HS Theory: from Main Ideas to Recent Developments

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Plan

General properties of HS theories

HS algebra

Some recent developments

HS Gauge Fields and the Role of AdS

HS gauge theories: theories of higher symmetries

All $m = 0$ HS fields are gauge fields Fronsdal 1978

$\varphi_{\nu_1 \ldots \nu_s}$ is a rank s symmetric tensor obeying $\varphi^{\rho}{}_{\rho}{}^{\mu}{}_{\mu\nu_5\ldots\nu_s} = 0$

Gauge transformation:

$$\delta\varphi_{\nu_1\ldots\nu_s} = \partial_{(\nu_1}\varepsilon_{\nu_2\ldots\nu_s)}\,, \qquad \varepsilon^{\mu}{}_{\mu\nu_3\ldots\nu_{s-1}} = 0$$

In 60th it was argued (Weinberg, Coleman-Mandula) that

HS symmetries cannot be realized in Minkowski space

In 70th it was shown by Aragone and Deser that HS gauge symmetries

are incompatible with GR if expanding around Minkowski space

Green light: Some HS interactions

A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984)

AdS background with $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \to 0$ is singular

HS Symmetries versus Riemann geometry

HS symmetries do not commute to space-time symmetries

$$[T^a\,, T^{HS}] = T^{HS}\,, \qquad [T^{ab}\,, T^{HS}] = T^{HS}$$

HS transformations map gravitational fields (metric) to HS fields

Consequence:

Riemann geometry is not appropriate for HS theory:

concept of local event may become illusive!

Way out: exterior algebra formalism of differential forms

Invariant de Rham differential

$$d = dx^\nu \frac{\partial}{\partial x^\nu}\,, \qquad d^2 = 0$$

Elaboration of this language in HS theory leads to new understanding

of fundamental concepts of space-time including its dimension

HS Gauge Theory and Quantum Gravity

Being in a certain sense maximal symmetry

HS symmetry cannot result from spontaneous breakdown of a larger symmetry:

HS symmetries are manifest at ultrahigh energies above any scale including Planck scale

- HS gauge theory should capture effects of Quantum Gravity:
  restrictive HS symmetry versus unavailable experimental tests
- Lower-spin theories as low-energy limits of HS theory:
  lower-spin symmetries: subalgebras of HS symmetry

HS AdS/CFT correspondence

General idea of HS duality Sundborg (2001), Witten (2001)

AdS_4 HS theory is dual to $3d$ vectorial conformal models

Klebanov, Polyakov (2002), Sezgin, Sundell (2005); Giombi and Yin (2009); Maldacena, Zhiboedov (2011,2012); MV (2012); Giombi, Klebanov, Pufu, Safdi, Tarnopolsky (2013) ...

AdS_3/CFT_2 correspondence Gaberdiel and Gopakumar (2010)

Analysis of HS holography helps to uncover the origin of AdS/CFT

Global HS Symmetry

HS symmetry in AdS_{d+1}:

Maximal symmetry of a d-dimensional free conformal field(s)=singletons

usually, scalar and/or spinor

Consider KG massless equation in Minkowski space

$$\Box C(x) = 0\,, \qquad \Box = \eta^{ab}\frac{\partial^2}{\partial x^a \partial x^b}$$

What are symmetries of KG equation beyond conformal?

Shaynkman, MV 2001 $3d$; Eastwood 2002 $\forall d$

Auxiliary problem

Consider equations

$$DC_A(x) = 0\,, \qquad D^2 = 0 \qquad (1)$$

$$D = d + \omega(x)\,, \qquad \omega_A{}^B(x) = \omega^\Omega(x) T_{\Omega\, A}{}^B$$

$\omega(x)$: flat connection on the space V of $\mathcal{C}_A$

(1) is invariant under the gauge transformation

$$\delta\mathcal{C}(x) = -\varepsilon(x)\mathcal{C}(x)\,, \qquad \varepsilon_A{}^B(x) = \varepsilon^\Omega(x) T_{\Omega\, A}{}^B(x)$$

$$\delta\omega(x) = D\varepsilon(x) := d\varepsilon(x) + \omega(x)\varepsilon(x) - \varepsilon(x)\omega(x)$$

Global symmetry: $gl(V)$

For a particular $\omega(x) = \omega_0(x)$, to keep the equations invariant demands

$$\delta_0\omega(x) \longrightarrow D_0\varepsilon^\Omega_{gl}(x) = 0$$

Since $D_0^2 = 0$, $\varepsilon^\Omega_{gl}(x)$ is reconstructed (locally) in terms of $\varepsilon^\Omega(x_0)$ $\forall x_0$

$\varepsilon^\Omega(x_0)$: global symmetry parameters of $D_0\mathcal{C}(x) = 0$

Solution: $\omega_0(x) = g^{-1}(x)dg(x)$, $\mathcal{C}(x) = g^{-1}(x)\mathcal{C}$, $\varepsilon_{gl}(x) = g^{-1}(x)\varepsilon_{gl}g(x)$.

