Beyond geometry

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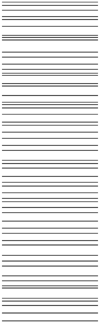
A quote

"The quest now is to understand what are the atoms of space..."

Joe Polchinski

Discrete energy spectrum

- Black holes have a discrete energy spectrum (large AdS...).
- In apparent tension with the smoothness of geometry.
- How far does geometry take us in understanding this discrete structure?

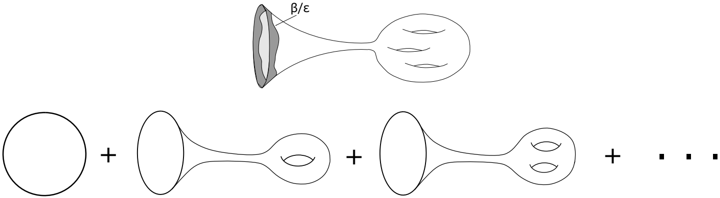


An ensemble of quantum systems

- Averaging over an ensemble of quantum systems seems to give geometry its best chance.
- For example, JT gravity is dual to a (random matrix) ensemble of quantum systems [Saad-SS-Stanford].

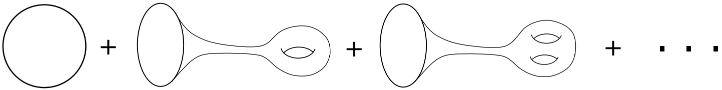
JT gravity

- Define JT gravity by the standard Lagrangian on surfaces of arbitrary topology (for $Z(\beta)$),



- These simple pictures are what I'll mean by "geometry."

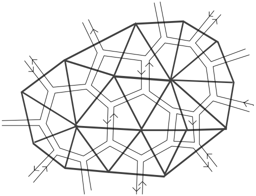
Genus expansion



- The JT action contains a topological term $S_0\chi$ which provides a genus counting parameter.
- $Z(\beta) = \sum_g Z^{(g)}(\beta) \times e^{(1-2g)S_0} = e^{S_0} \sum_g Z^{(g)}(\beta) \times (e^{-S_0})^{2g}$
- Looks like a perturbative string genus expansion (the "JT string").
- But here $g_s = e^{-S_0} \sim e^{-1/G_N} \sim e^{-N_{\text{SYK}}}$.
- These are nonperturbative effects in G_N. Joining and splitting of closed baby JT universes, a " third quantized" description.
- We need to sum them up.

- There are powerful techniques for summing over all topologies of certain types of 2D gravity coupled to “minimal” matter.
- These techniques (which originated in the 1980s) use the ‘t Hooft double-line diagram expansion of large rank (L) matrix integrals to describe triangulations of 2D surfaces,

$$\mathcal{Z} = \int dH e^{-L Tr V(H)}$$



- Surfaces with Euler character χ are weighted by L^{χ}.

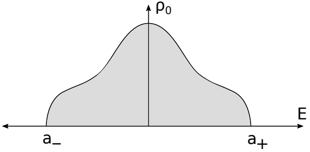
Minimal string versus JT string

$$\mathcal{Z} = \int dH e^{-L Tr V(H)}$$

- Such one matrix integrals, after double scaling, describe $(2, p)$ minimal models coupled to Liouville gravity – the "minimal" string.
- The JT string arises from a further limit, $p \to \infty$, combined with an energy rescaling.
- Some differences between the JT string and the minimal string:
  - $R = -2$ is a constraint, not just an equation of motion.
  - The metrical length of a boundary (macroscopic loop) is infinite, as in AdS/CFT, not finite.
  - The genus counting parameter g_s is e^{-1/G_N}, not $\sqrt{G_N}$.
  - The matrices represent second quantized boundary Hamiltonians, of (effective) rank e^S, not Yang-Mills fields of rank $\sqrt{S}$. A reflection of the third quantized description.

JT gravity as a matrix integral

- The JT string: tune V and L so that the large L density of states near the edge agrees with the leading order JT density of states $\rho_0(E) \sim e^{S_0} \sinh(\sqrt{E})$.



- Then we have

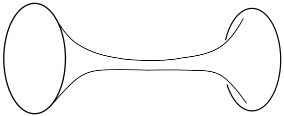
$$Z_{JT}(\beta) = \langle e^{-\beta H} \rangle_{\text{matrix}}$$

to all orders in the genus expansion, weighted by $e^{(1-2g)S_0}$.

