Han Ma

Ethan Lake

Michael Hermele

Wilbur Shirley

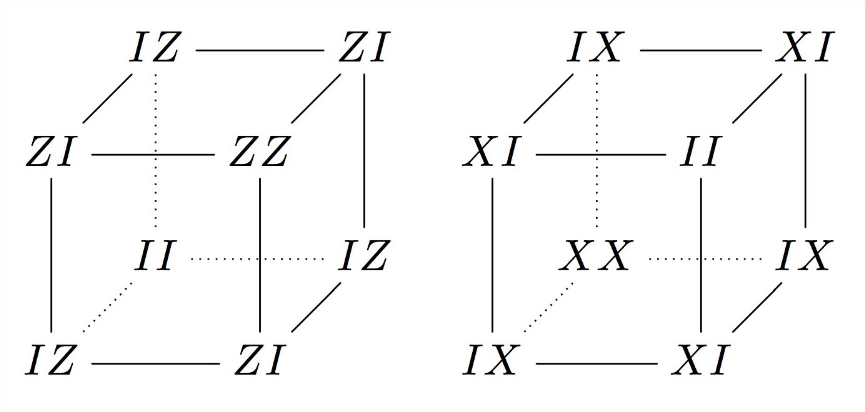
Fracton Topological Order

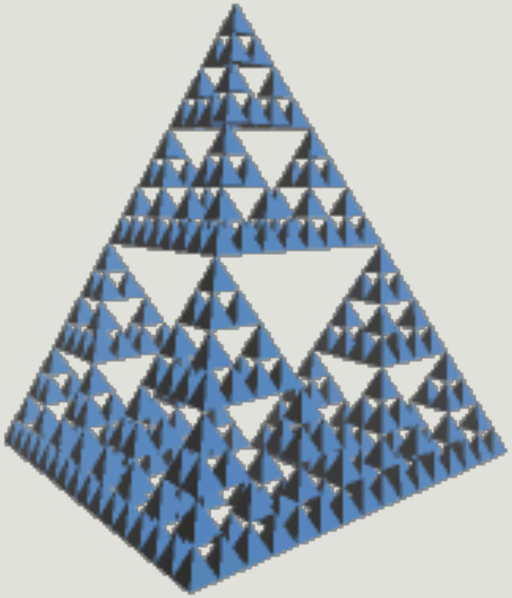
XIE CHEN, CALTECH

QINFO, OCT, 2017

arXiv:1701.00747, Han Ma, Ethan Lake, XC, Michael Hermele

Haah's code Quantum memory





Ground State Degeneracy

$$2[1 - 2q_2 + 2^{r+1}(q_2 + 12q_{15} + 60q_{63})]$$

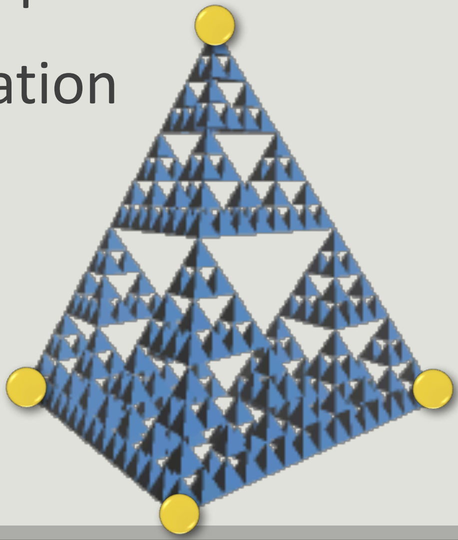
for $2 \leq L \leq 200$ $\quad q_n(L) = 1, \text{if } n \text{ divides } L, q_n(L) = 0, \text{otherwise}$

Haah, 2011

Haah's code vs. 3D anyons

- Free to move
- Appearing at ends of string
- Constant energy cost for generation and separation
- Significant thermal fluctuation

- Not free to move
- Appearing at corners of tetrahedral
- Log energy cost for generation and separation
- Thermal fluctuation suppressed



What is it?!

Where does it exotic properties come from?

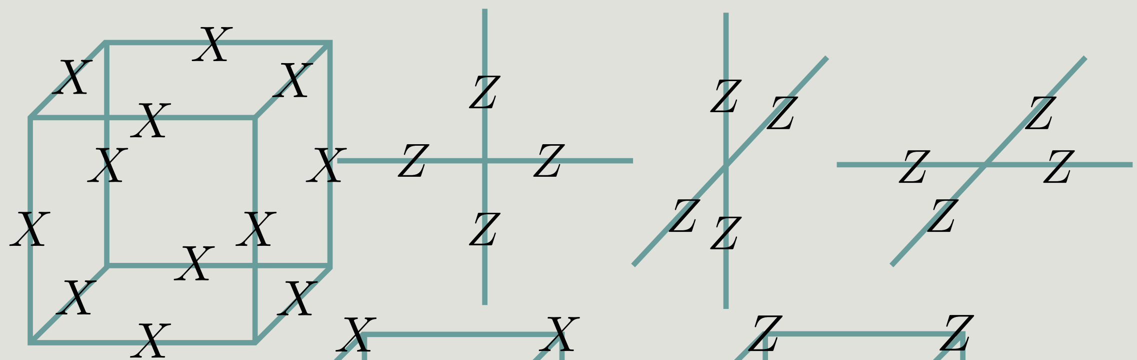
Is it related to things we already know?