Massless scalar unfolded

Minkowski space: $\omega(x) = e^a(x)P_a + \omega^{ab}(x)M_{ab}$

Cartesian coordinates: $\omega^{ab} = 0,\ e^a(x) = dx^a$

Introduce an infinite set of 0-forms

$$C_{a_1\dots a_n}(x) = C_{(a_1\dots a_n)}(x)\,, \quad \eta^{bc}C_{bca_3\dots a_n}(x) = 0$$

Unfolded KG equation

$$dC_{a_1\dots a_n}(x) = dx^b C_{a_1\dots a_n b}(x)$$

This system is consistent since $dx^b \wedge dx^c = -dx^c \wedge dx^b$

First two equations imply

$$\partial_a C(x) = C_a(x)\,, \qquad \partial_a C_b(x) = C_{ab}(x) \longrightarrow C_{ab}(x) = \partial_a\partial_b C(x)$$

Tracelessness of $C_{nm}(x)$:

$$\Box C(x) = 0.$$

All other equations:

$$C_{a_1\dots a_n}(x) = \partial_{a_1}\dots\partial_{a_n}C(x)$$

$C_{a_1\dots a_n}(x)$: set of all on-mass-shell nontrivial derivatives of $C(x)$

Conformal HS algebra

Conformal HS algebra in d dimensions: algebra of linear transformations of the infinite-dimensional space V of various traceless symmetric tensors $C, C_a, C_{ab} \ldots$, i.e., $h = gl(V)$

h was carefully defined by Eastwood in 2002 by different methods

Algebraic construction simplifies in $d = 3$ using spinor formalism most relevant in the context of AdS_4/CFT_3 HS holography

Spinorial form of $3d$ massless equations

$3d$ **Lorentz algebra:** $o(2,1) \sim sp(2,R) \sim sl_2(R)$. $3d$ **spinors are real**

$$\chi^\dagger_\alpha = \chi_\alpha\,, \qquad \alpha = 1,2$$

Antisymmetrization of $3d$ spinor indices is equivalent to contraction

$$A_{\alpha,\beta} - A_{\beta,\alpha} = \epsilon_{\alpha\beta} A_{\gamma,}{}^{\gamma}$$

$sp(2,R)$ **invariant tensor** $\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$ **relates lower and upper indices**

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta\,, \qquad \chi_\alpha = \chi^\beta \epsilon_{\beta\alpha}$$

IRREPS of Lorentz algebra: totally symmetric multispinors $A_{\alpha_1\ldots\alpha_n}$

$$A_{a_1\ldots a_m} \sim A_{\alpha_1\ldots\alpha_{2m}}\,, \qquad A^b{}_{ba_3\ldots a_m} = 0$$

Space V of all $3d$ traceless symmetric tensors is the space of (even) functions of commuting spinor variable y^α

$$C(y|x) = \sum_{n=0}^{\infty} C^{\alpha_1\ldots\alpha_{2n}}(x) y_{\alpha_1}\ldots y_{\alpha_{2n}}$$

Unfolded massless equations take the form

$$dx^{\alpha\beta}\left(\frac{\partial}{\partial x^{\alpha\beta}} + \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right) C(y|x) = 0$$

Shaynkman, MV (2001)

$3d$ HS symmetry

$3d$ conformal HS algebra is the algebra of various differential operators

$\epsilon(y,\frac{\partial}{\partial y})$ obeying $\epsilon(-y,-\frac{\partial}{\partial y}) = \epsilon(y,\frac{\partial}{\partial y})$

$$\delta C(y|x) = \epsilon(y,\frac{\partial}{\partial y}|x) C(y|x)$$

$$\epsilon(y,\frac{\partial}{\partial y}|x) = \exp\left[-x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right]\epsilon_{gl}(y,\frac{\partial}{\partial y})\exp\left[x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right]$$

For any polynomial $\epsilon_{gl}(y,\frac{\partial}{\partial y})$, $\epsilon(y,\frac{\partial}{\partial y}|x)$ is polynomial as well:

$\epsilon_{gl}(y,\frac{\partial}{\partial y})$ describes global HS transformations

