Reconstructing the black hole interior from the CFT

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based on work with Suvrat Raju and in progress with S. Banerjee, J. Brijan (Groningen) and P. Samantray (ICTS, Bangalore)

Does a big black hole in AdS have a smooth interior?

Can the CFT describe it?

Arguments against smooth interior:

Mathur - Almheiri, Marolf, Polchinski, Sully (AMPS): strong subadditivity paradox

AMPSS (AMPS+Stanford) and Marolf-Polchinski (MP): counting paradoxes in CFT

Explicit construction of interior from CFT (K.P. and Suvrat Raju)

Previous paradoxes can be resolved if we allow operators inside the black hole to be state-dependent

Recent progress (arXiv: 1502.06692 , 1503.08825)

Background

Black Hole Exterior

Consider big black hole in AdS. Expectation from bulk effective field theory (EFT) for a free scalar

$$\phi(t, r, \Omega) = \int_0^\infty d\omega \sum_{lm} b_{\omega lm} \, e^{-i\omega t} f_{\omega,l}(r) Y_{lm}(\Omega) + \text{h.c.}$$

where (dropping l, m indices) we have

$$[b_\omega, b^\dagger_{\omega'}] = \delta(\omega - \omega') \qquad [H, b_\omega] = -\omega b_\omega$$

and

$$\langle b^\dagger_\omega b_\omega \rangle \sim \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}}$$

How do we reconstruct this from the CFT?

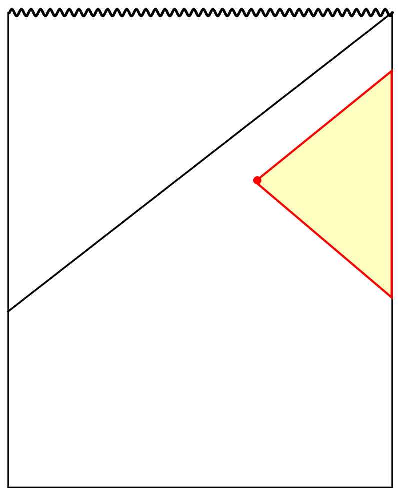
(BDHM, HKLL,...)

Consider single-trace operator $\mathcal{O}$ in CFT, dual to bulk field ϕ. Define Fourier modes

$$\mathcal{O}_{\omega lm} = \int dt d\Omega \; \mathcal{O}(t,\Omega)\, e^{i\omega t}\, Y^*_{lm}(\Omega)$$

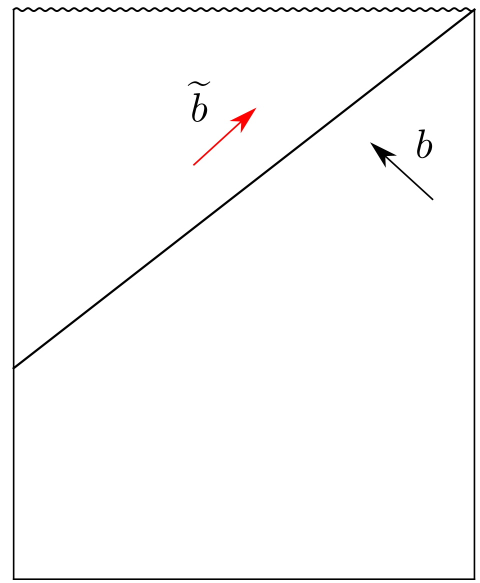
then we identify

$$b_{\omega lm} \;\propto\; \mathcal{O}_{\omega lm}$$



(interesting subtleties about large l modes, gauge invariance, 1/N corrections etc.)

Need for interior modes



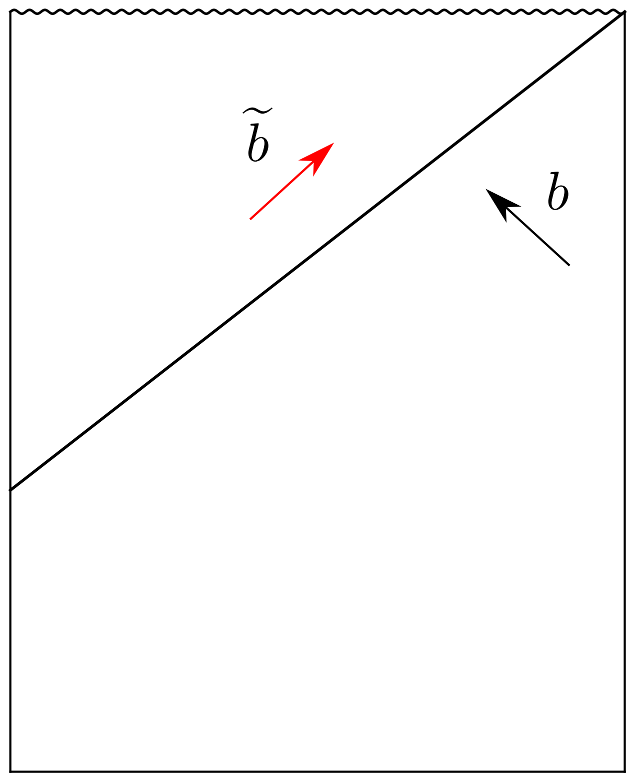
EFT: we need a new set of modes $\widetilde{b}$ which commute with b, and which are entangled with b.

We identified b with modes of $\mathcal{O}$ in CFT.

Central question:

Which CFT operators correspond to $\widetilde{b}$?

Counting argument, against existence of $\widetilde{b}$ operators in CFT (AMPSS)



$$[b, b^\dagger] = 1$$

$$[H, b^\dagger] = \omega b^\dagger$$

$$[\widetilde{b}, \widetilde{b}^\dagger] = 1$$

$$[H, \widetilde{b}^\dagger] = -\omega \widetilde{b}^\dagger$$

The required algebra between $\widetilde{b}, \widetilde{b}^\dagger, H$ is inconsistent with spectrum of states in CFT

$$[\widetilde{b}, \widetilde{b}^\dagger] = 1 \qquad \Rightarrow \qquad \widetilde{b}^\dagger = \text{"creation operator"}$$

$\Rightarrow \widetilde{b}^\dagger$ should not annihilate (typical) states of the CFT $\qquad (*)$.

