Based on earlier work by **de Saint-Exupéry**

The Python's Lunch: Geometric Obstructions to Decoding Hawking Radiation

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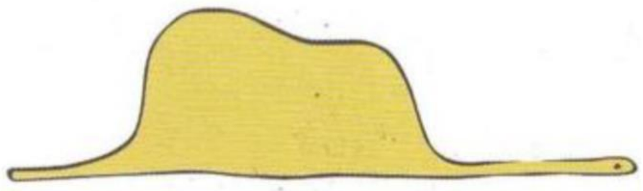


in principle

Previous three talks: how to see from semiclassical gravity that information that fell into the black hole can be reconstructed from the Hawking radiation

Hawking radiation encodes information (in an information-theoretic sense)

- Precise gravitational **calculations**

in practice

This talk: how to see from semiclassical gravity that information that fell into the black hole **can't** be reconstructed from the Hawking radiation

Reconstructing the information is ***exponentially complex***

- Some **non-gravitational** calculations
- A story about how gravity is **analogous** to these calculations

Complexity theory vs information theory

- Shannon single-handedly proved most of the fundamental theorems you would want to know about information theory (**noise channel coding theorem** etc.)
- We still don't know whether **P=NP**

Complexity theory is just **harder** than information theory

Restricted vs Unrestricted Complexity

- **Harlow-Hayden (2013)**: converting black hole + Hawking radiation into a simple state (in order to recover information/test the AMPS paradox) is **exponentially hard** ($e^{O(S_{BH})}$).
- **Susskind + collaborators (2014 – present):** state complexity is dual to **volume/action** in the bulk; for an evaporating black hole after the Page time, the volume/action is $O(S_{BH})$.

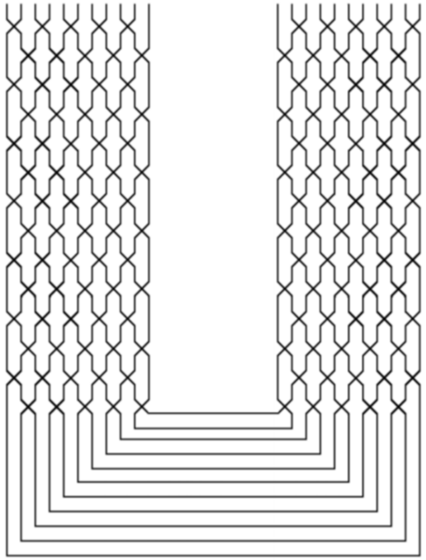
Contradiction?

Can make state in $O(S_{BH})$ time simply by creating and evaporating a black hole: state complexity is at most $O(S_{BH})$.

Different rules for allowed operations

Restricted vs Unrestricted Complexity

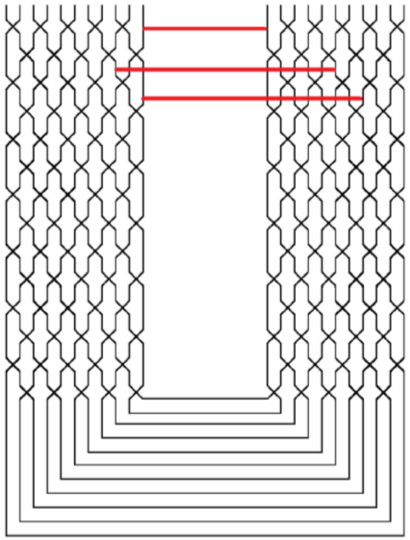
Restricted complexity *(Harlow-Hayden)*



Tells you how difficult it is to create a simple state (e.g. **TFD**) by acting only on the **Hawking radiation**

Start with a simple (entangled) state of a **bipartite system**. Then apply **separate** unitary circuits to each side to produce the desired state. (If **maximally entangled**, you only need to act on **one side**).

Unrestricted complexity *(volume/action)*



Coupling happens when **all** the degrees of freedom are **initially in the black hole** (before it evaporates)

Allow unitaries that couple the two systems together.

