Replica wormholes and the entropy of Hawking radiation

[part 2]

arXiv:1911.12333 with Almheiri, Maldacena, Shaghoulian, and Tajdini

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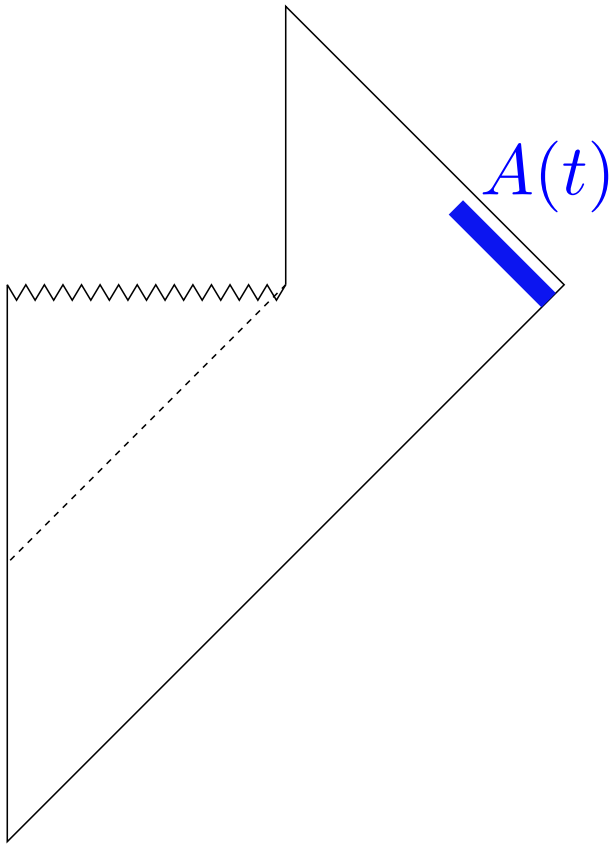
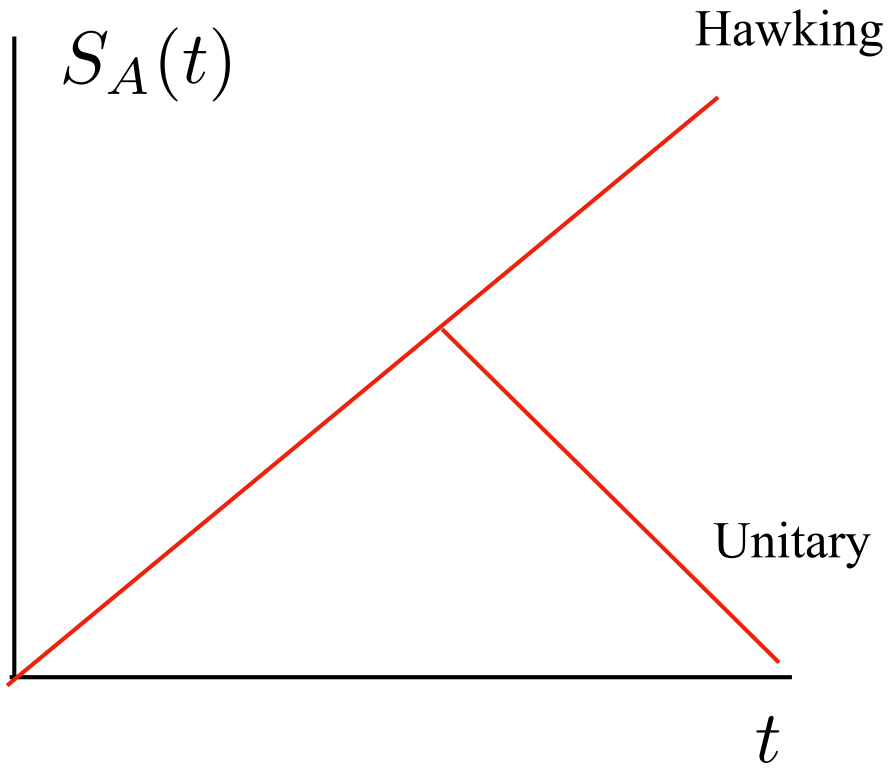
Geometry from the Quantum ✦ **KITP** ✦ **January 13, 2020**

An important part of the information paradox is a large error in the von Neumann entropy of Hawking radiation.

Flat spacetime



Page curve



$$\rho_A = \rho_{thermal} + \text{ perturbative } + \mathcal{O}(e^{-\#S})$$

e^{-S} corrections to each matrix element $(\rho_A)_{mn}$ are big enough to fix the entropy

$$S_A = -\text{tr}\rho_A \log \rho_A$$

"Paradox"

1. Local / perturbative corrections don't help
2. No known *mechanism* for information escape
3. Low-energy calculations of S_A itself, not just matrix elements of ρ_A

Goal:

Show that a direct calculation of S_A using the replica method gives the unitary Page curve.

Unitary curve is a non-perturbative effect from "replica wormholes".

