Black strings from TsT and irrelevant deformations

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Luis Apolo and WS, 1806.10127, 1907.03745 Luis Apolo, Stephane Detournay and WS, 1911.12359

Geometry from the Quantum, KITP, Jan 13-Jan 17, 2020

Which geometry from which quantum theory?



Overview

A class of toy models for non-AdS holography

🗖 dual QFT

supergravity TsT tranformations the vacuum black strings

string worldsheet



$T\bar{T}$ deformations

 $T\bar{T}$

[Zamolodchikov;Smirnov, Zamolodchikov;
Cavaglia, Negro, Szecsenyi, Tateo;
Cardy; Dubovsky, Flauger, Gorbenko;
Dubovsky, Gorbenko, Mirbabayi;
Conti, Iannella, Negro, Tateo; Frolov; ...]

$$\frac{\partial S_{QFT}}{\partial \mu} = \int dx^2 \left(T_{xx} T_{\bar{x}\bar{x}} - T_{x\bar{x}} T_{\bar{x}x} \right) \quad In$$

Instantaneous deformations

 $\frac{\partial S_{QFT}}{\partial \mu} = -4 \int J_{(1)} \wedge J_{(\bar{2})}$

universal form for $T\bar{T}, J\bar{T}, T\bar{J}, J\bar{J}$

$$J_{(1)} = T_{xx}dx + T_{x\bar{x}}d\bar{x}, \quad J_{(\bar{2})} = T_{\bar{x}x}dx + T_{\bar{x}\bar{x}}d\bar{x}.$$

$$d \star J_{(m)} = 0 \Longleftrightarrow \partial_{\mu}(\sqrt{-g}T_m{}^{\mu}) = 0 \Longleftrightarrow \nabla_{\mu}T_m{}^{\mu} = 0$$

• spectrum on a cylinder with $(x, \bar{x}) \sim (x + 2\pi R, \bar{x} + 2\pi R)$

$$E(\mu) = -\frac{R}{2\mu} \left(1 - \sqrt{1 + \frac{4\mu E}{R} + \frac{4\mu^2 J^2}{R^2}} \right), \quad J(\mu) = J$$

 $T\bar{T}$

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- complex spectrum at high energy
 cutoff AdS₃ [McGough, Mezei, Verlinde]
- real energy for the ground state
- no bounds for temperatures
- adding a Λ_2 flow \leftarrow patch of dS [Gorbenko, Silverstein, Torroba]

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6k: central charge of the seed CFT

- complex energy for the ground state if $\lambda \equiv \frac{k\mu}{R^2} > \frac{1}{2}$
- density of states $S_{T\bar{T}}(E_L(\mu), E_R(\mu)) = S_{Cardy}(E_L, E_R), \quad E_{L/R} = \frac{1}{2}(E \pm J)$
- Hagedorn growth at very high energy $E(\mu) \gg \frac{R}{\mu}$, $S_{T\bar{T}} \sim 2\pi \sqrt{2k\mu} E(\mu)$

• temperatures $T_{L/R} \equiv \left(\partial S_{T\bar{T}}/\partial E_{L/R}\right)^{-1}$, have a bound $T_L T_R \leq \frac{1}{8\pi^2 k\mu}$

 $T\bar{T}$

In a symmetric product theory $(\mathcal{M}_{6k})^p / S_p$, one can define a single trace version of $T\bar{T}$ deformation by

$$\frac{\partial S_{QFT}}{\partial \mu} = -4 \sum_{i=1}^{p} \int J_{(1)}^{i} \wedge J_{(\overline{2})}^{i}$$

The total entropy assuming the same temperature(or even distribution of energy) in each copy of the CFT

$$S_{T\bar{T}}(E_L, E_R) = \sum_{i=1}^{p} S_{T\bar{T}}^i(E_L^i, E_R^i)$$

= $2\pi \left[\sqrt{\frac{c}{6}RE_L(\mu) \left[1 + \frac{2\mu}{Rp}E_R(\mu)\right]} + \sqrt{\frac{c}{6}RE_R(\mu) \left[1 + \frac{2\mu}{Rp}E_L(\mu)\right]}\right]$

with total central charge of the seed CFT c = 6kp

and total energy

$$E_L = \sum_{i}^{p} E_L^i$$

Holographic dual to the single trace $T\bar{T}$

A holographic dual proposal

[Giveon, Itzhaki, Kutasov]

(the single trace version of) $T\bar{T}$ (deformed QFT) \iff LST (little string theory)

Evidence :

- long string spectrum on a geometry interpolating between zero mass BTZ and linear dilaton solution $\leftarrow \rightarrow$ the $T\bar{T}$ spectrum
- charged non-rotating black holes with 1-parameter $\leftarrow \rightarrow$ the single-trace $T\bar{T}$ entropy with zero angular momentum

Questions:

[Apolo, Detournay, WS]

- Can we find the general rotating black hole solutions?
- Can we find the bulk dual to the **ground state** in the deformed theory?
- For the superluminal deformation, can we find the **critical value** $\lambda_c = \frac{1}{2}$?
- Is there a **systematic way** to find such solutions?