On the other hand

$$[H, \widetilde{b}^\dagger] = -\omega \widetilde{b}^\dagger$$

implies that $\widetilde{b}^\dagger$ lowers the energy so it maps CFT states of energy E to $E - \omega$.

But in CFT, we have $S(E) > S(E - \omega)$.

$\Rightarrow$ if $\widetilde{b}^\dagger$ is an ordinary linear operator, it must have a nontrivial kernel.

Inconsistent with statement (*).

$\Rightarrow$ The CFT does not contain $\widetilde{b}$ operators and cannot describe the BH interior (?)

Previous counting argument can be made somewhat more precise (K.P and S.Raju)

Related argument $Tr[N_a] \neq 0$ (Bousso, Marolf-Polchinski)

Additional general argument: if $\widetilde{b}$ is a fixed, linear operator, it is hard to understand how $\mathbf{typical}$ CFT states can have the $\mathbf{particular, special}$ entanglement between $b, \widetilde{b}$ needed for smooth interior

These arguments against the existence of a smooth interior for a big black hole in AdS provide the most well-defined version of the firewall paradox

Is there a way out?

Construction of interior operators

Defining the "small Hilbert space $\mathcal{H}_\Psi$"

Suppose we have a typical BH microstate $|\Psi\rangle$ and bulk observer at $t = 0$.

Consider possible simple experiments the observer can perform within EFT.

To describe those, we do not need the entire Hilbert space of the CFT, but rather a smaller subspace.

If $\phi(x)$ is a bulk field, the states we need to use are

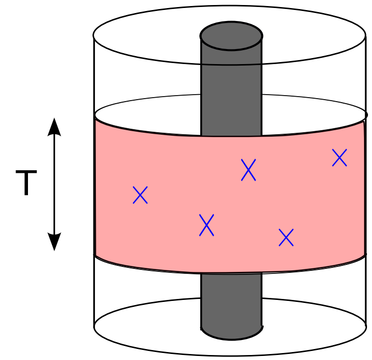
$$\phi(x)|\Psi\rangle$$

$$\phi(x_1)\phi(x_2)|\Psi\rangle, ...$$

$$\phi(x_1)...\phi(x_n)|\Psi\rangle, ...$$

and their linear combinations, where the number of insertions n does not scale with N and the points x_i are not too spread-out in time.

Defining the "small Hilbert space $\mathcal{H}_\Psi$"



In the CFT BH microstate → typical QGP state $|\Psi\rangle$

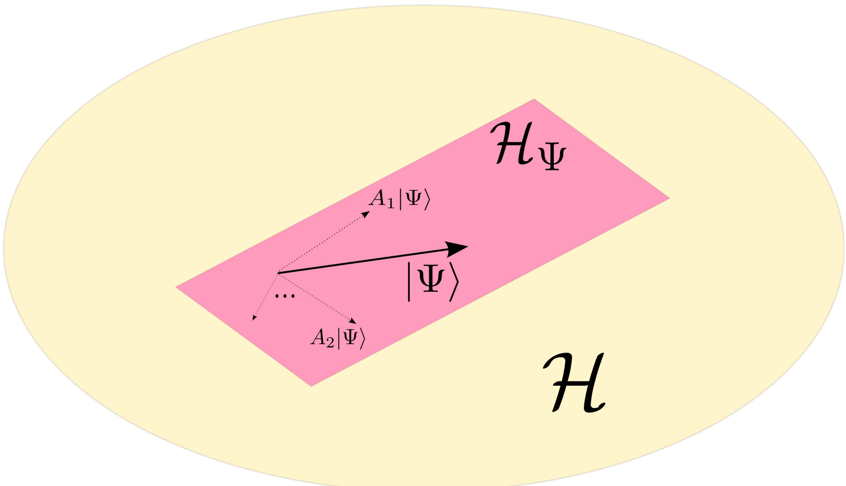
Bulk field ϕ related to boundary single-trace operator $\mathcal{O}$

$\mathcal{A}$= "algebra" of small products of single-trace operators

$$\mathcal{A} = \text{span of}\{\mathcal{O}(t_1, \vec{x}_1),\ \mathcal{O}(t_1, \vec{x}_1)\mathcal{O}(t_2, \vec{x}_2), ...\}$$

Here T is a long time scale and we also need some UV regularization.

Defining the "small Hilbert space $\mathcal{H}_\Psi$"



We define

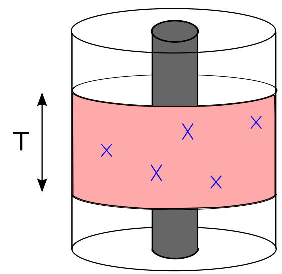
$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span of} : \mathcal{O}(t_1, \vec{x}_1)...\mathcal{O}(t_n, \vec{x}_n))|\Psi\rangle\}$$

Simple EFT experiments in the bulk, around BH $|\Psi\rangle$ take place within $\mathcal{H}_\Psi$

The interior operators $\widetilde{b}$ will be defined to act only on this subspace.

$\mathcal{H}_\Psi$ is similar to "code subspace"

Important point:



$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span of} : \mathcal{O}(t_1, \vec{x}_1)...\mathcal{O}(t_n, \vec{x}_n))|\Psi\rangle\}$$

already contains the states describing the BH interior! (i.e. states we would get in bulk EFT by acting with $\widetilde{b}$)

entanglement, compare with Reeh-Schlieder theorem in QFT

The CFT operators that will correspond to $\widetilde{b}$, will act within the subspace $\mathcal{H}_\Psi$

We will call them **mirror operators** and denote them by $\widetilde{\mathcal{O}}$. Notice that $\mathcal{O}, \widetilde{\mathcal{O}}$ must commute.

What is special when $|\Psi\rangle$ is a BH microstate, which allows the "small Hilbert space" $\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle$ to be big enough to accomodate the action of operators $\widetilde{\mathcal{O}}$ which commute with $\mathcal{O}$?