How to generalize this model?
Haah, Yoshida

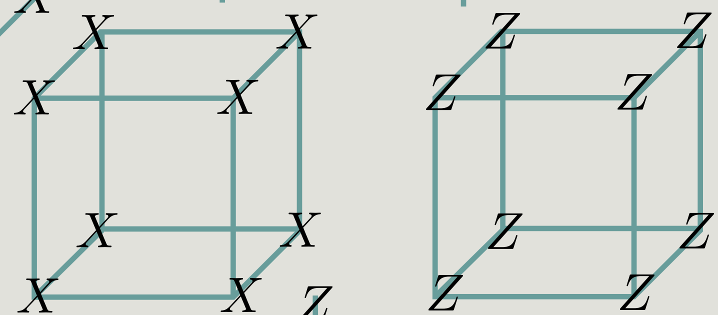
Are there other similar models with more exotic or less exotic properties?

More Models

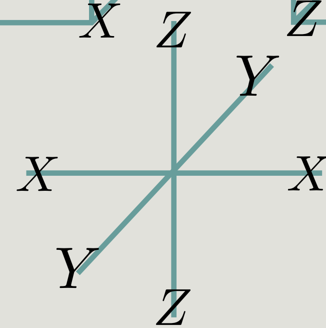
X-Cube



Checkerboard



Chamon’s model



Vijay, Haah, Fu, 15, 16; Chamon, 05

More Models

Common Properties

- "topological": ground state degeneracy cannot be lifted by local perturbation
- log(Degeneracy) ~ L
- has point excitations that does not move freely in 3D space -- fractons

Haah's code

Fracton I

Fracton II

Fracton III

- ❑ Better understanding of topological phases
- ❑ Better models for quantum memory

Known phases in 3D

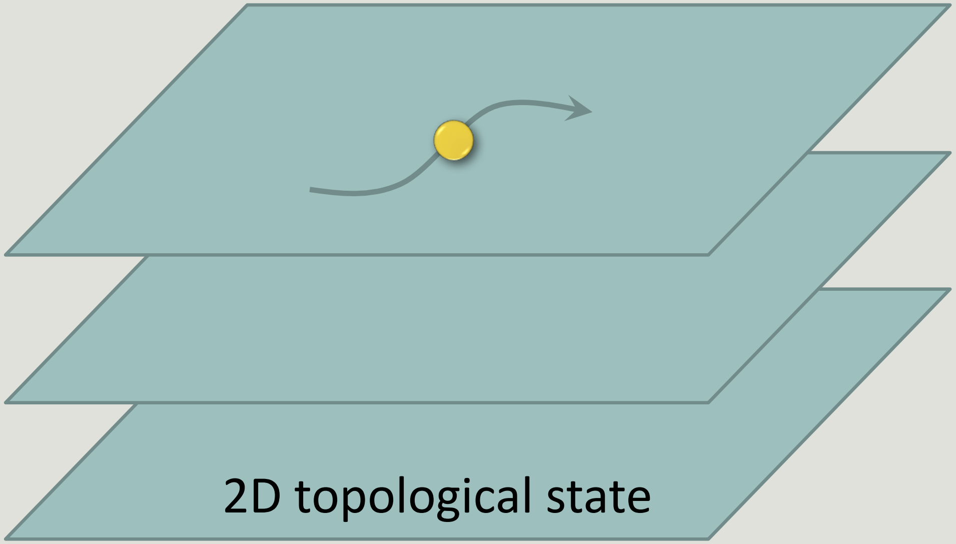
Different approaches

- Generalized gauge theory Vijay, Haah, Fu, 16
- Parton construction Hsieh, Halasz, 17
- Coupled layer construction
- Coupled chain construction Halasz, Hsieh, Balents, 17
- Higher rank gauge theory
- Glassy dynamics Prem, Haah, Nandikshore, 17
- Entanglement Shi, Lu, 17; Ma, Schmitz, Parameswaran, Hermele, Nandikshore, 17
- Field Theory Slagle, Kim, 17
- RG
- …

Coupled Layer Construction

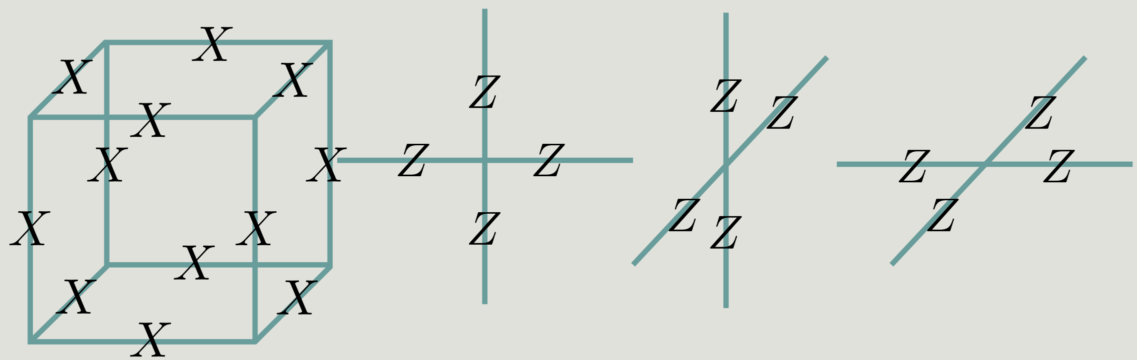
Decoupled layers of 2D Topological States

