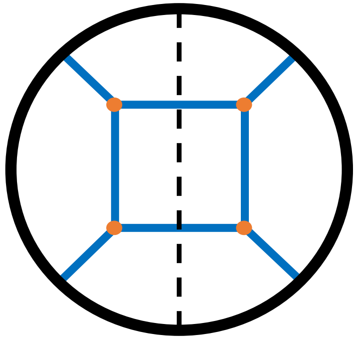

Geometry from the Quantum Corrections:

Loop Amplitudes and Extra Dimensions in AdS/CFT

Eric Perlmutter

Caltech,
Simons Collaboration on Nonperturbative Bootstrap

KITP, 1.16.20

The Lagrangian $\mathcal{N}$=4 SYM theory is dual to type IIB string theory on $AdS_5 \times S^5$.

$$L_{S^5} = L_{\text{AdS}}$$

From the perspective of $\mathcal{N}$=4 conformal data, why is the S^5 necessary?

The S^5 KK modes furnish a tower of protected superconformal multiplets, labeled by primaries

$$\mathcal{O}_2,\ \mathcal{O}_3,\ \mathcal{O}_4, \ldots$$

Only $\mathcal{O}_2$, in the stress tensor multiplet, is required by superconformal symmetry.

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Is the familiar $\mathcal{N}$=4 SYM theory the unique 4d $\mathcal{N}$=4 SCFT?

Does 5D maximal gauged SUGRA, with a “pure” AdS_5 vacuum, have a UV completion?

All fully-controlled examples of AdS/CFT involve large – i.e. AdS-scale – positive curvatures.

$$L_{\mathcal{M}} \sim L_{\rm AdS}$$

$AdS_5 \times S^5/T^{1,1}/Y^{p,q}/L^{p,q,r}$, $AdS_{4/7} \times S^{7/4}$, $AdS_4 \times CP^3$, $AdS_3 \times S^3 \times T^4$, $AdS_{3/2} \times S^{2/3} \times CY_3$, $AdS_2 \times S^2$...

This implies towers of light operators in the dual CFTs.

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Are large extra dimensions necessary, or is this a lamppost effect?

What are the statistics of AdS x Small solutions in the space of consistent AdS vacua?

From the large N CFT point of view, what are the bootstrap constraints on the low-energy EFT?

There is robust evidence for necessary conditions for AdS-scale bulk locality:

[Heemskerk, Penedones, Polchinski, Sully; El-Showk, Papadodimas; CEMZ; Maldacena, Shenker, Stanford; Caron-Huot]

Large N + Higher-spin gap $\Rightarrow$ Local AdS bulk

But in what dimension is the bulk local? = How sparse is the **low-spin** CFT spectrum?

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How does D appear in the 1/N expansion of the dual CFT?

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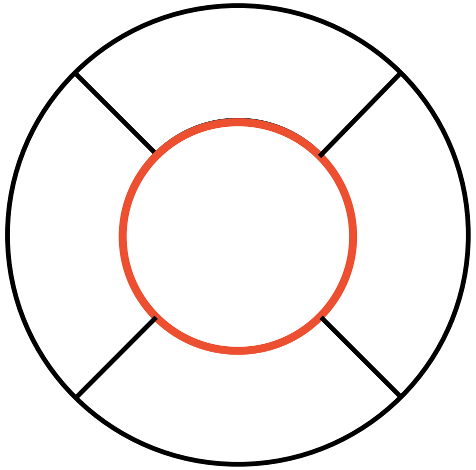

Why AdS loops probe the space of large N CFTs

In AdS/CFT, we are often concerned with sequences of CFTs admitting a large N limit.

Obvious: CFT consistency conditions – e.g. crossing symmetry – must be satisfied order-by-order in the 1/N expansion (assuming analyticity around infinite N).

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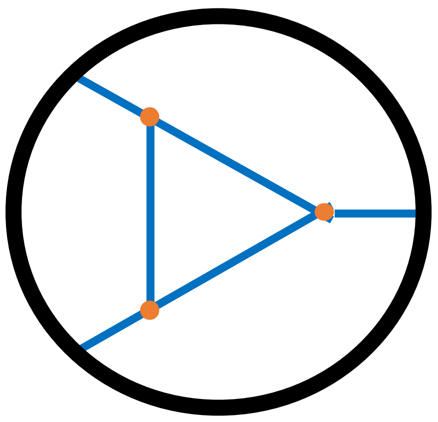
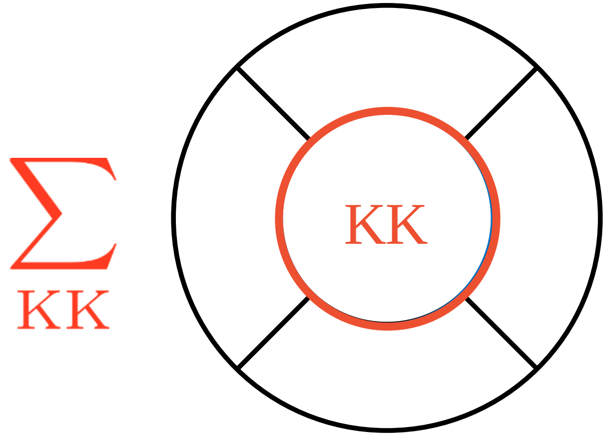
Not obvious (from the CFT point of view): different orders in 1/N are related.



Modulo details (e.g. renormalization effects) **all** orders are fixed by **planar** CFT data.

Why AdS loops probe extra dimensions

This is especially powerful for four-point functions, which depend on kinematics.



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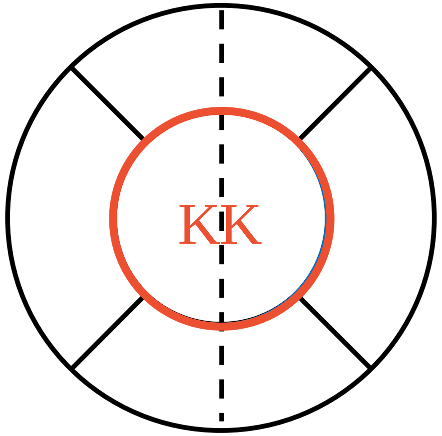
$\sum_{\text{KK}}$



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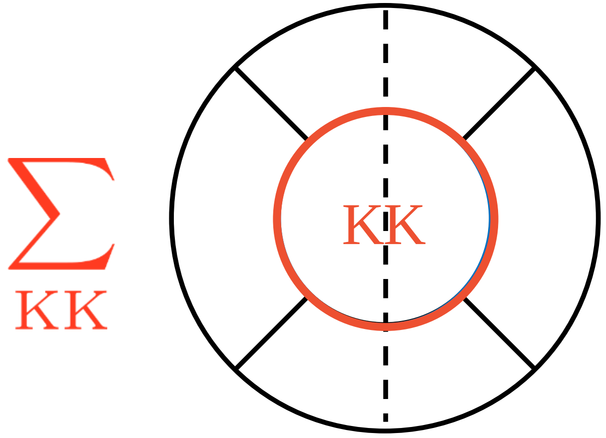
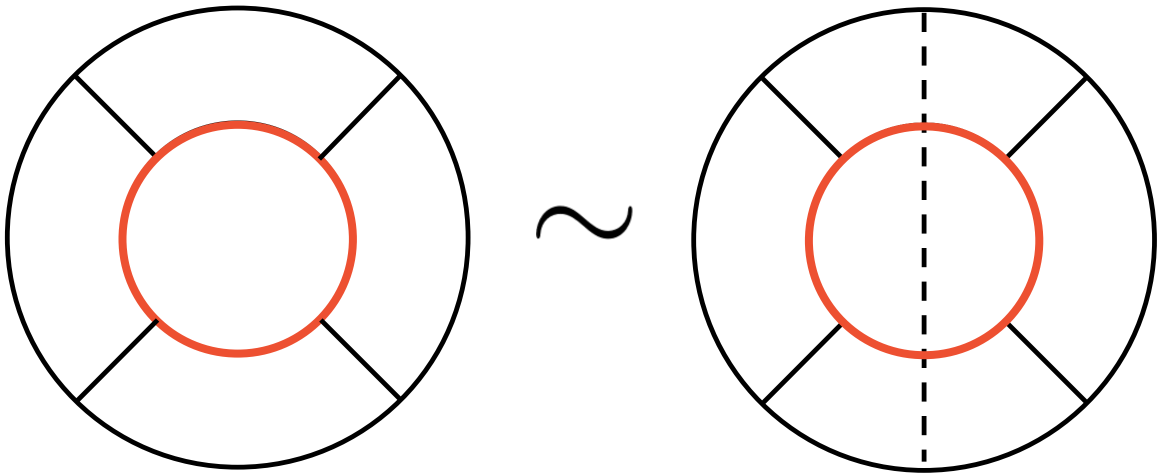