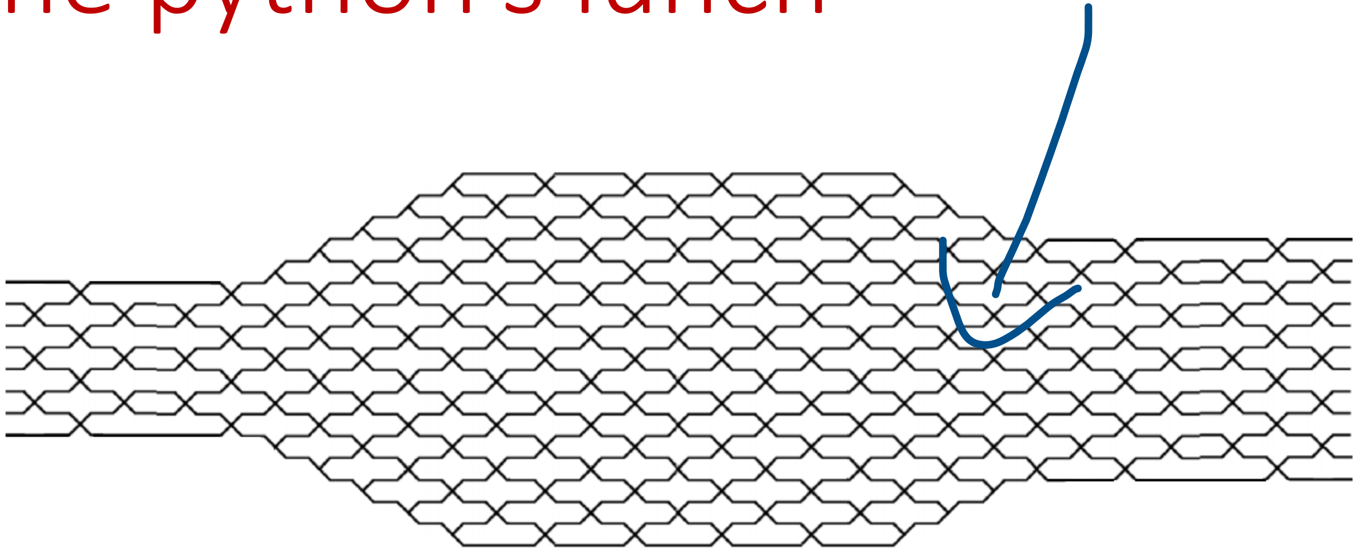
When is **restricted complexity** much larger than **unrestricted complexity**?

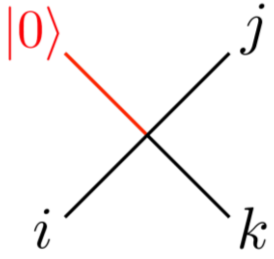
- Consider taking the **thermofield double state** and time evolving it for $O(S_{BH})$ time
- Restricted complexity $\sim$ Unrestricted complexity $\sim$ Volume/action $\sim$ $O(S_{BH})$
- What is the difference between this and an **evaporating black hole**?
- How can we see from the **semiclassical geometry** that the restricted complexity is exponentially large, even though the unrestricted complexity is comparatively small?

Answer: evaporating black holes contain a **python's lunch**

The python's lunch

Not an isometry (from left to right)