- The matrix integral gives a (non-unique) nonperturbative definition of the theory.

The double trumpet

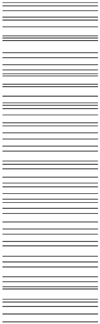
- Schematically the genus expansion comes from expanding in smooth fluctuations $\rho(E) = \rho_0(E) + \delta\rho(E)$: an ensemble of smooth functions $\delta\rho(E)$.
- For example consider the density-density correlator $\langle \delta\rho(E)\delta\rho(E')\rangle$ (a transform of $\langle Z(\beta)Z(\beta')\rangle$).
- $\langle \delta\rho(E)\delta\rho(E')\rangle \sim 1(E - E')^2$, to leading order in fluctuations.
- In JT gravity this is computed by the "double trumpet " geometry: $\chi = 0$, weighted by e^0.



- This corresponds to the "ramp" in the spectral form factor $\langle Z(\beta + it)Z(\beta - it)\rangle$, essentially the Fourier transform of $\langle \rho(E)\rho(E')\rangle$.
- Geometry captures some aspects of eigenvalue statistics.

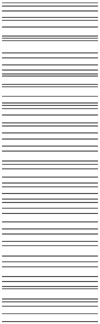
Discrete spectrum

- But actually each H drawn from the ensemble has a discrete spectrum. Each draw of $\rho(E)$ is a sum of delta functions.
- Does the sum over geometries contain complete information about the theory, including this discreteness?



Discrete spectrum, contd.

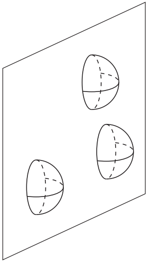
- The analog question in string theory – is the sum over string worldsheets enough to determine the theory?
- The lesson of the 1990's is no. One needs branes...
- Geometry provides internal evidence: the genus expansion diverges, like $(2g)!$, pointing to D-branes.
- The dynamics of D-branes in the minimal/topological string was worked out in the 2000s...
- The same technology applies to the JT string.
- The discrete eigenvalue structure is determined by D-branes in the JT string.



Worldsheets that end

- D-brane effects are described by arbitrary numbers of disconnected world sheets, ending on the D-brane.

$$e^{-c/g_s} = -\frac{c}{g_s} + \frac{1}{2}(\frac{c}{g_s})^2 + \dots$$

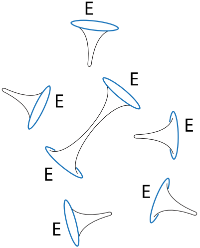


Spacetimes that end

- The eigenvalue structure in the JT string can be studied with a "probe FZZT brane."

$$\langle \psi(E) \rangle = e^{-LV(E)/2} \langle det(E-H) \rangle = e^{-LV(E)/2} \langle e^{Tr \log(E-H)} \rangle$$

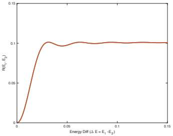
- Here the D-brane effects are described by arbitrary numbers of disconnected spacetimes. What they "end on" is not so clear... Beyond "simple" geometry.
- In the JT string $e^{-c/g_s} \sim e^{-ce^{S_0}}$, doubly exponential in G_N.



Computing without D-branes

- There are indications that it should be possible to compute these effects without invoking arbitrary numbers of boundaries.
- For example in SYK the spectral form factor $\langle Z(\beta + it) Z(\beta - it) \rangle$ Can be exactly rewritten as a two replica $G_{\alpha\beta}(t, t'), \Sigma_{\alpha\beta}(t, t')$ path integral.
- It would be interesting to understand how the D-brane effects are realized in such a description.

The density correlator

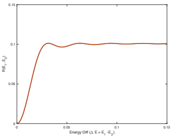


- We can compute the density-density correlator using D-branes:

$$\langle \rho(E)\rho(E')\rangle \sim e^{2S} - \frac{1}{2(\pi(E-E'))^2}(1-\cos(2\pi e^S(E-E'))).$$

- The $1/(E-E')^2$ term comes from the double trumpet.
- The $\cos(e^S(E-E')) \sim \mathrm{Re}\, e^{ie^S}$ term is a D-brane effect. Not small, but rapidly oscillating.
- The oscillations are a clear signal of discreteness, in an averaged system.

