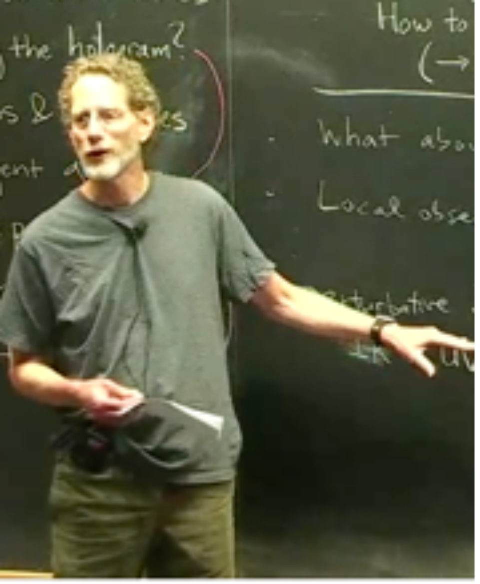
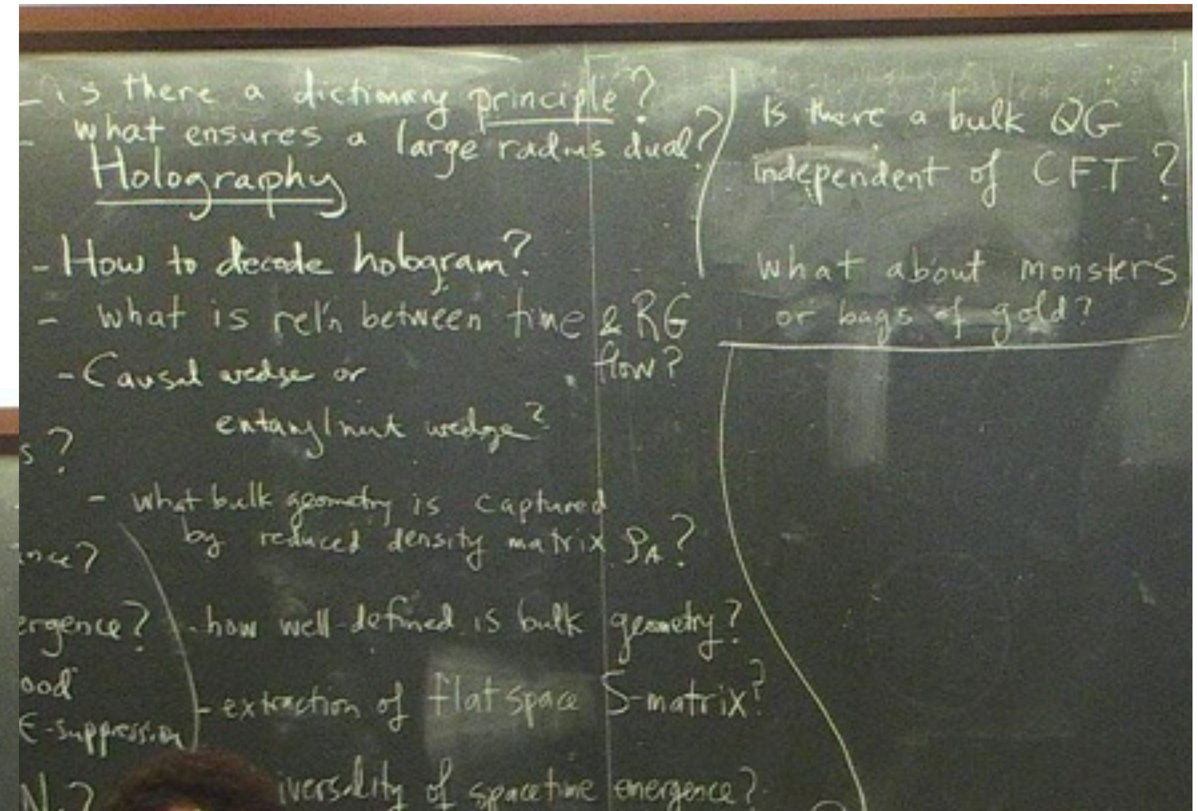
Finite boundary observables
and
white holes

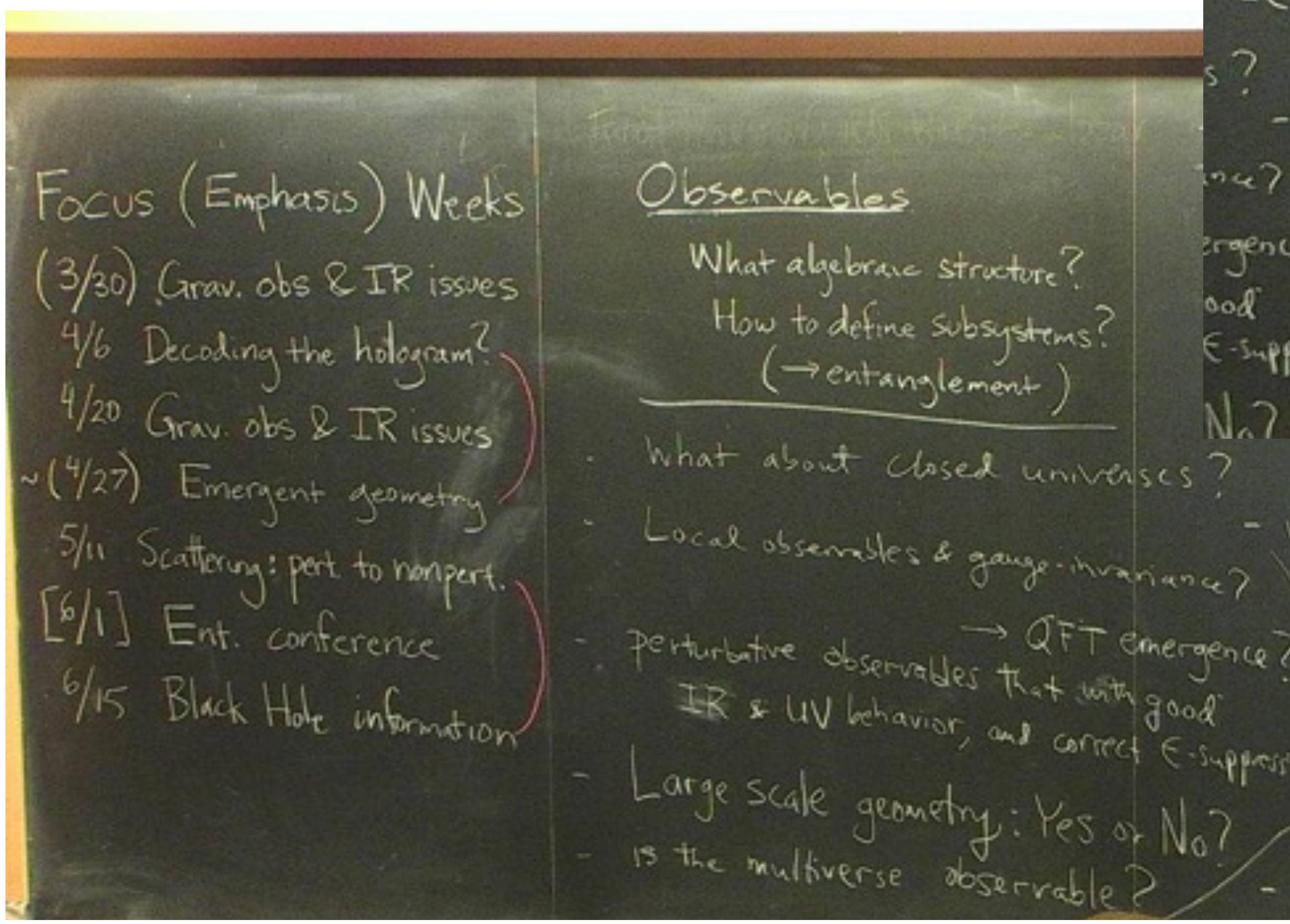
Carlo Rovelli

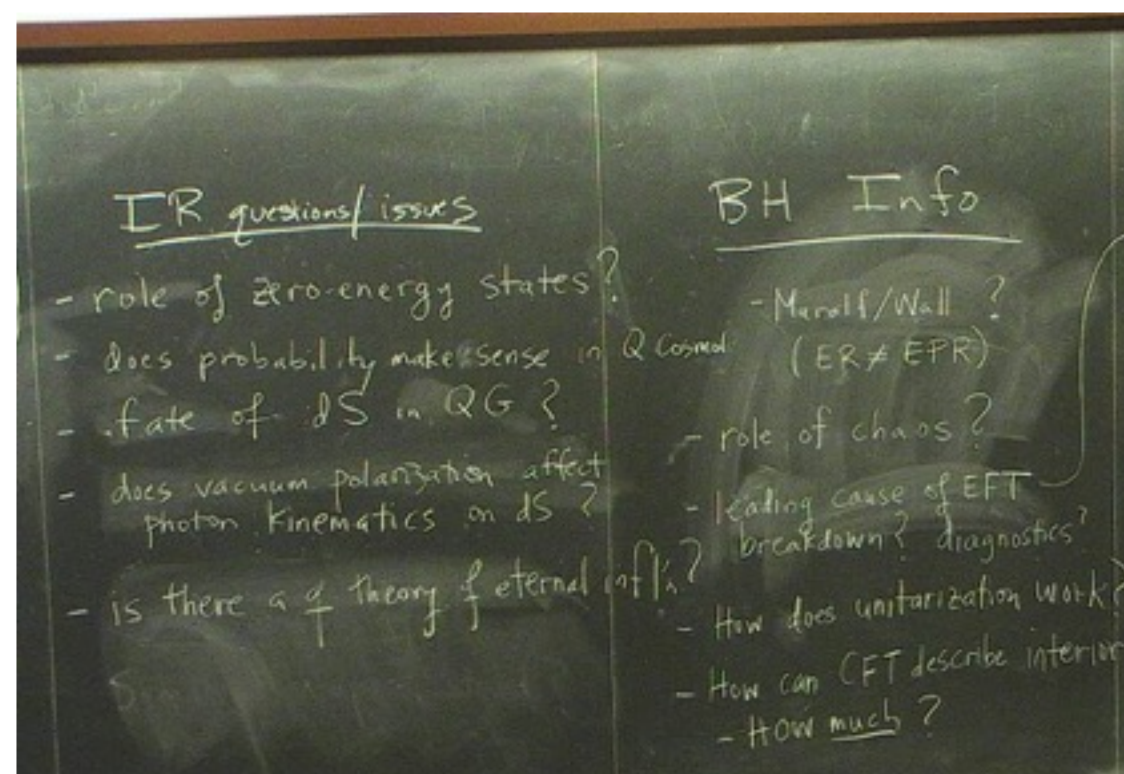
i. Finite boundary observables **[cfr: Don]**

iii. Concrete calculation of an observable:
black to white hole tunnelling time, and Fast Radio Bursts





