On M5 Branes

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Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, KM [arXiv:1307.7660] The general M5-brane superconformal Index

Hee-Cheol Kim, KM [arXiv:1210.0853] M5 brane theories on R x CP2

Hee-Cheol Kim, Seok Kim, Eunkyung Ko, KM [arXiv:1110.2175] On instantons as KK modes of M5 branes

Stefano Bolognesi, KM [arXiv:1105.5073] 1/4 BPS string junctions and N3 problem in 6-dim conformal field theories

6d (2,0) Superconformal Theories

- A, D, E type: type IIB on $R^{1+5} \times C^2/\Gamma_{ADE}$
  - A_{N-1}, D_N type on N M5 branes, N M5 (+OM5)
- superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times USp(4)_R$
- fields: B, $\Phi_{I\,(I=1,2,3,4,5)}$, Ψ : 3 + 5+ 8 d.o.f.
  - selfdual strength $H=dB={}^*H$, purely quantum $\hbar=1$
  - Weyl and symplectic-Majorana Ψ
- Nonabelian formulation?.
  - Sorokin, Chu, Ho, Lambert, Papageorgakis, Samtleben et.al, ...
  - Their 4d-reduction implies a local nonabelian field theory which has both electric and magnetic objects.
- N^3 degrees of freedom

Outline

- Goal: Calculate Something Concrete!
- 5d Approach: R^{1+4}
  - instantons
  - 1/4 BPS webs
  - index function of dyonic instantons
  - DLCQ
- 6d approach: $R \times S^5$
  - Twisting
  - Z_K-moding
- 5d approach: $R \times CP^2$
  - Minkowski and Euclidean Lagrangian
  - Index function
- Conclusion