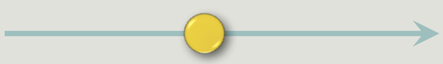
- log(Degeneracy) ~ L
- has point excitations that does not move freely in 3D space

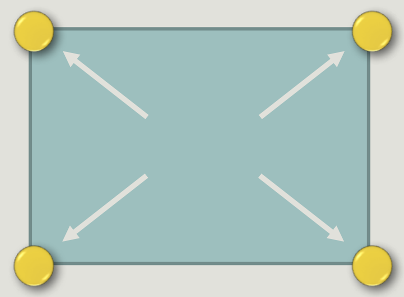


Coupled Layer Construction

X-Cube



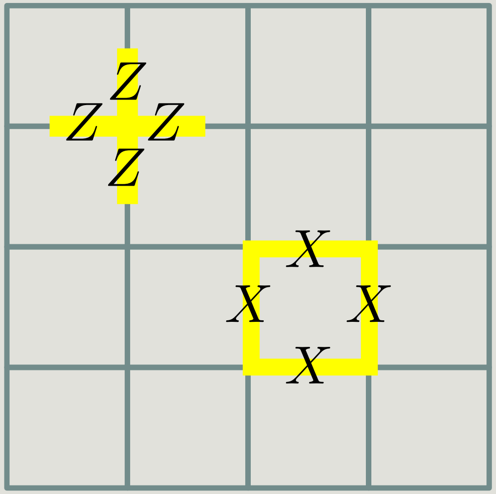


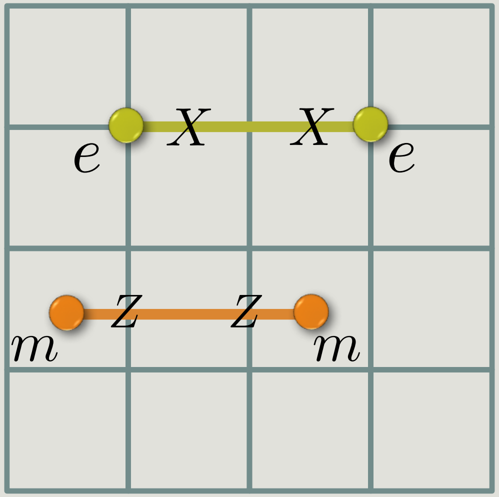


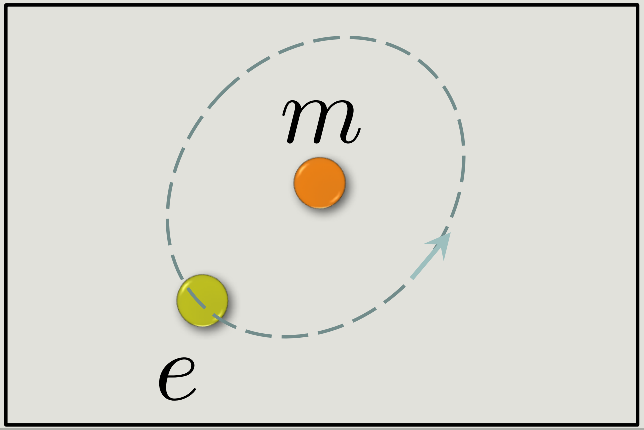
Restricted to move along a 1D line

Restricted to be at the corners of a rectangle