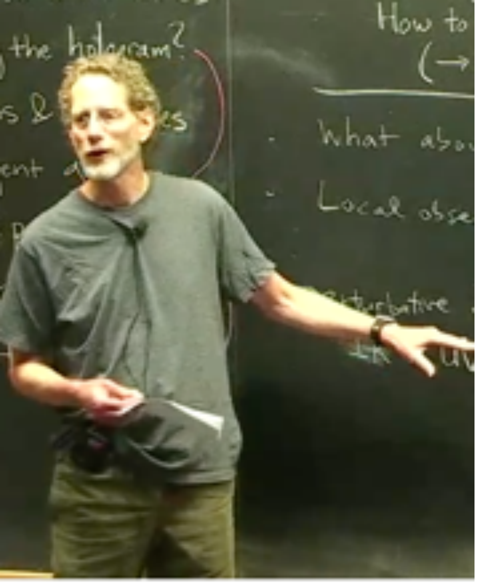


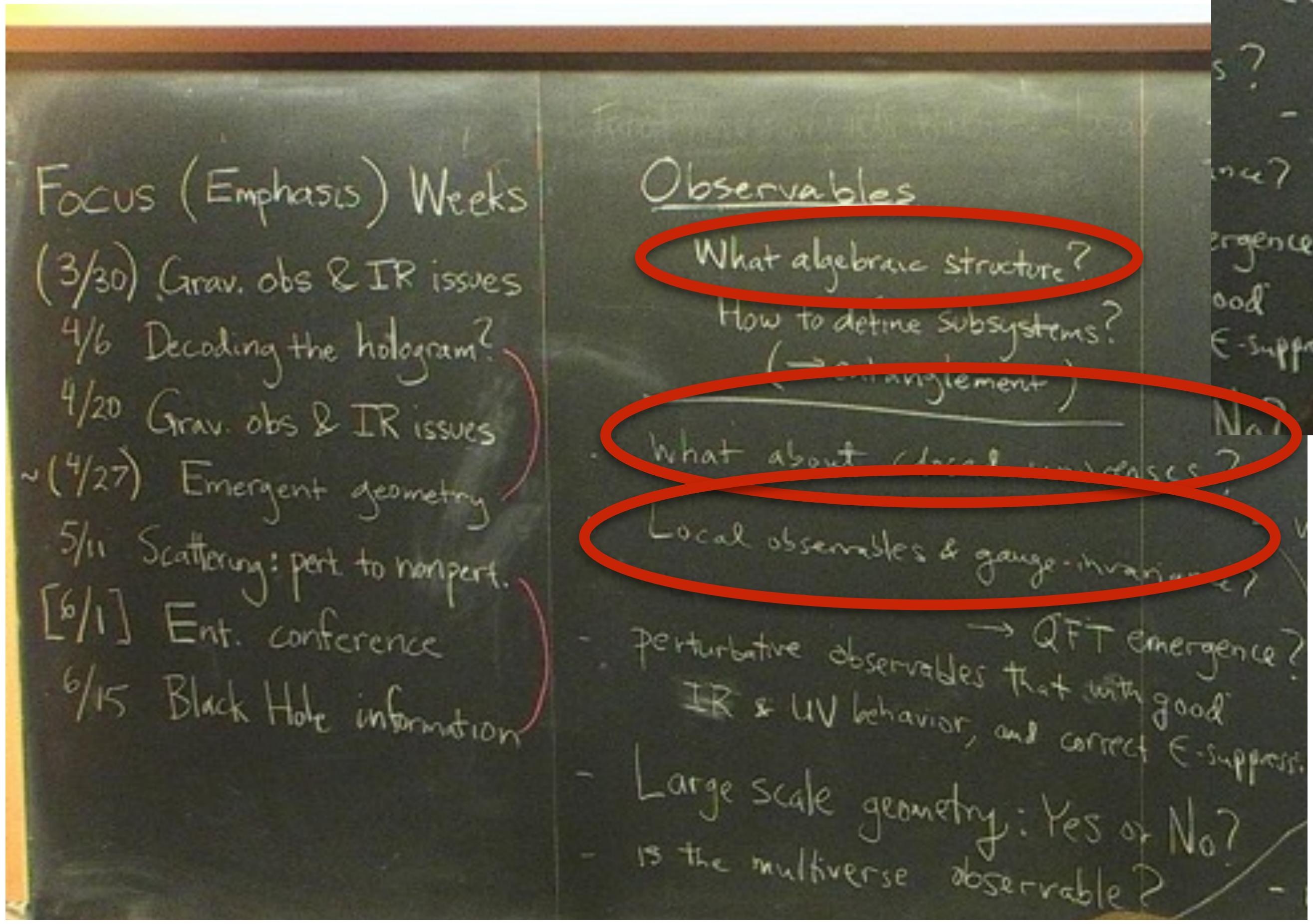


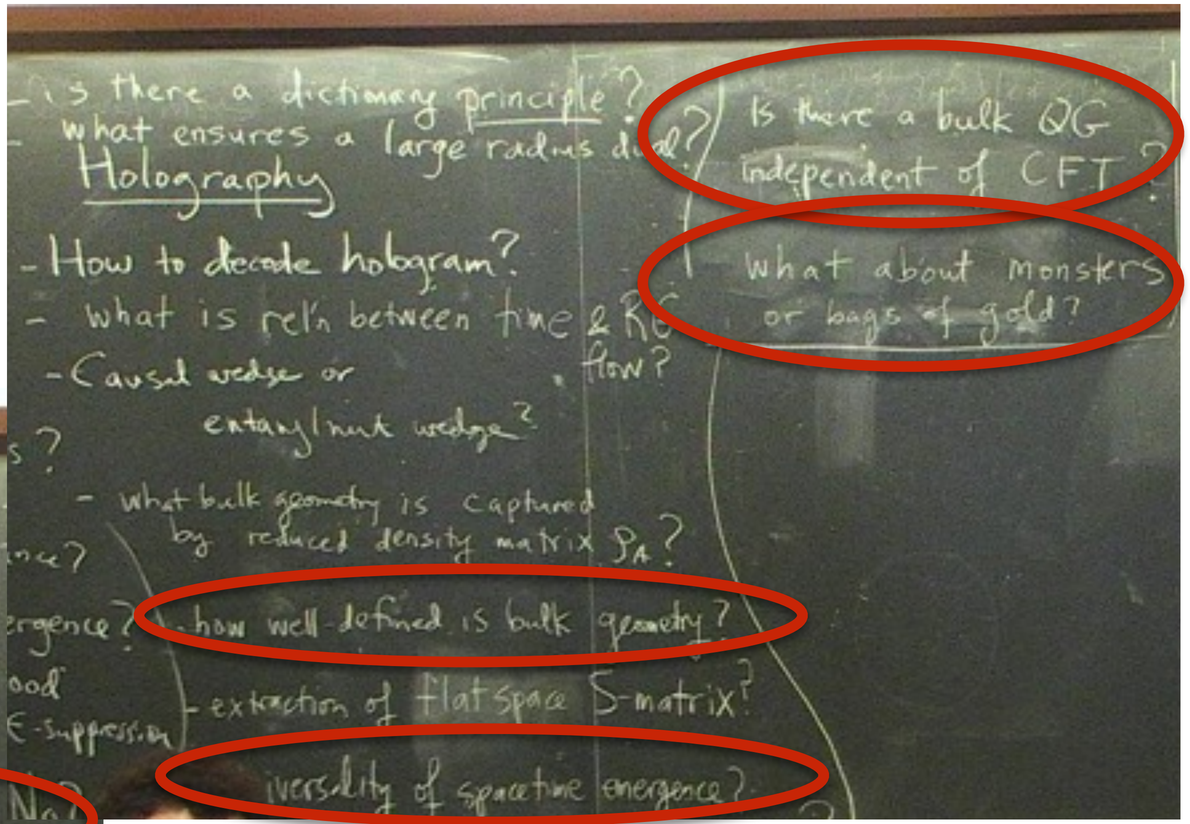
Facets of the problem of quantum gravity

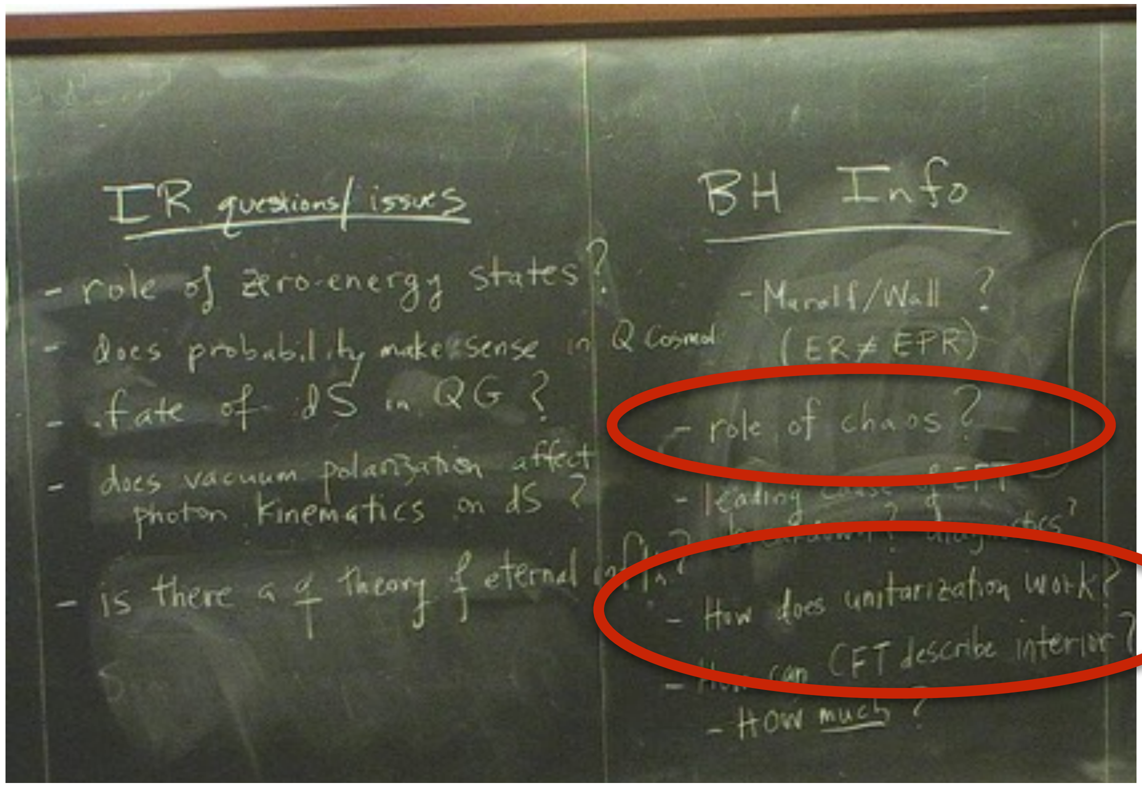
Mars 31st

Santa Barbara









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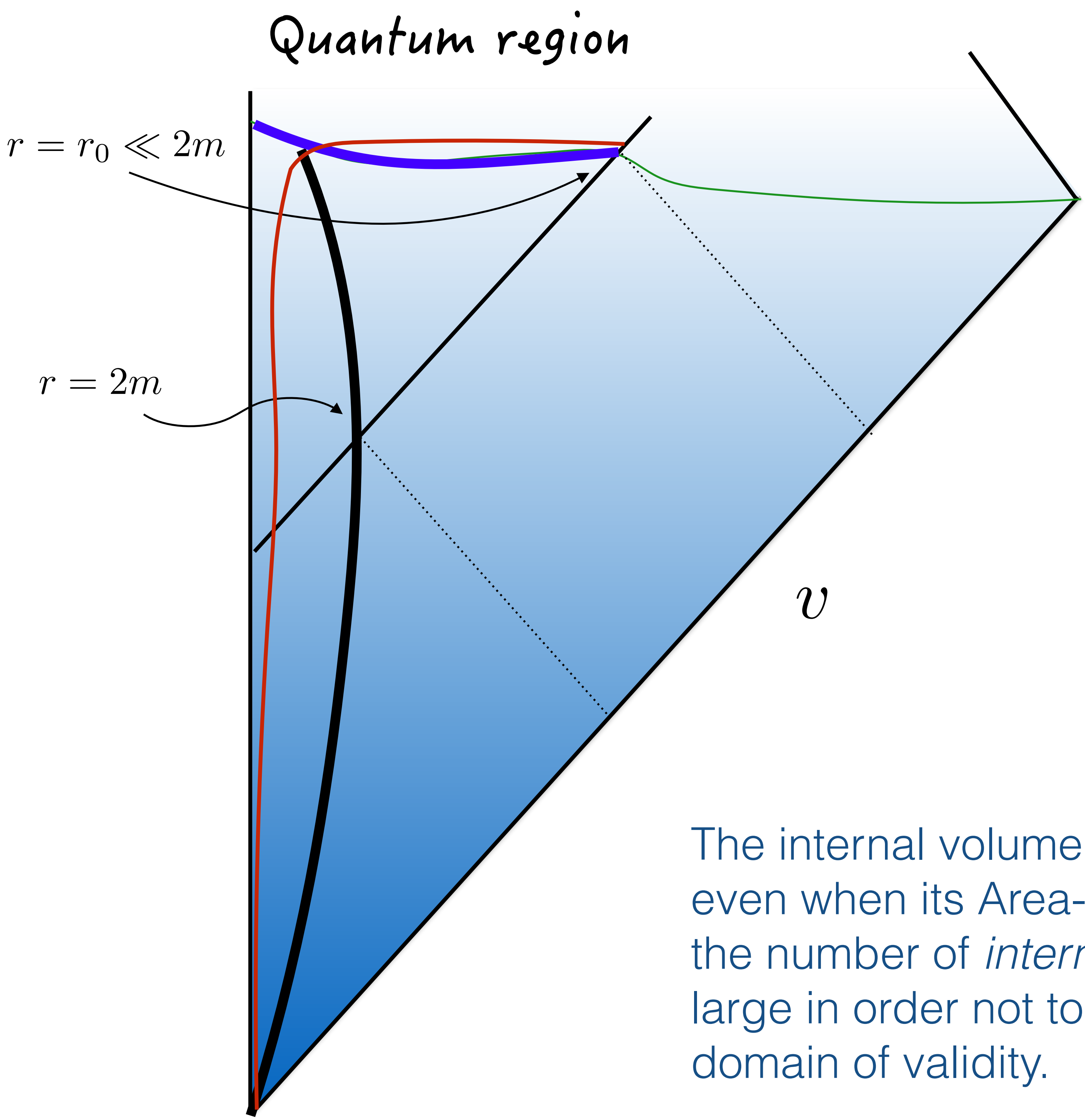

$$V \sim 3\sqrt{3}\,\pi\, m^2 v$$

How big is a black hole?
Marios Christodoulou, Carlo Rovelli.
Phys.Rev. D91 (2015) 6, 064046

After a time m[3]: $V \sim m^5$

At the end of the evaporation: $V \sim m^{\frac{7}{2}}$

The internal volume of a black hole is large even when its Area-Entropy is small: the number of *internal* degrees of freedom must be large in order not to violate local qft in its own domain of validity. … holography?

A quantum theory of gravity cannot be a **local** qft:

1. Local observables are not gauge invariant in gravity: diff invariance
2. Space-time discreteness, or similar (cfr: both loops and strings) **[cfr: Gabriele]**

Local qft: degrees of freedom localised with respect to classical entities.

Quantum gravity: degrees of freedom localised with respect to one another, as in classical general relativity.

i. General lessons from non perturbative QG

ii. partial observables (“local observables and gauge invariance”)

Quantum theory gives the probabilities of the outcomes of a (local) interaction of a quantum system with another system.

General relativity describes the (local) interaction of spacetime regions with one another.

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Quantum system
=
Spacetime region

Boundary

➜ **Hamilton function: S(q,t,q’,t’)**

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➜ **Hamilton function: S(q,t,q',t')**

➜ **Boundary formalism**

States: associated to **3d** boundaries of spacetime regions.

Transition amplitudes: associated to **4d** regions **[cfr: Bianca]**.

t', q'

t, q

Hamilton function $$S(q,t,q',t') = \int_t^{t'} d\tilde{t}\ L(\dot{q}_{q,t,q',t'}(\tilde{t}), q_{q,t,q',t'}(\tilde{t}))$$

Propagator $$W(q,t,q',t') = \langle q | e^{iH(t-t')} | q' \rangle \sim e^{\frac{i}{\hbar} S(q,t,q',t')}$$

$$= \int_{\substack{q(t)=q \\ q(t')=q'}} D[q]\ e^{\frac{i}{\hbar}\int dt L}$$

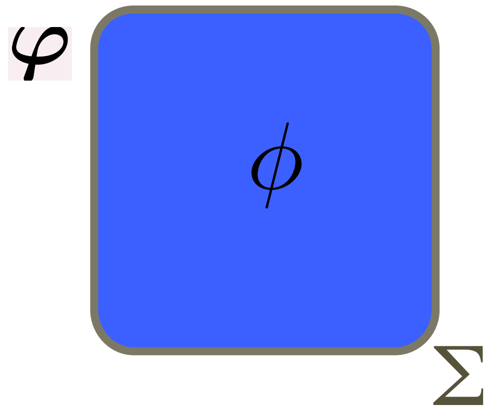
Systems evolving in observable time $\quad q(t) \quad$ $$L[q,\dot{q}] = \frac{m}{2}\left(\dot{q}^2 - \omega^2 q^2\right)$$

System evolving in parameter time $\quad q(\tau), t(\tau) \quad$ $$L'[q,\dot{q},t,\dot{t}] = \frac{m}{2}\left(\frac{\dot{q}^2}{\dot{t}} - \omega^2 \dot{t} q^2\right)$$

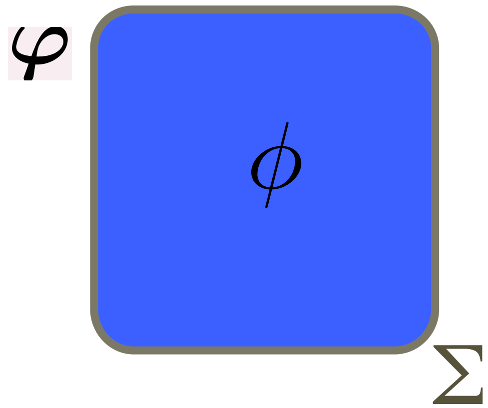
Hamilton function $$S(q,t,q',t',\tau,\tau') = \int d\tau\ L(\dot{q}_{q,t,q',t'}(\tau), q_{q,t,q',t'}(\tau), \dot{t}_{q,t,q',t'}(\tau), t_{q,t,q',t'}(\tau)) = S(q,t,q',t')$$

Propagator $$W(q,t,q',t') = \langle q,t | P | q',t' \rangle \sim e^{\frac{i}{\hbar} S(q,t,q',t')}$$

$$= \int_{\substack{q(\tau)=q \ \ q(\tau')=q' \\ t(\tau)=t \ \ t(\tau')=t'}} D[q,t]\ e^{\frac{i}{\hbar}\int dt L(q,\dot{q},t,\dot{t})}$$

