Large-D Gravity
Progress & Prospects

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Geometry from the Quantum - KITP UCSB

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Work in 2013-2019 with:

T Andrade, D Grumiller, K Izumi, D Licht,

R Luna, M Martínez, T Shiromizu,

R Suzuki, ***K Tanabe***, *T Tanaka*

Parallel program by

S Bhattacharyya, S Minwalla et al

Effective membrane theory of black holes

Same concepts but different implementation

$D \to \infty$

A technique to simplify calculations

A way to gain new insights

D as a parameter

$\boldsymbol{D < 4}$

UV regulator – good for quantum

But :(

No local dof's

No GWs

No BHs in $R_{\mu\nu} = 0$

D as a parameter

$\boldsymbol{D > 4}$

IR regulator – bad for quantum?

But :)

Has local dof's

GWs

BHs in $R_{\mu\nu} = 0$

Large N does simplify gauge theories

Reorganize meson dynamics into string worldsheets

Large D: does it simplify gravity?

YES

Reorganizes black hole dynamics into theory of membranes

Large D: does it simplify other aspects of gravity?

Probably yes

Large-D in General Relativity

$D^2 \sim$ # local degrees of freedom at a point

akin to Large-N SU(N) gauge theory

also large-c CFT, vector models, Potts models, SYK…

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$D \sim$ # connections between nearby points

