



# Measuring OTOCs with Trapped-Ions

Arghavan Safavi-Naini



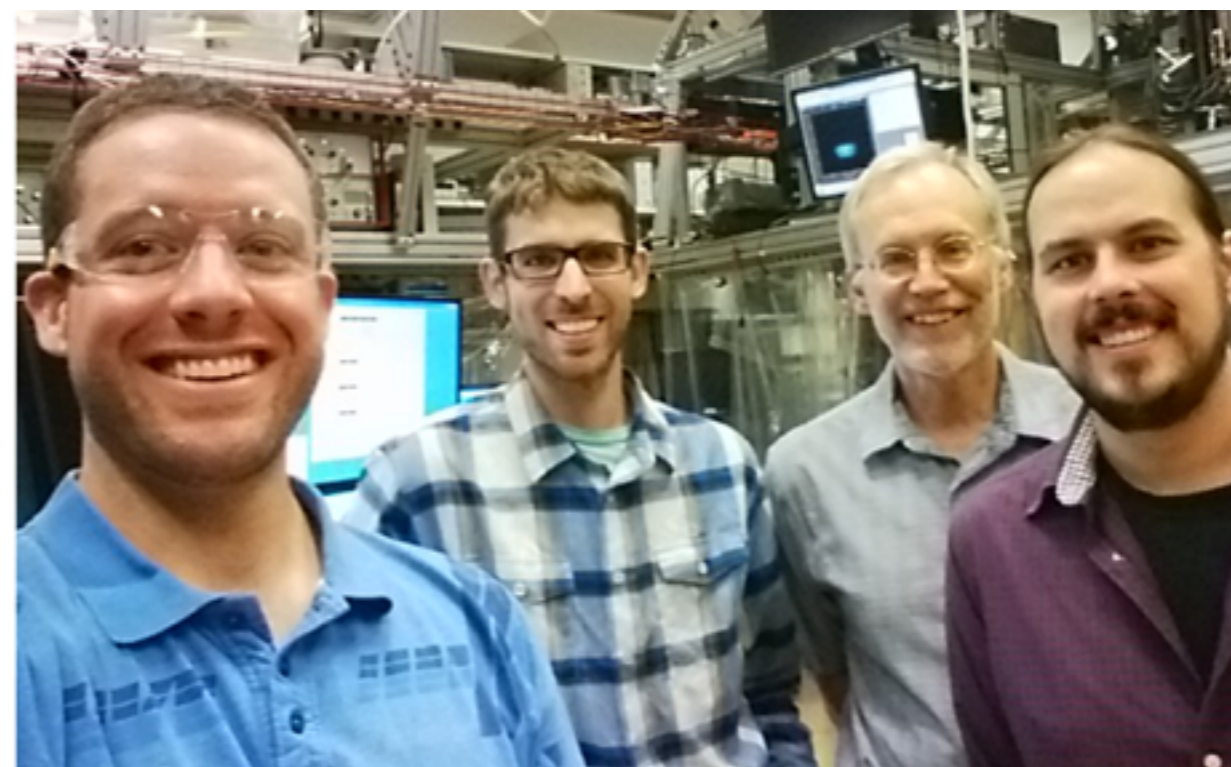
# Collaborators



Ana Maria Rey

## Experiments

John Bollinger



Joe Britton

Justin Bohnet

Brian Sawyer



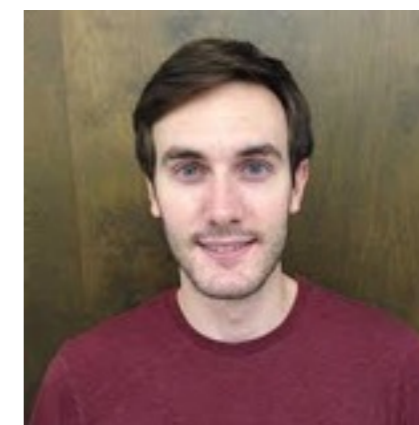
Michael Wall



Martin Garttner



Robert Lewis-Swan



Kevin Gilmore



Elena Jordan

## References

- Garttner, Bohnet, **ASN**, Wall, Gilmore, Bollinger, Rey '17
- Wall, **ASN**, Rey, '16, '17
- Garttner, Hauke, Rey '17
- **ASN**, Lewis-Swann, Garttner, Gilmore, Jordan, Rey, Bollinger, In preparation.

# Quantum Simulation

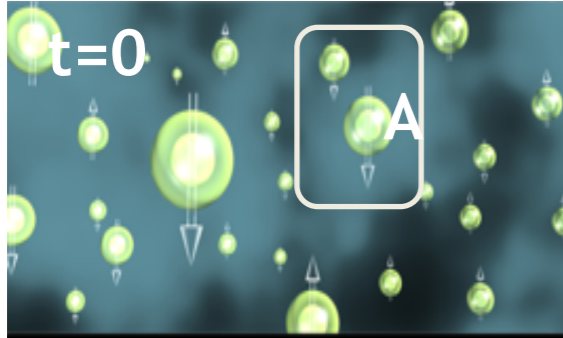
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- Quantum many-body problems intractable for classical computers.
- Use well-controlled quantum system to simulate (an idealized version of) another quantum system

# Quantum Simulation

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- Use well-controlled system of fully characterized quantum components to simulate another quantum system



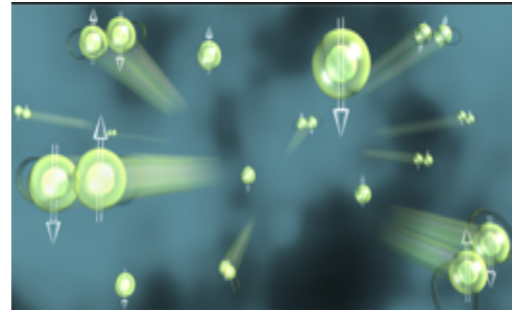
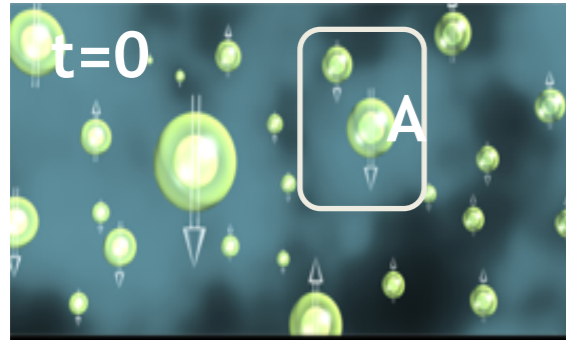
$$|\psi_{\text{in}}\rangle$$

Can prepare easily  
**separable**

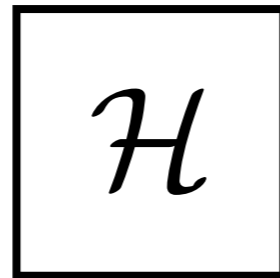
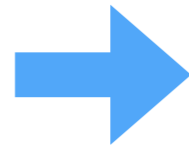
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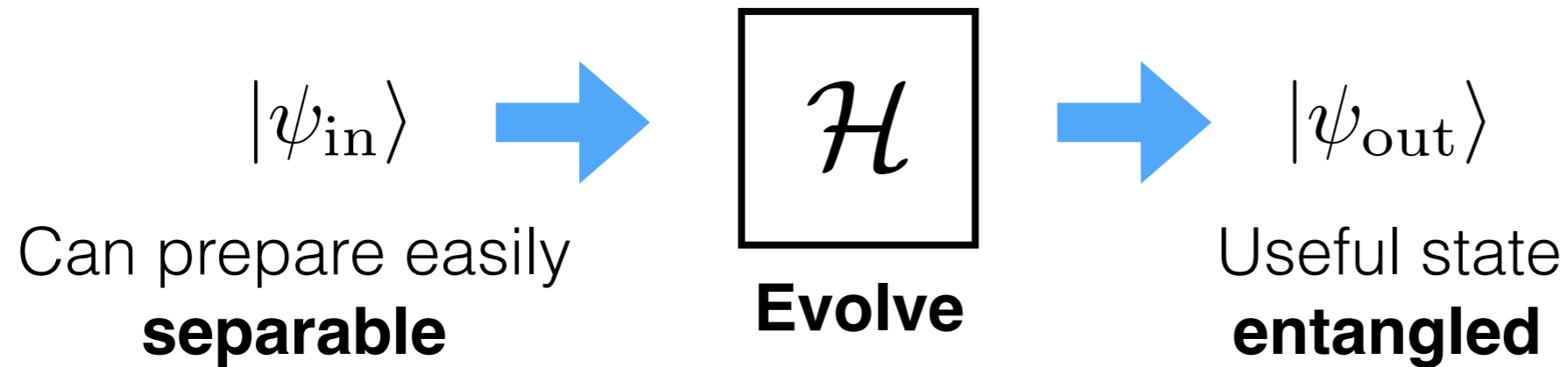
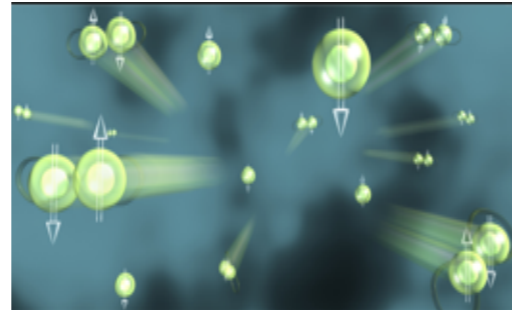
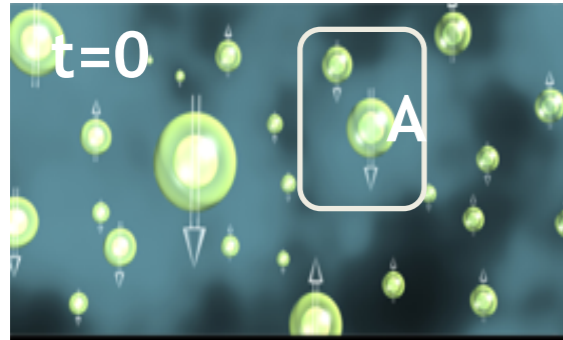
Can prepare easily  
**separable**