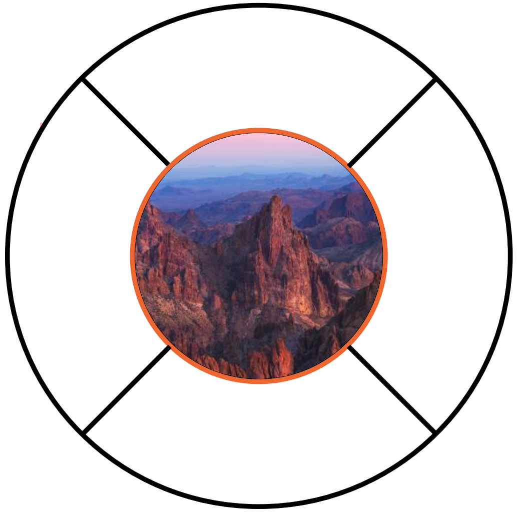
Cutting “sees” extra dimensions

→ Need **Unitarity Method for AdS Amplitudes**



Why AdS loops probe extra dimensions

Idea: leverage loop amplitudes to probe theory space.



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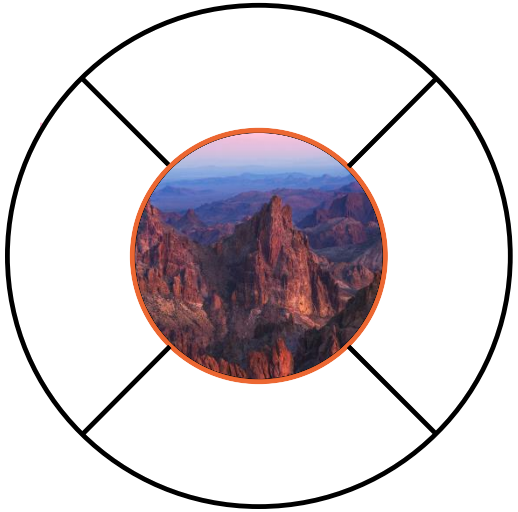
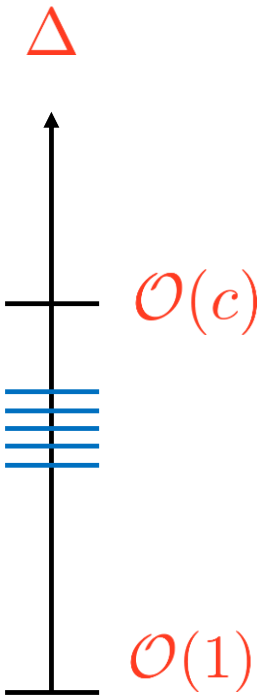


One lesson: in order to “discover” string theory, intermediate spectrum is important!

- **Extra dimensions**
- Existence and **dynamics of stringy d.o.f.**
- Existence and dynamics of non-perturbative (in g_s) d.o.f
- ...



Signatures should be visible in classical string theory/planar CFT.

Summary/Plan

1. Unitarity Methods in AdS/CFT

2. A dictionary for bootstrapping the landscape of AdS vacua

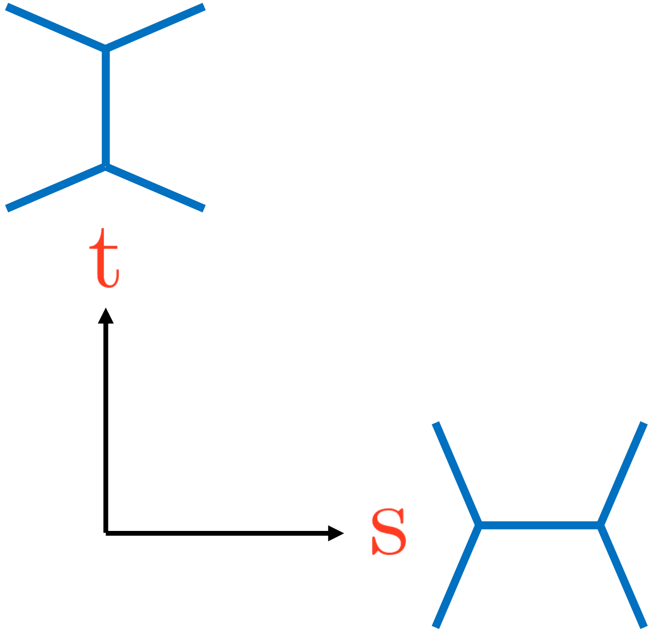
3. Universality in the String-String OPE

Based on:

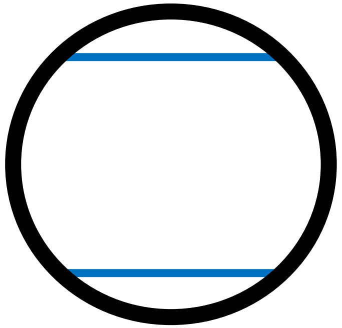
- 1612.03891, with O. Aharony, F. Alday, A. Bissi
- 1808.00612, with J. Liu, V. Rosenhaus, D. Simmons-Duffin
- 1906.01477, with F. Alday
- 1912.09521, with D. Meltzer, A. Sivaramakrishnan
- (WIP with D. Mazac, L. Rastelli)

(Notation:)

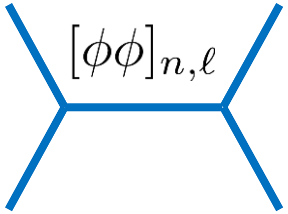
$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}} \mathcal{A}(z,\bar{z})$$



CFT decomposition of bulk amplitude <$\phi\phi\phi\phi$>.



$$= \sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty}$$



Double-trace composites:

$$[\phi\phi]_{n,\ell} \simeq \phi\Box^n\partial_{\mu_1}\ldots\partial_{\mu_\ell}\phi$$

$$\Delta_{n,\ell} = 2\Delta_\phi + 2n + \ell + \gamma_{n,\ell}$$

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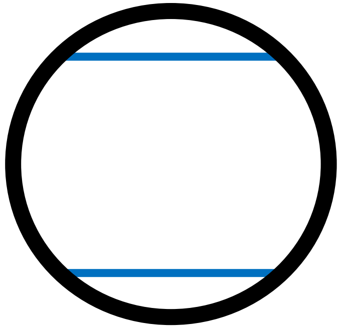

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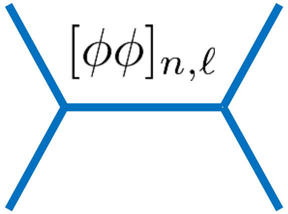

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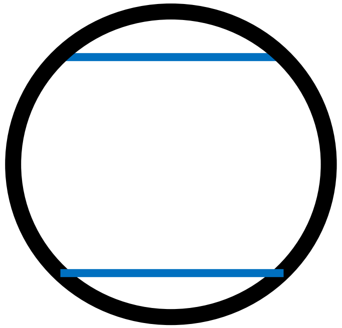

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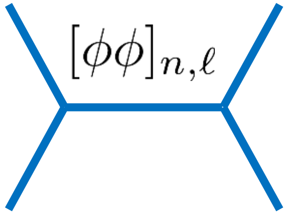

$$= \sum_{n=0}^{\infty} \sum_{\ell=0}^{\infty} p^{(0)}_{n,\ell}\, G^{(s)}_{n,\ell}$$