Random matrix statistics

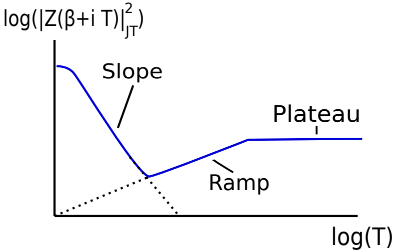


$$\langle \rho(E)\rho(E')\rangle \sim e^{2S} - \frac{1}{2(\pi(E-E'))^2}(1-\cos(2\pi e^S(E-E')))$$

- These effects are not limited to JT gravity.
- This is the "Sine kernel formula" for the eigenvalue correlations in (GUE) random matrix theory [Dyson; Gaudin; Mehta].
- Conjectured to be universal in quantum chaotic systems [Wigner; Dyson; Berry; Bohigas-Giannoni-Schmit; . . .].
- So these effects, including the doubly exponential oscillating ones, should be generic in (averaged) gauge/gravity dual systems.

Non-averaged systems

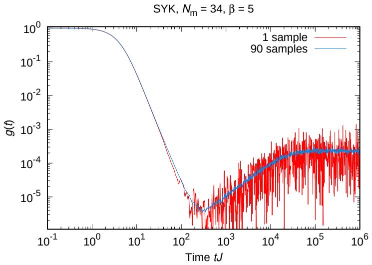
- Quantum systems that are not averaged, like SYM, pose additional challenges to a geometric bulk description.
- Diagnose with the spectral form factor $\langle Z(\beta + it)Z(\beta - it)\rangle$, the Fourier transform of the density-density correlator.



- The ramp is described by the double trumpet. The sharp transition to the plateau at exponentially late time is due to the oscillating D-brane effects.

Non-averaged systems, contd.

- For a non-averaged system the spectral form factor is very erratic. It is not self-averaging. [Prange].
- Universal (a consequence of random matrix statistics).
- What is the bulk explanation for this erratic behavior? $\star$

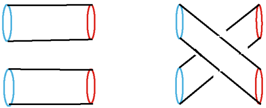


$$\langle Z(\beta + it) Z(\beta - it) \rangle$$

[CGHPSSST]

Moments from wormholes

- Can compute the size of fluctuations gravitationally (schematic).
- Compute the second moment, $\langle (Z(\beta + it) Z(\beta - it))^2 \rangle$.



- $\langle (ZZ^*)^2 \rangle = 2(\langle ZZ^* \rangle)^2$, and so the variance is given by $\langle (ZZ^*)^2 \rangle - (\langle ZZ^* \rangle)^2 = (\langle ZZ^* \rangle)^2$. Fluctuations are the same size as the signal.
- $\langle (ZZ^*)^k \rangle = k!(\langle ZZ^* \rangle)^k$.
- An exponential distribution.
- All simple smooth statistics, e.g. the time autocorrelation function, should be accessible this way, but not the actual erratic signal.

Analogies and models

- In the absence of a clear understanding of the "erratic red curve" the best we can do is offer analogies and models.
- A very interesting model addressing closely related issues has been developed by [Marolf-Maxfield]. Listen to Henry's talk!

Semiclassical quantum chaos

- An analogy: semiclassical chaos in an ordinary few body quantum mechanical systems, like a quantum billiard.
- Use the path integral (Gutzwiller trace formula), summing over periodic orbits (schematically)

$$Tr e^{-iHt/\hbar} \sim \sum_a e^{\frac{i}{\hbar} S_a}$$

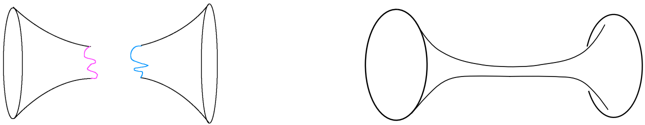
- In the analogy H is the boundary quantum system, the orbit sum is the microscopic bulk description.
- The spectral form factor becomes:

$$Tr e^{-iHt/\hbar} \; Tr e^{iHt/\hbar} \sim \sum_{ab} e^{\frac{i}{\hbar}(S_a - S_b)}$$

Semiclassical quantum chaos, contd.

$$\sum_{ab} e^{\frac{i}{\hbar}(S_a - S_b)}$$

- Long times $t \to$ long orbits $\to$ large phases $\to$ large fluctuations.
- But on averaging (over time, say) in the ramp region the only terms that survive are the ones where $a = b$, up to a time translation [Berry].
- There are e^t such paths, multiplied by an e^{-t} one loop determinant, giving a bulk microscopic derivation of the order one (times t) value of the ramp. The pattern of pairing – not the microscopics – seems analogous to the spacetime wormhole geometry. An effective description.