$3d$ conformal algebra $sp(4) \sim o(3,2)$

$$P_{\alpha\beta} = \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\,, \qquad K^{\alpha\beta} = y^\alpha y^\beta\,, \qquad M_{\alpha\beta} = y_\alpha \frac{\partial}{\partial y^\beta} + y_\beta \frac{\partial}{\partial y^\alpha}\,, \qquad D = y^\alpha \frac{\partial}{\partial y^\alpha} + 1$$

Weyl algebra and star product

Weyl algebra A_n: associative algebra of polynomials of oscillators $\hat{Y}_A$

$$[\hat{Y}_A\,,\hat{Y}_B]=2iC_{AB}\,,\qquad A,B,\ldots=1,\ldots 2n\,,\qquad C_{AB}=-C_{BA}$$

$3d$ **CHS algebra** $=$ AdS_4 **HS algebra is (even part of)** A_2

$$\hat{Y}_A=\begin{pmatrix} y^\alpha \\ \frac{\partial}{\partial y^\alpha}\end{pmatrix}$$

Weyl star–product language

$$[Y_A,Y_B]_*=2iC_{AB}\,,\qquad [a\,,b]_*=a*b-b*a$$

Weyl-Moyal formula

$$(f_1*f_2)(Y)=f_1(Y)\,\exp\,[i\overleftarrow{\partial^A}\overrightarrow{\partial^B}C_{AB}]\,f_2(Y)\ ,\qquad \partial^A\equiv\frac{\partial}{\partial Y_A}$$

AdS_4 HS theory via gauging of HS algebra

$3d$ Conformal HS symmetry $=$ AdS_4 HS symmetry

HS gauge fields: $\omega(Y|X)$

$Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$, $\alpha, \dot\alpha = 1,2$ **two-component spinor indices**

$$\omega(Y|X) = \sum_{n,m=0}^{\infty} \frac{1}{2in!m!} \omega_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_m}(X) y^{\alpha_1}\ldots y^{\alpha_n} \bar{y}^{\dot\alpha_1}\ldots\bar{y}^{\dot\alpha_m}$$

HS curvature and gauge transformation

$$R(Y|X) = d\omega(Y|X) + \omega(Y|X) * \wedge\omega(Y|X)$$

$$\delta\omega(Y|X) = D\epsilon(Y|X) = d\epsilon(Y|X) + [\omega(Y|X)\,,\epsilon(Y|X)]_*$$

$$[y_\alpha\,, y_\beta]_* = 2i\varepsilon_{\alpha\beta}\,, \qquad [\bar{y}_{\dot\alpha}\,,\bar{y}_{\dot\beta}]_* = 2i\varepsilon_{\dot\alpha\dot\beta}$$

Vacuum

Gravity sector is associated with $\omega \in sp(4) \sim o(3,2)$

$$\omega(Y|X) = \frac{1}{4i}(w^{\alpha\beta}(X)y_\alpha y_\beta + \bar{w}^{\dot\alpha\dot\beta}(X)\bar{y}_{\dot\alpha}\bar{y}_{\dot\beta} + 2\lambda h^{\alpha\dot\beta}(X)y_\alpha\bar{y}_{\dot\beta})$$

Equation of AdS_4: $R_0 = 0$ **for** $\omega_0 \in sp(4) \sim o(3,2)$

Fluctuations

$$\omega = \omega_0 + \omega_1 \,, \qquad R_1 = D_0\omega_1 = d\omega_1 + [\omega_0 \,, \omega_1]_*$$

Central On-Shell Theorem

The full unfolded system for free massless fields of all spins is formulated in terms of one-form $\omega(Y|X)$ and zero-form $C(Y|X)$ 1989

$$R_1^{ad}(y,\overline{y}\mid X)=\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}C(0,\overline{y}\mid X)+H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}C(y,0\mid X)\ ,$$

$$D_0^{tw}C(y,\overline{y}\mid X)=0\,,$$

where

$$H^{\alpha\beta}=h^{\alpha}{}_{\dot\alpha}\wedge h^{\beta\dot\alpha}\,,\qquad \overline{H}^{\dot\alpha\dot\beta}=h_{\alpha}{}^{\dot\alpha}\wedge h^{\alpha\dot\beta}\ ,$$

$$R_1^{ad}(y,\bar y\mid X)=D_0^{ad}\omega(y,\bar y\mid X)$$

$$D_0^{ad}=D^L-\lambda h^{\alpha\dot\beta}\left(y_\alpha\frac{\partial}{\partial\bar y^{\dot\beta}}+\frac{\partial}{\partial y^\alpha}\bar y_{\dot\beta}\right)\,,\qquad D_0^{tw}=D^L+\lambda h^{\alpha\dot\beta}\left(y_\alpha\bar y_{\dot\beta}+\frac{\partial^2}{\partial y^\alpha\partial\bar y^{\dot\beta}}\right)$$

$$D^LA=d_X-\left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta}+\bar\omega^{\dot\alpha\dot\beta}\bar y_{\dot\alpha}\frac{\partial}{\partial\bar y^{\dot\beta}}\right)$$