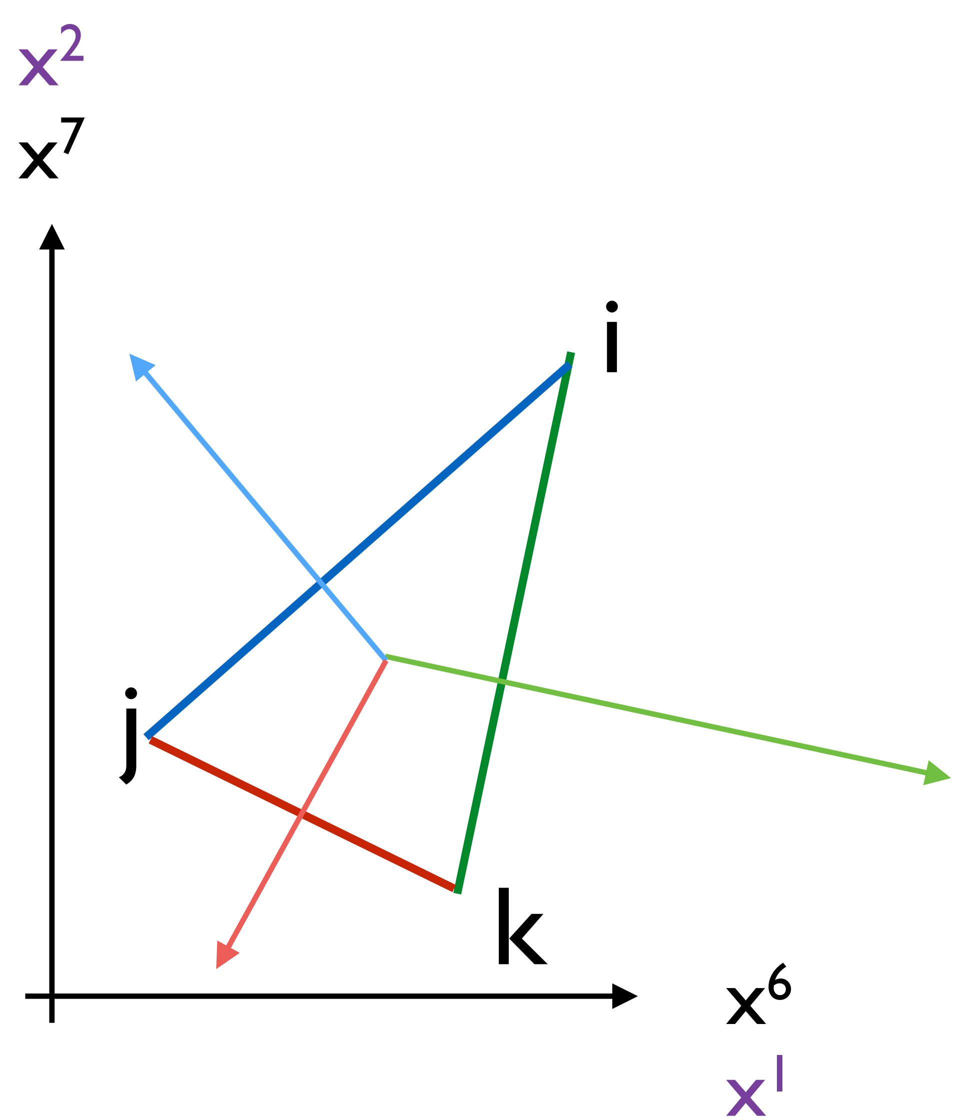
5-dim Approach

- Douglas, Lambert-Papageorgakis-Schmidt-Sommerfeld
- $x5 \sim x5 + 2\pi R$, 5d N=2 U(N) YM Theory
- Instantons= Kaluza-Klein modes , $4\pi / g^2 = 1/R$
  - threshold bound states of k instantons for Kaluza-Klein momentum k
  - strong coupling limit is 6d (2,0) theory
- perturbative approach: problem at 6-loop (Bern et.al. 1210): mysterious
  - incomplete? (Need higher order operators to complete the theory?)

1/4 BPS String Junctions

- Monopole strings, self-dual strings in Higgs phase
- count 1/2 BPS & 1/4 BPS objects: structure constants: f^{ijk} $f^{\alpha\text{-}\alpha h}$, $f^{\alpha\beta\gamma}$

$$\frac{N(N-1)}{2} + \frac{N(N-1)(N-2)}{6} = \frac{N(N^2-1)}{6} = h_G d_G$$



Dyonic Instantons in 5d N=2 SYM

- Index for BPS states with k instantons $Q = Q^{\dot{+}}_{\dot{+}}$ $\left.\begin{matrix} SU(2)_{2R} \\ SU(2)_{1R} \end{matrix}\right\} \Rightarrow SU(2)_R$

$$I_k(\mu^i, \gamma_1, \gamma_2, \gamma_3) = \mathrm{Tr}_k\left[(-1)^F e^{-\beta Q^2} e^{-\mu^i \Pi_i} e^{-i\gamma_1(2J_{1L}) - i\gamma_2(2J_{2L}) - i\gamma_R(2J_R)}\right]$$

adjoint hyper flavor

  - μ_i : chemical potential for $U(1)^N \subset U(N)_{\text{color}}$
  - $\gamma_1, \gamma_2, \gamma_R$: chemical potential for $SU(2)_{1L}$, $SU(2)_{2L}$, $SU(2)_R$

- calculate the index by the localization: $I(q, \mu^i, \gamma_{1,2,3}) = \sum_{k=0}^{\infty} q^k I_k$

- 5d *N=2** instanton partition function on $R^4 \times S^1$: $t \sim t + \beta$

- In $\beta \to 0$ and small chemical potential limit, the index becomes 4d Nekrasov instanton partition function :

$$a_i = \frac{\mu_i}{2} \qquad -\epsilon_1 = i\frac{\gamma_1 - \gamma_R}{2} \qquad \epsilon_2 = i\frac{\gamma_1 + \gamma_R}{2}, \quad m = i\frac{\gamma_2}{2} \qquad q = e^{2\pi i \tau}$$