A typical BH microstate $|\Psi\rangle$ cannot be annihilated by (nonvanishing) elements of the small algebra $\mathcal{A}$

This implies that the representation of $\mathcal{A}$ on $\mathcal{H}_\Psi$ has qualitative differences when $|\Psi\rangle$ is a BH microstate, compared to -say- when $|\Psi\rangle$ is the vacuum.

Physical interpretation:

The state $|\Psi\rangle$ appears to be entangled when probed by the algebra $\mathcal{A}$.

Consider the Hilbert space of two spins, and $\mathcal{A} =$ operators acting on the first. If the two spins are in the state

$$|\Psi\rangle = |\uparrow\uparrow\rangle$$

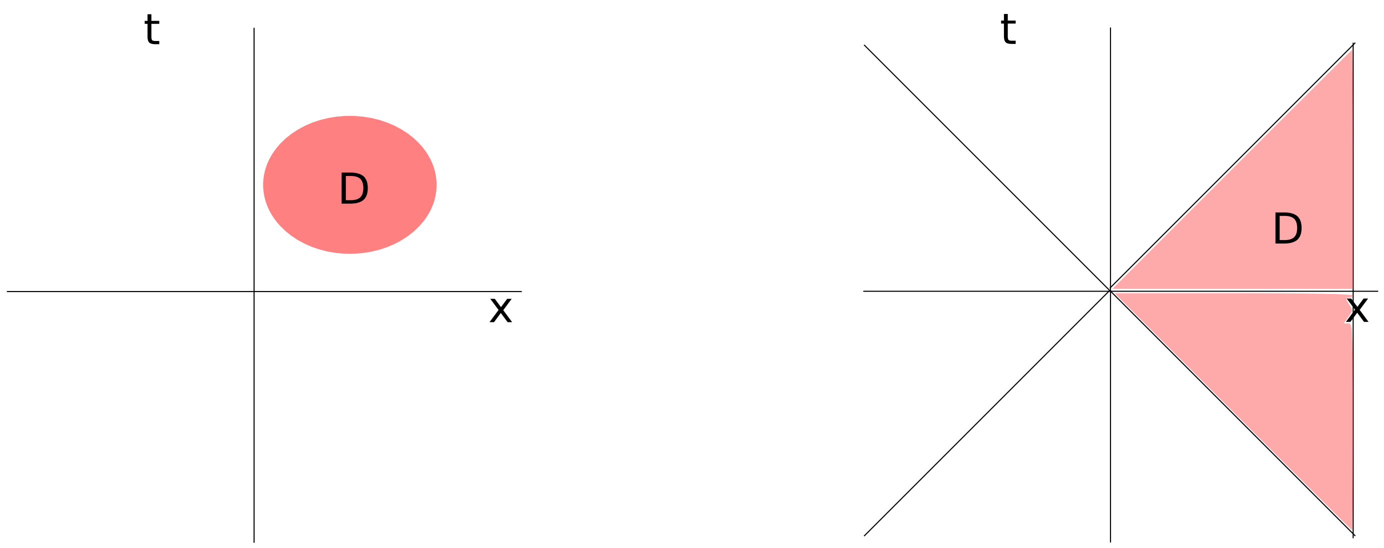
In this case there is no entanglement and indeed the previous condition is violated since

$$s_+^{(1)}|\Psi\rangle = 0 \qquad \text{while} \qquad s_+^{(1)} \neq 0$$

On the other hand consider the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Now there is entanglement and, relatedly, there is no non-vanishing operator acting on the first spin that annihilates the state.



Reeh-Schlieder theorem: Minkowski vacuum $|0\rangle_M$ cannot be annihilated by acting with local operators in D.

$$\Rightarrow$$

In $|0\rangle_M$ local operator algebras are entangled — (though, no proper factorization of Hilbert space due to UV divergences)

Algebras and Representations

We have the "small algebra" $\mathcal{A}$ acting on $\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle$, with the property that it cannot annihilate the state.

$\Rightarrow$ the representation of the algebra is reducible, and the algebra has a nontrivial commutant acting on the same space.

Proof: Define the antilinear map acting on $\mathcal{H}_\Psi$ by

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle \qquad A \in \mathcal{A}$$

any operator of the form

$$\hat{A} = SAS$$

commutes with all elements of the algebra $\mathcal{A}$

The hatted operators commute with those in $\mathcal{A}$:

$$\hat{B}A|\Psi\rangle = SBSA|\Psi\rangle = SBA^\dagger|\Psi\rangle = (BA^\dagger)^\dagger|\Psi\rangle = AB^\dagger|\Psi\rangle$$

and also

$$A\hat{B}|\Psi\rangle = ASBS|\Psi\rangle = AB^\dagger|\Psi\rangle$$

hence

$$[A, \hat{B}]|\Psi\rangle = 0$$

The operators $\hat{A} = SAS$ satisfy:

- Their algebra is isomorphic to $\mathcal{A}$ (since $S^2 = 1$)
- They commute with $\mathcal{A}$

they are almost the mirror operators, but not quite (the mixed A-$\hat{A}$ correlators are not "canonically" normalized)

Constructing the mirror operators (Tomita-Takesaki)

The mapping S is not an isometry. We define the "magnitude" of the mapping

$$\Delta = S^\dagger S$$

and then we can write

$$J = S\Delta^{-1/2}$$

where J is (anti)-unitary. Then the correct mirror operators are

$$\boxed{\widetilde{O} = JOJ}$$

The operator Δ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{“modular Hamiltonian''}$$

For entangled bipartite system $A \times B$ this construction would give $K_A \sim \log(\rho_A)$ i.e. the usual modular Hamiltonian for A.

