Euclidean vs. Lorentzian quantum gravity

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January 14, 2020

Introduction

In quantum mechanics there are two kinds of path integrals that we like:

- Euclidean path integral: thermal equilibrium, fixed-time correlators.
- Lorentzian path integral: scattering theory, out-of-equilibrium dynamics, chaos, etc.

For ordinary quantum systems there isn't much conceptual difference between them: either can be obtained from the operator formalism by repeatedly inserting complete sets of states, and they are related by a fairly simple analytic continuation.

In quantum gravity the situation is quite different. Indeed the Euclidean gravity path integral seems to know quite a few things that it doesn't have a right to:

- Black hole entropy formula Gibbons/Hawking
- Ryu-Takayanagi formula Lewkowycz/Maldacena, Dong/Lewkowycz/Rangamani, Dong/Lewkowycz
- Page curve! Penington, Almheiri/Engelhardt/Marolf/Maxfield, Penington/Shenker/Stanford/Yang, Almheiri/Hartman/Maldacena/Shaghoulian/Tajdini

How does it know these things?
I don't know, and this has bothered me for years.
What I *can* tell you is why it isn't obviously impossible for it to know them: because beyond perturbation theory the Euclidean gravity path integral is **not** related to the Lorentzian gravity path integral in any simple way!

Jackiw-Teitelboim gravity

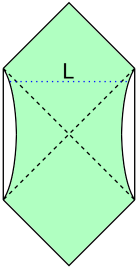
The distinction between Lorentzian and Euclidean gravity is particularly clear in two recent treatments of the asymptotically-AdS version of Jackiw-Teitelboim gravity, which has (Lorentzian) action

$$S = \Phi_0 \left(\int_M R + 2 \int_{\partial M} K \right) + \int_M \Phi(R+2) + 2 \int_{\partial M} \Phi(K-1)$$

and boundary conditions

$$\Phi|_{\partial M} = \frac{\phi_b}{\epsilon}$$
$$ds^2|_{\partial M} = -\frac{dt^2}{\epsilon^2}.$$

The first approach we can consider is canonical quantization, which for JT gravity can be done exactly Harlow/Jafferis.



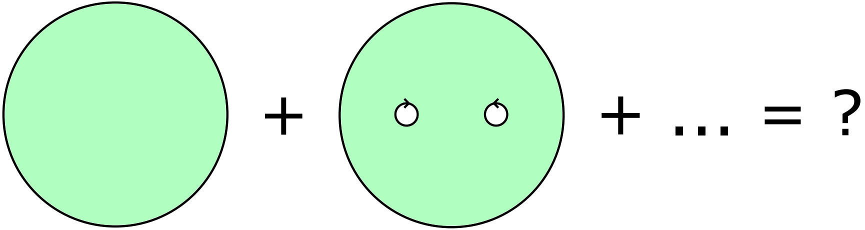
With two asymptotic boundaries there is a single gauge-invariant degree of freedom, which we can take to be the renormalized geodesic length L between the two boundaries, and the Hamiltonian is

$$H = \frac{P^2}{2\phi_b} + \frac{2}{\phi_b} e^{-L}.$$

The sum over topologies is trivial: the only connected globally hyperbolic manifolds obeying these BC have topology $\mathbb{R} \times I$.

The second approach is to begin with the Euclidean JT gravity path integral with a single boundary, including a nontrivial sum over topologies, and then to try to come up with some kind of quantum interpretation.

Saad/Shenker/Stanford,Stanford/Witten



This leads to the result that this sum over topologies can be re-interpreted as the *average* of a quantum partition function over an ensemble of Hamiltonians.

These two quantizations of the JT action are clearly inequivalent: the first produces a standard quantum mechanical system, while the second does not. What are we to make of this?

- Other than a technical contour ambiguity in option two, there does not seem to be anything wrong with either approach.
- The Euclidean path integral can be interpreted as preparing a state in the Hilbert space obtained by canonical quantization, which gives an "option one" interpretation of many of the calculations in option two.
- Expectation values of gauge-invariant operators on the canonical Hilbert space can be obtained by analytic continuation from option two in the limit $\Phi_0 \to \infty$, but this limit is not necessary for option one to make sense.

Perhaps the main point however is that although option one (the Lorentzian approach) is well-defined and easier to understand, it is also **boring**. With only one boundary the canonical Hilbert space is empty, so there are no black hole microstates. An average over black hole microstate ensembles may be confusing, but it's got to be more interesting than not having any at all!

Are we satisfied?

Despite the remarkable powers of Euclidean gravity, I think we should not be satisfied if it is the only way we have of computing something. It doesn't tell us what is actually going on!