**Evolve**

# Quantum Simulation

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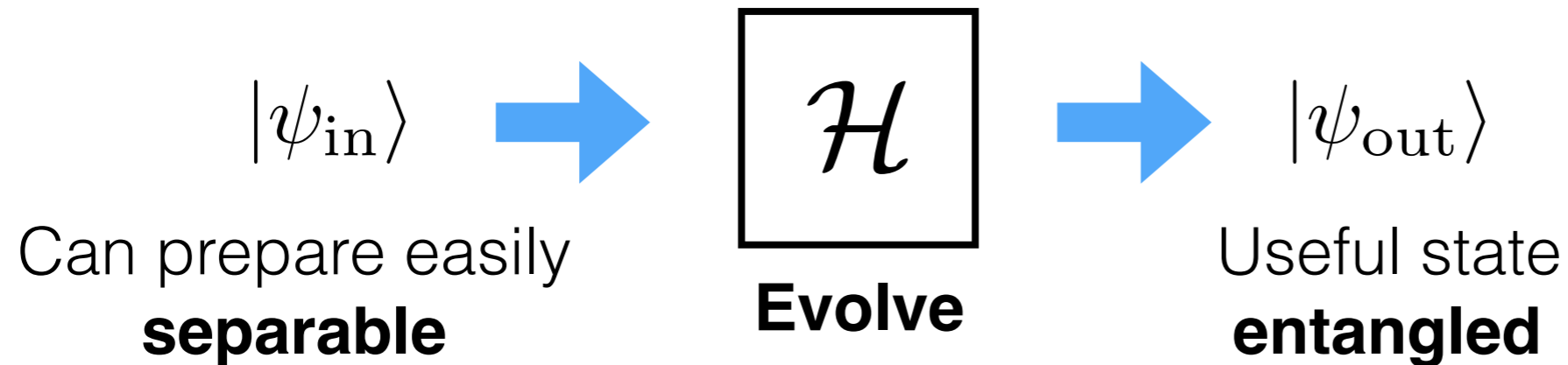
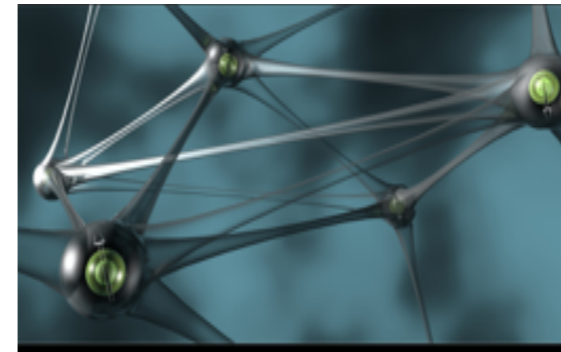
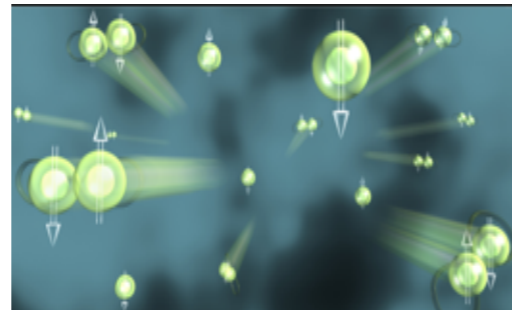
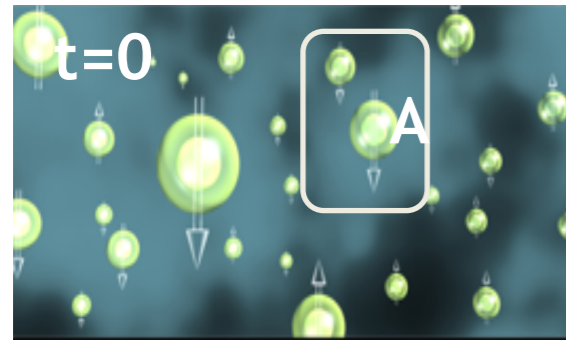
- Quantum many-body problems intractable for classical computers.
- Use well-controlled quantum system to simulate (an idealized version of) another quantum system



- Hard to characterize the state fully
  - Full-state tomography scales badly
  - Often limited to global observables in large systems
- Verifiable implementations
  - Choose Hamiltonians that can be benchmarked
  - Develop measures that are experimentally and theoretically viable

# Quantum Simulation

- Quantum many-body problems intractable for classical computers.
- Use well-controlled quantum system to simulate (an idealized version of) another quantum system



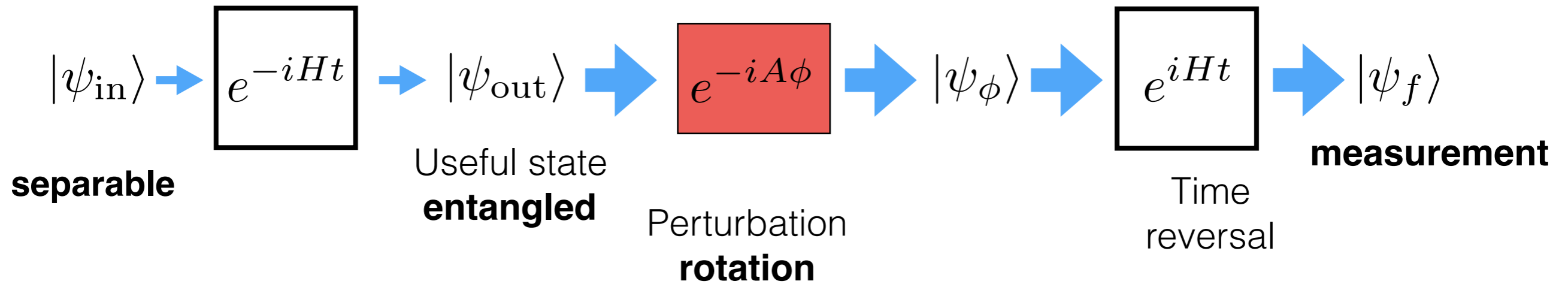
- Hard to characterize the state fully
  - Full-state tomography scales badly
  - **Often limited to global observables in large systems**

What is the best we can do with these limitations?

- Verifiable implementations
  - Choose Hamiltonians that can be benchmarked
  - Develop measures that are experimentally and theoretically viable

# Time-reversal to the Rescue

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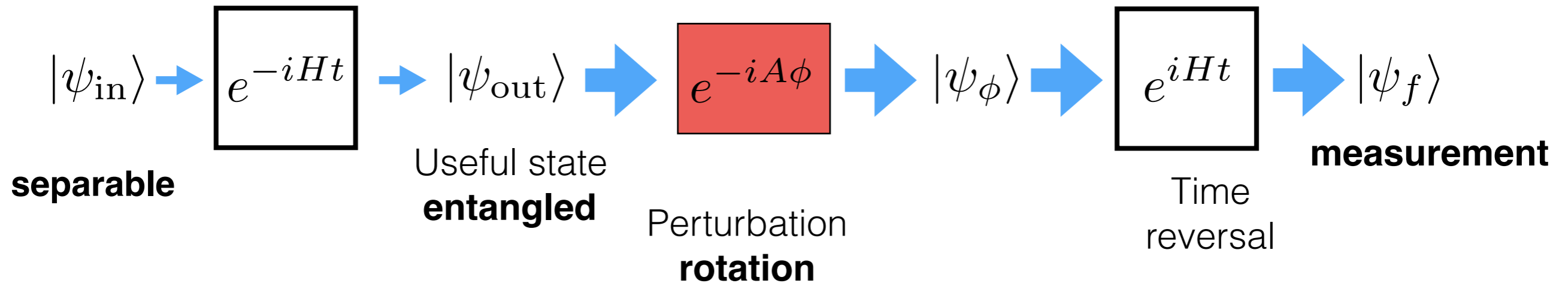


Time reversal allows to extract **more information** about the state using only **global observables**  $\langle \psi_f | \hat{A} | \psi_f \rangle$



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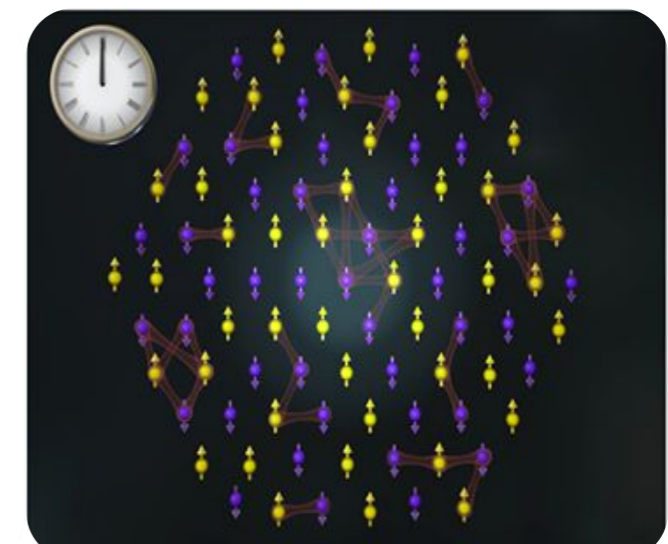
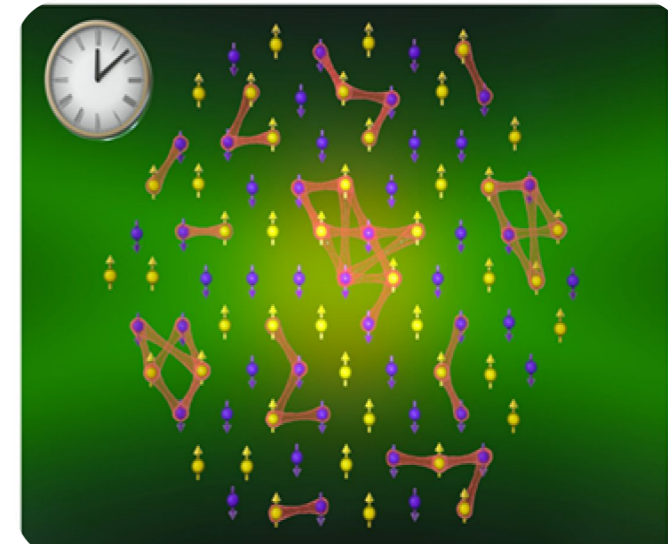
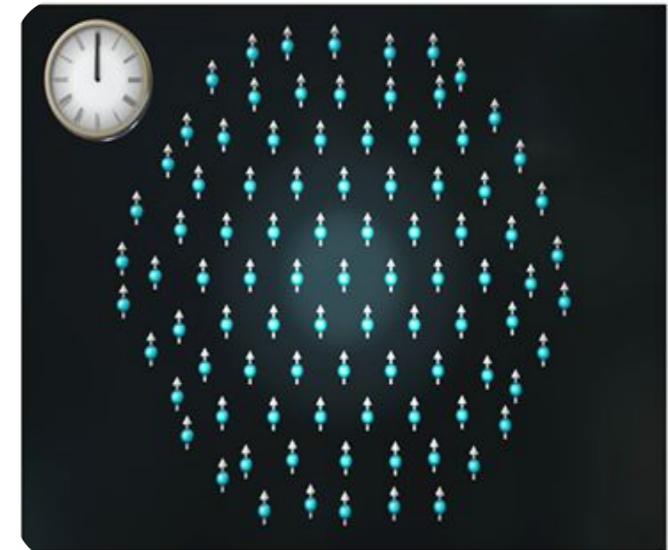


**Out of time order  
correlator**

# Overview

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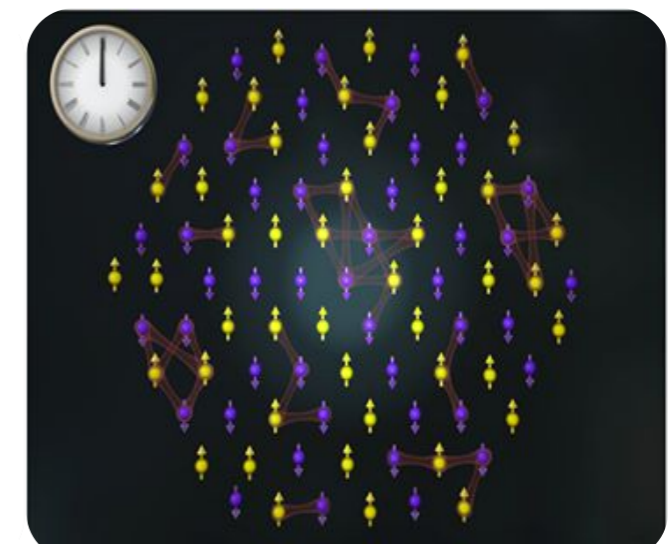
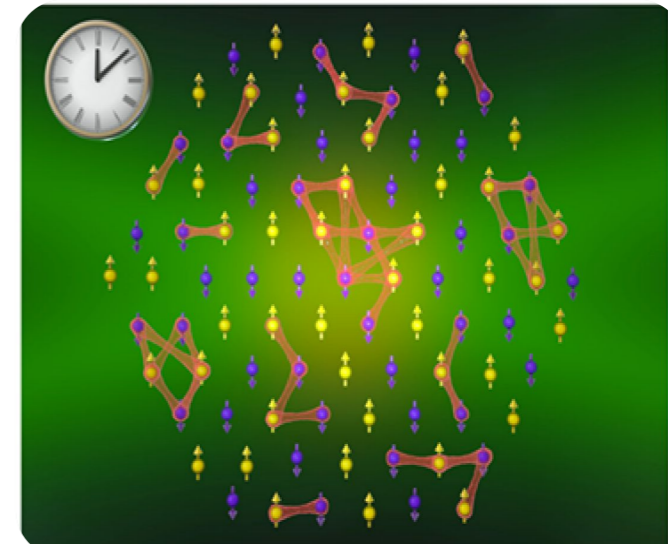
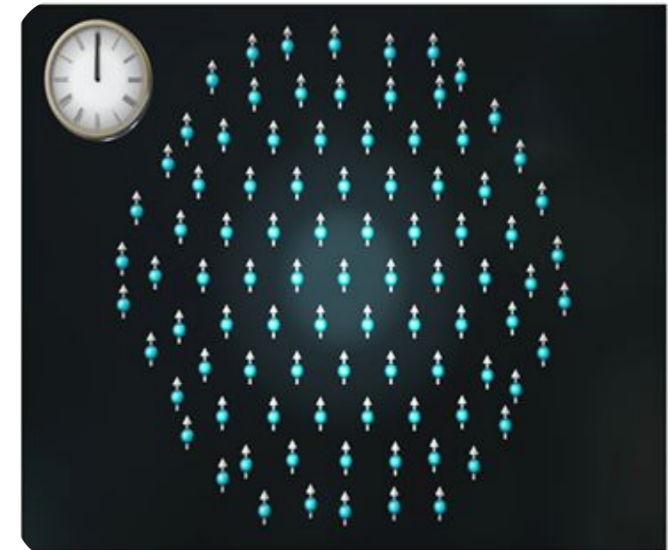
- Trapped ion quantum simulator of **Ising dynamics**