Squared OPE coefficients of MFT

S-channel conformal blocks

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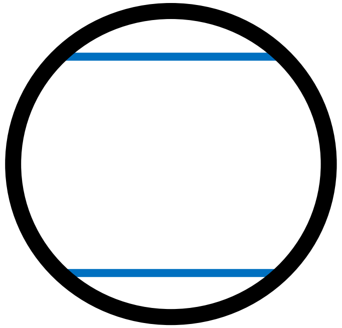

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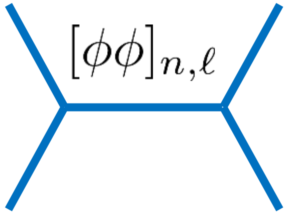

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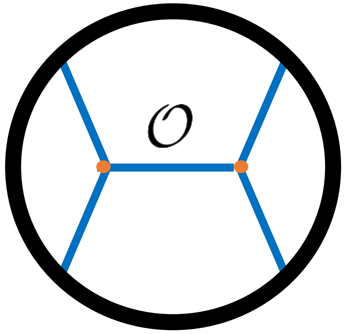
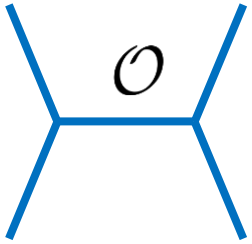


$$=$$



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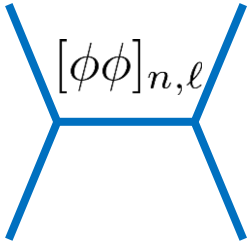


Single-trace

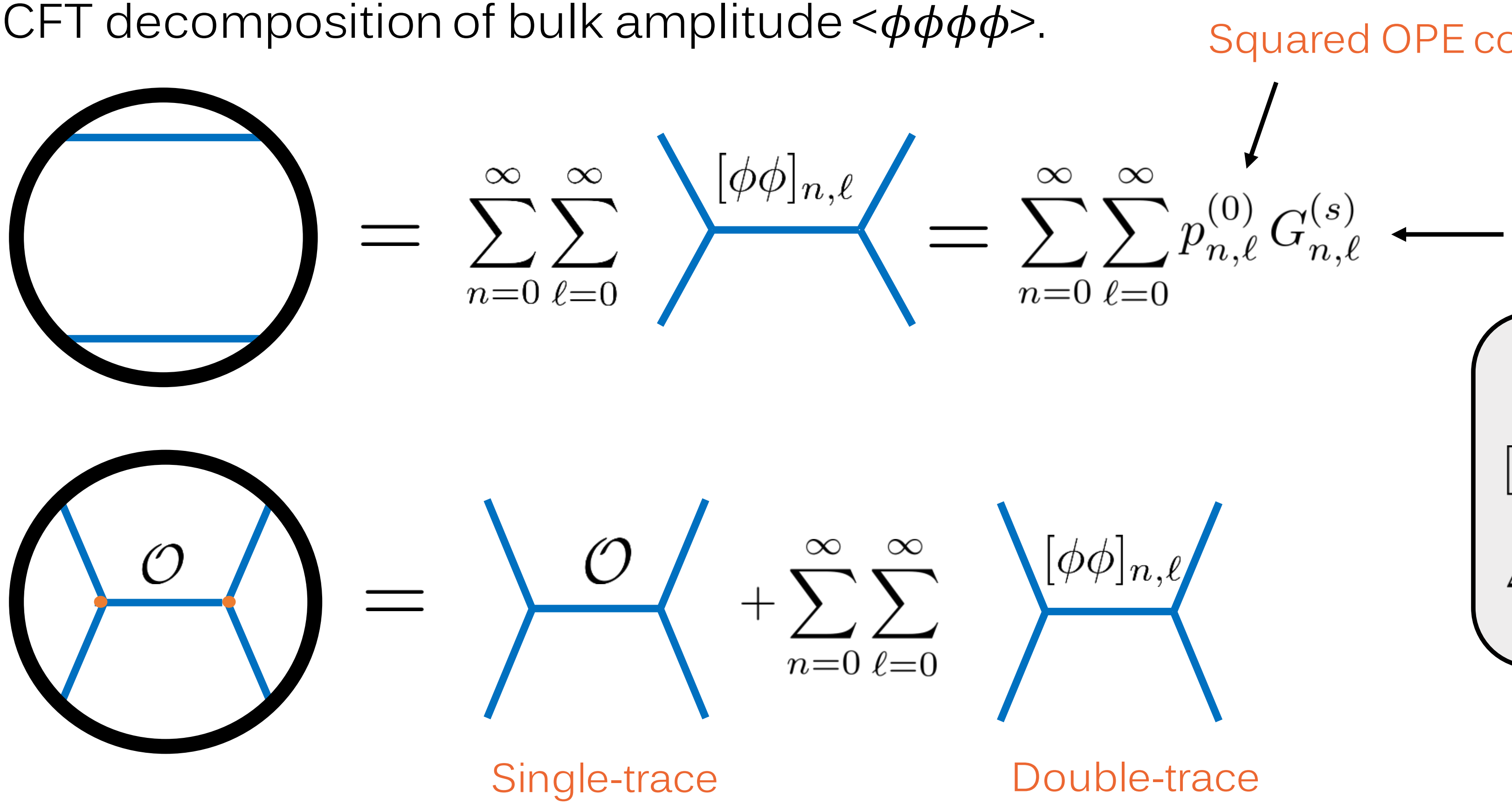
Double-trace

$[\phi\phi]$ anomalous dimension:

$$\gamma_{n,\ell} = \frac{\gamma^{(1)}_{n,\ell}}{c} + \frac{\gamma^{(2)}_{n,\ell}}{c^2} + \ldots$$

Tree-level
Fixed by single-trace data

CFT decomposition of bulk amplitude $\langle\phi\phi\phi\phi\rangle$.



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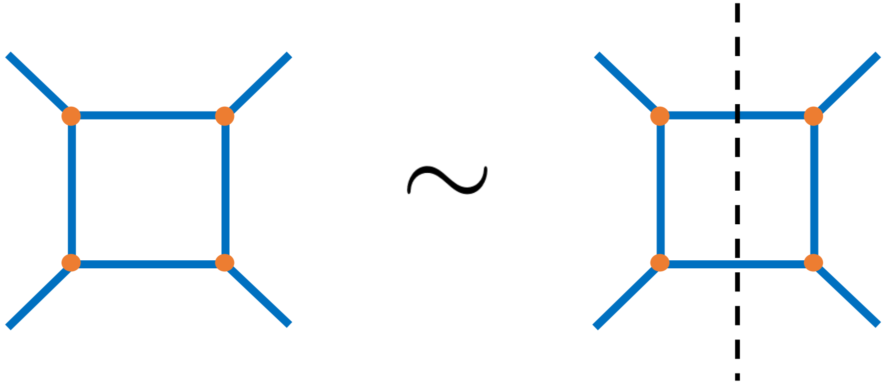
Tree-level
Fixed by single-trace data

1-loop
Fixed by **tree-level** data... how?

Unitarity methods build loop-level amplitudes from lower-order on-shell amplitudes.

$$S = \mathbf{1} + iT \quad \Longrightarrow \quad \mathrm{Disc}(T) = T^\dagger T$$



In AdS, no asymptotic states. But amplitudes compute CFT correlators.

Upshot: we have two (matching) prescriptions.

- **Boundary**: build non-planar OPE data from planar data.
- **Bulk**: build loop amplitudes directly from bulk cuts.