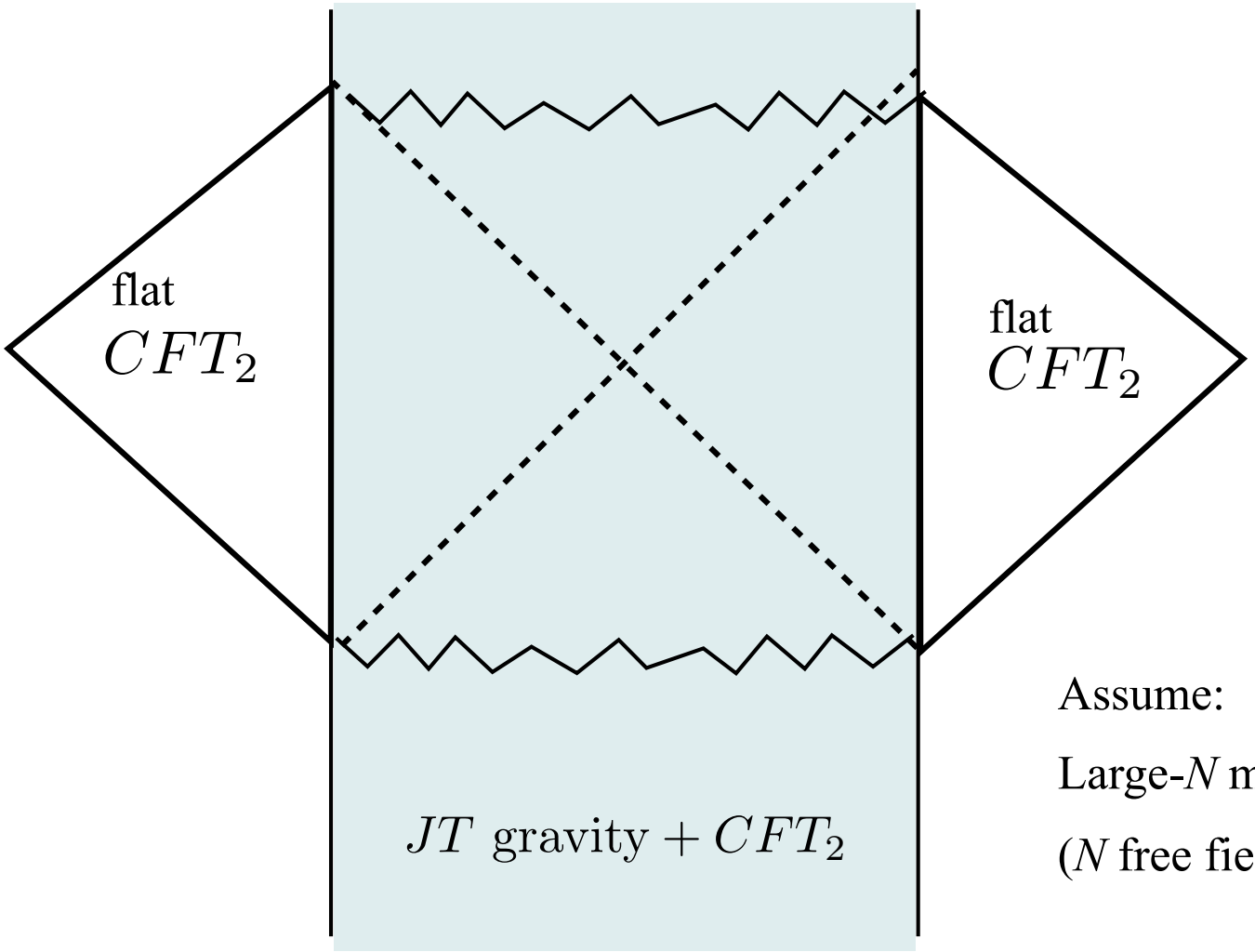
This is a semiclassical gravity calculation. It does not use holography.

It reproduces the holographic results of

[Almheiri, Engelhardt, Marolf, Maxfield '19]
[Penington '19]
[Almheiri, Mahajan, Maldacena, Zhao '19]
[Almheiri, Mahajan, Maldacena '19]

Info Paradox for an Eternal Black Hole

[Almheiri, Mahajan, Maldacena '19]



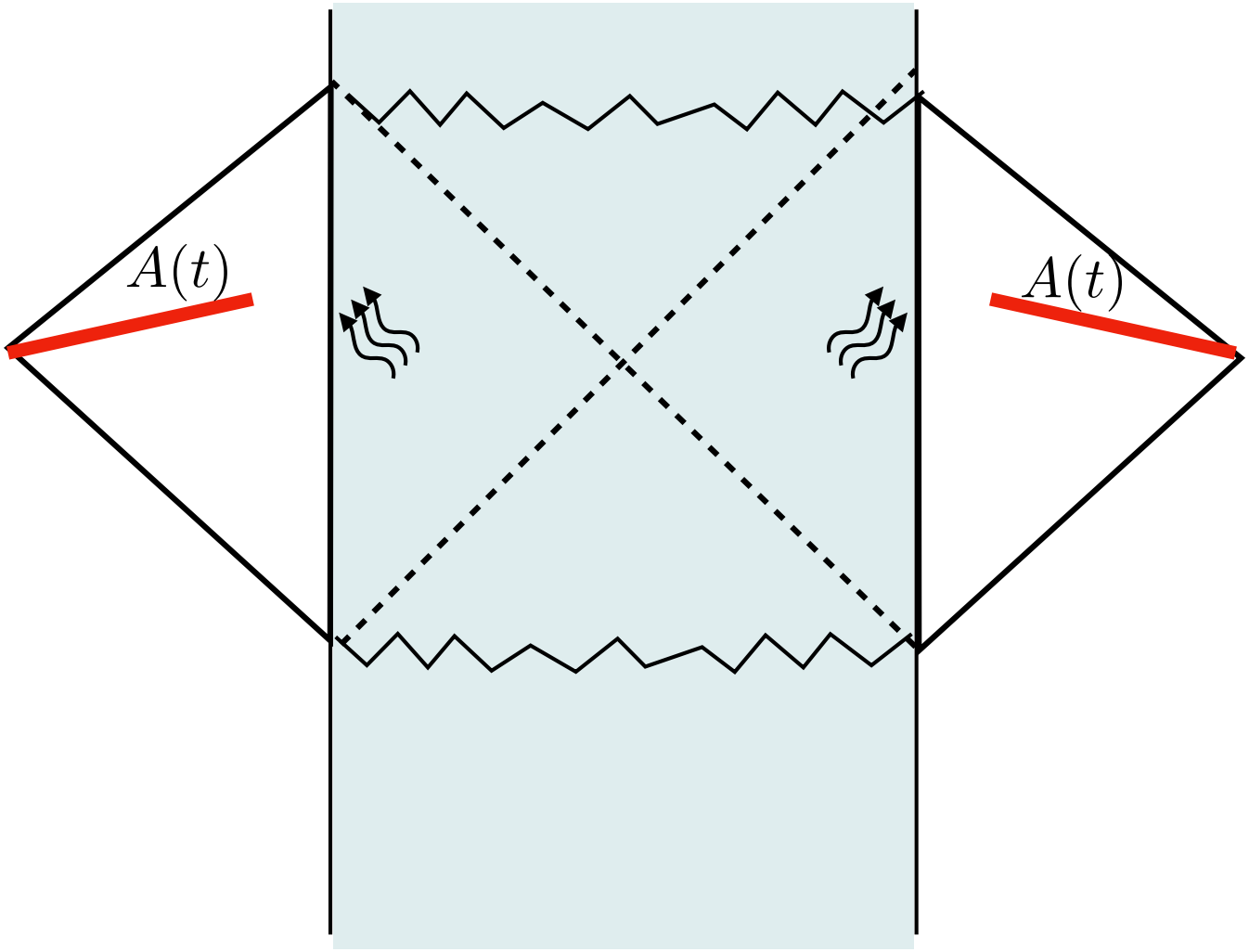
Assume:

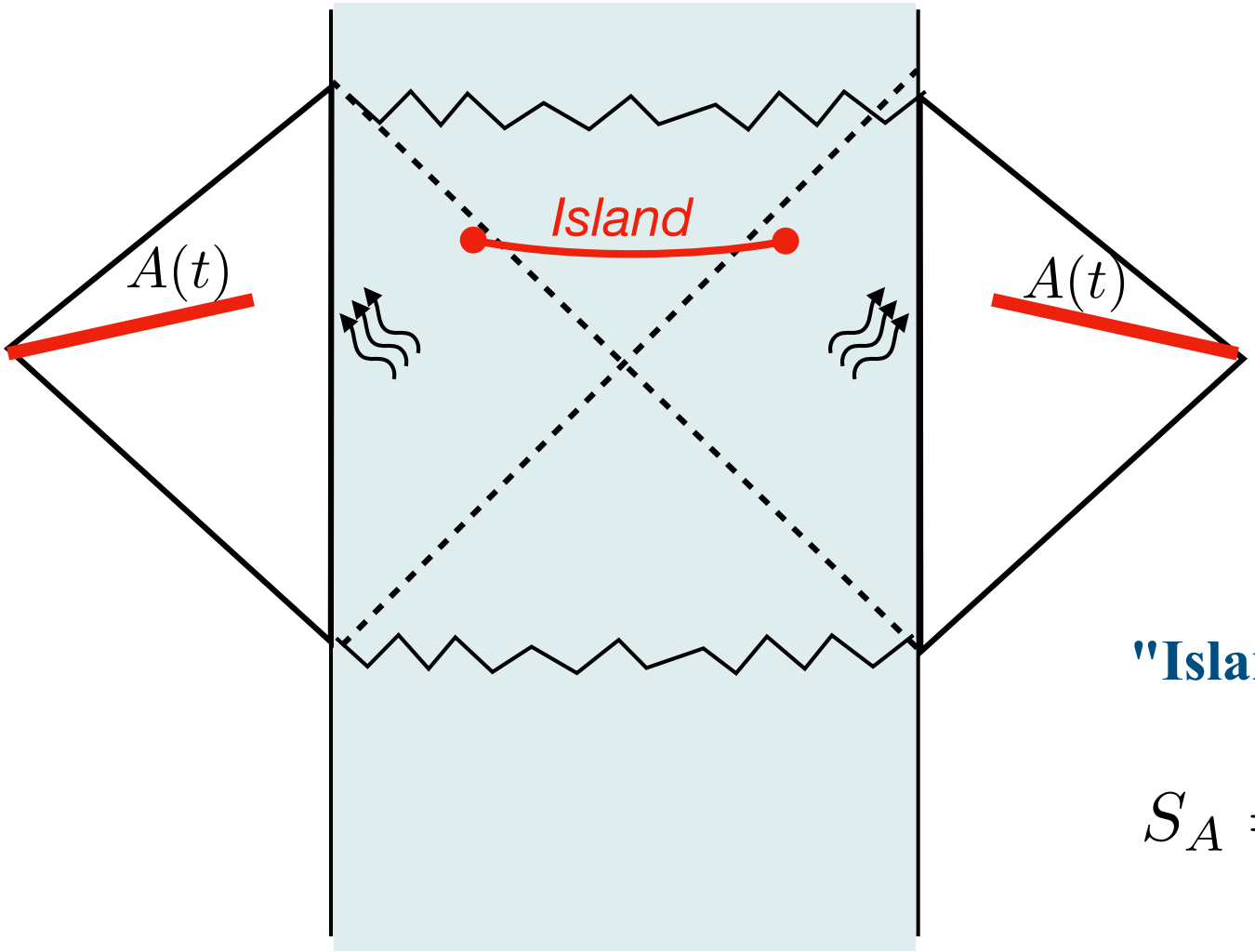
Large-N matter sector

(N free fields or holographic)

Info Paradox for an Eternal Black Hole

[Almheiri, Mahajan, Maldacena '19]