= directions out of a point

akin to Mean Field Theory limit in Stat Mech

average out/dilute long-distance effects

What we've found useful is

$D^2 \sim$ # local degrees of freedom at a point

akin to Large-N SU(N) gauge theory

$D \sim$ # connections between nearby points

= directions out of a point

Exploit large gradients of gravitational potential $\frac{1}{r^{D-3}}$

Main findings
about BHs @ large D

$\exists$ well-defined, universal near-horizon geometry

with dynamics that decouples from the far-region

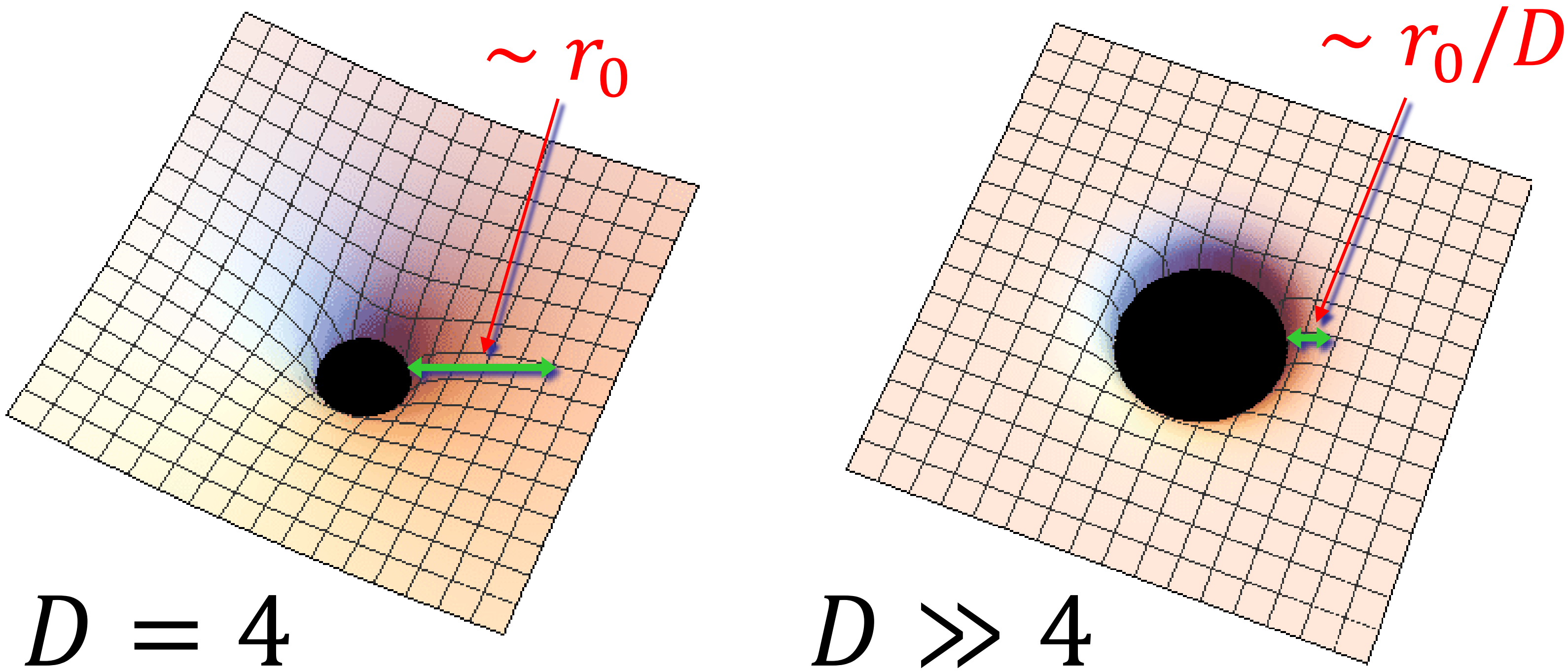
This allows to formulate an

effective theory of dynamical black holes

for slow fluctuations

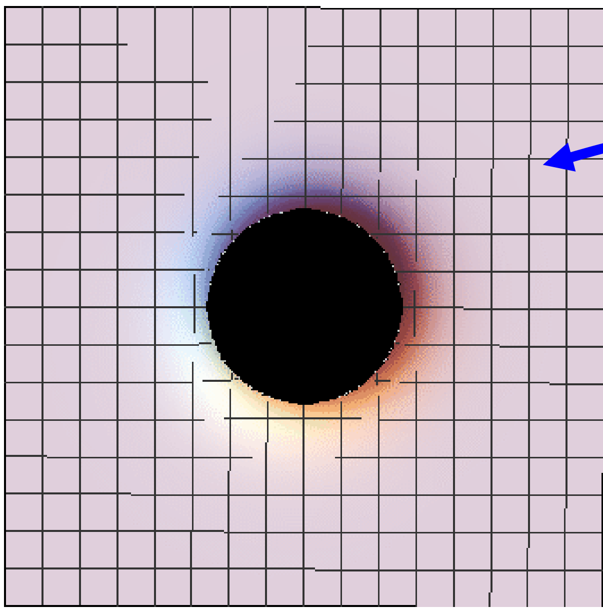
Similar to but not the same as

membrane paradigm, fluid/gravity



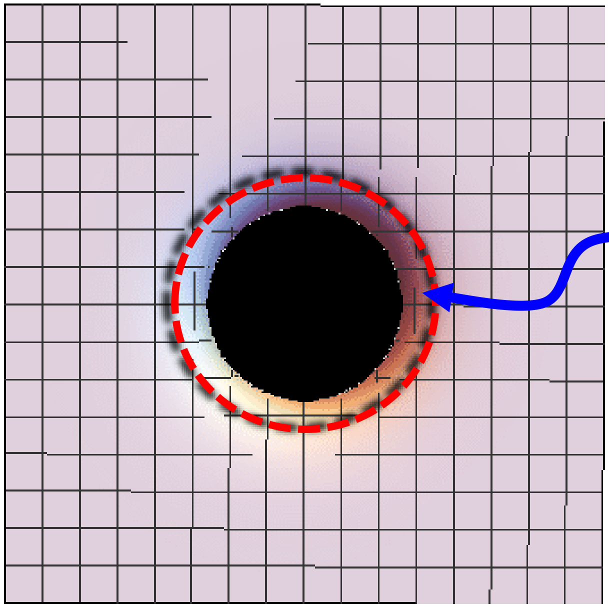
$\sim r_0$
$\sim r_0/D$
$D = 4$
$D \gg 4$

$$r > r_0 \quad \Rightarrow \left(\frac{r_0}{r}\right)^{D-3} \quad \to 0$$



Flat space

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \iff r - r_0 \lesssim \frac{r_0}{D}$$