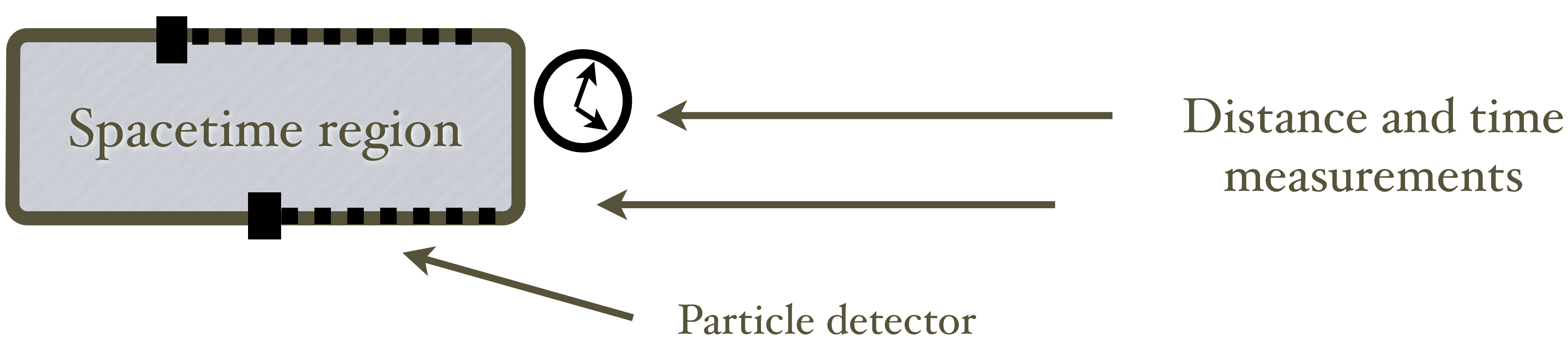


Boundary functional $$W[\varphi, \Sigma] = \int_{\phi|_\Sigma = \varphi} D\phi \; e^{iS[\phi]}$$



Boundary functional $$W[\varphi, \Sigma] = \int_{\phi|_\Sigma = \varphi} D\phi \; e^{iS[\phi]}$$

But in a generally covariant theory: $$W[\varphi, \Sigma] = W[\varphi]$$

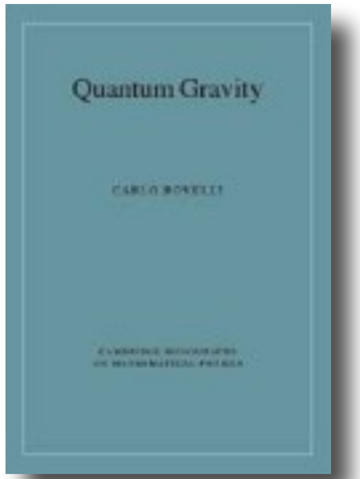


In a general relativistic theory, distance and time measurements are field measurements like the other ones: they are determined by the boundary data of the problem. **[cfr.: Jim]**

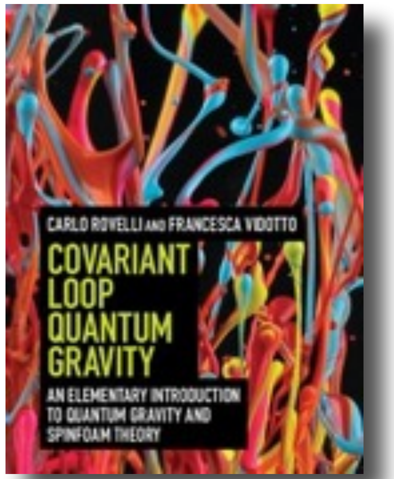
Transition amplitudes	$\langle\psi\vert e^{iH(t-t')}\vert\psi'\rangle$
Rep invariant	$\langle\psi\vert P\vert\psi'\rangle = \langle W\vert\psi\otimes\psi'\rangle$
Quantum gravity	$\langle W\vert\psi_{boundary}\rangle$

➜ The boundary determines the here and now of the interaction whose correlations are predicted by quantum theory.

➜ The physical relative position of the points on the boundary is determined by the (quantum) state of geometry on the boundary itself



Quantum gravity
Carlo Rovelli
CUP 2004



Introduction to Canonical Loop Quantum Gravity
Francesca Vidotto, Carlo Rovelli
CUP 2014

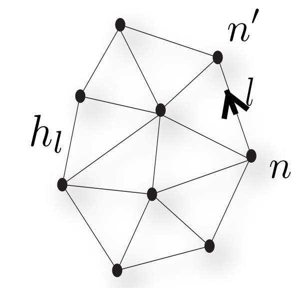
Let's make this concrete

Covariant loop quantum gravity. Full definition.

Kinematics **Boundary**

State space $$\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]_\Gamma \quad \ni \psi(h_l) \qquad \mathcal{H} = \lim_{\Gamma\to\infty} \mathcal{H}_\Gamma$$

Operators [cfr: Steve]: $$\vec{L}_l = \{L^i_l\}, i = 1,2,3 \qquad L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$$

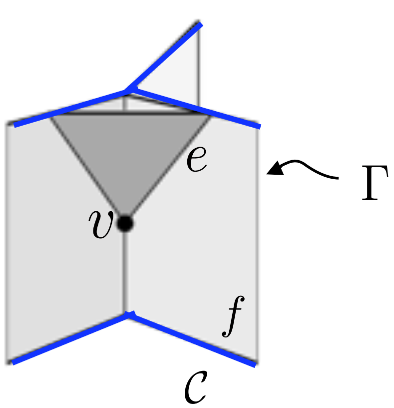


Γ spin network (nodes, links)

Dynamics **Bulk**

Transition amplitudes $$W_{\mathcal{C}}(h_l) = N_{\mathcal{C}} \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf}) \qquad W = \lim_{\mathcal{C}\to\infty} W_{\mathcal{C}} \qquad h_f = \prod_v h_{vf}$$

Vertex amplitude $$A(h_{vf}) = \int_{SL(2,\mathbb{C})} dg'_e \prod_f \sum_j (2j+1)\ D^j_{mn}(h_{vf}) D^{\gamma(j+1)\,j}_{jmjn}(g_e g_{e'}^{-1}) \qquad 8\pi\gamma\hbar G = 1$$



spinfoam
(vertices, edges, faces)

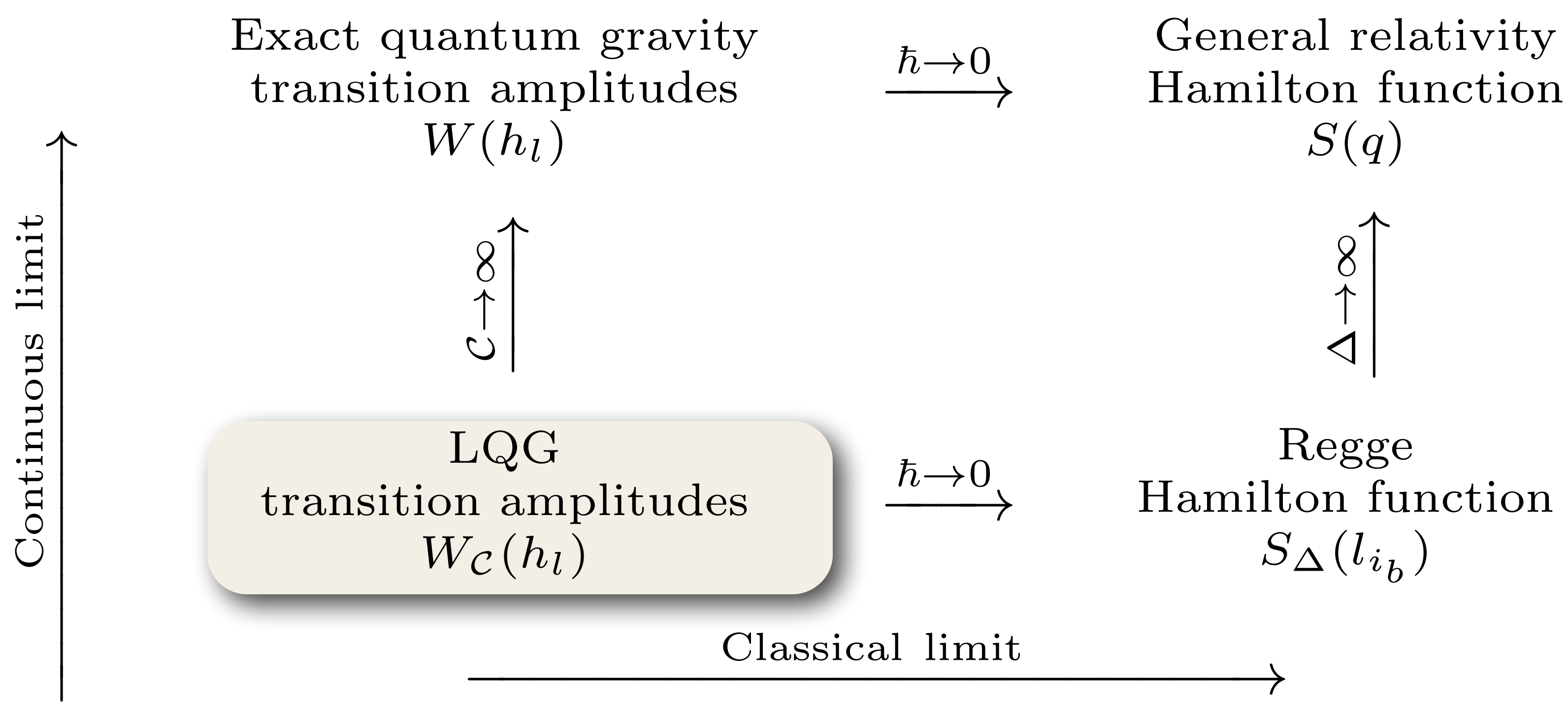
With cosmological constant:

SL(2,C) Chern-Simons Theory, a non-Planar Graph Operator, and 4D Loop Quantum Gravity with a Cosmological Constant
Hal M. Haggard, Muxin Han, Wojciech Kamiński, Aldo Riello.
arXiv:1412.7546

Main results: (i) Classical limit: GR

(ii) Spacetime discreteness

(iii) UV and IR finite amplitudes



Regime of validity of the expansion: $L_{Planck} \ll L \ll \sqrt{\frac{1}{R}}$

Why this works? "Ditt-invariance" (from B Dittrich)

Systems evolving in observable time

$$S = \frac{m}{2}\int dt \left(\left(\frac{dq}{dt}\right)^2 - \omega^2 q^2 \right)$$

Discretize $\quad a = t/N$

$$S_N = \frac{m}{2}\sum_n^N a \left(\left(\frac{q_{n+1}-q_n}{a}\right)^2 - \omega^2 q_n^2 \right)$$

$$W(q_f,t_f;q_i,t_i) = \lim_{\substack{\Omega\to 0\\ N\to\infty}} \mathcal{N} \int \overset{N}{dQ_n}\; e^{iS_{N,\Omega}(Q_n)} \qquad \Omega = a\omega$$

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$$W(q_f,t_f;q_i,t_i) = \lim_{N\to\infty} \int dQ_n\, dT_n\; e^{iS_N(Q_n,T_n)}$$

- Systems evolving in time: The continuum limit is obtained taking the number of steps to infinity and a coupling constant to a critical value.
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All these are empty formulas until we can do physics with them