# Overview

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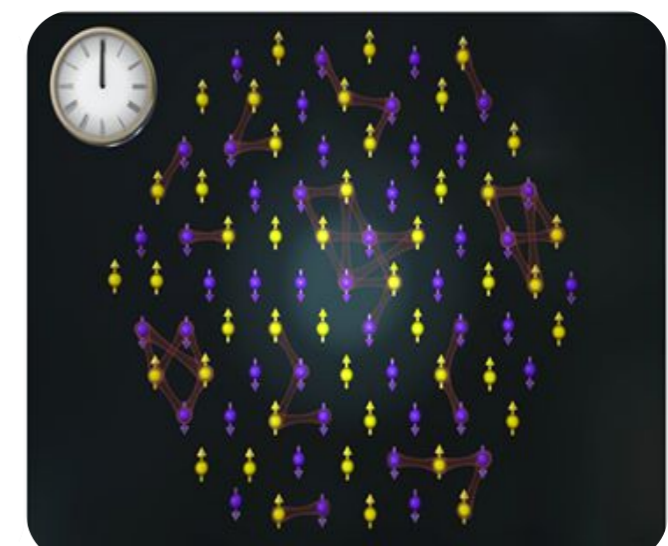
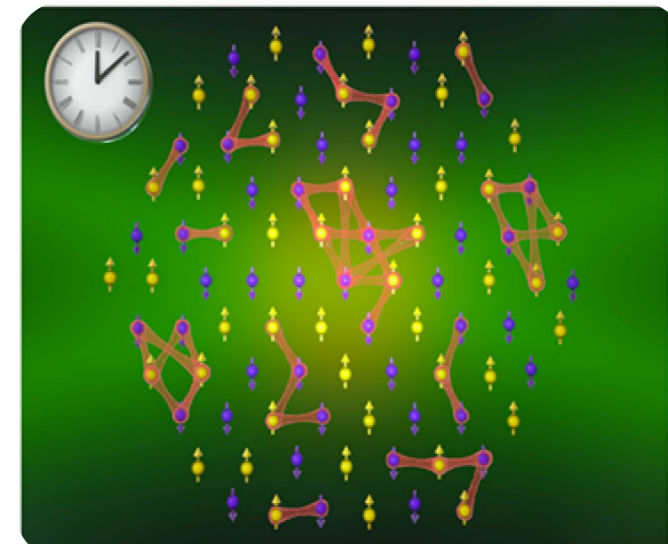
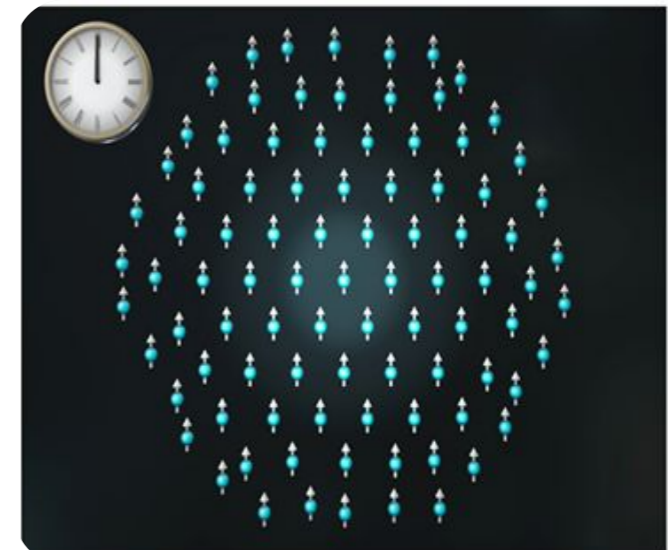
- Trapped ion quantum simulator of **Ising dynamics**
- **Fidelity** measurement
  - Multiple quantum coherences
  - Loschmidt echo
  - Quantum Fisher information
- **Magnetization** measurement
  - Buildup of correlations



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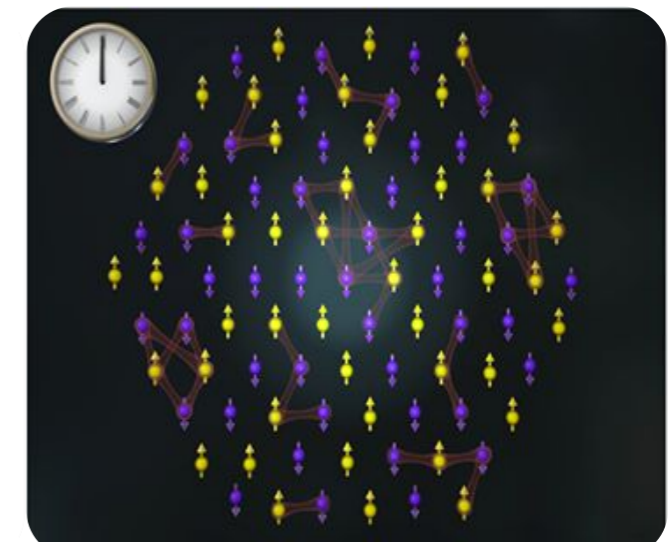
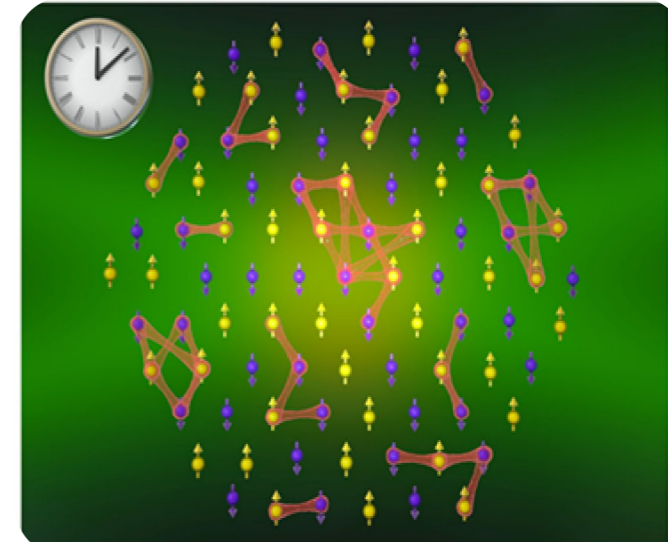
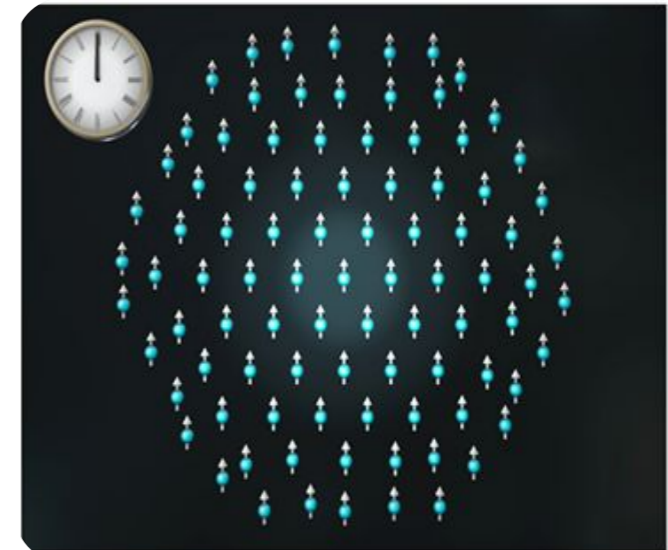
## OTOCs



# Overview

- Trapped ion quantum simulator of **Ising dynamics**
- **Fidelity** measurement
  - Multiple quantum coherences
  - Loschmidt echo
  - Quantum Fisher information
- **Magnetization** measurement
  - Buildup of correlations
- Adding complexity: **Dicke model**

