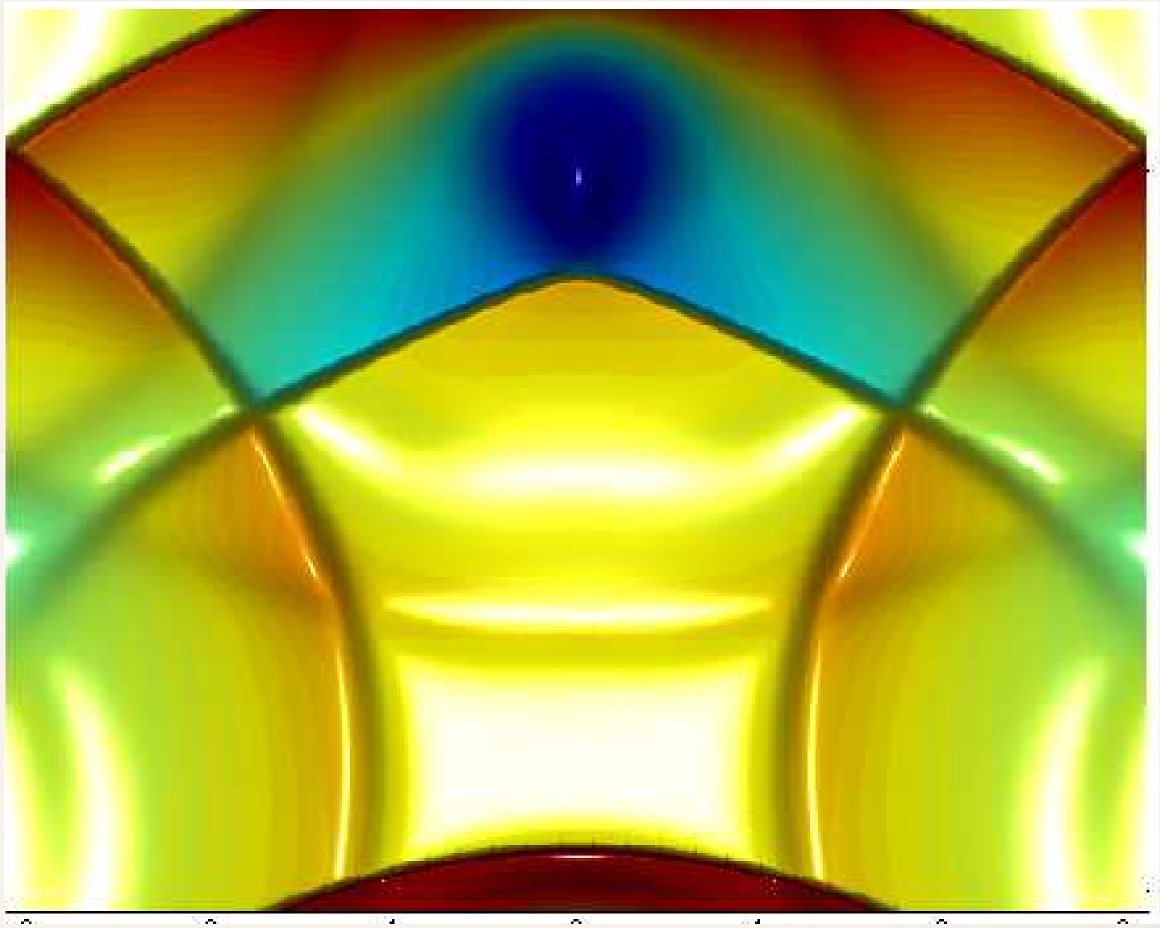




Losing Forward Momentum Holographically

Christopher Herzog (Stony Brook University)

January 7, 2014

Thanks to

- Koushik Balasubramanian
- Paul Chesler and Larry Yaffe
- my desktop computer

Strongly Interacting Fluids

- Through AdS/CFT, pure Einstein gravity with a negative cosmological constant describes a sector of many strongly interacting, conformal, relativistic, neutral fluids, e.g. the ABJM plasma.
- Traditionally, we have probed these systems near equilibrium.
- Beyond near equilibrium behavior.
  - What does turbulence look like?
  - What do shock waves look like?
  - When (if ever) is hydro no longer a good description?

Outline

- A couple of movies:
  - Evolving a state: The counter flow experiment. Reproduces results of others, Carrasco, Lehner, Myers et al. '12; Adams, Chesler, Liu '13
  - Turning on sources: Dragging a ball through the fluid. New.
- Some science from the movies: Translation breaking and roughness

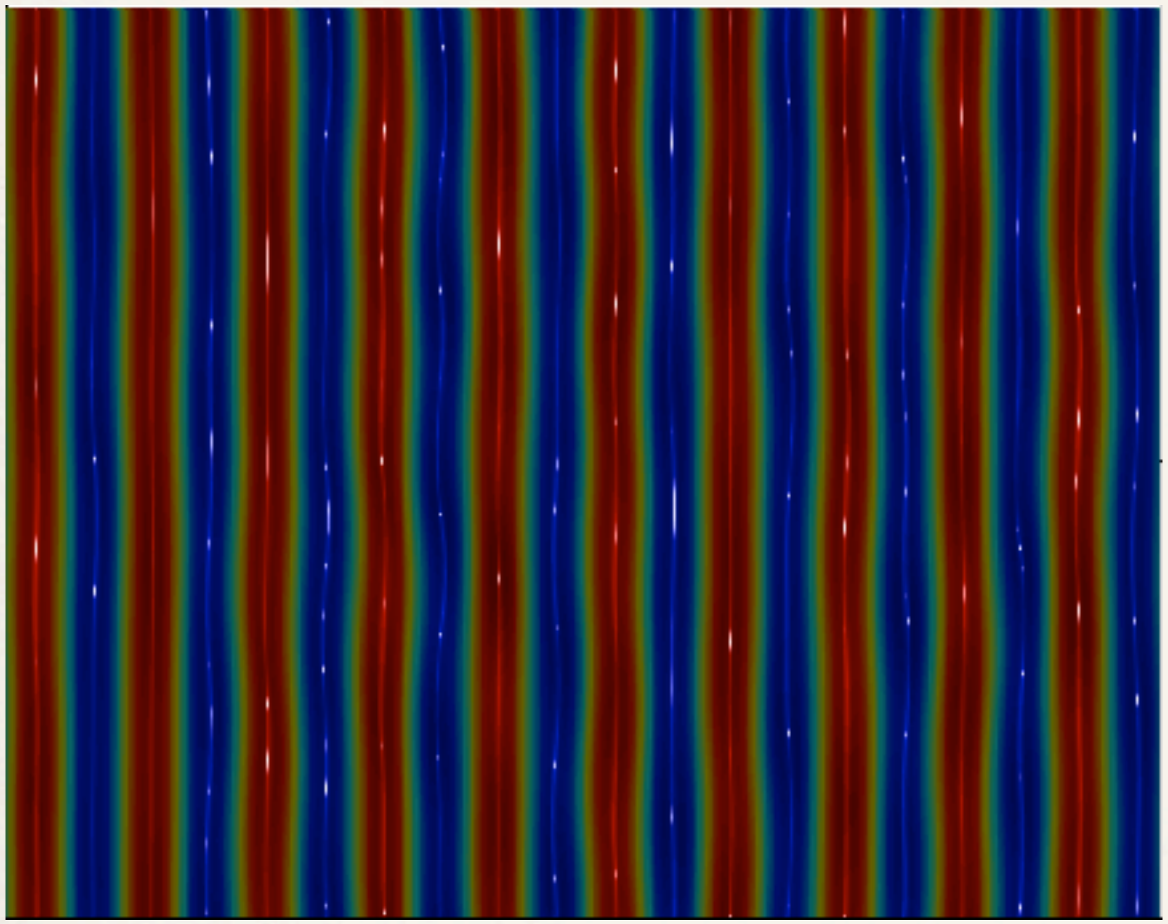
Counterflow

Sinusoidal initial velocity profile. Max speed that of sound. Noise term added to induce turbulence.

Carrasco, Lehner, Myers et al. '12; Adams, Chesler, Liu '13

ABJM plasma at second order in the gradient expansion: Vorticity as a function of time.

