Quantum Error Correction and the Black Hole Interior

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Geometry from the Quantum

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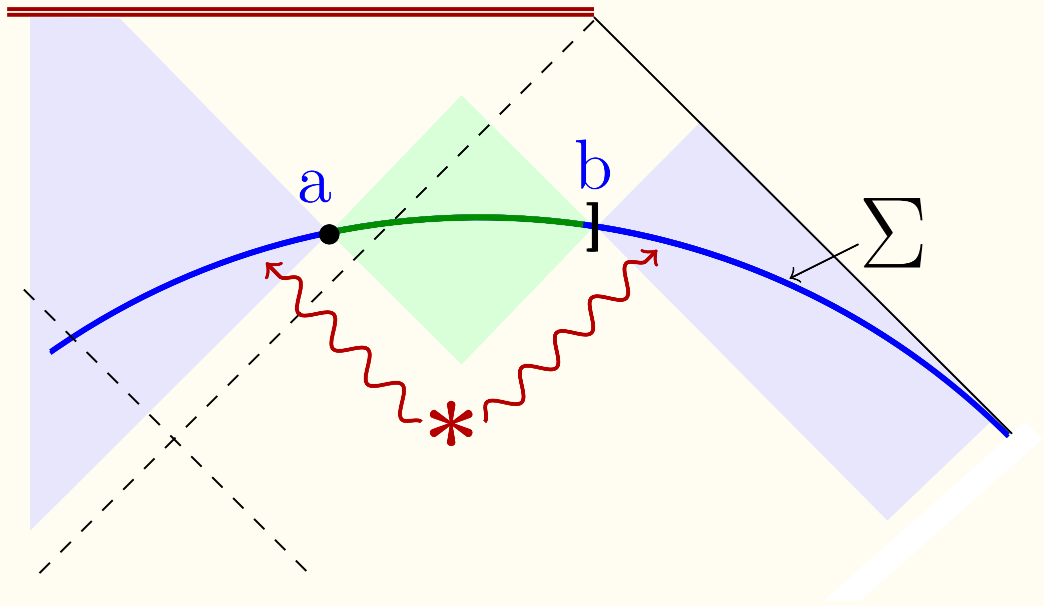
arXiv:2001.nnnn w/ E. Verlinde

Hayden, Preskill; Mathur; Papadodimas, Raju;Nomura, Maldacena, Susskind, Almheiri, Dong, Harlow; Yoshida, Kitaev; Qi; Penington; Shenker, Stanford, Yang; Wall, Engelhardt; Bousso; Hartman, Mahajan, Shagoulian;

The recent developments have shed important new light on the idea that, after the Page time, **interior operators** can be expressed as **acting on the radiation**

The space-time reason why this is *in principle* possible is that the interior and exterior regions are related via an exponentially large boost. ‘t Hooft

The fact that interior operators can *in principle* be expressed as radiation operators is equivalent to the statement of **black hole complementarity.**



$\Delta t > M \log M$

Different microscopic observables that are space-like separated on a Cauchy surface Σ, but have support on matter field configurations that, when propagated back in time, have collided with macroscopically large center of mass energies, are not simultaneously contained as commuting operators in the physical Hilbert space. Instead such operators are complementary.

\`Space Time Complementarity'

Kiem, VV

The recent developments have shed new light on the idea that, after the Page time, interior operators can be expressed as acting on the radiation.

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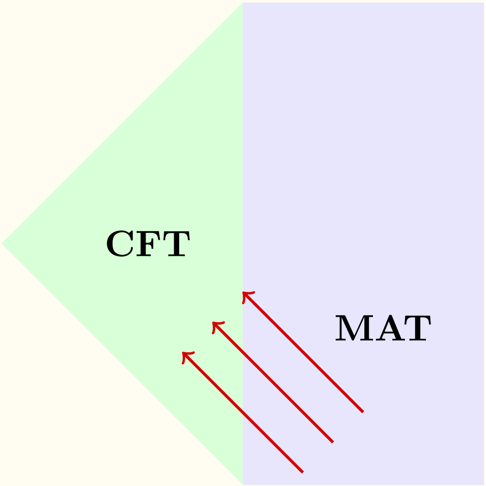
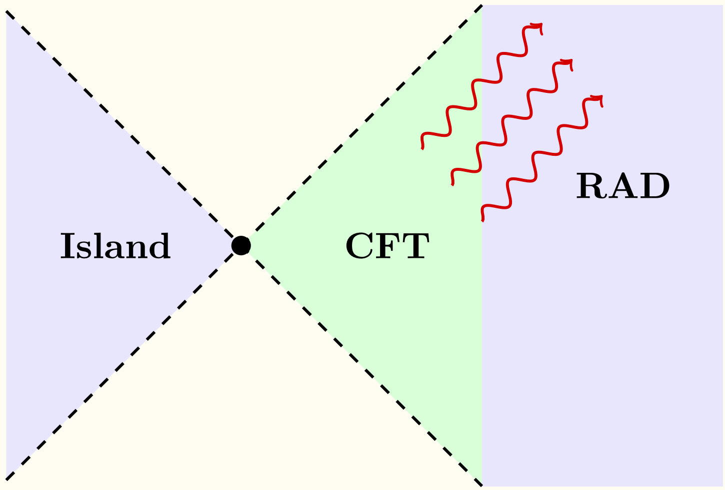
The fact that interior operators can in principle be expressed as radiation operators is equivalent to the statement of black hole complementarity.

