Microsoft

Extensions of the cubic code model

Jeongwan Haah
QuArC, Microsoft Research

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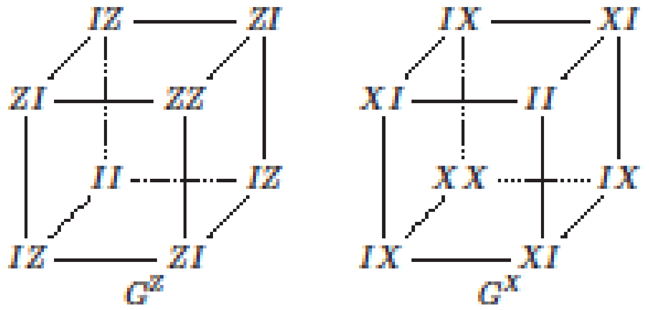
Cubic code model in 3D



$$H = -J \sum_{i \in \Lambda} \left(\sigma^x_{i,1} \sigma^x_{i,2} \sigma^x_{i+\hat{x},1} \sigma^x_{i+\hat{y},1} \sigma^x_{i+\hat{z},1} \sigma^x_{i+\hat{y}+\hat{z},2} \sigma^x_{i+\hat{z}+\hat{x},2} \sigma^x_{i+\hat{x}+\hat{y},2} \right.$$
$$\left. + \sigma^z_{i,1} \sigma^z_{i,2} \sigma^z_{i-\hat{x},2} \sigma^z_{i-\hat{y},2} \sigma^z_{i-\hat{z},2} \sigma^z_{i-\hat{y}-\hat{z},1} \sigma^z_{i-\hat{z}-\hat{x},1} \sigma^z_{i-\hat{x}-\hat{y},1} \right)$$

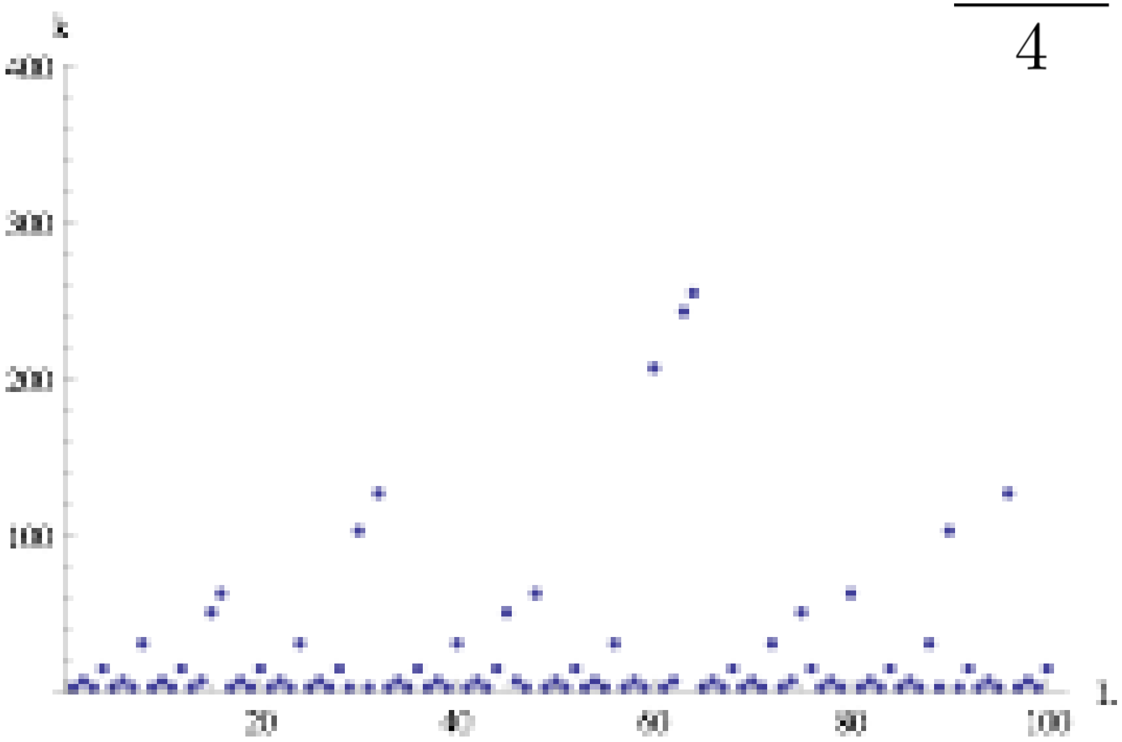
- Robustly degenerate ground state subspace.
- Any topological excitations are point-like and immobile.
- The immobility is also robust.
- G.S. has a branching MERA that live in 5-space.
  - Flat along 3D, Negatively curved along the emergent dimension.

Bravyi, Fu, JH, Kim, Nandkishore, Prem, Vijay, : 1101.1962, 1105.4159, 1310.4507, 1505.01480, 1603.04442, 1702.02952…

Degeneracy

Under periodic boundary conditions

$$2^k = \text{degeneracy}$$



$$\frac{k+2}{4} = \deg \gcd \begin{Bmatrix} 1+(1+t)^L, \\ 1+(1+\omega t)^L, \\ 1+(1+\omega^2 t)^L \end{Bmatrix}$$

$$= \begin{cases} L & \text{if } L = 2^p \\ 1 & \text{if } L = 2^p + 1 \end{cases}$$

arithmetic over $\mathbb{F}_4$

JH, 1204.1063

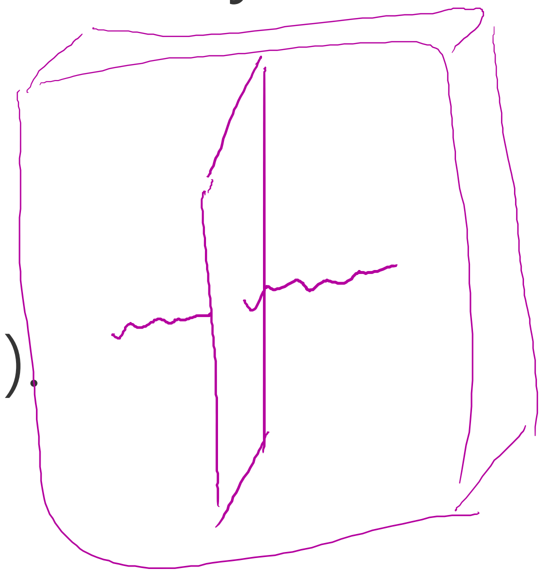
Phases of matter for thermal Q memory in 3D?