AdS loops literature review

Older (2007-2016): [Cornalba, Costa, Penedones; Penedones; Giddings, Gary; Fitzpatrick, Kaplan]

Boundary unitarity method: [Aharony, Alday, Bissi, EP]

Newer (2017-now):

- N=4 SYM/string amplitudes via boundary unitarity [Alday, Bissi; Aprile, Drummond, Heslop, Paul; Alday, Caron-Huot; Alday, Bissi, EP; Alday; Alday, Zhou; Drummond, Paul]
- N=4 SYM localization [Chester]
- O(N) correlators via boundary unitarity [Ponomarev, Sezgin, Skvortsov]
- Two-point functions [Giombi, Sleight, Taronna]
- Mellin bootstrap [Ghosh; Shyani]
- Regge exponentiation [Meltzer]
- AdS_2 [Mazac, Paulos; Beccaria, Tseytlin; Beccaria, Jiang, Tseytlin]
- Brute force [Cardona; Yuan; Bertan, Sachs, Skvortsov; Bertan, Sachs; Carmi, Di Pietro, Komatsu; Carmi]

Bulk unitarity method: [Meltzer, EP, Sivaramakrishnan]. (See also [Ponomarev].)

BOUNDARY UNITARITY METHOD

Boundary Unitarity Method

The CFT dispersion relation (Lorentzian inversion) is, schematically,

$$\mathcal{A}(z,\bar{z}) \approx \int dz' d\bar{z}' \, K(z,\bar{z};z',\bar{z}') \, \mathrm{dDisc}(\mathcal{A}(z',\bar{z}'))$$

("dDisc constructibility")

[Caron-Huot;
Carmi, Caron-Huot]

where

$$\mathrm{dDisc}_t(\mathcal{A}(z,\bar{z})) \equiv \frac{1}{2}\mathrm{Disc}^{\circlearrowleft}_{\bar{z}=1}(\mathcal{A}(z,\bar{z})) + \frac{1}{2}\mathrm{Disc}^{\circlearrowright}_{\bar{z}=1}(\mathcal{A}(z,\bar{z}))$$

For identical external scalar operators ϕ,

$$\mathrm{dDisc}_t(G^{(t)}_{\Delta,\ell}) = 2\sin^2\left(\frac{\pi}{2}\underline{\underline{(\Delta-\ell-2\Delta_\phi)}}\right) G^{(t)}_{\Delta,\ell}$$

→ Annihilates double-trace operators with $\gamma = 0$.

→ In the 1/c expansion,

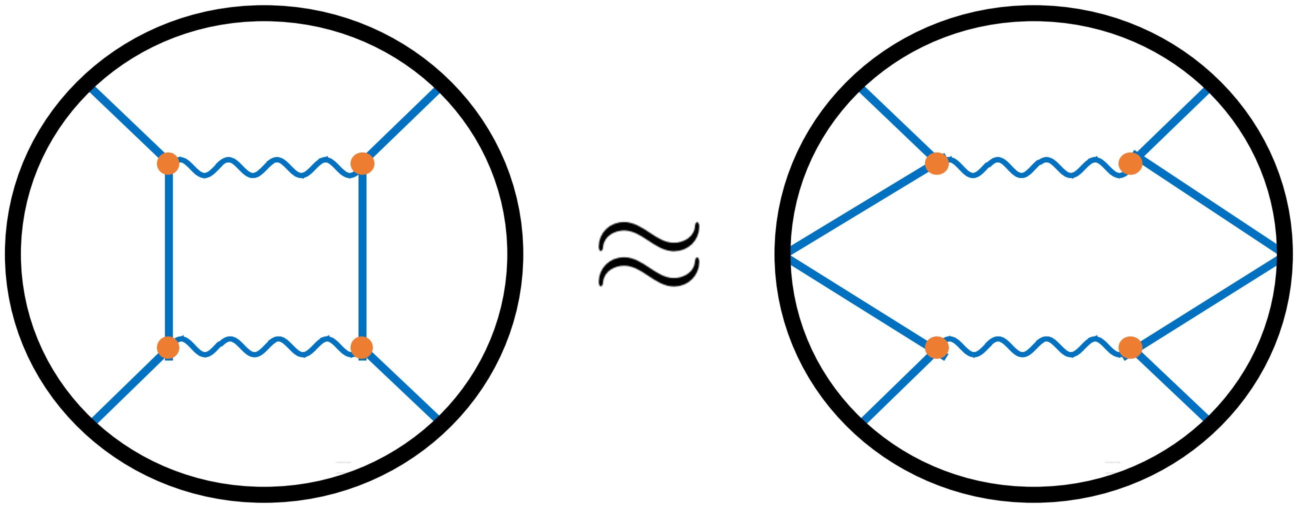
$$\mathrm{dDisc}_t(\mathcal{A}^{1-\mathrm{loop}}) \supset \frac{\pi^2}{2}\sum_{n,\ell} p^{(0)}_{n,\ell} \underline{\underline{(\gamma^{(1)}_{n,\ell})^2}} G^{(t)}_{n,\ell}$$

1-loop anomalous dimension does not appear = Fixed by tree-level

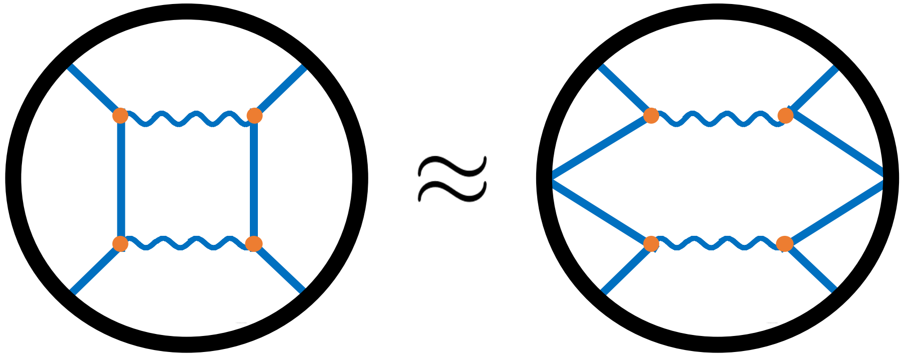
BULK UNITARITY METHOD

AdS Unitarity Method

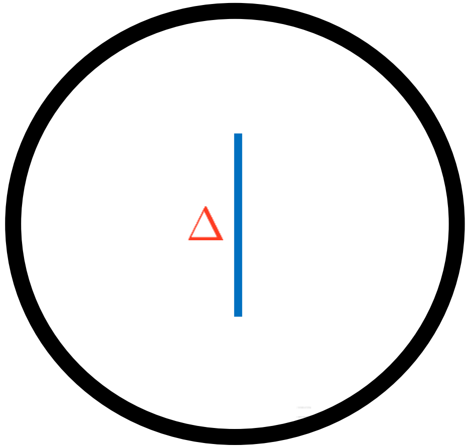
1-loop



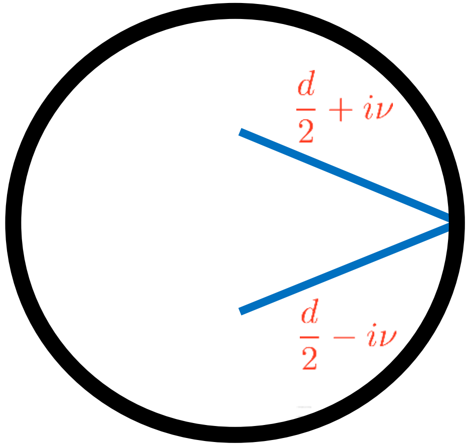
AdS Unitarity Method



- This picture can be made precise using CFT techniques (shadows etc)
- Because internal propagator is *off-shell*, correct procedure requires gluing a continuum.



$$= \int_{-i\infty}^{i\infty} d\nu \frac{\nu^2}{\nu^2 + (\Delta - \frac{d}{2})^2}$$