2D Toric Code

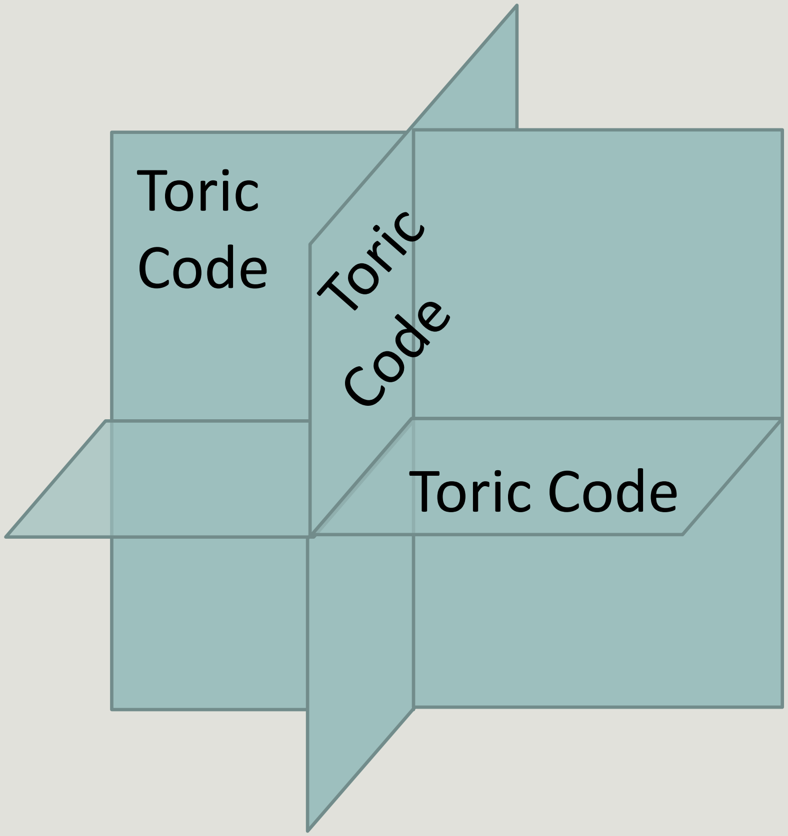


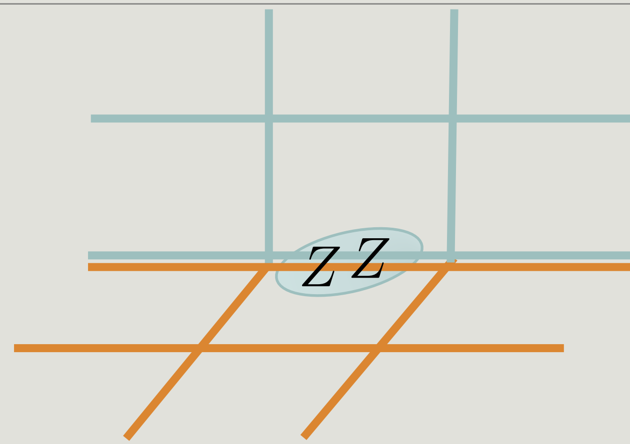




-1

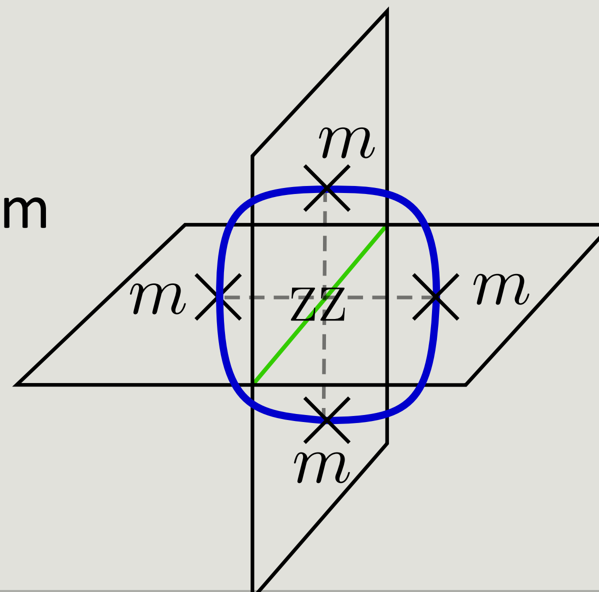
2D Toric Code to 3D X-Cube





Coupling along intersection line

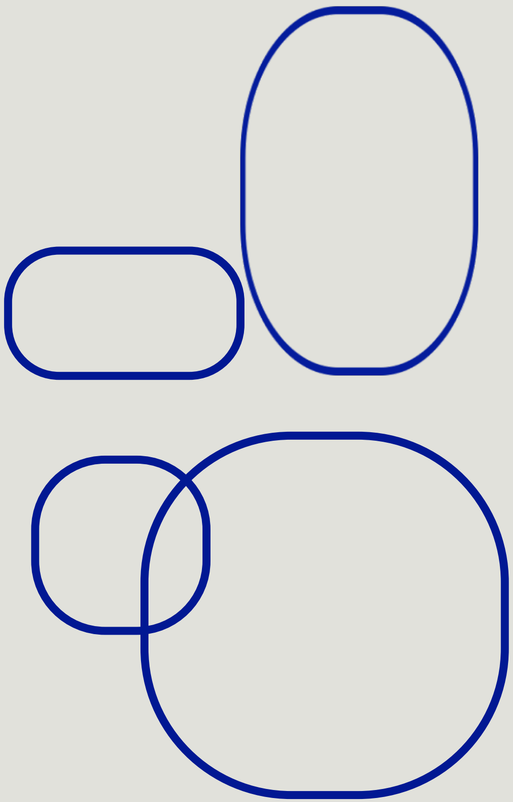
Condense m flux loops

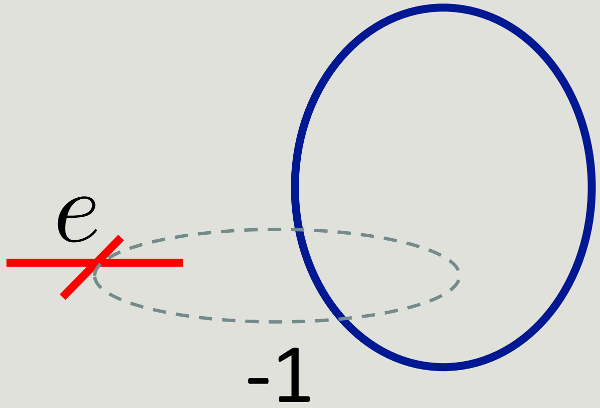


Three stacks of intersecting layers

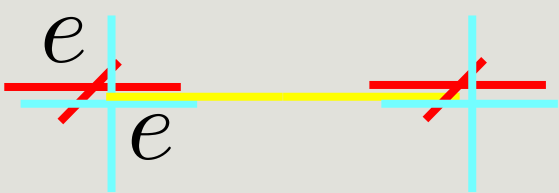