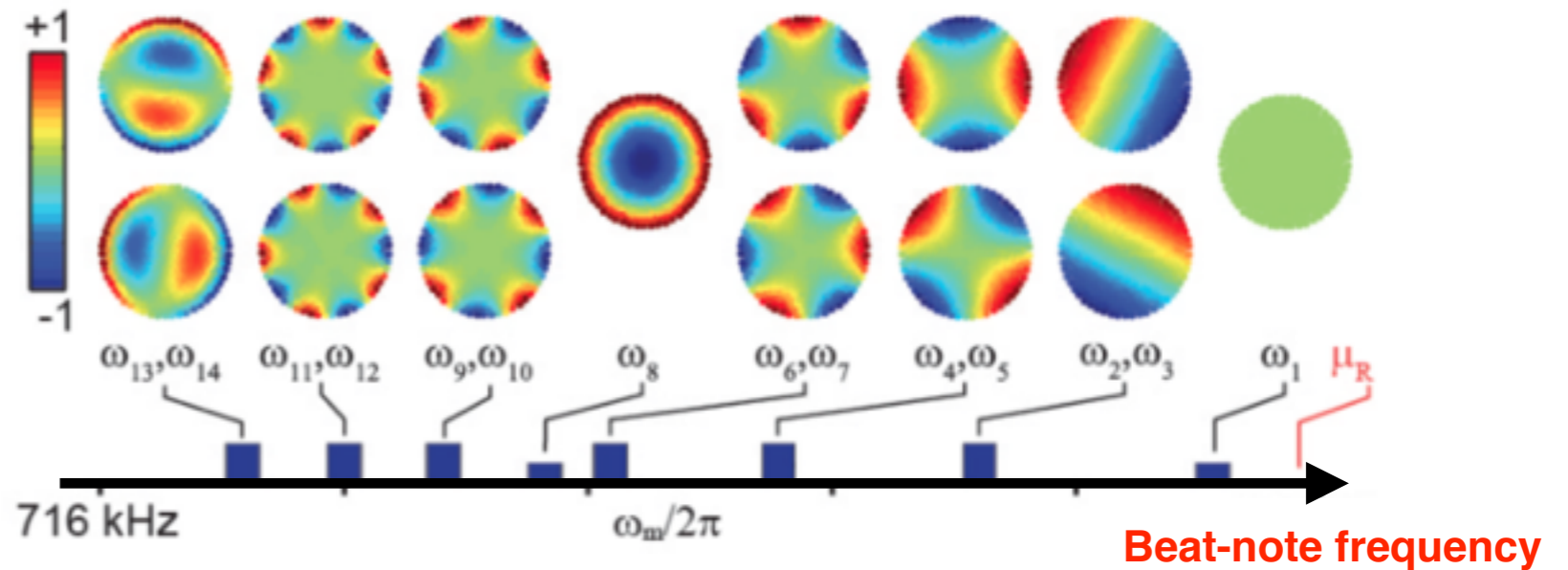
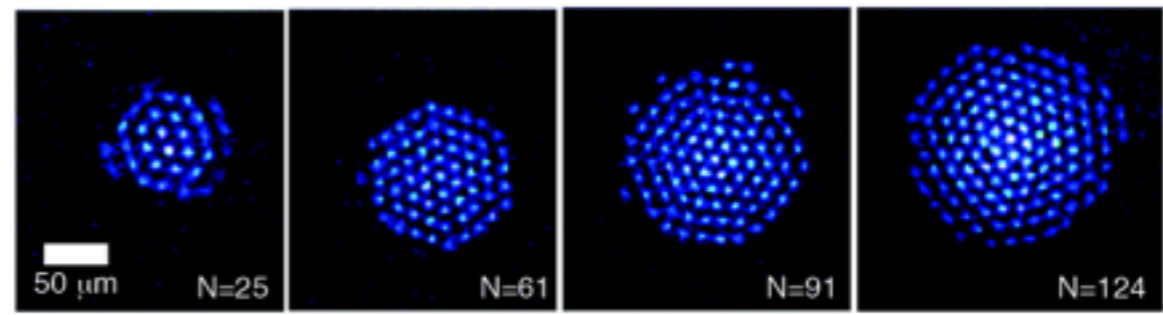
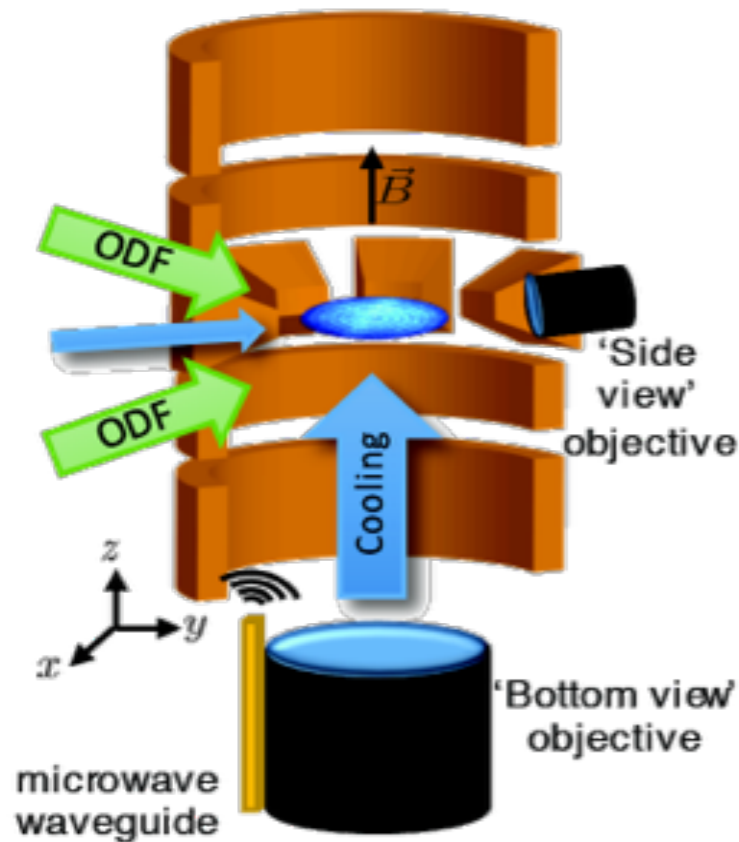
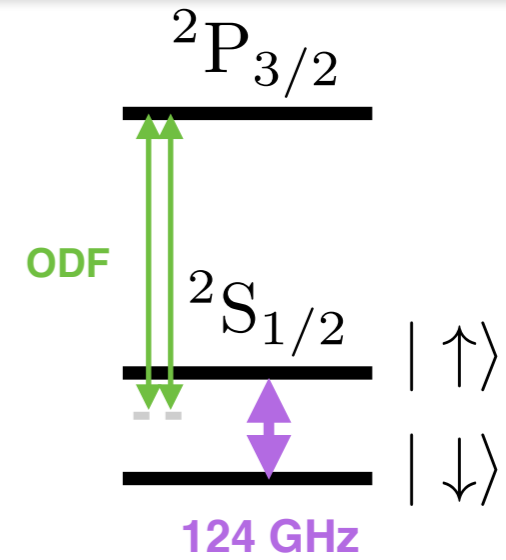
## OTOCs



# System

Our system:

- Triangular crystal of Be<sup>+</sup> ions
  - Stabilized due to the **Coulomb repulsion** between the ions and **external trapping**
- Use two hyperfine states of the ion to form the **spin**
- Couple the spin to the transverse modes of the crystal (**phonons**)

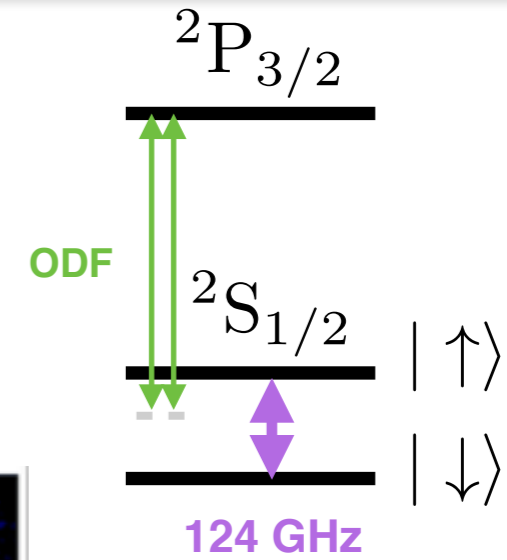
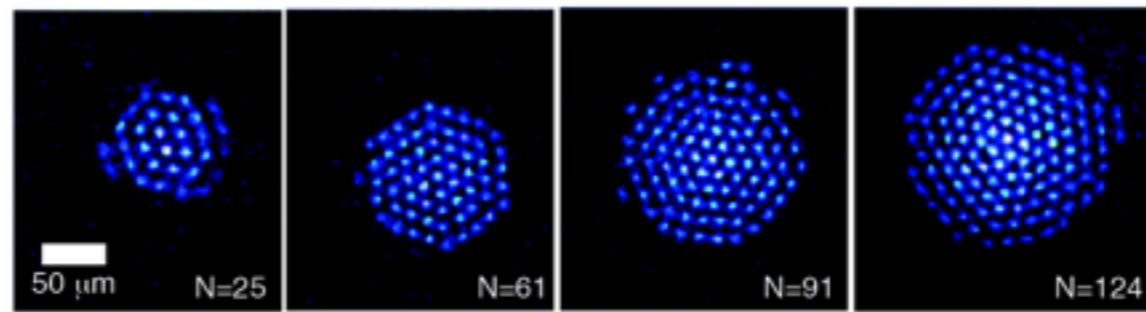
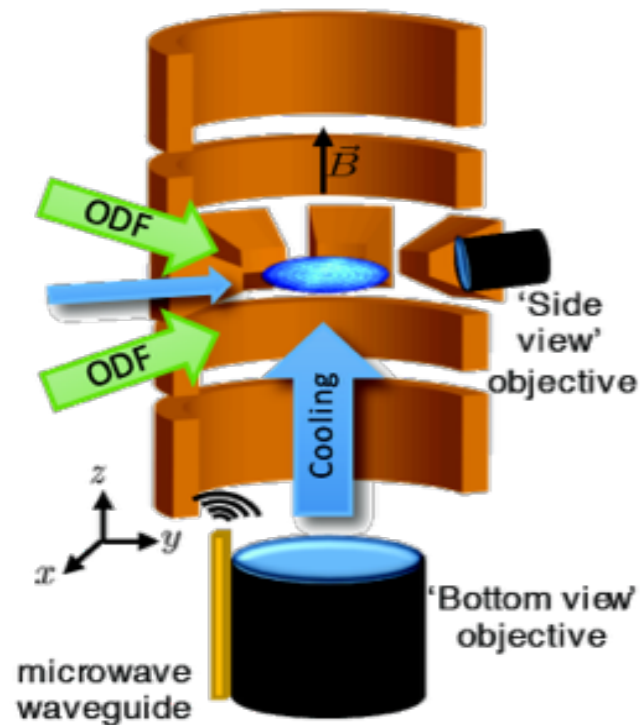


Frequency of the vibrational modes

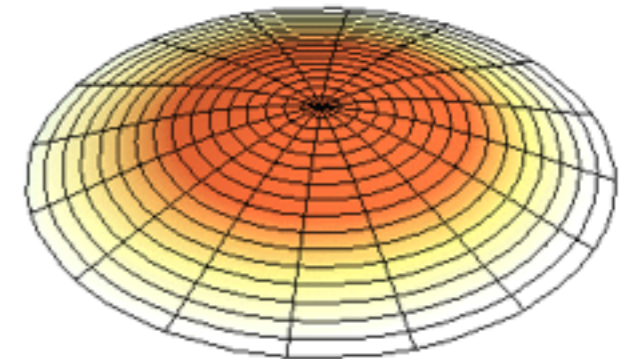
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**ONLY couple to COM**

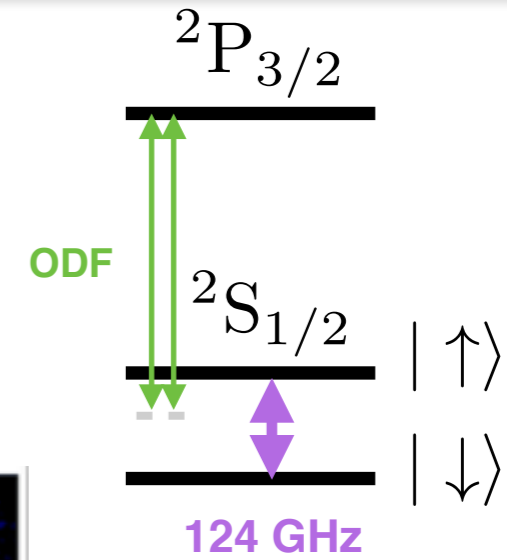
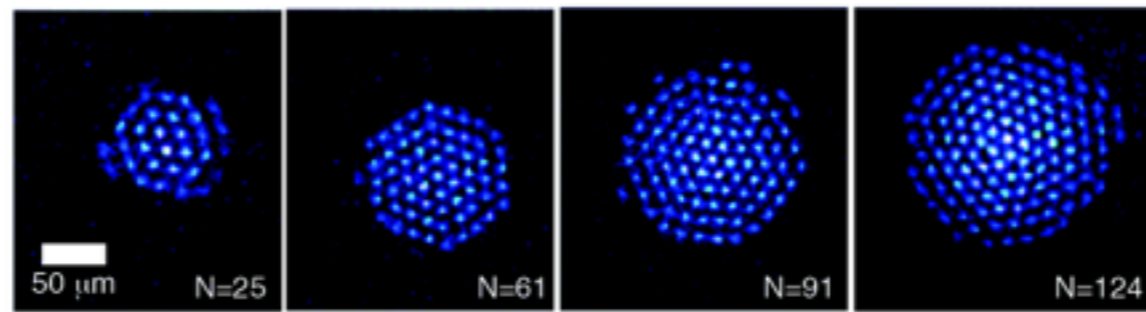
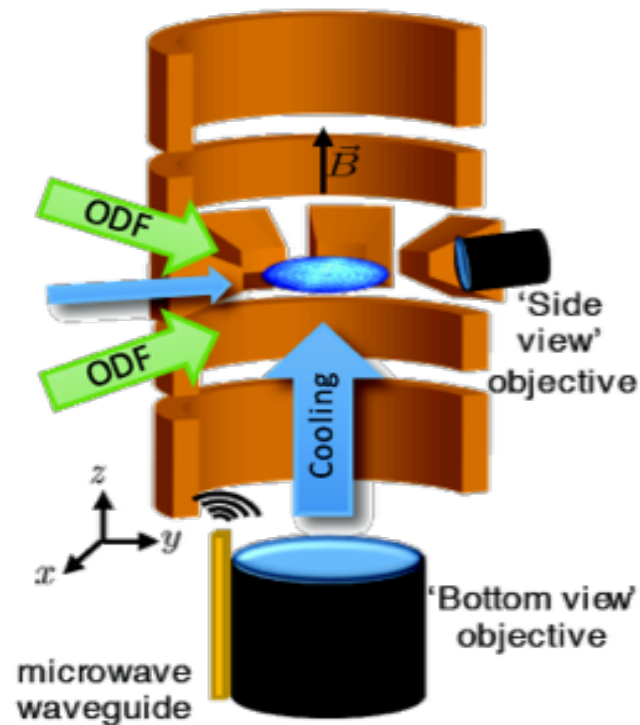