Pattern of massless equations

Gauge fields of different spins: homogeneous polynomials in Y

$$\omega^s(\nu y, \nu\bar{y}|X) = \nu^{2(s-1)}\omega(y,\bar{y}|X)\,, \qquad C^s(\nu y, \nu^{-1}\bar{y}|X) = \nu^{\pm 2s}C(y,\bar{y}|X)$$

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2, 5/2\ldots$

$$\omega^s_{\alpha_1\ldots\alpha_n\,,\dot{\beta}_1\ldots\dot{\beta}_m}: \quad n+m = 2(s-1)\,, \qquad C^s_{\alpha_1\ldots\alpha_n\,,\dot{\beta}_1\ldots\dot{\beta}_m}: \quad |n-m| = 2s$$

$C(Y|X)$: gauge invariant HS curvatures and spin-zero matter fields

Dynamical fields

Frame-like fields $\omega^s_{\alpha_1\ldots\alpha_n\,,\dot{\beta}_1\ldots\dot{\beta}_n}$ contain Fronsdal fields

$C(0,0|x)$: scalar

Other components are expressed via higher derivatives of the dynamical fields which come in combination

$$\lambda^{-1}\frac{\partial}{\partial x}\,, \qquad \lambda^2 = -\Lambda$$

Higher derivatives source nonanaliticity in Λ.

Examples

$s = 0:$ $C^0_{\alpha_1\dots\alpha_n\,,\dot\beta_1\dots\dot\beta_n} \sim C_{a_1\dots a_n}\,,$ $C^b{}_{ba_3\dots a_n} = 0$

$s = 2$: **Gauge fields: Lorentz connection** $\omega_{\alpha\beta}, \bar\omega_{\dot\alpha\dot\beta}$ **and vierbein** $\omega_{\alpha,\dot\beta}$

Zero-forms $C_{\alpha_1\alpha_2\alpha_3\alpha_4}(X)$ **and** $\bar C_{\dot\alpha_1\dot\alpha_2\dot\alpha_3\dot\alpha_4}(X)$: **Weyl tensor in terms of two-component spinors.**

Higher components $C^s_{\alpha_1\dots\alpha_n\,,\dot\beta_1\dots\dot\beta_m}$: $|n-m| = 4$: **all its derivatives**

$R_{\alpha,\dot\beta} = 0$: **expresses Lorentz connection via vierbein**

$R_{\alpha\beta} = H^{\gamma\delta}C_{\alpha\beta\gamma\delta},\ R_{\dot\alpha\dot\beta} = \bar H^{\dot\gamma\dot\delta}\bar C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$:

Riemann tensor = Weyl tensor $\Longrightarrow$ Ricci is zero

HS counterparts impose Fronsdal equations and express generalized Weyl tensors in terms of Fronsdal fields

From COST to nonlinear theory

$$R(y,\overline{y} \mid X) = \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\, C(0,\overline{y} \mid X) + H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\, C(y,0 \mid X) + \dots$$

$$D^{tw}C(y,\overline{y} \mid X) = O(C,\omega_1)$$

$$R(y,\overline{y} \mid X) = d\omega(y,\overline{y} \mid X) + \omega(y,\overline{y} \mid X) * \omega(y,\overline{y} \mid X)$$

$$D^{tw}C(y,\overline{y} \mid X) = dC(y,\overline{y} \mid X) + \omega(y,\overline{y} \mid X) * C(y,\overline{y} \mid X) - C(y,\overline{y} \mid X) * \omega(y,-\overline{y} \mid X)$$

Such field equations are unfolded: exterior differential of any field is expressed via the fields themselves

Problem: find gauge invariant nonlinear corrections

HS star product

$$(f\star g)(Z,Y)=\int dSdTf(Z+S,Y+S)g(Z-T,Y+T)\,\mathrm{exp}\,-iS_\nu T^\nu$$

$$[Y_A,Y_B]_\star=-[Z_A,Z_B]_\star=2iC_{AB}\,,$$ $Z-Y\,:\,Z+Y$ **normal ordering**

Inner Klein operators: $\kappa=\mathrm{exp}\,iz_\alpha y^\alpha\,,\qquad \bar\kappa=\mathrm{exp}\,i\bar z_{\dot\alpha}\bar y^{\dot\alpha}$

$$\kappa\star f=\tilde f\star\kappa\,,\qquad \kappa\star\kappa=1\,,\qquad \tilde f(z,y,\bar z,\bar y)=f(-z,-y,\bar z,\bar y)$$