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 • Is there a systematic way to find such solutions? Yes!

 $\mathrm{TsT} \leftrightarrow T\bar{T}/J\bar{T}(T\bar{J})/J\bar{J}$

TsT transformations with two global U(1) symmetries along X^1, X^2 :

T-duality along X^1 , then a shift $X^{\overline{2}} = X^{\overline{2}} - \#\lambda X^1$, and finally T-duality along X^1 .

TsT is a useful solution generating technique, and usually changes the solution locally. [Lunin-Maldacena]

In IIB string theory on $AdS_5 \times S^5$ with RR background flux,

TsT with two U(1)s both in AdS_5 / one in AdS_5 and the other in S^5 / both in S^5 non-commutative / dipole / β deformations

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A conjecture [Apolo, Detournay, WS] In IIB string theory $AdS_3 \times \mathcal{N}$ with NSNS background flux, patch of extremal Kerr [Chakraborty, Giveon, Kutasov;Apolo, WS] TsT with two U(1)s both in AdS_3 / one in AdS_3 and the other in \mathcal{N} / both in \mathcal{N} ingle trace $T\bar{T}/J\bar{T}(T\bar{J})/J\bar{J}$ deformations

Outline

dual QFT



supergravity analysis: review of AdS₃/CFT₂

A two parameter family of classical solutions in IIB SUGRA with NSNS fluxes in string frame

$$d\tilde{s}_{3}^{2} = \ell^{2} \left\{ \frac{dr^{2}}{4\left(r^{2} - 4T_{u}^{2}T_{v}^{2}\right)} + rdudv + T_{u}^{2}du^{2} + T_{v}^{2}dv^{2} \right\},\$$

$$e^{2\tilde{\Phi}} = \frac{k}{p} \longrightarrow \text{# of NS5 branes, magnetic charge} \qquad (u, v) \sim (u + 2\pi, v + 2\pi)$$

The holographic dictionary

• coordinates $u \leftrightarrow x$, $v \leftrightarrow \bar{x}$;

• Brown-Henneaux central charge c = 6pk

• T_u, T_v : parameters for the states

• gravitational Noether charge of $\partial_u \leftrightarrow$ Noether charge of ∂_x

$$\mathcal{Q}_{\partial_u} = \frac{1}{2}(M+J) = \frac{c}{6}T_u^2 \leftrightarrow E_L$$

- global AdS₃, $(T_u = T_v = \frac{i}{2}) \leftrightarrow$ ground state on the cylinder, NS vaccum
- zero mass BTZ($T_u = T_v = 0$) \leftrightarrow Ramond vacuum
- The Bekenstein-Hawking entropy \leftrightarrow Cardy formula in the dual CFT₂
- symmetric product CFT $(\mathcal{M}_{6k})^p / S_p$ [Argurio, Giveon, Shomer; Eberhardt and M. R. Gaberdiel]

stationary solutions dual to the single trace $T\bar{T}$ deformed CFT₂ can be obtained from the solution via the following TsT transformations:

T-dualize along u, shifting $v \to v - \frac{2\lambda}{k}v$, and T-dualizing along u once more

$$\frac{ds_3^2}{\ell^2} = \frac{dr^2}{4\left(r^2 - 4T_u^2 T_v^2\right)} + \frac{rdudv + T_u^2 du^2 + T_v^2 dv^2}{1 + 2\lambda r + 4\lambda^2 T_u^2 T_v^2} \qquad (u, v) \sim (u + 2\pi, v + 2\pi)$$
$$e^{2\Phi} = \frac{k}{p} \left(\frac{1 - 4\lambda^2 T_u^2 T_v^2}{1 + 2\lambda r + 4\lambda^2 T_u^2 T_v^2}\right) e^{-2\phi_0}$$

A quick look at the phase space

- TsT black strings for $T_u^2 \geq 0, T_v^2 \geq 0$, horizon $r_+ = 2T_uT_v$
- $T_u = T_v = 0, \phi_0 = 0$, the LST background of [Giveon, Itzhaki, Kutasov]
- conical defects for general $T_u^2 < 0, T_v^2 < 0$,
- smooth and geodesic complete solution exist

stationary solutions dual to the single trace TT deformed CFT₂ can be obtained from the solution via the following TsT transformations:

T-dualize along u, shifting $v \to v - \frac{2\lambda}{k}v$, and T-dualizing along u once more

arbitrary constant

• the B field is up to an exact term

A quick look at the phase space

• TsT black strings for
$$T_u^2 \ge 0, T_v^2 \ge 0$$
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- conical defects for general $T_{\mu}^2 < 0, T_{\nu}^2 < 0$,
- smooth and geodesic complete solution exist

The ground state can be obtained by assuming $T_u^2 = T_v^2 \equiv -\rho_0^2$, $\rho_0 > 0$ and requiring smoothness at the origin. After a change of coordinate, the metric is

$$ds_{3}^{2} = \ell^{2} \left\{ \frac{d\rho^{2}}{\rho^{2} + 1} + \frac{\rho^{2}d\varphi^{2} - (\rho^{2} + 1)dt^{2}}{1 + 2\lambda\rho^{2}} \right\} \qquad \rho \in [0,\infty), \quad \varphi \sim \varphi + 2\pi$$

$$\rho_{0} = \frac{1}{2\lambda}(1 - \sqrt{1 - 2\lambda})$$

$$B = -\frac{\ell^{2}(\rho^{2} + \rho_{o})}{2(1 + 2\lambda\rho^{2})}du \wedge dv, \quad e^{2\Phi} = \frac{k}{p}\frac{(1 - 2\lambda\rho_{o}^{2})}{2\rho_{o}(1 + 2\lambda\rho^{2})}e^{-2\phi_{0}}$$

- $\begin{array}{l} \lambda > \displaystyle \frac{1}{2} \text{ complex solution;} \\ \lambda_c = \displaystyle \frac{1}{2}, \ e^{2\Phi} \to 0 \text{ unless } \phi_0 \text{ is fine tuned, infinitely weak string coupling everywhere} \\ 0 < \lambda < \displaystyle \frac{1}{2}, \text{ smooth and real solution} \\ \mathrm{IR:} \ \rho \to 0, \ \mathrm{global} \ \mathrm{AdS}_3 \text{ , smooth, no horizons} \\ \mathrm{UV:} \ R^{1,1} \times S^1 \text{, locally flat with linear dilaton, infinitely weak coupled strings} \end{array}$
- $\lambda < 0$, CTC and curvature singularity at $\rho_c^2 = 1/2 |\lambda|$

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- $\lambda > \frac{1}{2} \text{ complex solution;} \qquad \begin{array}{l} \text{TsT}/T\bar{T} \text{ matching: } \ell \leftrightarrow R, \quad \lambda \ell_s^2 \leftrightarrow \mu \\ \text{Noether charges } \mathcal{Q}_{\partial_u} \leftrightarrow E_L^{vac} \end{array}$ $\cdot \lambda_c = \frac{1}{2}, \ e^{2\Phi} \to 0 \text{ unless } \phi_0 \text{ is fine tuned, infinitely weak string coupling everywhere}$
- $\lambda_c = \frac{1}{2}, e^{2\Phi} \to 0$ unless ϕ_0 is fine tuned, infinitely weak string coupling everywhere • $0 < \lambda < \frac{1}{2}$, smooth and real solution IR: $\rho \to 0$, global AdS₃, smooth, no horizons UV: $R^{1,1} \times S^1$, locally flat with linear dilaton, infinitely weak coupled strings
- + $\lambda < 0,$ CTC and curvature singularity at $\rho_c^2 = 1/2 \left| \lambda \right|$

supergravity analysis: $TsT/T\bar{T}$ matching

Black strings $T_u, T_v > 0$, asymptotic to $R^{1,1} \times S^1$ at large radius. Horizon at $r_+ = 2T_uT_v$, independent of λ Electric and magnetic charges $Q_e = pe^{2\phi_0}$, $Q_m = k$

TsT/ $T\bar{T}$ dictionary for $\phi_0 = 0$, i.e. fixed Q_e, Q_m

- Noether charges $\mathcal{Q}_{\partial_u} \leftrightarrow E_L$
- smooth Euclidean geometry at the horizon \leftrightarrow torus parameters $(u, v) \sim (u + i/\ell T_L, v i/\ell T_R)$
- Bekenstein Hawking entropy \leftrightarrow entropy of single trace $T\bar{T}$ $S_{TsT} = 2\pi \{ \sqrt{\mathcal{Q}_{\partial_u} \left(\mathcal{Q}_e \mathcal{Q}_m + 2\lambda \mathcal{Q}_{-\partial_v} \right)} + \sqrt{\mathcal{Q}_{-\partial_u} \left(\mathcal{Q}_e \mathcal{Q}_m + 2\lambda \mathcal{Q}_{\partial_u} \right)} \} = S_{T\bar{T}}$ • $T_u = T_v = \frac{i}{2\lambda} (1 - \sqrt{1 - 2\lambda}) \Leftrightarrow$ ground state, NS vacuum • $T_u = T_v = 0 \Leftrightarrow$ Ramond vacuum
 - upper bound for the temperatures by requiring real dilaton