2D Toric Code to 3D X-Cube

m loop condenses





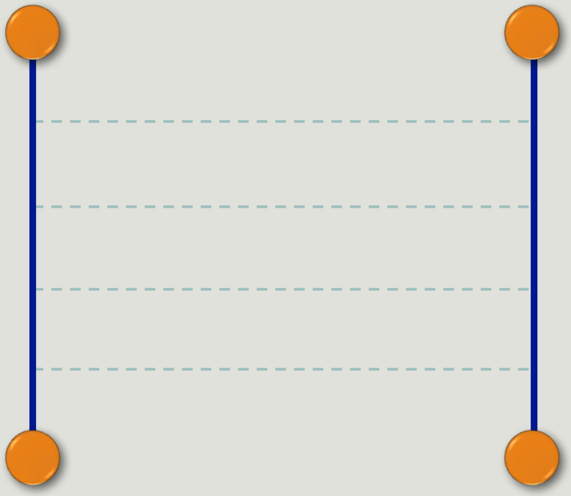
Single e particle is confined



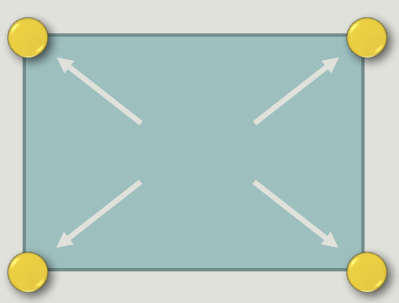
1D particle

Composite of two e particles remain

2D Toric Code to 3D X-Cube

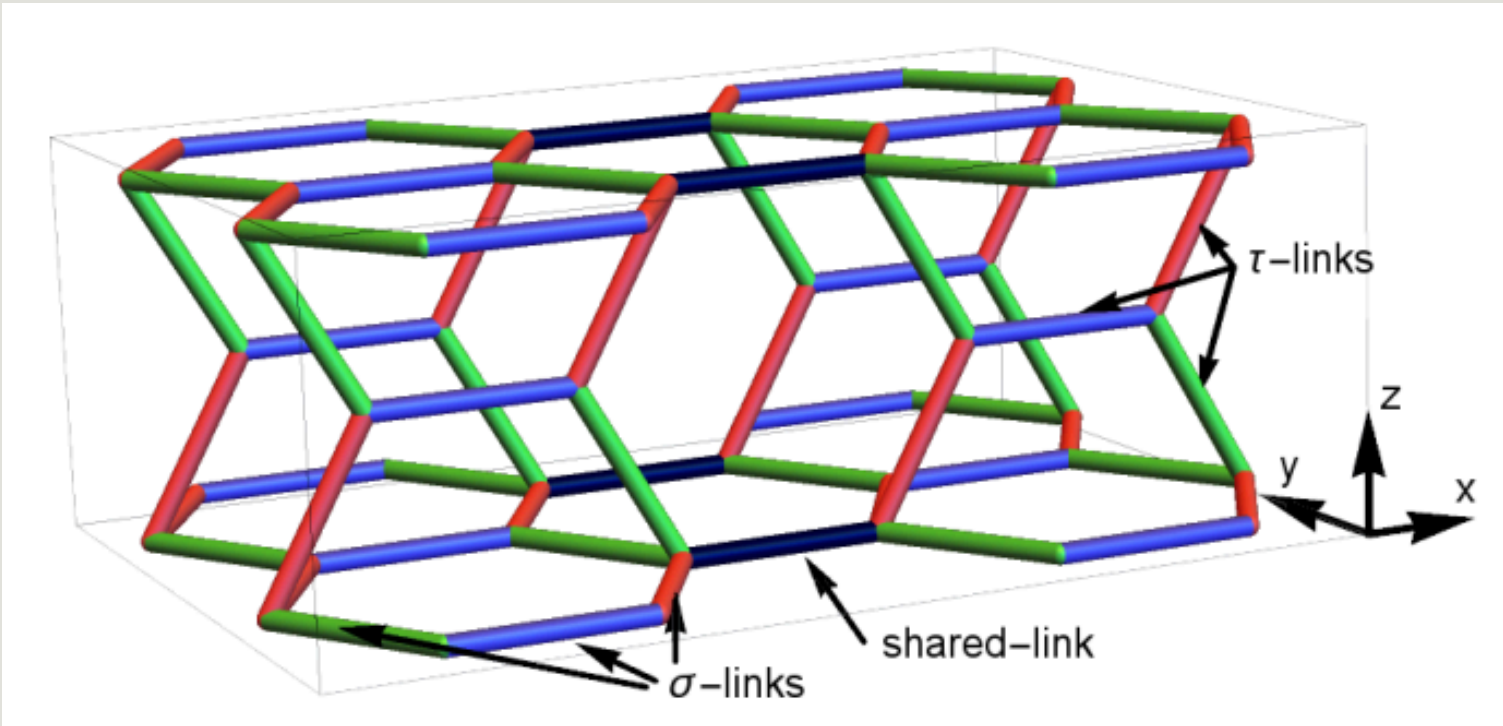


- In a condensate of m flux loop, excitations are ends of flux strings
- Flux string are created in pairs