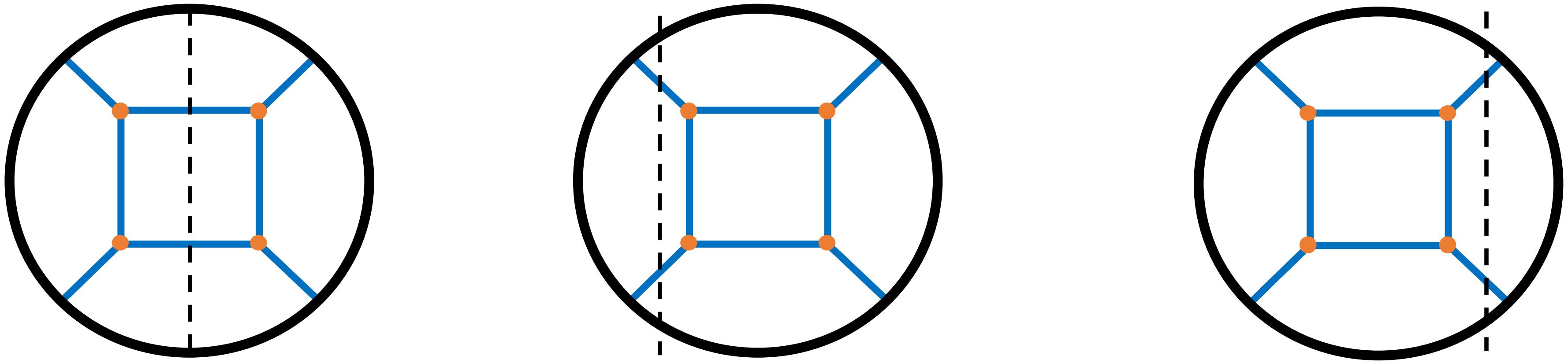
- We can define a **Cut** operator: this picks up the "single-trace pole" in the integrand.

Cut operator

Q: What operation implements an internal cut?

A in bulk: Putting the internal legs on-shell.

A in CFT: Isolating the respective double-trace operators in the CPW expansion.



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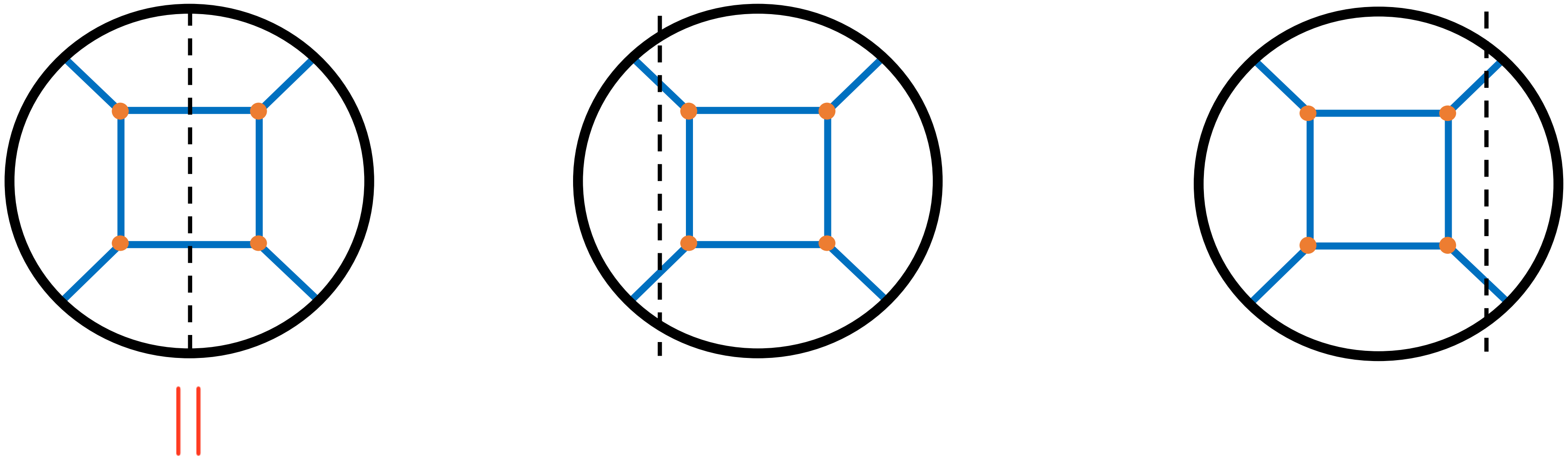
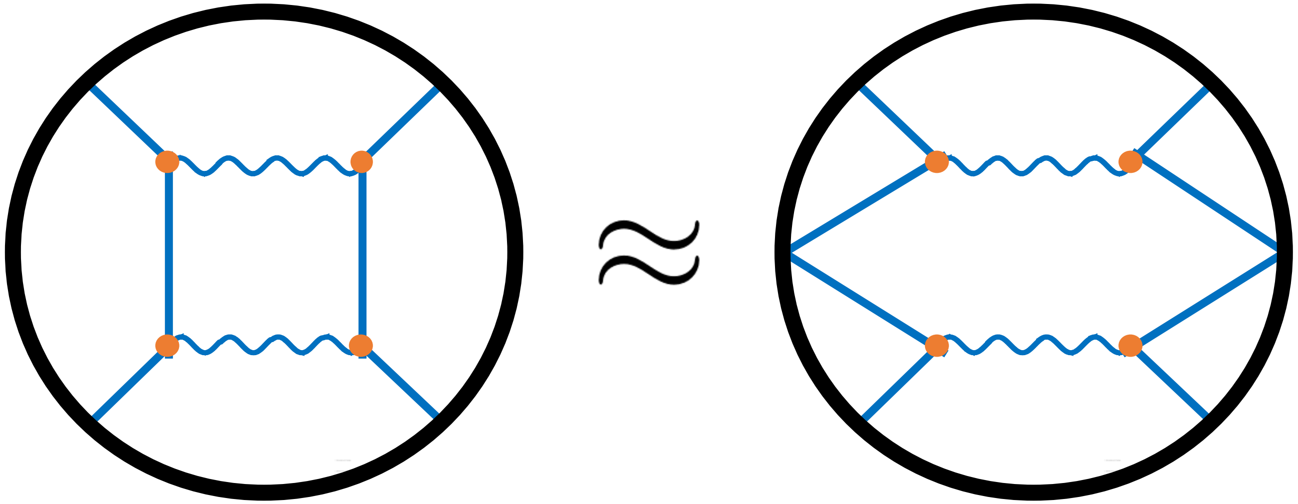


$\mathrm{dDisc}_t(\mathrm{box})$

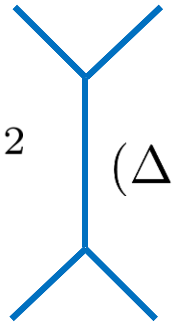
At 1-loop, dDisc is the internal cut operator. (For non-!PI, it's a weighted sum.)

6j symbols

When gluing diagrams, *6j symbols* for the conformal group appear.



$$\approx \sum_J \int d\Delta\,(6j)^2$$



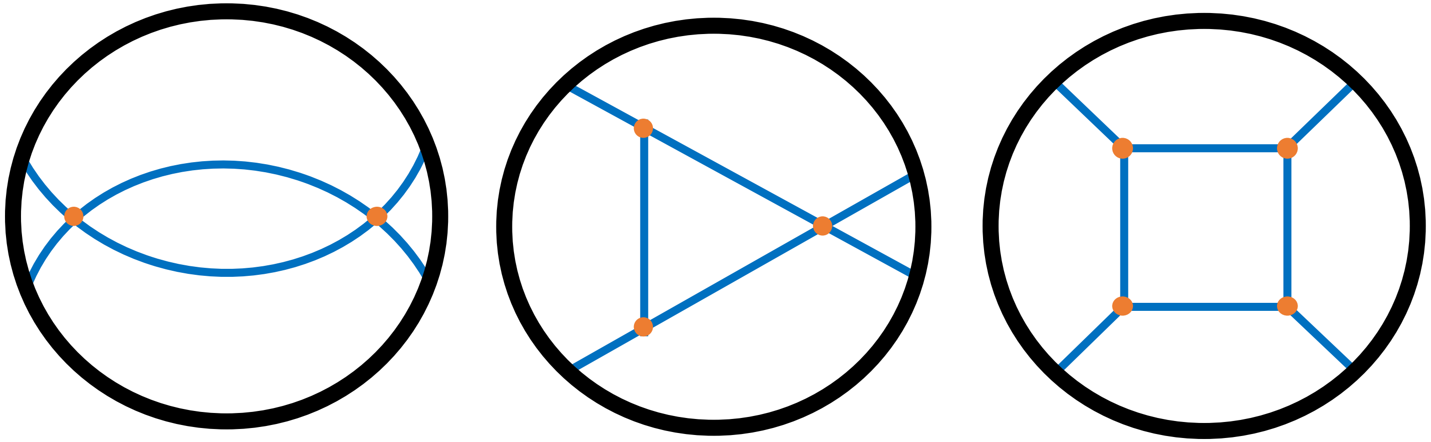
(Δ, J)

Every rung on a ladder generates a 6j symbol in the CPW decomposition.