The microscopic reason or why this is possible is that black evaporation is an **ergodic process** that produces an **random embedding** of the initial state into the Hilbert space of the radiation. Hayden, Preskill

Approximate quantum error correction techniques can be used to give an explicit construction of interior operators within a code subspace*.

VV, ADH, Yoshida, Penington





Let’s model the black hole formation and evaporation process via AdS/CFT + bath.
It maps the initial matter state into a randomly entangled state of CFT and Rad

$$\boldsymbol{U}|\Psi\rangle_{\text{Mat}} = \sum_a |\psi_a\rangle_{\text{CFT}} |a\rangle_{\text{Rad}} = \sum_n |n\rangle_{\text{CFT}} |\phi_n\rangle_{\text{Rad}}$$

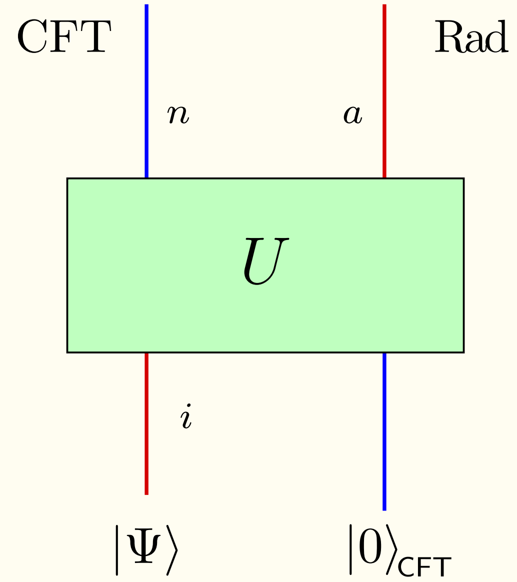
It is then possible to reconstruct the Island operators with exponential accuracy, provided that we are in the post-Page regime

$$\epsilon = e^{S_{\text{Mat}}+S_{\text{CFT}}-S_{\text{Rad}}} \ll 1$$

← Code subspace >> $\mathcal{H}_{QFT}$!

$$\boldsymbol{U}\,|\Psi\rangle_{\text{Mat}}\,|0\rangle_{\text{CFT}} \;=\; \sum_n \boldsymbol{C}_n\,|\Psi\rangle_{\text{Mat}}\,|n\rangle_{\text{CFT}}$$

$$|\Psi\rangle_{\text{Mat}} \;=\; \sum_i \alpha_i\,|\,i\,\rangle_{\text{Mat}}$$

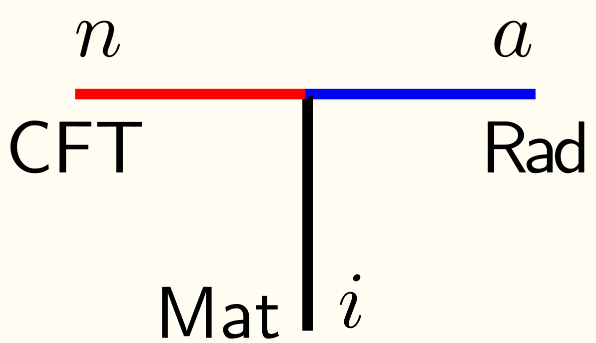


The $\boldsymbol{C}_n$ are called Krauss operators. They define a random embedding of Mat into Rad.

The crucial observation that allows for the reconstruction of interior operators is that

$$\boldsymbol{C}_m^\dagger \boldsymbol{C}_n \;\simeq\; p_n\,\delta_{mn}\,\mathbb{1}_{\text{Mat}} \qquad\qquad \sum_n p_n \;=\; 1$$

$$\boldsymbol{U}|i\rangle_{\text{Mat}} = \sum_{a,n} C^{i}_{an}\,|a\rangle_{\text{Rad}}\,|n\rangle_{\text{CFT}}$$



The Kraus matrices are microscopically determined* random matrices, drawn from a statistical matrix ensemble:

$$\sum_{b} C^{*i}_{nb}\,C^{j}_{bm} = p_n\,\delta_{nm}\,\delta^{ij} + s^{ij}_{nm}$$

$$\sum_{n} C^{i}_{an}\,C^{*j}_{nb} = q_a\,\delta_{ab}\,\delta^{ij} + r^{ij}_{ab}$$

$$\sum_{n} p_n = 1$$

$$\sum_{a} q_a = 1$$

The density matrices of CFT and Rad are given by the product of two $\boldsymbol{C}$-matrices

$$\rho_{\text{CFT}} = \boldsymbol{C}^\dagger \boldsymbol{C} \qquad ; \qquad \rho_{\text{Rad}} = \boldsymbol{C}\,\boldsymbol{C}^\dagger$$

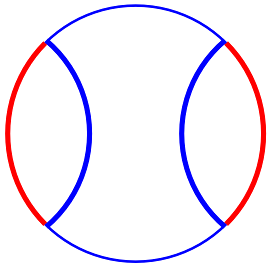
Coarse graining (phase or ensemble averaging) amounts to performing a Wick contraction

$$\overline{\rho_{\text{CFT}}} = \overbracket{\boldsymbol{C}^\dagger \boldsymbol{C}} = \sum_E p_E \, \mathbb{1}_{\text{CFT}}(M - E)$$