- Ends of flux strings appear at corner of rectangle

Two body Hamiltonian for X-Cube



Higher Rank Gauge Theory

Rank 1 (normal) gauge theory $E_i, \; A_i$

Gauss' Law

$$\partial_i E_i = \rho$$

Conservation Law

$$\int \rho \; d^3\mathbf{x} = 0$$

Charge Conservation

Higher Rank Gauge Theory

Rank 2 gauge theory $E_{ij}, \ A_{ij}$

Gauss' Law

$$\partial_i \partial_j E_{ij} = \rho$$

Conservation Law

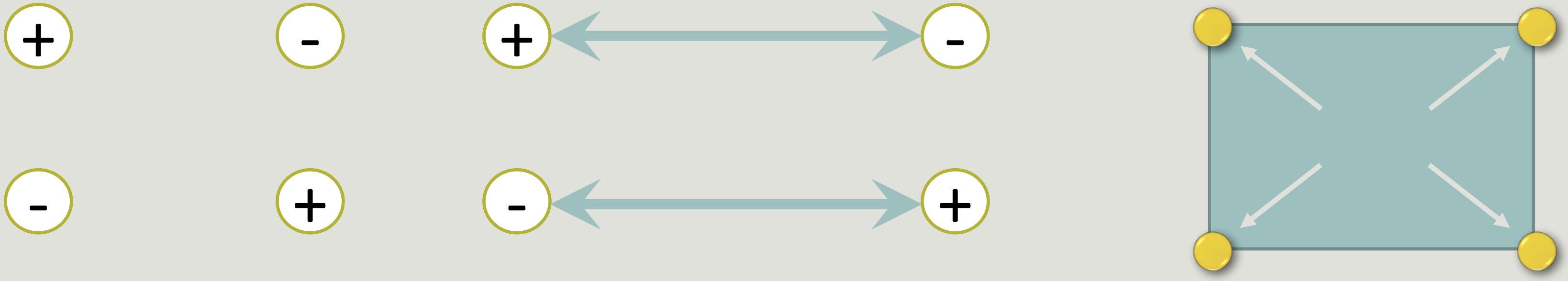
$$\int \rho \ d^3\mathbf{x} = 0$$

Charge Conservation

$$\int \rho \vec{x} \ d^3\mathbf{x} = 0$$

Dipole Conservation

Higher Rank Gauge Theory



- Charges in a rank 2 gauge theory are fractons!
- There are also gapless photon modes

  Gu, Wen, 12; Rasmussen, You, Xu, 2016;
- Gapped fracton phases by Higgsing to discrete gauge theory*

Renormalization Group

What is a fracton “phase”?

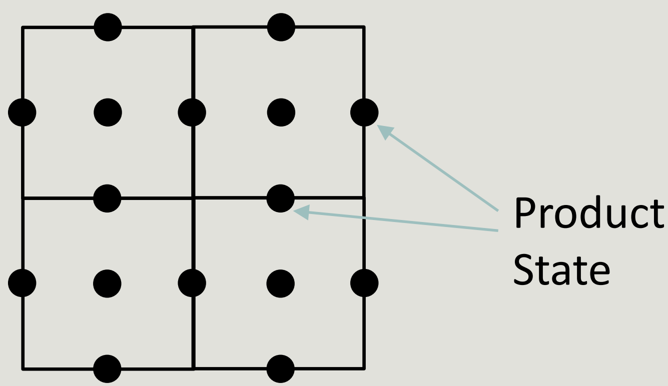
- We know a bunch of examples with exotic properties
- Which part of the properties is “universal”?
- How much of the story is lattice dependent?
- What is the criteria for saying two models have the same fracton order and belong to the same fracton phase?