Counterflow



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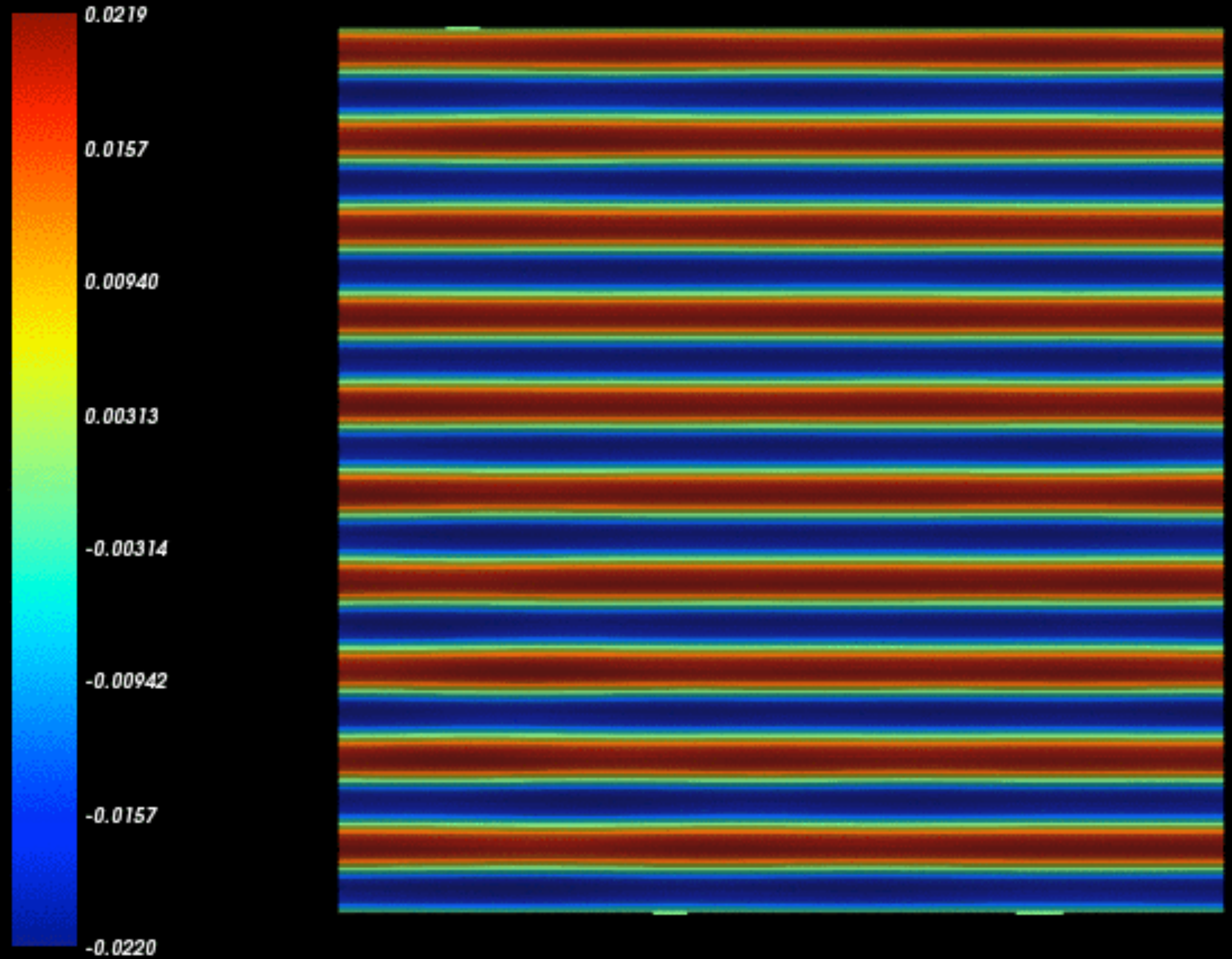
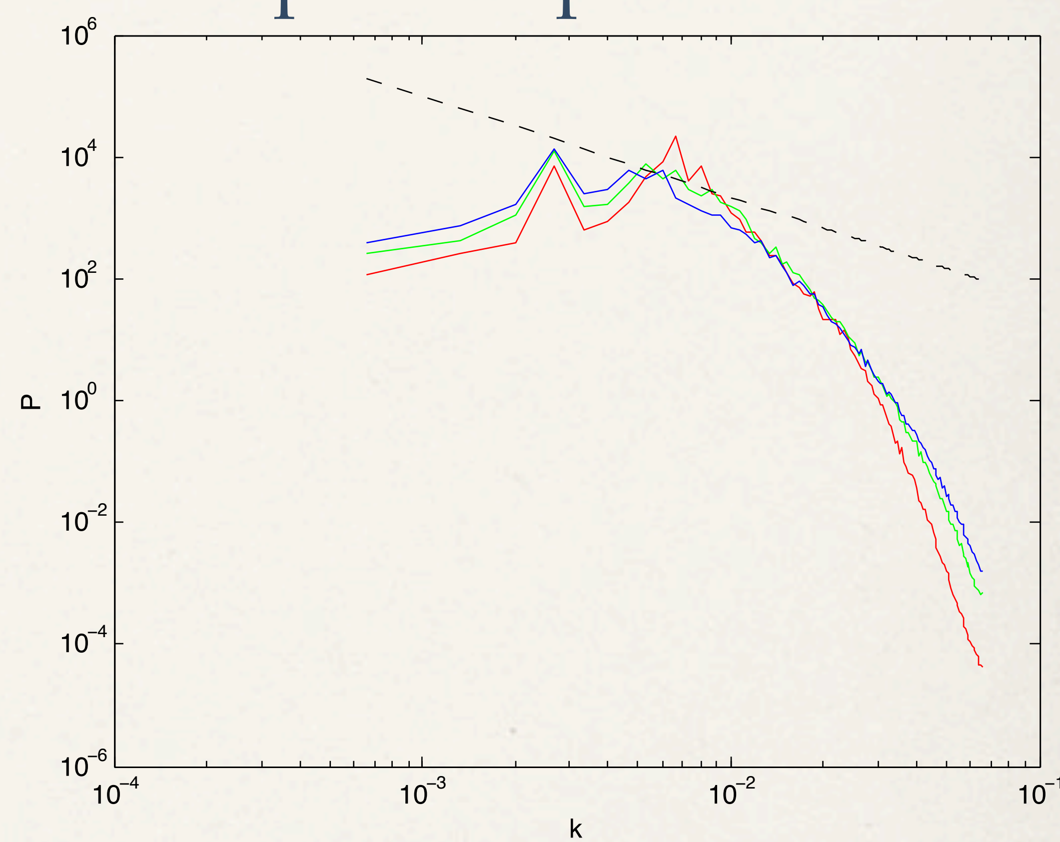

0.0219
0.0157
0.00940
0.00313
-0.00314
-0.00942
-0.0157
-0.0220

gravity simulation

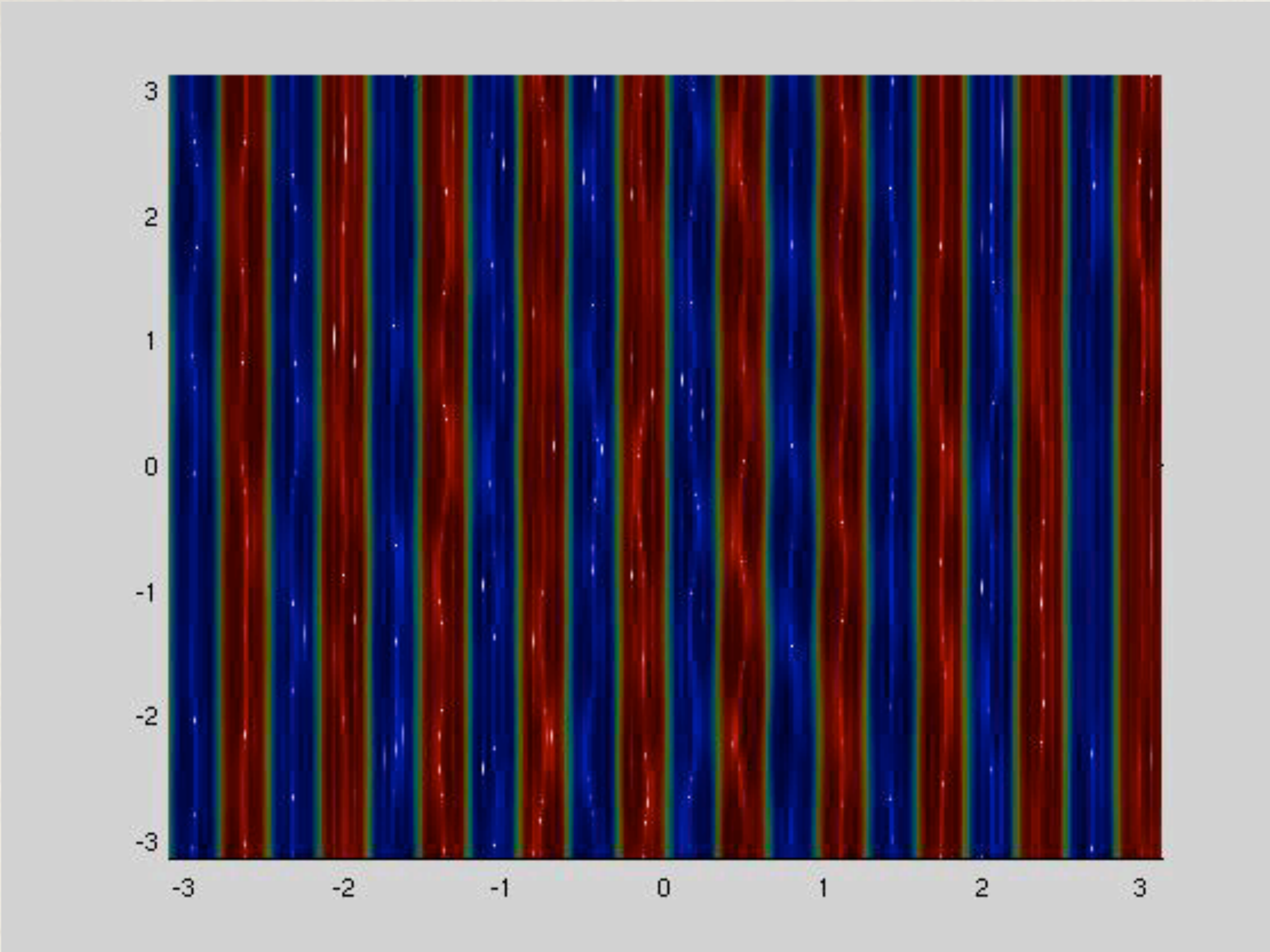
$$P(t,k) \equiv \frac{\partial}{\partial k}\int_{|k'|\leq k}\frac{d^2k'}{(2\pi)^2}|\tilde{u}(t,k')|^2$$

$\tilde{u}$ spatial Fourier transform of the four velocity u

power spectrum



red earliest
blue latest
dashed line: Kolmogorov scaling



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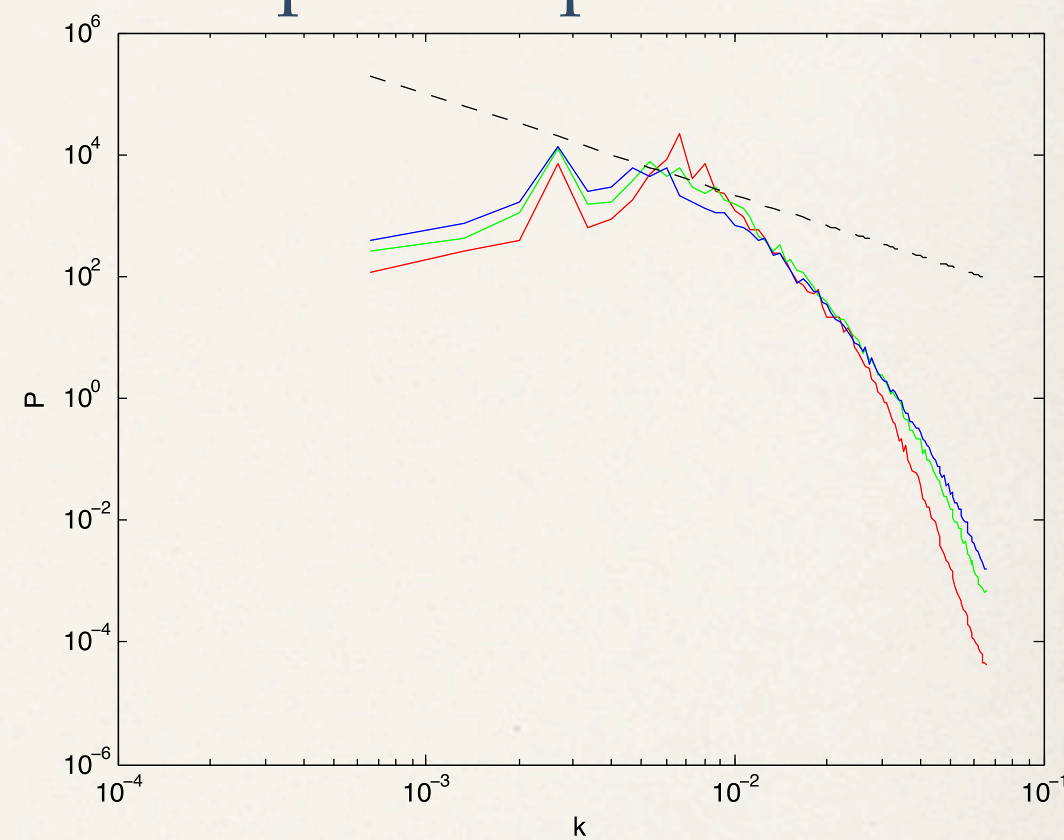