Fields

$$\mathcal{W}=(d+W)+S\,,\qquad B(Z;Y;K|X)$$

$$W(Z;Y;K|X)=dX^nW_n\,,\quad S(dZ;Z;Y;K|X)=dz^\alpha S_\alpha+d\bar z^{\dot\alpha}\bar S_{\dot\alpha}$$

Outer Klein operators: $K=(k,\bar k)$

$$k\star f=\tilde f\star k\,,\qquad k\star k=1\,,\qquad \tilde f(dz,z,y,k,\bar dz,\bar z,\bar y,\bar k)=f(-dz,-z,-y,k,\bar dz,\bar z,\bar y,\bar k)$$

$$k\star\kappa\star f(dz,z,y,k,\bar dz,\bar z,\bar y,\bar k)=f(-dz,z,y,k,\bar dz,\bar z,\bar y,\bar k)\star k\star\kappa$$

HS equations

$\star \qquad \mathcal{W} \star \mathcal{W} = i(dZ^A dZ_A + dz^\alpha dz_\alpha F(B) \star k \star \kappa + d\bar{z}^{\dot\alpha} d\bar{z}_{\dot\alpha} \bar{F}(B) \star \bar{k} \star \bar{\kappa})$

$\star \qquad \mathcal{W} \star B = B \star \mathcal{W}$

equivalent to

$$\begin{cases} dW + W \star W = 0 \\ dB + W \star B - B \star W = 0 \\ dS + W \star S + S \star W = 0 \\ S \star B - B \star S = 0 \\ S \star S = i(dZ^A dZ_A + dz^\alpha dz_\alpha F(B) \star k \star \kappa + d\bar{z}^{\dot\alpha} d\bar{z}_{\dot\alpha} \bar{F}(B) \star \bar{k} \star \bar{\kappa}) \end{cases}$$

The system is manifestly gauge invariant under

$$\delta \mathcal{W} = [\varepsilon, \mathcal{W}]_\star \,, \qquad \delta B = \varepsilon \star B - B \star \varepsilon \,, \qquad \varepsilon = \varepsilon(Z; Y; K|X)$$

Pattern of nonlinear HS equations

Nonlinear equations reconstruct Z-dependence in terms of "initial data"

$$W(0;Y;K|X)=\sum_{i,j=0,1}k^i\bar{k}^j\omega^{i,j}(Y|X)\,,\qquad B(0;Y;K|X)=\sum_{i,j=0,1}k^i\bar{k}^jC^{i,j}(Y|X)$$

In the lowest order with $F(B)=\eta B$, $\bar{F}(B)=\bar{\eta}B$ the system reproduces COMT for the doubled set of massless fields $\omega^{i,i}(Y|X)$ and $C^{i,1-i}(Y|X)$ and seemingly analogous system of linearized field equations

$$R_1^{tw}(y,\bar{y}\,|X):=D_0^{tw}\omega^{mod}(y,\bar{y}\,|X)=\overline{H}_{\dot{\alpha}\dot{\beta}}\bar{y}^{\dot{\alpha}}\bar{y}^{\dot{\beta}}C^{mod}(0,\bar{y}\,|X)+H_{\alpha\beta}y^{\alpha}y^{\beta}C^{mod}(y,0\,|X$$

$$D_0^{ad}C^{mod}(y,\bar{y}\,|X)=0$$

for the doubled set of moduli fields ω^{mod} and C^{mod} represented by $C^{i,i}(Y|X)$ and $\omega^{i,1-i}(Y|X)$

Due to exchange of the adjoint and twisted adjoint representations, at the linearized level the moduli sector contains an infinite set of subsystems each describing a finite number of degrees of freedom

Simplest examples: $BF-$like systems

$L=A_2dA_1$ or $L=A_3dA_0$: A_n are $n-$forms

Moduli fields

For odd $F(B)$: $F(-B) = -F(B)$ the full nonlinear system is invariant under the involutive map

$$\tau\mathcal{W}(Z,Y,K|X) = \mathcal{W}(Z,Y,-K|X)\,, \qquad \tau B(Z,Y,K|X) = -B(Z,Y,-K|X)$$

allowing to truncate away ω^{mod}, C^{mod}. The truncated AdS_4 HS theory mostly considered in the literature is a single (most symmetric) point in the infinite-dimensional space of moduli fields to be interpreted as couplings of the larger theory

In AdS_3 HS theory (no dotted spinors $\bar{y}_{\dot\alpha}$) the model with $B = \nu = const$ describes HS interactions of massive matter fields Prokushkin, MV (1998)