In the large N gauge theory and using the KMS condition for correlators of single-trace operators we find that for equilibrium states

$$K = \beta(H_{CFT} - E_0)$$

To summarize, we have

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle$$

and

$$\Delta = e^{-\beta(H_{CFT} - E_0)}$$

We define the J by

$$J = S\Delta^{-1/2}$$

Finally we define the mirror operators by

$$\boxed{\widetilde{O} = JOJ}$$

Constructing the mirror operators

Putting everything together we find that the mirror operators are given by the following set of linear equations

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}^\dagger_\omega|\Psi\rangle$$

and

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

$$[H,\widetilde{\mathcal{O}}_\omega]\mathcal{O}....\mathcal{O}|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle$$

These conditions are self-consistent because $A|\Psi\rangle \neq 0$, which in turns relies on

1. The algebra $\mathcal{A}$ is not too large
2. The state $|\Psi\rangle$ is complicated (this definition would not work around the ground state of CFT)

Reconstructing the interior

Using the $\mathcal{O}_\omega$'s and $\widetilde{\mathcal{O}}_\omega$'s we can reconstruct the black hole interior by operators of the form

$$\phi(t,r,\Omega) = \sum_m \int_0^\infty d\omega \Big[\mathcal{O}_{\omega,m}\, e^{-i\omega t} Y_m(\Omega) g^{(1)}_{\omega,m}(r) + \text{h.c.}$$

$$+\widetilde{\mathcal{O}}_{\omega,m}\, e^{-i\omega t}\, Y_m(\Omega)\, g^{(2)}_{\omega,m}(r) + \text{h.c.}\Big]$$

Low point functions of these operators reproduce those of effective field theory in the interior of the black hole

$\Rightarrow$

$\exists$ Smooth interior

Nothing dramatic when crossing the horizon

Realization of Complementarity

The operators $\widetilde{\mathcal{O}}$ seem to commute with the $\mathcal{O}$'s

This is only approximate: the commutator $[\mathcal{O}, \widetilde{\mathcal{O}}] = 0$ only inside low-point functions i.e. in the "small Hilbert space" $\mathcal{H}_\Psi$

If we consider N^2-point functions, then we find that the construction cannot be performed since we will violate

$$A|\Psi\rangle \neq 0, \qquad \text{for} \qquad A \neq 0$$

or in a sense we will find that $[\mathcal{O}, \widetilde{\mathcal{O}}] \neq 0$ inside complicated correlators.

Relatedly, we can express the $\widetilde{\mathcal{O}}$'s as very complicated combination of $\mathcal{O}$'s.

Simple vs Complicated experiments and decomposition of Hilbert space in interior$\times$ exterior

State-dependence as a central issue

State dependence of construction

The operators $\widetilde{\mathcal{O}}$ are defined as linear operators acting only on the "small Hilbert space" around any given state.

Different microstate — different "small Hilbert space"— different linear operators $\widetilde{\mathcal{O}}$

Can we stitch them together into globally defined (linear) operators?

NO , e^S states, overlaps between $\mathcal{H}_\Psi$'s too large, counting arguments of AMPSS, MP

(but generically can be done for small subsets of states)

How state dependence resolves firewall arguments

1. Counting argument of AMPSS about $\widetilde{b}^\dagger$ lowering energy

2. $\mathrm{Tr}(N_a) \neq 0$ argument

3. Explains how we get correct entanglement for typical states since $\widetilde{b}$ operators (partly) "selected by entanglement"

Other examples of state dependence in QM? BH exterior?

Consistency of state dependence with Quantum Mechanics

1. Linear superpositions of states in a CFT.

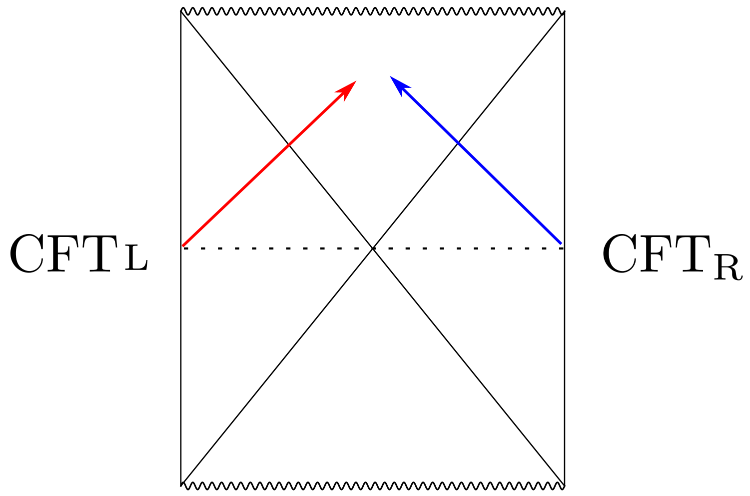
2. CFT is entangled with external system.

In recent paper we showed that for a large class of examples, which could potentially lead to problems, there is no observable violation of the linearity of QM.

We have a disagreement with Don and Joe about the existence of a class of states for which, if they exist, we would have to perform additional checks of linearity — we believe these states do not exist

Eternal black hole

(evidence for state dependence, 1502.06692)



$$H_{\text{total}} = H_L + H_R$$

$$|\text{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_L \otimes |E_i\rangle_R$$

$\exists$ general agreement that eternal BH has smooth interior

In some conventions the wormhole is centered at $t_L = t_R = 0$. We call the corresponding CFT state $|\text{TFD}\rangle$.

$$\langle \text{TFD}| \, \mathcal{O}_L(t_L = 0) \, \mathcal{O}_R(t_R = 0) |\text{TFD}\rangle \sim O(1)$$

The $|\mathrm{TFD}\rangle$ has the exact symmetry

$$(H_R - H_L)|\mathrm{TFD}\rangle = 0$$

which implies

$$e^{i(H_R - H_L)t}|\mathrm{TFD}\rangle = |\mathrm{TFD}\rangle$$

On the other hand

$$e^{i(H_R + H_L)t}|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} \, e^{2iE_i t} \, |E_i\rangle_L \otimes |E_i\rangle_R$$

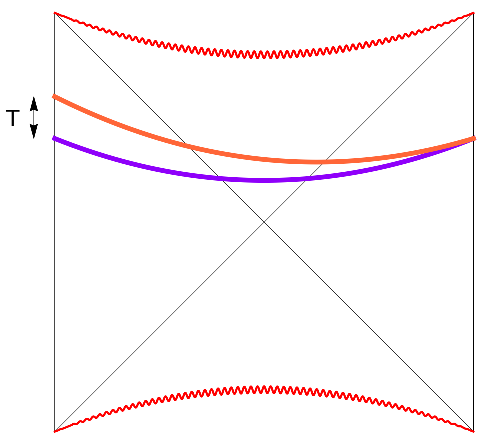
is a genuinely new state due to the phases (large diff in bulk).