In this sense, 6j symbol "=" AdS ladder kernel.

[Liu, EP, Rosenhaus, Simmons-Duffin]

We applied this prescription to the following scalar diagrams



+ some non-1PI diagrams.

BULK

=

BOUNDARY

The boundary unitarity method reconstructs the crossing-symmetric amplitude.

To make contact with the bulk, we want to adapt it to reconstruct specific diagrams.

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$$\mathrm{dDisc}_t(\mathcal{A}^{1-\mathrm{loop}}) \supset \frac{\pi^2}{2}\sum_{n,\ell} p^{(0)}_{n,\ell}(\gamma^{(1)}_{n,\ell})^2 G^{(t)}_{n,\ell}$$

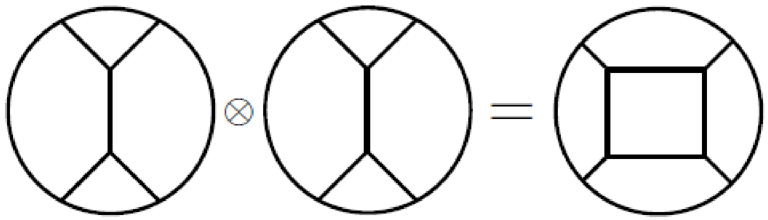
Split this as

$$\gamma^{(1)}_{n,\ell} = \gamma^{(1),\mathbf{s}}_{n,\ell} + \gamma^{(1),\mathbf{t}}_{n,\ell} + \gamma^{(1),\mathbf{u}}_{n,\ell}$$

Then each term corresponds to the bulk gluing in the respective channels.

AdS | CFT

Cuts compute dDisc



$$\gamma^{\mathbf{t},\phi^3}\gamma^{\mathbf{t},\phi^3}$$

Plug into dDisc formula

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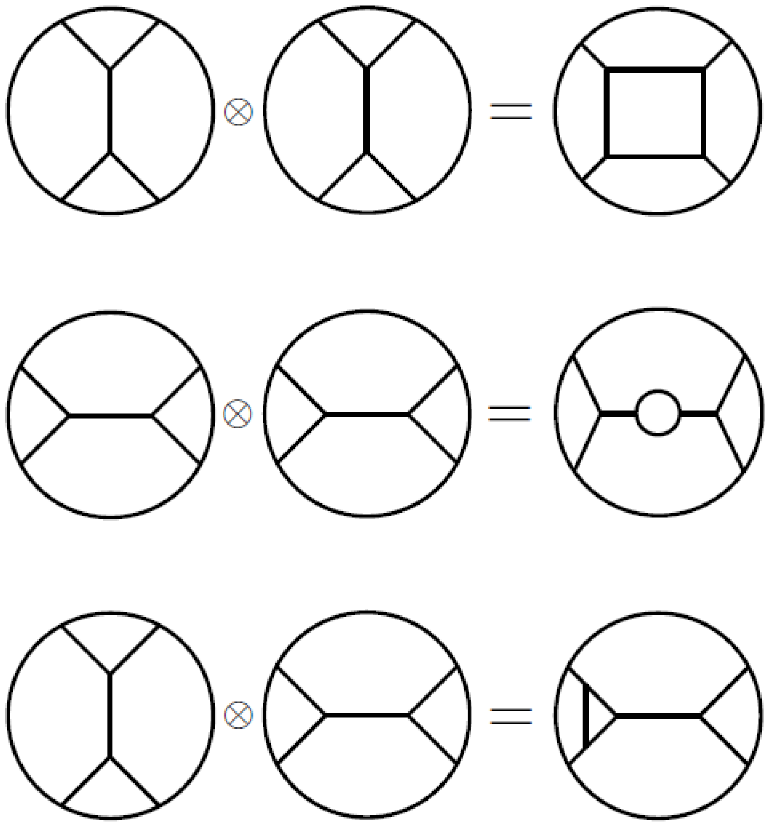

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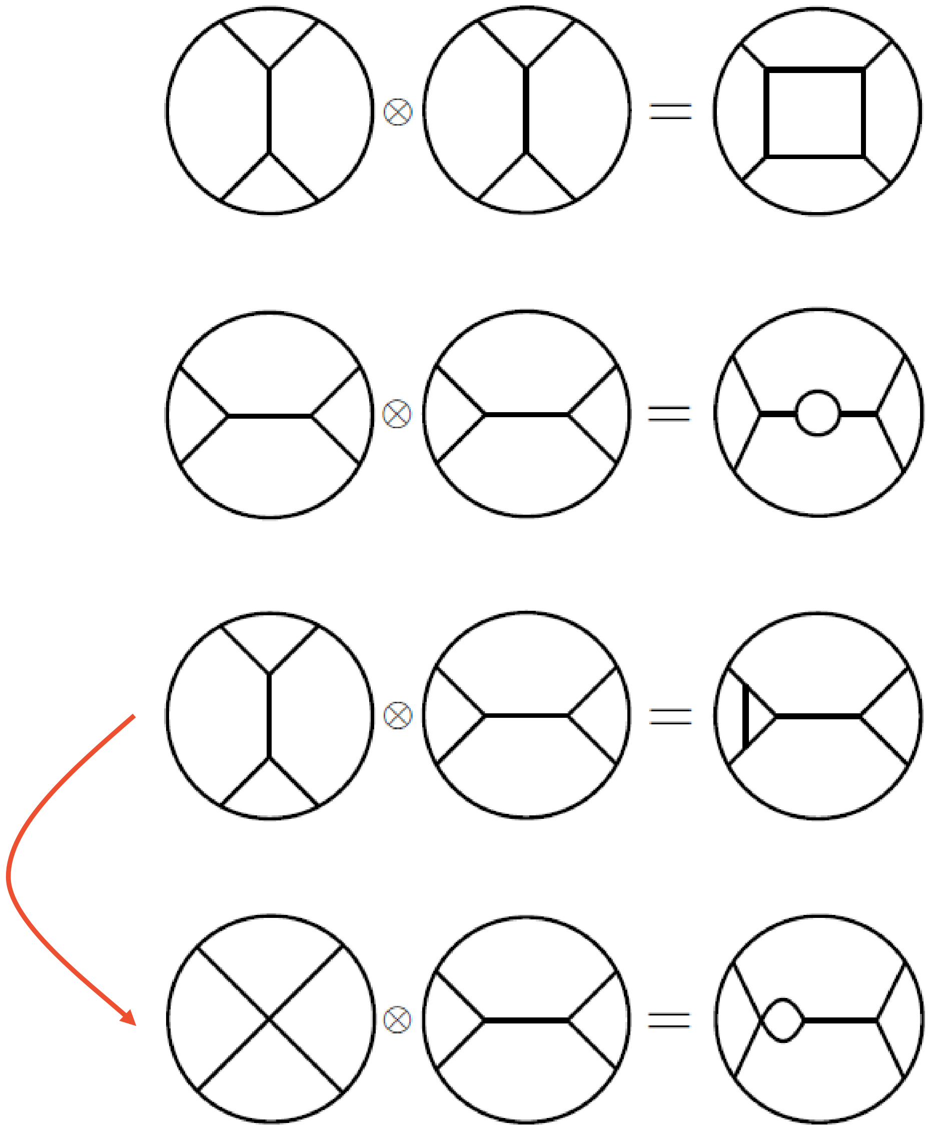
Cuts compute dDisc