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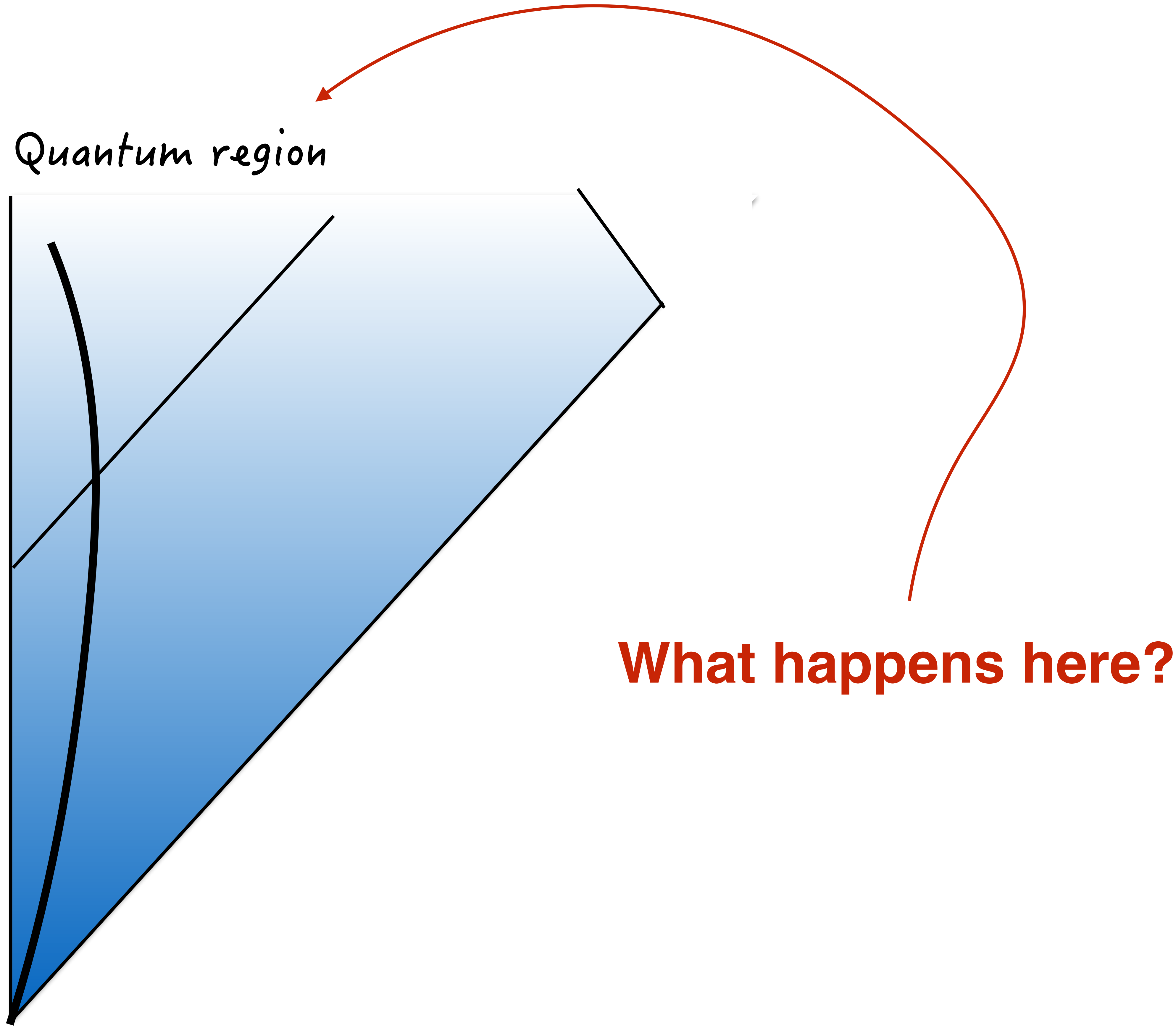
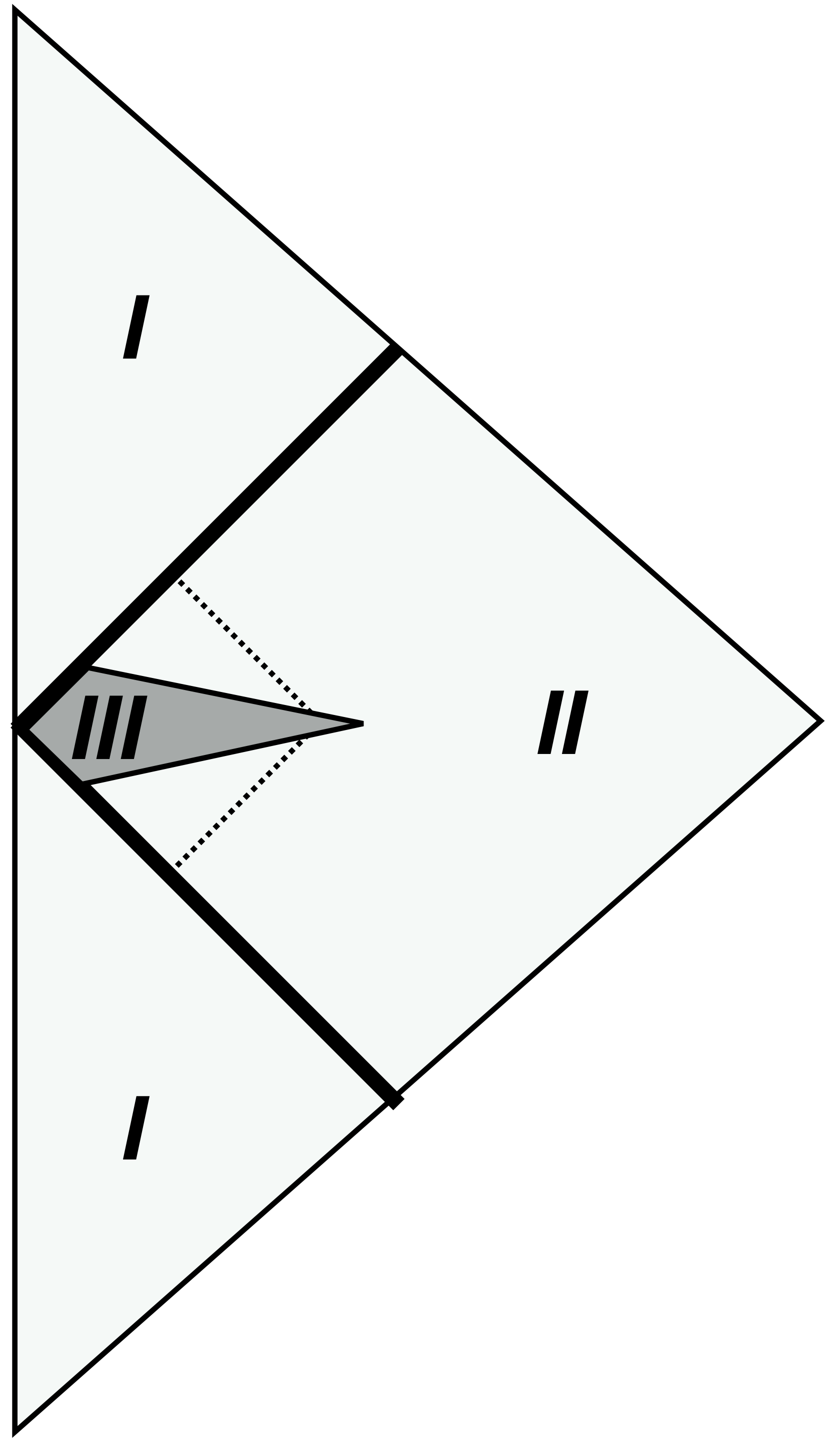
ii. A concrete calculation of an observable:
Fast Radio Bursts and black to white hole tunnelling time


Quantum region
What happens here?



A technical result in classical GR:

The following metric is an exact vacuum solution, plus an ingoing and outgoing shell, of the Einstein equations outside a finite spacetime region (grey).

$$ds^2 = -F(u,v)dudv + r^2(u,v)(d\theta^2 + sin^2\theta d\phi^2)$$

Region I $\quad F(u_I, v_I) = 1, \qquad r_I(u_I, v_I) = \frac{v_I - u_I}{2}.$

$v_I < 0.$

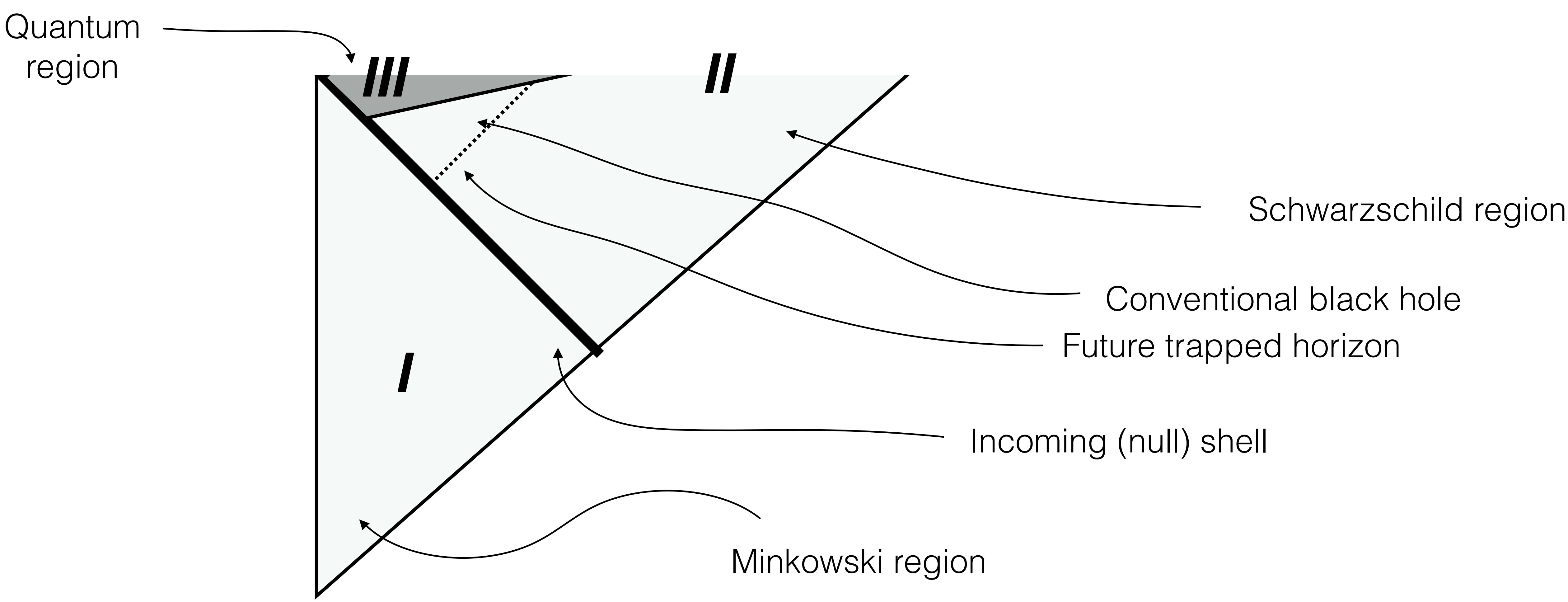
Region II $\quad F(u,v) = \frac{32m^3}{r} e^{\frac{r}{2m}} \qquad \left(1 - \frac{r}{2m}\right) e^{\frac{r}{2m}} = uv.$

Matching: $\quad r_I(u_I, v_I) = r(u,v) \longrightarrow u(u_I) = \frac{1}{v_o}\left(1 + \frac{u_I}{4m}\right) e^{\frac{u_I}{4m}}.$

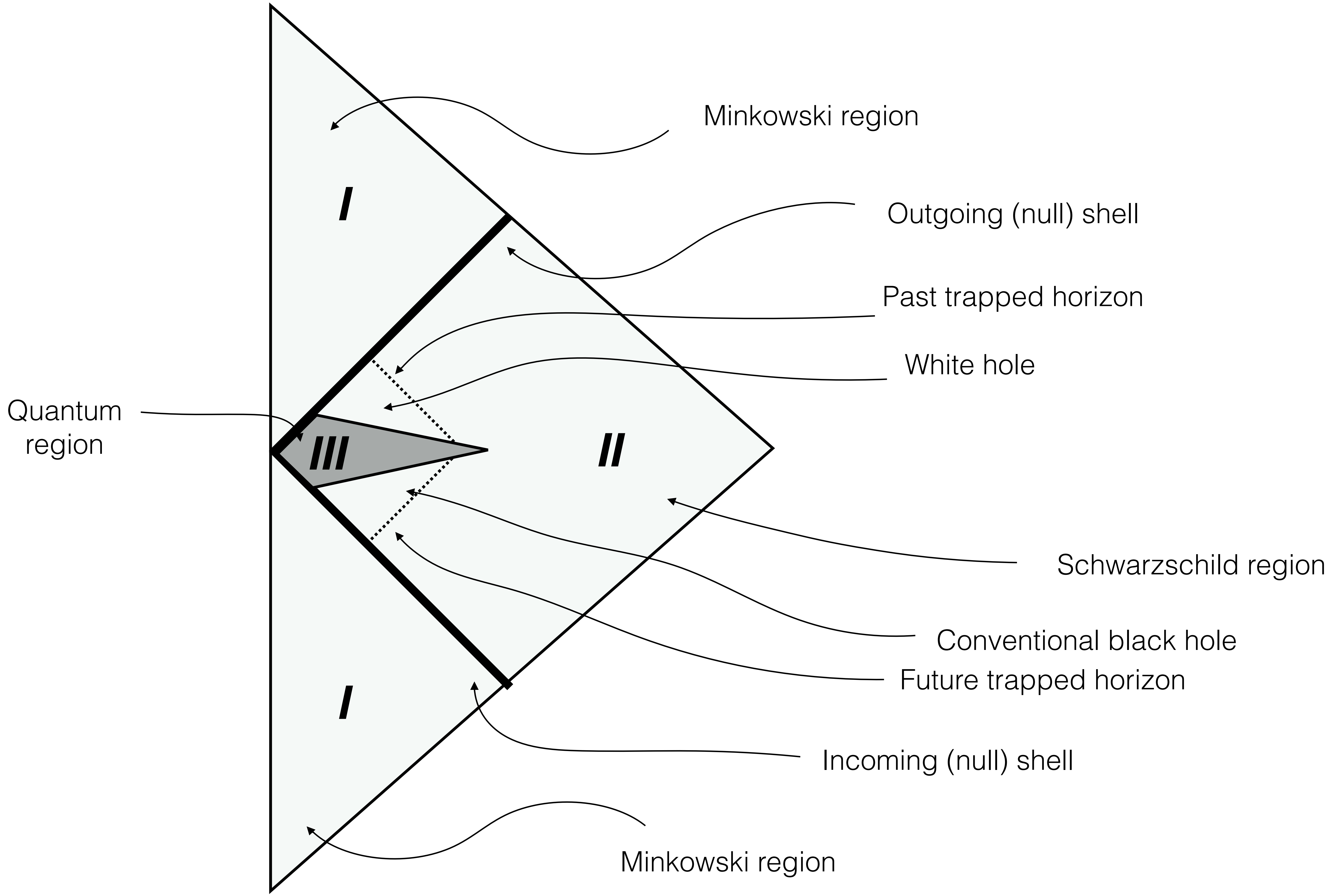
Region III $\quad F(u_q, v_q) = \frac{32m^3}{r_q} e^{\frac{r_q}{2m}}, \qquad r_q = v_q - u_q.$

Black hole fireworks: quantum-gravity effects outside the horizon spark black to white hole tunneling
Hal M. Haggard, Carlo Rovelli
arXiv:1407.0989

The metric is determined by three constants: m, ϵ, δ



Quantum
region
III
II
I
Schwarzschild region
Conventional black hole
Future trapped horizon
Incoming (null) shell
Minkowski region