$$r - r_0 \sim \frac{r_0}{D}$$

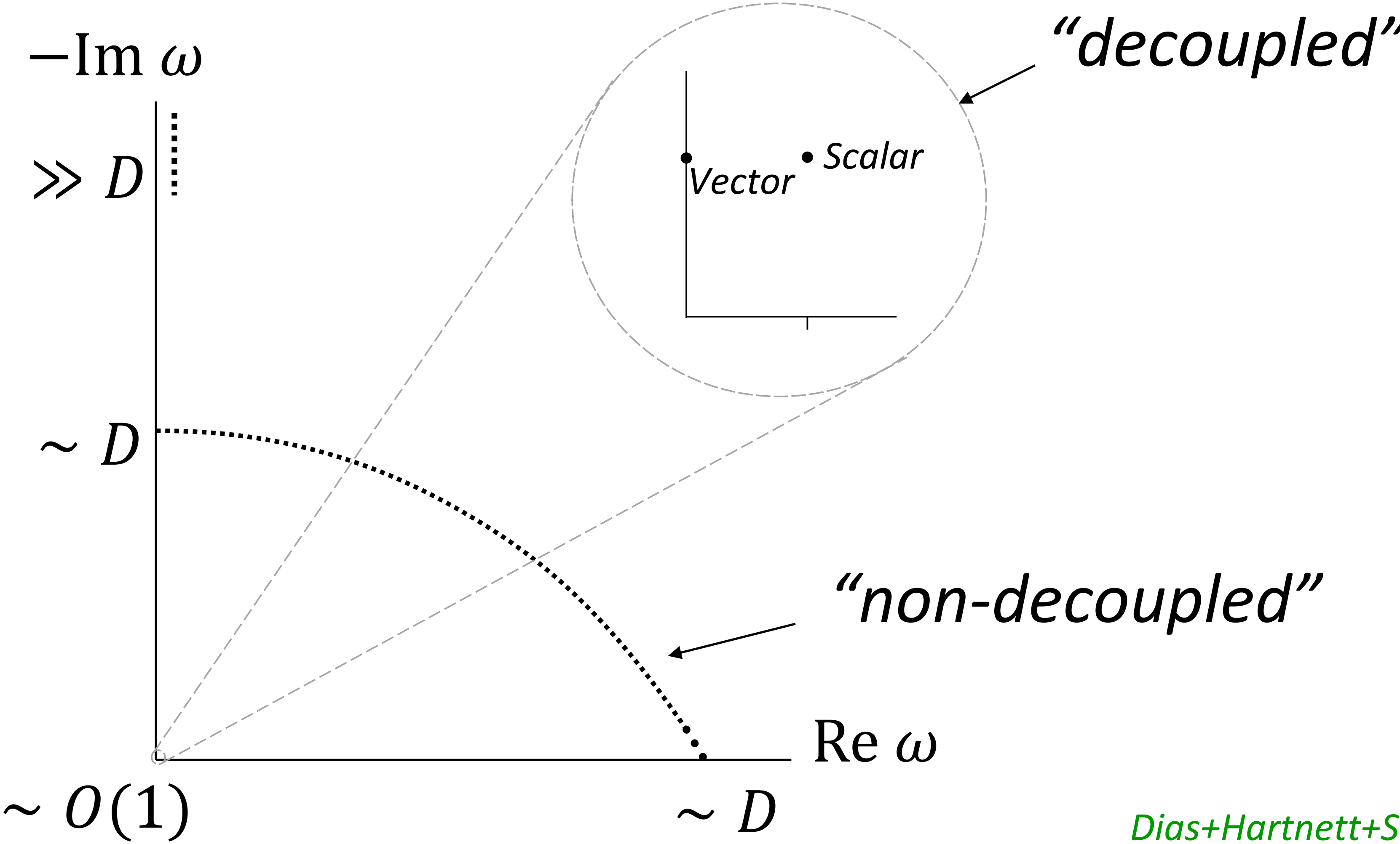
non-trivial
gravitational field

Two kinds of BH fluctuations

- **Slow,** localized near horizon, **decoupled** from far zone

- **Fast,** everywhere, **non-decoupled**

Quasinormal frequencies of Schwarzschild$_{D\gg 1}$



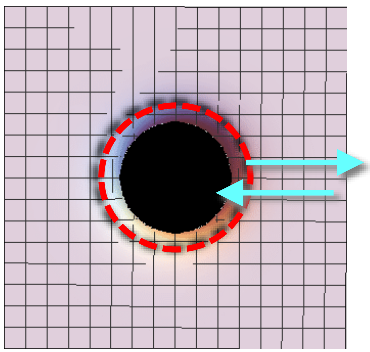
Dias+Hartnett+Santos