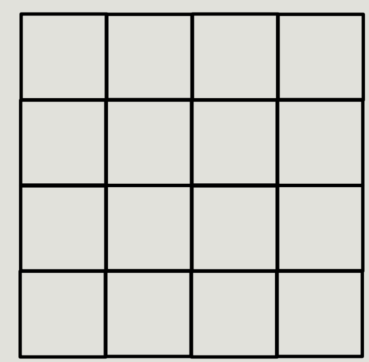
Gapped phases

H_1 Gapped Path H_2

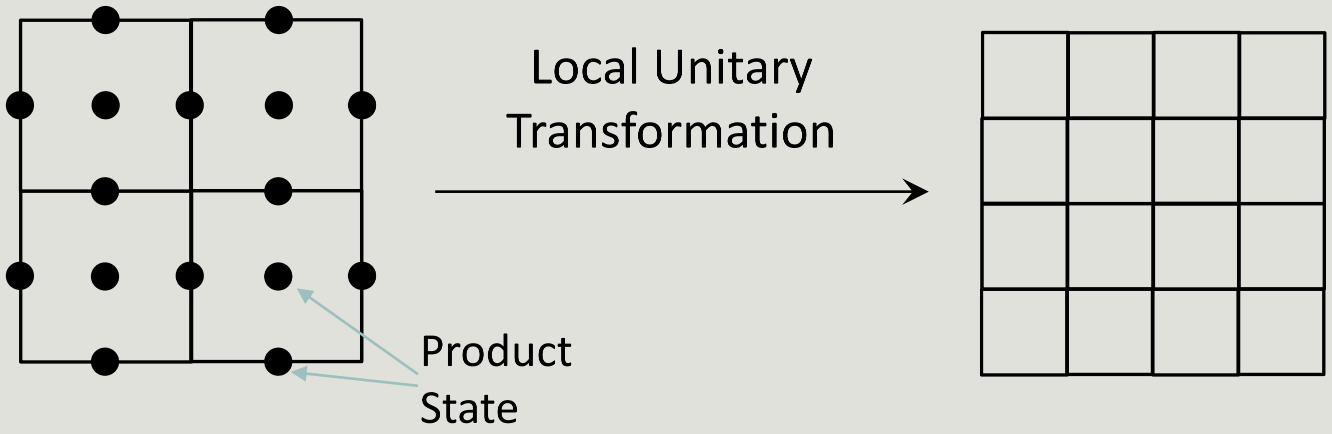
$|\psi_1\rangle$ $|\psi_2\rangle$

Local Unitary Transformation



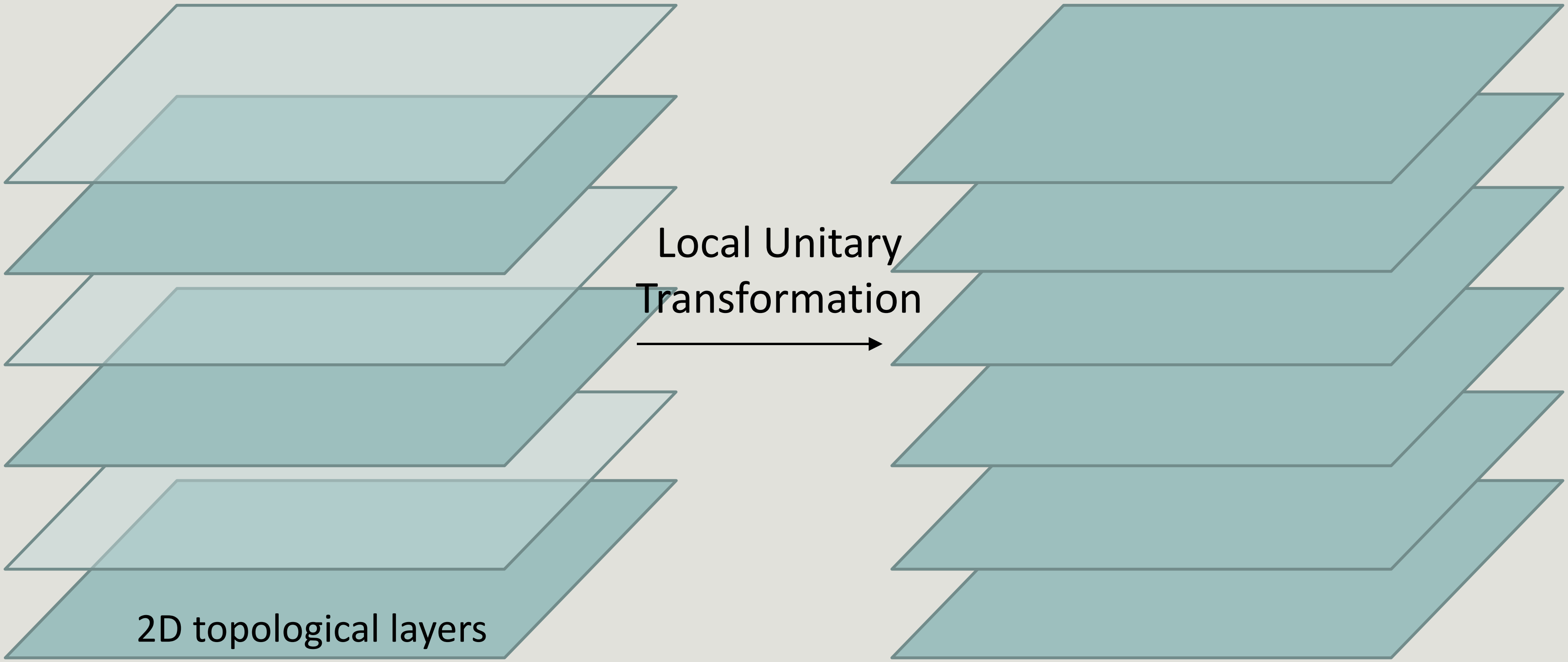


Renormalization Group



- Ground state degeneracy does not change
- Does not cover fractons

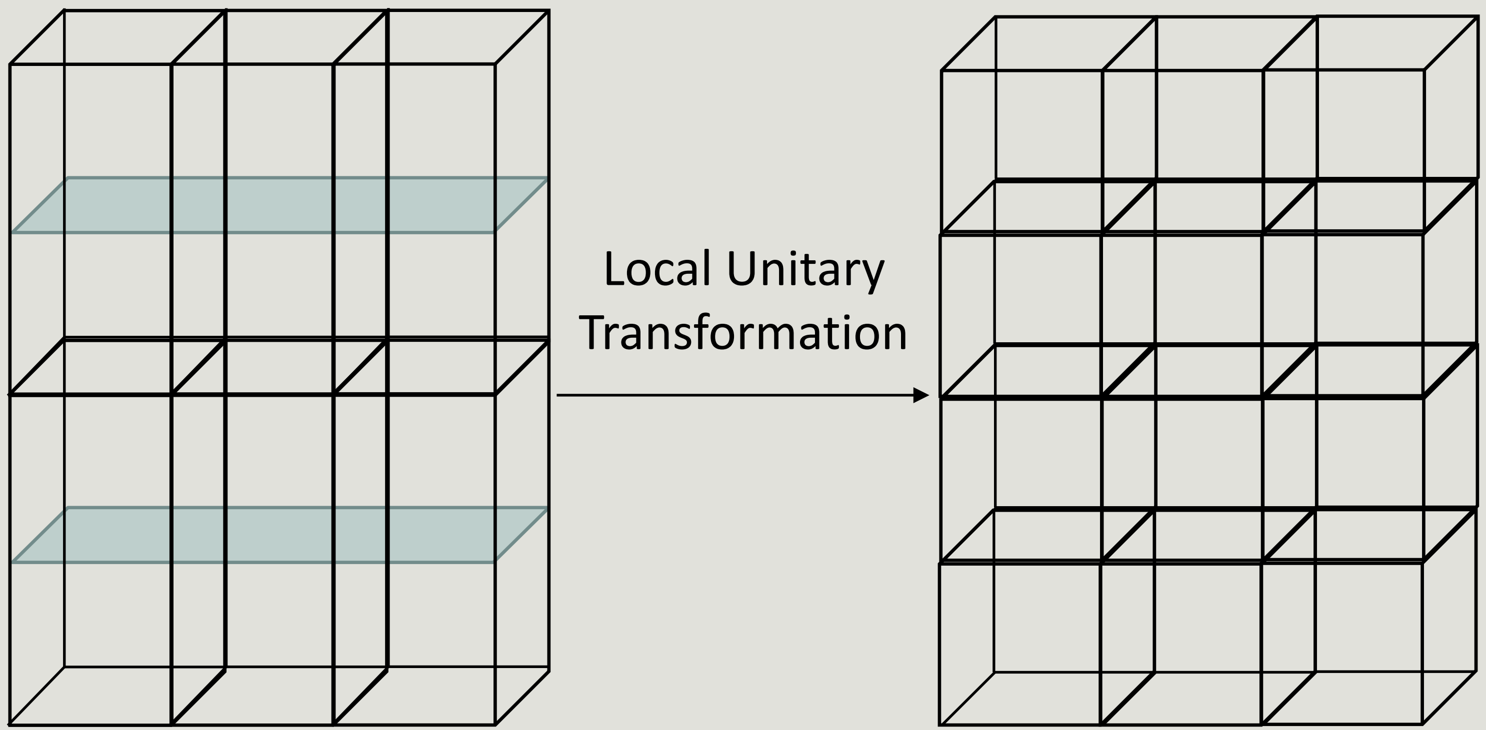
RG for fractons



2D topological layers as free resource

log(Degeneracy) ~ L

