Adam Brown, Stanford University

KITP QGravity15

Hot Bubbles of Nothing

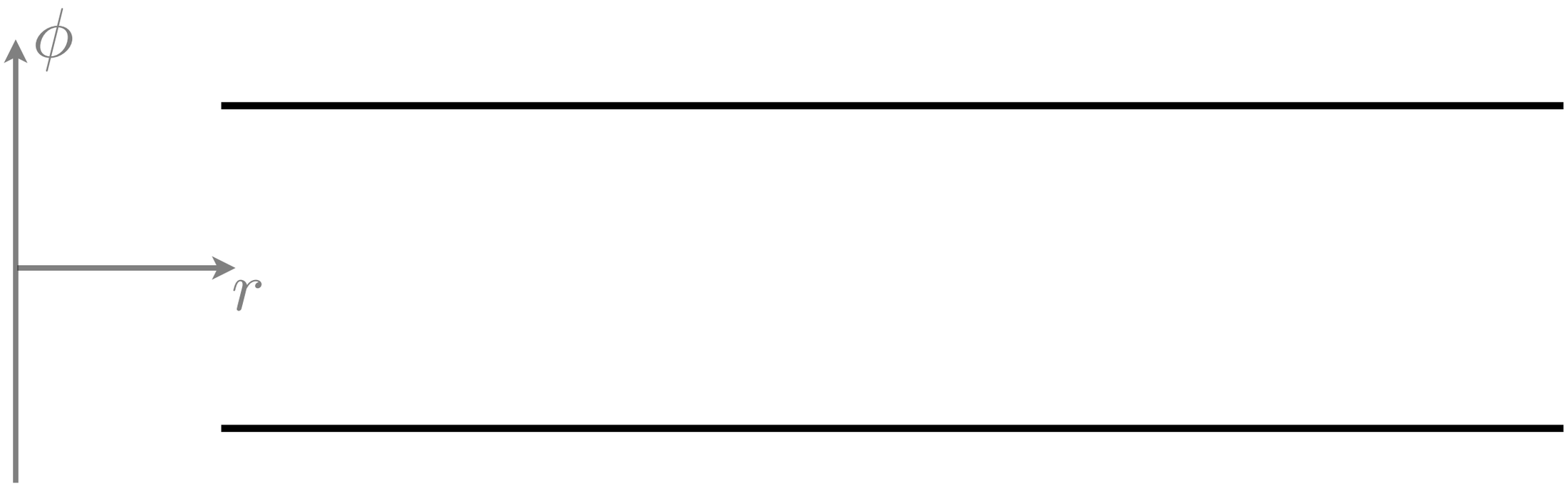
Jun 04, 2015

based on my paper, *The Decay of Hot KK Space*, arXiv:1408.5903

Bubble of nothing

(Witten, 1982)

instability of $R^{3,1} \times S^1$



$$ds^2 = dr^2 + r^2 d\Omega_2^2 + d\phi^2$$

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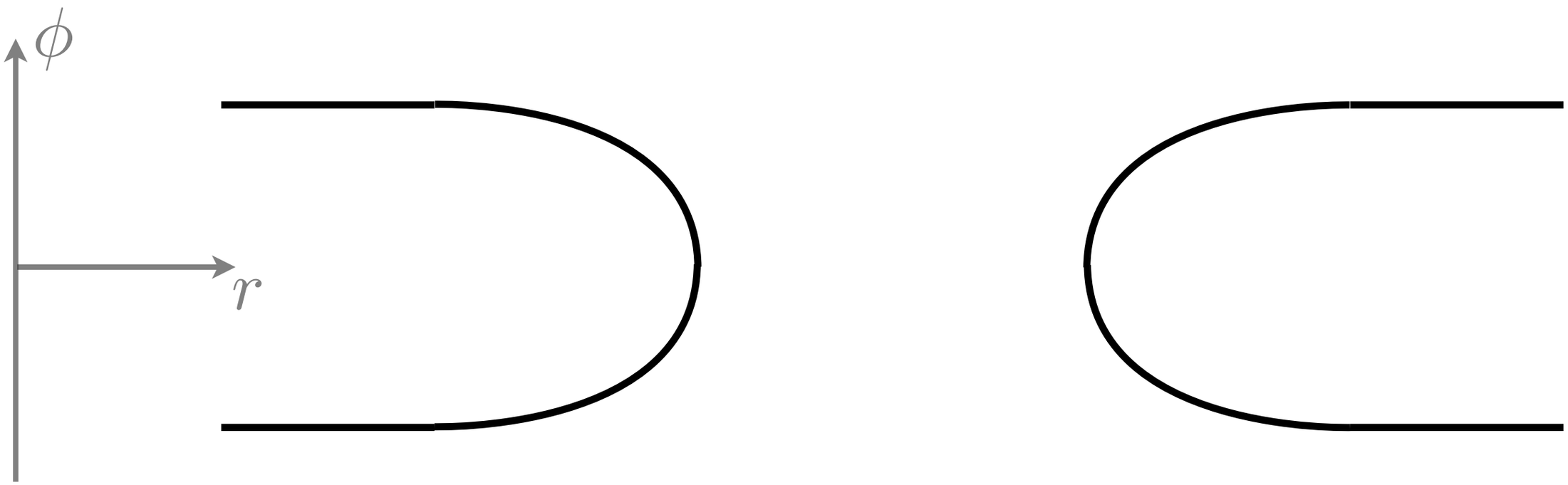

$$ds^2 = \frac{dr^2}{1 - 1/r^2} + r^2 d\Omega_2^2 + \left(1 - \frac{1}{r^2}\right) d\phi^2$$

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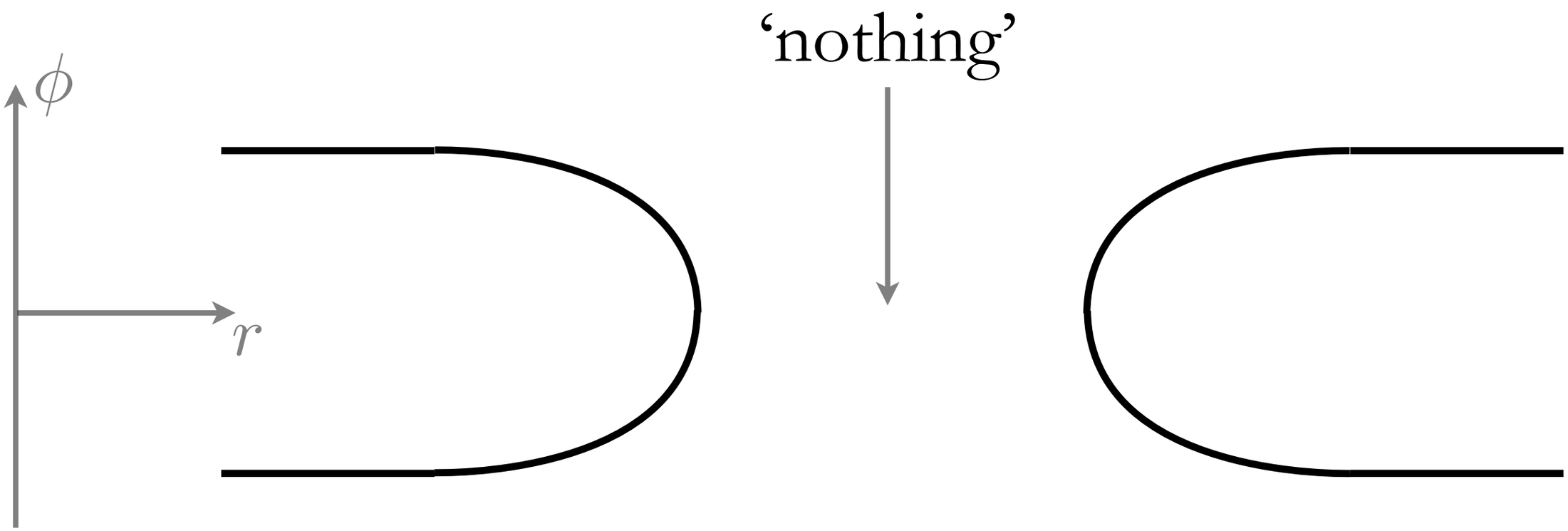

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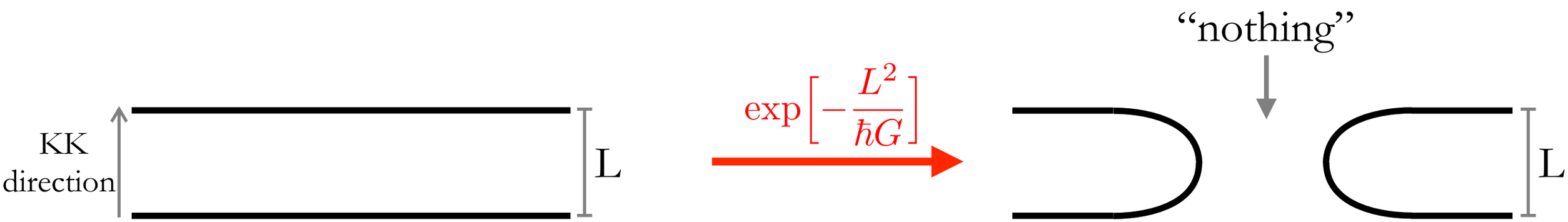

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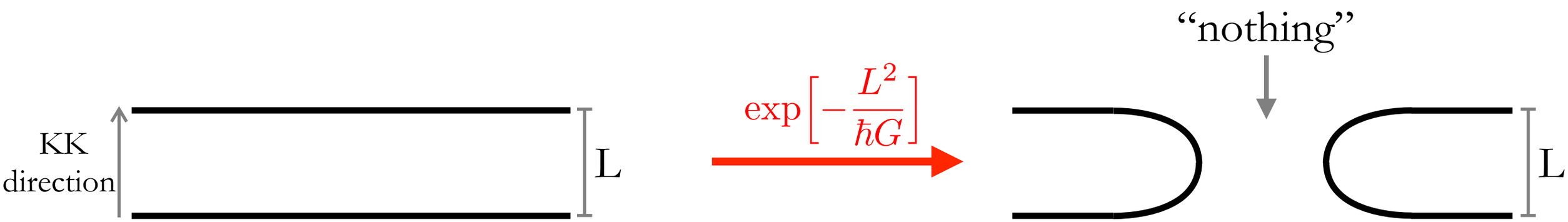

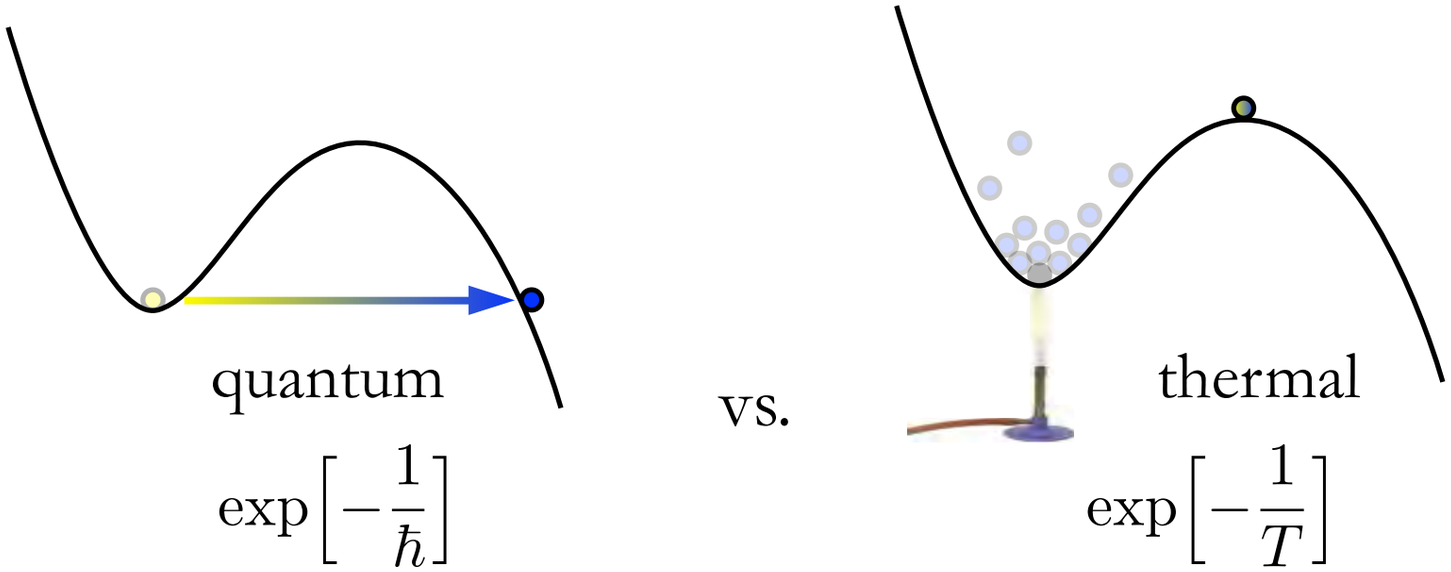

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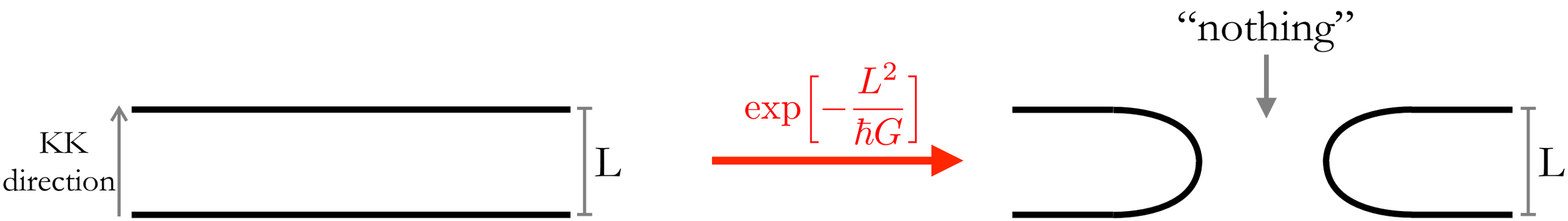
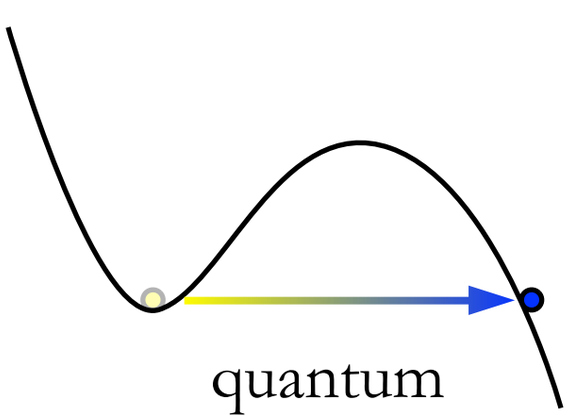
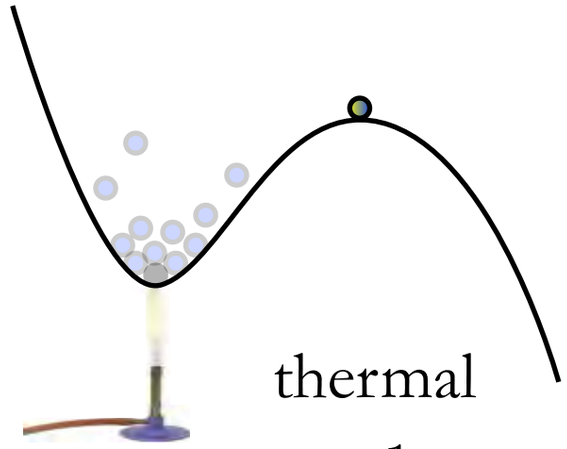




quantum

$\exp\left[-\frac{1}{\hbar}\right]$

vs.



thermal

$\exp\left[-\frac{1}{T}\right]$

is there a thermal bubble of nothing?