Minkowski region
Outgoing (null) shell
Past trapped horizon
White hole
Quantum region
I
III
II
Schwarzschild region
Conventional black hole
Future trapped horizon
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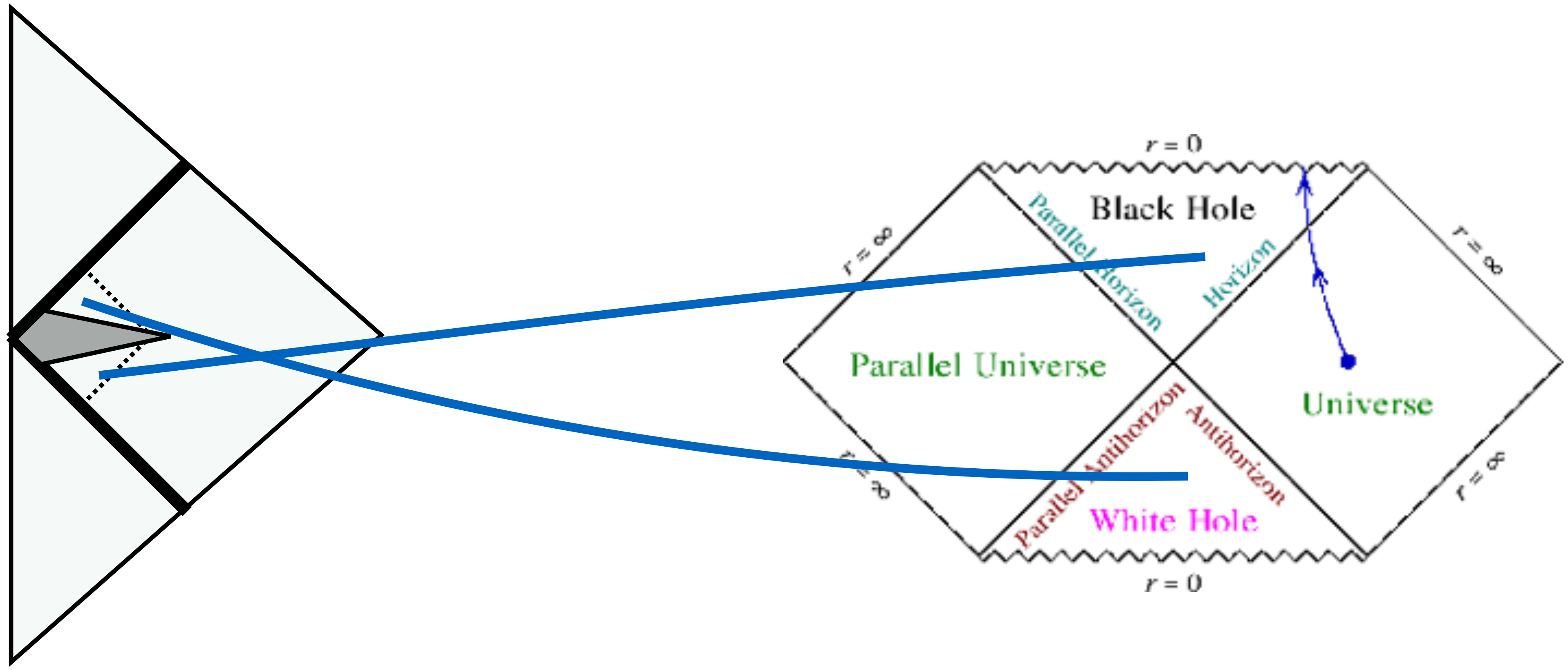
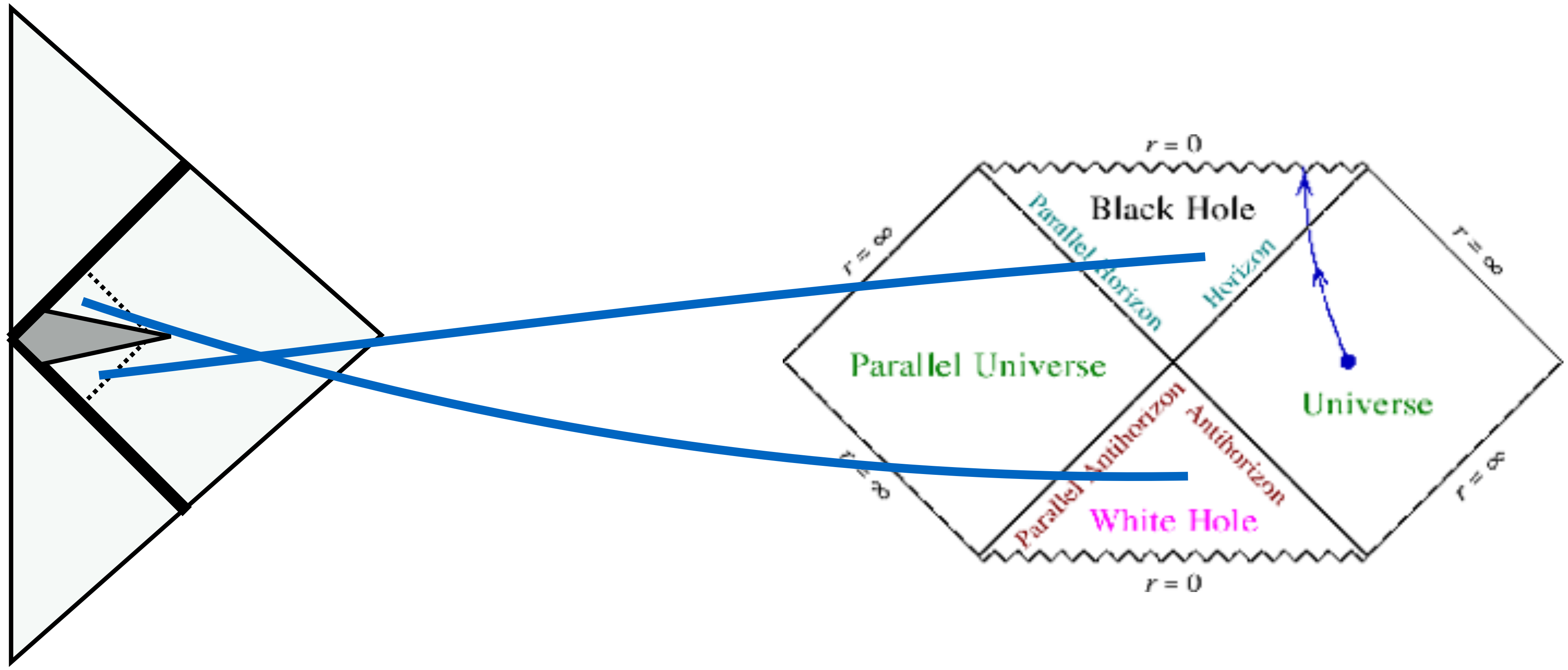
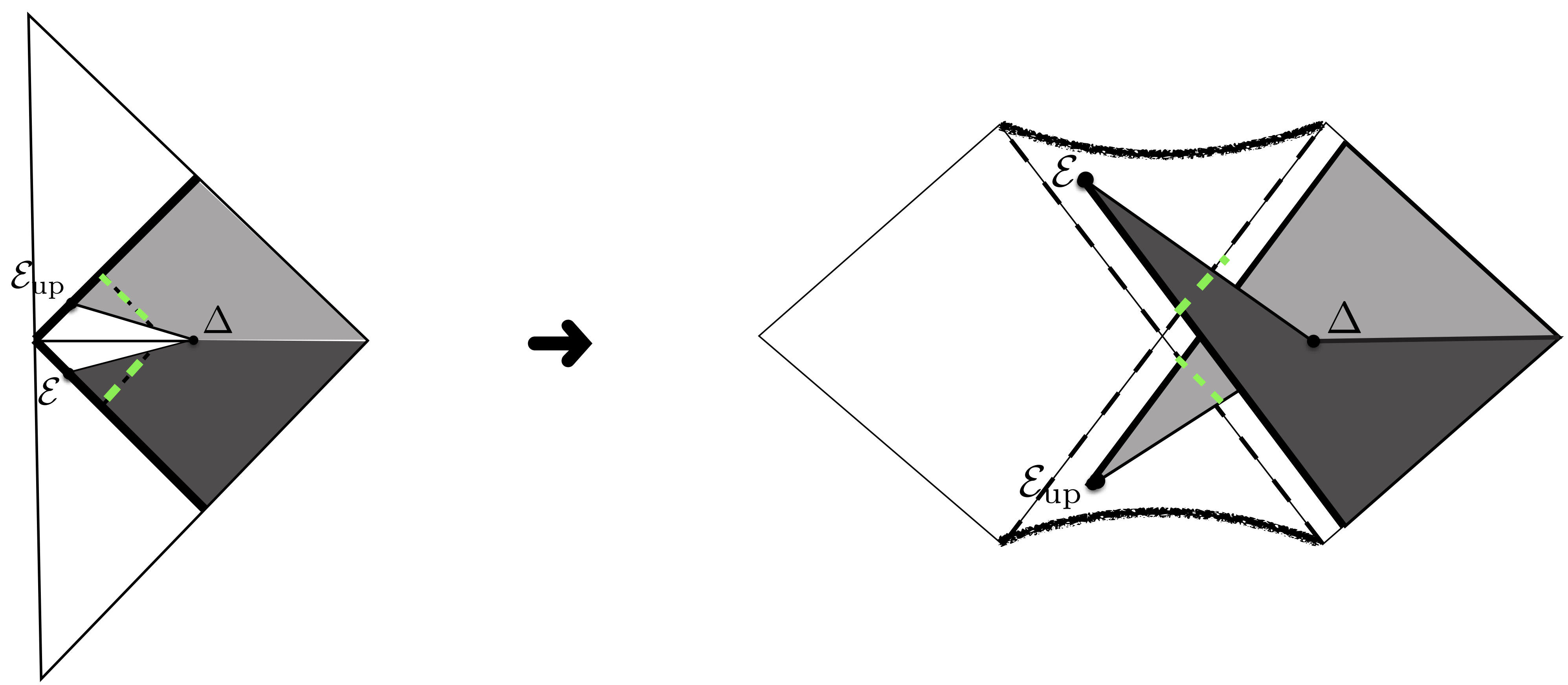
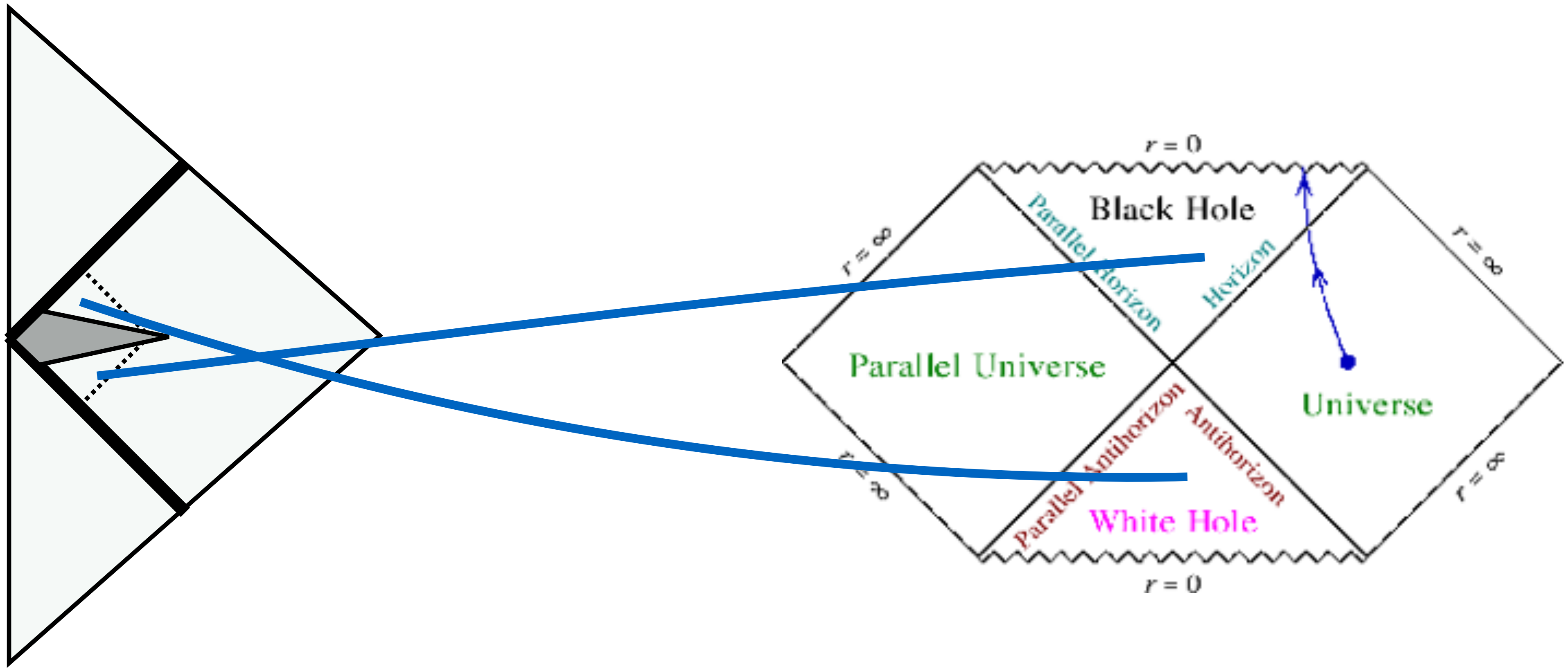
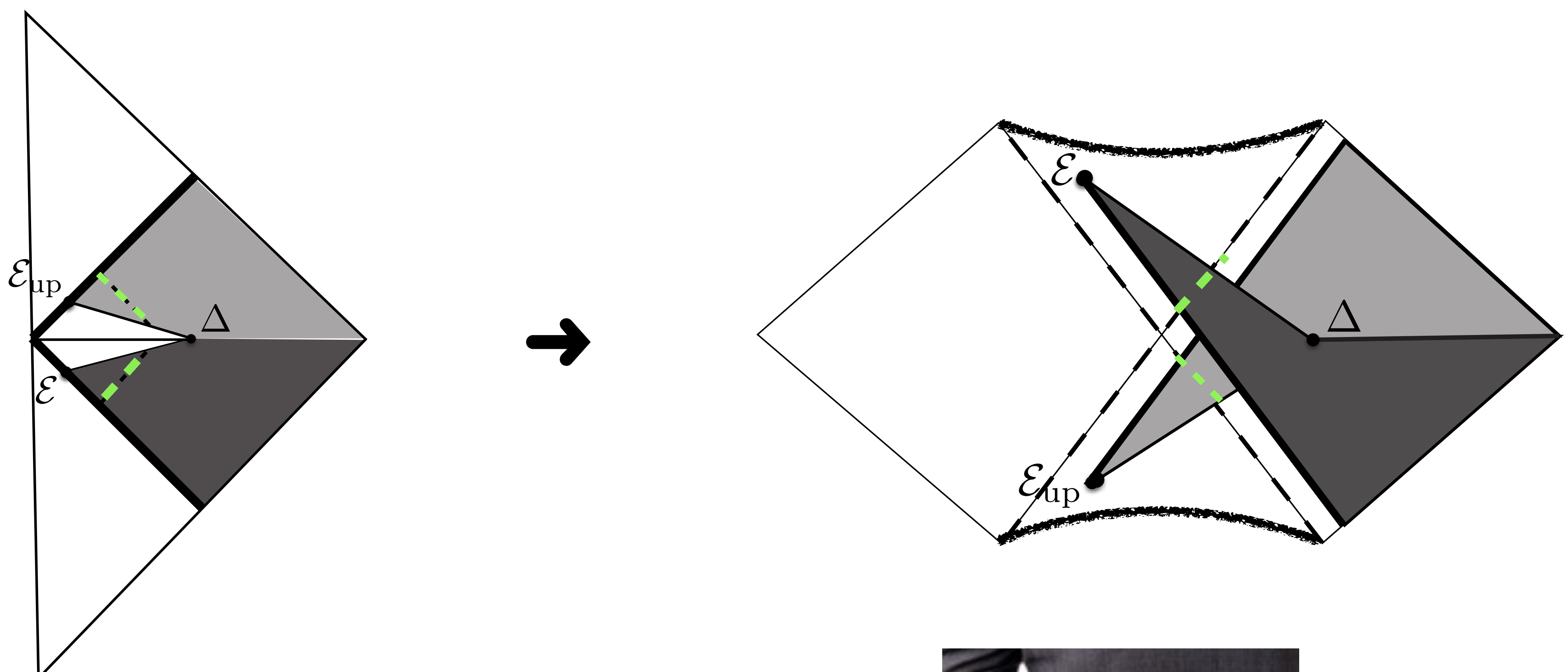
Incoming (null) shell
Minkowski region


r = 0
Black Hole
Parallel Horizon
Horizon
r = ∞
Parallel Universe
Universe
Parallel Antihorizon
Antihorizon
White Hole
r = ∞
r = ∞
r = ∞
r = 0


r = 0
Black Hole
Parallel Horizon
Horizon
r = ∞
r = ∞
Parallel Universe
Universe
Antihorizon
Parallel Antihorizon
r = ∞
r = ∞
White Hole
r = 0


$\mathcal{E}_{\rm up}$
Δ
$\mathcal{E}$
$\mathcal{E}$
Δ
$\mathcal{E}_{\rm up}$





The Fingers Crossed

The metric is determined by three constants:

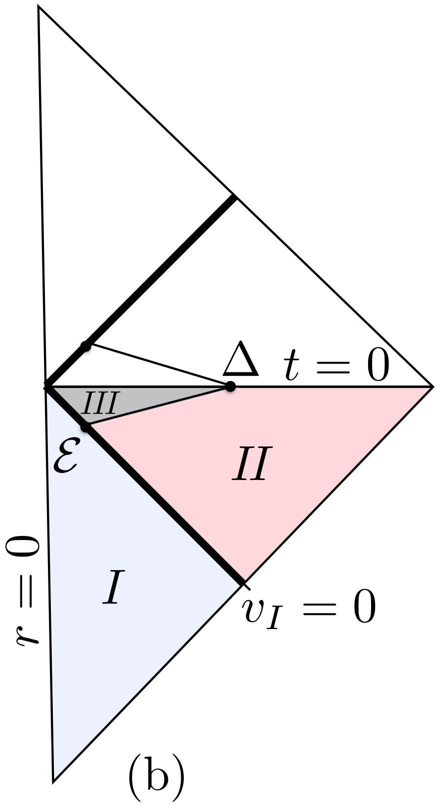
- m is the mass of the collapsing shell.