red earliest

blue latest

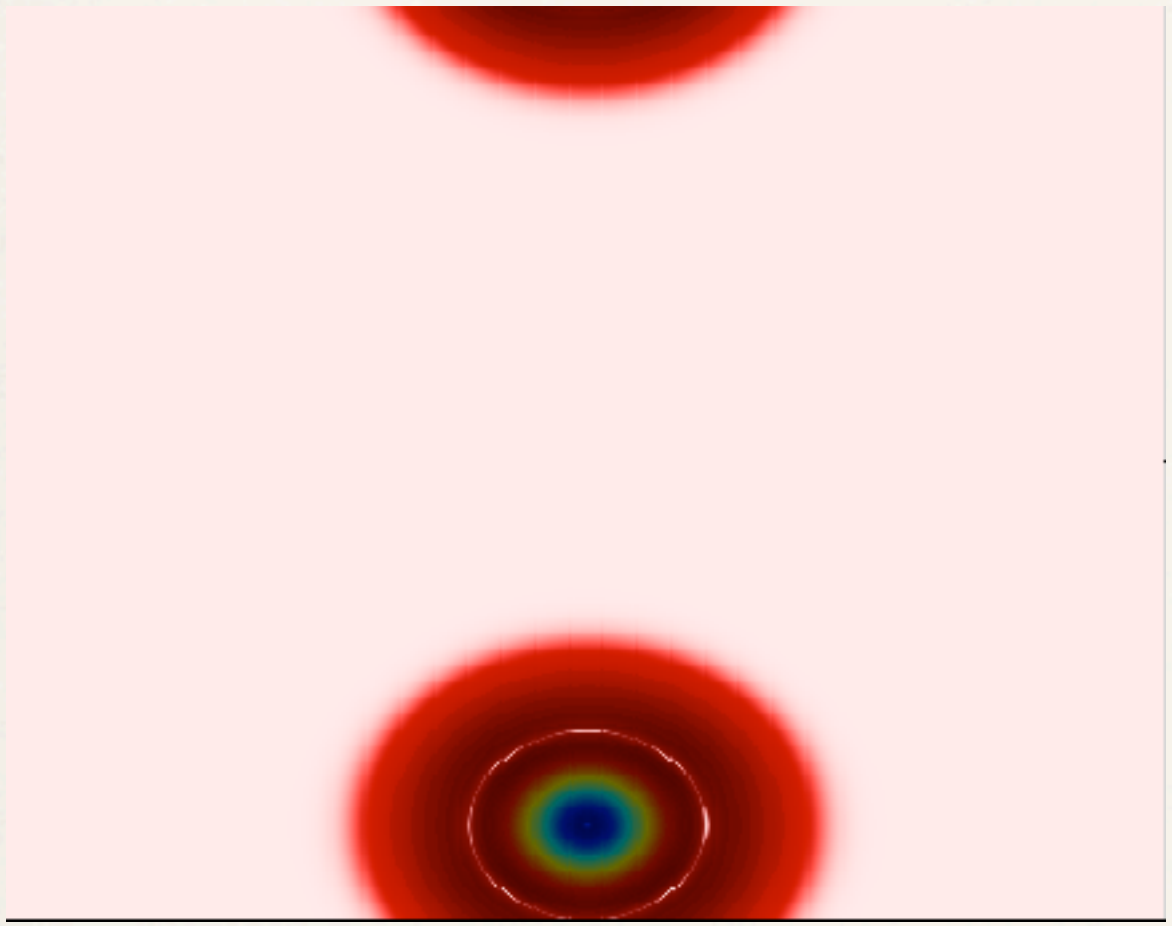
dashed line: Kolmogorov scaling

Dragging a ball

Source moving close to the speed of sound. The metric source produces a “cold ball”.

ABJM plasma at second order in the gradient expansion: Temperature response for a moving circular g_{tt} source.

Dragging a ball



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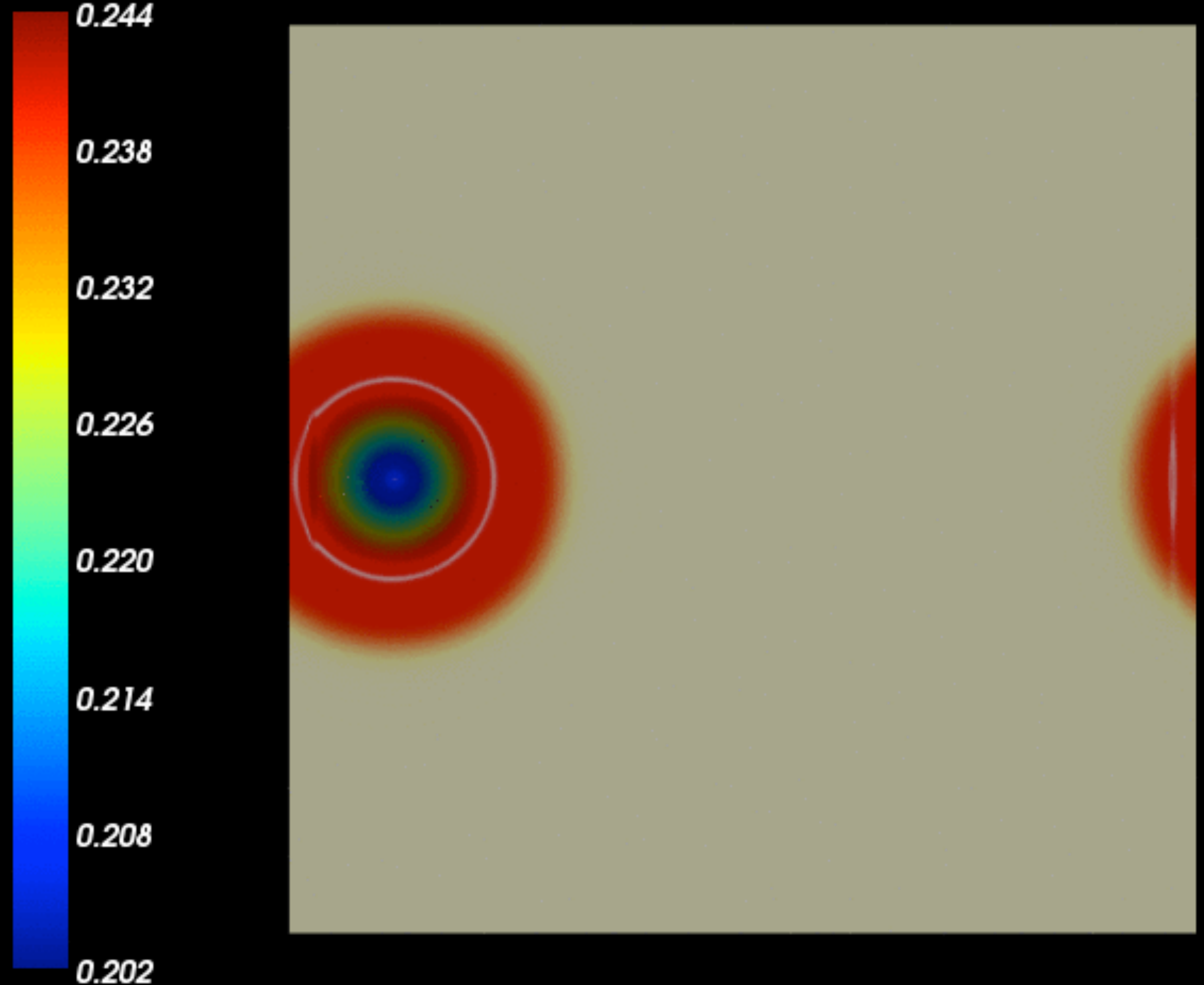
0.244
0.238
0.232
0.226
0.220
0.214
0.208
0.202

Relativistic Hydro Code

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} + \Pi^{\mu\nu}$$

1st and 2nd order dissipative terms

energy density

metric

pressure

four velocity

Relativistic Hydro Code

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} + \Pi^{\mu\nu}$$

$$\begin{aligned}\Pi^{\mu\nu} &= -\eta\sigma^{\mu\nu} - \tau_\Pi \left[D\Pi^{\langle\mu\nu\rangle} + \frac{d}{d-1}\Pi^{\mu\nu}(\nabla\cdot u)\right] \\ &\quad + \kappa\left[R^{\langle\mu\nu\rangle} - (d-2)u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta\right] \\ &\quad + \frac{\lambda_1}{\eta^2}\Pi^{\langle\mu}{}_\alpha\Pi^{\nu\rangle\alpha} - \frac{\lambda_2}{\eta}\Pi^{\langle\mu}{}_\alpha\Omega^{\nu\rangle\alpha} + \lambda_3\Omega^{\langle\mu}{}_\alpha\Omega^{\nu\rangle\alpha} \ .\end{aligned}$$

Baier et al. '07

In 3d λ_1 and λ_3 vanish.

$\Omega^{\mu\nu}$ is the vorticity.

$u_\mu\Pi^{\mu\nu} = 0$ and $\Pi^\mu_\mu = 0$ mean $\Pi^{\mu\nu}$ has only two independent components.

For the ABJM plasma, the first and second order transport coefficients can be looked up (for example in Raamsdonk 08)

Relativistic Hydro Code

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} + \Pi^{\mu\nu}$$

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Numerical Strategy for 3d Hydro

- Take T, u^x, u^y, $\Pi^{xx} - \Pi^{yy}$, and Π^{xy} as the independent variables X.
- Write stress tensor conservation and the definition of $\Pi^{\mu\nu}$ as a 5x5 matrix equation

$$\frac{dX}{dt} = M(X)$$

- Propagate using your favorite numerical scheme.