Bubble of nothing

(Witten, 1982)

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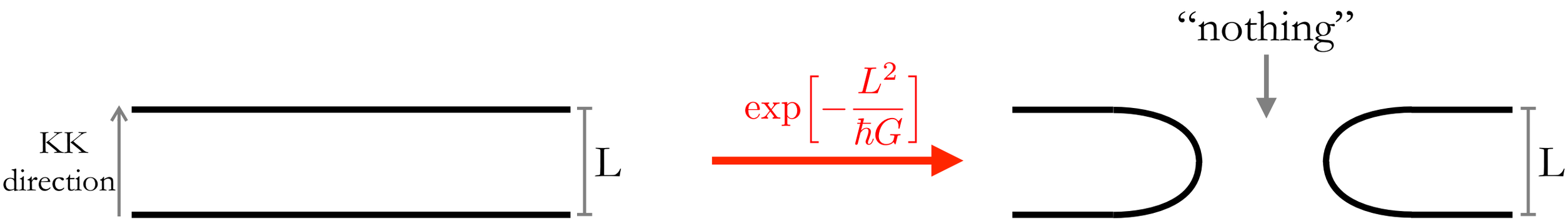

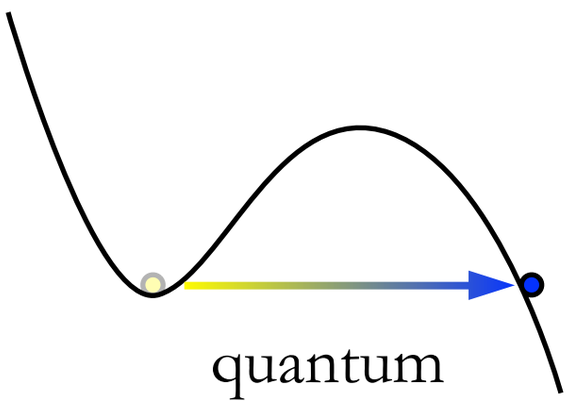

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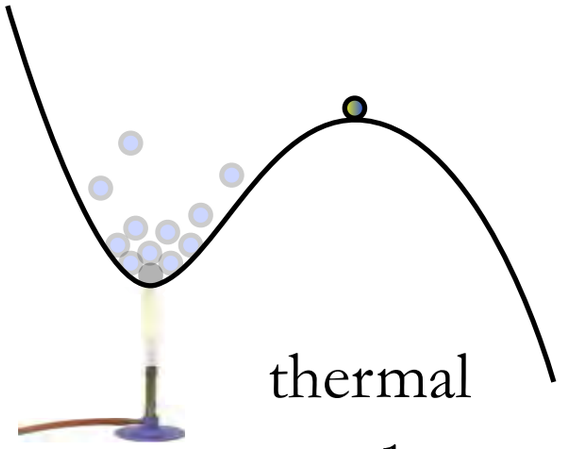

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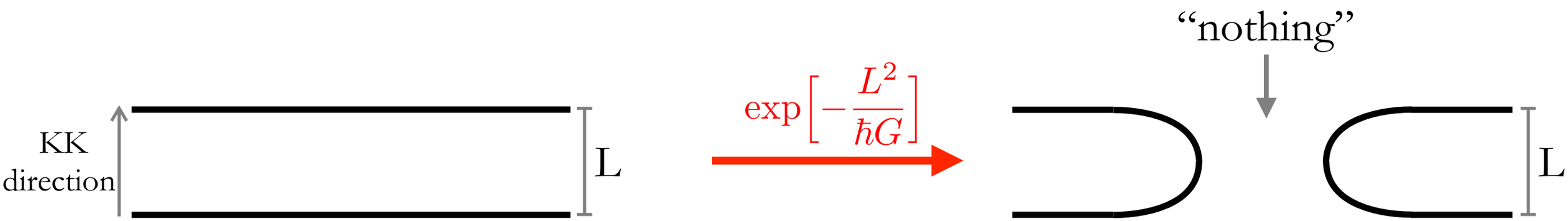

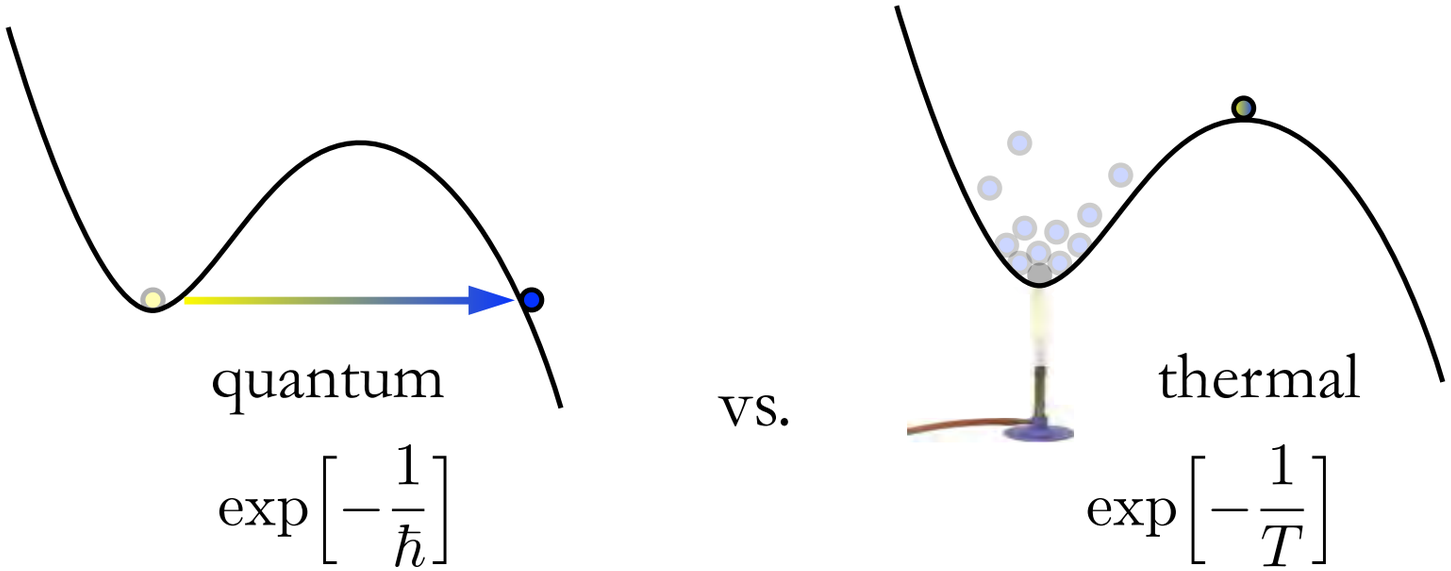

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what are the decays of hot KK space?

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3,1 1

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(Gross-Perry-Yaffe, 1982)

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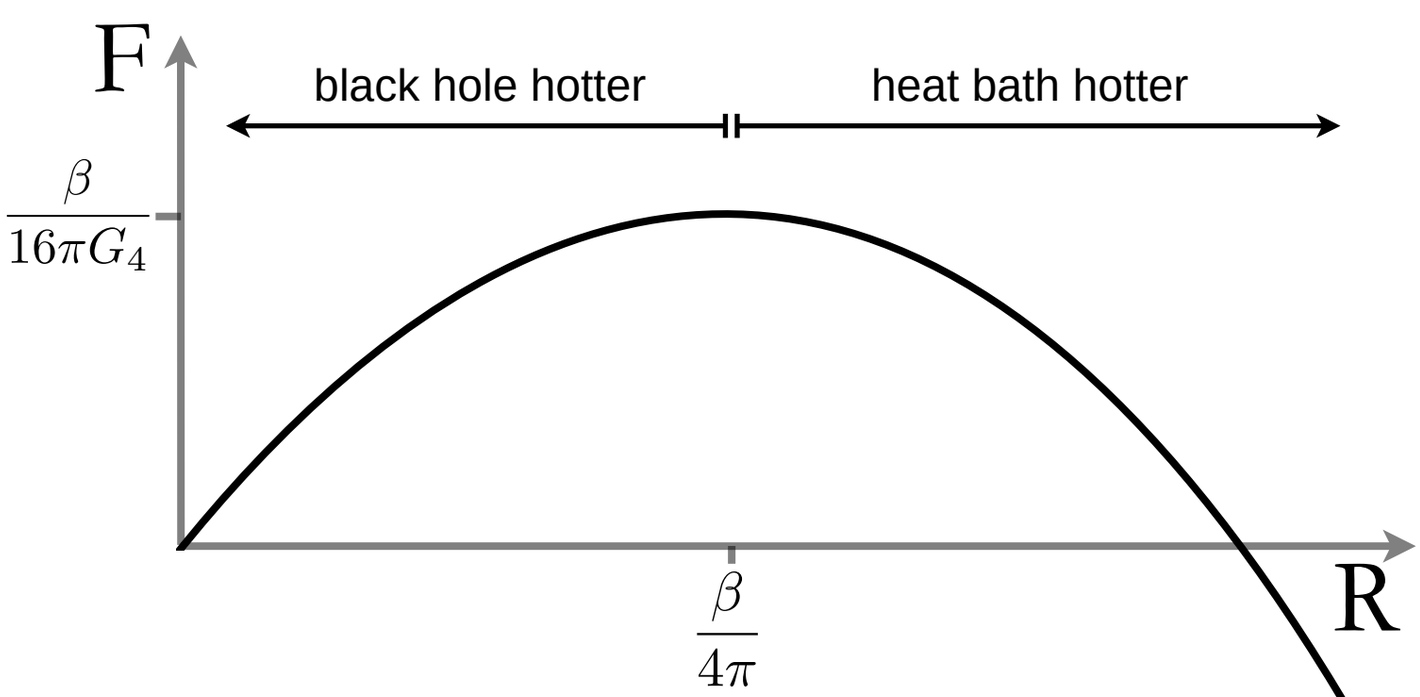
$$F \equiv m - TS = \frac{1}{2G}\left(R - \frac{2\pi T R^2}{\hbar}\right)$$



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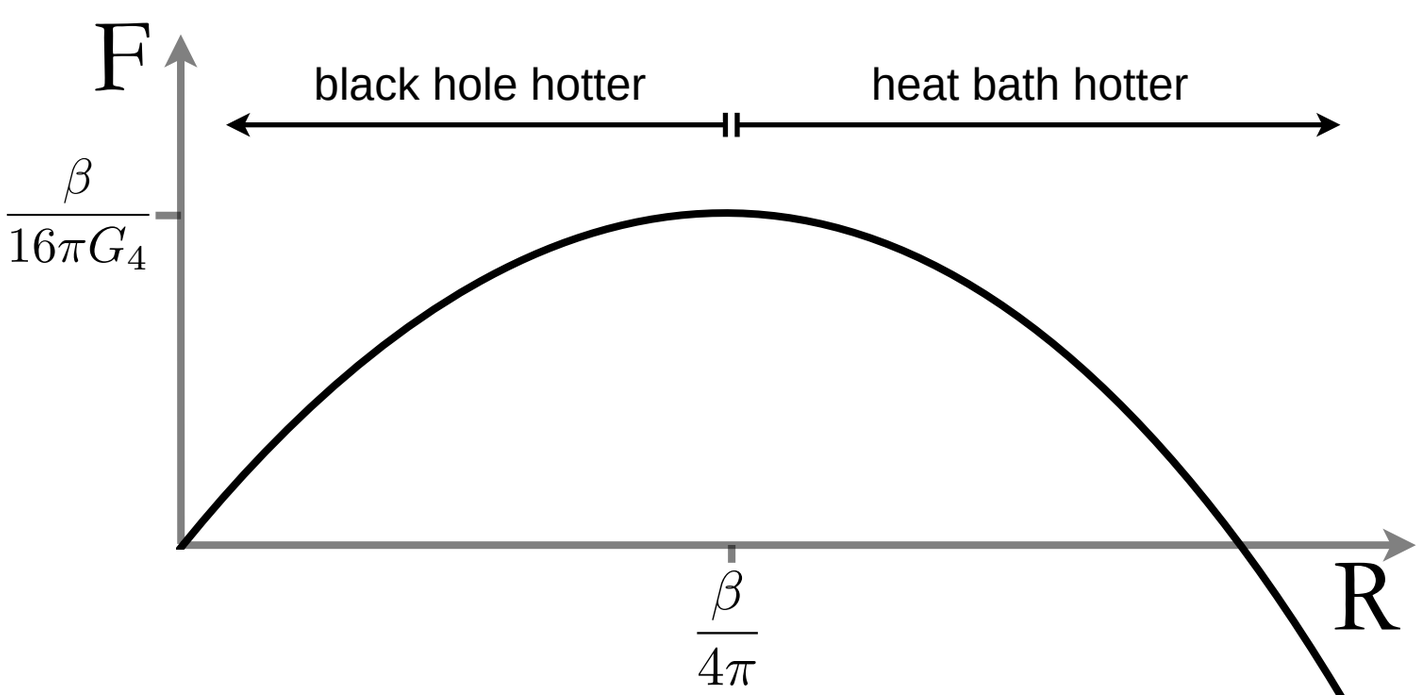

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$$R_{\rm BH} \sim \beta$$

$$R_{\rm Jeans} \sim \beta^2/\ell_4$$

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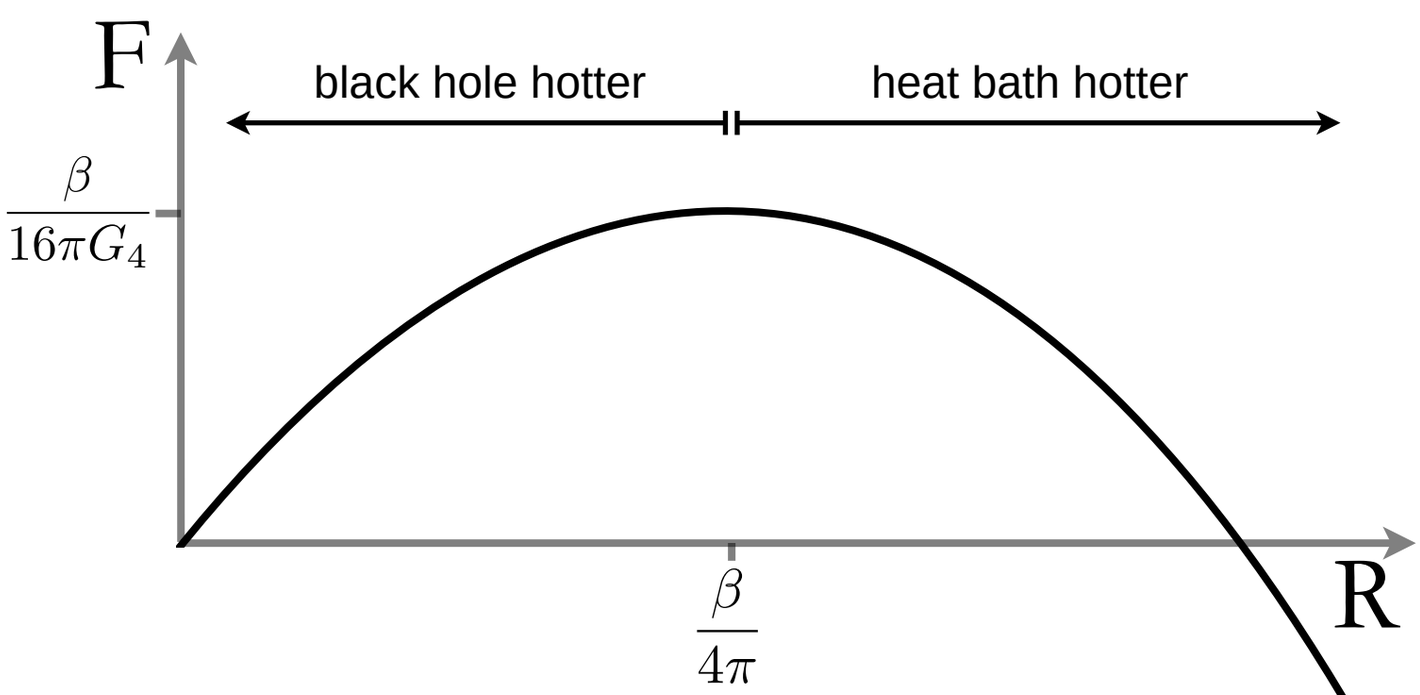