- Want to remove point-like mobile particles.
  - Thermally excited deconfined particles may destroy encoded qubit
- Only mobility removed, not the particle!
- Minimal energy barrier for any logical operator is only $\log L$.

- WISH:
  - (1) Higher energy barrier,
  - (2) Higher dimensionality of excitations

"Dimensional duality"

- Operators that act on the ground space have duality in dimension.
  - In 2D, conj. op. of a string op. (1) is a string (1).
  - In 3D, conj. op. of a string op. (1) is a membrane (2).
  - In 4D, conj. op. of a membrane op. (2) is a membrane (2).



- Operator's min. dim. $\leq \left\lfloor \frac{D}{2} \right\rfloor$
- Excitation's min. dim. $\leq \left\lfloor \frac{D}{2} \right\rfloor - 1$

Results

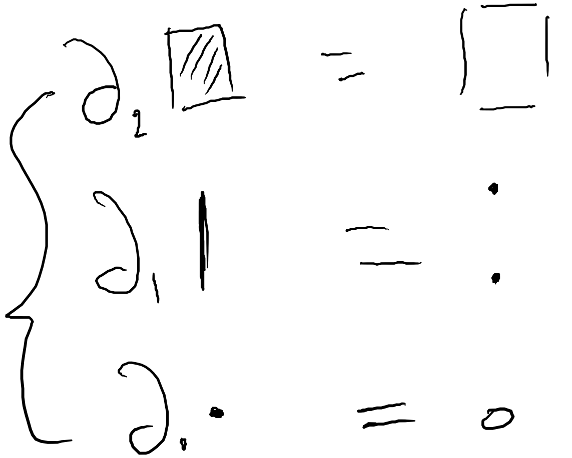
- In 5D, there exists a qubit gapped model without any point-like or string-like excitation, which must've existed under "dimensional duality."

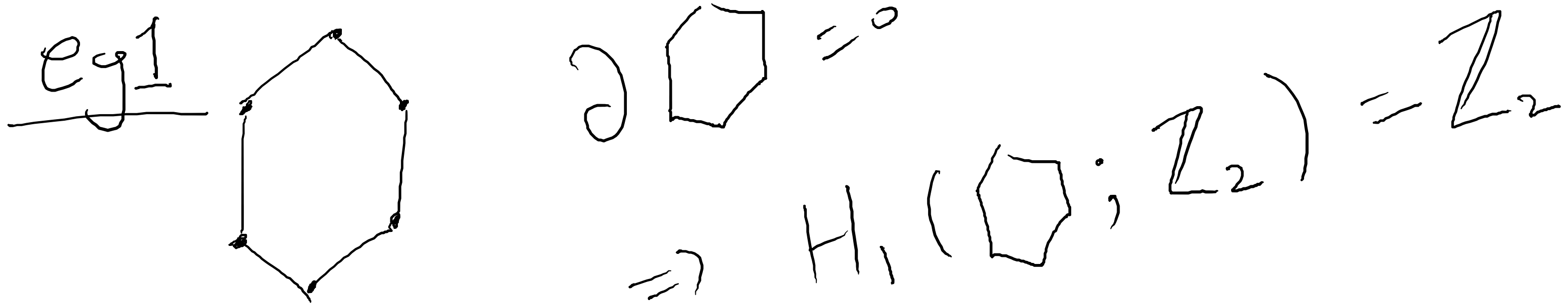
- In 3D, there exists a rotor model where the energy barrier of isolating a charge is exponential** in the separation distance $\boldsymbol{\ell}$. ("Flux" can be arbitrarily small.)

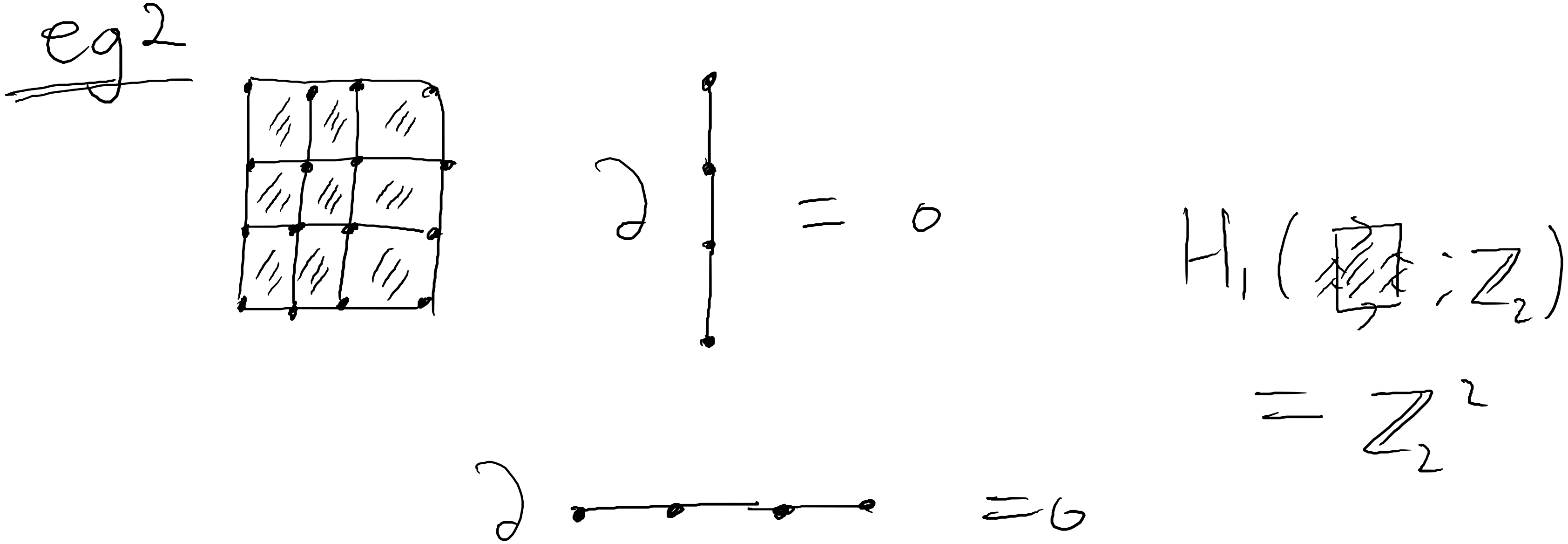
  - ** if the anti-particle is contained in a proper cone, and a charge q has energy $q^{\alpha>0}$.
  - ** if a charge q has energy $\log(1+q)$, then energy barrier is $\geq \Omega(\ell)$.