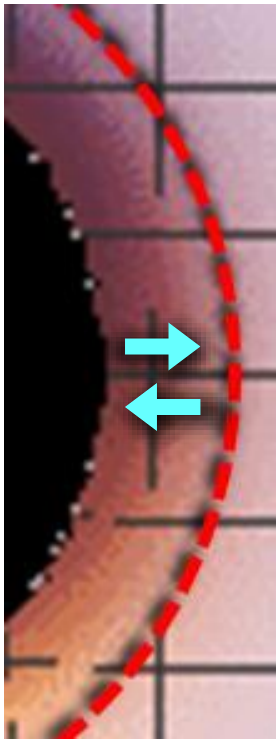
RE+Suzuki+Tanabe

Fast, non-decoupled

$$\omega \sim D/r_0$$

Characteristic crossing time of near-horizon





Slow, decoupled

$$\omega \sim 1/r_0$$

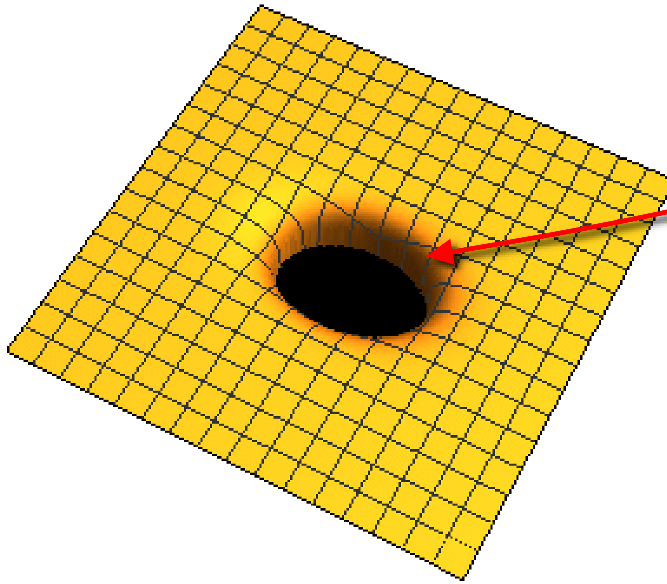
Almost static in near-horizon

Decoupled from far zone

Non-linear dynamics

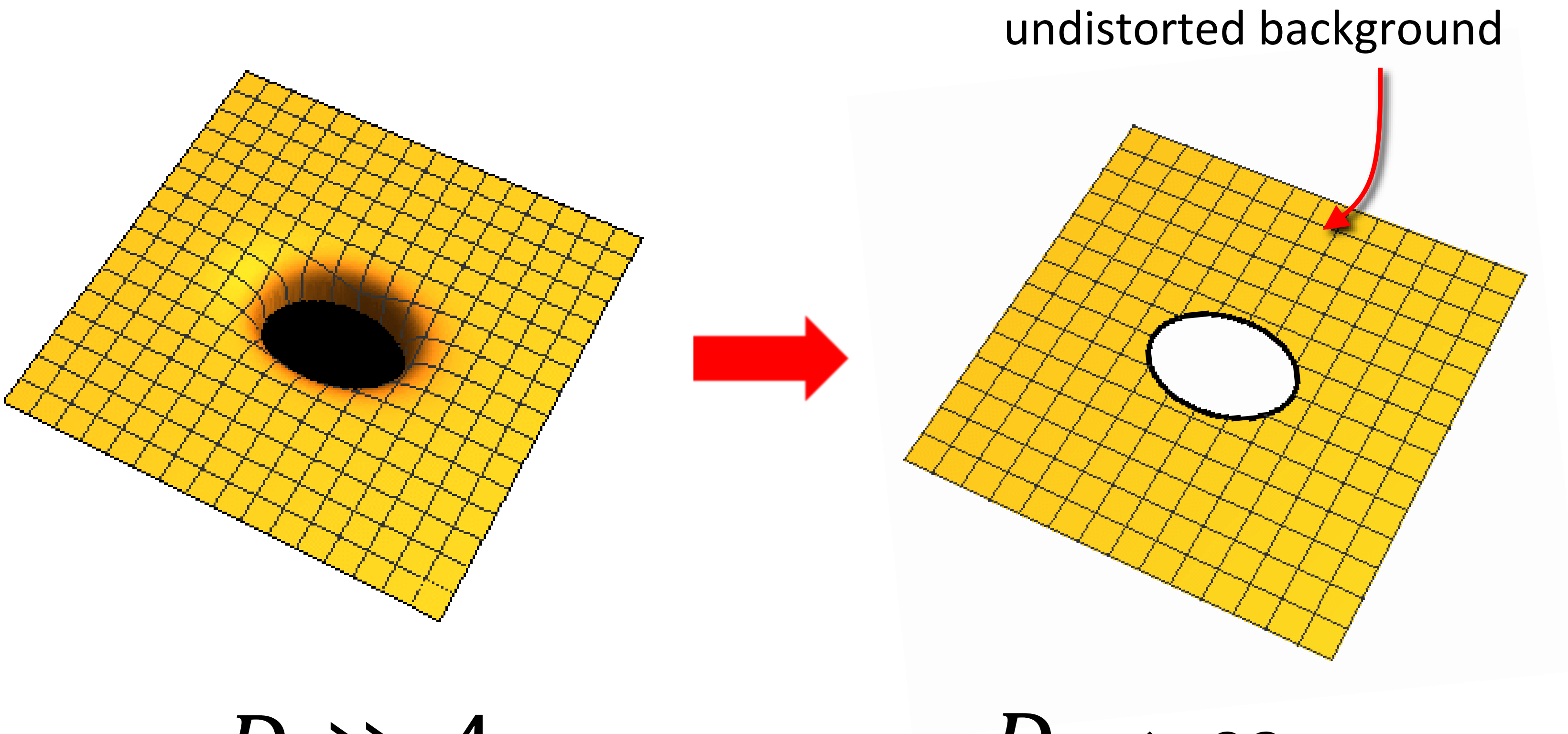
Effective theory of slow decoupled fluctuations