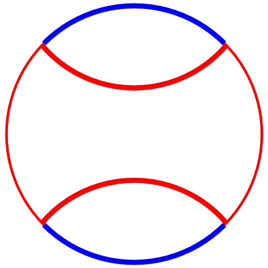
$$\overline{\rho_{\text{Rad}}} = \overbracket{\boldsymbol{C}\,\boldsymbol{C}}^\dagger = \sum_E q_E \, \mathbb{1}_{\text{Rad}}(E)$$

$$\mathrm{tr}\left(\rho^2_{\text{CFT}}\right) = \mathrm{tr}\left(\boldsymbol{C}^\dagger\boldsymbol{C}\,\boldsymbol{C}^\dagger\boldsymbol{C}\right) = \boldsymbol{C}^\dagger\boldsymbol{C}\,\boldsymbol{C}^\dagger\boldsymbol{C} + \boldsymbol{C}^\dagger\boldsymbol{C}\,\boldsymbol{C}^\dagger\boldsymbol{C}$$

$$=$$



$$+$$

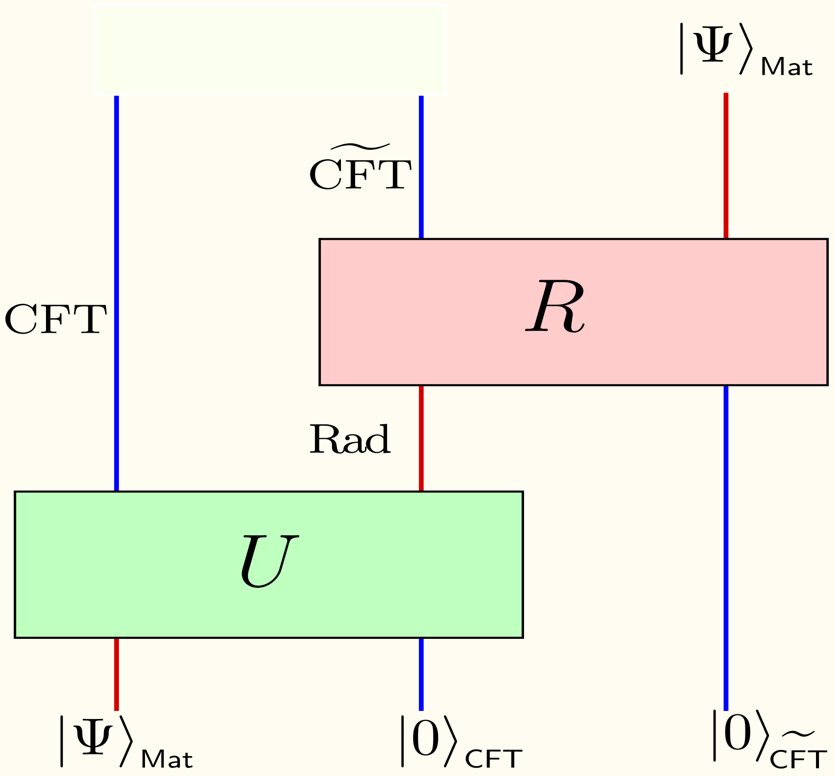


$$= \sum_E \left(q_E^2 + q_E p_E\right)$$

$$\mathrm{tr}\left(\rho^n_{\text{CFT}}\right) = \sum_E \sum_{m=0}^{n-1} N(n,m)\, q_E^{n-m} p_E^m$$

The active ingredient in the construction of interior operators is the recovery operator ***R***.
It acts on the radiation and on an ancillary CFT, and effectively inverts the time evolution.

Importantly, it does so by employing an **entanglement swap:** after the recovery, the CFT and ancilla CFT are in a thermo-field double state:

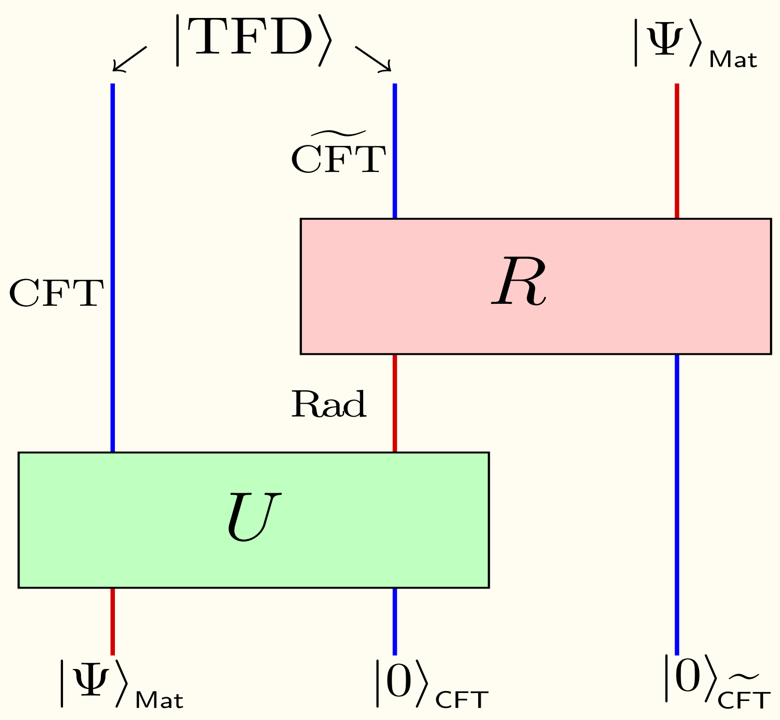


In formulas:

$$\boldsymbol{R}\,|\Phi\rangle_{\text{Rad}}\,|0\rangle_{\widetilde{\text{CFT}}} \;=\; \sum_n \boldsymbol{R}_n\,|\Phi\rangle_{\text{Rad}}\,|n\rangle_{\widetilde{\text{CFT}}}$$

$$\boldsymbol{R}\,\boldsymbol{U}\,|\Psi\rangle_{\text{Mat}}\,|0\rangle|\widetilde{0}\rangle \;\simeq\; |\Psi\rangle_{\text{Mat}}\,|\text{TFD}\rangle$$

$$|\text{TFD}\rangle \;=\; \sum_n \sqrt{p_n}\,|n\rangle_{\text{CFT}}\,|n\rangle_{\widetilde{\text{CFT}}}$$



- How do we construct this recovery operator?
- And what is it good for?