Romatschke

Setting up a gravity code

$$ds^2 = -\left(e^{2\beta}Vz - \frac{h_{AB}U^AU^B}{z^2}\right)dt^2 - \frac{2e^{2\beta}}{z^2}dt\,dz$$
$$- \frac{2h_{AB}U^B}{z^2}dt\,dx^A + \frac{h_{AB}}{z^2}dx^A\,dx^B \ .$$

$$h = e^{2\chi}\begin{pmatrix} e^{\alpha}\cosh\theta & \sinh\theta \\ \sinh\theta & e^{-\alpha}\cosh\theta \end{pmatrix} \ .$$

Characteristic Formulation: Inspired by a Living Review article by Winicour. Very close to the scheme used by Chesler and Yaffe. While they set β=0, we fix the form of χ.

More on Numerical Schemes

- As pointed out by Winicour, the metric ansatz allows for a nested solution scheme.
- In hydro, we had to propagate all five variables at once.
- Here, we only need propagate h_{AB} and the boundary values of U^A in time. That's two bulk fields and three boundary fields instead of eight bulk fields! At any given time step, the remaining five bulk fields are reconstructed by solving ODEs in the radial direction.

What we've done

- No sources. We can reproduce the counterflow experiment. (Carrasco, Lehner, Myers, et al. '12; Adams, Chesler, and Liu '13). In this experiment, hydro is close to gravity by the fluid gravity correspondence (Hubeny, Minwalla, Rangamani, et al. '07).
- Sources turned on. We can now also turn on a source for g_{tt}. Allows us to reproduce several classic hydro experiments: flow over a rough surface, flow past a compact object, driven turbulence, etc.

From Movies to Science

- While the movies are nice, what science can we extract?
- One interesting problem is breaking translation symmetry. How to get a finite DC conductivity.
- Many have focused on ionic lattices and scalar fields.
- We can of course also consider sourcing the energy density through g_{tt}.

Memory Function Formalism

- Described in Forster's book on hydrodynamics.
- Introduced to AdS/CFT by Hartnoll, Kovtun, Mueller, Sachdev '07. Way of calculating a momentum relaxation time.
- Later uses in AdS/CFT context include Hartnoll and H '08, Hartnoll and Hofman '12.

$$\delta S = g \int \mathcal{O}(x) e^{ikx} dx \ .$$

$$\frac{1}{\tau} = \frac{g^2 k^2}{\epsilon + p} \lim_{\omega \to 0} \frac{\operatorname{Im} G^R_{\mathcal{O}\mathcal{O}}(\omega, k)}{\omega} \Bigg|_{g=0}$$

Memory Method, Conductivity, and Hydro

Stress tensor conservation with Joule heating term.

$$\partial_\mu T^{\mu i} = F^{i\mu} J_\mu$$

stress tensor

EM field

charge current

Memory Method, Conductivity, and Hydro

Stress tensor conservation with Joule heating term.

$$\partial_\mu T^{\mu i} = F^{i\mu} J_\mu$$

For small velocities, in the presence of a relaxation time, one finds

$$(\epsilon + p)\dot{v} \approx -\frac{1}{\tau}(\epsilon + p)v + En$$

energy density

pressure

velocity

electric field

charge density

Memory Method, Conductivity, and Hydro

Stress tensor conservation with Joule heating term.

$$\partial_\mu T^{\mu i} = F^{i\mu} J_\mu$$

For small velocities, in the presence of the lattice, one then finds

$$(\epsilon + p)\dot{v} \approx -\frac{1}{\tau}(\epsilon + p)v + En$$

Comparing with Ohm's Law, one deduces the Drude like form

$$\sigma_{DC} = \frac{n^2 \tau}{\epsilon + p}$$

Metric Lattice

εε correlator has a universal form (e.g. Kovtun '12)

$$G^R_{\epsilon\epsilon} = \frac{k^2(\epsilon + p)}{k^2(c_s^2 + i\Gamma\omega) - \omega^2} + \epsilon$$

speed of sound

$$\Gamma = \frac{1}{\epsilon + p}\left(\frac{2(d-1)}{d}\eta + \xi\right)$$

shear viscosity

bulk viscosity

Metric Lattice

εε correlator has a universal form (e.g. Kovtun '12)

$$G^R_{\epsilon\epsilon} = \frac{k^2(\epsilon + p)}{k^2(c_s^2 + i\Gamma\omega) - \omega^2} + \epsilon$$

From which we may deduce

metric coupling

cosine

$$\frac{1}{\tau} = \frac{g^2 k^2}{4sTc_s^4}\left(\frac{2(d-1)}{d}\eta + \xi\right)\cdot\frac{1}{4}\cdot 2$$

Becomes small in incompressible limit.

entropy density

Metric Lattice

εε correlator has a universal form (e.g. Kovtun '12)

$$G^R_{\epsilon\epsilon} = \frac{k^2(\epsilon + p)}{k^2(c_s^2 + i\Gamma\omega) - \omega^2} + \epsilon$$

From which we may deduce

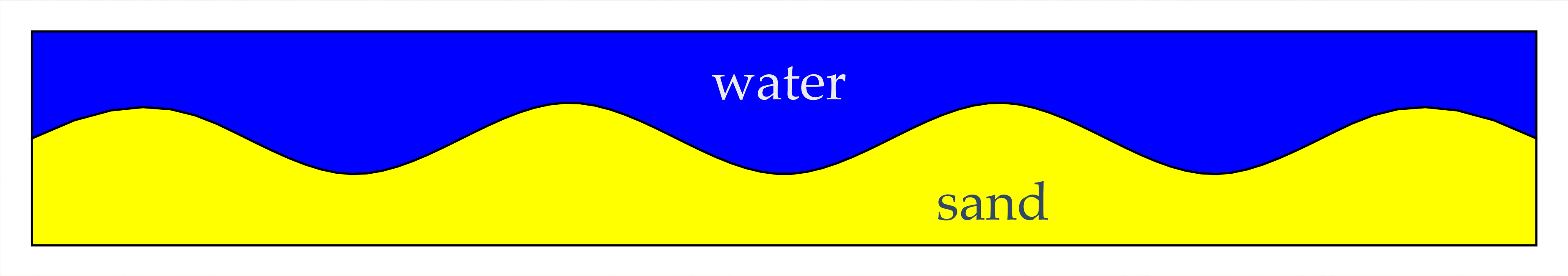
$$\frac{1}{\tau} = \frac{g^2 k^2}{4sTc_s^4}\left(\frac{2(d-1)}{d}\eta + \xi\right) \cdot \frac{1}{4} \cdot 2$$

For AdS/CFT (conformal case)

$$\frac{1}{\tau} = \frac{g^2 k^2}{8\pi T}\frac{d(d-1)}{2}$$

Metric Lattice

$$\frac{2\pi}{k}$$

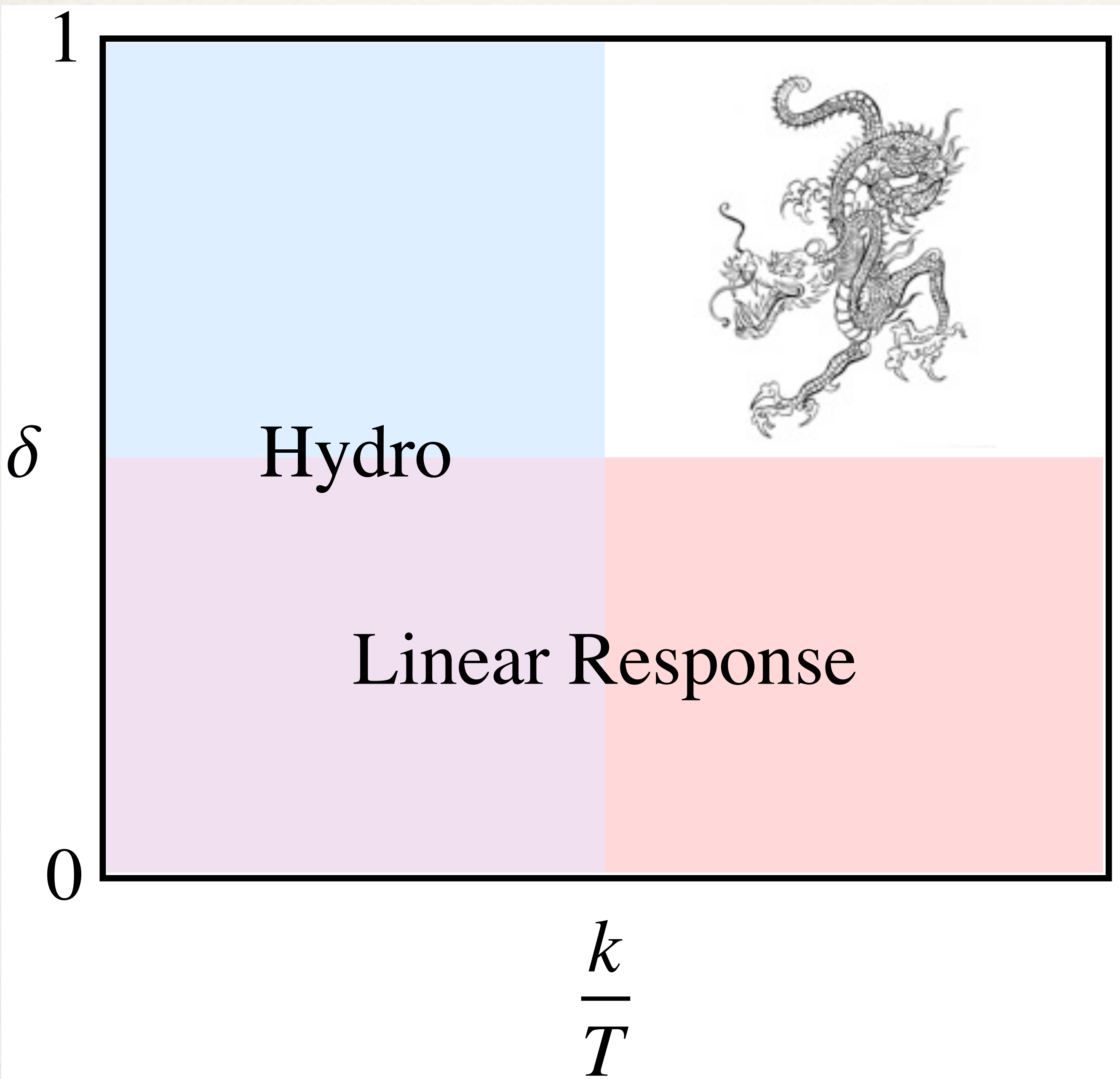


- Take $g_{tt} = -1 + \delta \cos(kx)$. Like a shallow sea with a varying depth.
- $|g_{tt}| > 1$ is "shallower": less density, colder temperatures