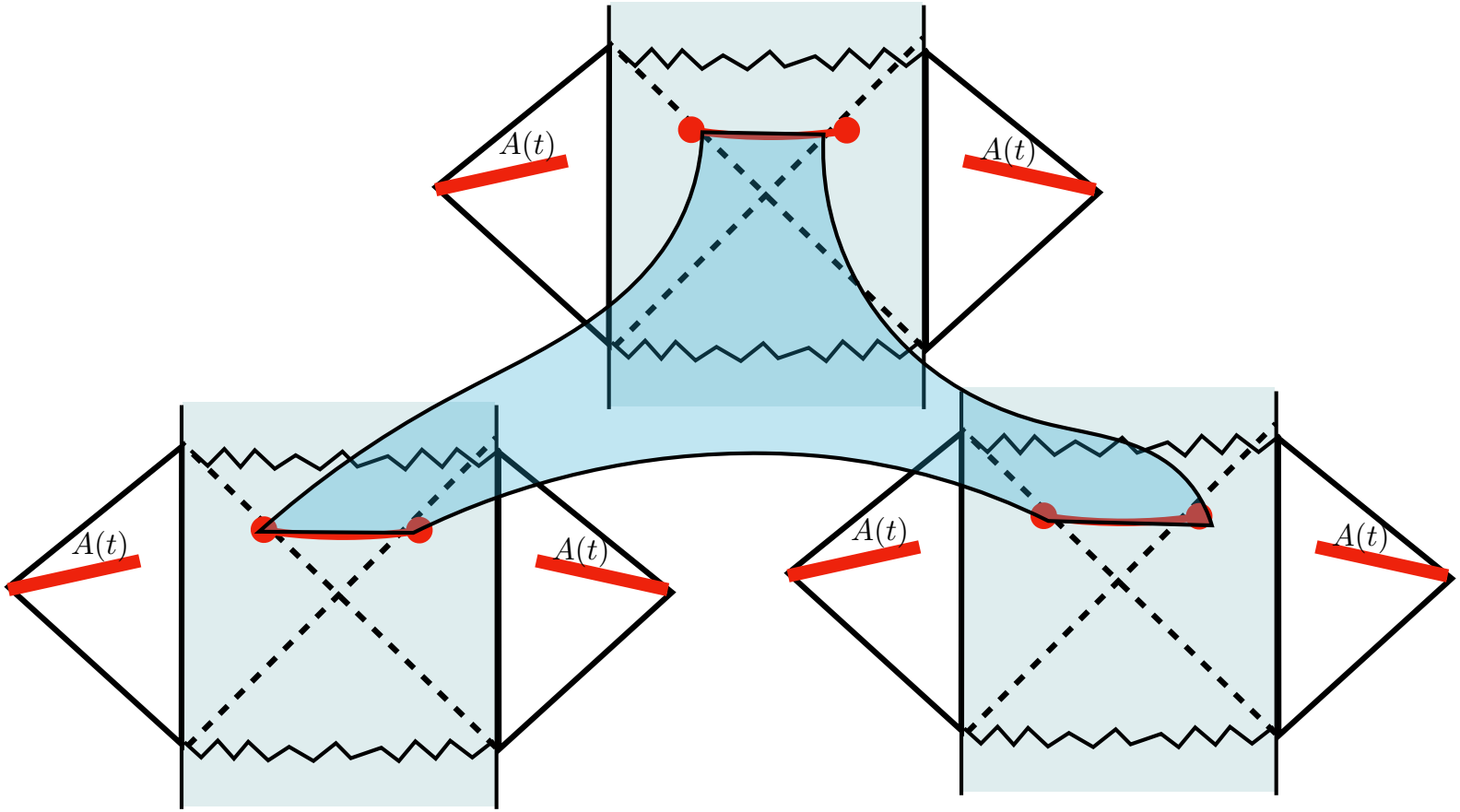


"Island" appears at late times

$$S_A = \mathrm{ext}_I S_{gen}(A \cup I)$$

[Almheiri, Engelhardt, Marolf, Maxfield '19]
[Penington '19
[Almheiri, Mahajan, Maldacena, Zhao '19]

Basic idea



In the replica method, dynamical wormholes appear connecting the black hole interiors.

These are complex saddles, with no symmetries, that we can construct by explicit solution of the EOM for gravity+matter in certain simple cases.

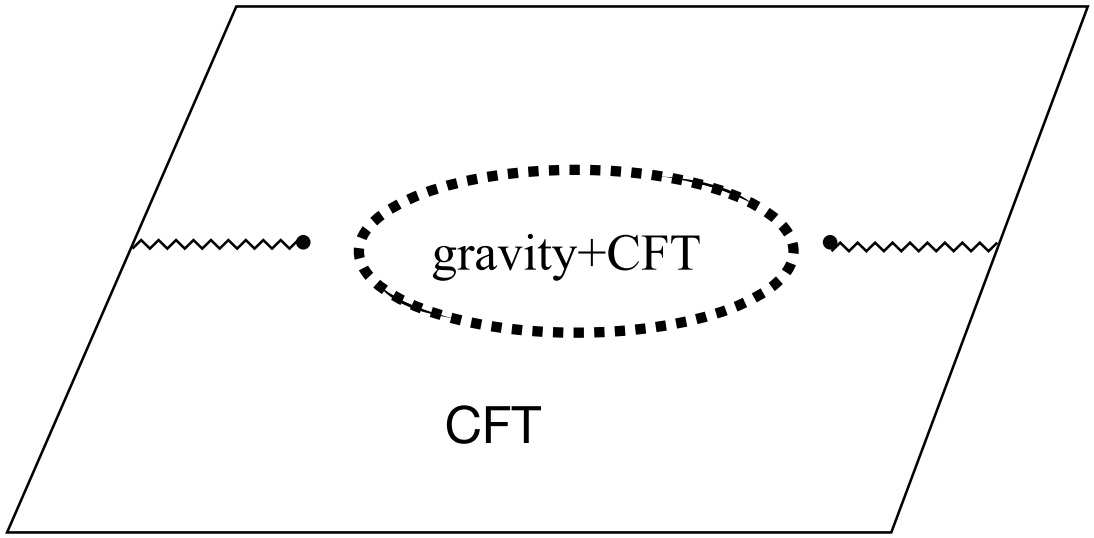
In the replica limit, these saddles leave an imprint on certain observables, including the von Neumann entropy.

Replica Calculation

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A$$
$$= -\partial_n \mathrm{Tr}\ (\rho_A)^n|_{n=1}$$