Scalar Vev

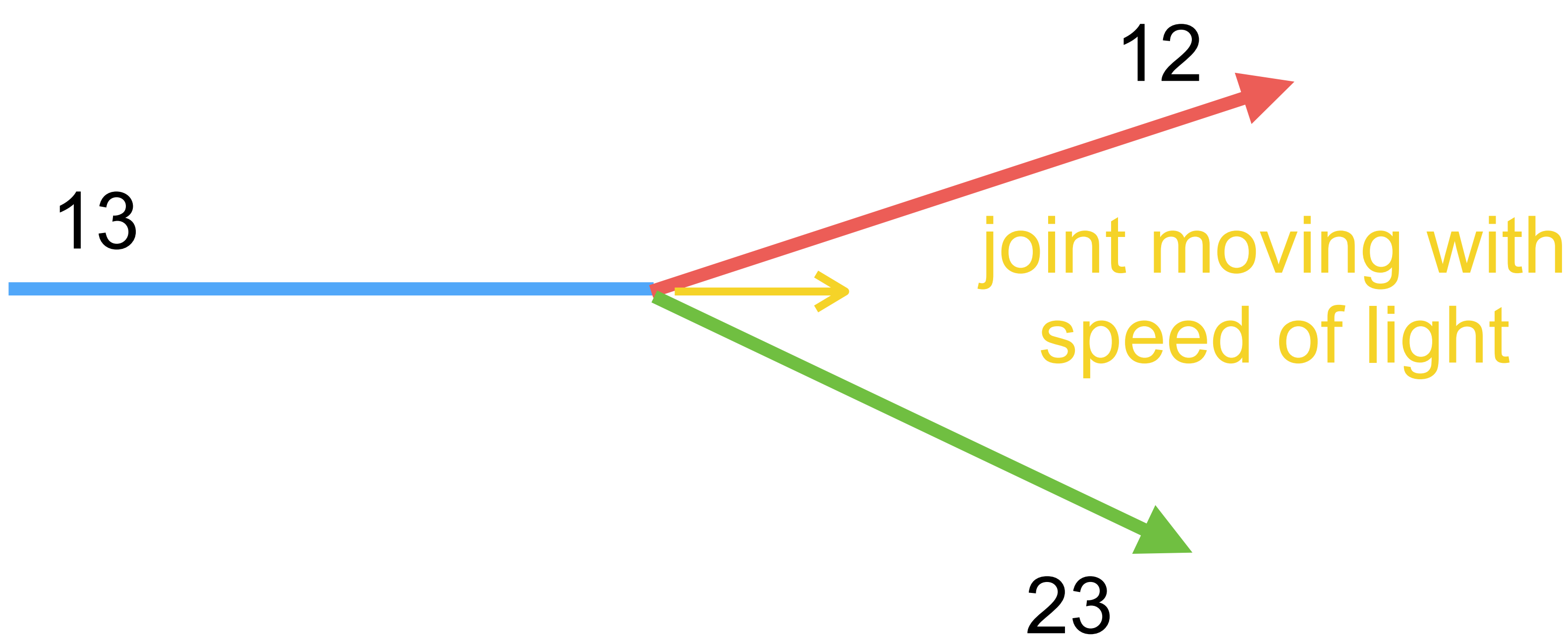
Omega deformation parameter

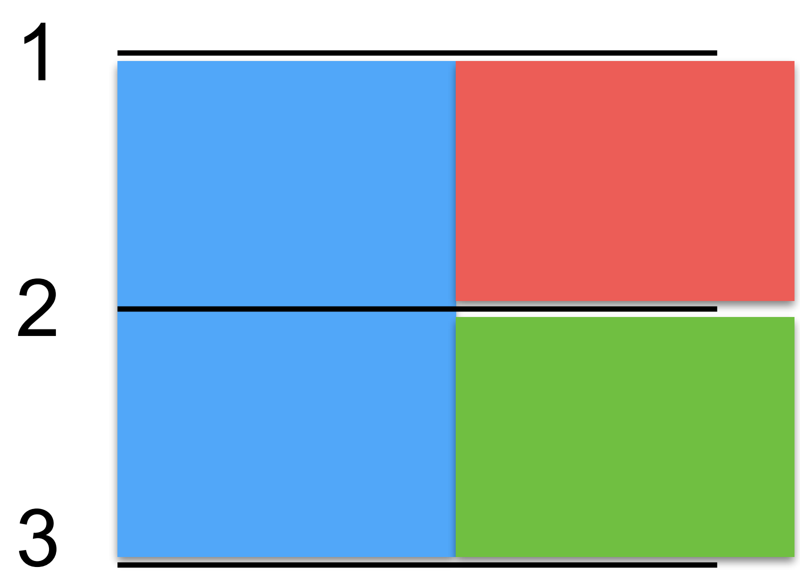
Adj hypermultiplet mass

instanton fugacity

Most striking aspect

- Additional States appearing in dyonic instanton calculations
- Can be regarded as the degenerate limit of junctions
- There is a threshold bound states of W-boson for 13 and instantons on 2nd M5 branes