In formulas and pictures:

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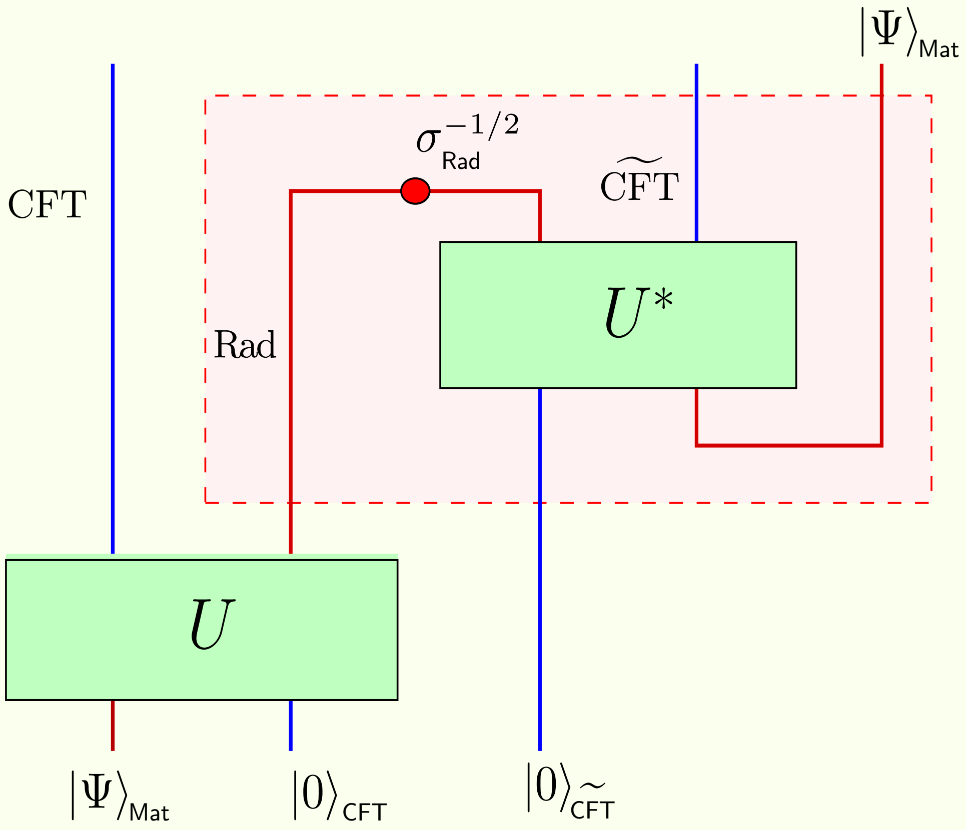
$$R|\Phi\rangle_{\text{Rad}}\,|0\rangle_{\widetilde{\text{CFT}}} = \sum_n R_n|\Phi\rangle_{\text{Rad}}\,|n\rangle_{\widetilde{\text{CFT}}}$$

$$R_n = \frac{1}{\sqrt{p_n}}\,C_n^\dagger \qquad R_n \simeq C_n^\dagger\,\sigma_{\text{Rad}}^{-1/2}$$

$$\sigma_{\text{Rad}} = \sum_n C_n\,\sigma_{\text{Mat}}\,C_n^\dagger \qquad R_n C_m^\dagger \simeq \delta_{nm}\,1_{\text{Mat}} \qquad \sum_n C_n C_n^\dagger$$



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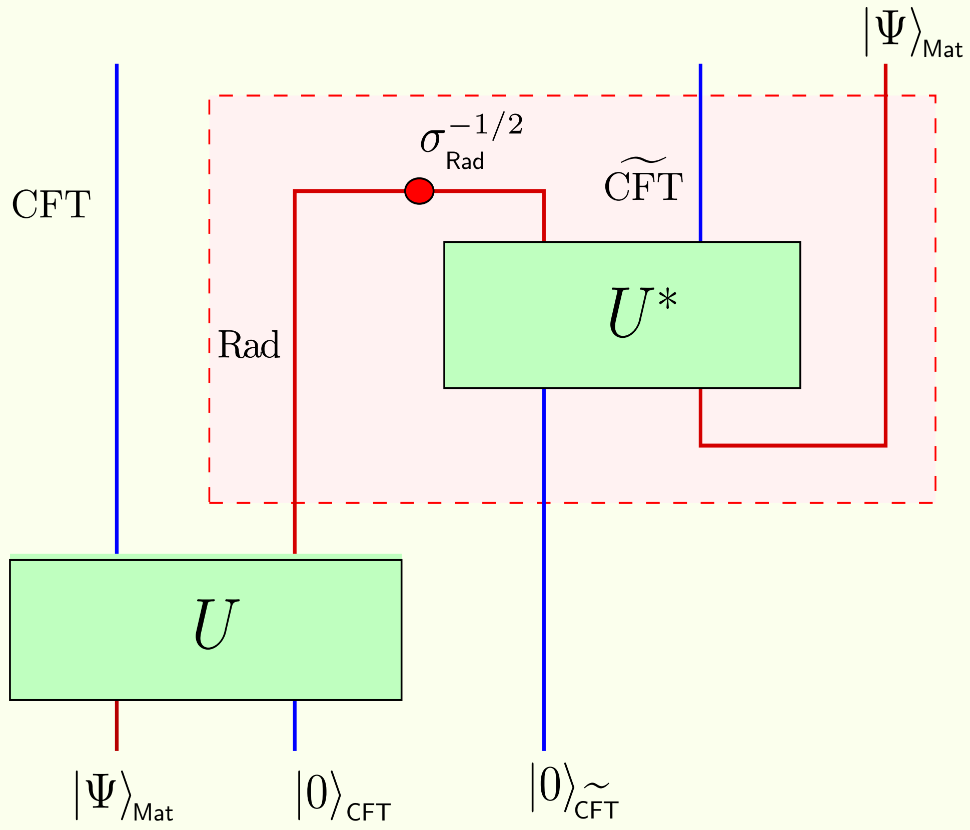
$$\boldsymbol{R}_n \;\simeq\; \frac{1}{\sqrt{p_n}}\,\boldsymbol{C}_n^{\dagger}$$