- ϵ is the radius where quantum effect start on the shell: $$\epsilon \sim \left(\frac{m}{m_P^3}\right)^{\frac{1}{3}} l_P.$$

Quantum pressure causes the bounce

Planck stars
Carlo Rovelli, Francesca Vidotto
Int.J.Mod.Phys. D23 (2014) 12, 1442026

- δ is related to the distance from the horizon where the theory is entirely classical



(b)

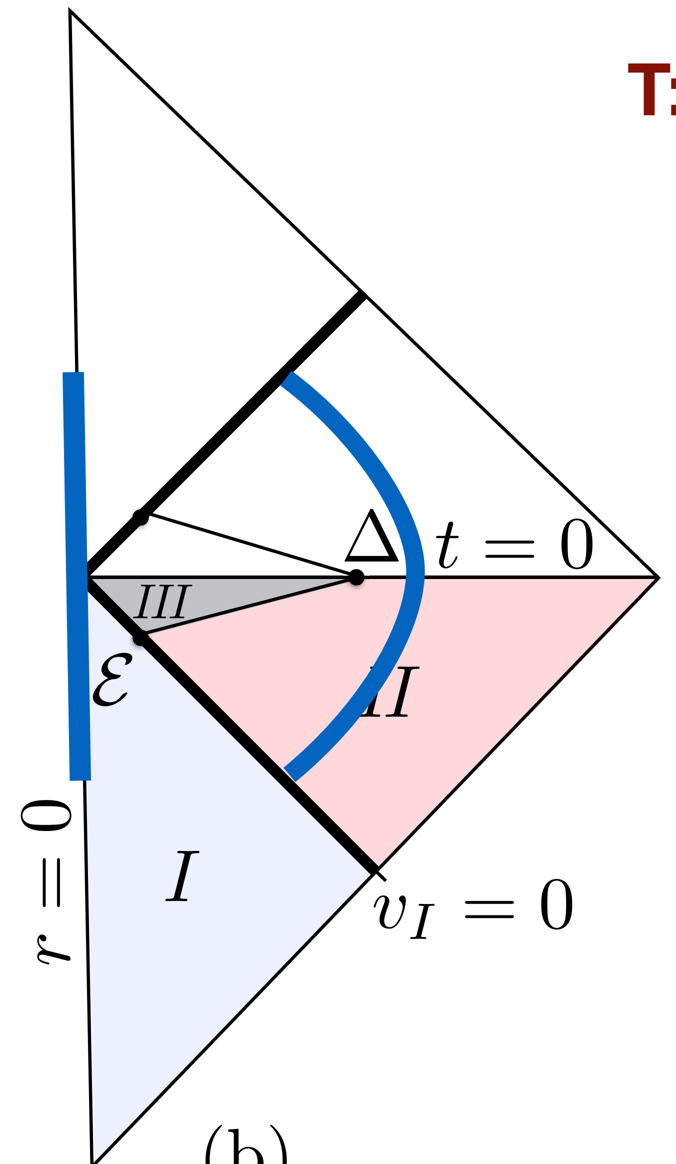
What does δ represent and what determines it?

Time dilation

$$\tau_R = 2R - m\ln(\delta/m)$$

T: bounce time (very large)

$\tau_{internal} \sim m \sim 1ms$



(b)

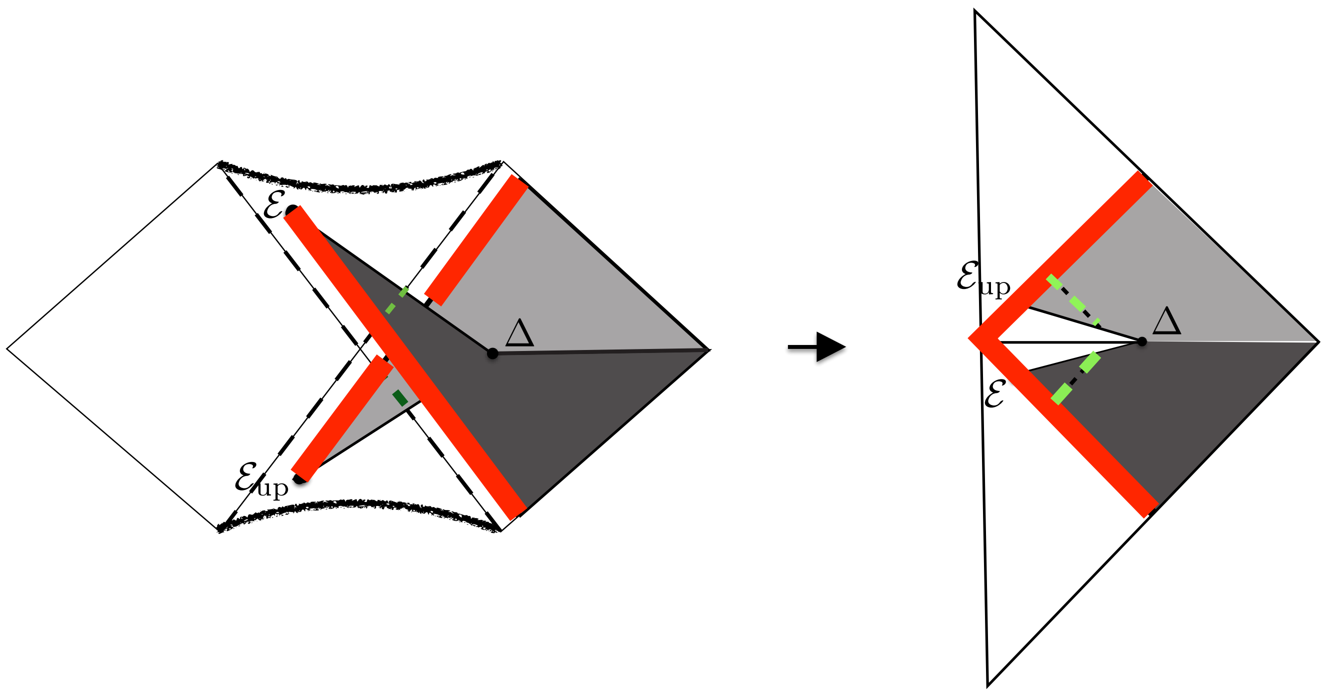
$\tau_{external} \sim m^2 \sim 10^9 years$

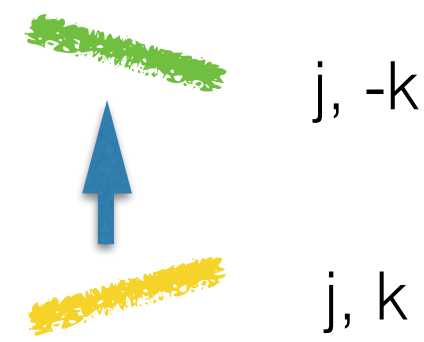
“A black hole is a short cut to the future”

What do we expect of the bouncing time:

$$T = \begin{cases} \sim e^{m} & \text{Naive expectation from analogy with tunnelling in space} \\ \sim m^{3} & \text{Page time. Requiring that AMPS firewall are avoided} \\ \sim m^{2} & \text{Minimal failure of local qft: } RT > L_{Planck}^{-1} \\ \sim m \ln m & \text{Calculation from LQG, first contribution (too short!)} \end{cases}$$

- **Covariant loop quantum gravity. Calculation of T(m).**





$$|A(j,k)|^2 \sim 1$$

$$T \sim m \ \ln m$$

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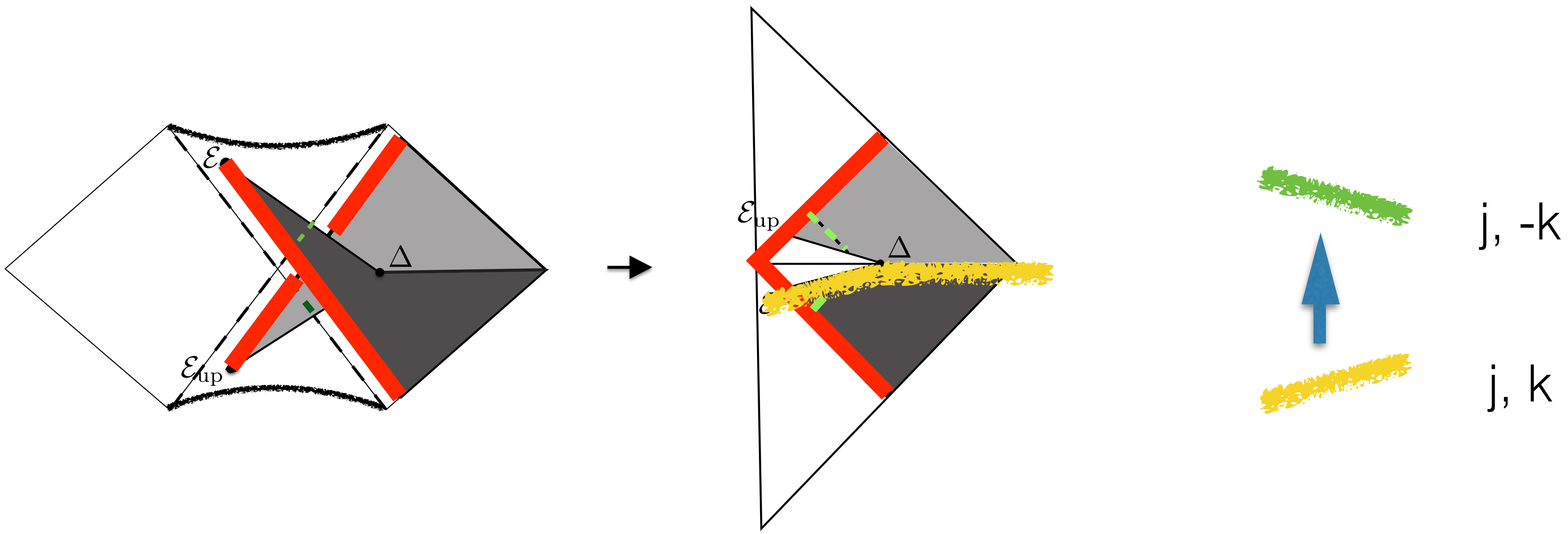

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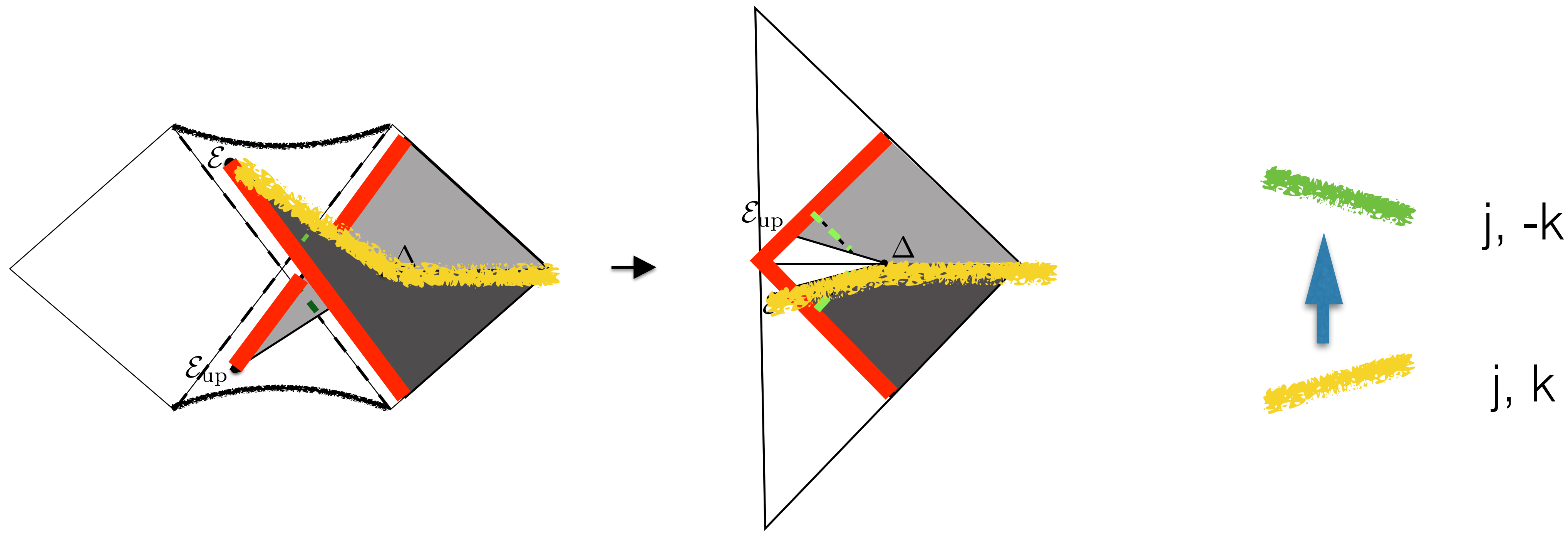