Polynomial Representation and Extensions of Models

Cellular homology







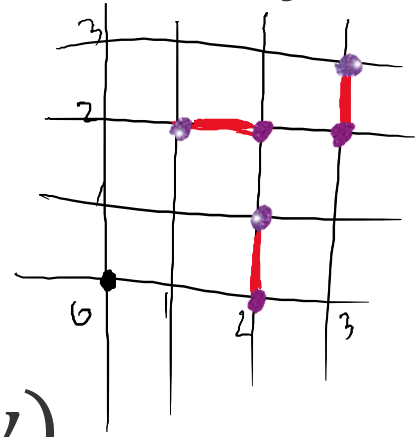
$$H_k(M) = \frac{\ker \partial_k}{im\ \partial_{k+1}}$$

Blind calculation of homology

- Polynomial representation of cells and boundary maps (all over $\mathbb{Z}/2\mathbb{Z}$)

$$C_0 = (x^2 + x^2 y + x y^2 + x^2 y^2 + x^3 y^2 + x^3 y^3)$$

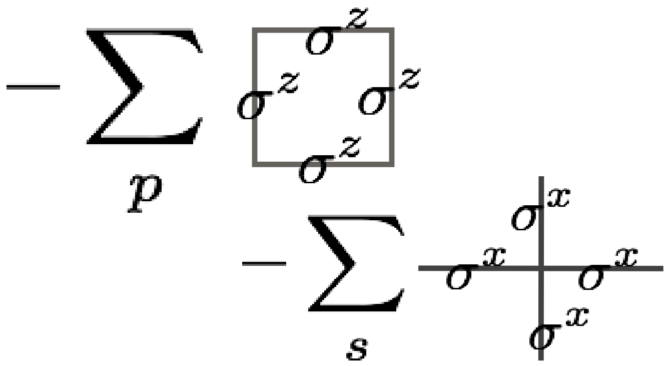
$$C_1 = \begin{pmatrix} x y^2 \\ x^2 + x^3 y^2 \end{pmatrix}$$



$$\partial_1 = (1 + x,\ 1 + y) \qquad \partial_2 = \begin{pmatrix} 1 + y \\ 1 + x \end{pmatrix}$$

subject to boundary conditions $x^L = 1, y^L = 1$

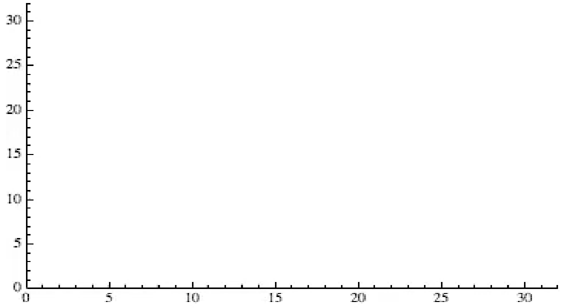
Hamiltonians realizing chain complexes



$$\partial_1 = (1+x \;\; 1+y),\;\; \partial_2 = \begin{pmatrix} 1+y \\ 1+x \end{pmatrix}$$

- Plaquette term is $\boldsymbol{\partial_2}$.
- $\boldsymbol{\partial_1}$ describes the star-term violation upon action by $\boldsymbol{\sigma^z}$.

- $\dfrac{\mathbf{ker}\,\boldsymbol{\partial_1}}{\boldsymbol{im}\,\boldsymbol{\partial_2}} = \mathbf{0}$ without boundary
  ⇔ No local observable on G.S.
  ⇔ Error correcting code