New conjecture: $4d$ HS theory with $B = \nu = const$ describes the HS dual of $3d$ boundary theory of massive matter fields (work in progress)

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

 # degrees of freedom = # of dynamical variables

Unfolded dynamics: multidimensional generalization

$$\frac{\partial}{\partial t} \to d\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p} W^\Omega_{n_1 \ldots n_p}(x)$$

$$dW^\Omega(x) = G^\Omega(W(x))\,, \qquad d = dx^n \partial_n$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Lambda_1 \ldots \Lambda_n} W^{\Lambda_1} \wedge \ldots \wedge W^{\Lambda_n}$$

Covariant first-order differential equations

$d > 1$: **Compatibility conditions**

$$G^\Lambda(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Lambda} \equiv 0$$

Properties

- General applicability
- Manifest (HS) gauge invariance under the gauge transformation

$$\delta W^{\Omega} = d\varepsilon^{\Omega} + \varepsilon^{\Lambda}\frac{\partial G^{\Omega}(W)}{\partial W^{\Lambda}},$$

  gauge parameter $\varepsilon^{\Omega}(x)$ is a $(p_{\Omega} - 1)$-form

- Invariance under diffeomorphisms

  Exterior algebra formalism

- Interactions: nonlinear deformation of $G^{\Omega}(W)$
- Degrees of freedom are in 0-forms $C^{i}(x_0)$ at any $x = x_0$ (as $q(t_0)$)

  infinite-dimensional module dual to the space of single-particle states

Unfolded dynamics provides a tool to control unitarity in presence of higher derivatives

Space-time metamorphoses

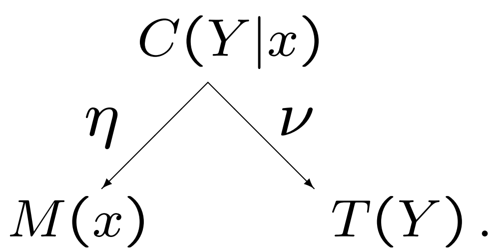
Independence of ambient space-time: geometry is encoded by $G^{\Omega}(W)$

Key observation: unfolded equation makes sense in any space-time

$$dW^{\Omega}(x) = G^{\Omega}(W(x))\,, \quad x \to X = (x,z)\,, \quad d_x \to d_X = d_x + d_z\,, \quad d_z = dz^u \frac{\partial}{\partial z^u}$$

X-dependence is reconstructed in terms of fields $W^{\Omega}(X_0) = W^{\Omega}(x_0, z_0)$ at any X_0 To take $W^{\Omega}(x_0, z_0)$ in space M_X with coordinates X_0 is the same as to take $W^{\Omega}(x_0)$ in the space $M_x \in M_X$ with coordinates x

Classes of holographically dual models: different G defining a twistor-like transform



$W^{\Omega}(Y|x)$ are functions on the "correspondence space" C.

Space-time M : coordinates x. Twistor space T : coordinates Y.

Holographic duality: different space-times M for the same T (2012)

HS Theory and String theory

HS theories: $\Lambda \neq 0$, $m = 0$

symmetric fields $s = 0, 1, 2, \ldots \infty$

String Theory: $\Lambda = 0$, $m \neq 0$ **except for a few zero modes**

mixed symmetry fields $\vec{s} = 0, 1, 2, \ldots \infty$

String theory has much larger spectrum:

HS Theory: first Regge trajectory

Pattern of HS gauge theory is determined by HS symmetry

What is a string-like extension of a global HS symmetry underlying a string-like extension of HS theory?

- **String Theory as spontaneously broken HS theory?!** Gross (1989)

  Singleton String Engquist, Sundell (2005, 2007)

  Recent conjecture (Chang, Minwalla, Sharma and Yin (2012)**):**

  String Theory = Quantum HS theory?!

Current operator algebra

$3d$ conformal equations

Conformal invariant massless equations in $d = 3$

$$\left(\frac{\partial}{\partial x^{\alpha\beta}} \pm i \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right) C^{\pm}_j(y|x) = 0\,, \qquad \alpha, \beta = 1, 2\,, \quad j = 1, \dots \mathcal{N}$$

Generalization to matrix space: $\alpha, \beta = 1, 2, \dots M$.