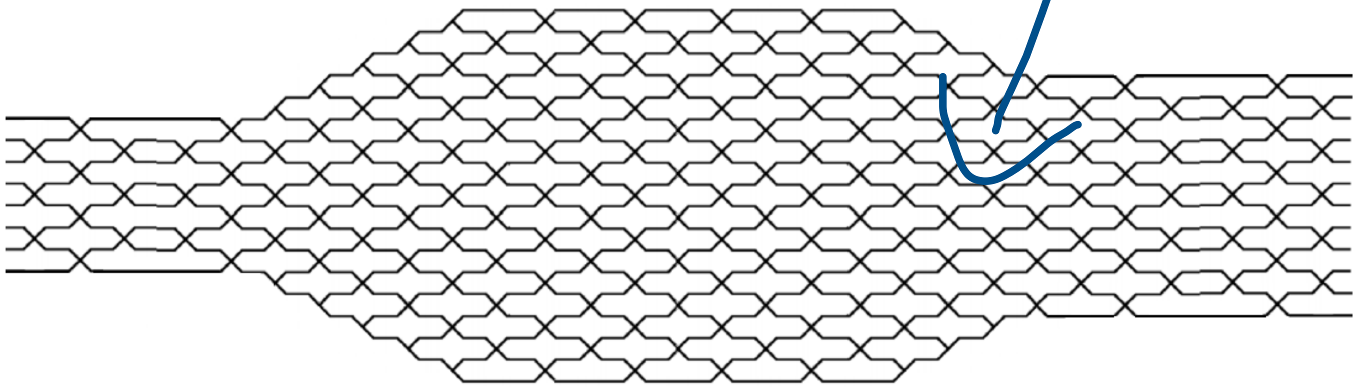


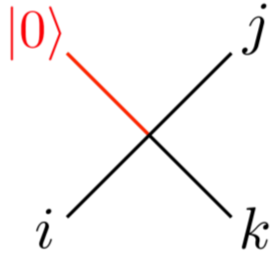
All tensors either *unitaries* or *isometries*

- **Generically**, the entire network will be an **isometry** (up to a very small error) from **left to right**
- However, even though it is a **simple tensor network**, that does **not** mean that it describes a **simple isometry** because the individual **steps** are **not all unitary**

The python's lunch

Not an isometry (from left to right)





All tensors either *unitaries* or *isometries*

More explicitly,

Simple

Input

$$|\psi\rangle \propto \langle 0|^{\otimes m_R}\, U_{TN}|I\rangle\, |0\rangle^{\otimes m_L}$$

Output

Postselection

Ancilla

NOT UNITARY

$$|\psi\rangle\, |0\rangle^{m_R} = U_{PL}\, |I\rangle |0\rangle^{m_L}$$

Simple or not?

How hard is it to bypass postselection?

- Naïve approach (if input state can be prepared many times and measurements are allowed): keep trying until you get lucky and measure the correct state
- Estimated time is $O(2^{m_R})$ (**exponentially hard**). Also still not really unitary
- Better method: **Grover search**
- First apply U_{TN}. Then apply a *phase of* (-1) if all m_R ancilla qubits in zero state. Apply $U_{TN}^{\dagger}$. Apply *phase of* (-1) if all m_L ancilla qubits in zero state. Repeat $2^{m_R/2}$ times.
- **Still exponentially hard**

Could there be a more efficient way?

- **Maybe**. Complexity theory is hard
- Grover search is the **optimal search strategy**
- However, in this case, we know in advanced what we're searching for so that could mean more efficient approaches exist
- Very strong reasons to think that it cannot generally be done in polynomial time
- This would imply BQP = PostBQP = PP

Incredibly powerful

- If you suggest that this is true to **Scott Aaronson** he will laugh at you

A restricted complexity conjecture

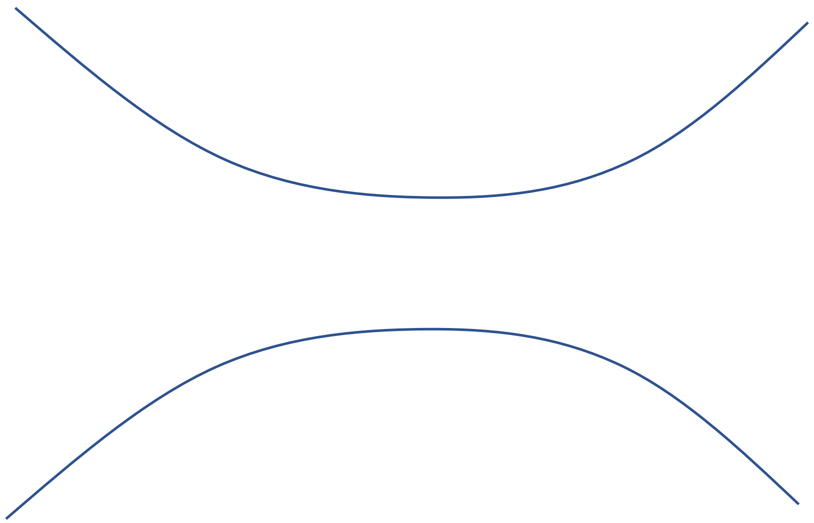
The *restricted complexity* of the bipartite state produced by a **python's lunch** tensor network is generically