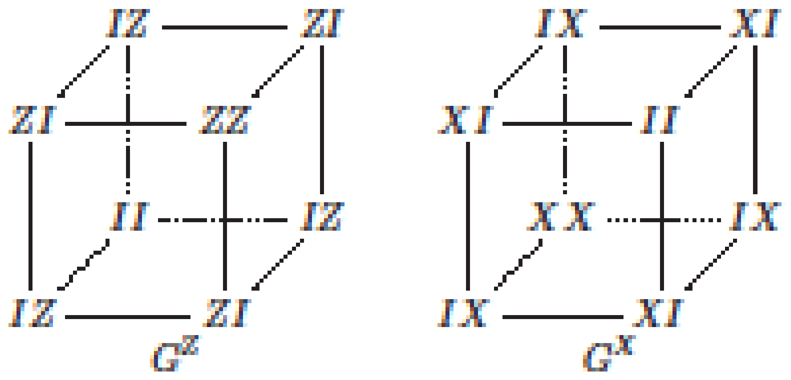


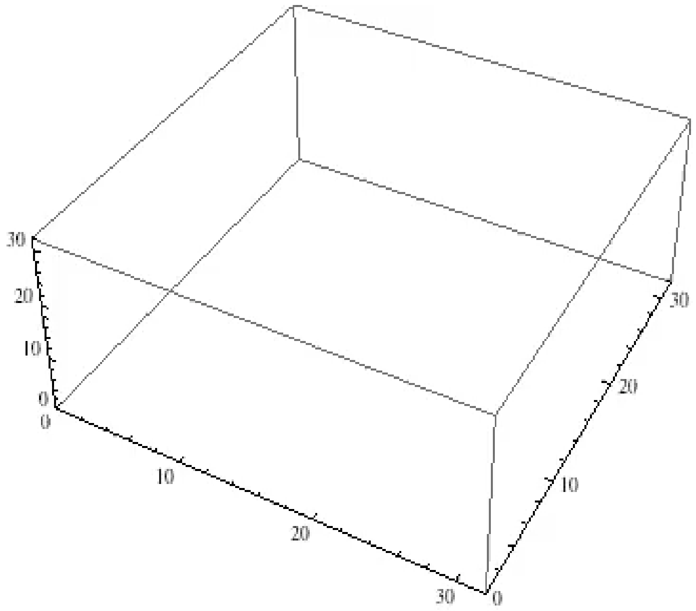
Hamiltonians realizing chain complexes

$$\partial_1 = (1 + x + y + z\,, 1 + xy + yz + zx),$$

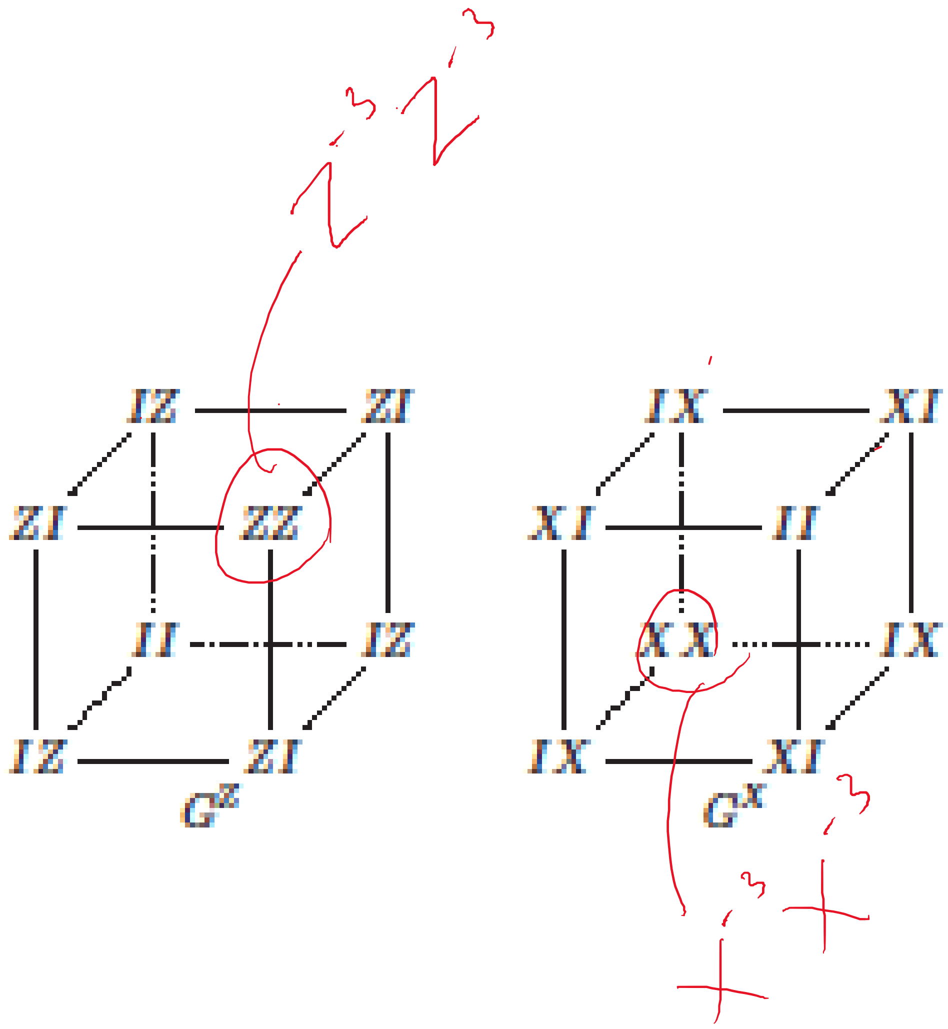
$$\partial_2 = \begin{pmatrix} 1 + xy + yz + zx \\ 1 + x + y + z \end{pmatrix}$$

- $\mathbf{ker}\,\partial_1 = im\,\partial_2$ without boundary





H from chain complex with coeff. in $\mathbb{F}_p$



$$\partial_1 = (x + y + z - 3\,, xy + yz + zx - 3),$$

$$\partial_2 = \begin{pmatrix} xy + yz + zx - 3 \\ x + y + z - 3 \end{pmatrix}$$

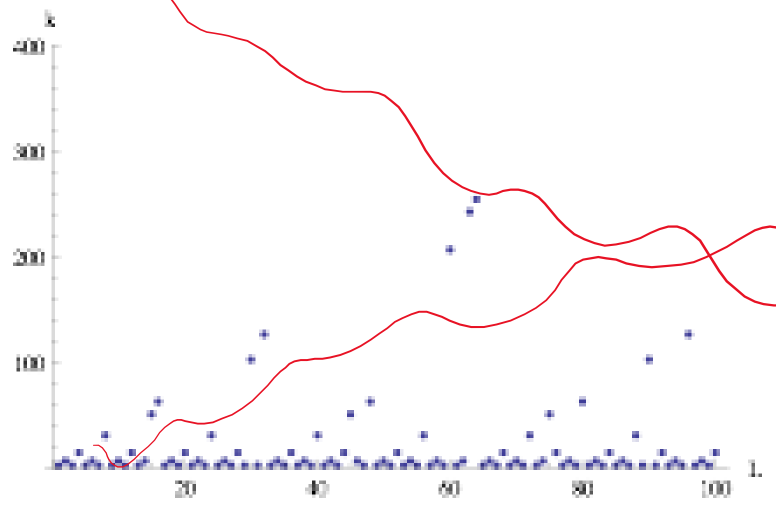
$$X = \sum |j+1\rangle\langle j|$$
$$Z = \sum e^{2\pi i j/p} |j\rangle\langle j|$$

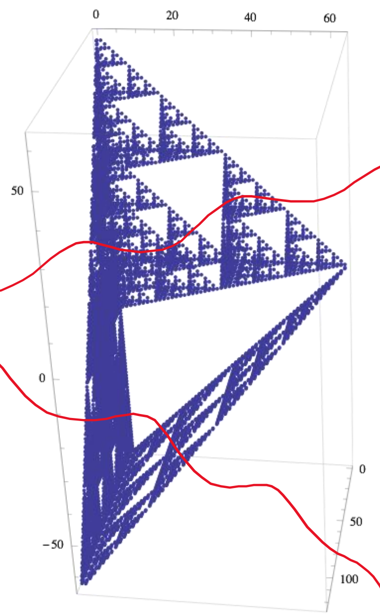
Degeneracy = p^{2k}

- The formula is uniform over p. Unless $p|L$, we have

$$\frac{k+1}{2} = \deg_t \gcd\left[(1+t)^L - 1,\ (1+\omega t)^L - 1,\ (1+\omega^2 t)^L - 1\right]$$

= 1 for all sufficiently large p





for large p

Hence, in $p \to \infty$, G.S. consists of two rotors.