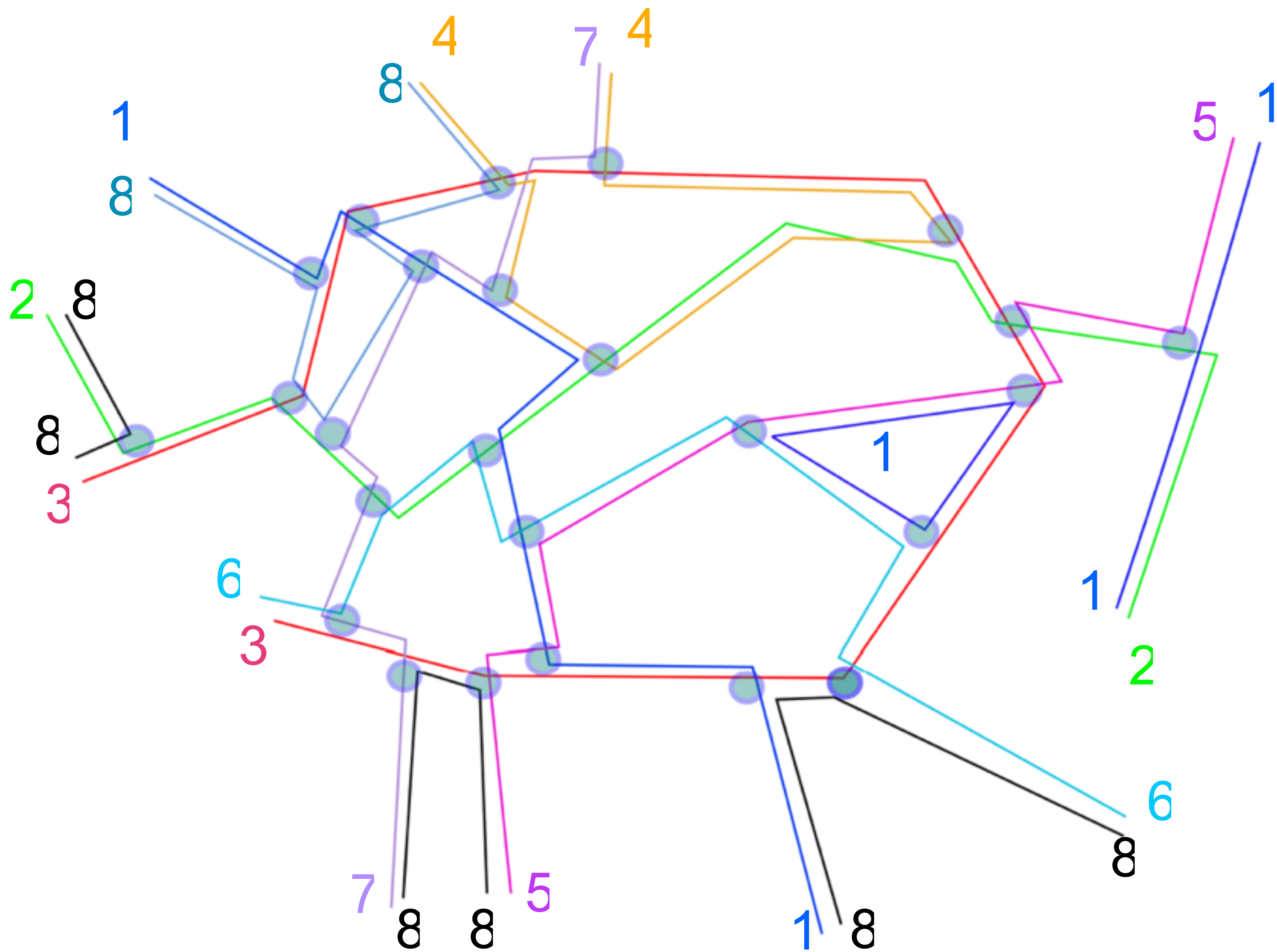


High Temperature in Coulomb Phase

- Micro-canonical
- Massless on N M5 branes: O(N)
- Loops of self-dual strings excitations: $O(N^2)$
- Beyond the Hagedorn temperature
- Webs of junctions and anti-junctions: $O(N^3)$
- Higher order of junctions can be decomposed to elementary junctions
- Excitations of webs of tensionless strings with junctions act as atoms.

- N^3 degrees of freedom

Nonzero Temperature in Symmetric Phase



DLCQ

[Aharony-Berkooz-Seiberg 97]
Berkooz, Rozali, Seiberg 1997, Berkooz, Douglas 1996

* To get the index in symmetric phase, integrate over $\mu_i = i\, a_i$ with Haar measure
* DLCQ on null circle: Nonrelativistic superconformal symmetry
  * P_- on the null circle = instanton number
  * Superalgebra: $2i\{Q, S\} = iD \mp (4J_{2R} + 2J_{1R}) \ \rightarrow \ iD \geq \pm(4J_{2R} + 2J_{1R})$
* Nonrelativistic superconformal index

$$I_{SC} = \mathrm{Tr}\left[(-1)^F e^{-\beta\{\hat{Q},\hat{S}\}} e^{-2i\gamma_R J_R} e^{-2i\gamma_1 J_{1L} - 2i\gamma_2 J_{2L}} e^{-i\alpha_i \Pi_i}\right]$$

* In the limit $\beta \to 0$, this superconformal index becomes the integration over the Coulomb phases of our index.
* For single instanton with $t = e^{-i\gamma_R}$

$$I_{k=1} = \frac{e^{i\gamma_2} + e^{-i\gamma_2} - e^{i\gamma_1} - e^{-i\gamma_1}}{(1 - te^{i\gamma_1})(1 - te^{-i\gamma_1})}\left[t + \sum_{n=1}^{N-1}(e^{in\gamma_2} + e^{-in\gamma_2})t^{n+1} - \chi_{\frac{N-2}{2}}(\gamma_2)t^{N+1}\right]$$

* Large N

$$I_{N\to\infty,k=1} = \frac{e^{i\gamma_2} + e^{-i\gamma_2} - e^{i\gamma_1} - e^{-i\gamma_1}}{(1 - te^{i\gamma_1})(1 - te^{-i\gamma_1})} \frac{t - t^3}{(1 - te^{i\gamma_2})(1 - te^{-i\gamma_2})}$$

AdS7 x S4 calculation confirm it.