COM

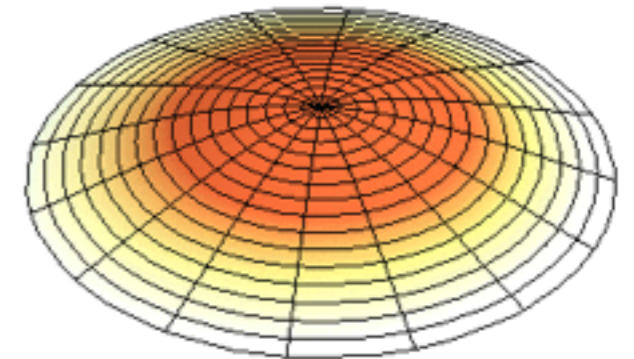
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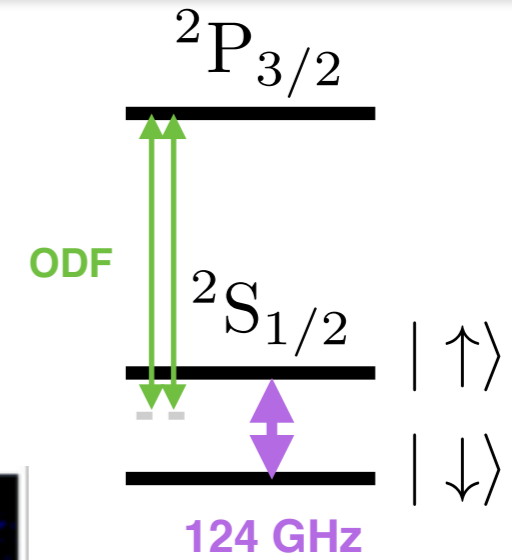
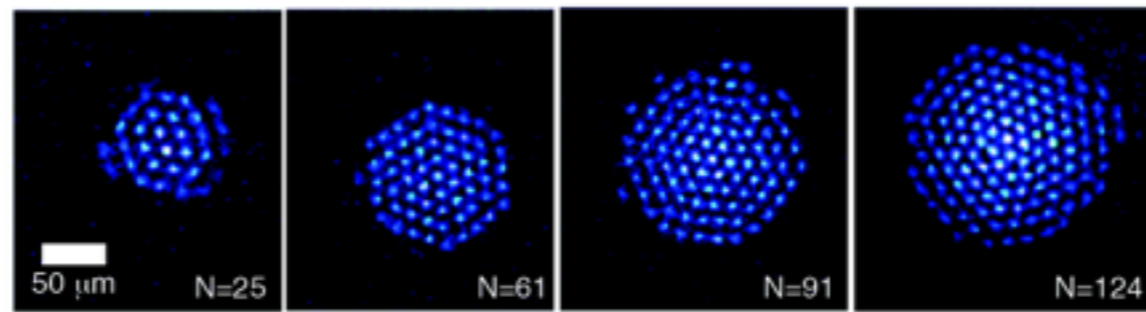
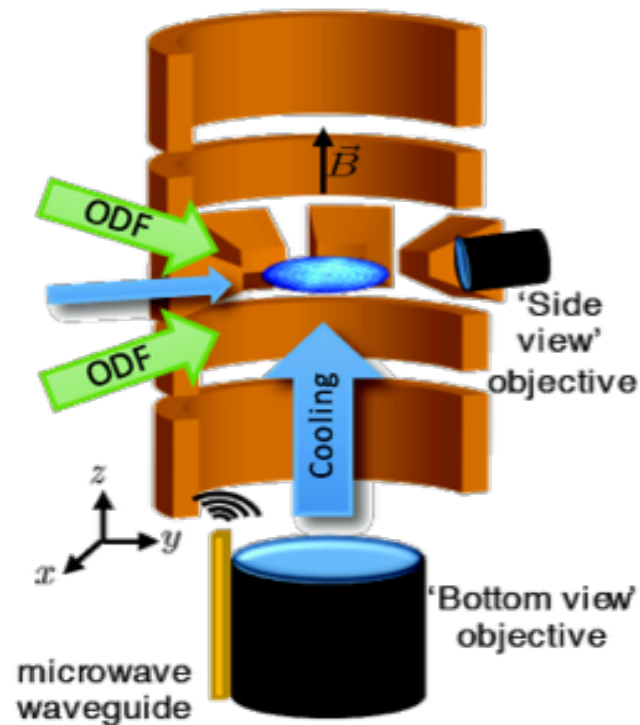
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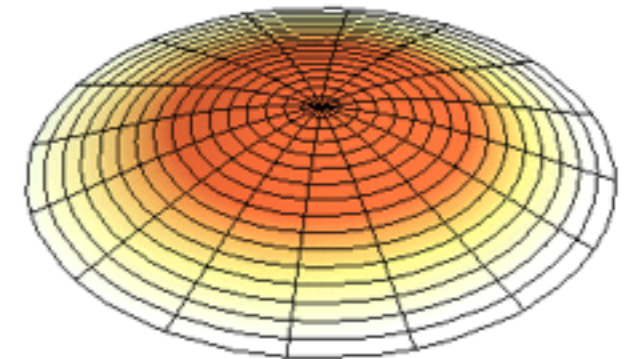
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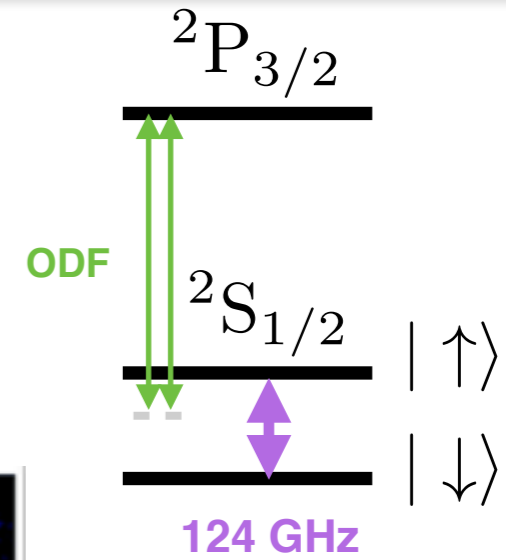
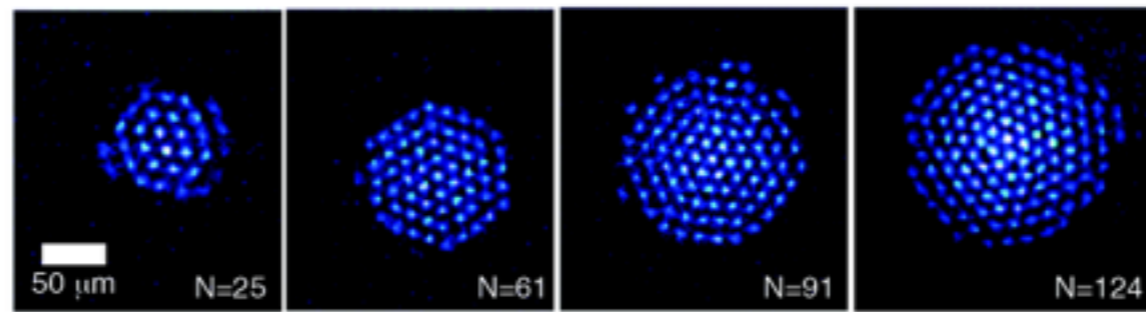
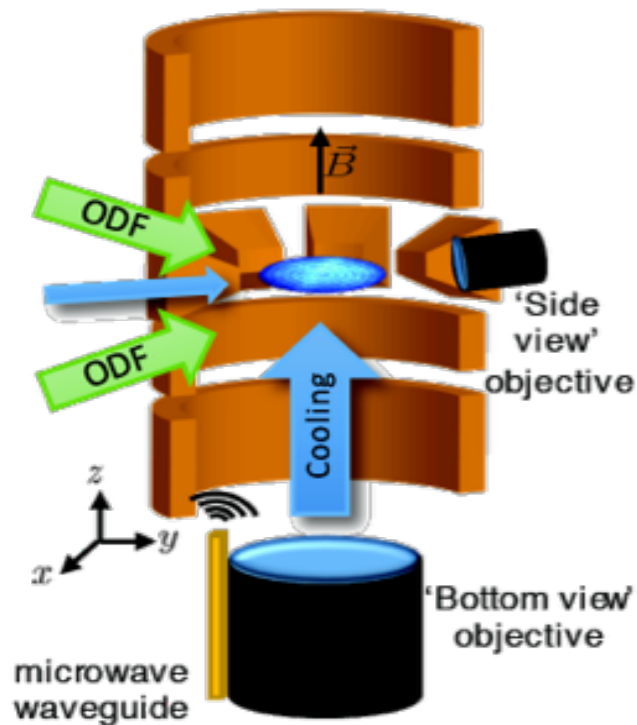