$$O(C_{TN} \times 2^{m_R/2})$$

Number of postselected qubits

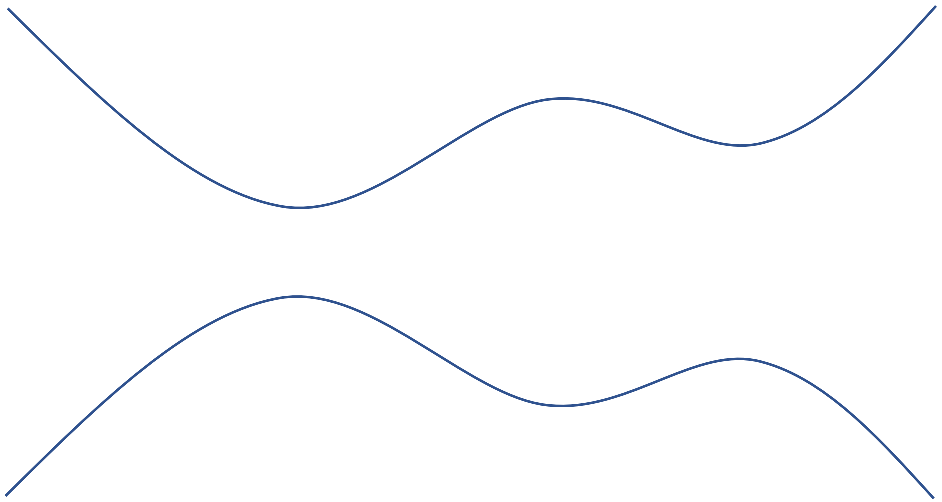
Complexity of U_{TN}

Python's lunches in gravity



An ordinary wormhole
(e.g. **time-evolved TFD**)