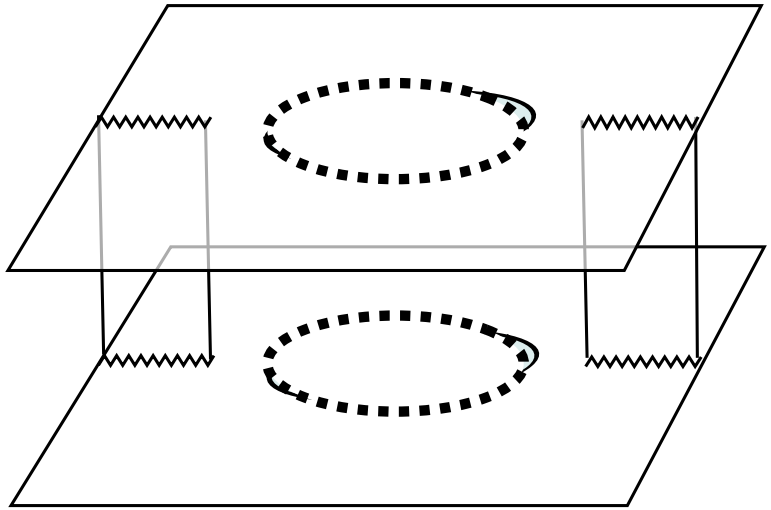
n=1 replica

$\mathrm{Tr}\rho_A =$



n=2 replicas

$$\mathrm{Tr}\rho_A^2 =$$



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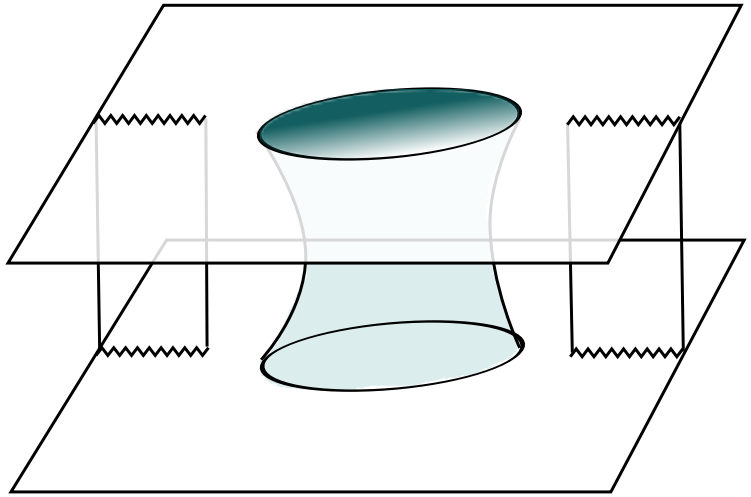


"Hawking" saddle

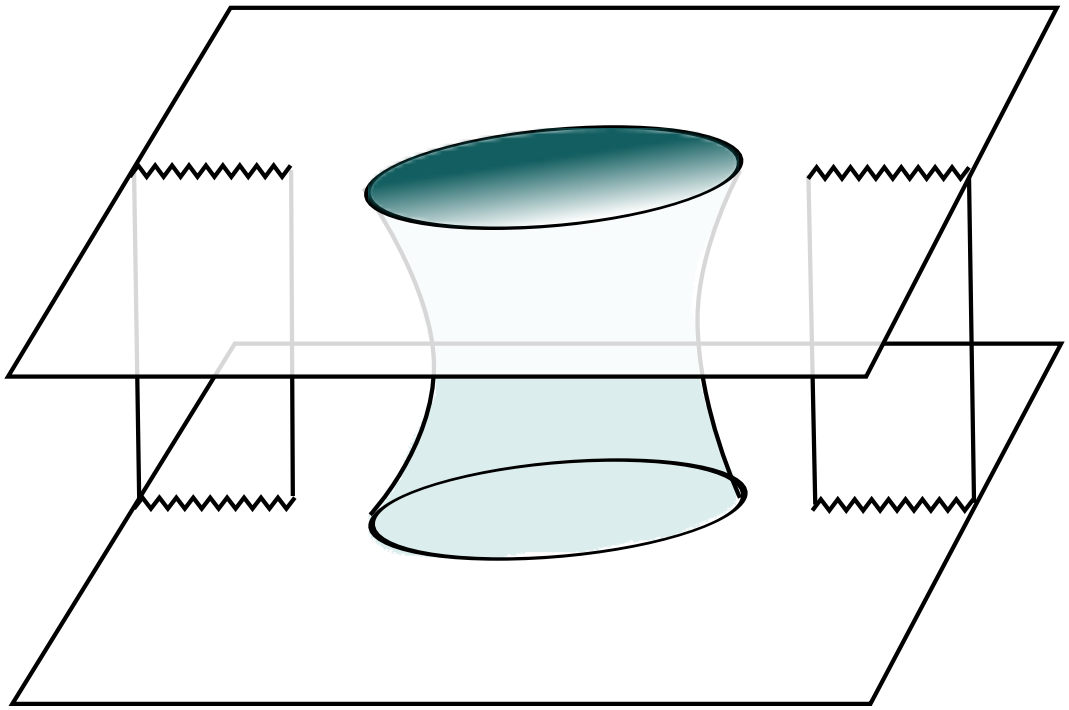
Replica wormhole

$$= \quad e^{-S_2^{\mathrm{Hawking}}} \quad + \quad e^{-S_2^{\mathrm{Wormhole}}} \quad + \quad \cdots$$