- Gibbons and Hawking told us how to compute black hole entropy, but it took twenty more years for us to begin counting black hole microstates in string theory and AdS/CFT.
- Lewkowycz and Maldacena told us how to compute the von Neumann entropy of a boundary subregion using bulk gravity, but the Lorentzian interpretation of the Ryu-Takayanagi formula through quantum error correction is still a work in progress. Harlow, Dong/Harlow/Marolf, Akers/Rath,...
- Recent work has taught us how to compute (analogues of) the Page curve, but we still do not know what the bulk dynamics are by which information gets out. We also don't know whether or not there are firewalls in typical black hole microstates.

I will spend the rest of this talk describing a toy model which attempts to provide a Lorentzian interpretation of the Page curve. Akers/Engelhardt/Harlow

A simple holographic model of black hole evaporation

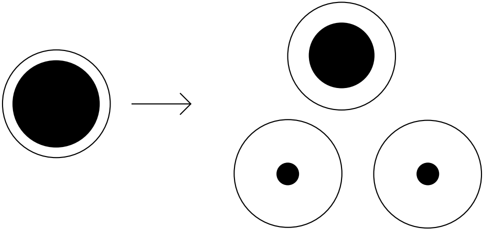
In the recent Page curve papers, there is an annoyance and a puzzle:

- The annoyance: in most of the models, it is essential to use the full Engelhardt/Wall version of the Ryu-Takayanagi formula. This includes bulk entropies which are somewhat challenging to handle, especially in greater than two spacetime dimensions.
- The puzzle: the calculation of the Page curve seems to rely on an assumption that from the bulk point of view the quantum state of the radiation is mixed. But why should such an assumption play any role in a theory where the radiation is pure?

With Akers and Engelhardt, we set out to provide a model simpler than those of Penington,Almheiri/Engelhardt/Marolf/Maxfield, which would avoid the annoyance and clarify the puzzle. Moreover our dynamics are explicit (albeit artificial); there is no role for Euclidean gravity magic (although we still use it to prepare states).

A new evaporation channel

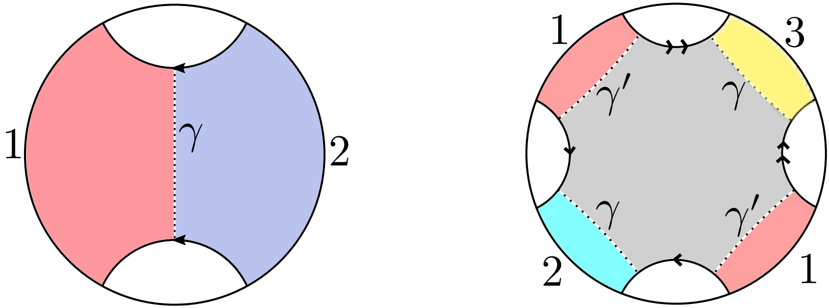
The idea of our model is to consider a big AdS black hole which evaporates not into soft quanta but instead into smaller black holes, each of which ends up in its own asymptoticly-AdS universe:



This may seem unnatural, but what it buys us is a completely explicit holographic description.
Also note that many of the BH paradoxes are about the structure of the state at a fixed time, so the details of the dynamics are often not important.

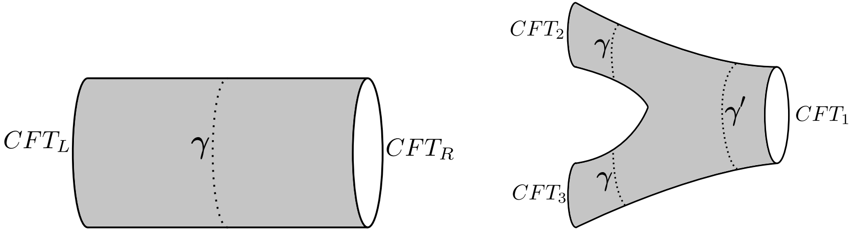
Multiboundary wormholes

Our detailed model is constructed using multiboundary (spatial) wormholes. These are simple to understand in $2+1$ dimensions, so that is where we will work.



The classical solutions are constructed by quotients AdS_3, which we will take to be time-reflection symmetric. The spatial geometry at the symmetric slice is a quotient of the Poincare disk.