Time-shifted wormhole

Consider the state

$$|\Psi_T\rangle \equiv e^{iH_L T}|\mathrm{TFD}\rangle$$

In the bulk, the state $|\Psi_T\rangle$ is related to $|\mathrm{TFD}\rangle$ by a **large diff** (which acts as a time translation on the left boundary, and as identity on the right boundary).



An observer jumps from the right CFT at $t_R = 0$ into the state $|\Psi_T\rangle$. Does he/she experience a smooth horizon?

Right-relational observables invariant under left-sided large diffs.

This implies that the state

$$|\Psi_T\rangle \equiv e^{iH_L T}|\mathrm{TFD}\rangle$$

should appear to be smooth to the observer who jumps from the right CFT at $t_R = 0$.

Moreover, this should be true for all T, even it if $T \sim e^S$

We have a class of states $|\Psi_T\rangle$, which all appear to be smooth to the observer who jumps from the right CFT at $t_R = 0$.

We will now prove that this can only happen if we allow state-dependence.

Equivalently, we will assume that this behavior can be reproduced by state-independent operators, and we will run into a contradiction.

This will provide us with a strong argument in favor of state-dependence

(similar arguments, different conclusion Marolf and Wall)

Time-shifted wormhole

Suppose that the observer jumps from the right CFT at $t_R = 0$ and measures the "occupation number" of some infalling mode behind the horizon. Suppose this occupation number is described by a state-independent, linear operator N. Then from the previous arguments we have the prediction that

$$\langle \Psi_T | N | \Psi_T \rangle = 0 \qquad \text{for all } T$$

or

$$\sum_E \frac{e^{-\beta E}}{Z} \langle E, E | N | E, E \rangle + \sum_{E \neq E'} \frac{e^{-\frac{\beta(E+E')}{2}}}{Z} \langle E', E' | N | E, E \rangle e^{i(E-E')T} \approx 0$$

Take long time average to kill off-diagonal terms

$$\sum_E \frac{e^{-\beta E}}{Z} \langle E, E | N | E, E \rangle \approx 0$$

$$\langle E, E|\mathbf{N}|E, E\rangle = 0$$

so $|E, E\rangle$ is smooth.

According to the AMPSS/MP counting arguments, for a single CFT, $|E\rangle$ is singular. How can it be that $|E, E\rangle$ is now smooth? Why does one CFT care about the existence of another copy if there is no entanglement?

More precise argument: let us **demand** as a condition, that if there is no entanglement between two systems there should be no wormhole.

This implies, that for any unitary U_L on the left CFT (even a complicated one) we should have

$$\langle E, E|U_L^\dagger\, N\, U_L|E, E\rangle = \langle E, E|N|E, E\rangle = 0$$

since N was right-relationally defined.

No wormhole condition $\Rightarrow$ all $|E', E\rangle$ are smooth, for any E, E'.

This is too strong to be consistent with state-independence. Easy to show that we again run into AMPSS, MP counting arguments.

Conclusion: We assumed eternal black hole is smooth and that the interior operators are state-independent and we found a contradiction

Possibilities:

1. Eternal black hole has a firewall

2. AdS $\neq$ CFT (maybe related to superselection sectors of Marolf-Wall, ...)

3. State-dependence

Other developments

Superposition principle

For given $|\Psi\rangle$ we define $\widetilde{\mathcal{O}}$ on $\mathcal{H}_\Psi$. So these are sparse operators

For another state $|\Psi'\rangle$ we have different $\widetilde{\mathcal{O}}$ on $\mathcal{H}_{\Psi'}$

For a small number of such subspaces, the operator $\widetilde{\mathcal{O}}$ can be promoted to an operator defined on the direct sum of the subspaces.

In superpositions of small numbers of states, the infalling observer cannot detect any state-dependence (or violations of QM) within EFT

Ambiguities-Unitaries behind the horizon (see 1503.08825)

ER=EPR from the mirror construction

(Maldacena, Susskind: entanglement = wormhole)

CFT entangled with other system $\mathcal{S}$

$$|\Psi\rangle = \sum_{ia} c_{ia} |\Psi_i\rangle_{\text{CFT}} \otimes |\phi_a\rangle_{\mathcal{S}}$$

$\mathcal{S}$ could be

-Small system (qubit)

-Large system (possibly same CFT), generic entanglement

-Same CFT, special entanglement (for example TFD state)

How do we describe BH interior for observer falling from the CFT? Is there a wormhole, is it geometric?

Mirror operators for entangled systems

$\mathcal{A}_{\rm CFT}$ = small algebra of simple operators in CFT

$\mathcal{A}_{\mathcal{S}}$ = (relevant) algebra of operators in system $\mathcal{S}$.

We define

$$\mathcal{A}_{\rm product} \equiv \mathcal{A}_{\rm CFT} \otimes \mathcal{A}_{\mathcal{S}}$$

Now the "small Hilbert space" $\mathcal{H}_{\Psi}$ is defined as

$$\mathcal{H}_{\Psi} = \mathcal{A}_{\rm product}|\Psi\rangle$$

In general this is larger than just $\mathcal{A}_{\rm CFT}|\Psi\rangle$ (but in cases with special entanglement, like the TFD, the two spaces may be the same)

In the general case the Hilbert space

$$\mathcal{H}_\Psi = \mathcal{A}_{\text{product}}|\Psi\rangle$$

will decompose into a direct sum

$$\mathcal{H}_\Psi = \bigoplus_j \mathcal{H}_\Psi^j$$

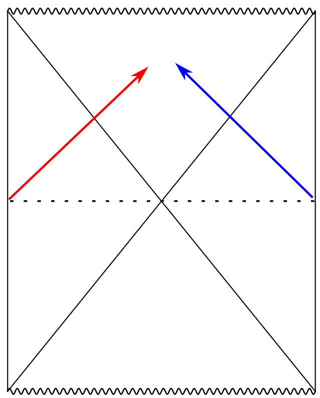
of subspaces $\mathcal{H}_\Psi^j$, each of which are closed under the algebra $\mathcal{A}_{\text{CFT}}$

We identify an equilibrium state $|\Psi\rangle_j$ within each of the $\mathcal{H}_\Psi^j$. Notice that $\mathcal{H}_\Psi^j = \mathcal{A}_{\text{CFT}}|\Psi\rangle_j$

We define the $\widetilde{\mathcal{O}}$ to act within each $\mathcal{H}_\Psi^j$ as before.