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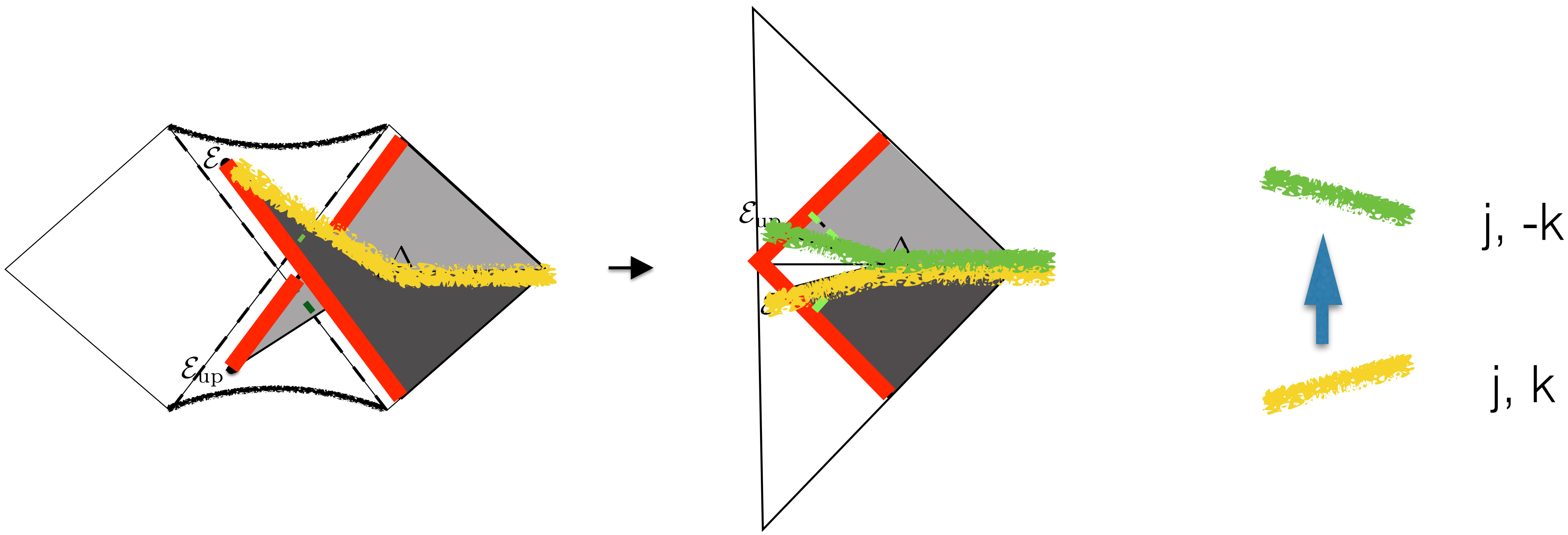

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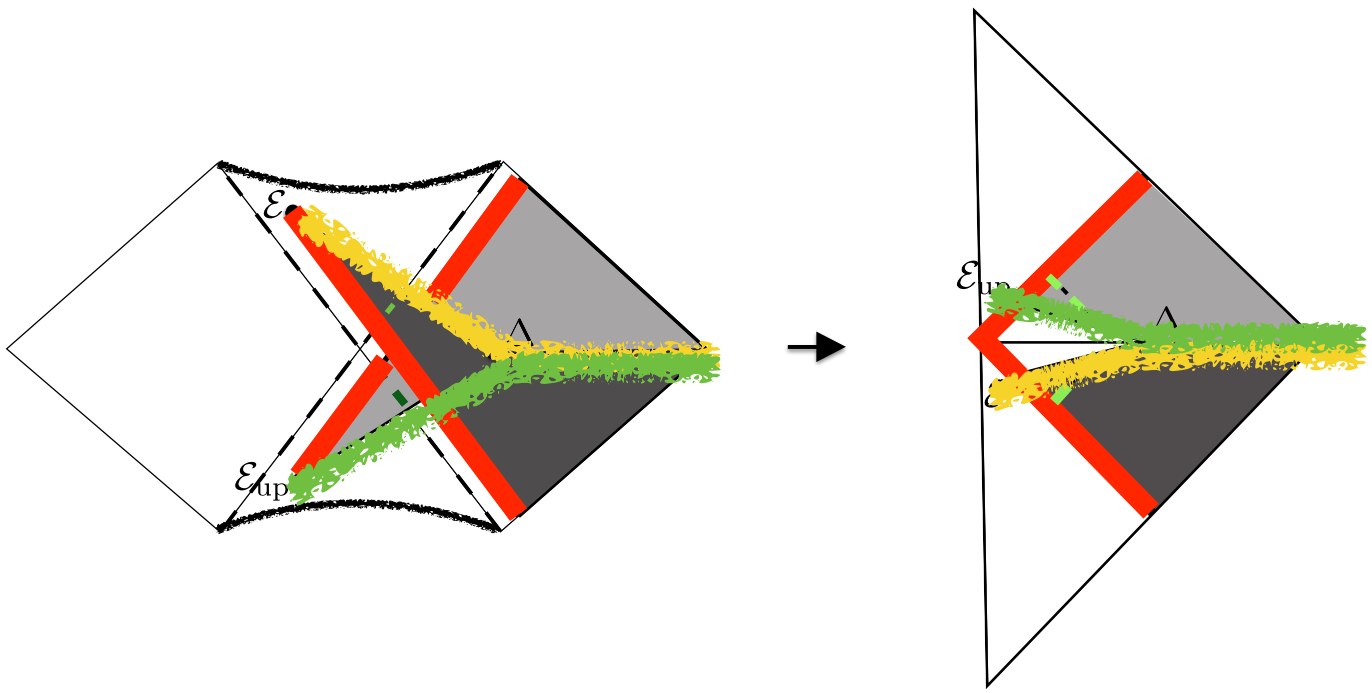

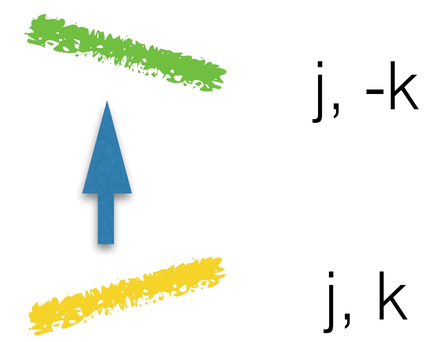

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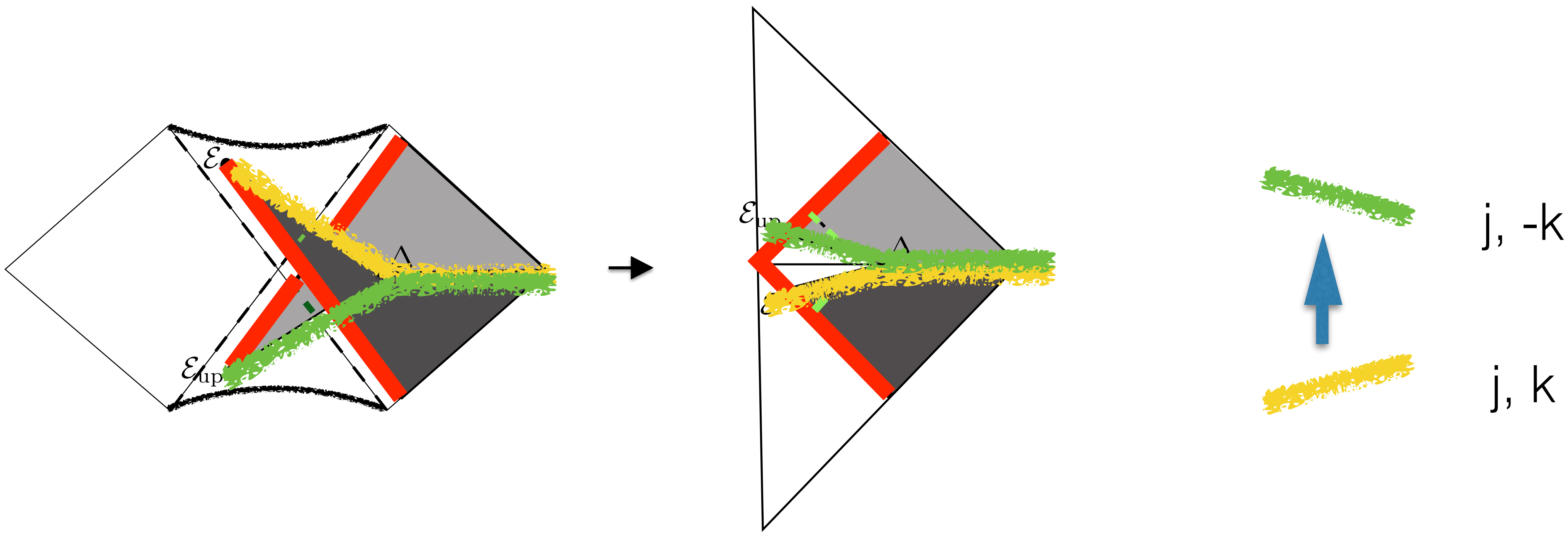

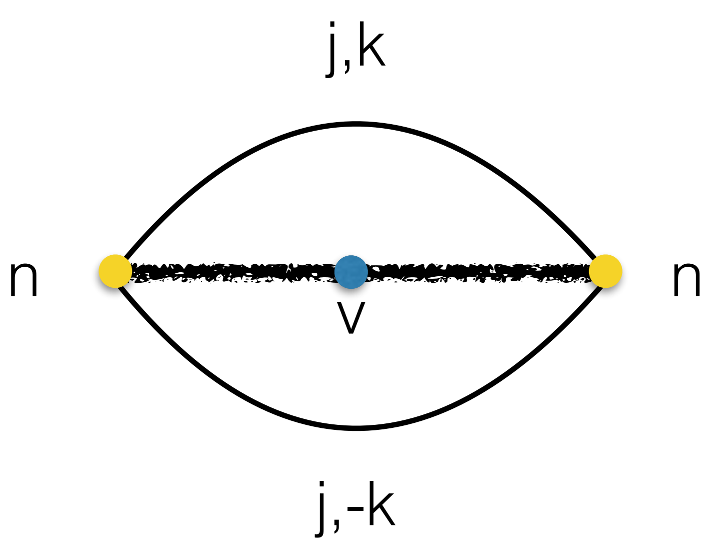

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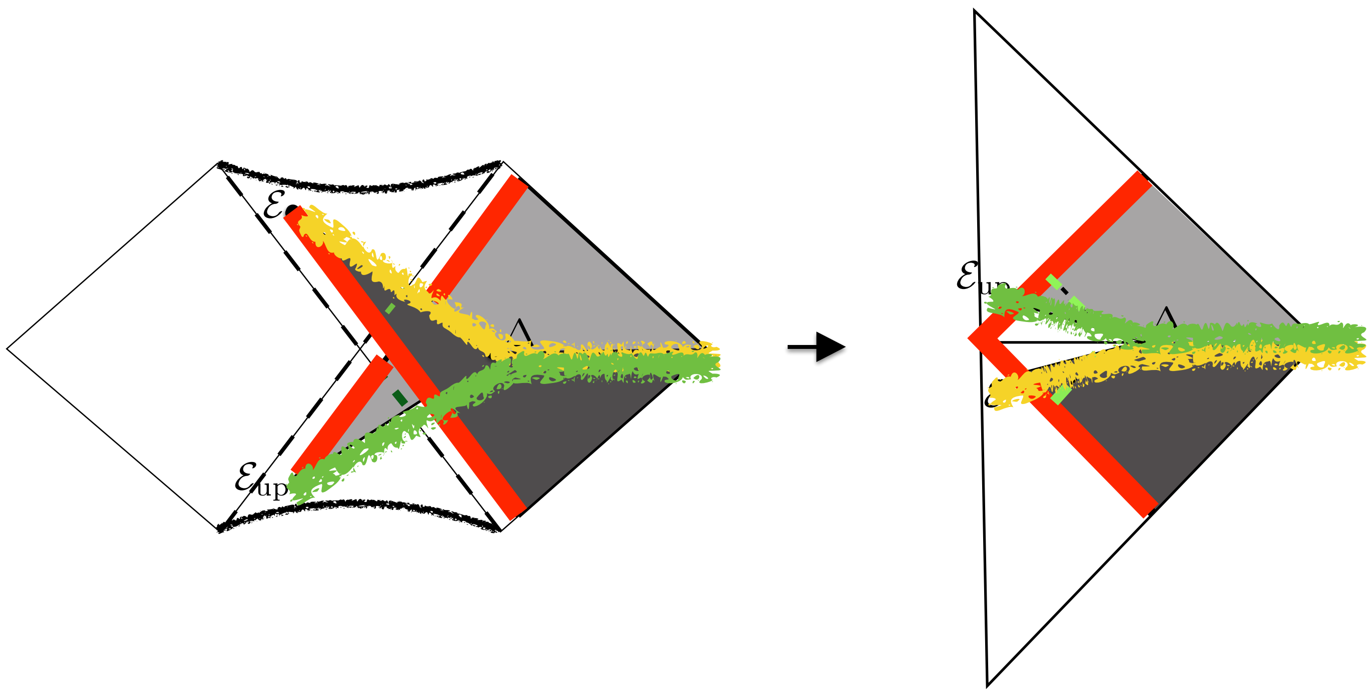

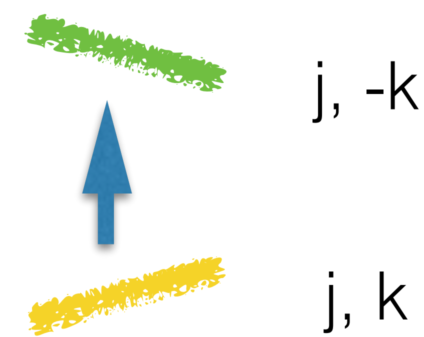

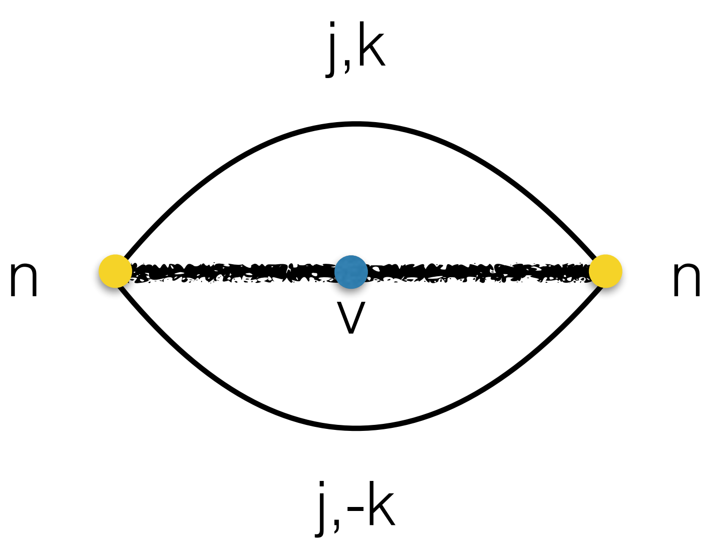

$$A(j,k) = \int_{SL(2C)} dg \int_{SU2} dh_- \int_{SU2} dh_+ \sum_{j_+ j_-} e^{-(j_+-j)^2} e^{-(j_+-j)^2}$$
$$Tr_{j_+}[e^{k\sigma_3} Y^\dagger g Y] Tr_{j_-}[e^{k\sigma_3} Y^\dagger g Y]$$