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$$S_{TST} = 2\pi \{ \sqrt{\mathcal{Q}_{\partial_u} (\mathcal{Q}_e \mathcal{Q}_m + 2\lambda \mathcal{Q}_{-\partial_v})} + \sqrt{\mathcal{Q}_{-\partial_u} (\mathcal{Q}_e \mathcal{Q}_m + 2\lambda \mathcal{Q}_{\partial_u})} \} = S_T$$

• $T_u = T_v = \frac{i}{2\lambda} (1 - \sqrt{1 - 2\lambda}) \iff \text{ground}$
state, NS vacuum
• $T_u = T_v = 0 \iff \text{Ramond vacuum}$

• upper bound for the temperatures by requiring real dilaton

Horne-Horowitz black string

• reparameterization of the radial coordinates

$$\lambda = \frac{1}{2}, T_u = T_v, \phi_0 = 0$$

$$B_{HH} = B_{TsT} + \frac{\ell^2}{2} du \wedge dv$$

The effect is a shift of the zero energy point proportional to the the electric charge

supergravity analysis: $TsT/T\bar{T}$ matching

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Horne-Horowitz black string $\lambda = \frac{1}{2}, T_u = T_v, \phi_0 = 0$ $B_{HH} = B_{TsT} - \frac{\ell^2}{2} du \wedge dv$ The effect is a shift of the zero energy point proportional to the the electric charge

Giveon-Itzhaki-Kutasov black string $\lambda = \frac{1}{2}, T_u = T_v$ $B_{GIK} = B_{TsT} + \frac{\ell^2}{2} du \wedge dv$ ϕ_0 depends on temperature $Q_{\partial_t} \leftrightarrow E_L$ Q_e varies with the mass, and the $T\bar{T}$ temperature is not the Hawking temperature supergravity analysis

How about $\lambda < 0$?

- CTC & conical singularity at large radius
- real dilaton \leftrightarrow real spectrum $T_u T_v \le \frac{1}{2|\lambda|} \leftrightarrow E(\mu) \le \frac{pR^2}{2|\mu|}$

"=" when the horizon coincides with the CTC & conical singularity

• the vaccum is always a real solution with CTC & conical singularity at $\rho^2 = 1/2 |\lambda|$

Outline

✓dual QFT



TsT transformations: T-duality along X^1 , then a shift $X^{\overline{2}} = X^{\overline{2}} - 2\lambda \tilde{\lambda} X^1$, and finally T-duality along X^1

string worldsheet deformation
$$\frac{\partial S_{WS}}{\partial \tilde{\lambda}} = -4 \int j_{(1)} \wedge j_{(\bar{2})}$$
 -true for any consistent background -not necessarily chiral/anti-chiral $j_{(1)}$, $j_{(\bar{2})}$ are worldsheet Noether 1-forms associated to ∂_{X^1} , and $\partial_{X^{\bar{2}}}$
Noether charges $p_{(m)} \propto \oint j_m$

dual quantum field theory

$$\frac{\partial S_{QFT}}{\partial \mu} = -4 \sum_{i=1}^{p} \int J_{(1)}^{i} \wedge J_{(\overline{2})}^{i}$$

 $J_{(1)}$, $J_{(\overline{2})}$ are the **boundary spacetime** Noether 1-forms associated to ∂_{X^1} , and $\partial_{X^{\overline{2}}}$ Noether charges $P_{(m)} \propto \oint J_m$

TsT on the worldsheet

After the TsT, string spectrum on a cylinder can be obtained by two observations:

1. Before the TsT, string spectrum on a cylinder with $\oint \partial_{\sigma} u = 2\pi w$, $\oint \partial_{\sigma} v = 2\pi w$ can be obtained from zero winding by "spectral flow" with parameter *w* [Maldacena,Ooguri]

2. TsT on the WS \Leftrightarrow field redefinition: [Alday] string solutions on new background with periodic b.c. \Leftrightarrow strings on the old background with twisted boundary conditions depending on the momentum $p_{(1)}, p_{(\bar{2})}$. *assuming* $j_{(1)}/j_{(\bar{2})}$ *to be chiral/antichiral up to total derivative terms (satisfied for the WZW model)* \Leftrightarrow momentum dependent "spectral flow"

The final string spectrum on a cylinder after TsT can hence be obtained from string spectrum on a cylinder before TsT with a momentum dependent "spectral flow". Comparing the Virasoro constraints can give us the relation between the spectrum before and after the TsT. This relation is just the $T\bar{T}/J\bar{T}(T\bar{J})$ spectrum [Apolo, WS;Apolo, WS; Apolo,Stephane,WS;] Summary

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