Bosons (fermions) are even (odd) functions of y: $C_i(-y|x) = (-1)^{p_i} C_i(y|x)$

For normalizable $C^{\pm}_j(y|x)$ **a sign** $\pm$ **does matter distinguishing between positive and negative frequencies**

Unfolding induces quantization

Full field

$$\Phi_j(y|x) = C^{+}_j(y|x) + i^{p_j} C^{-}_j(iy|x)\,, \qquad \overline{\Phi}_j(y|x) = C^{-}_j(y|x) + i^{p_j} C^{+}_j(iy|x)$$

$$\left(\frac{\partial}{\partial x^{\alpha\beta}} + i \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right) \Phi_j(y|x) = 0\,, \qquad \left(\frac{\partial}{\partial x^{\alpha\beta}} - i \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right) \overline{\Phi}_j(y|x) = 0$$

Currents

Rank-two equations: conserved currents

$$\left\{ \frac{\partial}{\partial x^{\alpha\beta}} - \frac{\partial^2}{\partial y^{(\alpha}\partial u^{\beta)}} \right\} J(u,\, y|x) = 0$$

Gelfond, MV (2003)

$J(u,\, y|x)$: generalized stress tensor. Rank-two equation is obeyed by

$$J(u,\, y\,|x) = \sum_{i=1}^{\mathcal{N}} \overline{\Phi}_i(u+y|x)\, \Phi_i(y-u|x)$$

Rank-two fields: bilocal fields in the twistor space.

Primaries: $3d$ currents of all integer and half-integer spins

$$J(u,0|x) = \sum_{2s=0}^{\infty} u^{\alpha_1} \dots u^{\alpha_{2s}} J_{\alpha_1 \dots \alpha_{2s}}(x)\,, \qquad \tilde{J}(0,y|x) = \sum_{2s=0}^{\infty} y^{\alpha_1} \dots y^{\alpha_{2s}} \tilde{J}_{\alpha_1 \dots \alpha_{2s}}(x)$$

$$J^{asym}(u,y|x) = u_\alpha y^\alpha J^{asym}(x)$$

$$\Delta J_{\alpha_1 \dots \alpha_{2s}}(x) = \Delta \tilde{J}_{\alpha_1 \dots \alpha_{2s}}(x) = s+1 \qquad \Delta J^{asym}(x) = 2$$

Differential equations: conservation condition

$$\frac{\partial}{\partial x^{\alpha\beta}} \frac{\partial^2}{\partial u_\alpha \partial u_\beta} J(u,0|x) = 0\,, \qquad \frac{\partial}{\partial x^{\alpha\beta}} \frac{\partial^2}{\partial y_\alpha \partial y_\beta} \tilde{J}(0,y|x) = 0$$

HS CFT_3/AdS_4

Extension of $3d$ **space-time:** $x \to X = (x, z)$ **with**

$$\left\{ \frac{\partial}{\partial x^{\alpha\beta}} - \frac{\partial^2}{\partial y^{(\alpha}\partial u^{\beta)}} \right\} J(u,\, y|x) = 0 \qquad \longrightarrow \qquad dX^{\alpha\dot\beta}\Big(\frac{\partial}{\partial X^{\alpha\dot\beta}} + \frac{\partial^2}{\partial y^{\alpha}\partial \bar{y}^{\dot\beta}}\Big)C(Y|X) = 0\,,$$

$3d$ **current equations get replaced by** $4d$ **massless equations** (2012)

Quantization

Operator fields obey

$$[\hat{C}^-_j(y|x)\,,\hat{C}^+_k(y'|x')] = \frac{1}{2i}\delta_{jk}\Big(\mathcal{D}^-(y-y'|x-x') + (-1)^{p_j p_k}\mathcal{D}^-(y+y'|x-x')\Big)$$

$$\mathcal{D}^\pm(y|x) = \pm\frac{i}{4\pi}\,\mathrm{exp}\pm\frac{i\pi I_x}{4}|det|x||^{-1/2}\,\mathrm{exp}[-\frac{i}{4}x^{-1}_{\alpha\beta}y^\alpha y^\beta]$$

Since

$$\mathcal{D}^\pm(y|0) = \mp i\delta^M(y)$$

commutation relations make sense at $x = x'$

$$[\hat{C}^-_j(y|x)\,,\hat{C}^+_k(y'|x)] = \frac{1}{2}\delta_{jk}\Big(\delta(y-y') + (-1)^{p_j p_k}\delta(y+y')\Big)$$

Singularity at $(y,x) = (y',x')$ does not imply singularity at $x = x'$.