Suppressed by topology $e^{S_0\chi}$

Suppressed by large entanglement of radiation with interior

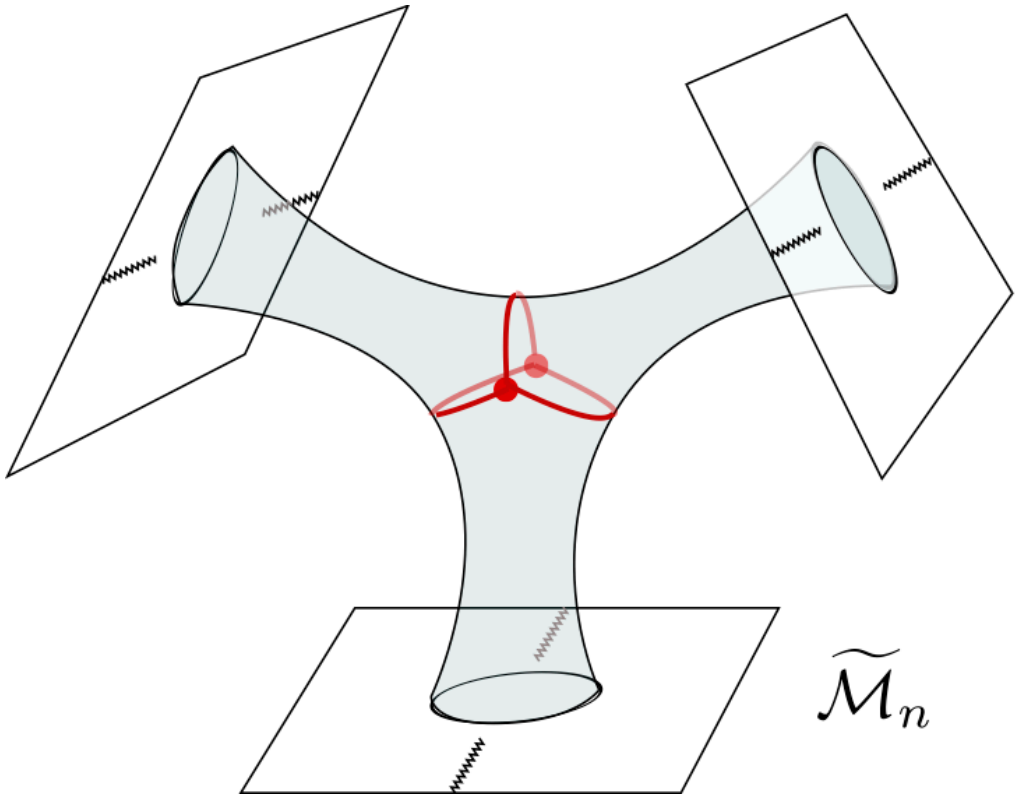
Comments on Ensemble Averaging



Wormholes connecting *decoupled* QFTs violate factorization, $\langle Z(\beta)^2 \rangle = Z(\beta)^2$

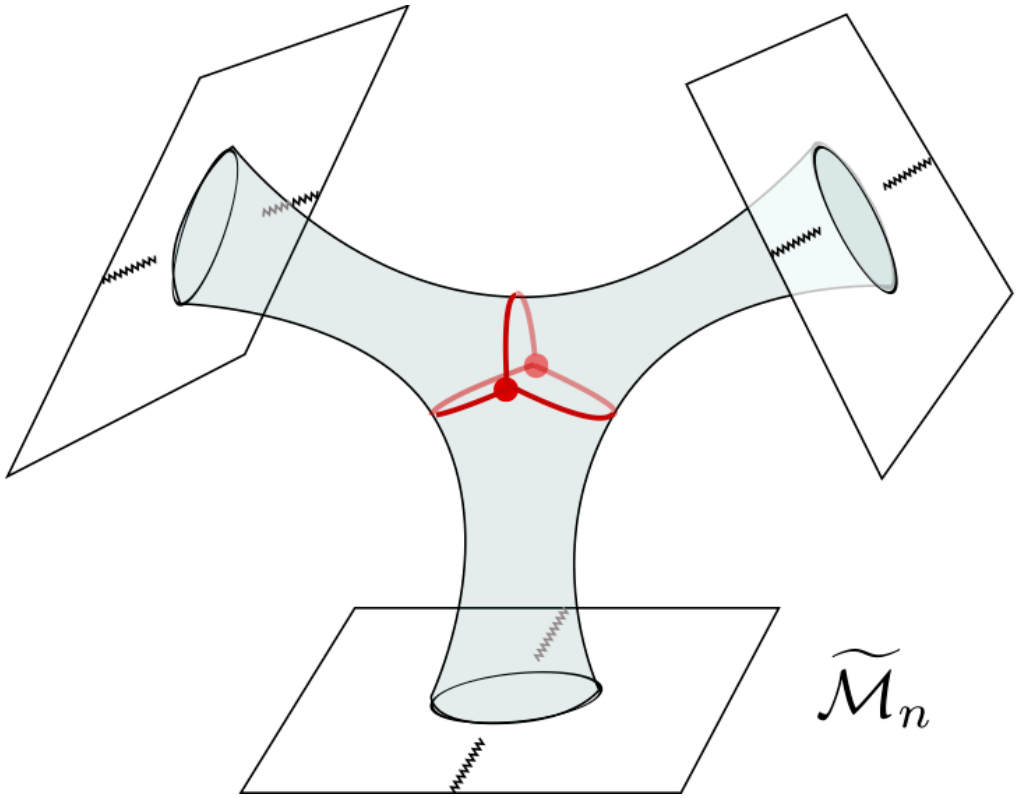
The gluing in region *A* explicitly couple the two sheets, so this paradox does not arise here.

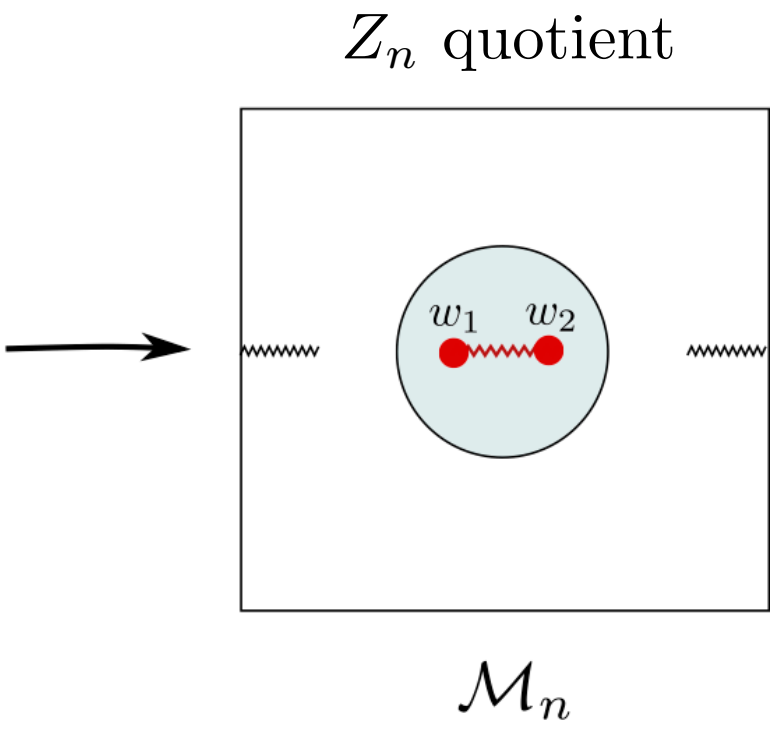
Higher replicas



Assume Z_n replica symmetry

Higher replicas



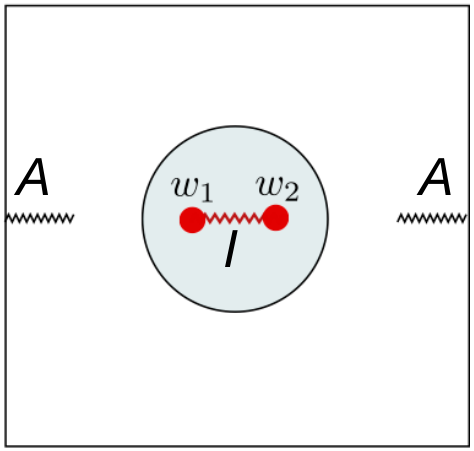


Twist operators appear *dynamically* in the gravity region.