All the black hole dynamics is concentrated here



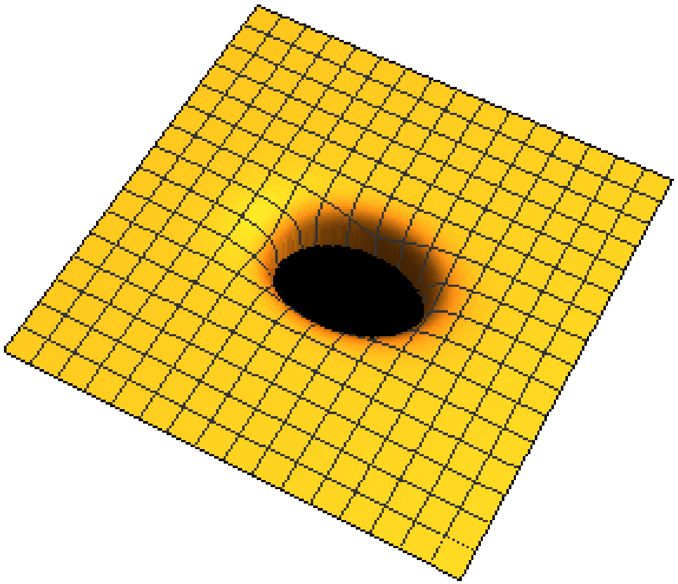
$$D \gg 4$$

Replace bh → Surface (‘membrane’)

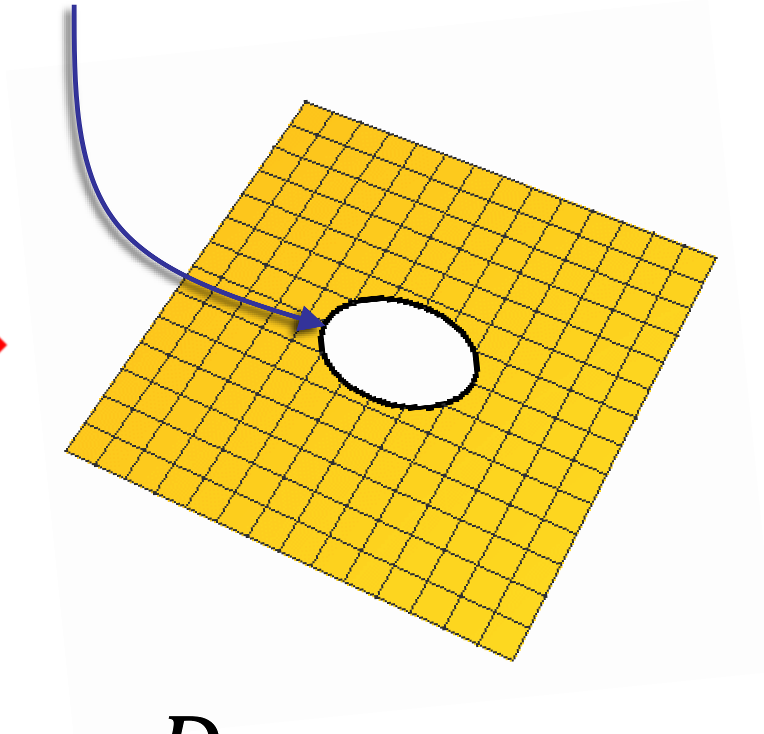


$D \gg 4$ $\quad$ $D \to \infty$

What's the dynamics of this membrane?