Only if $p \ll L$.

Isolating a charge in U(1) model

- $H = \sum_c \left(\sum_{i=1}^{8} -i \frac{\partial}{\partial \theta_{s_{c,i}}} \right)^{\alpha} - \sum_c \cos(\sum_{i=1}^{8} \theta_{\hat{s}_{c,i}}) + \lambda \sum_s L_s^2$
- Charge = violation of the "divergence" term (first term)
- "Anti"-charge at distance $\boldsymbol{d}$ has energy $\mathbf{exp}\, \boldsymbol{d}$.
- Proof sketch: Energy = $\sum_{a,b,c} n_{a,b,c}^{\alpha}$
  1. Overall configuration is created by some finitely supported operator.
  2. $1 + \sum_{a,b,c} n_{a,b,c,} x^a y^b z^c = u(x+y+z-3) + v(xy+yz+zx-3)$
  3. There is a zero of RHS such that if all charges are contained in a cone,
  $$1 \leq \sum_{a,b,c} |n_{a,b,c}| \cdot |x_0^a y_0^b z_0^c| \leq (\#terms)(\max n)\,(A>1)^{-d}$$
  4. $\max(\#terms, \max n) \geq A^d$.

Step-back: *Backbone* chain complex

- $(a\ b) \circ \begin{pmatrix} b \\ -a \end{pmatrix}$: 2D toric code/gauge theory
- $(a\ b\ c) \circ \begin{pmatrix} 0 & c & b \\ c & 0 & -a \\ -b & -a & 0 \end{pmatrix} \circ \begin{pmatrix} a \\ b \\ c \end{pmatrix}$: 3D toric code/gauge theory
- $(a\ b\ c\ d) \circ \begin{pmatrix} 0 & 0 & 0 & d & c & b \\ 0 & d & c & 0 & 0 & -a \\ d & 0 & -b & 0 & -a & 0 \\ -c & -b & 0 & -a & 0 & 0 \end{pmatrix} \circ M_{6\times 4} \circ M_{4\times 1}$:
  4D gauge theory, two versions.

- As long as the entries are algebraically independent with coefficients in $\mathbb{F}_p$,
  the spin model under the prescription has robust ground state subspace below a gap.
- Shape of excitation largely depends on the backbone.
- Detail of entries determine whether excitations are *fractons*.

e.g., 5D model without string excitation

- Fill "4D-complex"
  with symmetric polynomials in 5 variables.
- $(a\ b\ c\ d) \circ \begin{pmatrix} 0 & 0 & 0 & d & c & b \\ 0 & d & c & 0 & 0 & -a \\ d & 0 & -b & 0 & -a & 0 \\ -c & -b & 0 & -a & 0 & 0 \end{pmatrix} \circ M_{6\times 4} \circ M_{4\times 1}$

- a,b,c,d=$sym_{1,2,3,4}(x-1, y-1, z-1, v-1, w-1)$.

- Model from 1st $\circ$: Immobile point-like excitations
- Model from 2nd $\circ$: No point exc. & No line exc.

Summary – Type II fracton phases

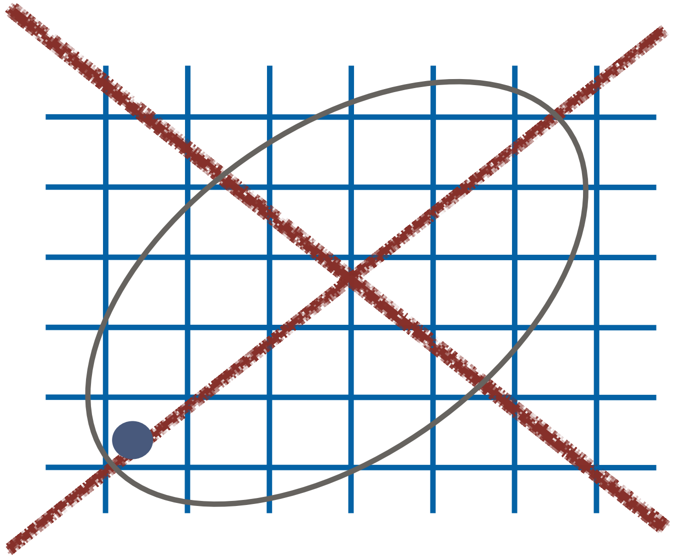
- In 3+1D or higher, there exist robust gapped phases that are not captured by (existing) TQFT.
  - Point-like charges may not be mobile.
  - Not always dimensional duality between Wilson operators
  - Minimal dimension of excitations can be $> \left\lfloor \frac{space\ dim.}{2} \right\rfloor - 1$.
  - Should redefine what "phase" is, mathematically.
- Some U(1) analog contains dynamically heavy anticharges.
  - Exp. in separation distance when in a cone.
    Similar to confinement, but totally different reason.
  - MBL in a translation-invariant system?
  - Do charge insertion op.s have finite energy, generically?

Misc.