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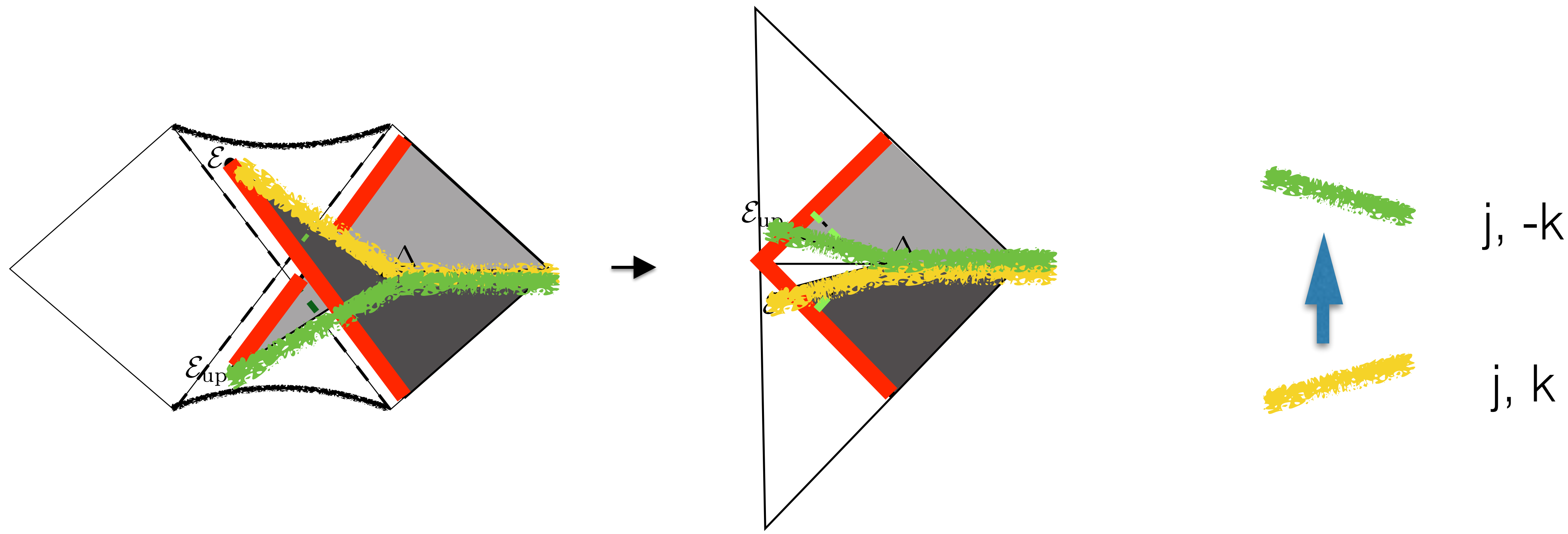
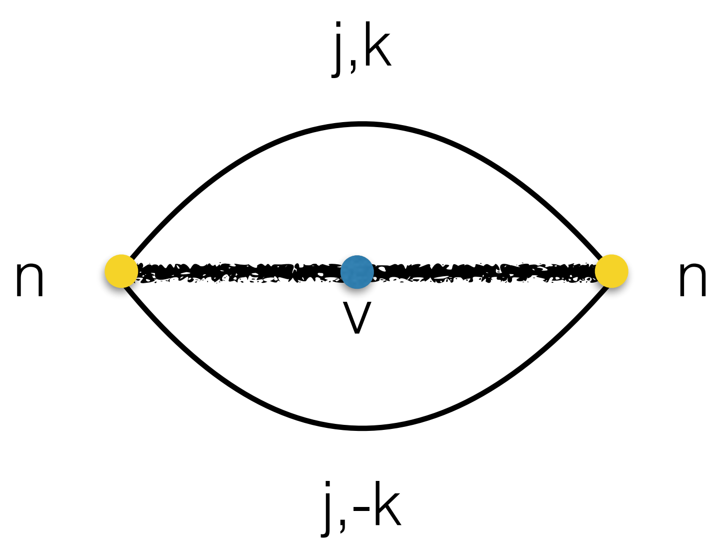
$$T \sim m \ \ln m$$

Covariant loop quantum gravity. Calculation of T(m).





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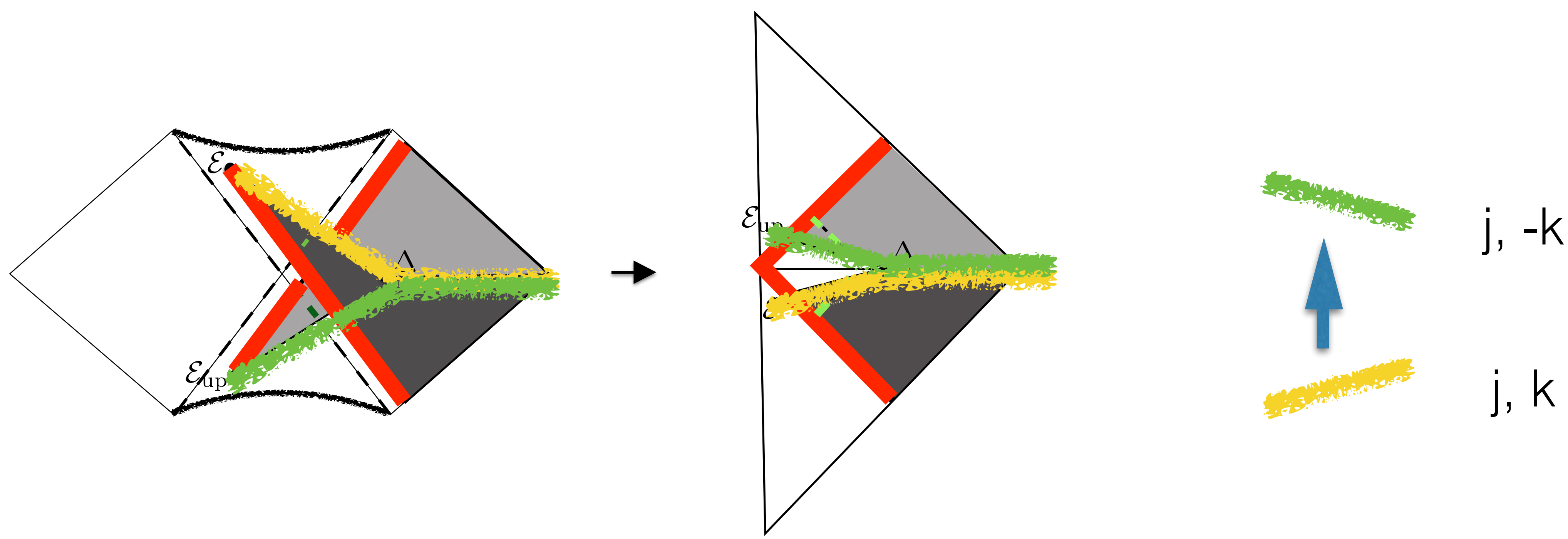
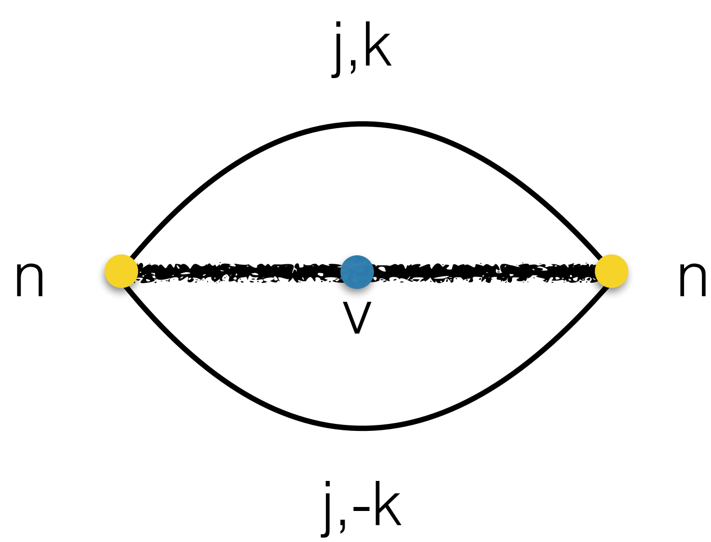
Covariant loop quantum gravity.   Calculation of T(m).





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$$T \sim m \ \ln m$$

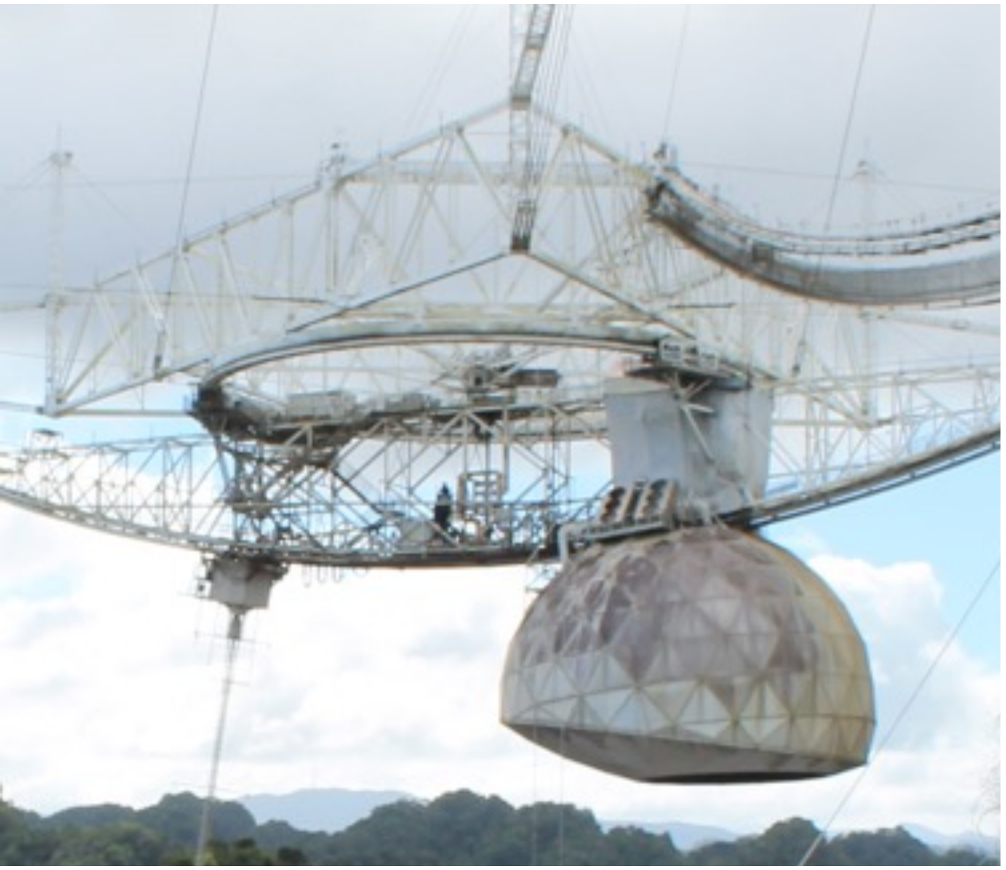
Detectable?
Already detected?

For T~m^3 primordial black hole give signals in the cosmic ray spectrum

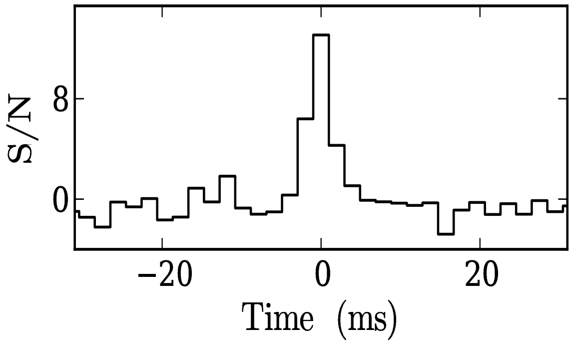
Planck star phenomenology
Aurelien Barrau, Carlo Rovelli.
Phys.Lett. B739 (2014) 405

For T~m^2 primordial black hole give signals in the radio: Fast Radio Bursts?

Fast Radio Bursts and White Hole Signals
Aurélien Barrau, Carlo Rovelli, Francesca Vidotto.
Phys.Rev. D90 (2014) 12, 127503

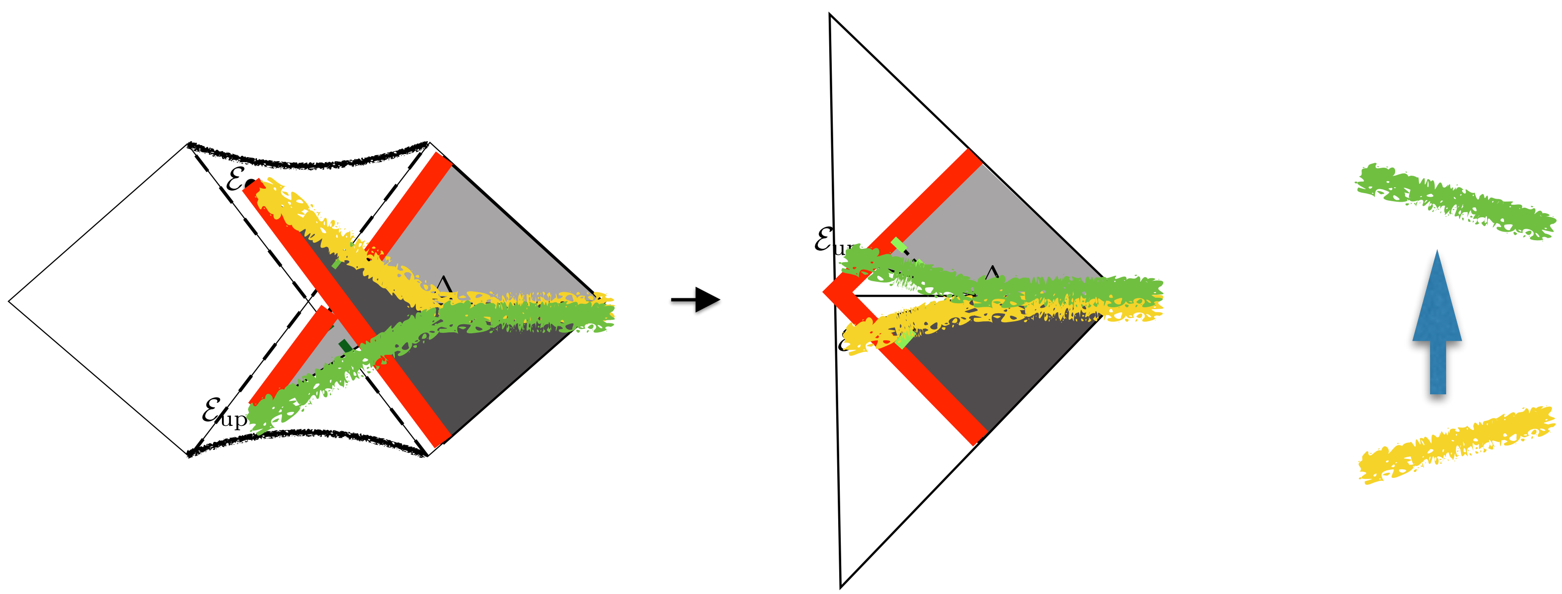


Fast Radio Bursts



- Duration: ~ milliseconds
- Frequency: 1.3 GHz
- Observed at: Parkes, Arecibo
- Origin: Likely extragalactic
- Estimated emitted power: 10^{38} erg
- Physical source: unknown.

Main message: A quantum-gravity transition amplitude calculation can be done in the bulk.



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