RG for fractons



X-cube, checkerboard

S-sourcery by Swingle, McGreevy

$$A(2L) \to A(L) + B(L)$$

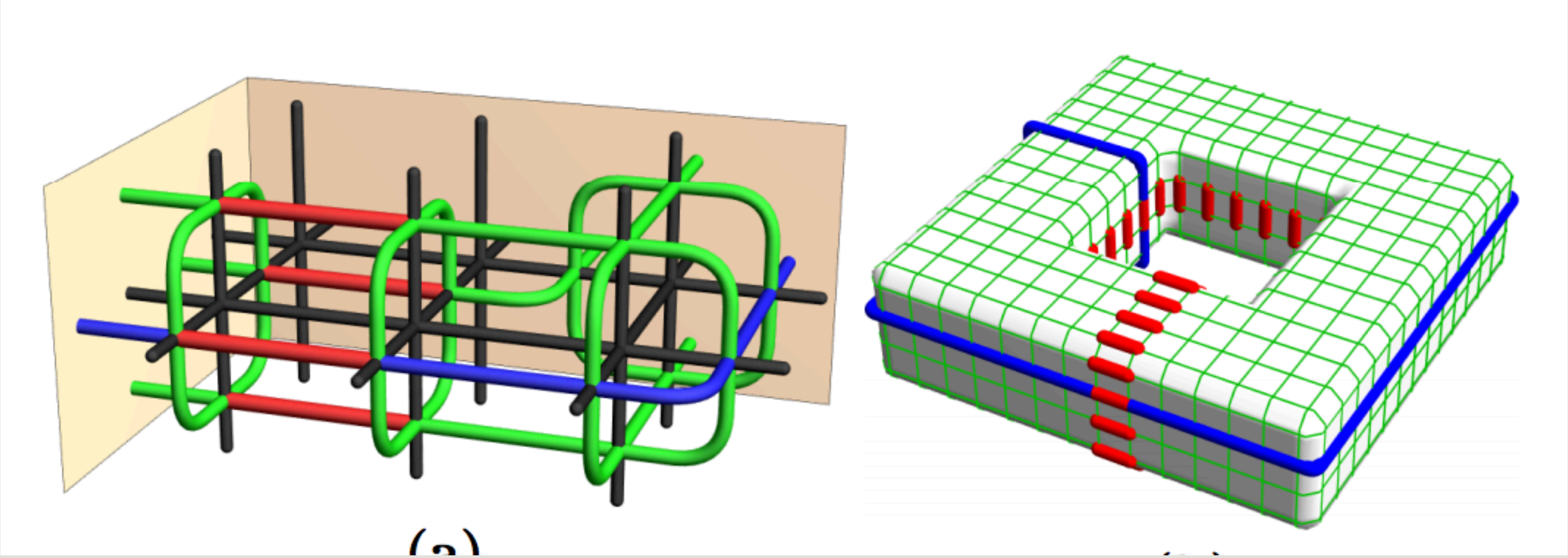
Fracton Fracton 2D Topo Layers

$$B(2L) \to 2B(L)$$

2D Topo Layers 2D Topo Layers

Haah’s code follows the same RG rule (Haah, 14)

Geometry dependent degeneracy



Open questions

- How general does this hold?
- How is this RG process related to the existence of fracton?
- Type I and Type II?