Small restricted complexity



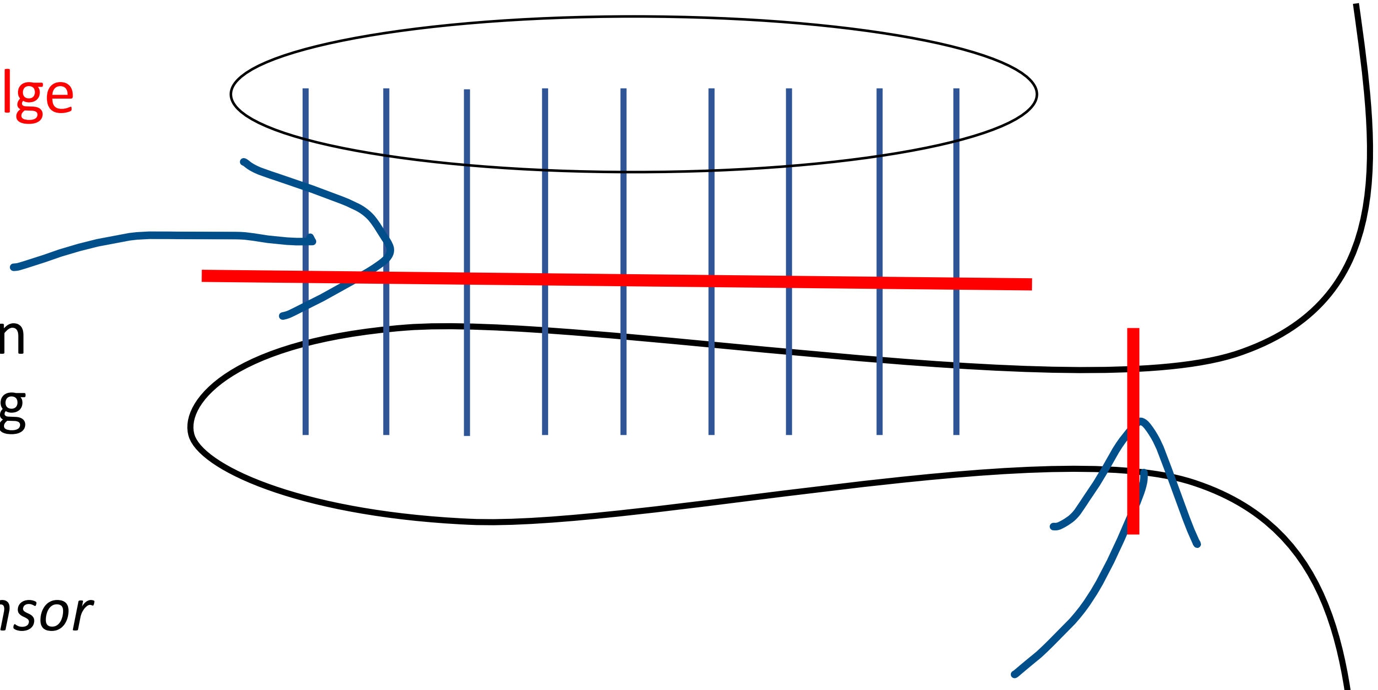
A python's lunch wormhole

Exponentially large
restricted complexity?

An evaporating black hole

For the moment, we consider a single, ‘nice’ Cauchy slice that sticks close to the event horizon (we will define everything **covariantly** later)

Two constrictions with a bulge in the middle



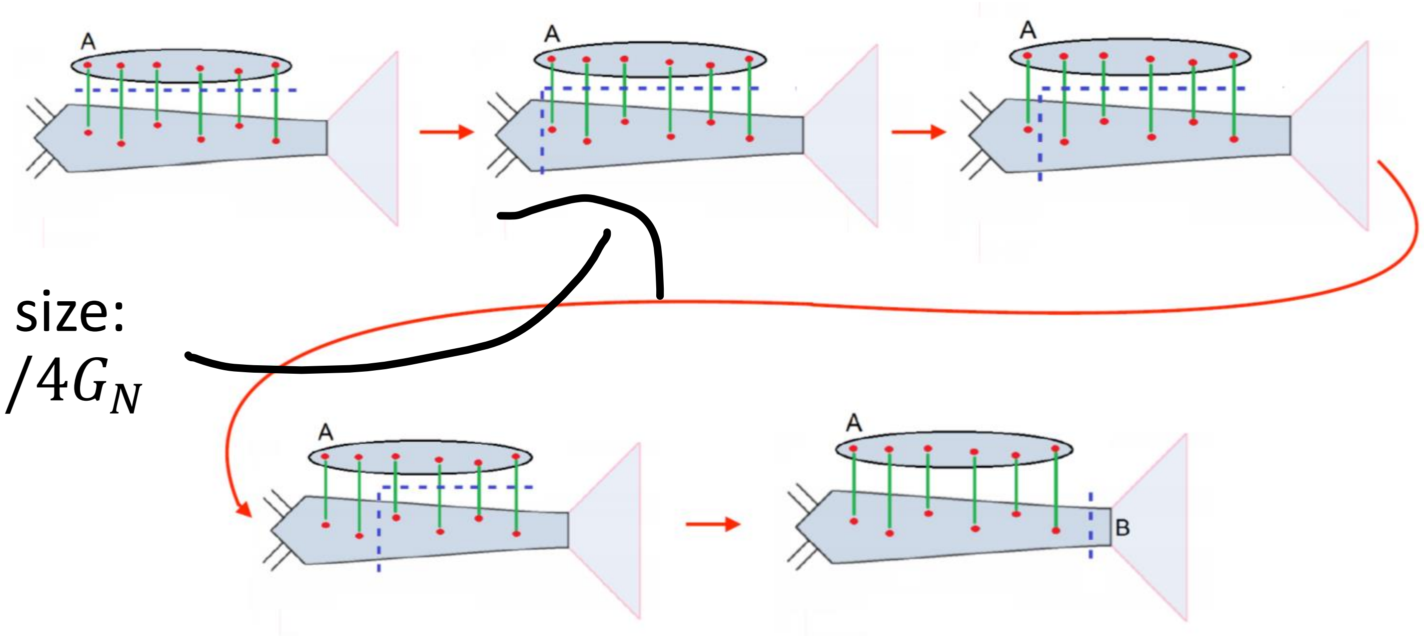
Bulk entanglement between interior modes and Hawking radiation. Equivalent to classical area *(ER=EPR, Engelhardt-Wall, HaPPY tensor networks)*

Wormhole narrowest at horizon (deeper into wormhole = further back in time)

Python’s lunch

How big is the python’s lunch?