Finally, the $\widetilde{\mathcal{O}}$ on the entire $\mathcal{H}_\Psi$ are the sum of individual $\widetilde{\mathcal{O}}$ for each $\mathcal{H}_\Psi^j$.

Example 1: TFD



$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_L \otimes |E_i\rangle_R$$

In this case we have $\mathcal{A}_{\mathrm{CFT}} =$ simple operators in CFT$_R$ and $\mathcal{A}_{\mathcal{S}} =$ simple operators in CFT$_L$.

Because of special entanglement of TFD we have relations like

$$\mathcal{O}_{L,\omega}|\mathrm{TFD}\rangle = e^{-\frac{\beta\omega}{2}} \mathcal{O}^\dagger_{R,\omega}|\mathrm{TFD}\rangle$$

and more generally

$$A_L|\mathrm{TFD}\rangle = e^{-\frac{\beta H_R}{2}} A^\dagger_R e^{\frac{\beta H_R}{2}} |\mathrm{TFD}\rangle$$

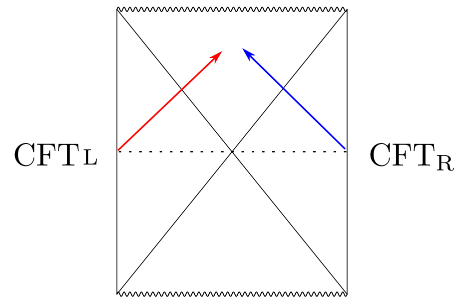
Example 1: TFD

In this case $\mathcal{A}_{\rm product}|\Psi\rangle = \mathcal{A}_{\rm CFT}|\Psi\rangle$ so we simply define the mirror operators $\widetilde{\mathcal{O}}$, as usual, relative to the state $|{\rm TFD}\rangle$

$$\widetilde{\mathcal{O}}_\omega A_R|{\rm TFD}\rangle = e^{-\frac{\beta\omega}{2}} A_R \mathcal{O}^\dagger_\omega |{\rm TFD}\rangle$$

$$[H_R, \widetilde{\mathcal{O}}_\omega] A_R |{\rm TFD}\rangle = \omega \widetilde{\mathcal{O}}_\omega A_R |{\rm TFD}\rangle$$

This reproduces the geometric wormhole between the two CFTs, which is expected for the TFD.



$$[\mathcal{O}_L, \widetilde{\mathcal{O}}] \neq 0 \qquad , \qquad [H_L, \widetilde{\mathcal{O}}] = 0$$

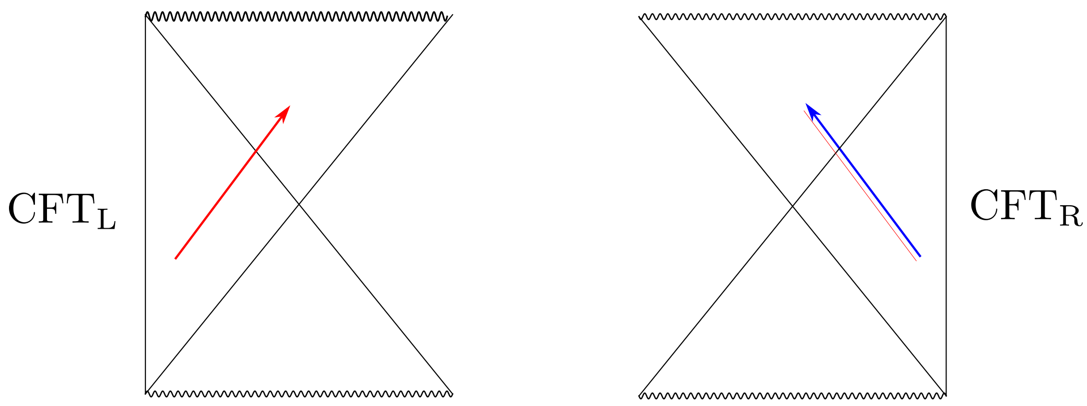
Example 2: generic entangled state of 2 CFTs

$$|\Psi\rangle = \sum_{ij} c_{ij} |E_i\rangle_L \otimes |E_j\rangle_R$$

Typical state with same amount of entanglement as TFD (but random pattern of entanglement). In this case the states $\mathcal{O}_R|\Psi\rangle$ and $\mathcal{O}_L|\Psi\rangle$ are linearly independent. So now

$$\mathcal{A}_{\rm product}|\Psi\rangle \sim \left(\mathcal{A}_{\rm CFT}|\Psi\rangle\right)^2$$

Example 2: generic entangled state of 2 CFTs



No (short) geometric wormhole

However, $\widetilde{\mathcal{O}}$ operators for right CFT **DO** have support on left CFT