$D \gg 4$



$D \to \infty$

Solve Einstein equations in near-horizon

→ *Effective membrane theory*

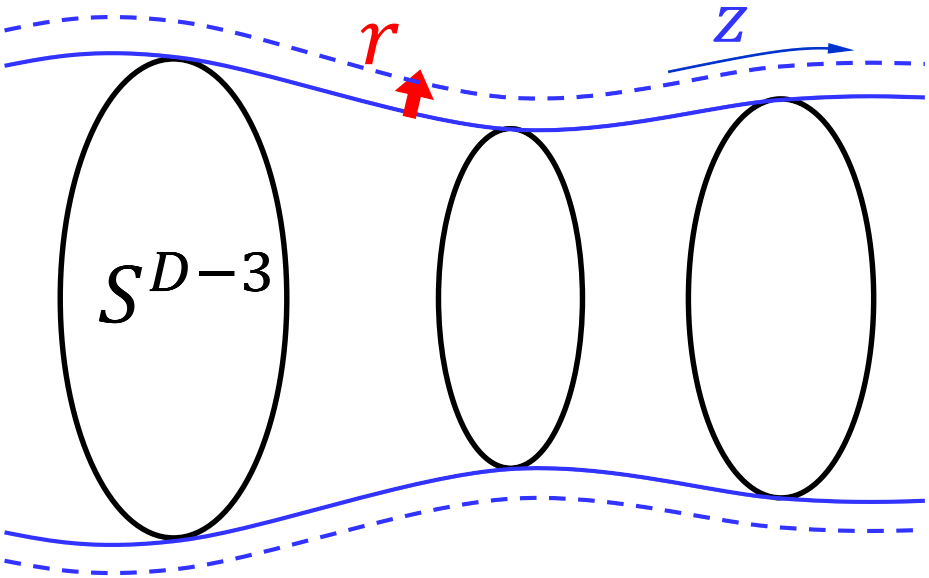
Non-linear effective theory of lightest

quasinormal modes

Gradient hierarchy

$\perp$ Horizon: $\partial_r \sim D$

$\parallel$ Horizon: $\partial_z \sim 1$ (or $\sim \sqrt{D}$)



Effective equations

Most general & elegant formulation by

Bhattacharyya+Minwalla et al

$$\left(\frac{\nabla^2 u}{\mathcal{K}} - \frac{\nabla \mathcal{K}}{\mathcal{K}} + u \cdot K - (u \cdot \nabla) u\right) \cdot \mathcal{P} = 0$$

$$\mathcal{K} = \eta^{AB} K_{AB}$$
$$\nabla \cdot u = 0\,, \qquad n \cdot u = 0$$
$$\mathcal{P}_{AB} = \eta_{AB} - n_A n_B + u_A u_B$$

n, K_{AB}: normal & **extrinsic curvature** of membrane

u: **velocity** field on membrane

Simplifies

–conceptually and technically–

in two important cases:

1. Stationary black holes

2. Black branes, AdS or AF

Stationary solution

Soap-bubble equation (redshifted)

$$\boxed{K = 2\gamma\kappa}$$

$K =$ trace **extrinsic curvature** of membrane

$\gamma =$ **redshift** factor on membrane

$\kappa =$ **surface gravity**

Effective equations for fluctuating black brane

effective fields

$m(t, z^i)$: mass and area density of black brane

$p_i(t, z^j)$: pressure/momenta along brane

$$\partial_t m - \nabla^2 m = -\nabla_i p^i$$

$$\partial_t p_i - \nabla^2 p_i = \pm \nabla_i m - \nabla_j \left(\frac{p_i p^j}{m} \right)$$

AF/AdS

$$\partial_t m - \nabla^2 m = -\nabla_i p^i$$

$$\partial_t p_i - \nabla^2 p_i = \pm \nabla_i m - \nabla_j \left(\frac{p_i p^j}{m} \right)$$

Black brane: $m = 1, p_i = 0$

Fluctuate: $m(t, z^i), p_i(t, z^j)$

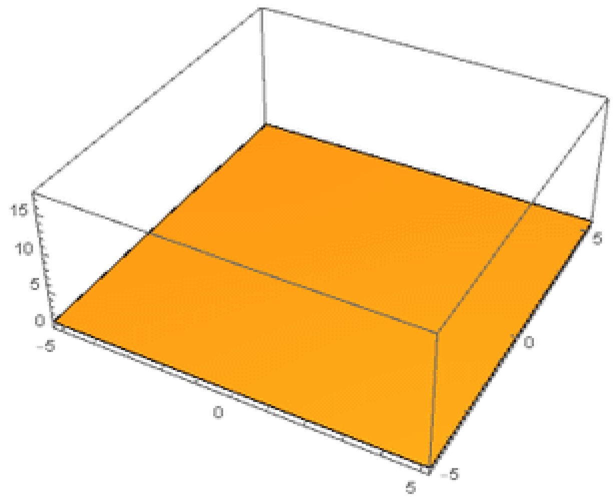
$$\partial_t m - \nabla^2 m = -\nabla_i p^i$$

$$\partial_t p_i - \nabla^2 p_i = \pm \nabla_i m - \nabla_j \left(\frac{p_i p^j}{m} \right)$$

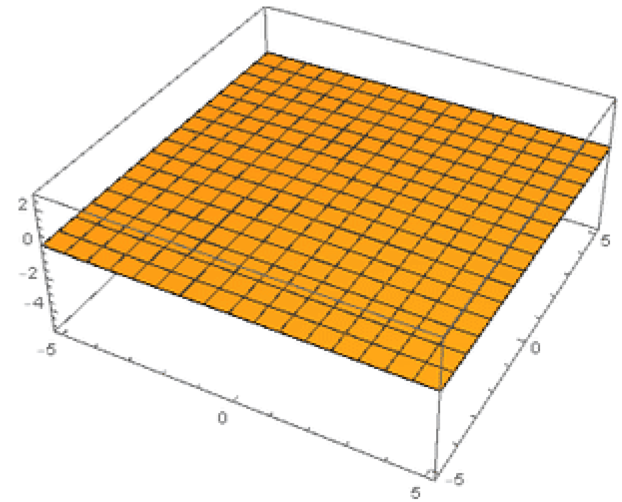
almost linear diffusion equations, except for