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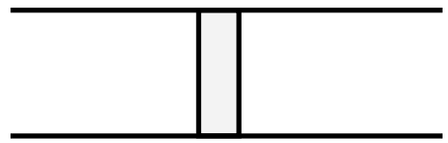
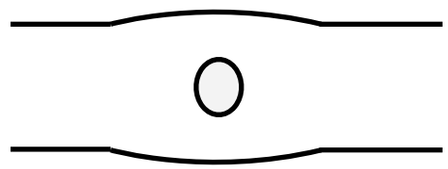
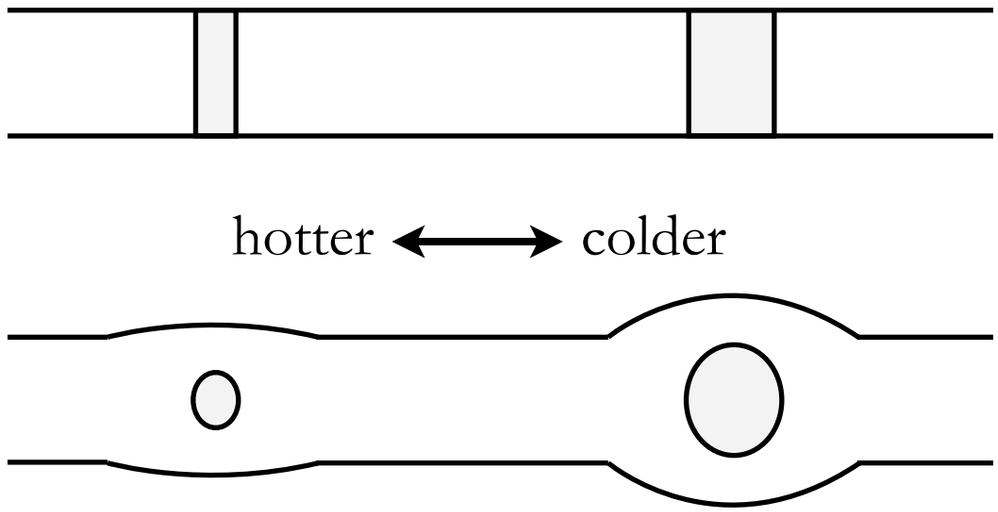
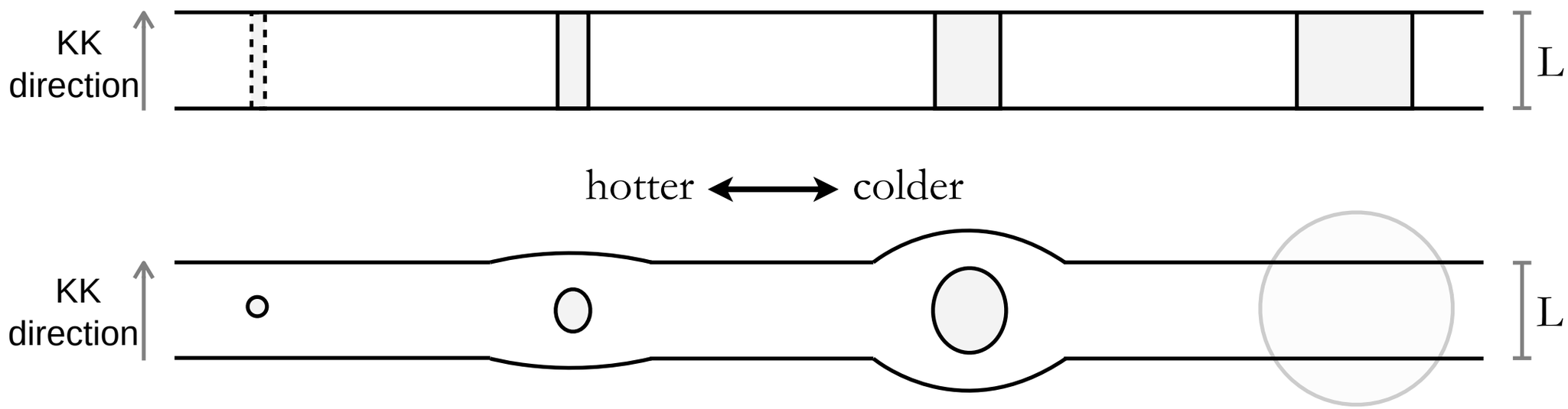


black string



‘caged’ black hole


hotter
colder


KK
direction
L
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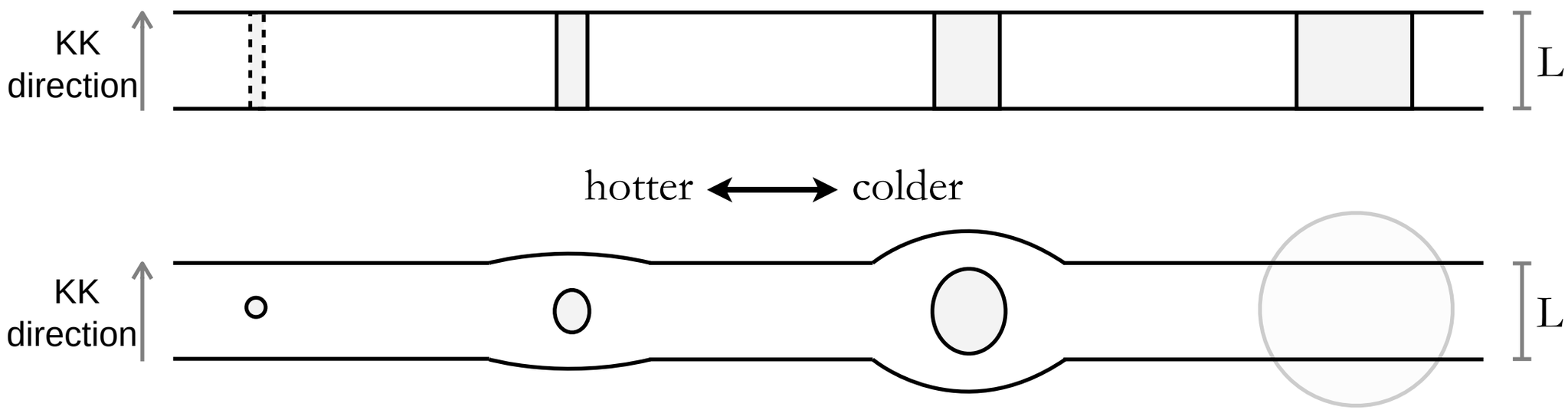
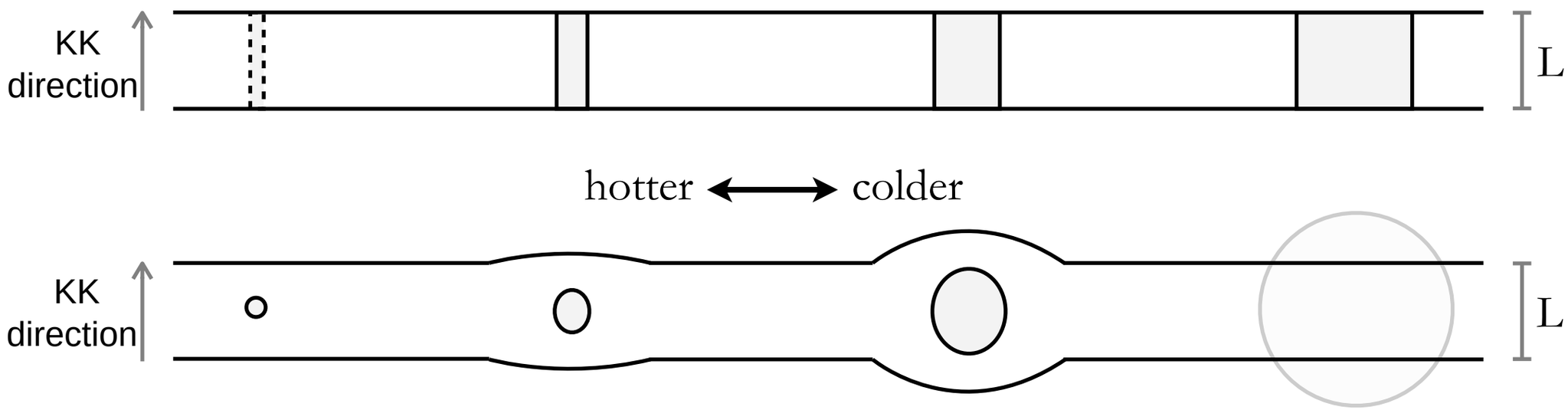
KK
direction
L



$$F(\text{black string}) \equiv m - TS = \frac{1}{16\pi}\frac{\beta L}{G_5}$$

$$F(\text{'caged' black hole}, L < \infty) = \frac{1}{32\pi}\frac{\beta^2}{G_5}\left(1 - \frac{\beta^2}{16L^2} + \frac{\beta^4}{128L^4} + \ldots\right)$$

Kol, Harmark, etc.



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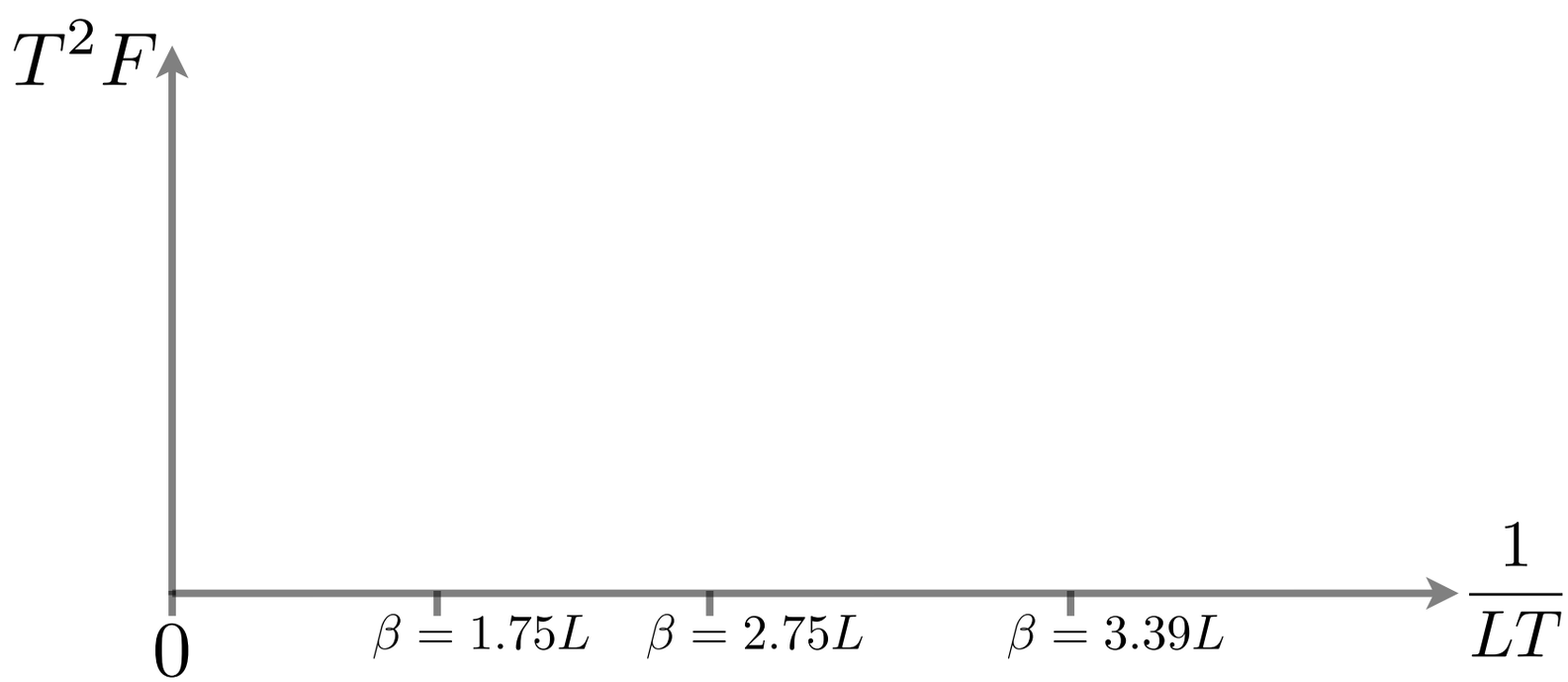
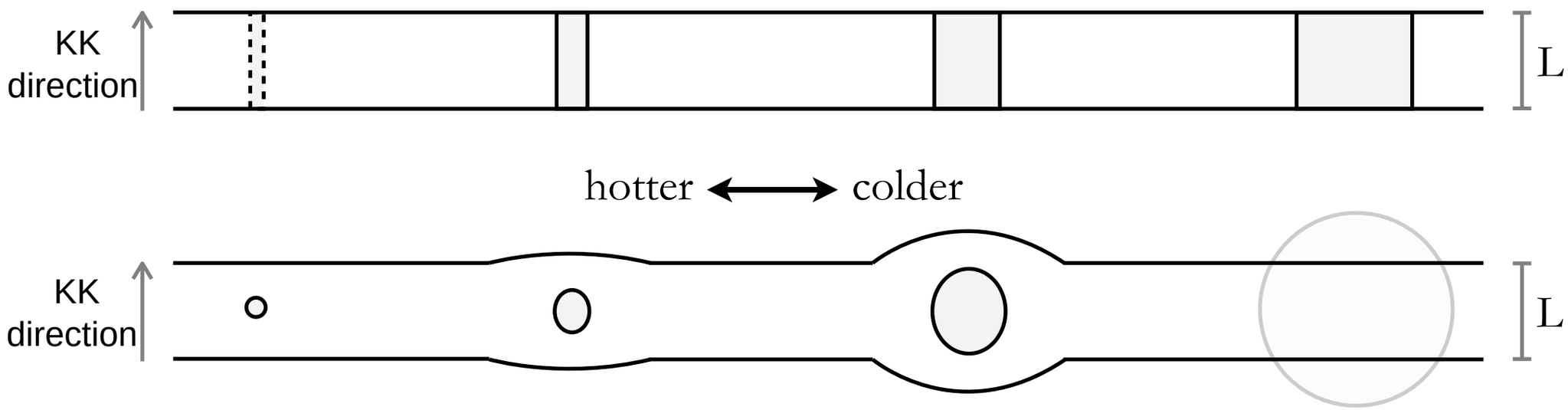


Wiseman, Kitchen Headrick, Kol, Piran, Horowitz, Elvang, etc.