6d (2,0) Theories

- Difficulties with nonabelianization of the B field and its strength H=dB
- Generalize ABJM to M5 brane theory
  - Mode out by R^8/Z_K
  - Weak coupling limit
  - No fixed point
- R^{1+5}/Z_K has a fixed point
- Consider the 6d (2,0) theory on $RxS^{5:}$ TheRadial Quantization
- S^5 = a circle fibration over CP^2
  - $ds^2_{S5} = ds^2_{CP2} + (dy + V)^2$, $dV = 2J$, $J = -*J$, $y \sim y + 2\pi$

- $AdS_7 \times S^4/Z_K$ (Tomasiello, 2013): 6d theory with D6 and <u>D6</u> : still 6d theory with H=*H

Index Function on $S^1 \times S^5$

- Supercharge $$Q^{R_1,R_2}_{j_1,j_2,j_3} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}},\ S = Q^\dagger$$

- BPS bound: $$E = j_1 + j_2 + j_3 + 2(R_1 + R_2)$$

- 6-dim index function:

$$I = \mathrm{Tr}\left[(-1)^F e^{-\beta'\{Q,S\}} e^{-\beta\left(E - \frac{R_1+R_2}{2} - m(R_1-R_2) + aj_1 + bj_2 + cj_3\right)}\right],\ a+b+c=0$$

- Euclidean Path Integral of (2,0) Theory on $S^1 \times S^5$
  - $S^5 = S^1$ fiber over CP^2 : $-i\,\partial_y$ = KK modes

$$k \equiv j_1 + j_2 + j_3$$

- Z_K modding keeps only k/K=integer modes

6d Abelian Theory (Fermion+ Scalar)

- on R x S^5, ……. (Could Include H=dB)

$$-\frac{i}{2}\bar{\lambda}\Gamma^M\hat{\nabla}_M\lambda-\frac{1}{2}\partial_M\phi_I\partial^M\phi_I-\frac{2}{r^2}\phi_I\phi_I$$

- gamma matrices Γ^M, ρ^a
- Symplectic Majorana $\lambda = -BC\,\lambda^*$, $\varepsilon = BC\,\varepsilon^*$
- Weyl: $\Gamma^7\,\lambda=\lambda$, $\Gamma^7\,\varepsilon = -\varepsilon$
- 32 supersymmetry

$$\begin{aligned}\delta\phi_I &= -\bar{\lambda}\rho_I\epsilon=+\bar{\epsilon}\rho_I\lambda,\\ \delta\lambda &= +\frac{i}{6}H_{MNP}\Gamma^{MNP}\epsilon+i\partial_M\phi_I\Gamma^M\rho_I\epsilon-2\phi_I\rho_I\tilde{\epsilon},\\ \delta\bar{\lambda} &= -\frac{i}{6}H_{MNP}\bar{\epsilon}\Gamma^{MNP}+i\partial_M\phi_I\bar{\epsilon}\Gamma^M\rho_I-2\bar{\tilde{\epsilon}}\rho_I\phi_I.\end{aligned}$$

- additional condition on Killing spinor:

$$\hat{\nabla}_M\epsilon=\frac{i}{2r}\Gamma_M\tilde{\epsilon},\quad \Gamma^M\hat{\nabla}_M\tilde{\epsilon}=2i\epsilon,\qquad \tilde{\epsilon}=\pm\Gamma_0\epsilon.$$

Twisting & Dimensional Reduction to R x CP^2

Killing spinor eq $\nabla_M \epsilon_\pm = \pm \frac{1}{2r}\Gamma_M \Gamma_\tau \epsilon_\pm$