BH collisions
and
Cosmic Censorship violation
in Hi-D

“Black hole blob” in a black membrane

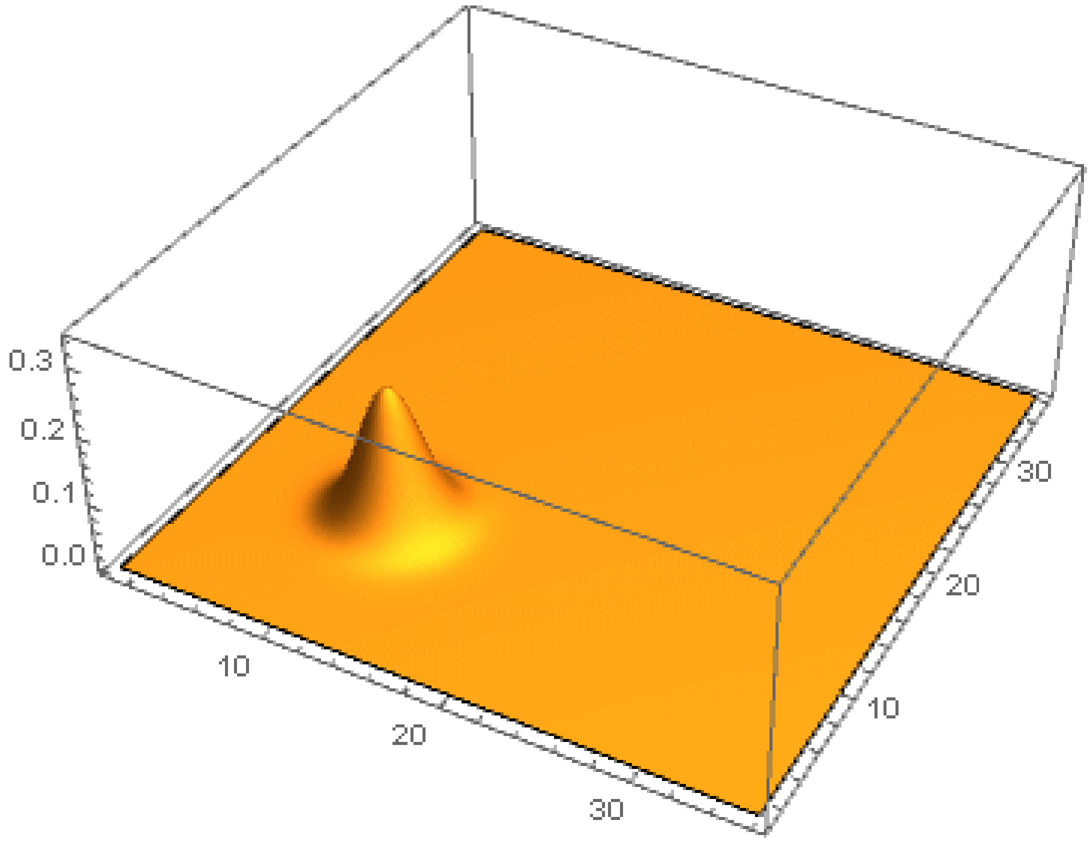


Area density
m



Area radius
$r_H = \ln m$

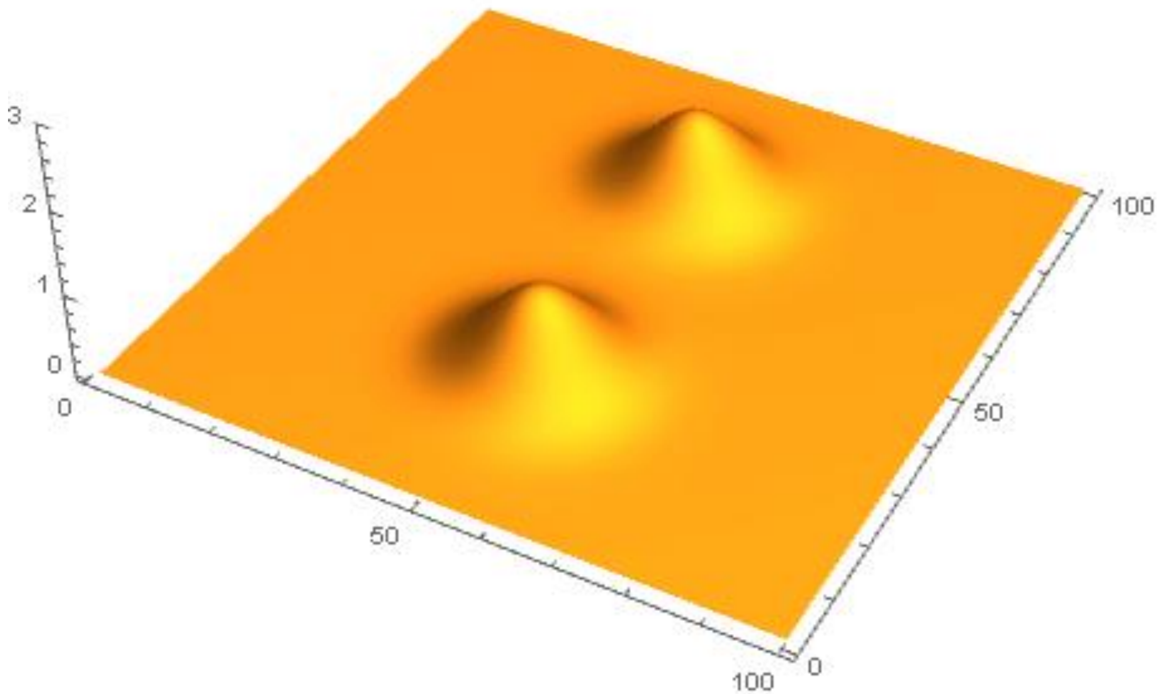
Moving black hole



Collisions of black hole blobs

Brane acts as a “regulator”:
continuous horizon

BHs never really merge nor split:
smooth evolution



3
2
1
0
0
50
100
0
50
100

Large D and the Quantum

Large D gravity and the Quantum

Still at preliminary stage

A few studies, some general observations

Not yet a systematic understanding nor
"effective theory"

– Short-distance quantum fluctuations strongly enhanced

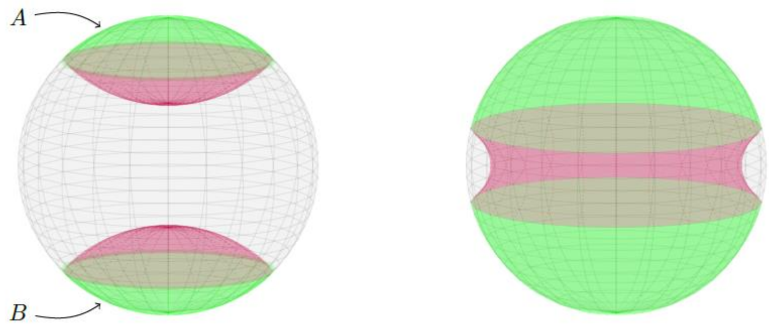
– Long-distance quantum fluctuations average out

Holo-dual to behavior of gravitational field at large D

Entanglement structure

Colin-Ellerin, Hubeny, Niehoff, Sorce 2019

Phase transition in holographic mutual information



Analytical solution @ large D

Entanglement structure

At $D \to \infty$

Entanglement between neighbors vanishes

too many neighbors

Spatial decoupling of holographic correlations

Entanglement between neighbors vanishes at $D \to \infty$

In the bulk this does not involve actual large field gradients

RT/HRT surfaces explore full dimensionality