Example 3: CFT entangled with small qubit system

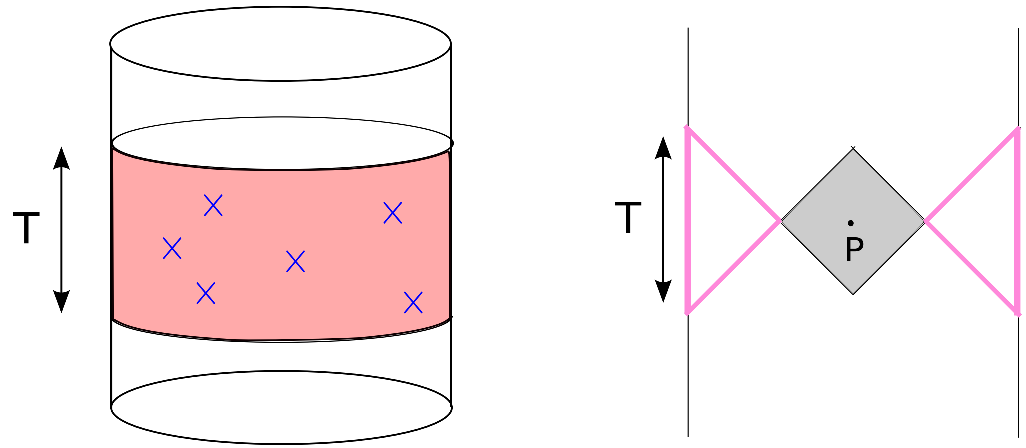
Consistency of construction

Toy model for complementarity

(in progress: with S. Banerjee, J. Brijan and S.Raju)

A toy model for complementarity

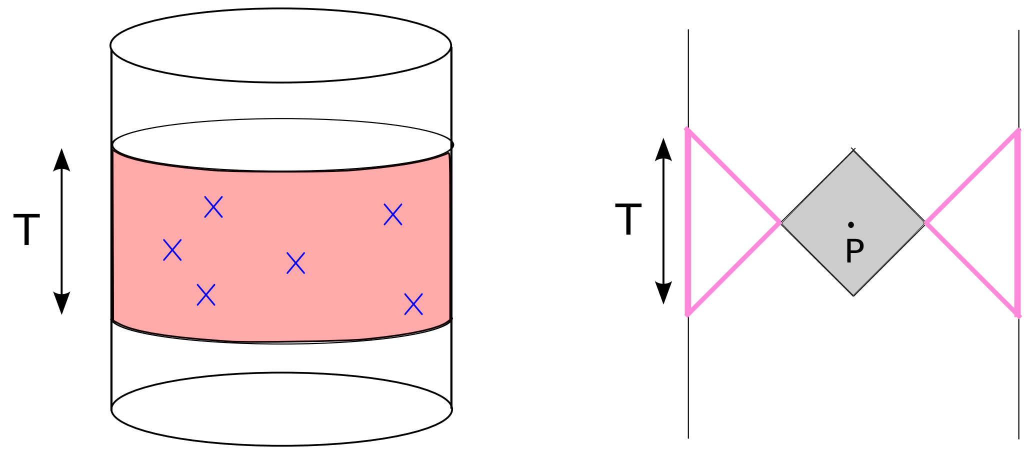
Consider a holographic CFT on $S^{d-1} \times \mathrm{time}$ in the ground state



Consider a time-band on the boundary of length $T < \pi R$ (AdS light-crossing time)

The gray causal diamond around center of AdS is spacelike relative to time-band on boundary

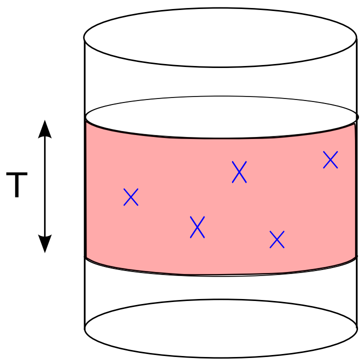
According to EFT a local operator $\phi(P)$ should commute with local operators in time-band (up to Gauss-law tails)



Two related questions:

1. The gray causal diamond seems to be behind a horizon for observers in complementary annular wedge. Can we define "mirror operators" to reconstruct the interior of the gray diamond?

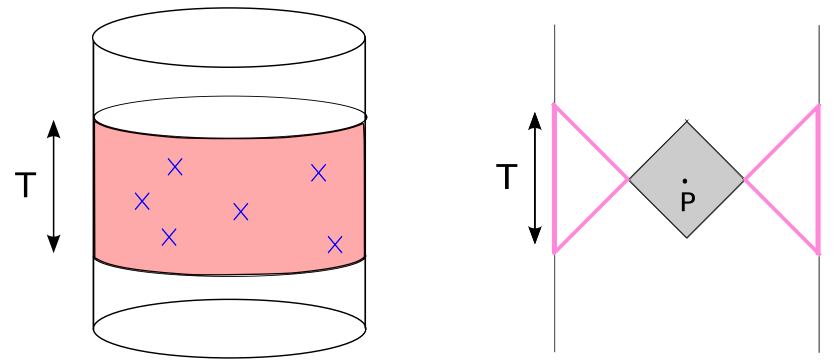
2. A puzzle which has been discussed before in this meeting: $\phi(P)$ seems to commute with all CFT local operators within time-band, but then "time-slice axiom" $\rightarrow \phi(P) \sim \mathrm{identity}$ (?)



Simple vs Complex operators

If we act with arbitrary numbers of local operators in the band, we (presumably) get the full algebra of operators in the CFT. Clearly $\phi(P)$ cannot really commute with the full algebra of (complicated) operators in the time band

Bulk EFT requires $\phi(P)$ to commute only within simple correlation functions of local operators in the band

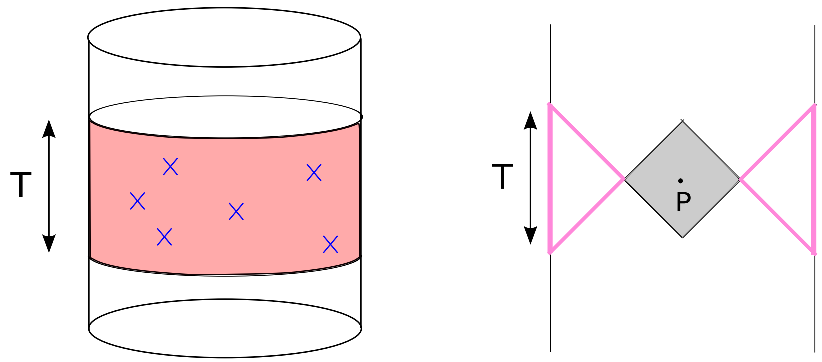


Define the "small algebra" generated by a small number of local operators in the time band

$$\mathcal{A} = \text{span of}\{\mathcal{O}(t_1, \vec{x}_1),\ \mathcal{O}(t_1, \vec{x}_1)\mathcal{O}(t_2, \vec{x}_2), ...\}$$

and the "small Hilbert space" as $\mathcal{H}_{|0\rangle} = \mathcal{A}|0\rangle$. Because the ground state is entangled this space already contains states with excitations inside the gray diamond.