- Killing spinors: SO(1,5)=SU(2,2) chiral spinor and 4-dim of Sp(2)=SO(5)$_R$
- 32 Killing spinors = 3x8 (SU(3) triplet) +1x 8 (SU(3) singlet) under SU(3) isometry of CP^2:
  - (I) $\varepsilon_+ \sim$ exp(-it/2 +3i y/2)… : singlet
  - (II) $\varepsilon_+ \sim$ exp(-it/2 - i y/2)… : triplet
- Twisting

$$
\begin{aligned}
\epsilon_{old} &= e^{-\frac{y}{4}M_{IJ}\rho_{IJ}}\epsilon_{new},\\
\lambda_{old} &= e^{-\frac{y}{4}M_{IJ}\rho_{IJ}}\lambda_{new},\\
(\phi_1 + i\phi_2)_{old} &= e^{+(3+p)iy/2}(\phi_1+\phi_2)_{new}\\
(\phi_4 + i\phi_5)_{old} &= e^{+(3-p)iy/2}(\phi_4+i\phi_5)_{new}.
\end{aligned}
$$

$$
M_{12} = -M_{21} = \frac{3+p}{2},\ M_{45} = -M_{54} = \frac{3-p}{2}
$$
$$
p = \cdots, -5, -3, -1, 1, 3, 5, \cdots .
$$

$$
\partial_y \to \partial_y + \frac{3i}{2}(R_1+R_2) + \frac{ip}{2}(R_1 - R_2)
$$

$$
k \equiv j_1 + j_2 + j_3 + \frac{3}{2}(R_1+R_2) + \frac{p}{2}(R_1-R_2),\ p = \text{odd integer}
$$

Singlets ε_+, ε_- for $Q = Q^{++}_{---}, S = Q^{--}_{+++}$

5d Lagrangian

$$Q = Q^{++}_{---},\ S = Q^{--}_{+++}$$

- Lagrangian on R x CP² with 2 supersymmetries for any p:

$$\begin{aligned} S \ = \ & \frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\sqrt{|g|}\ \mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta-\frac{2i}{3}A_\rho A_\sigma A_\eta\Big) \\ & -\frac{1}{2}D_\mu\phi_I D^\mu\phi_I+\frac{1}{4}[\phi_I,\phi_J]^2-2\phi_I^2-\frac{1}{2}(M_{IJ}\phi_J)^2-i(3-p)[\phi_1,\phi_2]\phi_3-i(3+p)[\phi_4,\phi_5]\phi_3 \\ & -\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda-\frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda]-\frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn}+\frac{1}{8}\bar\lambda M_{IJ}\rho_{IJ}\lambda\Big], \end{aligned} \tag{2.27}$$

- Supersymmetry Transformation

$$\begin{aligned} \delta A_\mu &= +i\bar\lambda\gamma_\mu\epsilon = -i\bar\epsilon\gamma_\mu\lambda, \quad \delta\phi_I = -\bar\lambda\rho_I\epsilon = \bar\epsilon\rho_I\lambda, \\ \delta\lambda &= +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon - 2\phi_I\rho_I\tilde\epsilon - M_{IJ}\phi_I\rho_J\epsilon. \end{aligned}$$

- p/2=-1/2 : $k = j_1+j_2+j_3+R_1+2R_2$
    - additional supersymmetries: Total 8 supersymmetries

$$Q^{+-}_{-++},\ Q^{+-}_{+-+},\ Q^{+-}_{++-} \quad \text{conjugates}$$

Coupling Constant Quantization

- Instanton number on CP2

$$\nu = \frac{1}{8\pi^2}\int_{\mathrm{CP}^2} \mathrm{Tr}(F\wedge F) = \frac{1}{16\pi^2}\int_{\mathrm{CP}^2} d^4x\sqrt{|g|}\ \mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

- Instantons represents the momentum K or energy K:

$$\frac{1}{g_{YM}^2} = \frac{K}{4\pi^2 r}$$

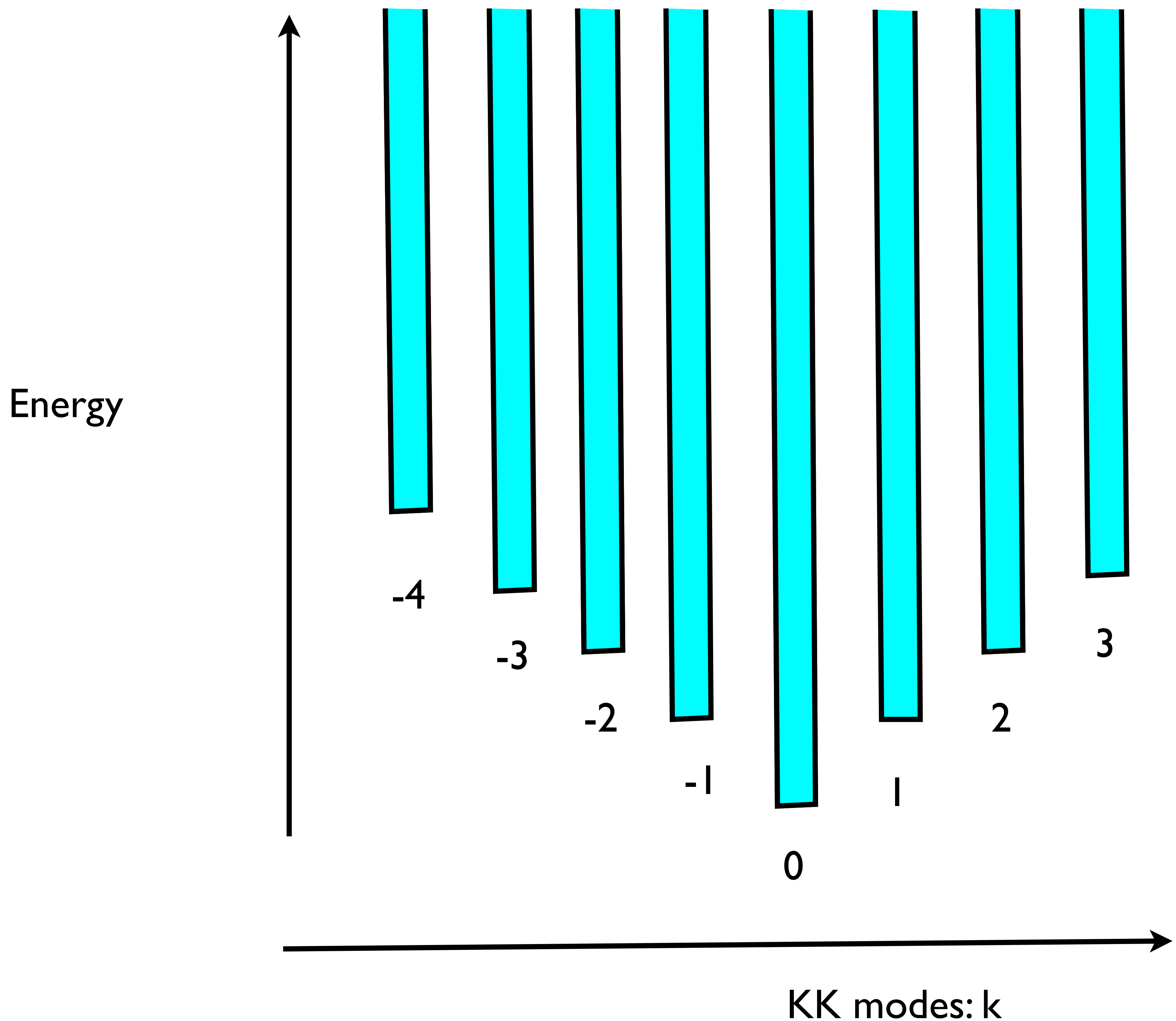
- Another approach to quantization: F=2J: 2π flux on a cycle, 1/2 instanton for abelian theory

$$\frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\ \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\partial_\rho A_\sigma A_\eta \ \Rightarrow K\int dt\ A_0$$

- ‘t Hooft coupling constant: λ= N/K

- Large K => Free Theory

Diluting degrees of freedom with Z_K modding+ Twisting



Expected Enhanced Supersymmetries

- Killing spinors with $p/2=-1/2$, $k = j_1+j_2+j_3+ R_1+ 2R_2$
  - $k=0$: 8 kinds
  - $k= \pm 1$: 14 kinds
  - $k= \pm 2$: 8 kinds
  - $k= \pm 3$: 2 kinds
- # of supersymmetries
  - $K \geqq 4$: 8 supersymmetries
  - $K=3$: 10 supersymmetries
  - $K=2$: 16 supersymmetries
  - $K=1$: 32 supersymmetries

the index function on $S^1 \times S^5$

* 5d SYM on S^5 Hee-Cheol Kim, Seok Kim:1206.6339; Hee-Cheol Kim, Joonho Kim, S.K. 1211.0144, Minahan-Nedelin-Zabzine, 1207.3763
  * S-dual version of the index
  * Vacuum energy:

$$(\epsilon_0)_{index} = \lim_{\beta' \to 0} \mathrm{Tr}\left[(-1)^F \frac{E-R}{2} e^{-\beta'(E-R_1)}\right]$$
$$= \frac{N(N^2-1)}{6} + \frac{N}{24}$$

* $S^1 \times CP^2$ path integral off-shell
* Stationary phase: $D^1=D^2=0$, $F = 2s\,J$, $\varphi + D^3 = 4s$, $s = \mathrm{diag}(s_1, s_2, \ldots, s_N)$
  * analogous to 3-dim Monopole operator
* Path Integral: Off-shell, localization (K=1 case_

$$\sum_{s_1,s_2,\cdots s_N=-\infty}^{\infty} \frac{1}{|W_s|} \oint \left[\frac{d\lambda_i}{2\pi}\right] e^{\frac{\beta}{2}\sum_{i=1}^N s_i^2 - i\sum_i s_i\lambda_i} Z^{(1)}_{\rm pert} Z^{(1)}_{\rm inst} \cdot Z^{(2)}_{\rm pert} Z^{(2)}_{\rm inst} \cdot Z^{(3)}_{\rm pert} Z^{(3)}_{\rm inst} \cdot$$