Including these saddles in the replica calculation gives the island rule for the entropy.

cf. [Lewkowycz and Maldacena '13]
[Dong, Lewkowycz, Rangamani '16]
[Dong, Lewkowycz '17]

$$\mathrm{Tr}\ (\rho_A)^n = \int DgD\phi\, e^{-S_{JT}[g,\phi]} Z_{CFT}[g]$$



Expand n ~ 1

$$Z_{CFT} \approx e^{-(n-1)S_{CFT}(A\cup I)}$$

$$S_{JT} \approx (n-1)\frac{\mathrm{Area}(I)}{4}$$

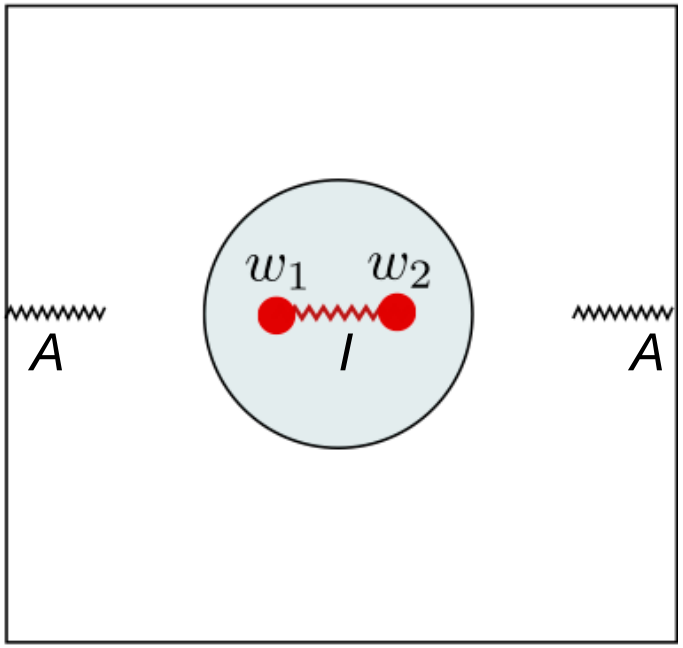
$$\mathrm{Tr}\ (\rho_A)^n \approx \int D(\mathrm{moduli}) e^{-(n-1)S_{gen}(A\cup I)}$$

[Here we assumed large N matter]

$$S_A = \mathrm{ext}_I S_{gen}(A \cup I)$$

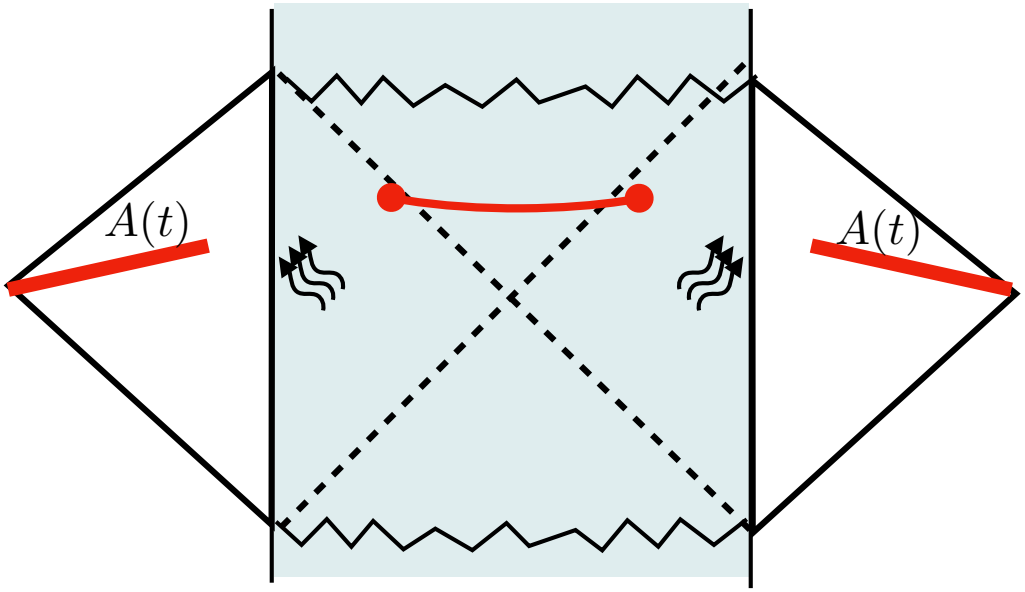
This gives the unitary Page curve for the eternal or evaporating black hole.

Comments



Note that we are calculating the entropy of region A.

This is the entropy of a *QFT-only* density matrix, despite the island region appearing in the entropy formula.



We inserted defect operators at the edge of the QFT region A.