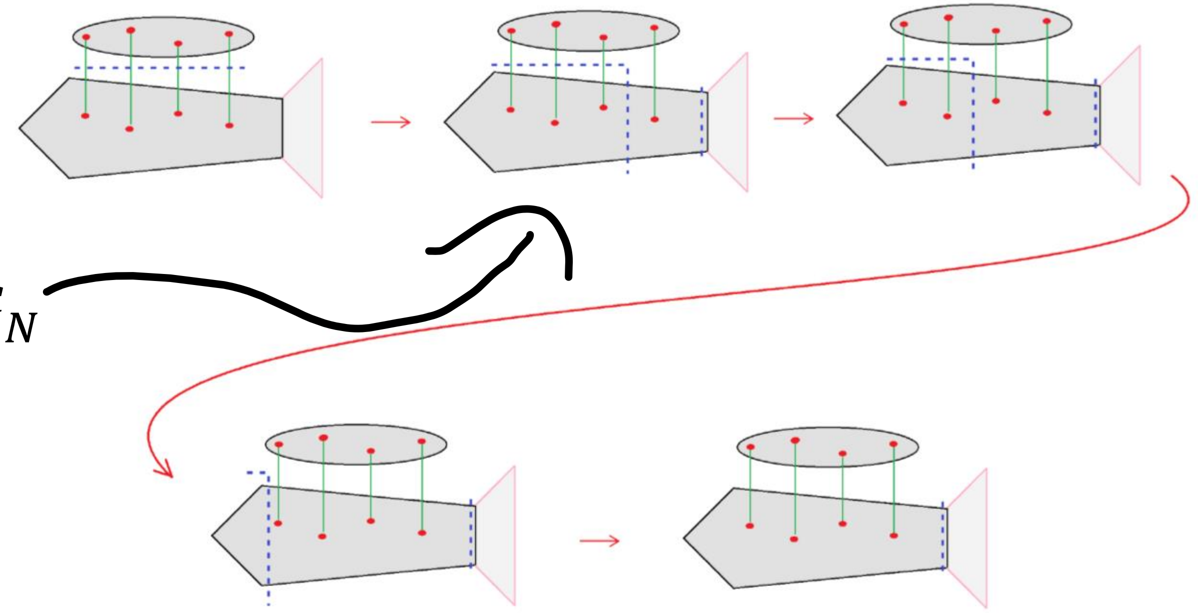
- The **maximum size** of the lunch depends on how you slice it from one constriction to the other
- We want to choose the slicing that **minimizes** this maximum size (this corresponds to the **most efficient** Grover search protocol)
- One option: start at end of the wormhole and move **forwards** along it



Maximum size:
$S_{rad} + A_0/4G_N$

How big is the python's lunch?

- Alternative option: start with **double cut** near the horizon, and the move one cut **backwards** along the wormhole



Maximum size:
$S_{rad} + 2A_{hor}/4G_N$

More efficient when $A_{hor} < A_0/2$. Note that this transition happens **strictly after** the Page time (defined by $S_{rad} = A_{hor}/4G_N$).

A restricted complexity conjecture for evaporating black holes

Intuition from tensor networks: restricted complexity is

Amount of postselection required

$$O\left(C_{TN} \exp\left(\frac{1}{2}\left[S^{gen}_{max} - S^{gen}_{larger\ min}\right]\right)\right)$$