Can this be exploited/encoded in some effective theory?

Holographic renormalization

RE+Suzuki

UV structure of holographic CFTs (planar)

simplifies at $D \to \infty$

Fefferman-Graham expansion

$$ds^2 = \frac{1}{z^2}(dz^2 + g_{ij}(z,x)dx^i dx^j)$$

$$g(z,x) = g^0(x) + z^2 g^1(x) + \cdots + z^{D-1}\langle T(x)\rangle + \cdots$$

UV, state-independent

Counterterms

Quantum effective action

IR, state-dependent

vevs

Fefferman-Graham expansion

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UV and IR decoupled at all 1/D orders

IR: effective theory of black branes

UV: boundary expansion

Bulk reconstruction at $D \to \infty$

$$ds^2 = \frac{1}{z^2}(dz^2 + g_{ij}(z,x)dx^i dx^j + d\Sigma_{D-p-1})$$

Bulk is reconstructed as Ricci flow from boundary geometry

$$\frac{\partial g_{\mathrm{ij}}}{\partial \log z} = 2R_{ij}$$

Counterterm action truncates finitely at each order in $1/D$

Large D gravity and the Quantum

Scales in Hawking radiation

emission time, scrambling time, evaporation time

Hod

Holdt-Sørensen, McGady, Wintergerst

String theory @ large D: an old story

Alvarez, Ambjorn, Gubser…

Large-N + large-D tensor model X_{μ}^{ij} *Ferrari*

Large D gravity and the Quantum

still lacking basic concepts

what problems are good?

much to explore...

Thank you