$$\epsilon \sim T^d$$

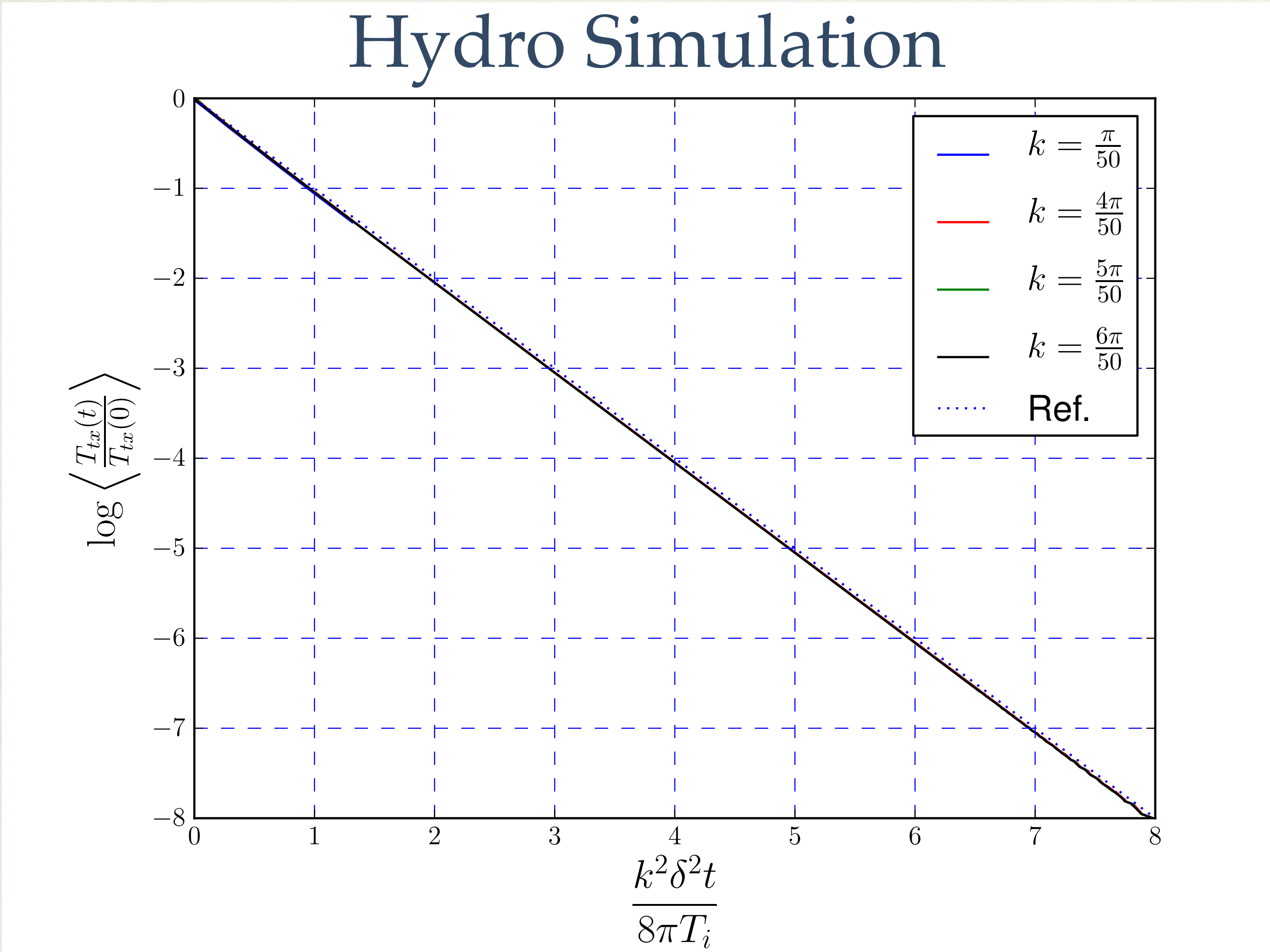
- $|g_{tt}| < 1$ is "deeper": more density, hotter temperatures
- Set up an experiment with a constant flow at time $t=0$. Four regimes, based on relative values of k/T and δ.

Four Regimes



$g_{tt} = -1 + \delta \cos(kx)$

Linear Response and Hydro Valid

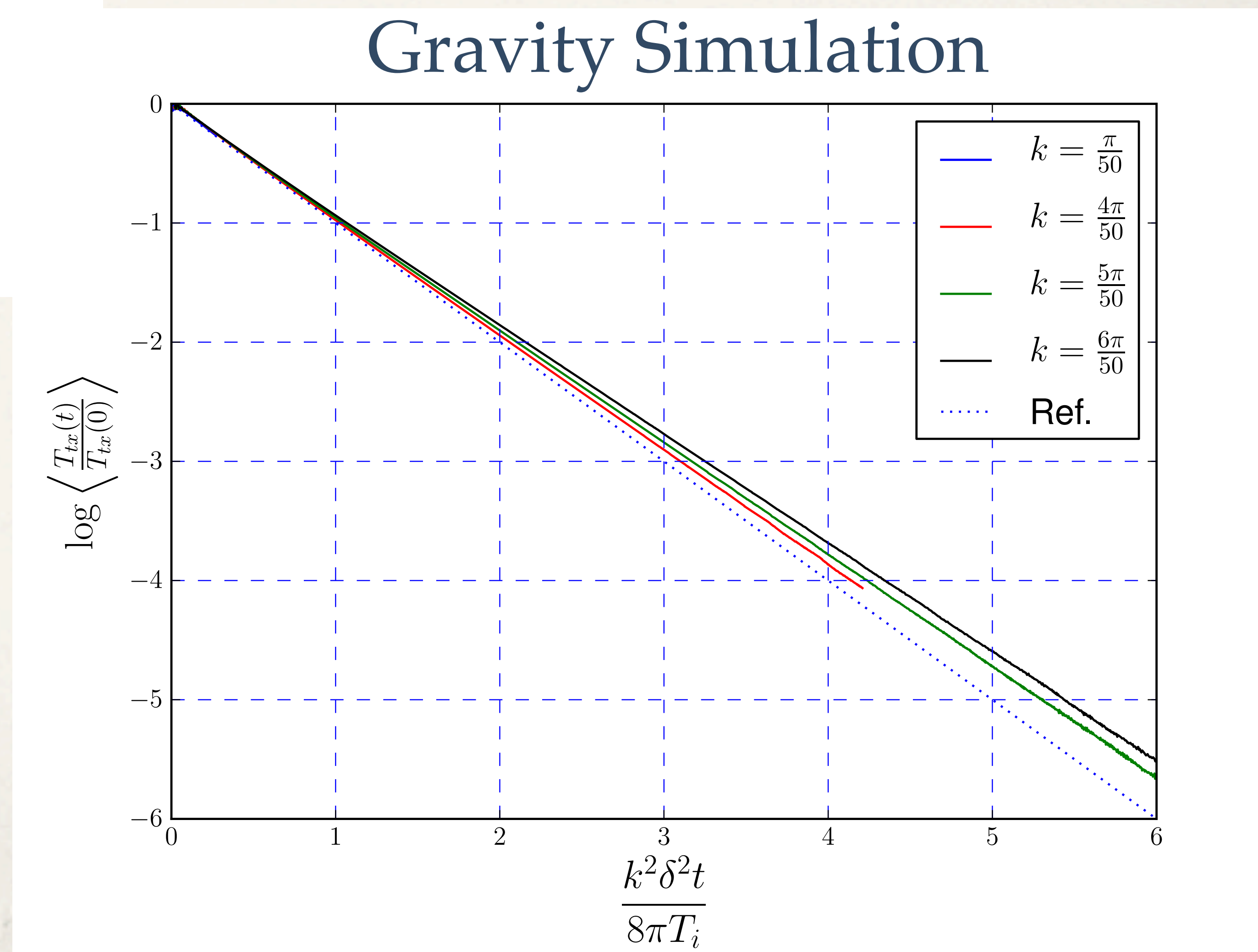


Dashed line predicted by hydro in the linear response regime.