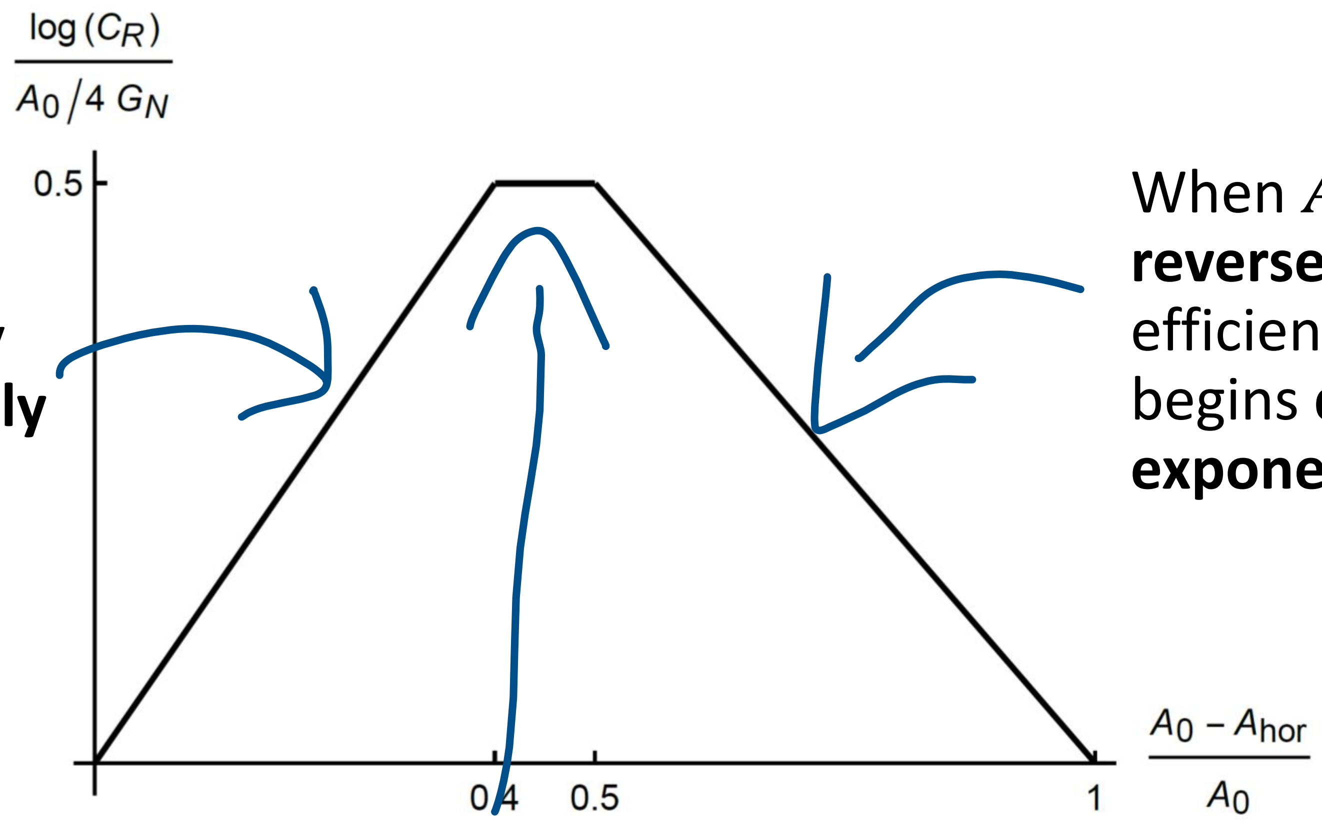
Volume/action = $O(t)$

Maximum generalized entropy in the most efficient slicing (**minimax surface**)

Generalised entropy of the larger of the two minima

A restricted complexity conjecture for evaporating black holes

Complexity initially **grows exponentially**



When $A_{hor} < A_0/2$, the **reverse sweep** becomes more efficient and the complexity begins **decreasing exponentially**

After Page time, larger minimum becomes the **empty surface**. Complexity grows **linearly**

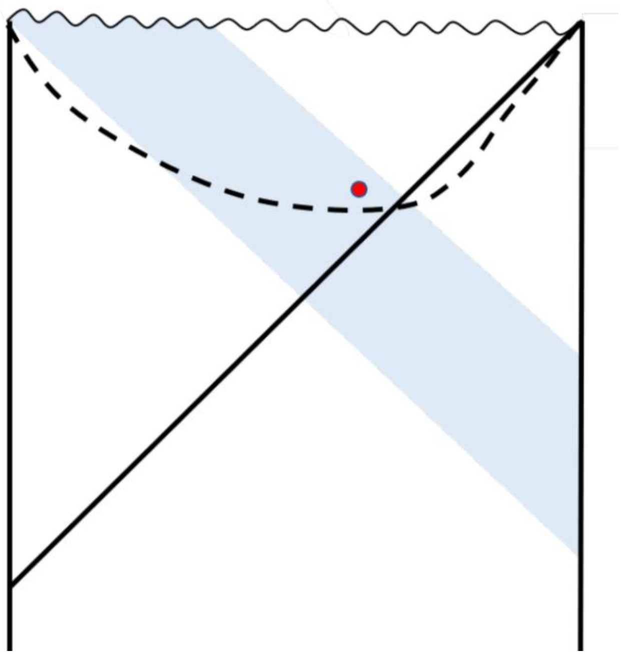
After the evaporation is complete, **restricted complexity = unrestricted complexity =** $\boldsymbol{O(S_{BH})}$**.**