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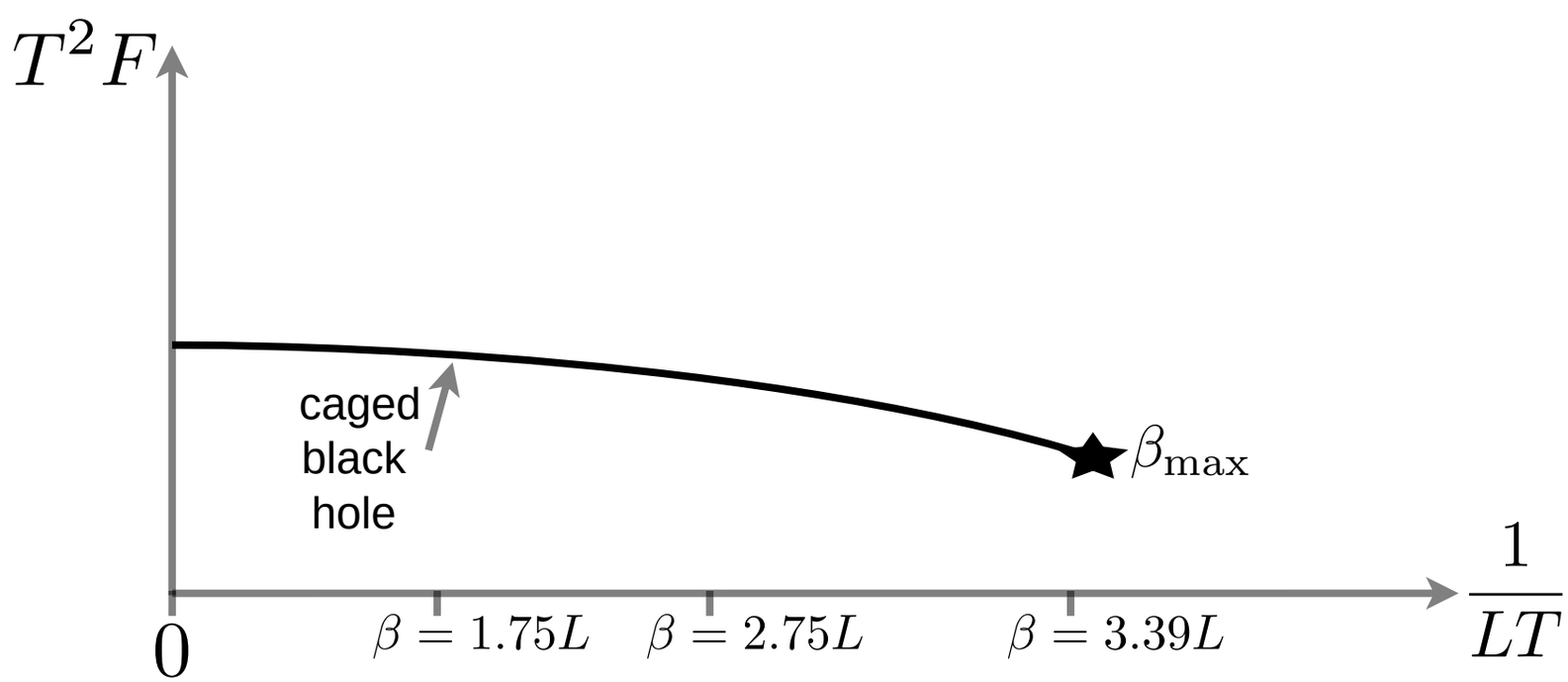

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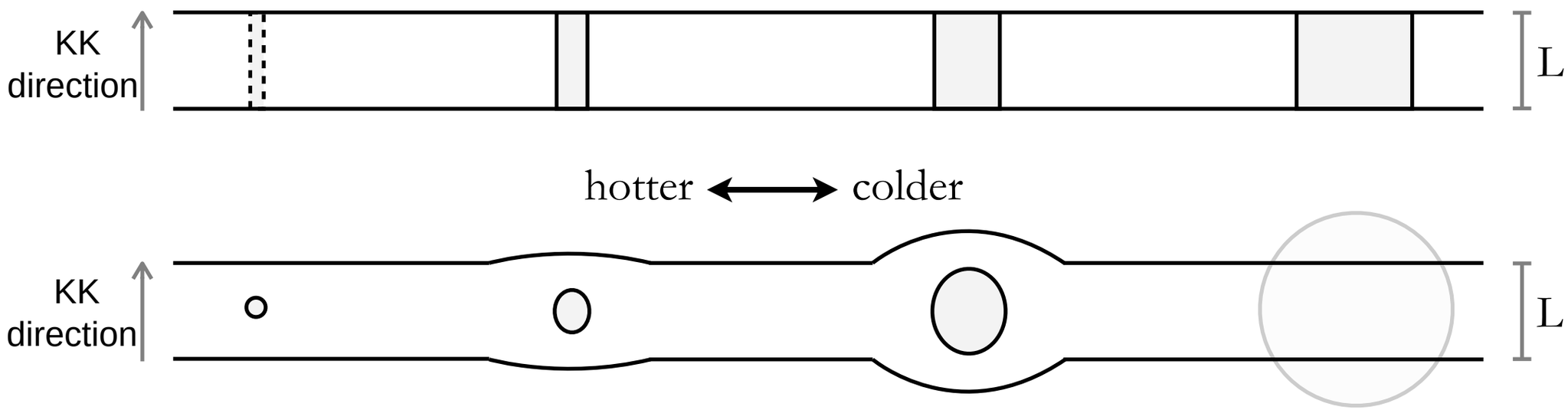

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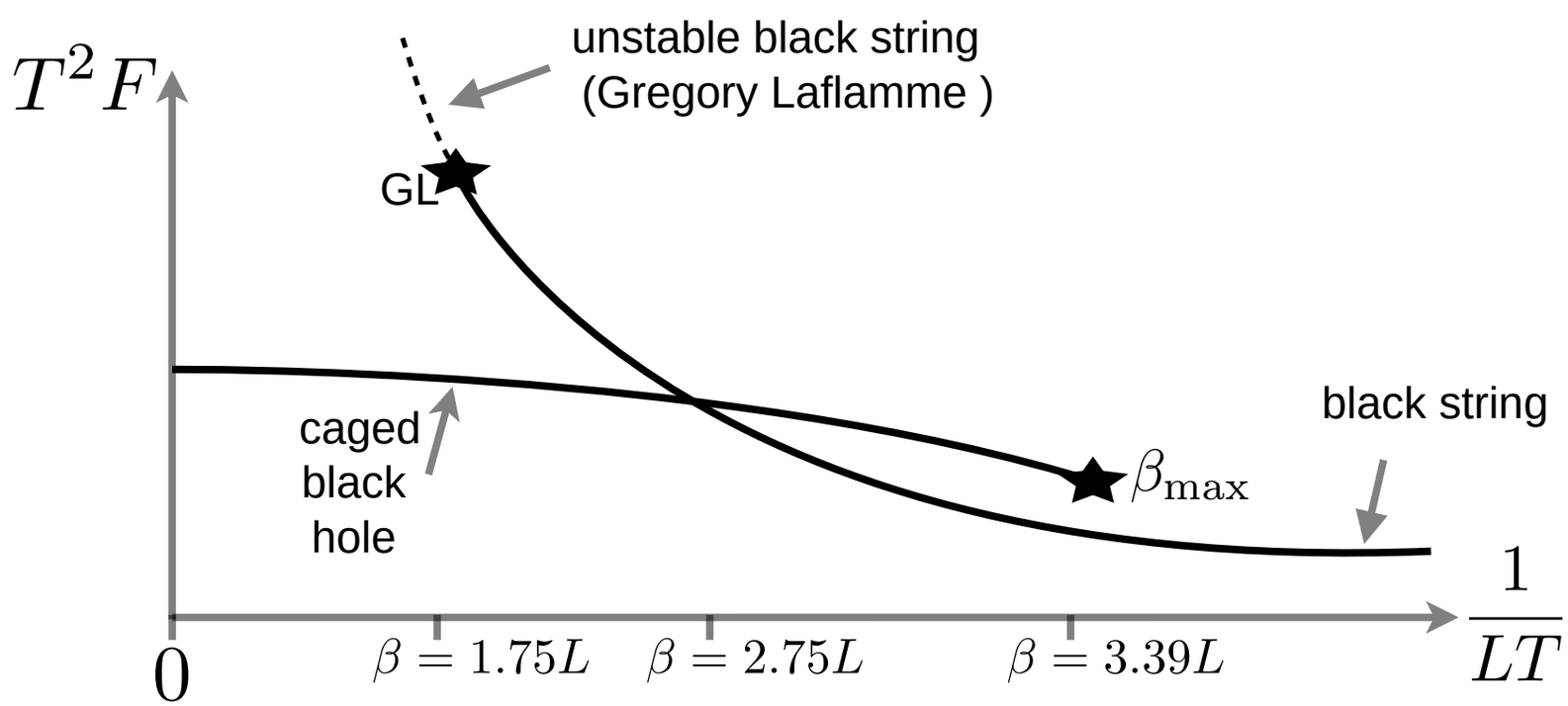

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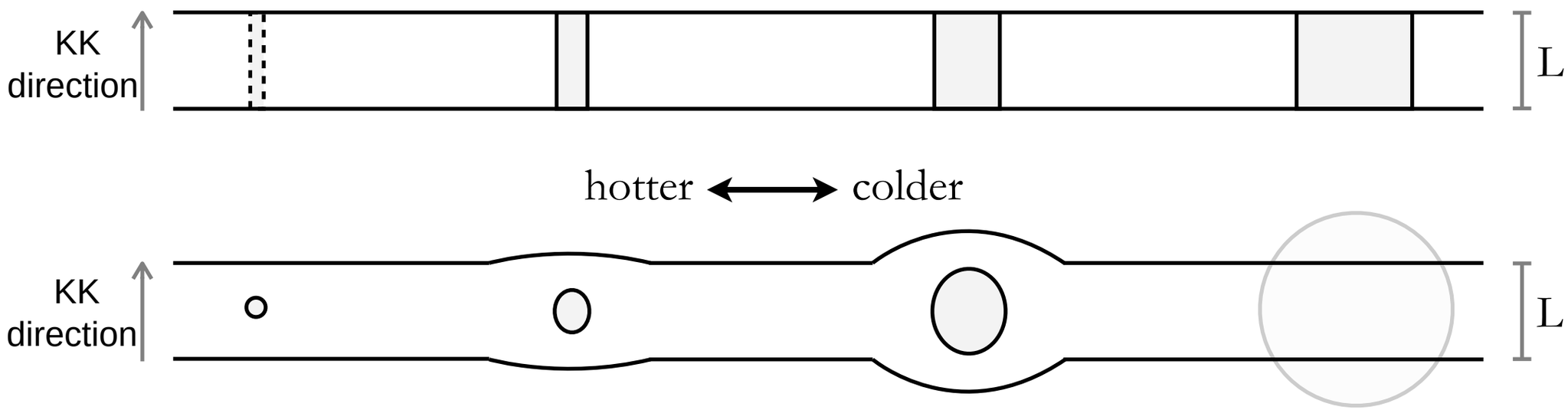

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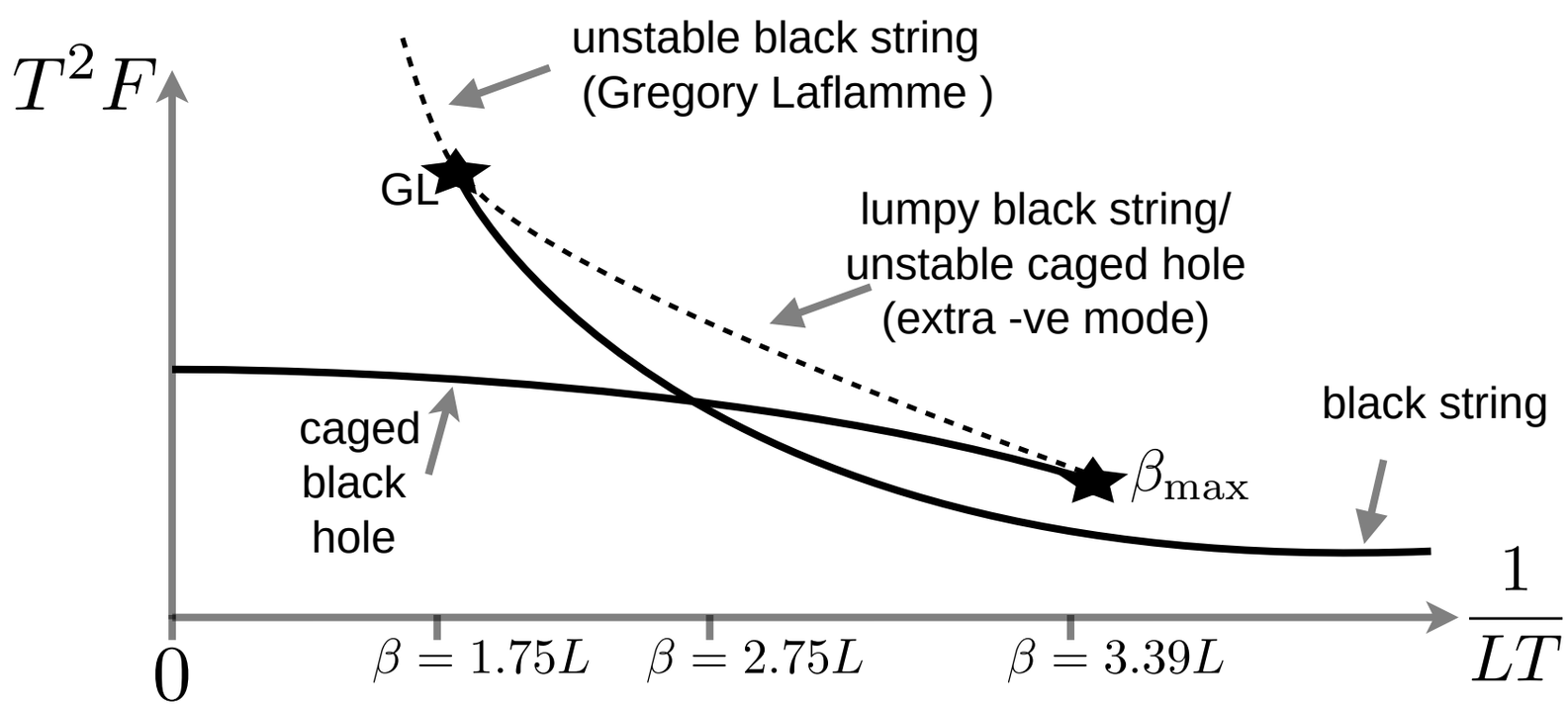