$\delta = 0.2$

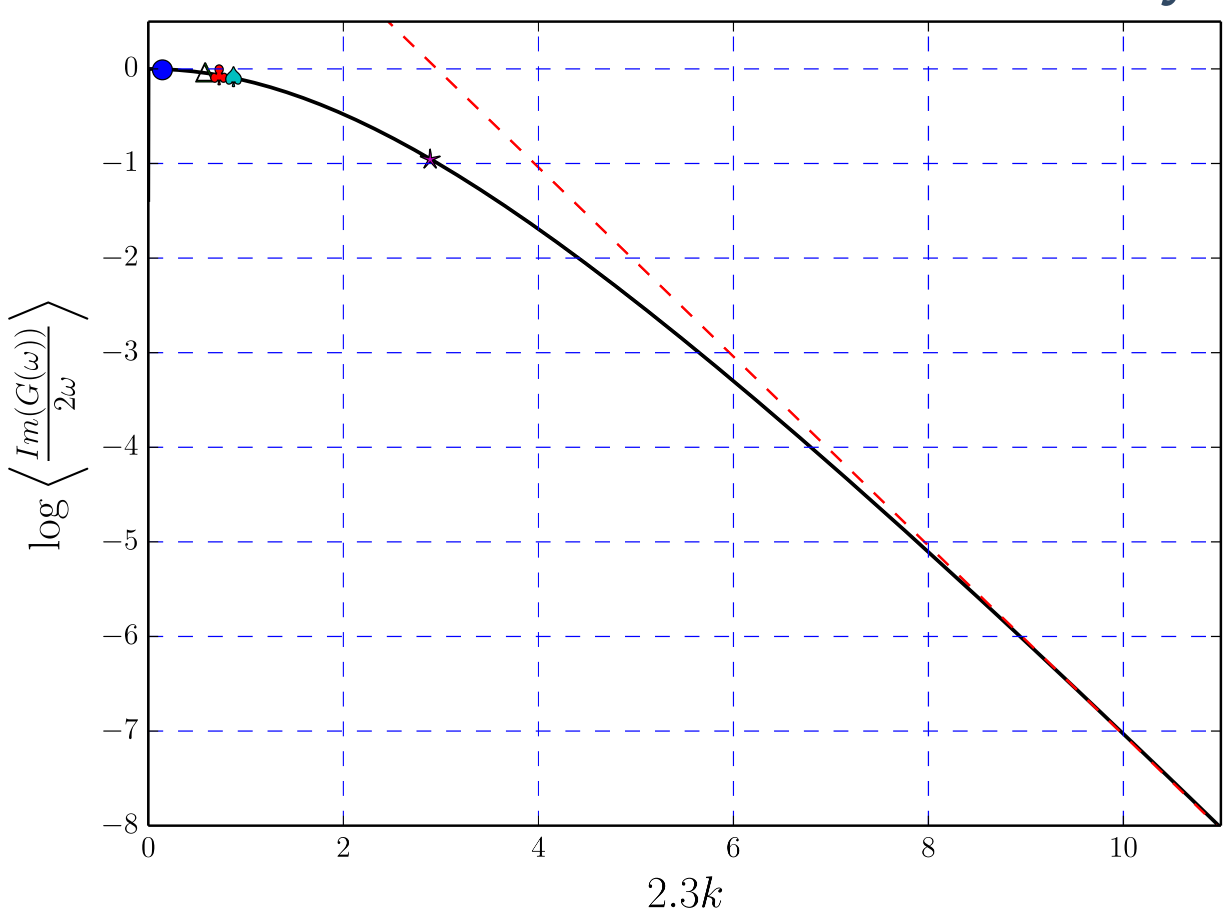
$v = 0.2$

$k/T_i = 0.015, 0.06, 0.075, 0.09$



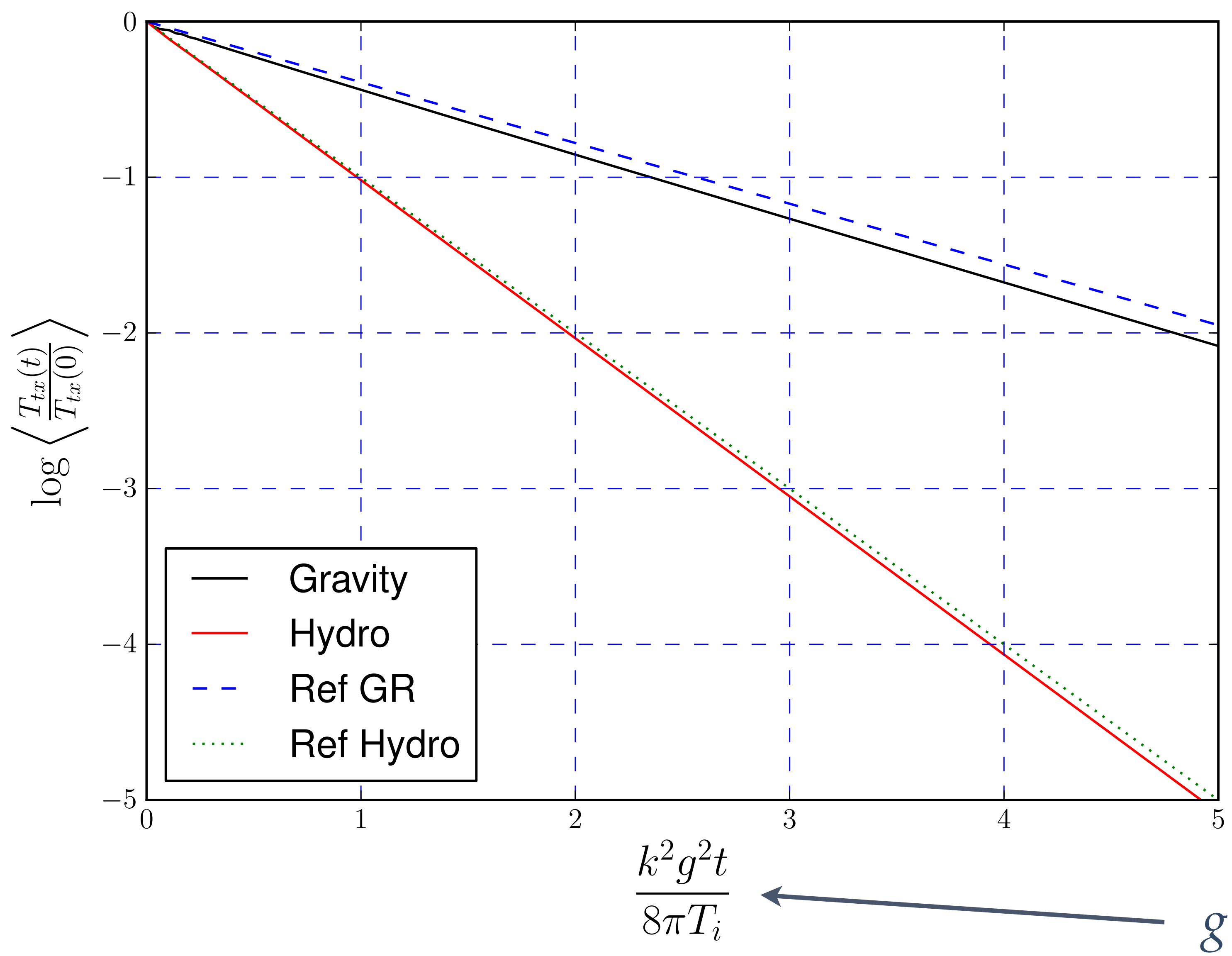
Linear Response and Gravity

Green's Function from Gravity



- The first four points come from the slopes of the curves on the previous slide.
- The solid line is the prediction from AdS/CFT and the memory function formalism.
- One could also ask what happens for larger *k*/*T*.