$$\boldsymbol{R}_n\boldsymbol{R}_m^{\dagger} \;\simeq\; \delta_{nm}\,\mathbb{1}_{\text{Mat}}$$



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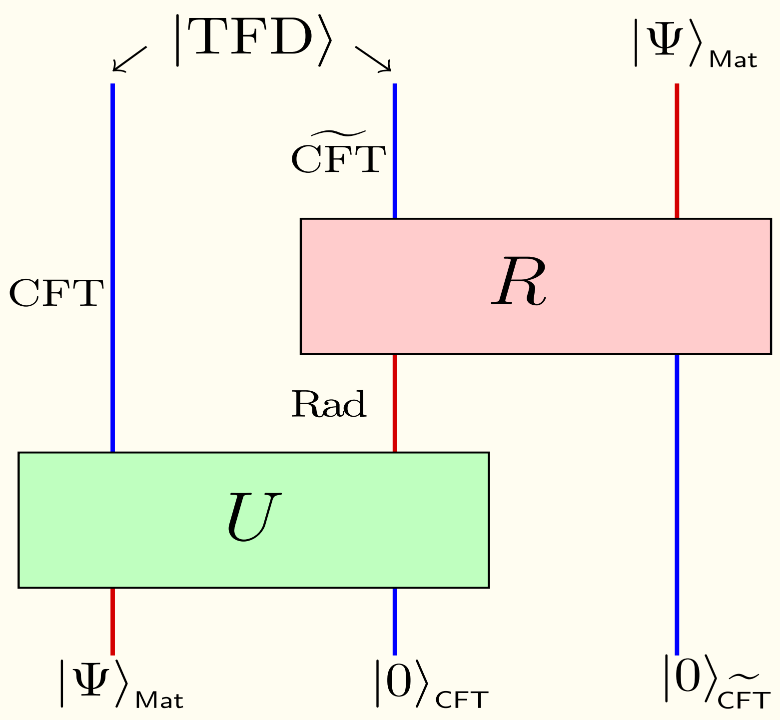


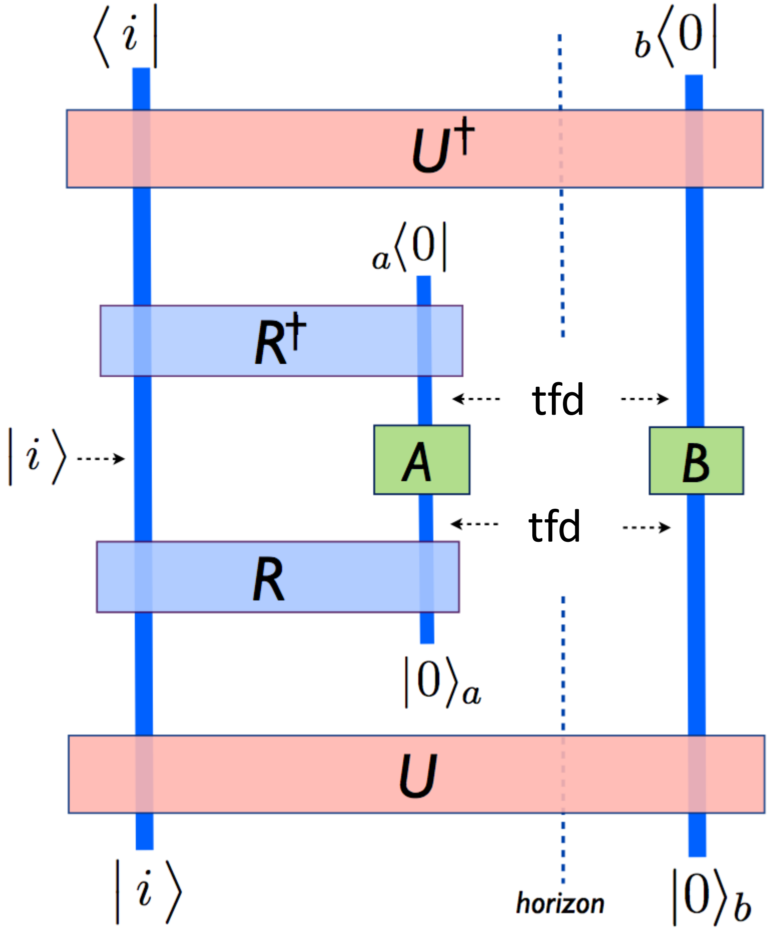
Reconstruction of interior operators:

$$O^{\text{R}}_{\text{Mat}} \;=\; \langle\widetilde{0}|\,\boldsymbol{R}^\dagger\, O_{\text{Mat}}\,\boldsymbol{R}\,|\widetilde{0}\rangle \;=\; \sum_n \boldsymbol{R}_n^\dagger\, O_{\text{Mat}}\,\boldsymbol{R}_n$$

$$O^{\text{R}}_{\text{QFT}} \;=\; \langle\widetilde{0}|\,\boldsymbol{R}^\dagger\, \widetilde{O}^{\text{HKLL}}_{\text{QFT}}\;\boldsymbol{R}\,|\widetilde{0}\rangle \;=\; \sum_{n,m} \boldsymbol{R}_n^\dagger\,(\widetilde{O}^{\text{HKLL}}_{\text{QFT}})_{nm}\,\boldsymbol{R}_m$$

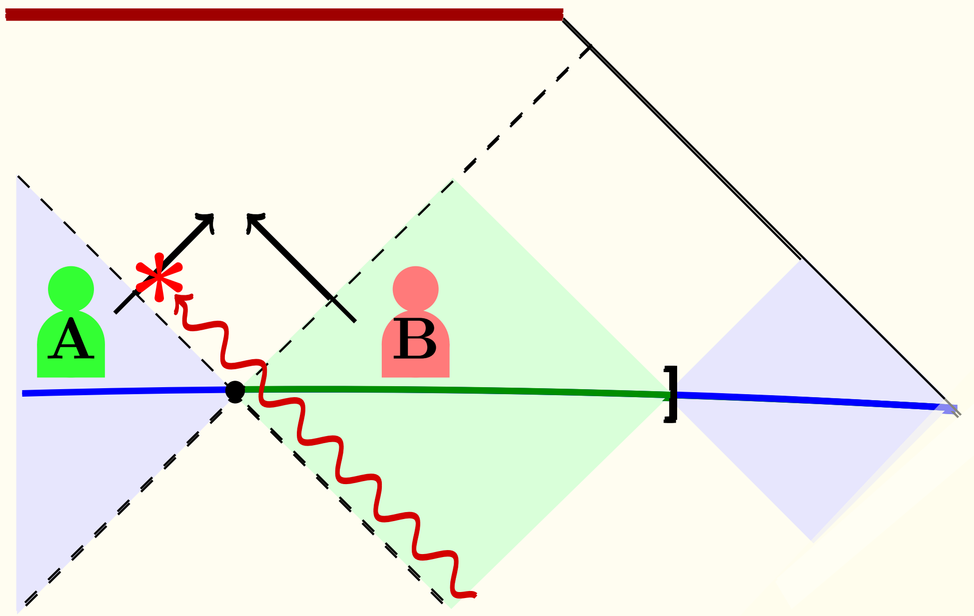
What is it good for?

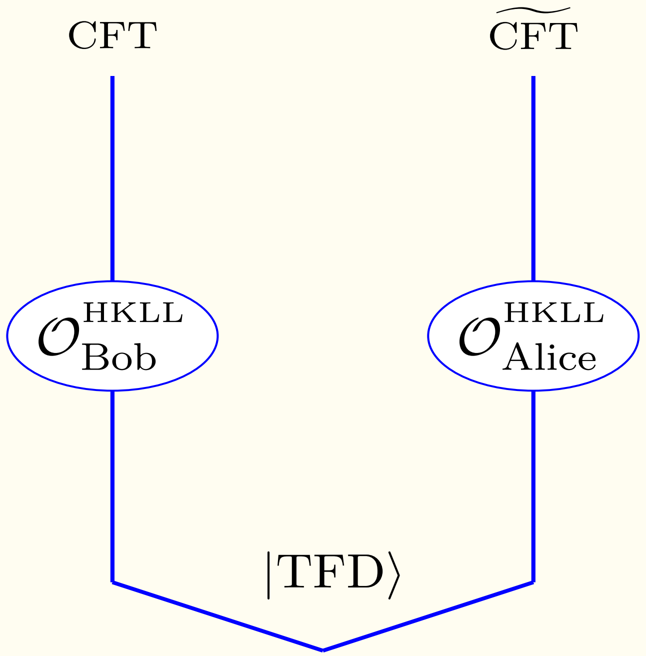
In pictures:



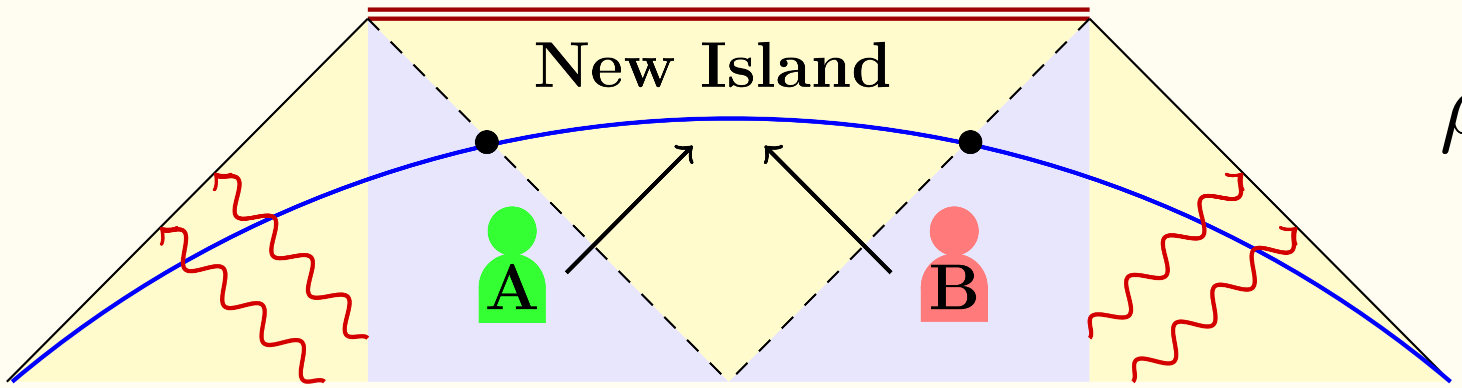
$$\mathrm{tr}\big(\rho\, \boldsymbol{\phi_a}(y)\phi_b(x)\big) = \big\langle \mathrm{tfd}\,\big|\phi_a(y)\phi_b(x)\big|\,\mathrm{tfd}\big\rangle,$$

Suppose we make the ancilla real. Alice and Bob want to arrange a Rendez-Vous on the Island with the help of the powerful quantum supercomputer owned by Bob's uncle Charlie. Will they succeed?





Charlie is still worried: the black hole still evaporates and the TFD state decoheres.

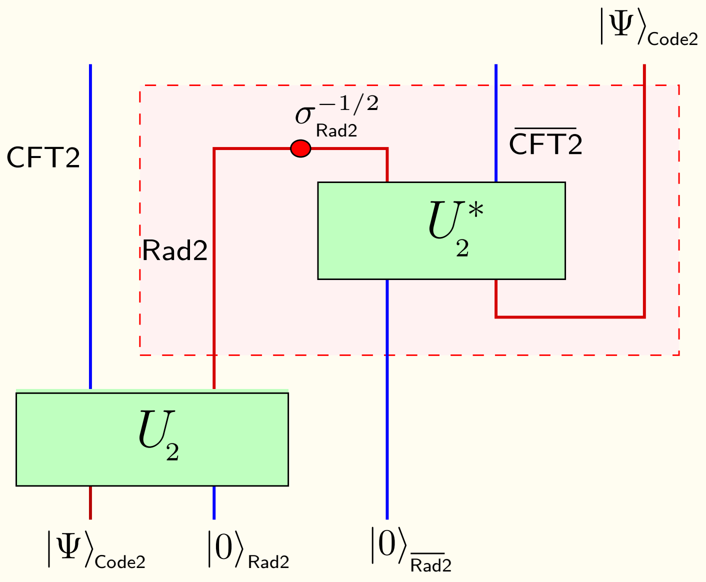


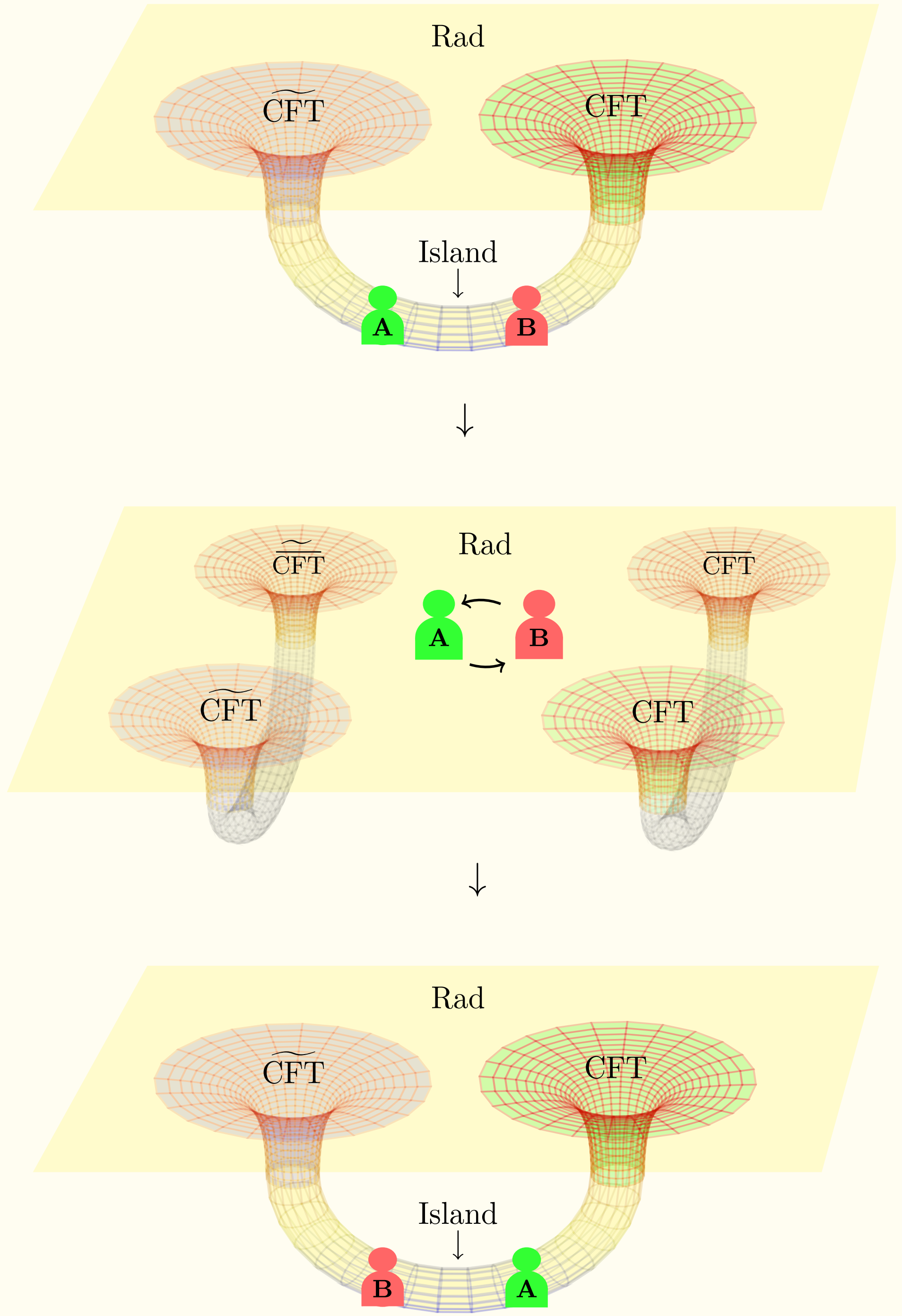
$$\rho_{\mathsf{CFT}} \otimes \rho_{\widetilde{\mathsf{CFT}}}$$