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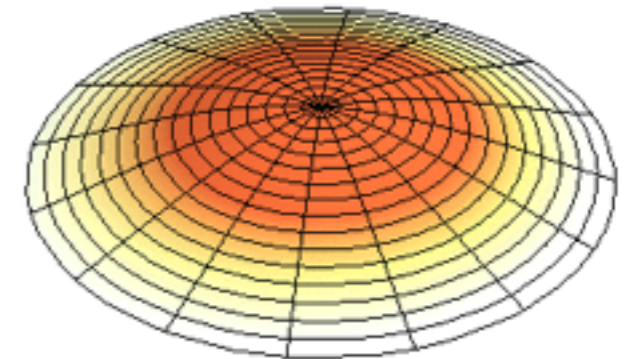
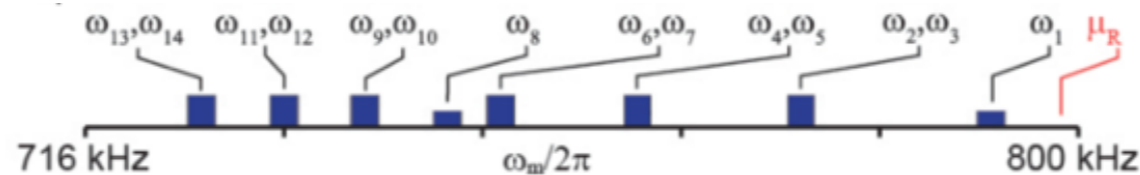
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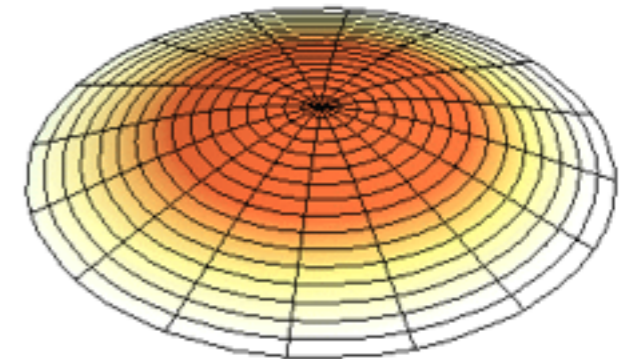
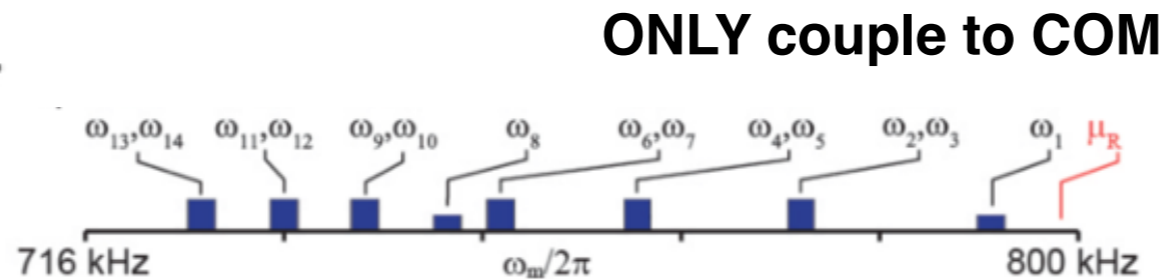
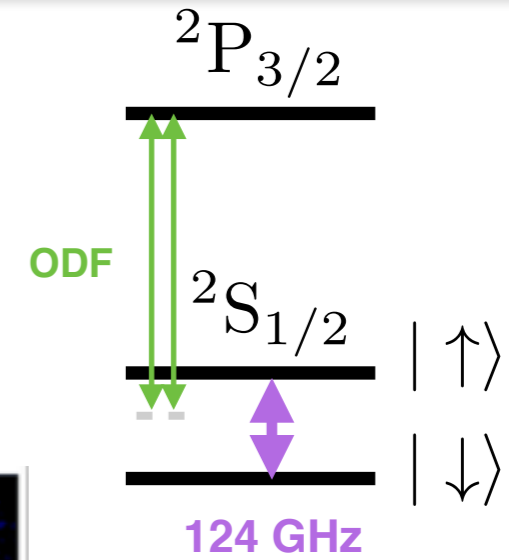
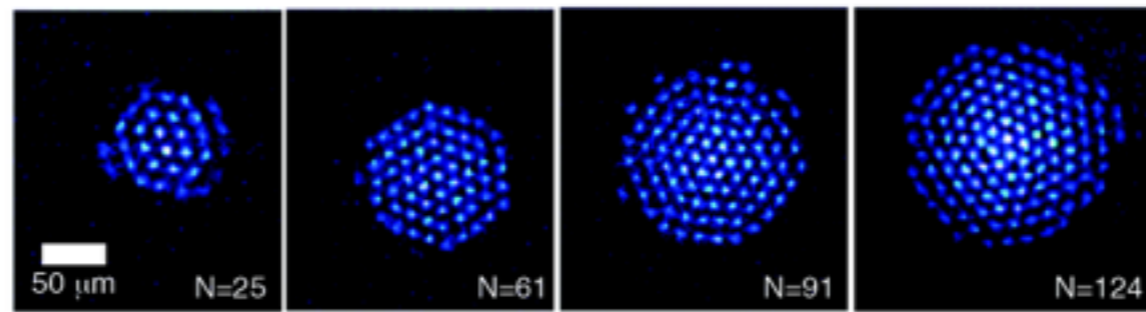
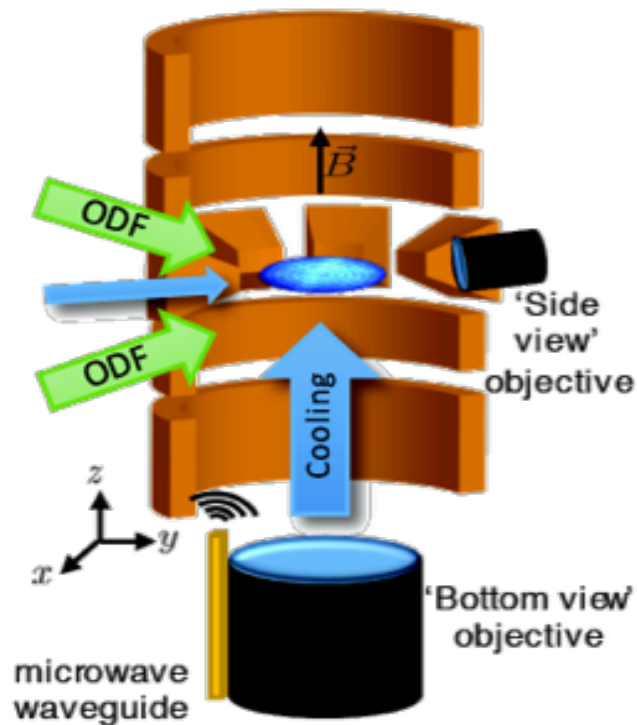
COM

$$H(t) = -g(\mu_R, t) \hat{z} \hat{S}_z + \omega_0 n$$

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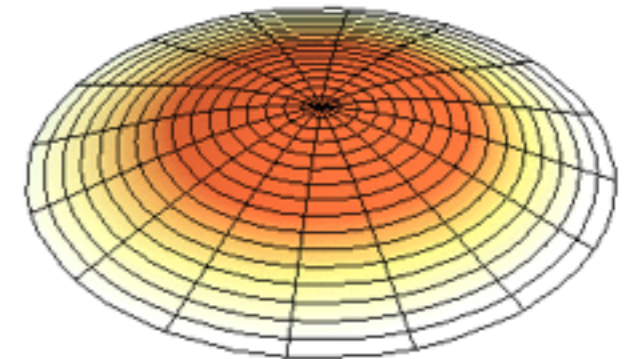
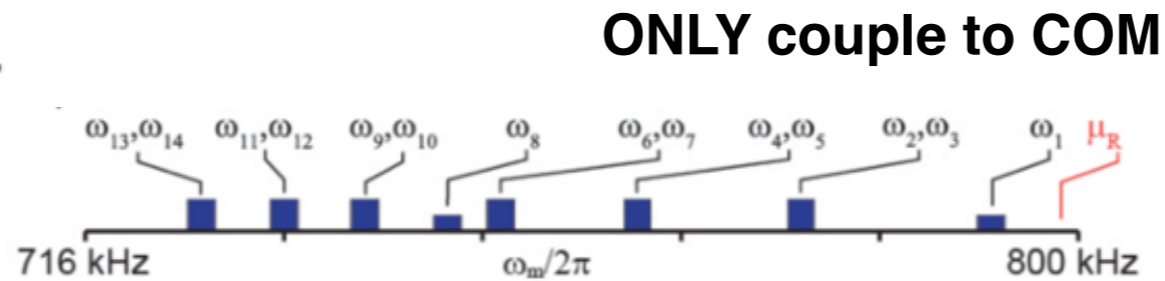
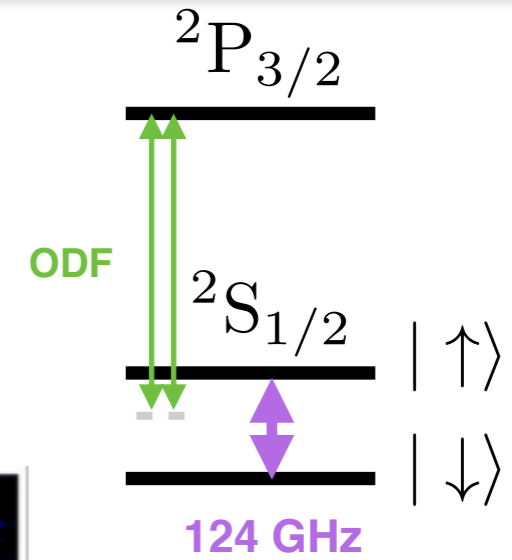
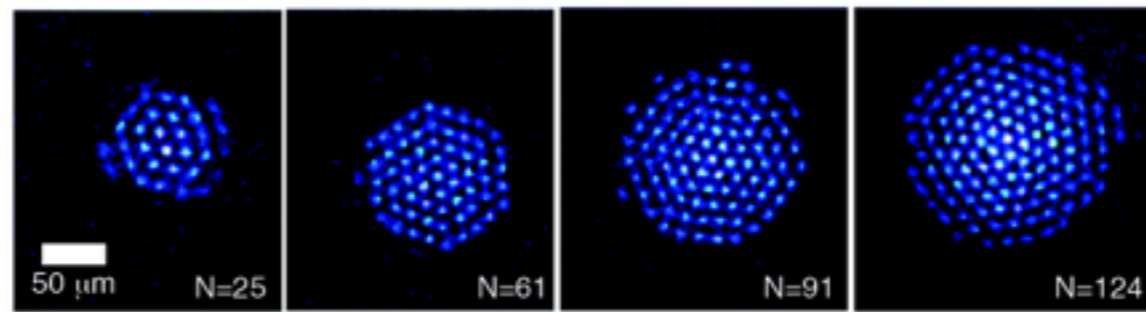
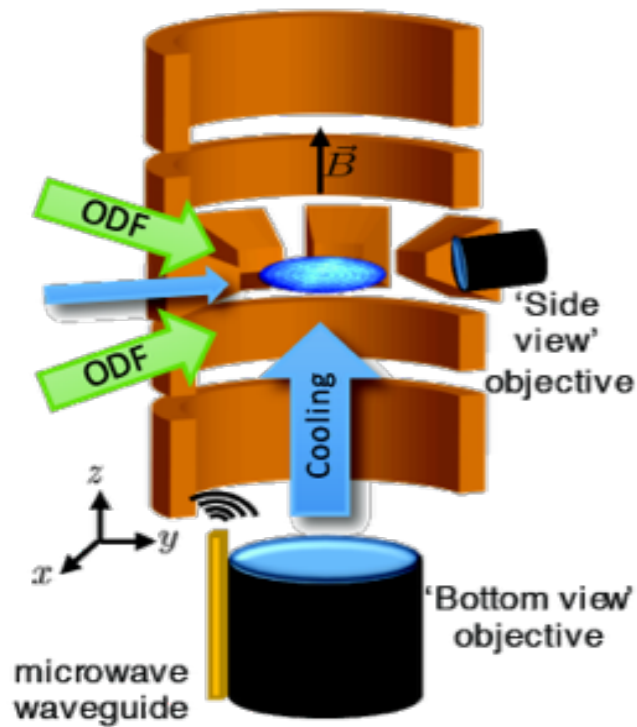
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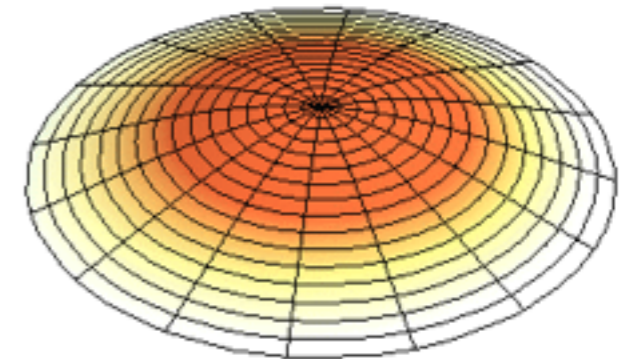
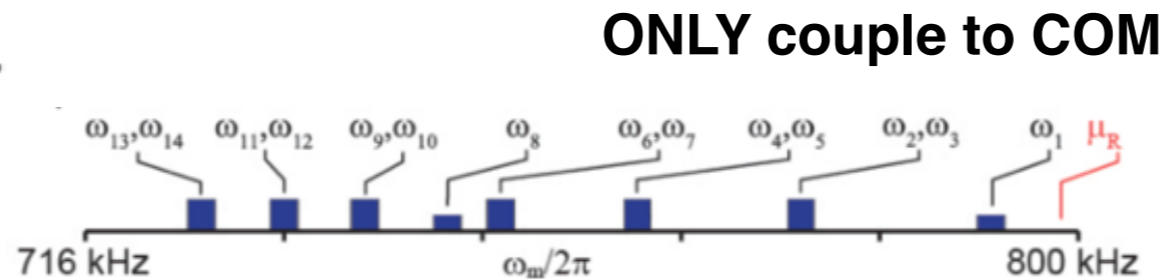
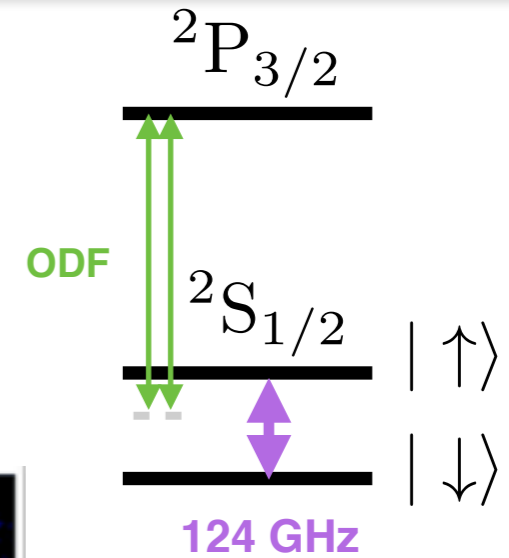
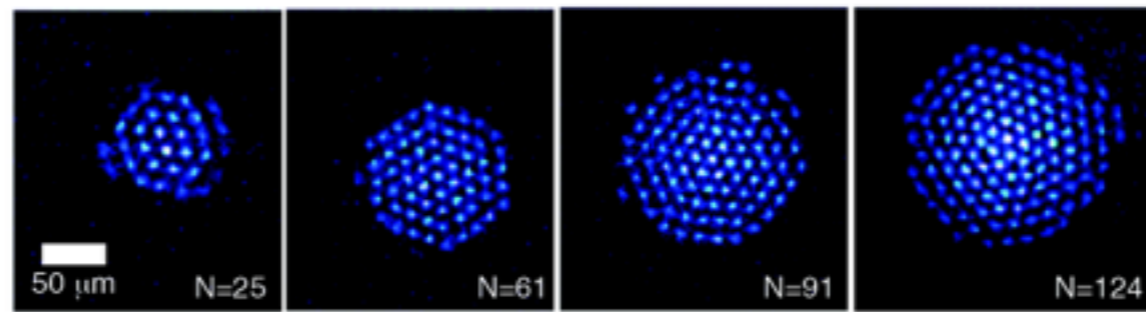
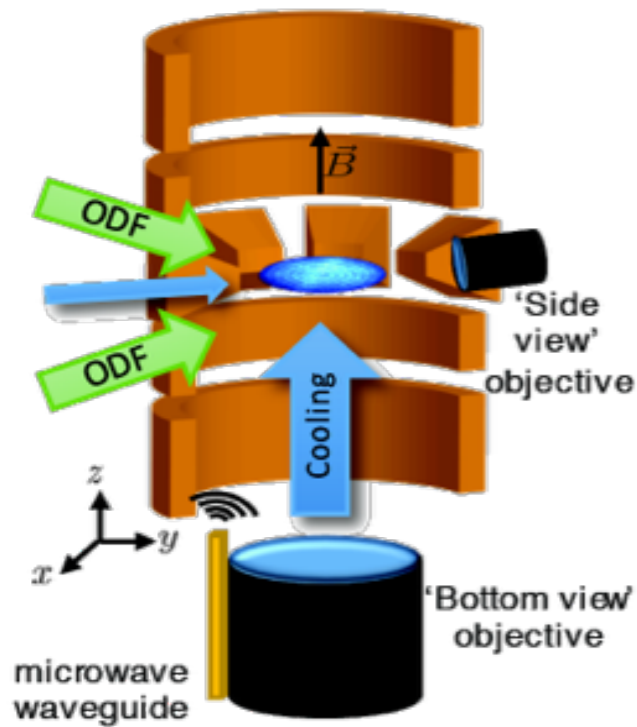
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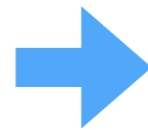