* For K=1, well-confirmed for k ≤ N with N=1,2,3 with the AdS/CFT calculation

Strange Vacua

- K=1, F=2sJ background

$$U(2)\ (1,-1)$$
$$U(3)\ (2,0,-2),(2,-1,-1),(1,1,-2),(1,0,-1)$$
$$U(4)\ (3,1,-1,-3),(3,1,-2,-2),(2,2,-1,-3),(3,0,-1,-2),$$
$$(2,1,0,-3),(2,0,0,-2),(2,0,-1,-1),(1,1,0,-2),(1,0,0,-1)$$

- the Lowest one $s_G = 2\rho \cdot H$ with negative energy $-2\rho^2$, where ρ = Weyl vector
- Ground State for Index: $K \leq N$ (Strong 't Hooft coupling $\lambda = N/K$)

K	$U(2)$	$U(3)$	$U(4)$	$U(5)$	$U(6)$	$U(7)$	$U(8)$	$U(9)$	$U(N)$
1	-1	-4	-10	-20	-35	-56	-84	-120	$-\frac{N(N^2-1)}{6}$
2	0	-1	-2	-5	-8	-14	-20	-30	
3		0	-1	-2	-3	-6	-9	-12	
4			0	-1	-2	-3	-4	-7	
5				0	-1	-2	-3	-4	
6					0	-1	-2	-3	
7						0	-1	-2	
8							0	-1	
9								0	

Table 1: Vacuum energies divided by K, at general $\mathbb{Z}_K$ (and fluxes)

Check with AdS/CFT

- E.g. k = N = 3: (all results multiplied by vacuum energy factor & $e^{-3\beta}$) $y_i = e^{-\beta a_i}$, $y = e^{\beta(m-\frac{1}{2})}$

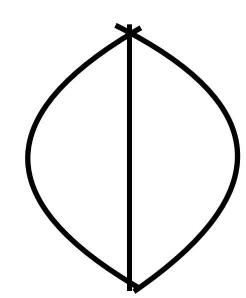
$$
\begin{aligned}
Z_{(2,0,-2)} &= 3\left[y^2(y_1+y_2+y_3)+y(y_1^2+y_2^2+y_3^2)+y^{-1}(y_1+y_2+y_3)-(1+\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots)+y^3\right] \\
&\quad +6y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right]+y^3 \\
Z_{(2,-1,-1)}+Z_{(1,1,-2)} &= -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -4y^3-4y^2(y_1+y_2+y_3)-2y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)+2\left(\frac{y_1}{y_2}+\frac{y_2}{y_3}+\cdots\right)-2y^{-1}(y_1+y_2+y_3) \\
Z_{(1,0,-1)} &= y^3+y^2(y_1+y_2+y_3)-y(y_1^{-1}+y_2^{-1}+y_3^{-1})+1
\end{aligned}
$$

add all

$$
Z_{SUGRA} = 3y^3+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^3-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)-\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots\right)+y^{-1}(y_1+y_2+y_3)
$$

- Non-zero flux states contributing to the index
  - s=(N-1,N-3,…,-(N-1)) = s_0 : SU(N) Weyl vector
  - index vacuum energy:

$$
E_0 = -\frac{N(N^2-1)}{6}
$$



$$
f^{abc} f^{abc}
$$

Kim &Kim: 2-loop effect on S^5 SYM

SU(2) Case

- BPS Eq. for Homogeneous Configuration with instanton number n^2

$$A = V\mathrm{diag}(n,-n),\ \ F = 2J\mathrm{diag}(n,-n),$$

- homogeneous bosonic solutions possible only with n=+1,-1
- but gauss law is violated
- for one of the constant bps solutions, the homogeneous fermionic zero mode is possible.
- gauss law can be satisfied with fermionic contribution for K=1 but not for K>1.
- energetic is more complicated to due to zero-point contribution to the classical one,...

Conclusion

- New 5d supersymmetric theories for M5 are found with discrete coupling constant and a weak coupling limit

- Index Function of 6d A_N (2,0) is partially obtained.
- highly nontrivial vacuum structure in the strong coupled regime
- UV finite? How rigid is the theory with eight supersymmetries.
- Enhanced supersymmetry to K=1,2,3?
- Wilson-loop can be easily included.

- Near BPS objects? perturbative approach?
- Rastelli et.al on conformal bootstrap
- 6-dim (1,0) theories