We can represent these spatial geometries more heuristically:

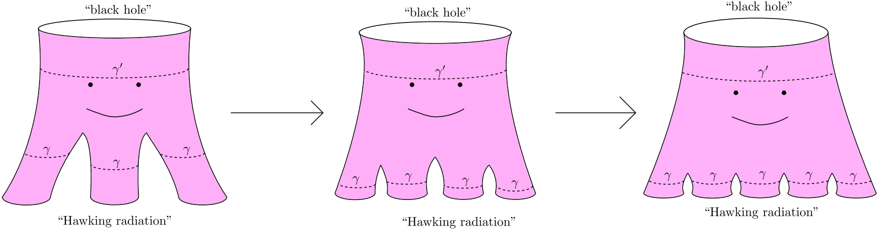


We will need three properties of these geometries:

- The dotted lines are spacetime geodesics, and thus candidate HRT surfaces.
- For greater than two exits we can adjust their lengths independently.
- We can prepare these as CFT states by cutting a Riemann surface.

Dynamics

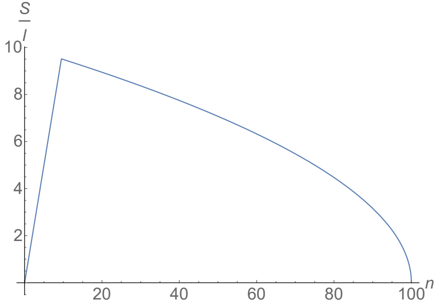
The dynamics of our model are then that we begin with a holographic CFT on a large number of circles, with the state being a three-exit wormhole with one “large” exit and two “small” exits together with the vacuum on the remaining exits.We then choose a (time-dependent) Hamiltonian to gradually mix in more of the CFTs, leading to:



The Page curve arises from the competition between the “headband” and “ankle bracelet” candidates for the RT surface.

The Page curve

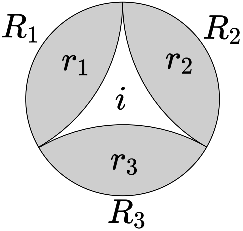
The initial RT surface of the Hawking radiation is the ankle bracelets γ'. As the evaporation proceeds, the length of γ' increases and the length of γ decreases (we take the total energy to be conserved), so at some "Page time" there is a transition between them. We thus have a Page curve:



At the same time the entanglement wedge of the radiation jumps to include the interior, so any further information thrown in to the black hole can be quickly recovered as in Hayden/Preskill. Quite simple!

Quantum error correction and "islands"

The "body" of the octopus above is an example of what was called an "island" in Almheiri/Mahajan/Maldacena/Zhao. Indeed these "islands" are a standard part of the story of quantum error correction in AdS/CFT:



Mathematically they should be understood via an isometric embedding

$$\mathcal{H}_{code} = \mathcal{H}_{r_1} \otimes \mathcal{H}_{r_2} \otimes \mathcal{H}_{r_3} \otimes \mathcal{H}_i \mapsto \mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_{R_3},$$

and physically they are a part of the shared dual of $\mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_{R_3}$ that arises due to restrictions we place on $\mathcal{H}_{code}$.

This clarifies the role of the "mixed radiation" appearing in the calculations of Penington, Almheiri/Engelhardt/Marolf/Maxfield, ... This can be explained precisely in terms of the "island formula", which in Almheiri/Hartman/Maldacena/Shaghoulian/Tajdini is written as

$$S(\rho_R) = \mathrm{ext}_Q \left[\frac{\mathrm{Area}(Q)}{4G} + S(\widetilde{\rho}_{I \cup R}) \right].$$

The problem with this formula is that the symbol R means two different things on the two sides of this equation: on the right side at late times $\widetilde{\rho}_R$ a mixed state, while on the left side at late times ρ_R a pure state. But why should there be two different states for the same thing? There aren't of course, and this formula should really be written as

$$S(\widetilde{\rho}_R) = \mathrm{ext}_Q \left[\frac{\mathrm{Area}(Q)}{4G} + S(\widetilde{\rho}_{i \cup r}) \right].$$

Here i is the island and r is a bulk region which in these examples is the causal wedge of the boundary subregion R (more generally it should be the union of the entanglement wedges of pieces of R?). On both sides $\widetilde{\rho}$ is the same CFT (or CFT + bath) state, it is really the algebras that differ in the two entropies.

Conclusions

- In gravity the low-energy Euclidean and Lorentzian path integrals are inequivalent. The former knows more about the UV completion, but to really understand what is going on we need a Lorentzian picture.
- We've realized the essential features of the Page curve calculations (exchange of dominance, islands, etc) in a very simple geometric model.
- The "radiation" which is mixed in the discussions of Penington, Almheiri/Engelhardt/Marolf/Maxfield, or equivalently the "R" appearing on the right-hand side of the "island formula", is really a bulk region which in our model is the "causal wedge" of the actual radiation, which removes an apparent contradiction.
- Our evaporation model is rather crude, I think that it is a reasonable caricature of how things should actually work but to go further will require a better understanding of the non-perturbative degrees of freedom of quantum gravity. Perhaps it is possible in tensor networks?

Thanks!