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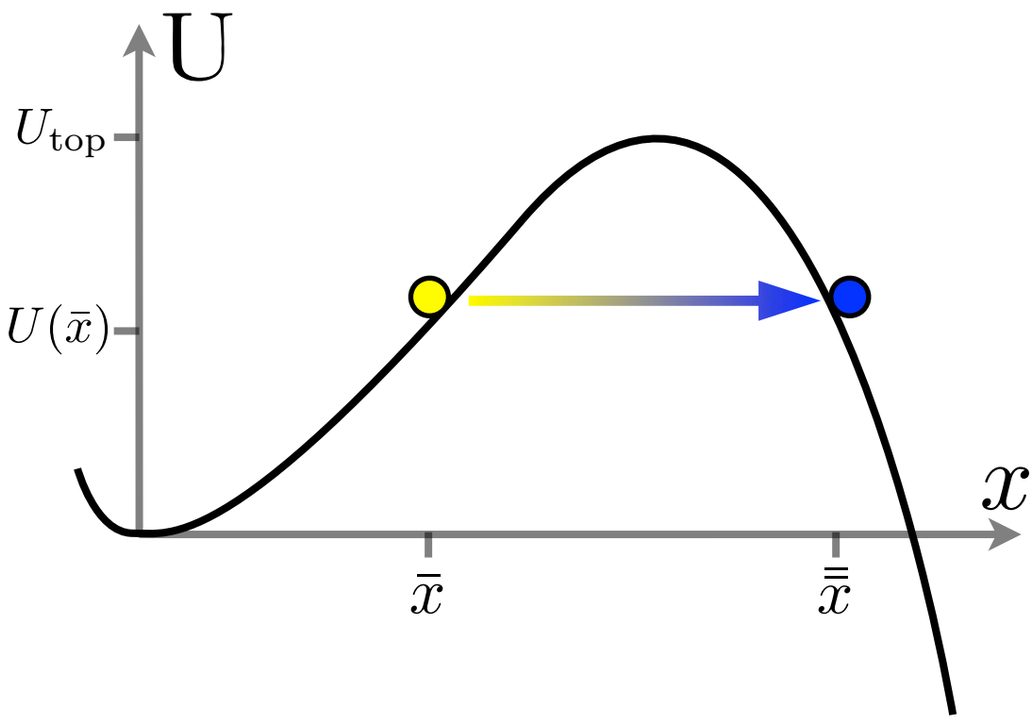
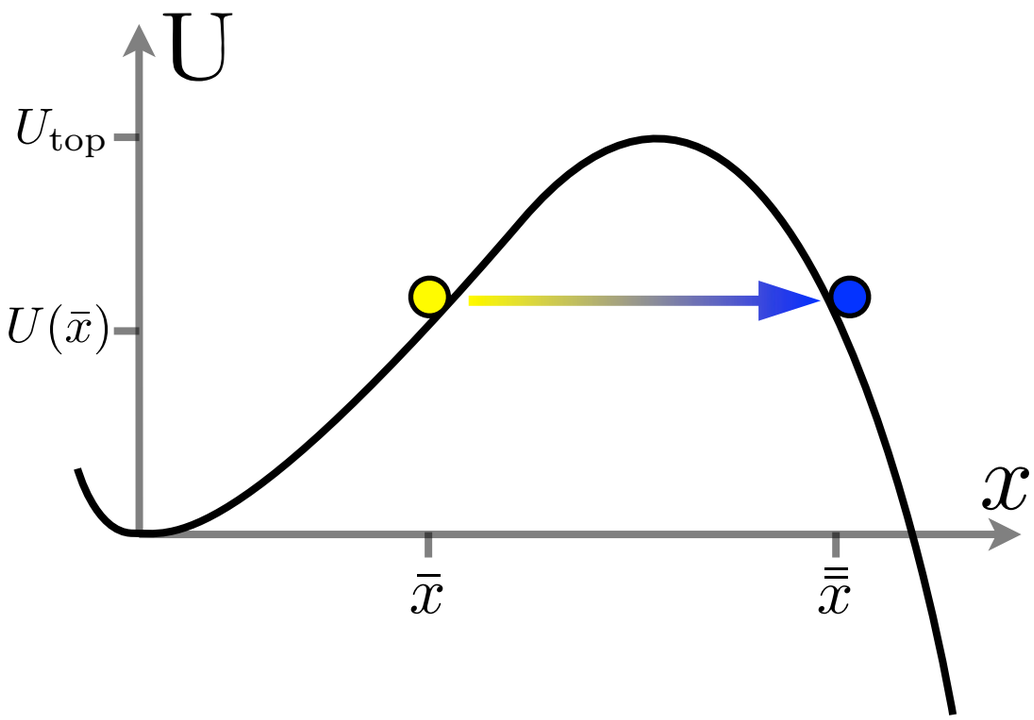

U
U_{top}
$U(\bar{x})$
x
$\bar{x}$
$\bar{\bar{x}}$



$$\sim \exp\left[-\frac{1}{T}U(\bar{x}) - \frac{2}{\hbar}\int_{\bar{x}}^{\bar{\bar{x}}} dx\sqrt{2[U(x)-U(\bar{x})]}\right]$$

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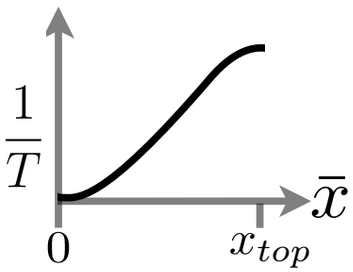

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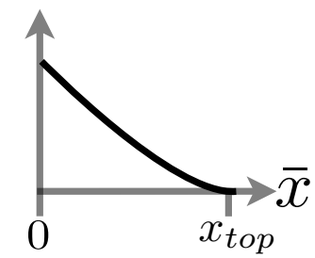
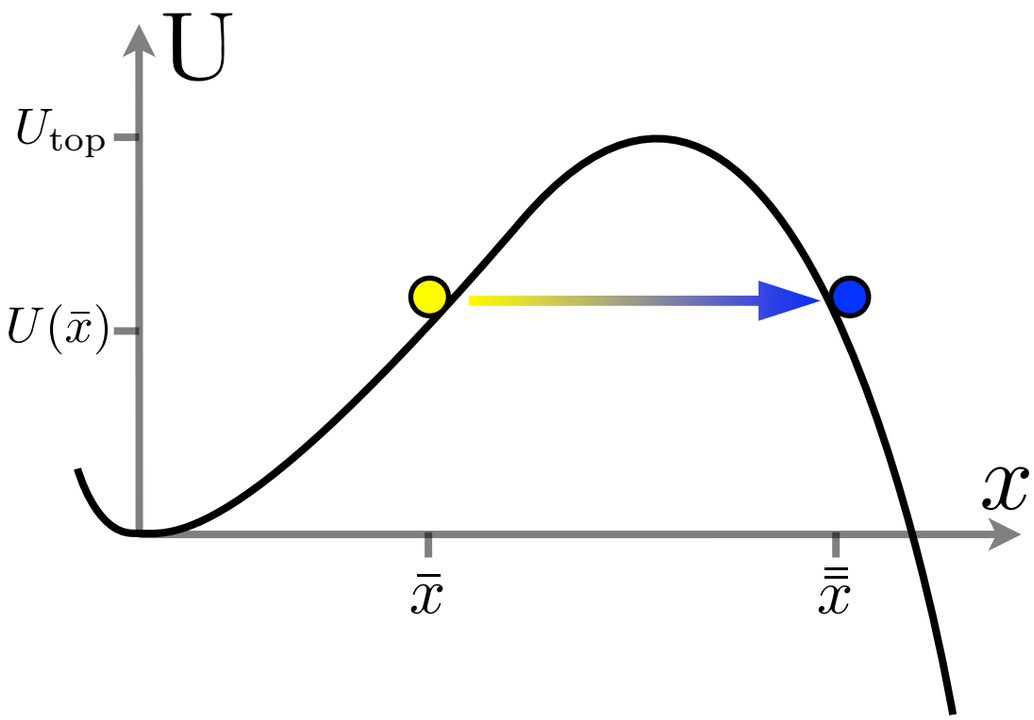


$$\Bigg]$$

$$\sim \exp\left[-I_E\right]$$



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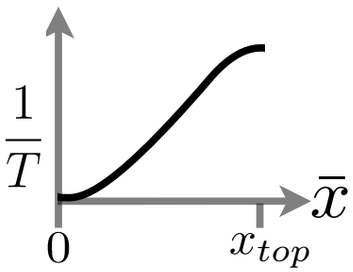

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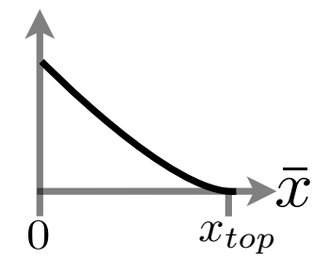

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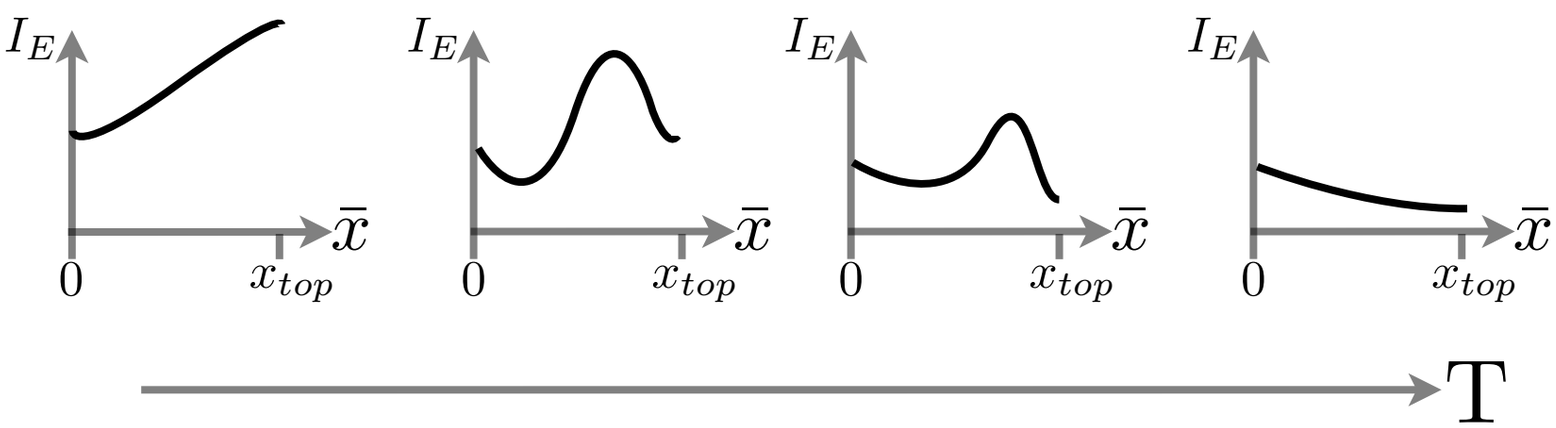

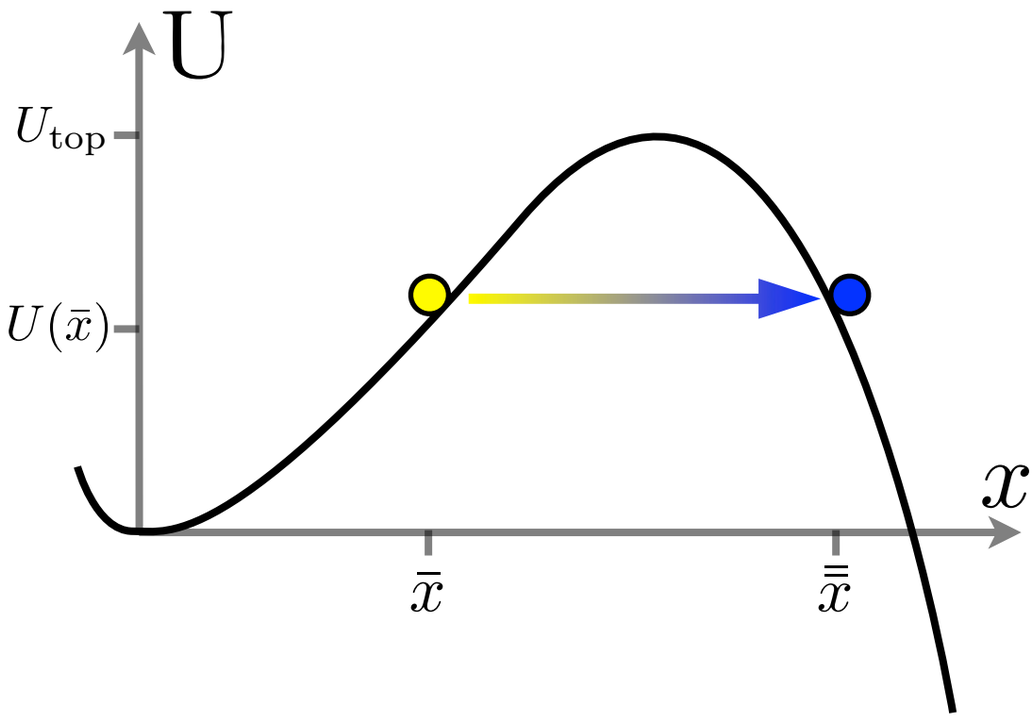

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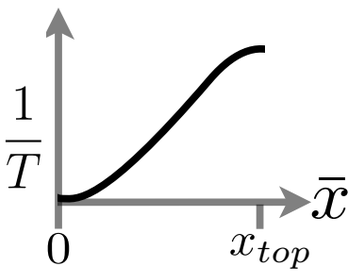
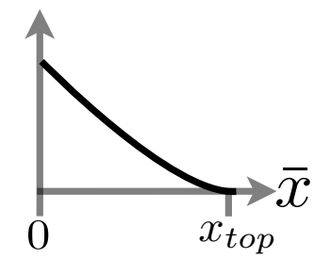
$\sim \exp\left[-\frac{1}{T}\right.$  $-$  $\left.\right]$

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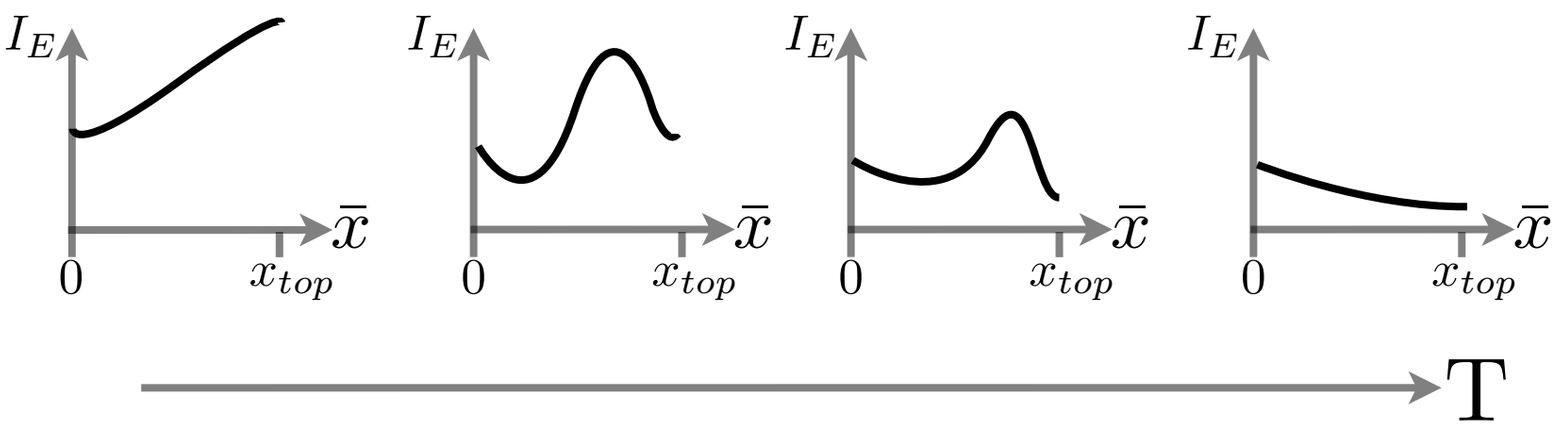
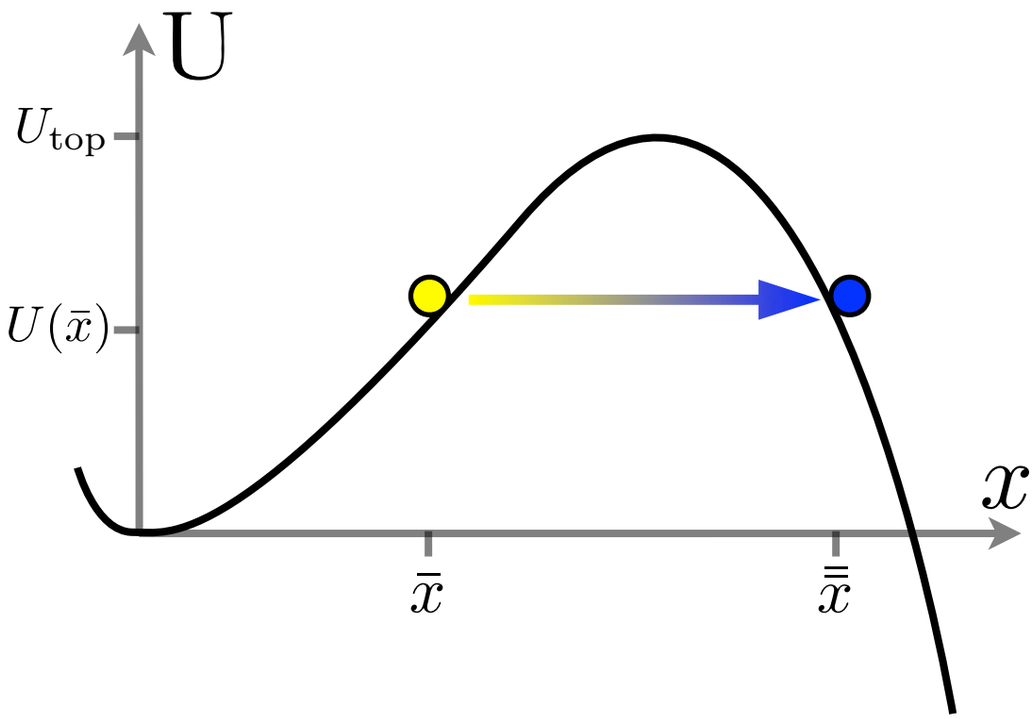