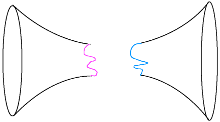


Factorization

- Wormholes conflict with factorization [Maldacena-Maoz].
- The non-averaged spectral form factor obviously factorizes, because of the double sum.

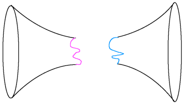
$$\sum_{ab} e^{\frac{i}{\hbar}(S_a - S_b)}$$

- But because averaging picks out the diagonal terms $a = b$, it destroys factorization. It makes the wormhole connection [Coleman; Giddings-Strominger].
- How to restore factorization? Don't get rid of the wormhole. *Add back in the off-diagonal terms* [Maldacena-Maoz]. These are responsible for the erratic behavior.

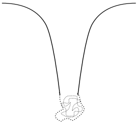


- What is the bulk realization of these contributions?

Restoring factorization in the bulk

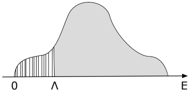


- Microscopic phase space semiclassically determines all the microstates of the system.
- What is the bulk description of the black hole microstates? The Fuzzball program [Mathur...]
- Can they be described geometrically, or are other degrees of freedom, strings, branes etc., necessary?
- Whatever the description is, it must produce random matrix statistics – perhaps by some chaotic bulk dynamics of strings, branes, etc....

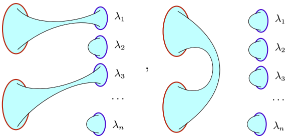


Eigenbranes

- A simple model: "Eigenbranes"
  [Blommaert-Mertens-Verschelde, 1911.11603]
- Freeze a subset of the matrix integral eigenvalues to mock up some non-averaged microstates.

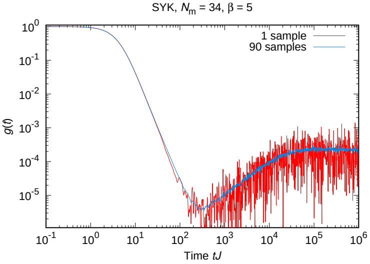


- Trumpets can end on eigenvalues (FZZT branes)



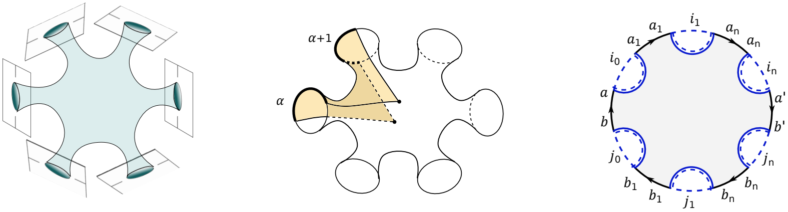
- Get an erratic contribution to the red curve...

Erratic behavior at early times



- Erratic behavior begins quite early, at the Thouless time $\sim \log S$.
- Does not involve fine grained energy statistics. Should be enough to treat density as smooth.
- Instead of summing over an ensemble of smooth $\delta\rho(E)$, take one representative.
- Should produce the erratic behavior.
- Perhaps there is a simpler bulk interpretation of this.

Averaging and replica wormholes



[Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini; Penington-SS-Stanford-Yang]

- Replica wormhole pinwheels compute $Tr\rho^n$ and the Page curve geometrically. In a non-averaged theory are there other "off-diagonal" non-geometrical contributions to $Tr\rho^n$ that cause large fluctuations?
- Assume the system is part of an ensemble and compute its variance:

$$\langle Tr\rho^n Tr\rho^n \rangle - \langle Tr\rho^n \rangle \langle Tr\rho^n \rangle .$$

Variance of pinwheels

$$\langle Tr\rho^2\, Tr\rho^2 \rangle - \langle Tr\rho^2 \rangle \langle Tr\rho^2 \rangle$$

- A handle connecting two pinwheels – of relative magnitude e^{-2S_0}. The relative variance is small!
- We say $Tr\rho^n$, and hence the Page curve, is a *self-averaging* quantity. A single element of the ensemble, a *non-averaged* system, gives a result close to the pinwheel value.
- But there may be an alternate bulk description of the non-averaged theory, perhaps involving microstates, for which the pinwheel is at best an effective description.

Going forward

"The quest now is to understand what are the atoms of space. That's what we're doing today – that's where the fun is."

Joe Polchinski

Thank you

Thank You