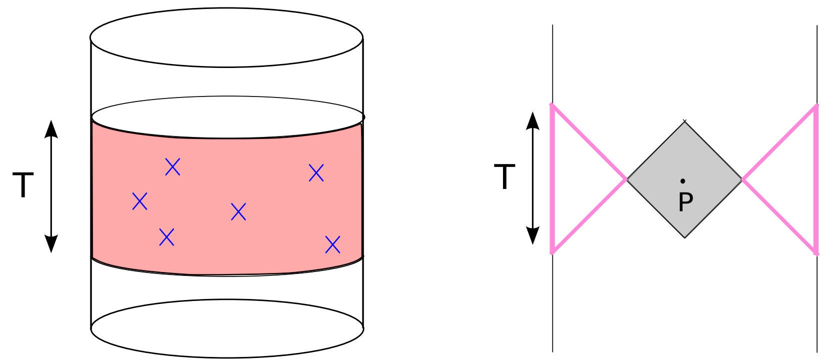
We should be able to define "mirror operators" acting on this space, and which can be used to reconstruct $\phi(P)$



These mirror operators approximately commute with single-trace operators within the time-band.

This can only be approximate due to time-slice axiom. The mirror operators in this case can be re-expressed as very complicated combinations of operators within the time-band.

Toy model of black hole complementarity (without black holes...)



Consider small algebra $\mathcal{A}$ and small Hilbert space $\mathcal{H}_{|0\rangle} = \mathcal{A}|0\rangle$.

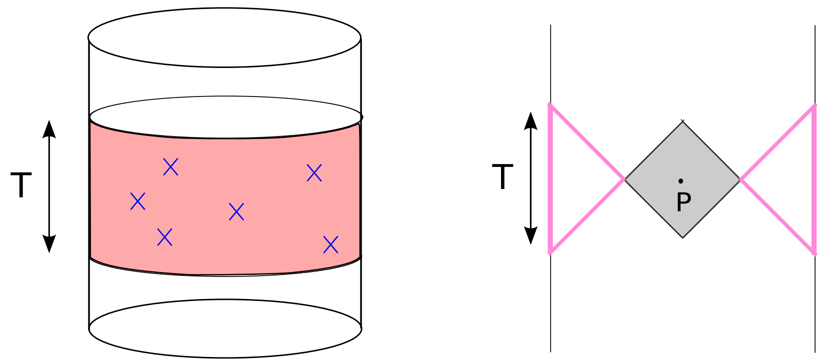
As in case of BH, the representation of $\mathcal{A}$ on $\mathcal{H}_{|0\rangle}$ is reducible and there is nontrivial commutant.

This follows from the fact that: if $T < \pi R$ we cannot annihilate $|0\rangle$ with simple local operators in time-band.

We can define

$$SA|0\rangle = A^{\dagger}|0\rangle$$

and then the commutant of the algebra is $\hat{A} = SAS$.

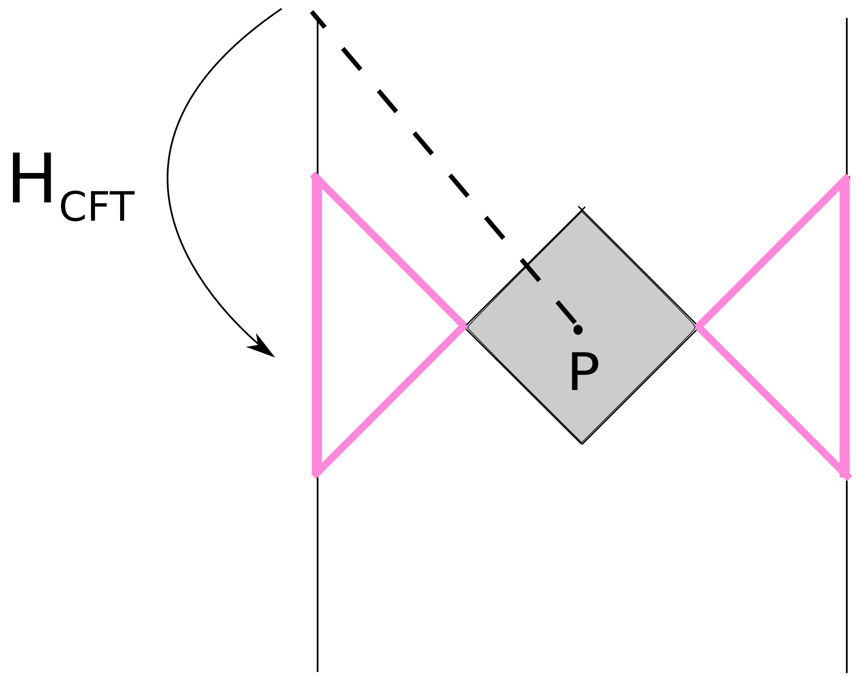


What is the modular hamiltonian? $\log(S^\dagger S) = ?$

From bulk point of view, it seems that it may act non-locally in the diamond.

Different from AdS-Rindler wedge decomposition (or from causal diamond in flat-space CFT) where modular Hamiltonian is symmetry generator

Relation to "precursors"



Reconstructing point P via mirror operators vs precursors ?

Conclusions

Summary

Reconstructing the BH interior is an outstanding problem in AdS/CFT

Proposed approach: the mirror operator construction

State-dependence plays a key role

Outlook

1. Measurement theory for infalling observer: uniqueness, selection of $\widetilde{\mathcal{O}}$, dynamics
2. Robustness of the construction: coarse graining to define “small algebra”, 1/N corrections
3. Possible importance of “precursors“ for describing the BH interior
4. Probing the singularity

THANK YOU