Plug into dDisc formula



$$\gamma^{\mathbf{t},\phi^3}\gamma^{\mathbf{t},\phi^3}$$

$$\gamma^{\mathbf{s},\phi^3}\gamma^{\mathbf{s},\phi^3}$$

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Substitution

$$\gamma^{\phi^4}\gamma^{\mathbf{s},\phi^3}$$

III. The String Landscape and Extra Dimensions in AdS/CFT

There *are* attempts at constructing AdS x Small solutions in string/M-theory. e.g.:

1. Large Volume Scenario (non-SUSY AdS_4, IIB) [Balasubramanian, Berglund, Conlon, Quevedo]
2. KKLT (SUSY AdS_4, IIB) [Kachru, Kallosh, Linde, Trivedi]
3. DGKT (SUSY AdS_4 IIA) [DeWolfe, Giryavets, Kachru, Taylor]
4. Polchinski-Silverstein (SUSY AdS_4, AdS_5, F-theory)

+ more recent variants

Such constructions involve various assumptions or approximations. (Applicability of EFT, pert. and non-pert. effects in α' and/or g_s, backreaction of localized sources, existence of hierarchical Calabi-Yau manifolds, compatibility of ingredients.)

These arguments may well be correct.

Our goal: use different machinery – the large N conformal bootstrap – capable of making rigorous, quantitative statements.

Challenges for the large N bootstrap

This question seems tailor-made for the bootstrap. Why is it hard?

Challenges for the large N bootstrap

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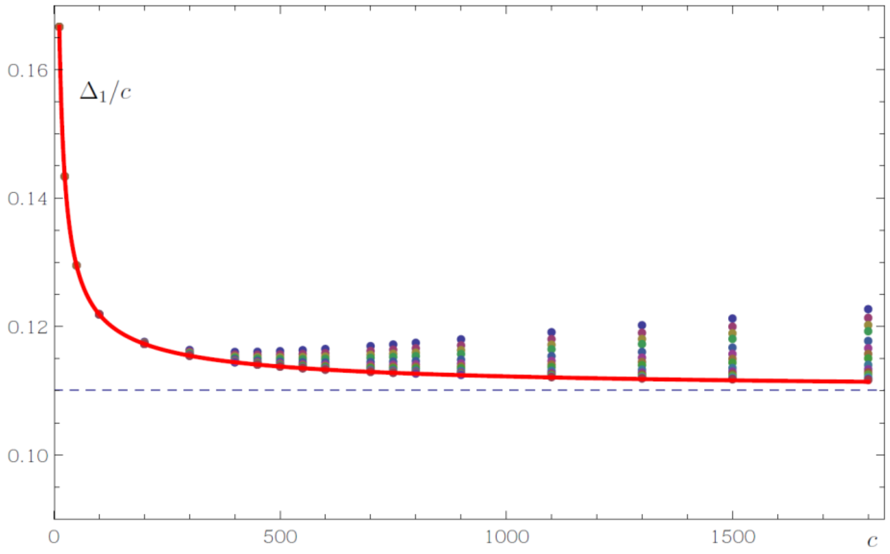
1. "The AdS EFT Problem"

2. Non-perturbative constraints may not apply in 1/N perturbation theory
   - e.g. Regge growth

3. Large N competes with approx. solutions of crossing

4. "The Asymptoticity Problem"



[Afkhami-Jeddi, Hartman, Tajdini]

A 1-loop sum rule for D

Consider a D-dimensional two-derivative theory of gravity.

$$A_D(s,t) = G_N A_D^{\text{tree}}(s,t) + G_N^2 A_D^{1-\text{loop}}(s,t) + \mathcal{O}(G_N^3)$$

Suppose there exists an $\text{AdS}_{d+1} \times \mathcal{M}_{D-d-1}$, with $L_{\mathcal{M}} \sim L$ (= safe regime for EFT).

Define $$A_{d+1}(s,t) \equiv \frac{A_D(s,t)}{\text{Vol}(\mathcal{M})}$$

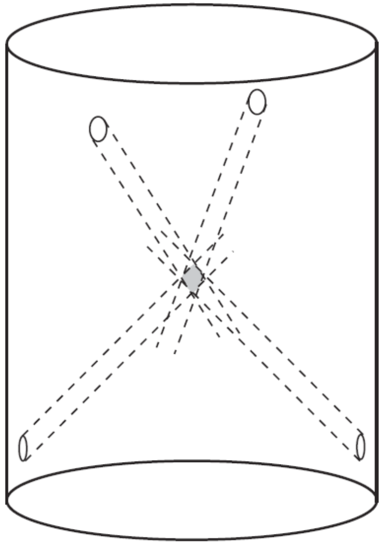
At high-energy $s,t \gg 1$ and fixed-angle $\cos\theta = 1 + \frac{2t}{s}$,

$$A_{d+1}(s \gg 1, \theta) = \frac{(L\sqrt{s})^{d-1}}{c} f_{d+1}^{\text{tree}}(\theta) + \frac{(L\sqrt{s})^{D+d-3}}{c^2} f_{d+1}^{1-\text{loop}}(\theta) + \mathcal{O}(c^{-3})$$

Goal: match to flat-space limit of CFT correlator.

A 1-loop sum rule for D

[Okuda,Penedones; Gary, Giddings, Penedones; Penedones; Fitzpatrick, Katz, Poland, Simmons-Duffin; Maldacena, Simmons-Duffin, Zhiboedov]



To match to flat space, we can match partial wave coefficients. At 1-loop,

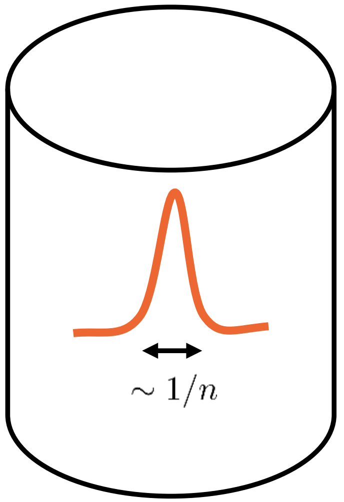
$$\mathrm{dDisc}_t(\mathcal{A}^{1-\mathrm{loop}}) = \sum_{n,\ell} \beta^{1-\mathrm{loop}}_{n,\ell} \, \underbrace{p^{(0)}_{n,\ell} \, G^{(t)}_{n,\ell}}$$

$\downarrow$ $L_{\mathrm{AdS}} \to \infty$

Partial waves

The dictionary between OPE data and flat space momentum is known to be

$$L\sqrt{s} \sim n$$



Recalling the previous formula, matching in flat space limit n>>1 yields

$$\beta^{1-\mathrm{loop}}_{n\gg1,\ell} \sim n^{D+d-3}$$

A 1-loop sum rule for D

$$\beta^{1-\text{loop}}_{n\gg 1,\ell} \sim n^{D+d-3}$$

The power comes from determining $\beta^{\text{1-loop}}$ in terms of planar data via unitarity.