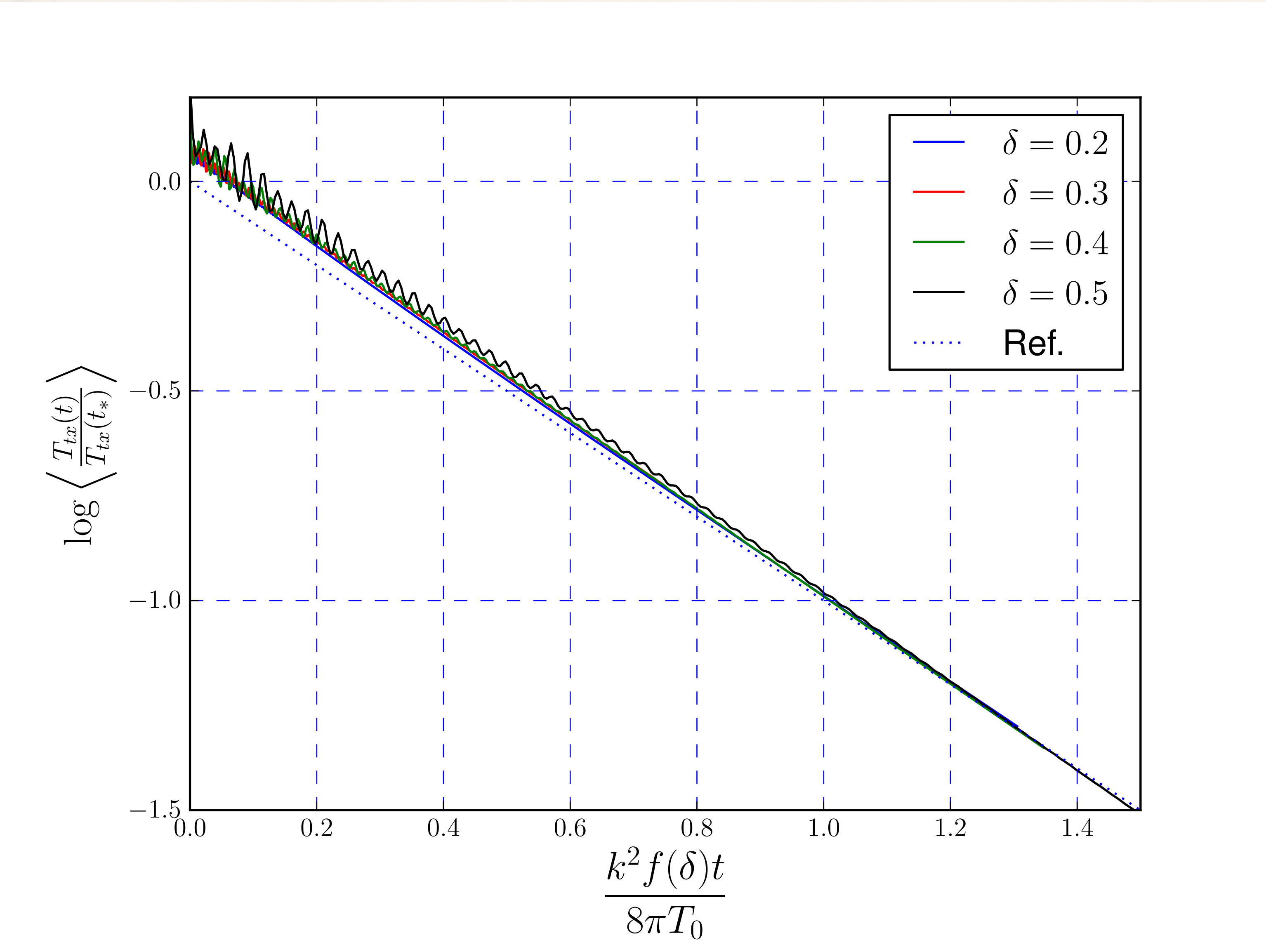
Linear Response Valid, Hydro Invalid



$k/T_i = 0.3$

g should be δ

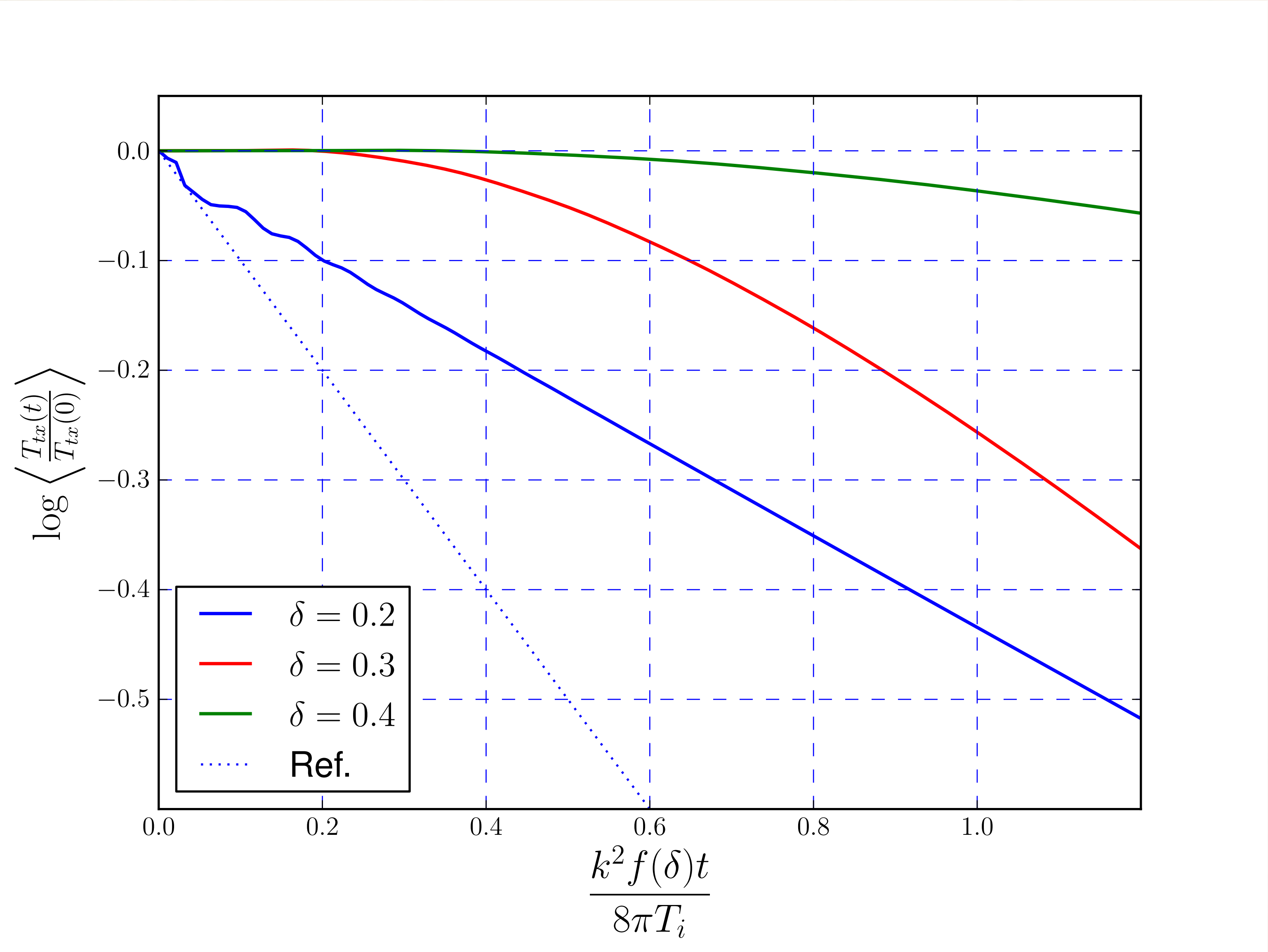
Hydro Valid, Linear Response Invalid



Hydro predicts

$$f(\delta) = 2 - 2\sqrt{1 - \delta^2}$$

Hydro and Linear Response Invalid



Gravity simulations with $k/T_i = 0.3$ and $v = 0.2$

Dashed line is the hydro prediction.

Additional time scale from time dependence of g_{tt}: blue curve 0.21, red curve 0.48, green curve 0.88

Summary of results

- Memory method valid at small δ and arbitrary *k*. An AdS/CFT Green's function calculation indicates $\log(\tau) \sim k$ for large *k*.

- Hydro valid for small k and arbitrary δ. For larger δ, the linear response dependence $\delta^2/2$ in relaxation time is replaced by

$$1 - \sqrt{1 - \delta^2}$$

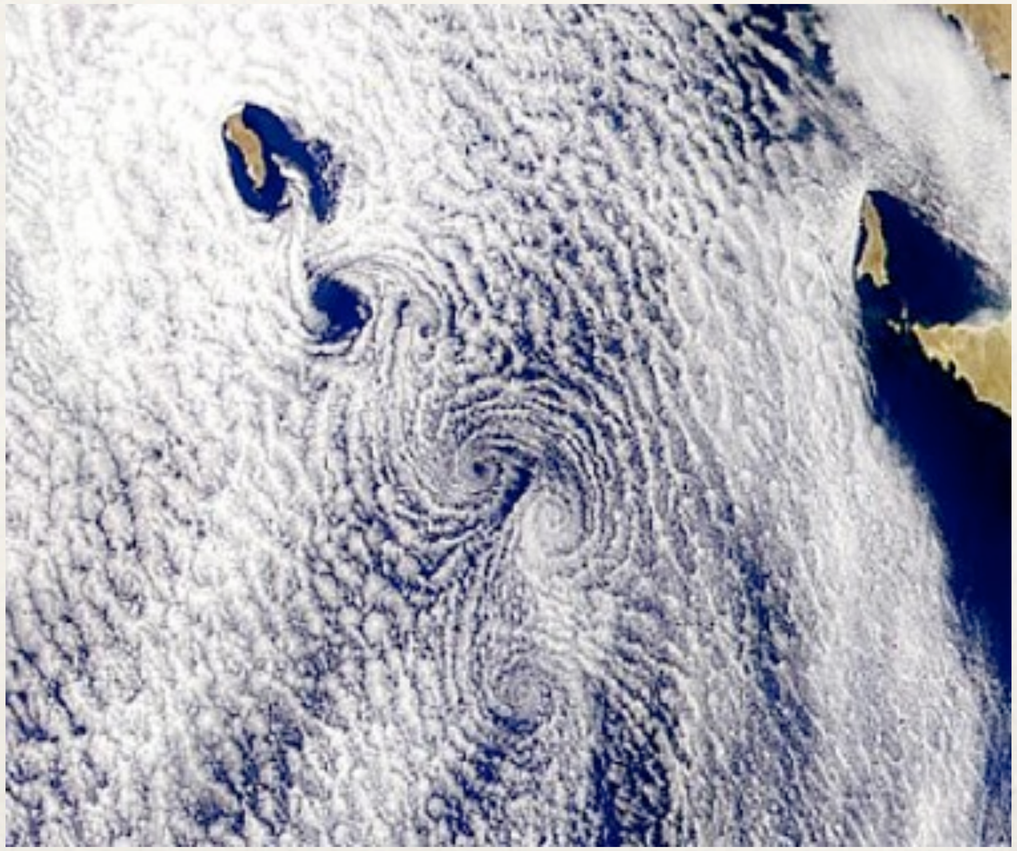
- In hydro limit, long time temperature dependence approaches

$$T(x) = \frac{T_0}{\sqrt{|g_{tt}|}}$$

- For large *k* and δ, need our full numerical gravity code.

What's next

- Random metric perturbations and impurities.
- Time dependent metric perturbations and driven turbulence.
- Moving objects, shock waves, and vortices; the role of roughness.



Ionic Lattice and Hydro

$$\lim_{\omega\to 0} \mathrm{Im}\, \frac{G^R_{nn}(\omega,k)}{\omega} = -\frac{1}{w^2}\left[\left(\frac{\partial}{\partial\mu}\frac{s}{n}\right)^2 \frac{n^4T^2}{k^2\sigma} + (\mathcal{D}n)^2\left(\frac{2(d-1)}{d}\eta+\xi\right)\right]$$

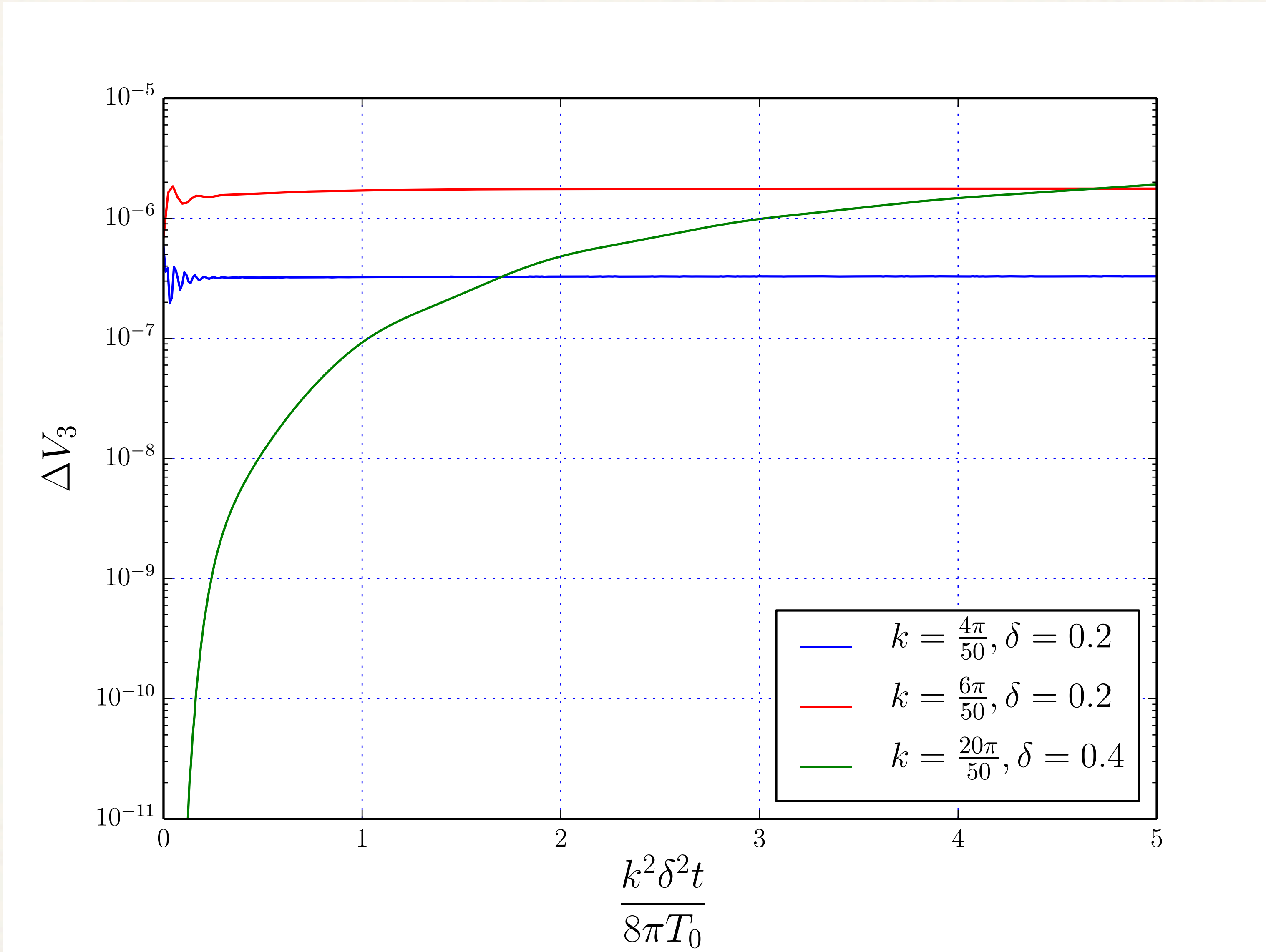
$$\lim_{\omega\to 0} \mathrm{Im}\, \frac{G^R_{\epsilon\epsilon}(\omega,k)}{\omega} = \frac{1}{w^2}\left[\left(\mathcal{D}\frac{s}{n}\right)^2 \frac{n^4T^3}{k^2\sigma} + (\mathcal{D}\epsilon)^2\left(\frac{2(d-1)}{d}\eta+\xi\right)\right]$$