COM

$$H(t) = -g(\mu_R, t) \hat{z} \hat{S}_z + \omega_0 n$$

$$U(t) = U_{SP}(t) U_{SS}(t)$$

**spin-phonon    spin-spin**



$$\hat{H}_{SS} = \frac{J}{N} \hat{S}_z^2$$

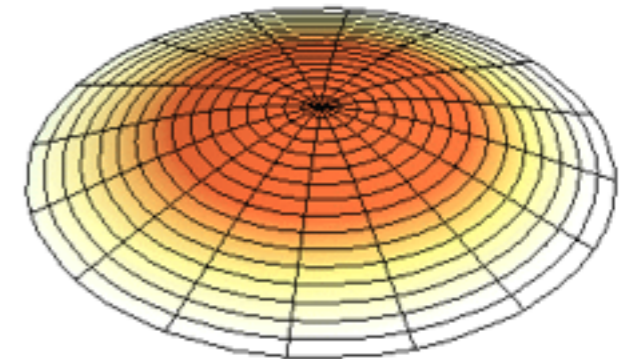
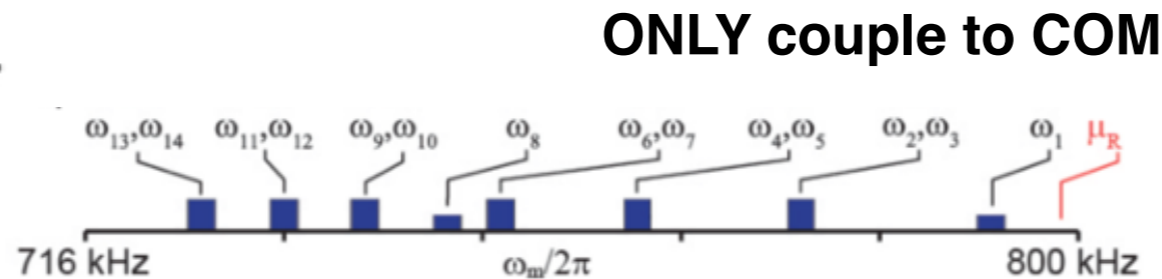
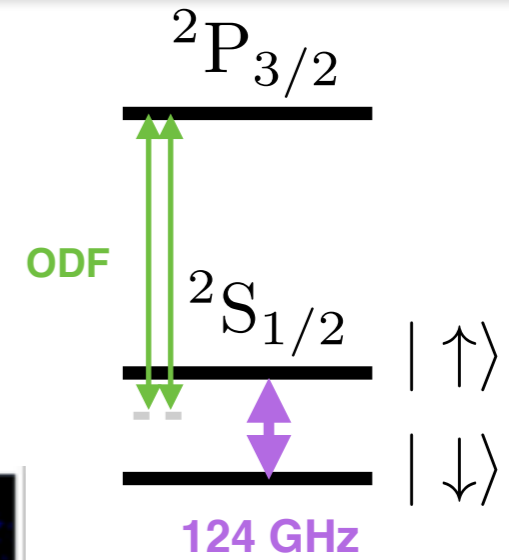
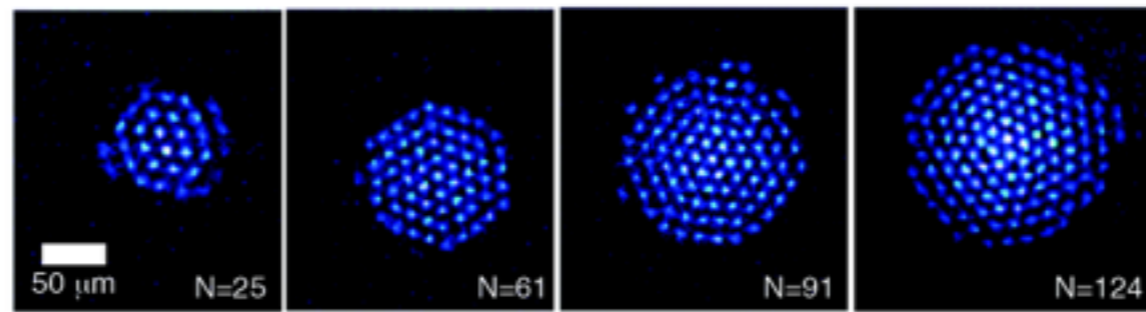
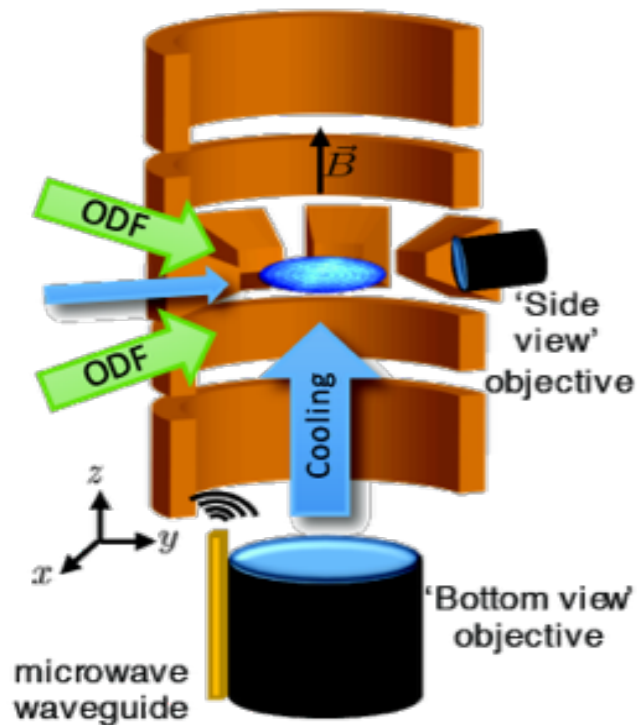
$$J \propto \frac{g^2}{\delta}$$

Britton et al. Nature 2012  
 Bohnet et al. Science 2015  
 Wall, ASN, Rey, '16 '17

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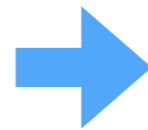