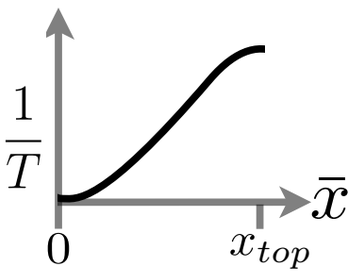
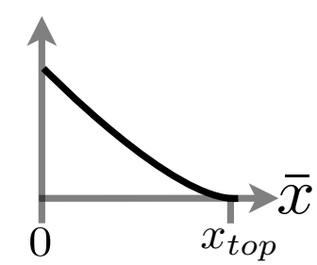
$$\frac{\partial \Gamma}{\partial \bar{x}} = 0$$

$$\frac{1}{T} = \frac{2}{\hbar}\int_{\bar{x}}^{\bar{\bar{x}}} \frac{dx}{\sqrt{2[U(x)-U(\bar{x})]}}$$

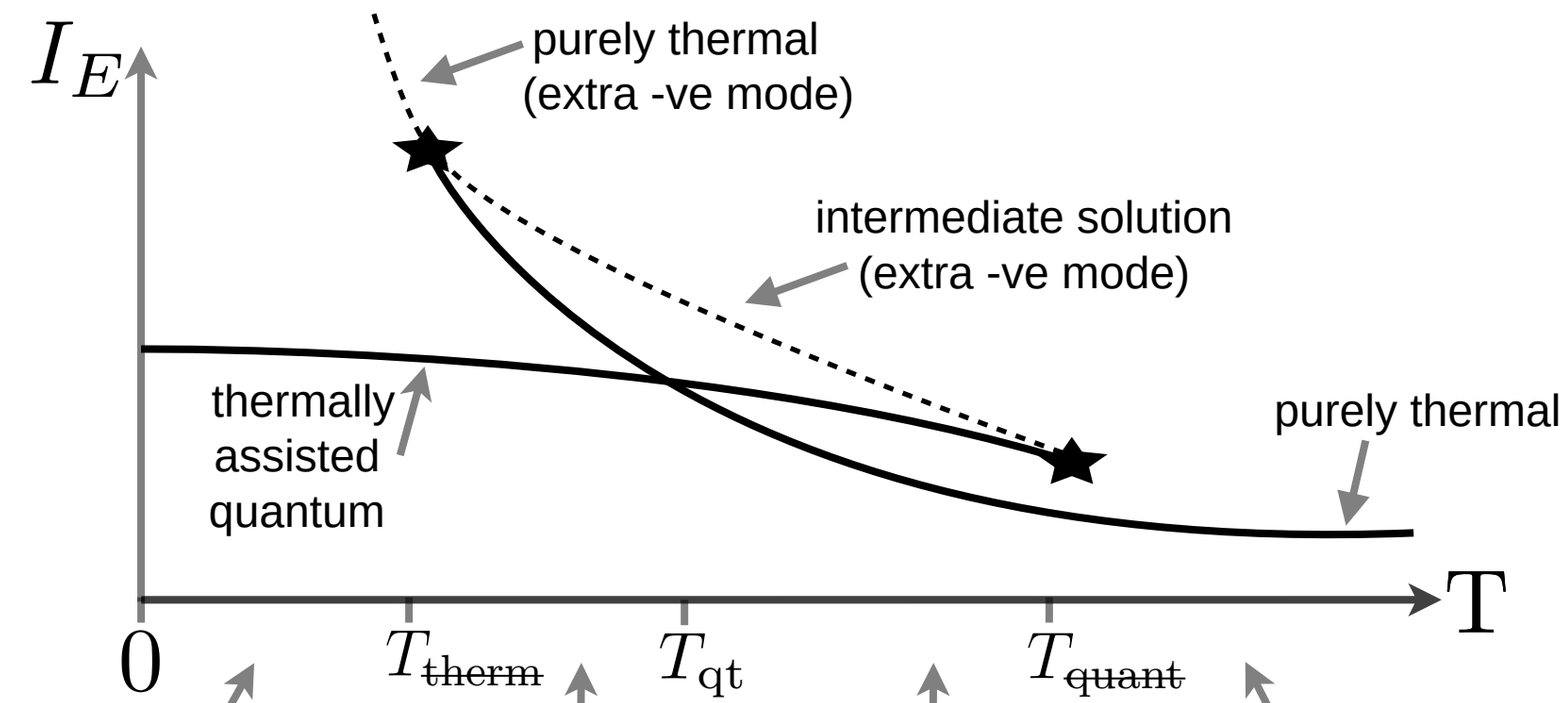
= Euclidean period

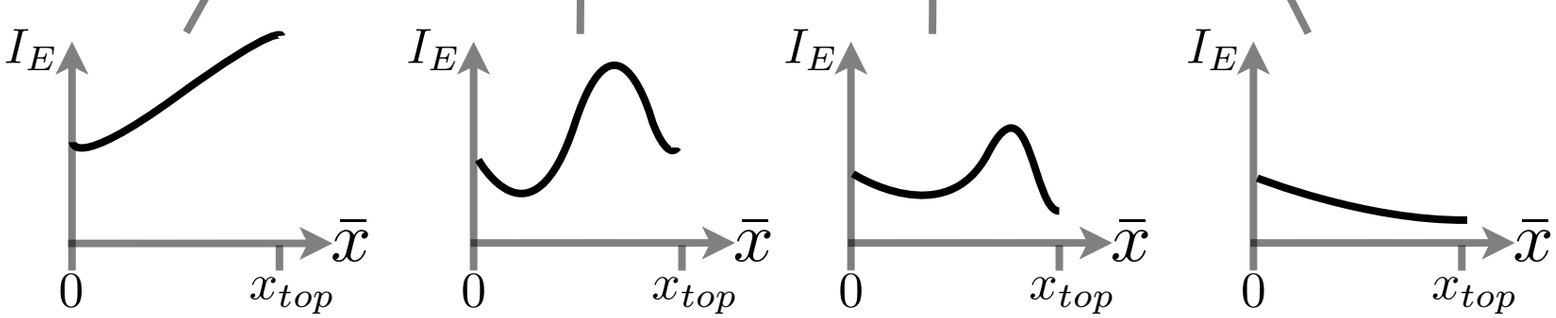


$$\sim \exp\left[-\frac{1}{T}U(\bar{x}) - \frac{2}{\hbar}\int_{\bar{x}}^{\bar{\bar{x}}} dx\sqrt{2[U(x)-U(\bar{x})]}\right]$$

$\sim \exp\Big[-\frac{1}{T}$  $-$  $\Big]$

$$\sim \exp\left[-I_E\right]$$



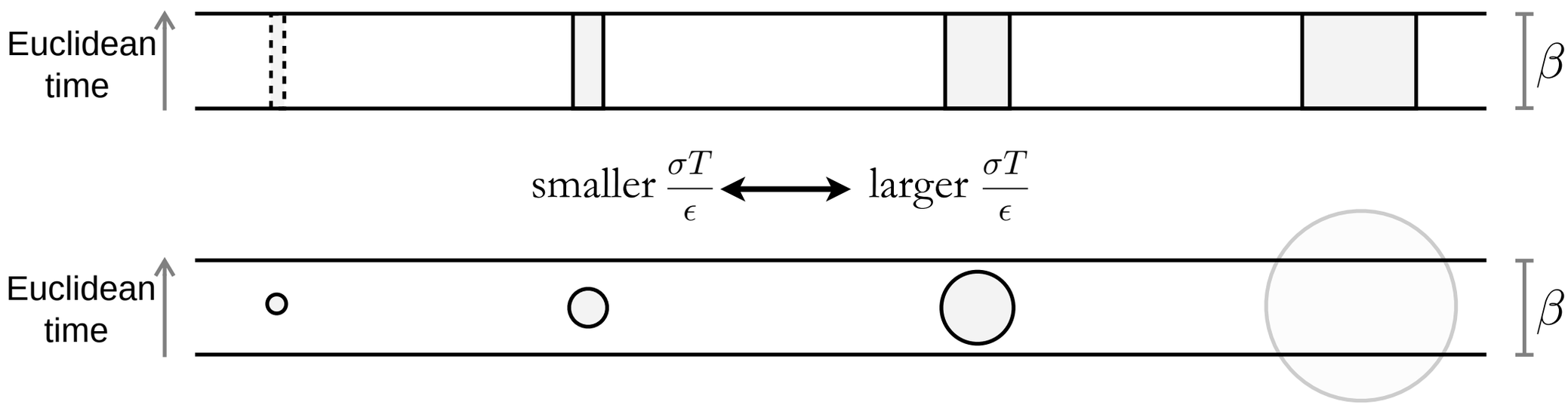


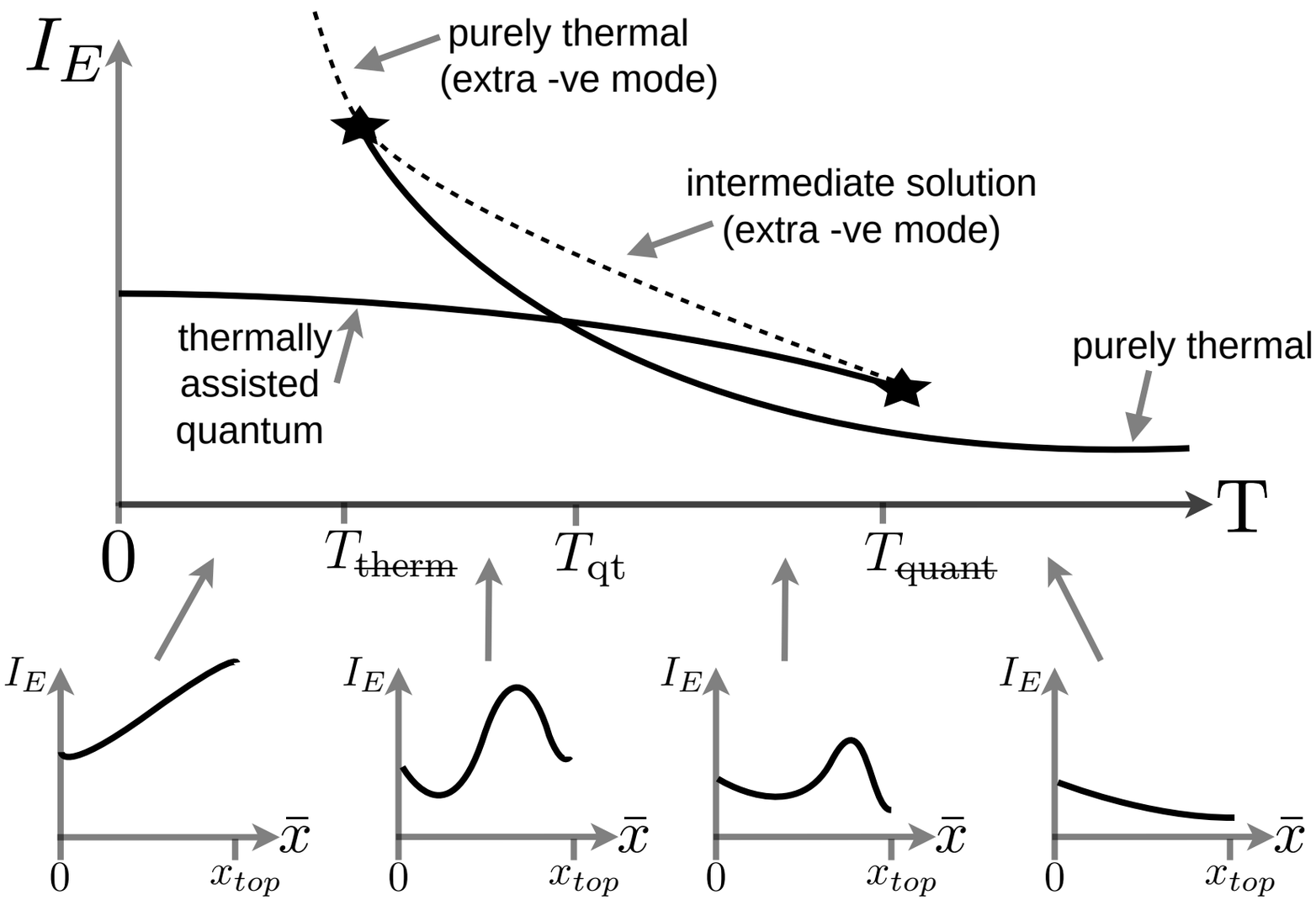
$$\frac{\partial\Gamma}{\partial\bar{x}} = 0$$