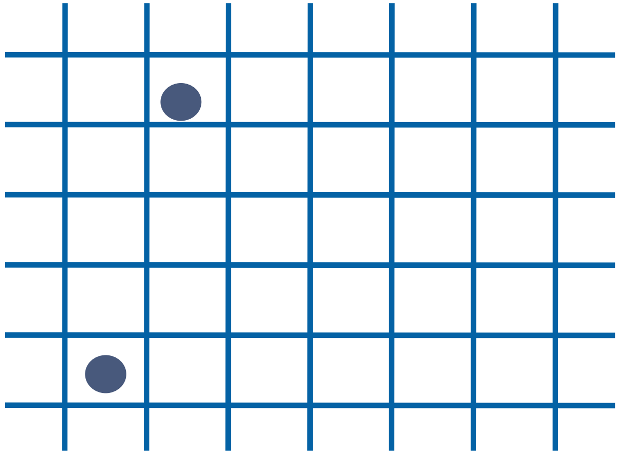
Immobile Excitations

$$\langle \psi_1 | O T^n | \psi_1 \rangle = 0 \quad n \geq 1$$

T is a translation along any direction.
O is supported on a ball that does not touch the boundary of the system

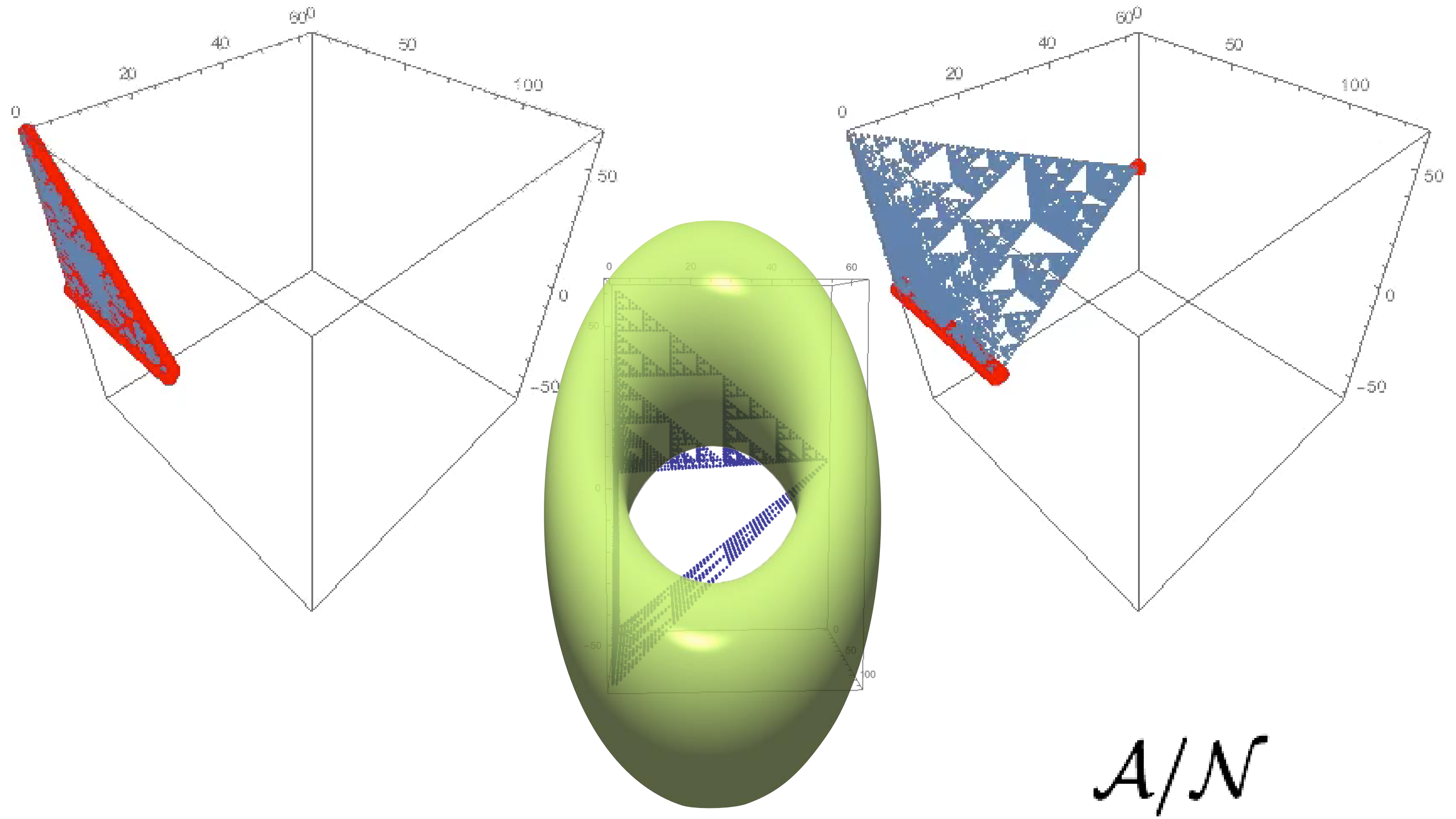


perhaps allowed



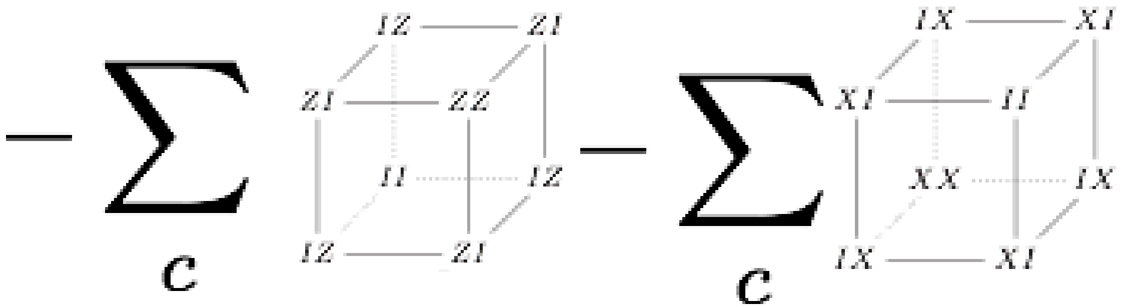
- Interaction-driven localization [Kim, JH, 1505.01480]