where $\mathcal{D} \equiv T\partial_T + \mu\partial_\mu$

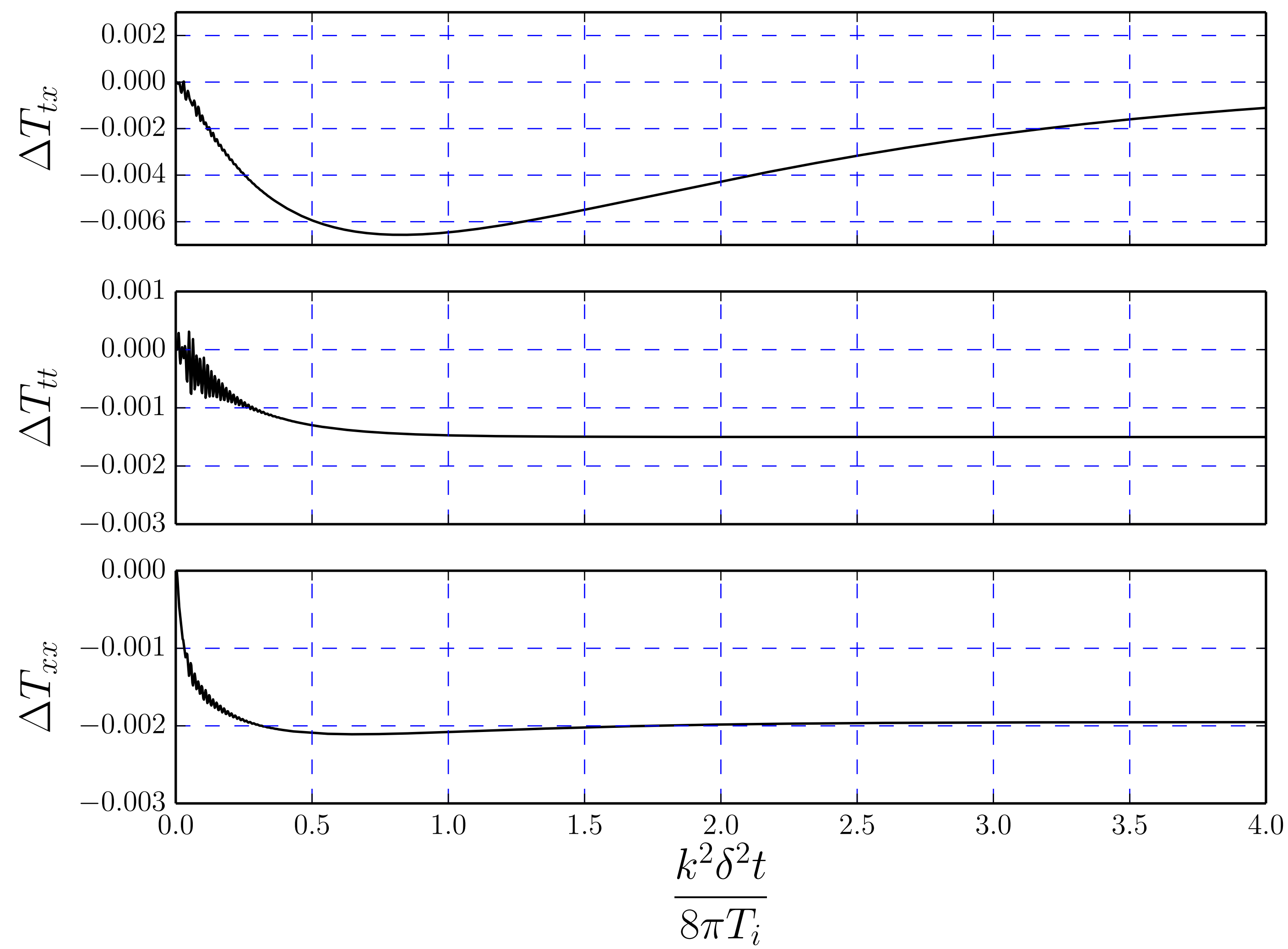
- Relativistic analog of a result from Andreev, Kivelson, Spivak '12.
- k^2 term likely contaminated by second order gradients in current.
- However, for a CFT, the leading term in $G^R_{\epsilon\epsilon}(\omega,k)$ vanishes, leaving

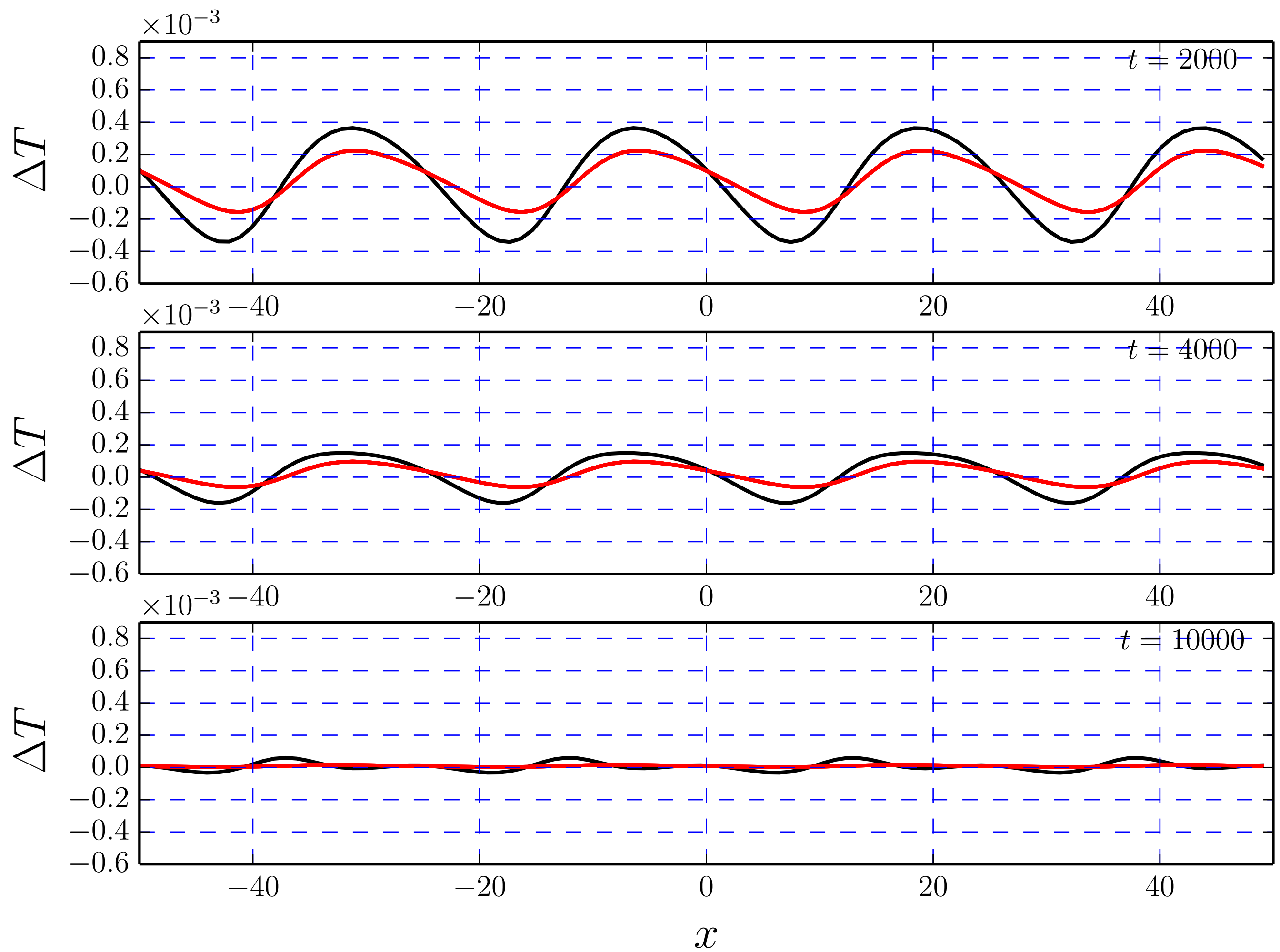
$$\lim_{\omega\to 0} \mathrm{Im}\, \frac{G^R_{\epsilon\epsilon}(\omega,k)}{\omega} = 2d(d-1)\eta$$

see also Davison, Schalm, Zaanen, '13



10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
ΔV_3
0
1
2
3
4
5
$\frac{k^2\delta^2 t}{8\pi T_0}$
$k = \frac{4\pi}{50}, \delta = 0.2$
$k = \frac{6\pi}{50}, \delta = 0.2$
$k = \frac{20\pi}{50}, \delta = 0.4$


0.002
0.000
−0.002
−0.004
−0.006
ΔT_{tx}
0.001
0.000
−0.001
−0.002
−0.003
ΔT_{tt}
0.000
−0.001
−0.002
−0.003
ΔT_{xx}
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
$\frac{k^2\delta^2 t}{8\pi T_i}$



$\times 10^{-3}$
$t = 2000$
$t = 4000$
$t = 10000$
ΔT
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−40
−20
0
20
40
x