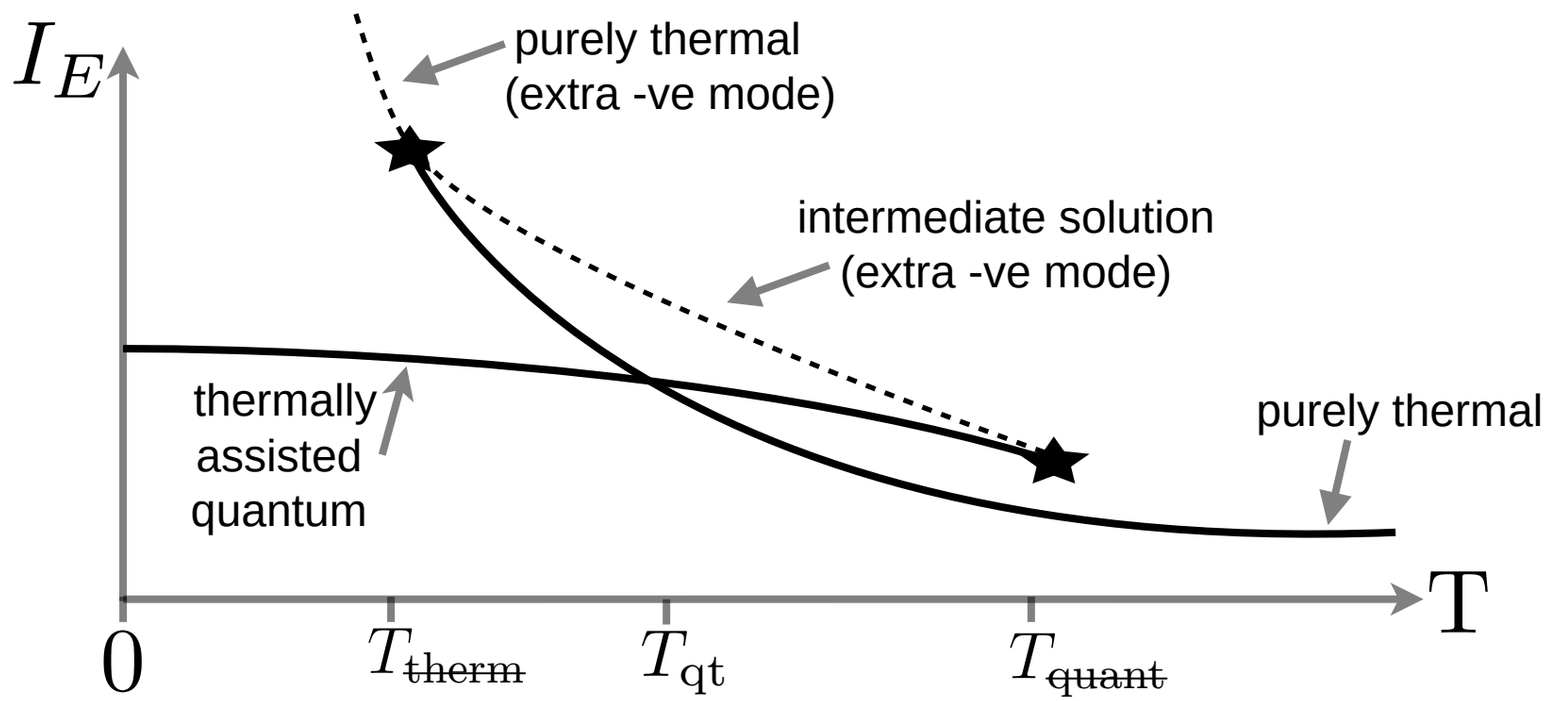
$$\frac{1}{T} = \frac{2}{\hbar}\int_{\bar{x}}^{\bar{\bar{x}}} \frac{dx}{\sqrt{2[U(x)-U(\bar{x})]}}$$

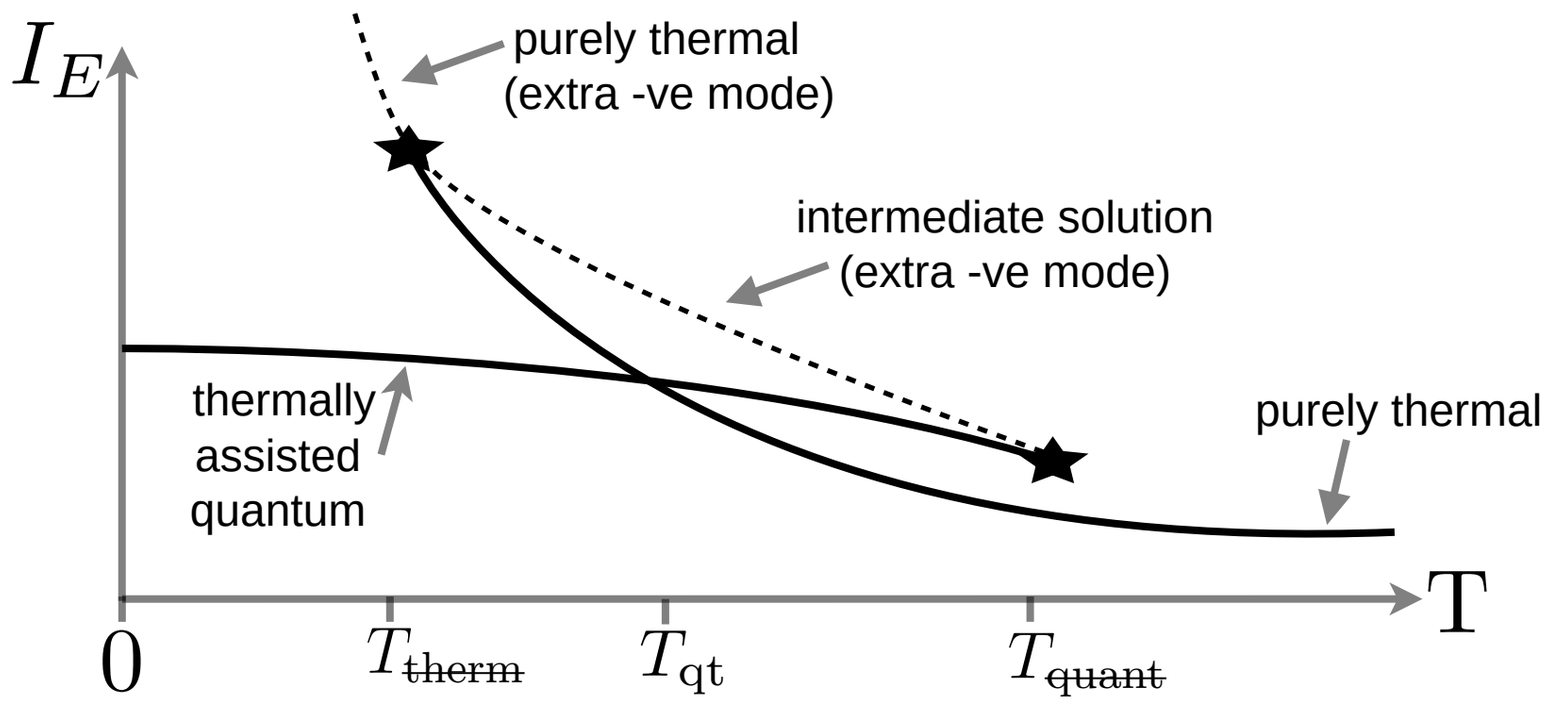
= Euclidean period

thermal QFT has same phase diagram as thermal QM (Linde, 1981):






I_E
purely thermal
(extra -ve mode)
intermediate solution
(extra -ve mode)
thermally
assisted
quantum
purely thermal
T
0
$T_{\cancel{\text{therm}}}$
T_{qt}
$T_{\cancel{\text{quant}}}$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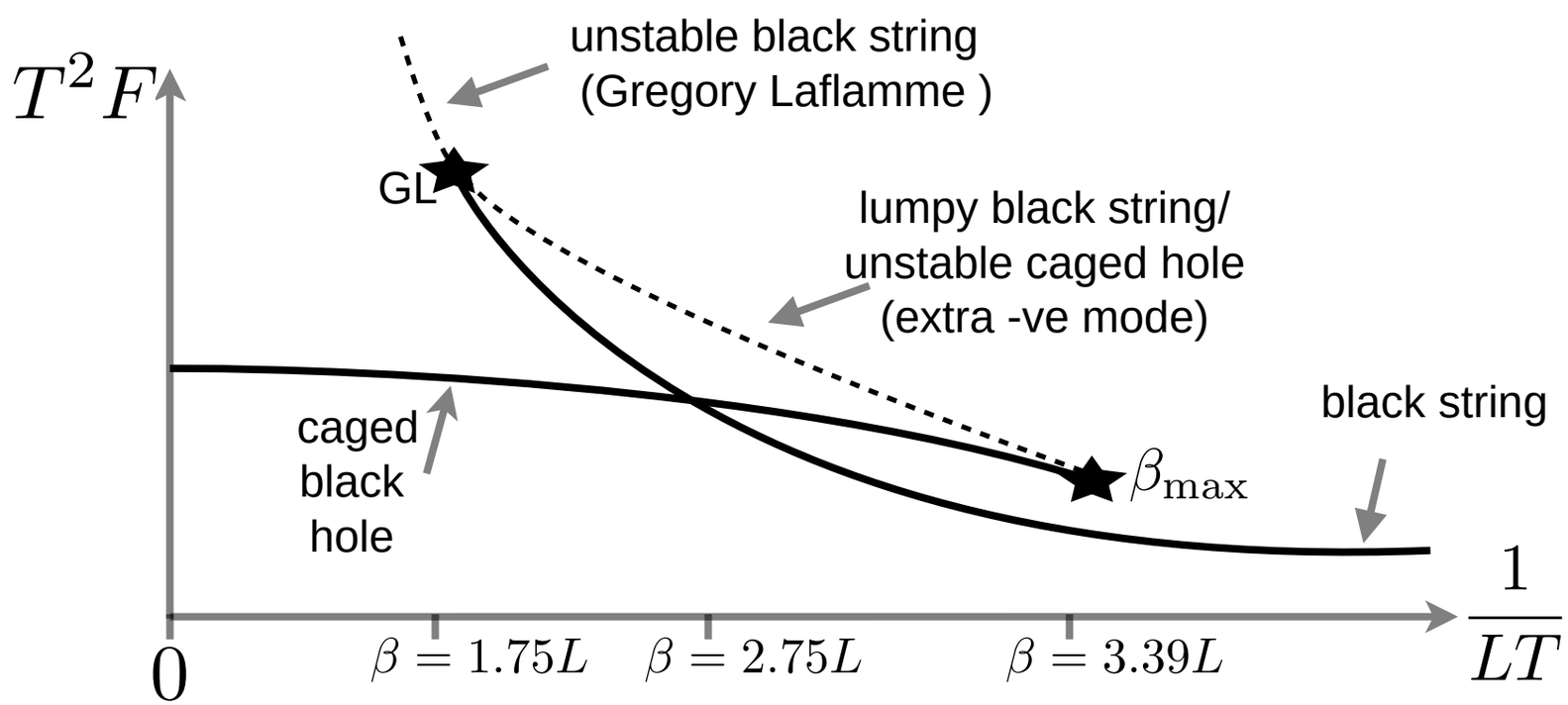
$T^2 F$
unstable black string
(Gregory Laflamme)
GL
lumpy black string/
unstable caged hole
(extra -ve mode)
caged
black
hole
$\beta_{\max}$
black string
$\frac{1}{LT}$
0
$\beta = 1.75L$
$\beta = 2.75L$
$\beta = 3.39L$

what are the decays of hot KK space?

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instanton asymptotically $R^3 \times S^1 \times S^1$ ← length β

length L

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length L

Euclidean black string

$$ds^2 = (1 - \frac{R}{r})dw^2 + \frac{dr^2}{1 - \frac{R}{r}} + r^2 \left(d\theta^2 + \cos^2\theta d\phi^2\right) + dz^2$$

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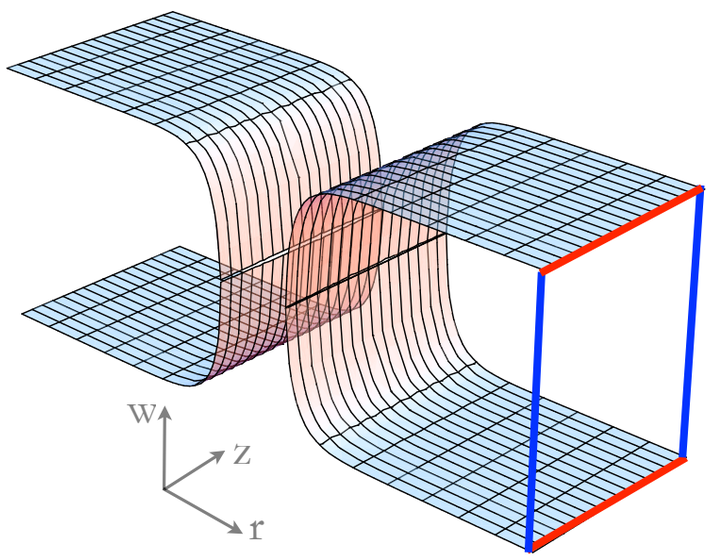

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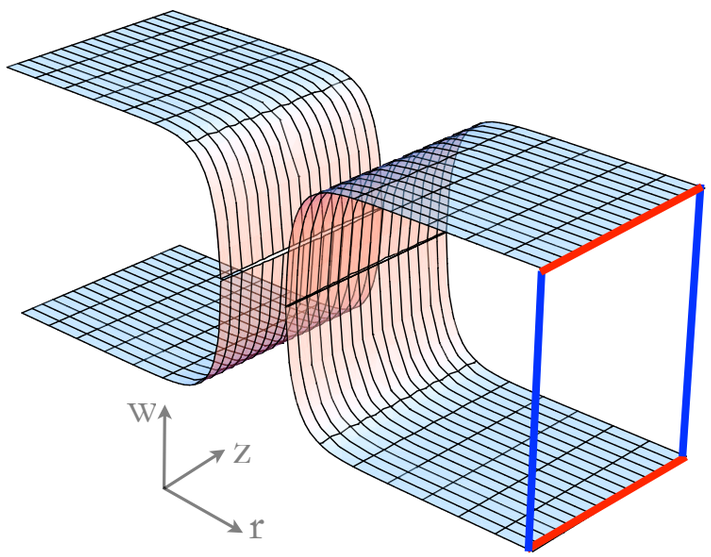
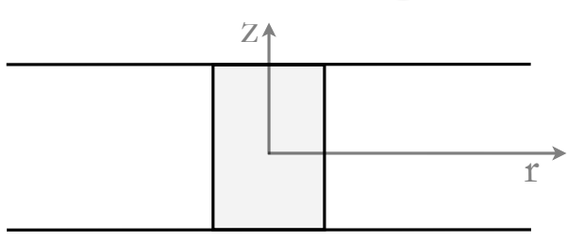


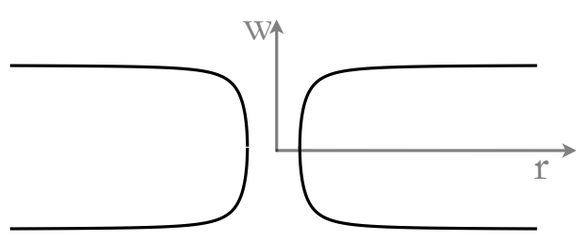
$w \to it$

black string



$z \to it$

thermal BoN



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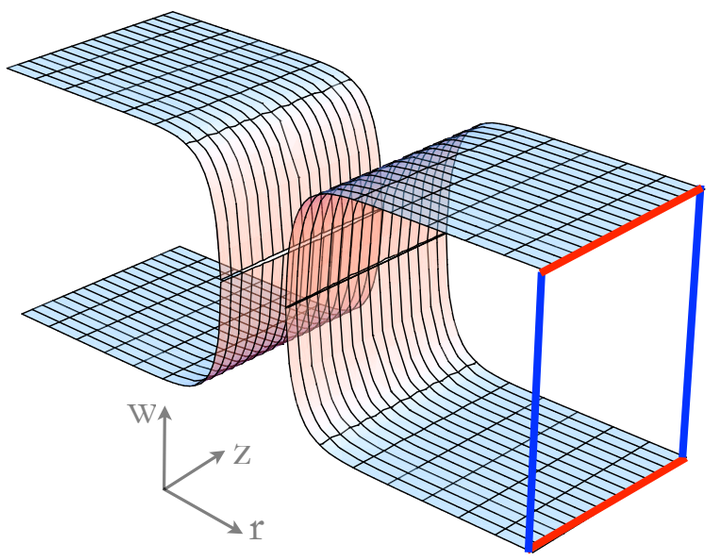
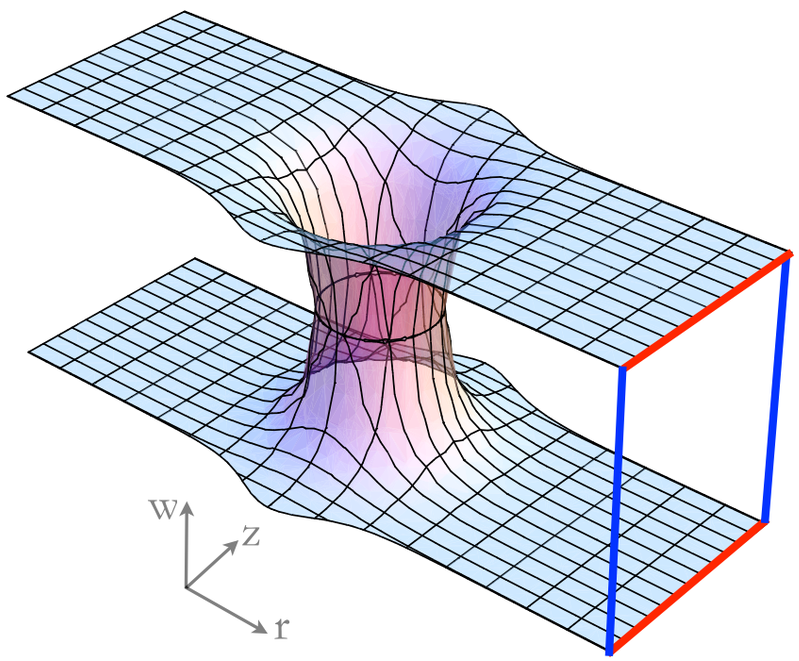
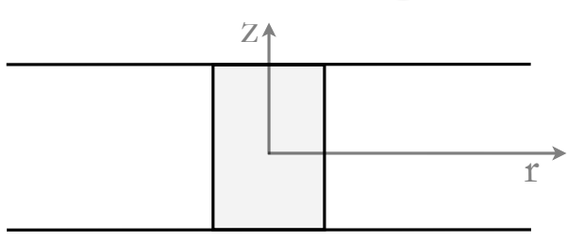