Braiding of extended charges



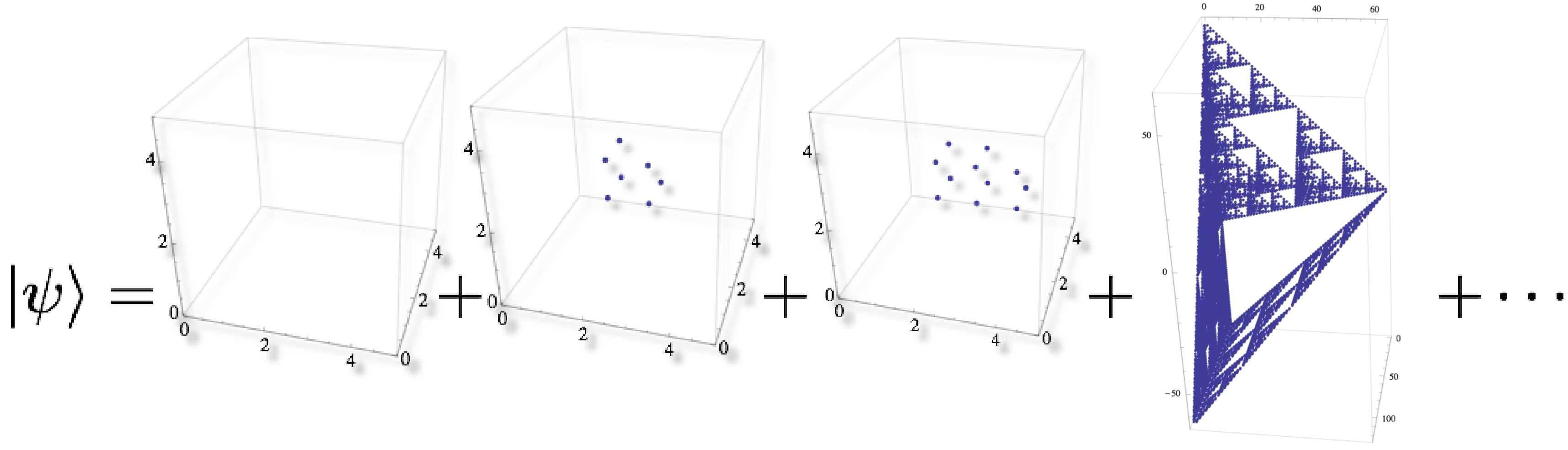
$\mathcal{A}/\mathcal{N}$

Wave function



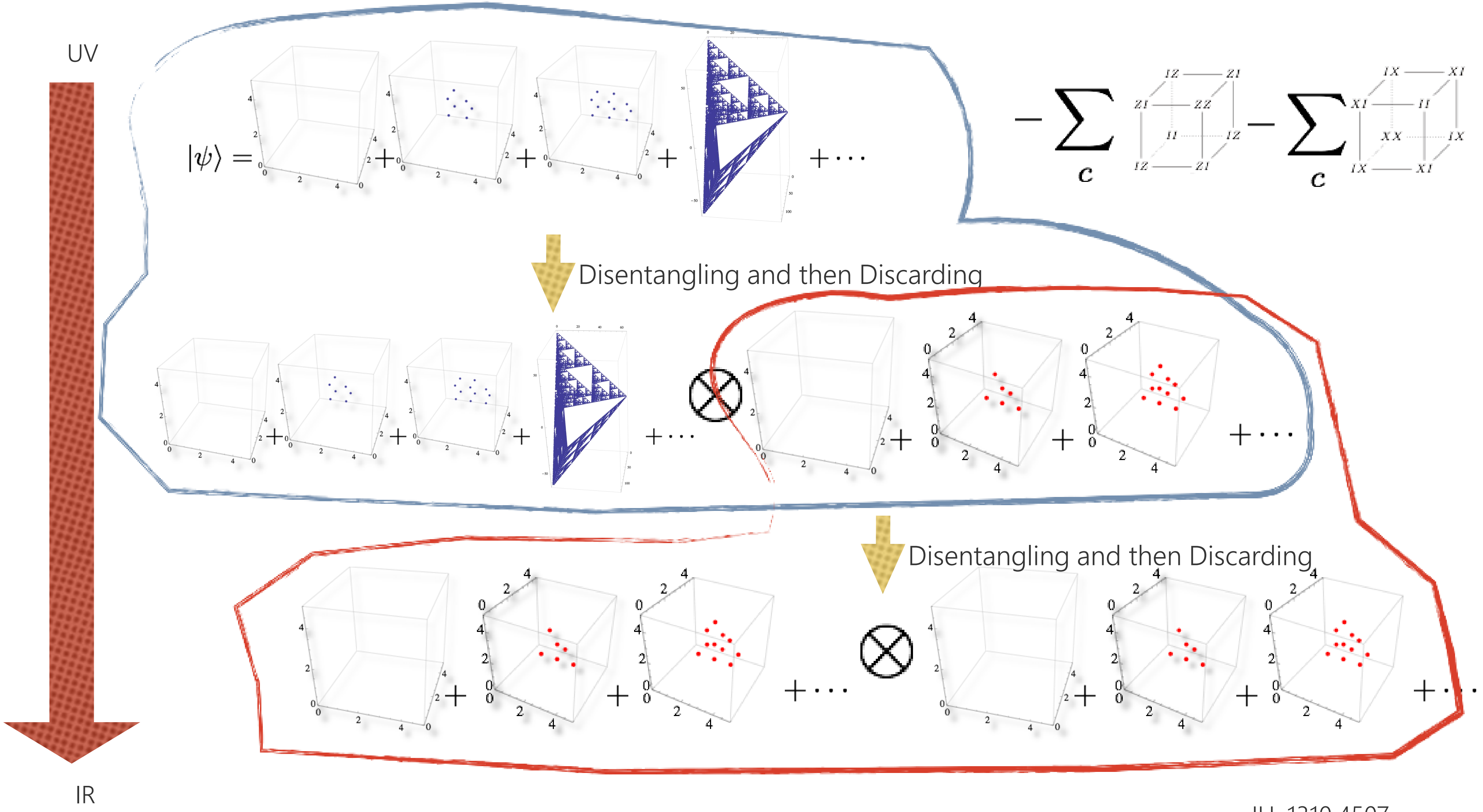
σ^x = +

σ^x = -



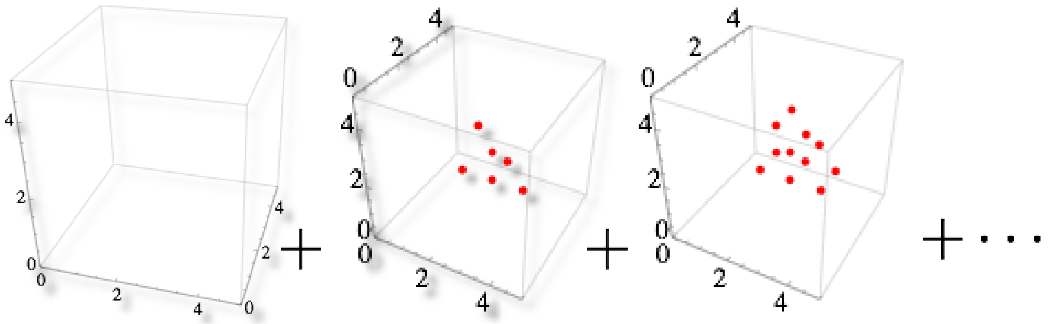
Ground state is a condensate of “fractals” or “objects.”