The general result is

$$\beta^{1-\text{loop}}_{n,\ell} \equiv 2\sum_{\mathcal{O}} \rho_{\text{ST}}(\Delta_{\mathcal{O}}) \left(\frac{\pi^2}{4} \langle \gamma^{(1)}_{n,\ell}(\mathcal{O}) \rangle^2 + \sin^2(\pi(\tau_{\mathcal{O}} - \Delta_\phi)) \| C^2_{\phi\phi[\mathcal{O}\mathcal{O}]_{n,\ell}} \| \right)$$

The flat space matching thus yields a **1-loop sum rule for D.**

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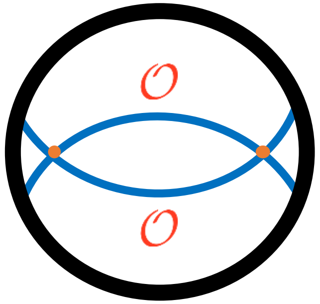
Non-degenerate double-traces

$$\Delta_{\mathcal{O}} - \Delta_\phi \notin \mathbb{Z}$$



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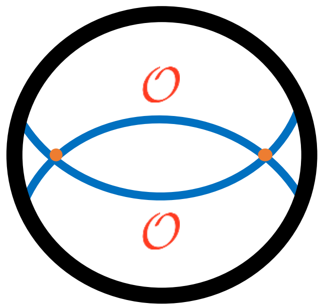

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Single-trace density of states

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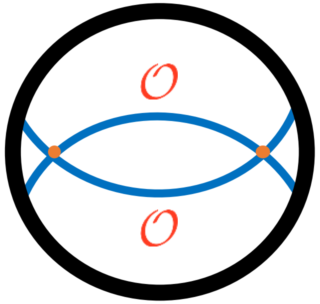

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$$\beta_{n\gg 1,\ell}^{1-\text{loop}} \sim n^{D+d-3}$$

Comments:

1. **Positive-definite**, term-by-term

$$\beta_{n,\ell}^{1-\text{loop}} \equiv 2\sum_{\mathcal{O}} \rho_{\text{ST}}(\Delta_{\mathcal{O}}) \left(\frac{\pi^2}{4} \langle \gamma_{n,\ell}^{(1)}(\mathcal{O}) \rangle^2 + \sin^2(\pi(\tau_{\mathcal{O}} - \Delta_\phi)) \| C^2_{\phi\phi[\mathcal{O}\mathcal{O}]_{n,\ell}} \| \right)$$

N.B. dDisc crucial!

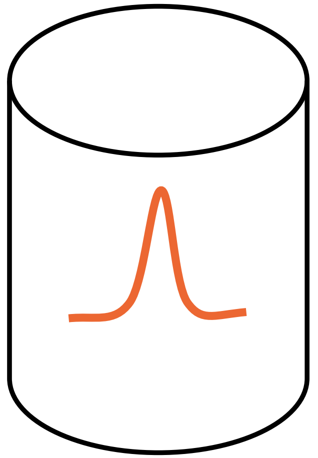
2. Trees are insensitive to D. $\beta_{n\gg 1,\ell}^{\text{tree}} \sim n^{d-1}$ [Cornalba, Costa, Penedones]

That's good: consistent truncations exist!

Non-degenerate operators

Suppose we have a power law density of non-degenerate single-trace operators:

$$\rho_{\rm ST}(\Delta_{\mathcal{O}} \gg 1) \sim \Delta_{\mathcal{O}}^{\mathtt{x}-1}$$



Sum dominated by large double-trace dimensions, $1 \ll n \sim \Delta_{\mathcal{O}} \ll \Delta_{\rm gap}$

$$\beta^{\rm 1-loop}_{n\gg 1,\ell} \sim n^{2d+\mathtt{x}-1} \quad \Rightarrow \quad D = d+1+\mathtt{x}$$

→ x large extra dimensions.

Converse: Weyl's law growth of eigenvalues λ on compact manifold $\mathcal{M}$ with smooth $\partial\mathcal{M}$.

Parameterizing $\lambda \sim \Delta^2$,

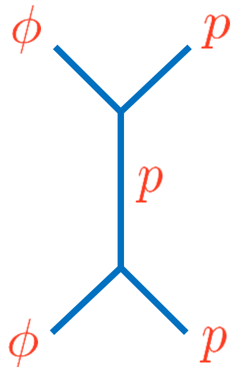
$$\int^{\Delta_* \gg 1} d\Delta\, \rho_{\mathcal{M}}(\Delta) \sim \frac{\mathrm{vol}(\mathcal{M})}{(4\pi)^{\frac{\dim(\mathcal{M})}{2}}\Gamma\left(\frac{\dim(\mathcal{M})}{2}+1\right)} \Delta_*^{\dim(\mathcal{M})}$$

Degenerate operators

Now suppose there is a tower of operators degenerate with ϕ modulo integers,

$$\Delta_p = \Delta_\phi + p - 2 \ , \ \text{where } p = 2, 3, \ldots$$

Assuming a cubic coupling ϕpp,



$\xrightarrow{\text{(Crossing)}}$ $\langle \gamma^{(1)}_{[\phi\phi]_{n,\ell}}(p) \rangle$

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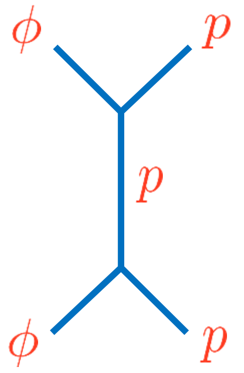

$\longrightarrow$ (Crossing) $\langle \gamma^{(1)}_{[\phi\phi]_{n,\ell}}(p) \rangle$

Result:

$$\langle \gamma^{(1)}_{[\phi\phi]_{n,\ell}}(p) \rangle \Big|_{1 \ll n \sim p} \sim n^{d-3} \cdot C^2_{\phi pp} \Big|_{p \gg 1}$$

$\longleftarrow$ Depends on OPE asymptotics

If $C_{\phi pp}\Big|_{p \gg 1} \sim \dfrac{p^{1+\frac{\alpha}{4}}}{\sqrt{c}}$ then the sum rule implies $D = d + 2 + \alpha$

Planar OPE universality

In familiar cases $\phi = T_{\mu\nu}, \mathcal{L}$, the OPE coefficient $C_{\phi pp}$ is exactly linear (α=0).

Also, taking a tower of operators in rank-p symmetric traceless irreps of so(n) gives D=d+n+α (using *dim*(rank-p>>1) ~ p^{n-2}). So α=0 suggests $AdS_{d+1} \times S^{n-1}$

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Conjecture: For single-trace operators ϕ and ϕ_p with

$$\Delta_\phi \ll \Delta_{\text{gap}} \quad \text{and} \quad \Delta_\phi \ll \Delta_p \propto p \ll c^{\#>0}$$

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  - Superconformal primaries in maximal SCFT in d=2,3,4,6 [Lunin, Mathur; Lee, Minwalla, Rangamani, Seiberg; Bastianelli, Zucchini]
  - Extremal correlators in ABJM 't Hooft limit [Hirano, Kristjansen, Young]
  - Short ($p = \lambda^{1/4}$), semiclassical ($p = \lambda^{1/2}$) strings in N=4 SYM [Zarembo; Minahan, Pereira; Bargheer, Minahan, Pereira; Roiban, Tseytlin]

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- Rough sketch of proof: worldsheet CFT + 2d HHL

Bounding holographic spectra

Reverse the logic: ***assume*** string/M-theory dual with D ≤ 10/11.

What does this imply about single-trace spectrum of planar CFT?

1. Density of states: $\rho_{\rm ST}(\Delta \gg 1) \lesssim \Delta^{8-d}$ (string)

$\rho_{\rm ST}(\Delta \gg 1) \lesssim \Delta^{9-d}$ (M)

2. If ϕ_p furnish sequence of irreps R_p of a global symmetry, with asymptotics

$$\dim(R_{p\gg 1}) \sim p^{\mathbf{r}_p}$$

then since D = d + 2 + r_p, the above inequalities **bound** $\boldsymbol{r_p}$.

Can we prove these bounds *from CFT*? If so, why are they true?

A final speculation

What is required for AdS x Small?

A possible Holographic Hierarchy Conjecture:

Large Higher-Spin Gap + No Global Symmetries $\Rightarrow$ Local AdS dual with D = d+1

This generalizes arguments of [Polchinski, Silverstein]

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Let’s see.

Summary

Hands-on Unitarity Methods in AdS/CFT

Probing extra dimensions from CFT data

Universality in the OPE of planar CFT

Future directions

Unitarity methods in AdS/CFT

Systematic exploration. e.g. a 1-loop basis?

What is the L-loop function space? Transcendentality properties?

AdS landscape

What bootstrap constraints must loop amplitudes obey?

Planar OPE universality

Prove refined properties needed to infer bulk string/M-theory?