Rank-one twistor to boundary evolution

$$C^\pm(y|x) = \mp i\int d^2y'\mathcal{D}^\mp(y'-y|x'-x)C^\pm(y'|x')\,.$$

Bulk extension via twistor-to-bulk $\mathcal{D}$-function

$$\mathcal{D}(y|X)\,,\qquad X = (x,z)\,,\qquad D_0\mathcal{D}(y|X) = 0\,,\qquad \mathcal{D}^\pm(y|0) = \mp i\delta^2(y)$$

Quantum currents

$J_{jk}(y_1, y_2|x) =: \hat{\Phi}_j(y_1|x)\overline{\hat{\Phi}}_k(y_2|x):$

Generating function J_g^2 with a test function g

$$J_g^2 = \int dw_1 dw_2\, g^{mn}(w_1, w_2) J_{mn}(w_1, w_2|0)\,,$$

$$J_{j\,k}(w_1, w_2|x) = \sum_{a,b=+,-} (\kappa_1^a)^{p_j} (\kappa_2^b)^{p_k} J_{j\,k}^{ab}(\kappa_1^a y_1, \kappa_2^b p_2|x)\,,$$

$$J_{j\,k}^{a\,b}(w_1, w_2|x) =: \hat{C}_j^a(w_1|x) \hat{C}_k^b(w_2|x):$$

$$\kappa_1^+ = \kappa_2^- = 1\,, \qquad \kappa_2^+ = \kappa_1^- = i$$

where

$$J_g^2(x) = \int dw_1 dw_2\, g_{ab}^{mn}(w_1, w_2) J_{mn}^{ab}(w_1, w_2|x) = J_{g(x)}^2$$

x**-dependence of** $g_{ab}^{mn}(x)$ $(a, b = \pm)$ **is reconstructed by $\mathcal{D}$-functions**

Twistor current algebra

Elementary computation gives

$$J^2_g J^2_{g'} =: J^2_g J^2_{g'} : + J^2{}_{[g\,,g']_\star} + \mathcal{N} tr_\star(g \star g') J^0\,, \qquad J^0 = Id$$

Convolution product $\star$ is related to HS star-product via half-Fourier transform

$$\tilde{g}(w,v) = (2\pi)^{-1} \int d^2u \exp[i w_\alpha u^\alpha] g(v+u, v-u)$$

Star product of AdS_4 HS theory results from OPE of boundary currents

Full set of operators

$$J^{2m}_g =: \underbrace{J^2_g \ldots J^2_g}_{m} : \qquad J^0_g = Id$$

What is the associative twistor operator algebra?!

Since

$$J^2_{g_1} J^2_{g_2} - J^2_{g_2} J^2_{g_1} = 2 J^2_{[g_1\,,g_2]_\star}$$

universal enveloping algebra $U(h)$ of the HS algebra h Gelfond, MV 2013

Explicit construction of multiparticle algebra

Being maximal symmetry $gl(V)$ HS algebra is associated with associative algebra $End(V)$

Universal enveloping algebra $U(l(A))$ of a Lie algebra $l(A)$ associated with an associative algebra A allows explicit description

Let $\{t_i\}$ be some basis of A

$$a \in A: \quad a = a^i t_i\,, \qquad t_i \star t_j = f_{ij}^k t_k\,, \qquad t_i \sim J^2\,, \qquad a^i \sim g(w_1, w_2)$$

$U(l(A))$ is algebra of functions of α_i (commutative analogue of t_i)

Explicit composition law of $M(A)$ 2012

$$F(\alpha) \circ G(\alpha) = F(\alpha) \exp\left(\frac{\overleftarrow{\partial}}{\partial \alpha_i} f_{ij}^n \alpha_n \frac{\overrightarrow{\partial}}{\partial \alpha_j}\right) G(\alpha)$$

where derivatives $\frac{\overleftarrow{\partial}}{\partial \alpha_i}$ and $\frac{\overrightarrow{\partial}}{\partial \alpha_j}$ act on F and G, respectively.

Associativity of $\star$ of A implies associativity of $\circ$ of $M(A)$

Theories with different $\mathcal{N}$: different frames of the same algebra!

$U(h)$ possesses different invariants generating different n-point functions

What are models associated with different frame choices?!

Multiparticle algebra as a symmetry of a multiparticle theory

$l(U(h))$

- contains h as a subalgebra
- admits quotients containing up to k^{th} tensor products of h:

  k Regge trajectories?!
- Acts on all multiparticle states of HS theory

Promising candidate for a HS symmetry algebra of HS theory with mixed symmetry fields like String Theory

String Theory as a theory of bound states of HS theory

Chang, Minwalla, Sharma and Yin (2012)

Conclusion

Unfolding supports

- democracy between space-times of different dimensions:
  clarifies the origin of holographic duality

- quantization: distinguishes between particles and antiparticles

Moduli fields describe an infinite variety of deformed HS theories

A multiparticle theory: quantum HS theory and String Theory

Multiparticle algebra is a Hopf algebra.
Relation with integrable structures underlying both String Theory and analysis of amplitudes?!