He wants to find out if Alice and Bob were able to meet:

$$|\Psi\rangle_{\mathsf{Code}} = \tilde{\mathcal{O}}_{\mathrm{Alice}}\, \mathcal{O}_{\mathrm{Bob}} |\mathrm{tfd}\rangle_{\mathsf{QFT}}$$

$$\boldsymbol{R}\, \boldsymbol{U}\, |\Psi\rangle_{\mathsf{Code}} |0\rangle_{\mathsf{Rad}} |0\rangle_{\overline{\mathsf{Rad}}} \simeq |\Psi\rangle_{\mathsf{Code}} \otimes |\mathrm{TFD}\rangle \otimes |\widetilde{\mathrm{TFD}}\rangle$$





Rad
$\widetilde{\text{CFT}}$
CFT
Island
A
B
Rad
$\overline{\widetilde{\text{CFT}}}$
$\overline{\text{CFT}}$
A
B
$\widetilde{\text{CFT}}$
CFT
Rad
$\widetilde{\text{CFT}}$
CFT
Island
B
A

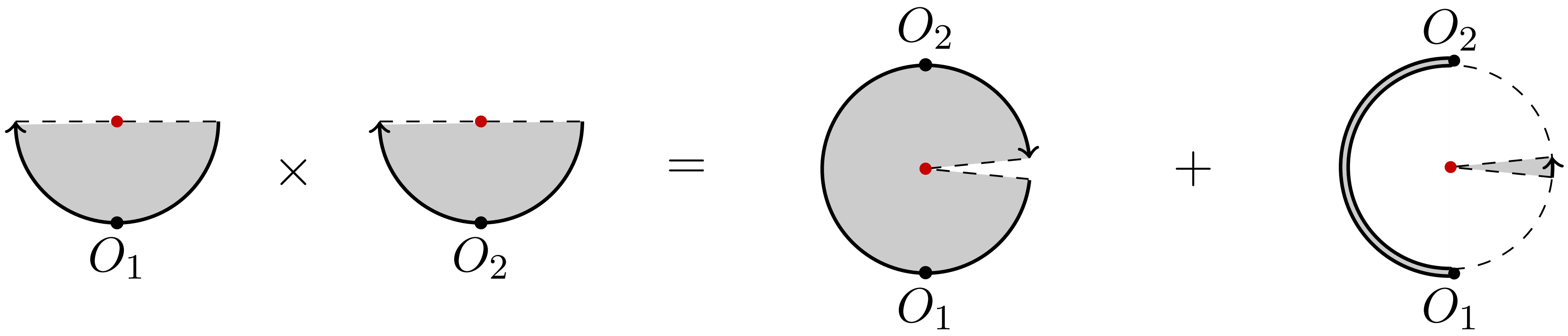
$$(O_1)^{\mathsf{R}} (O_2)^{\mathsf{R}} = (O_1 O_2)^{\mathsf{R}} + \text{Error}$$

$$\text{Error} = \epsilon \, \mathrm{tr}_{\mathsf{CFT}} \left(O_1 O_2 \, e^{\beta H} \right)$$

The reconstructed Island operator algebra has a finite `unrecoverable error’.

These error terms are responsible for the transfer of information to the exterior.

$$(\rho^{1/4} O_1 \rho^{1/4})^{\mathsf{R}} \; (\rho^{1/4} O_2 \rho^{1/4})^{\mathsf{R}} = (\rho^{1/4} O_1 \rho^{1/2} O_2 \rho^{1/4})^{\mathsf{R}} + \epsilon \, \mathrm{tr}(O_1 \rho^{1/2} O_2 \rho^{-1/2})$$



- Reconstruction of interior operators in terms of the radiation uses that black evaporation produces an **random embedding** of the initial state into the Hilbert space of the radiation.
- This construction is largely state independent, but requires the introduction of a code subspace with entropy S_{code} such that

$$S_{\rm Mat} + S_{\rm QFT}(t) \lesssim S_{\rm Code} < S_{\rm Rad}(t) - S_{\rm CFT}(t)$$

- No ensemble average is needed for reconstruction of interior operators: gravity only knows the ensemble, because it's a coarse grained description.
- Replica wormholes may provide crucial insight into how QFT breaks down and into the possible mechanism for information transfer to the exterior.