Euclidean black hole



$w \to it$

black string



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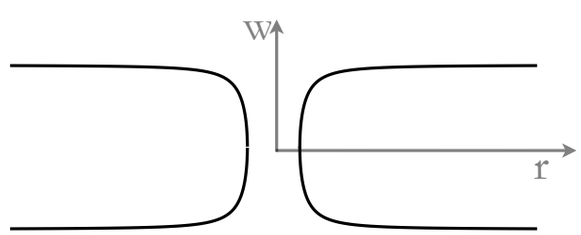

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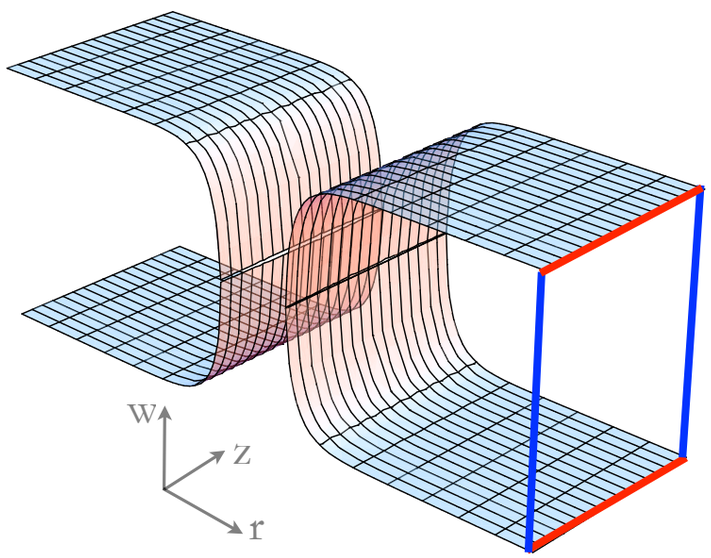

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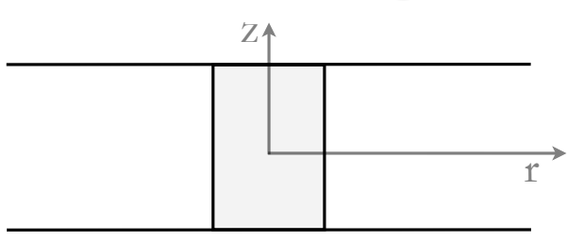

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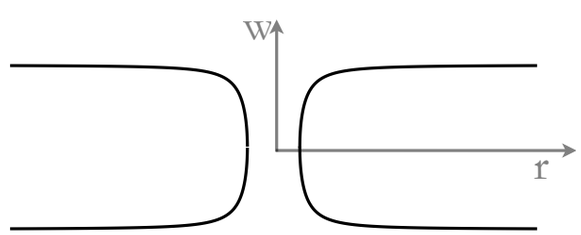

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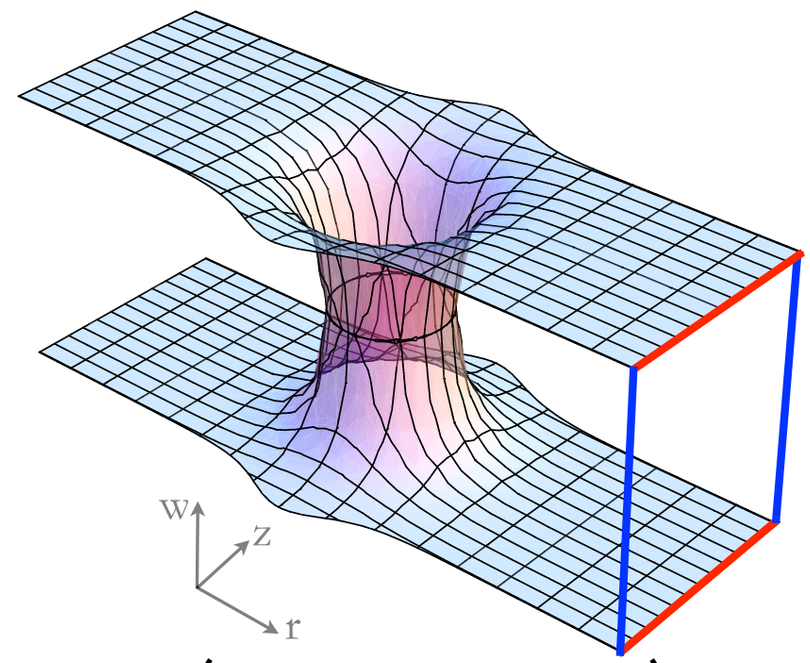
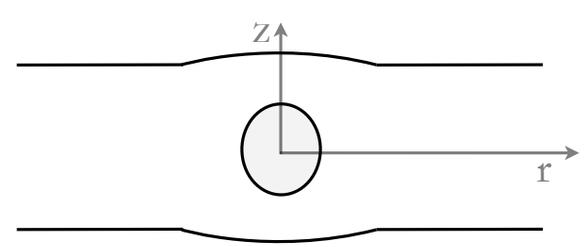


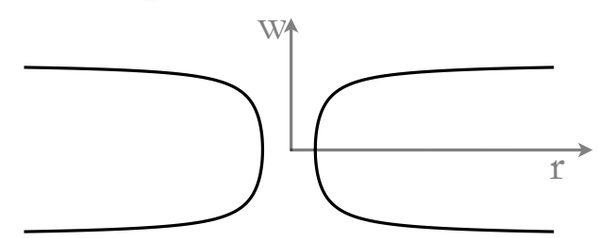
$w \to it$

black hole



$z \to it$

quantum BoN



$$\frac{\beta}{L} \longleftrightarrow \frac{L}{\beta}$$

	T = 0	T > 0
no extra dimension	stable	black hole (GPY, 1982)
compact extra dimension	quantum BoN (Witten, 1982)	thermal BoN ↔ black string

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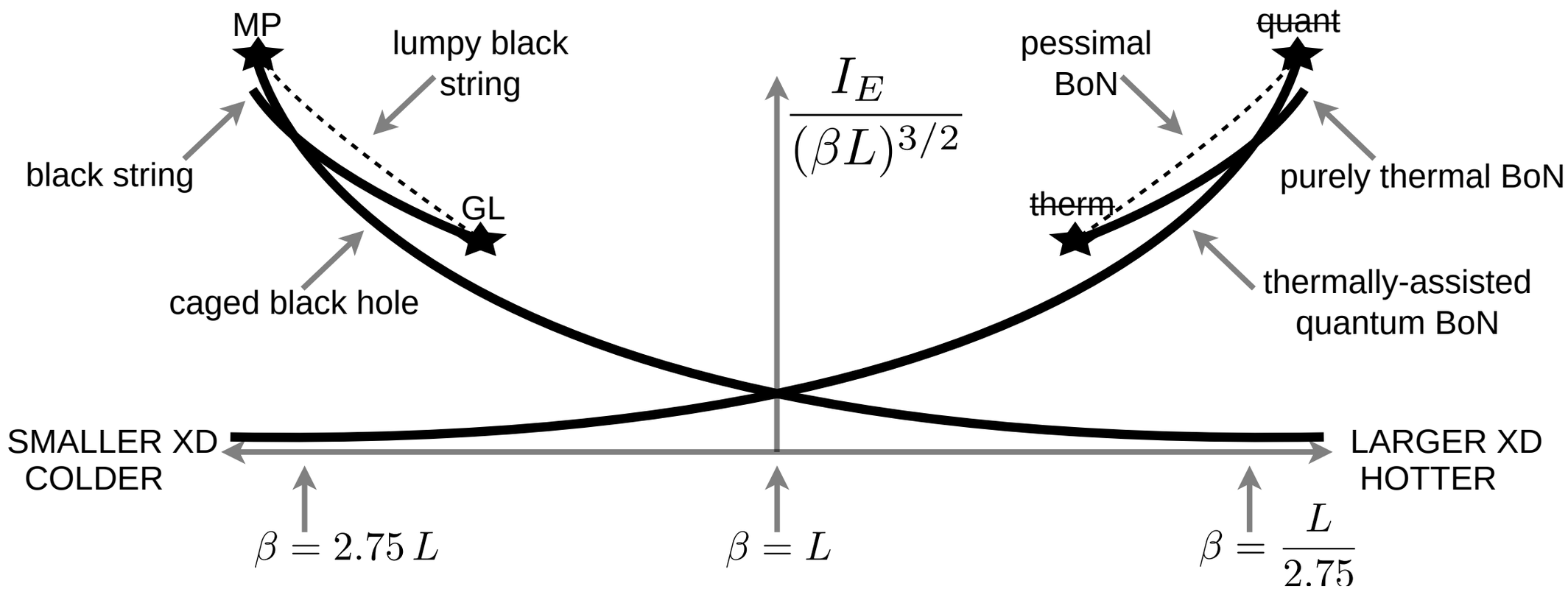
decay	instanton	rate
'quantum' bubble of nothing	black hole	$\exp\left[-\frac{1}{32\pi}\frac{L^3}{\ell_5^3}\left(1-\frac{L^2}{16\beta^2}+\frac{L^4}{128\beta^4}+\dots\right)\right]$
'thermal' bubble of nothing	black string	$\exp\left[-\frac{1}{16\pi}\frac{\beta L^2}{\ell_5^3}\right]$
4D black hole	black string	$\exp\left[-\frac{1}{16\pi}\frac{\beta^2 L}{\ell_5^3}\right]$
5D black hole	black hole	$\exp\left[-\frac{1}{32\pi}\frac{\beta^3}{\ell_5^3}\left(1-\frac{\beta^2}{16L^2}+\frac{\beta^4}{128L^4}+\dots\right)\right]$



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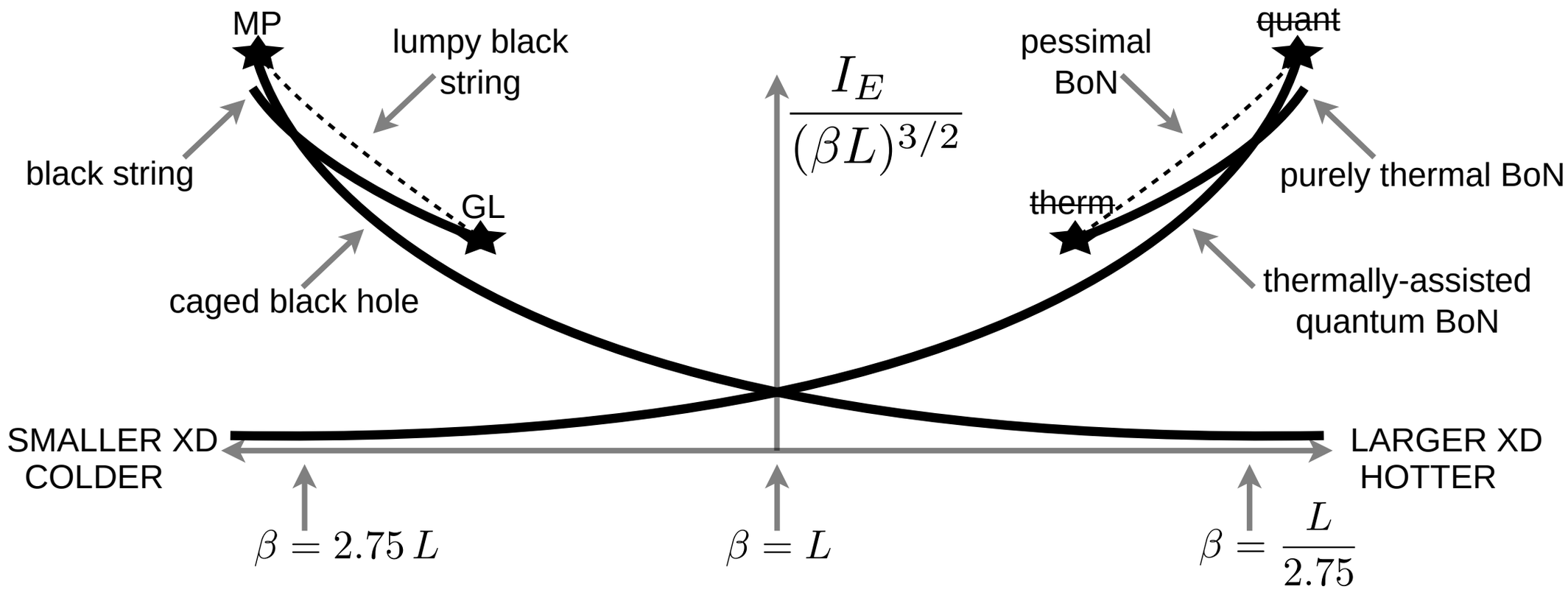

mechanical instability of thermal BoN	thermodynamic instability of black string in heat bath
thermal 'assist' of quantum BoN	attraction between image black holes
quantum BoN disappears at $T_{\cancel{\text{quant}}}$ $L = (3.39\ldots)\beta$	black hole ceases to exist at Merger Point $\beta = (3.39\ldots)L$
exchange of dominance between thermal and quantum BoN $L = (2.75\ldots)\beta$	exchange of dominance between black string and black hole $\beta = (2.75\ldots)L$
thermal BoN no longer locally optimal path across barrier $L = (1.7524\ldots)\beta$	Gregory-Laflamme instability of black string $\beta = (1.7524\ldots)L$

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- quantum gravity is hard
- simple toy systems are useful
- quantum black holes well studied
- bubbles of nothing next? entropy?

	black string	thermal BoN
ADM mass	$\frac{1}{8\pi}\frac{\beta L}{G_5}$	$\frac{1}{16\pi}\frac{L^2}{G_5}$
entropy	$\frac{1}{16\pi}\frac{\beta^2 L}{\hbar G_5}$	0
action	$\frac{1}{16\pi}\frac{\beta^2 L}{G_5}$	$\frac{1}{16\pi}\frac{\beta L^2}{G_5}$

Bubble of nothing

(Witten, 1982)

instability of
3+1+1 Minkowski