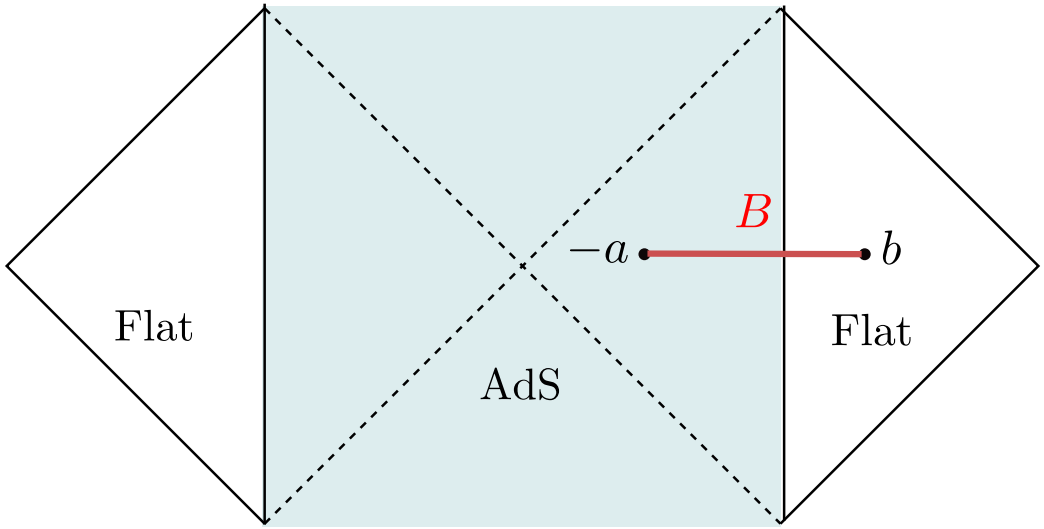
At late times, dynamical defects are pair-created at the edge of the island.

$$\text{cost} \sim \frac{\text{area}}{4G_N}$$

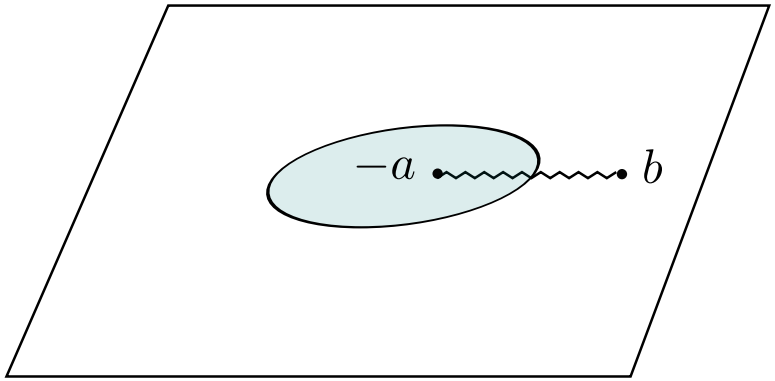
Explicit construction of a wormhole geometry

for single interval

Lorentzian

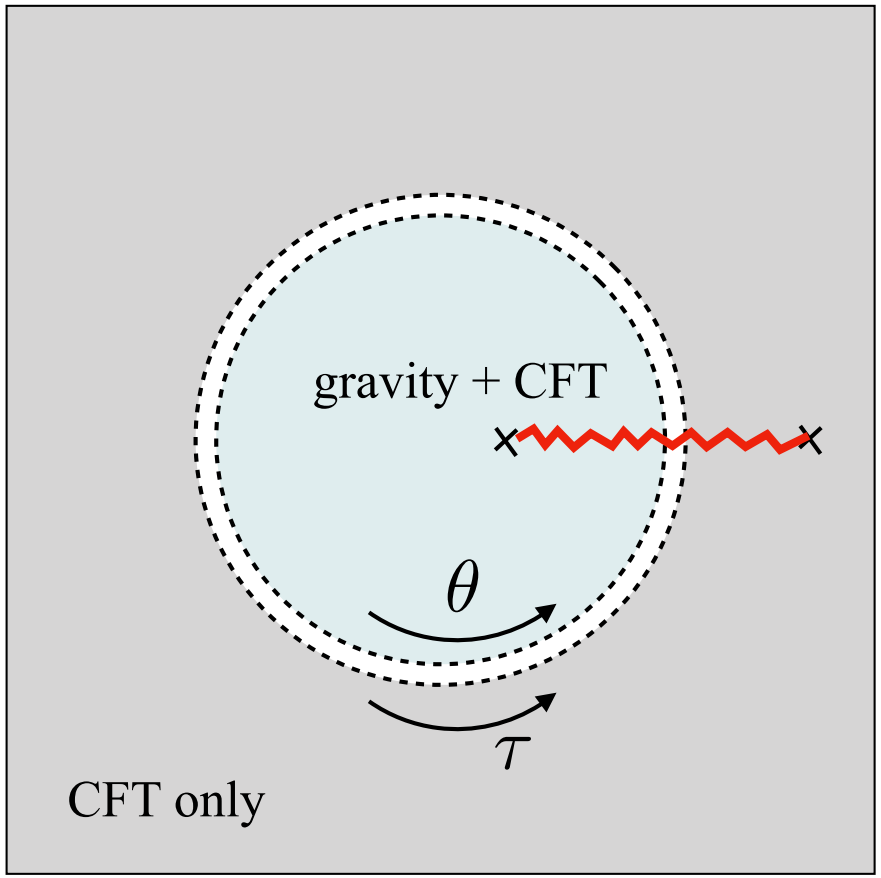


Euclidean



(The 2-interval case should factorize into two copies of this wormhole at late times.)

The only dynamical d.o.f. in JT gravity is the boundary "gluing function" $\theta(\tau)$



EOM with the standard hyperbolic metric

$$\partial_\tau\{e^{i\theta(\tau)},\tau\} = i(T-\overline{T})$$

With an *n*-fold twist defect there's an extra term

$$\partial_\tau\{e^{i\theta},\tau\}+(1-\frac{1}{n^2})\partial_\tau R[\theta] = i(T-\overline{T})$$

Here *T* is the CFT stress tensor on the glued manifold, which depends on $\theta(\tau)$ in a complicated way.

"conformal welding"

This e.o.m. is on the quotient manifold, so at this point, we can analytically continue to non-integer replica number *n*.

The equation can be solved analytically for

1) $\beta \to 0$, any n (high-temperature limit)

This gives explicit, finite-*n* wormhole solutions.

2) $n \sim 1$, any β (entanglement entropy limit)

In this case the equation of motion imposes the quantum extremal surface condition on the location of the dynamical twist defect,

$$\partial_{\pm} S_{gen} = 0$$

Conclusion

The replica calculation of the Page curve has contributions from replica wormholes.

These lead to an entropy compatible with unitarity.

Questions

What about small N?

What is the Lorentzian interpretation?

What does this tell us about holography in Minkowski spacetime?