Entanglement RG



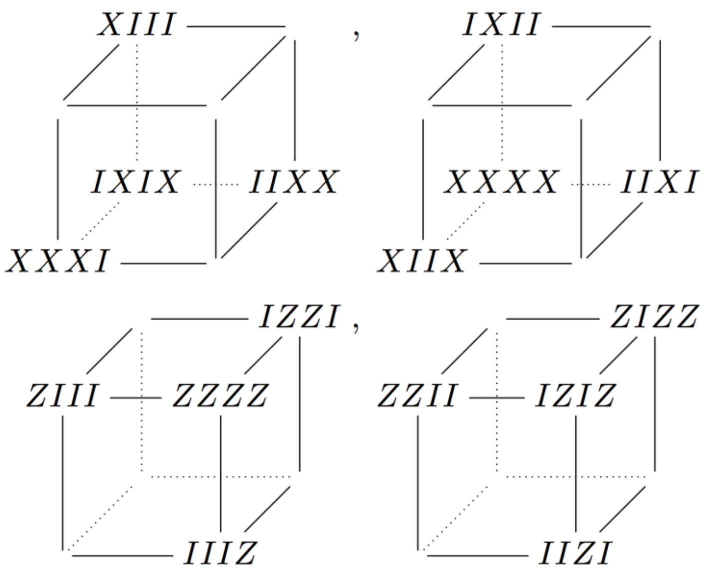
JH, 1310.4507

Entanglement RG fixed point

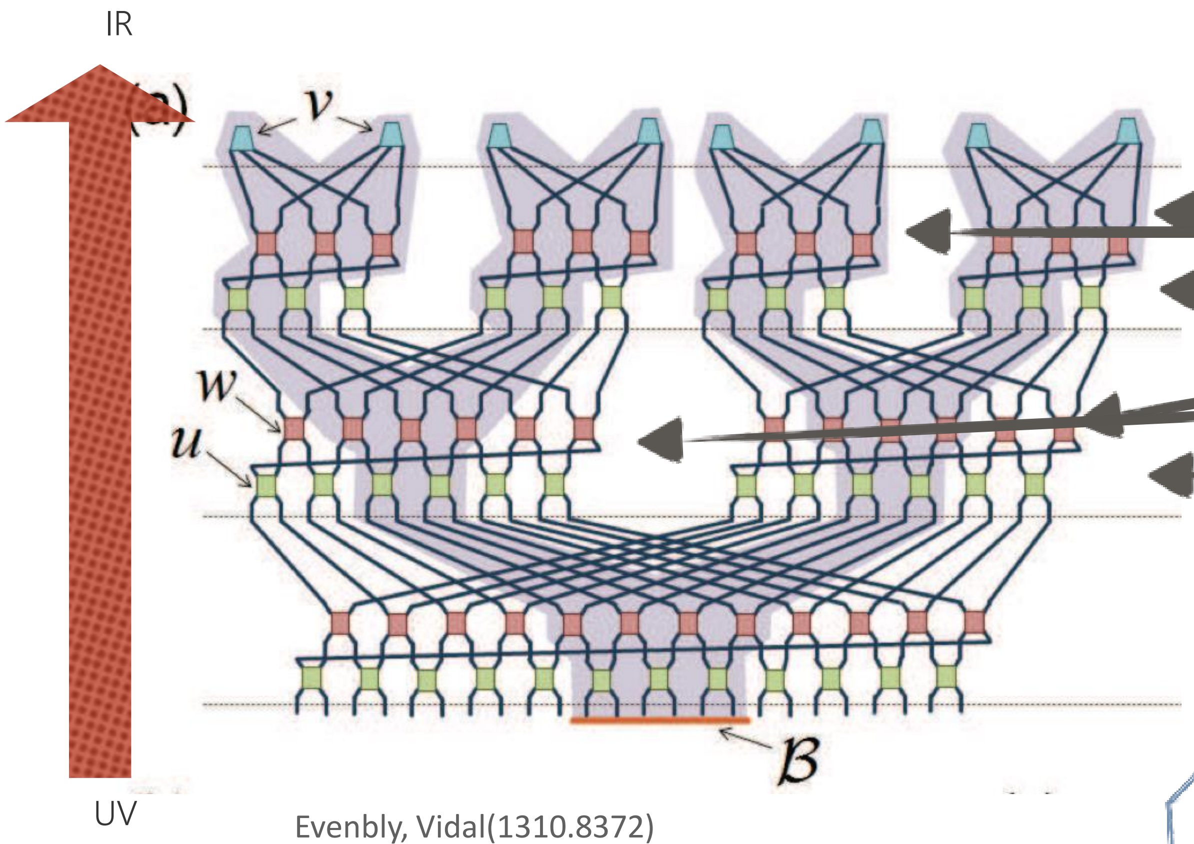


is a ground state of

$$H = -\sum_{c}$$



Branching MERA



Start with qubits

Apply quantum circuit of constant depth

Expand the lattice

Apply quantum circuit of constant depth

Expand the lattice

Evenbly, Vidal(1310.8372)

$$S = \begin{cases} \mathcal{O}(L^{d-1}) & \text{if } b < 2^{d-1} \\ \mathcal{O}(L^{d-1}\log L) & \text{if } b = 2^{d-1} \\ \mathcal{O}(L^{d-1+\log_2(b/2^{d-1})}) & \text{if } b > 2^{d-1} \end{cases}$$