Covariant python’s lunches

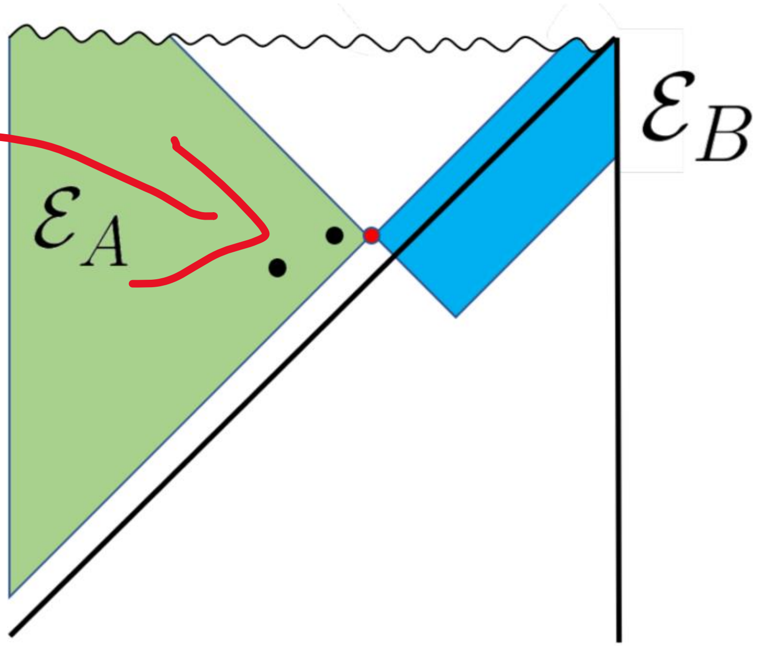
- The covariant surface that corresponds to the **minimal cut** in a tensor network is the **minimal quantum extremal surface** *(Engelhardt, Wall 2014)*.
- This can also be found using a **maximin prescription**: first find the minimal generalised entropy surface within a Cauchy slice, then maximise over all Cauchy slice *(Wall 2012, Akers, Engelhardt, GP, Usatyuk 2019)*.
- Other end of the lunch = a second larger QES
- What is the covariant definition of the maximum size of the lunch?
- For a tensor network, it was a **minimax surface** (minimize the maximum slice over all ways of slicing from one end to the other)
- **Our conjecture:** covariant definition is the **maximinimax surface** (maximise the minimax surface over all Cauchy slices containing the two ends of the lunch)
- Assuming everything is well behaved, this should also be a **quantum extremal surface**.

Covariant python's lunches for evaporating black holes

We can explicitly find quantum extremal surfaces that give the maximum bulge size in the **forwards and reverse sweeps**



Only explicitly calculable in JT gravity + free fermions



Forwards sweep: QES is a **sphere** inside the shell of collapsing matter that formed the black hole

Reverse sweep: QES is the union of **two spheres**, just inside the minimal QES

Final comments:

- **Non-minimal** quantum extremal surfaces matter too!
- Still lots to learn from the semiclassical geometry of an evaporating black hole (we all shouldn't **only** move on to thinking about **microstates**)
- Evaporating black holes where the Hawking radiation has been **measured** in a simple basis provide an example of a state with large **unrestricted** unitary circuit complexity, but small volume/action (geometry is a **one-sided python's lunch**)
- Suggests that volume/action is dual to size of minimal tensor network **not** unitary circuit complexity
- Finally, it suggests that entanglement wedge reconstruction should be much **easier** when the bulk operator is not behind a non-minimal QES, even if it's not in the causal wedge. Maybe possible to find reconstructions in this case that don't need to make use of **modular flow/the Petz map** etc.

Thank you!