COM

$$H(t) = -g(\mu_R, t) \hat{z} \hat{S}_z + \omega_0 n$$

$$U(t) = U_{SP}(t) U_{SS}(t)$$

**spin-phonon    spin-spin**



$$\hat{H}_{SS} = \frac{J}{N} \hat{S}_z^2$$

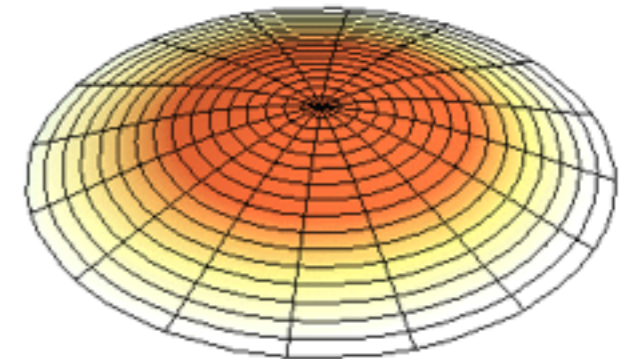
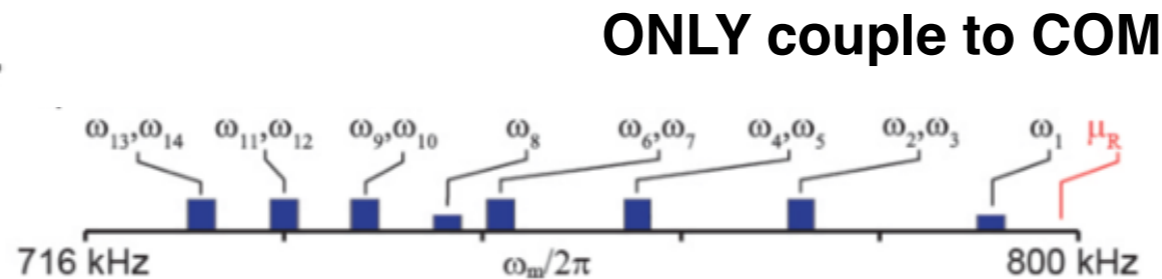
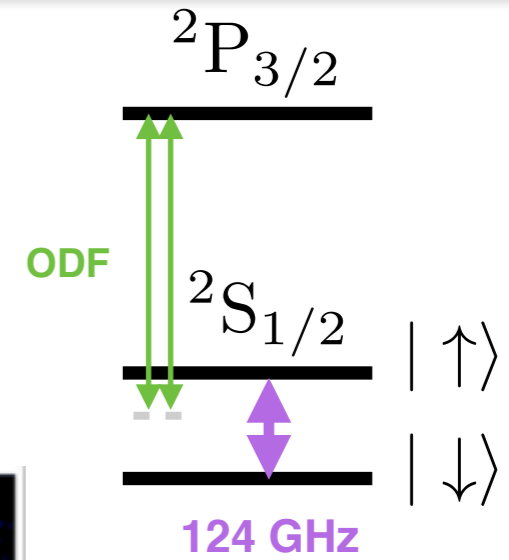
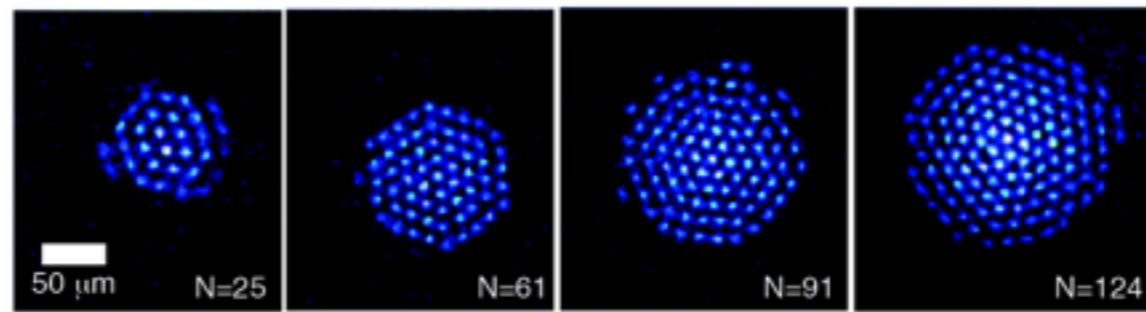
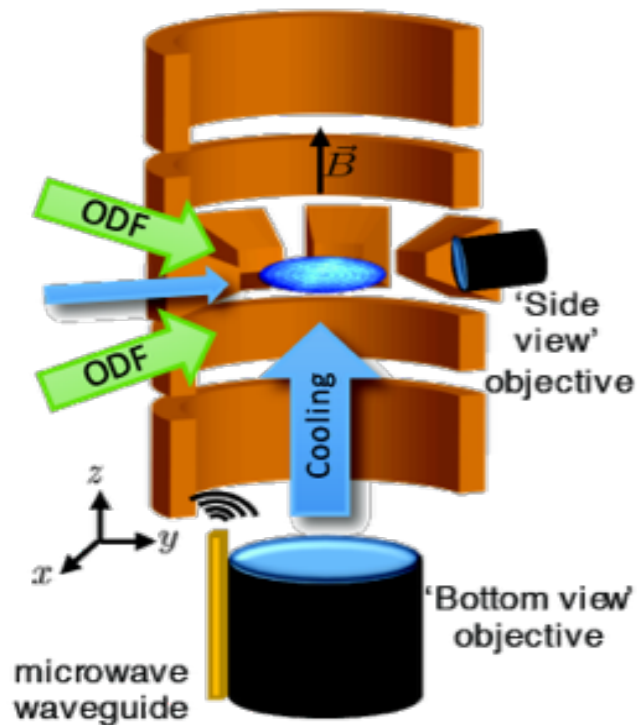
$$J \propto \frac{g^2}{\delta}$$

Britton et al. Nature 2012  
 Bohnet et al. Science 2015  
 Wall, ASN, Rey, '16 '17

# System

Our system:

- Triangular crystal of Be<sup>+</sup> ions
  - Stabilized due to the **Coulomb repulsion** between the ions and **external trapping**
- Use two hyperfine states of the ion to form the **spin**
- Couple the spin to the transverse modes of the crystal (**phonons**)

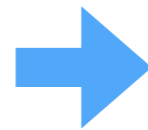


COM

$$H(t) = -g(\mu_R, t) \hat{z} \hat{S}_z + \omega_0 n$$

$$U(t) = U_{SP}(t) U_{SS}(t)$$

**spin-phonon    spin-spin**

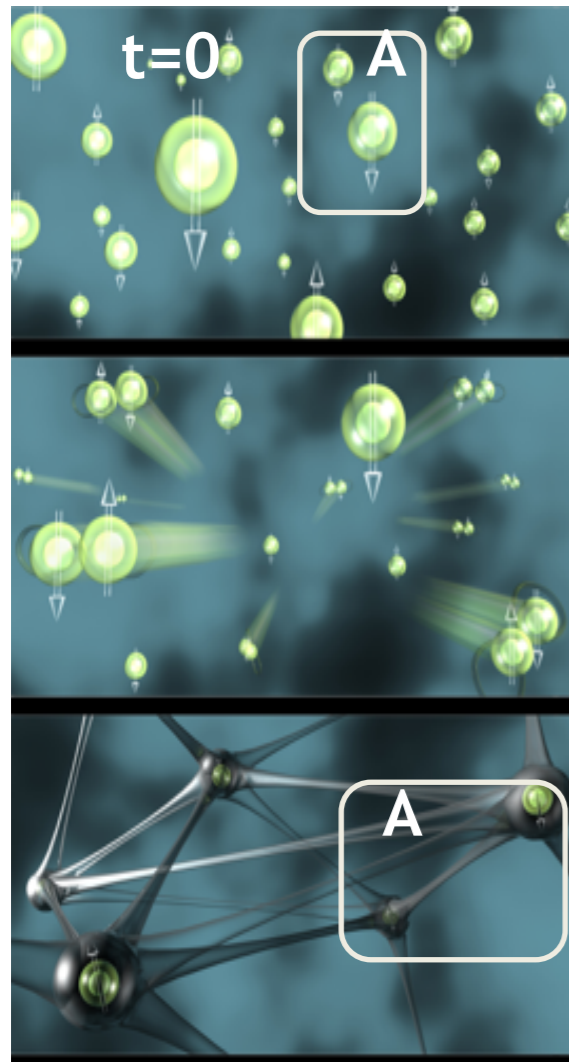


$$\hat{H}_{SS} = \frac{J}{N} \hat{S}_z^2$$

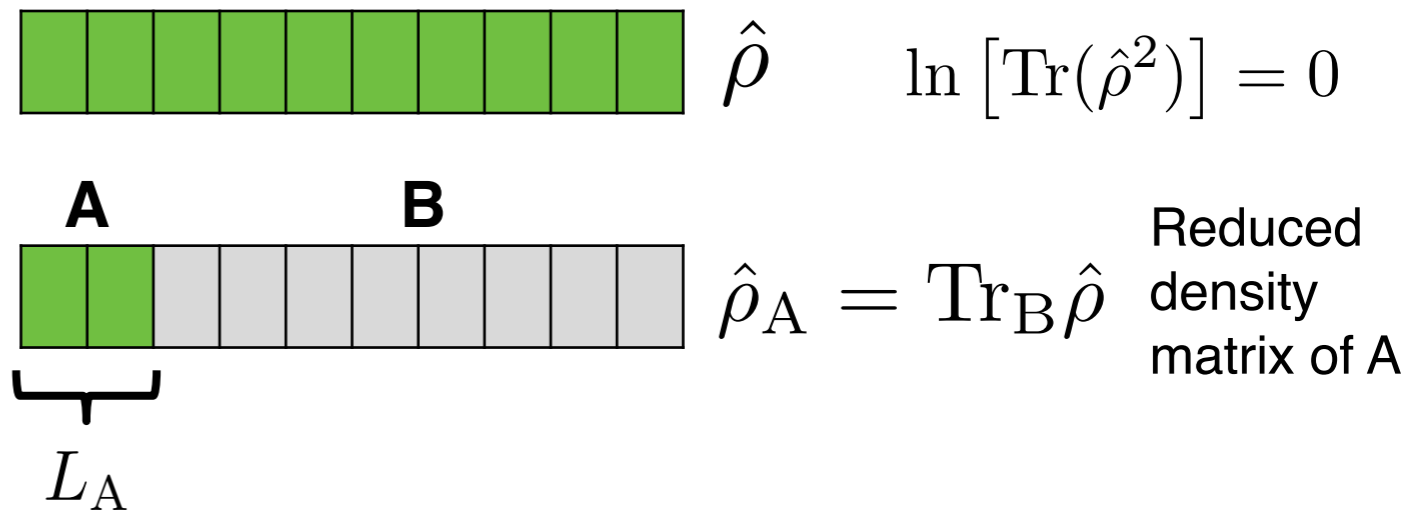
$$J \propto \frac{g^2}{\delta}$$

Britton et al. Nature 2012  
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 Wall, ASN, Rey, '16 '17

# Entanglement Entropy



**Global  
Unitary  
dynamics**

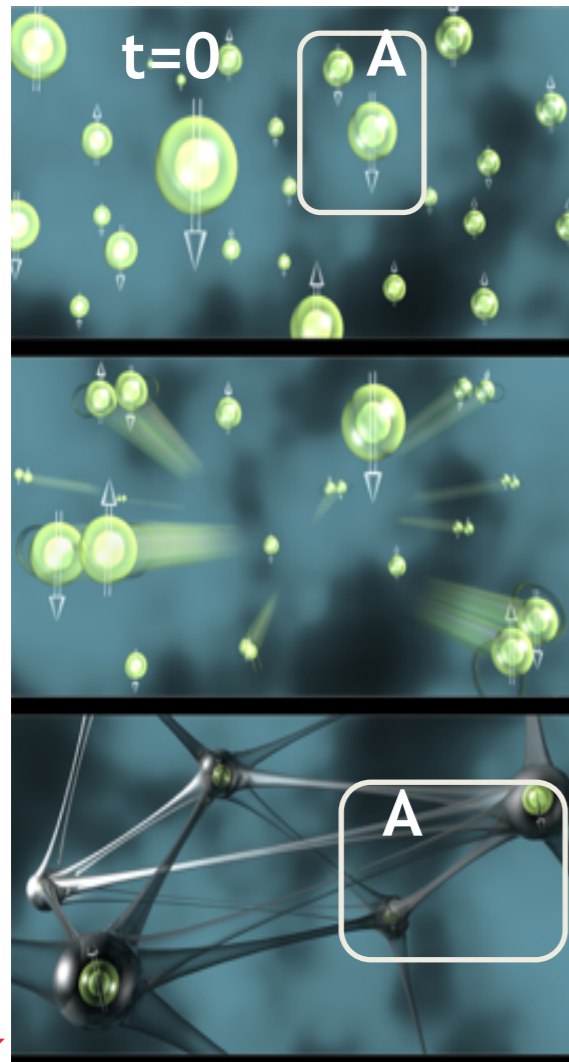


**Renyi entropy:** Purity of the A subsystem

$$S_A = -\ln [\text{Tr}(\hat{\rho}_A^2)]$$



# Entanglement Entropy



$S_A = 0$   
Product state

**Global  
Unitary  
dynamics**

$S_A \rightarrow L_A$   
Highly  
entangled

$\hat{\rho}$   $\ln [\text{Tr}(\hat{\rho}^2)] = 0$

**A** **B**  
 $\hat{\rho}_A = \text{Tr}_B \hat{\rho}$  Reduced density matrix of A  
 $L_A$

**Renyi entropy:** Purity of the A subsystem

$$S_A = -\ln [\text{Tr}(\hat{\rho}_A^2)]$$

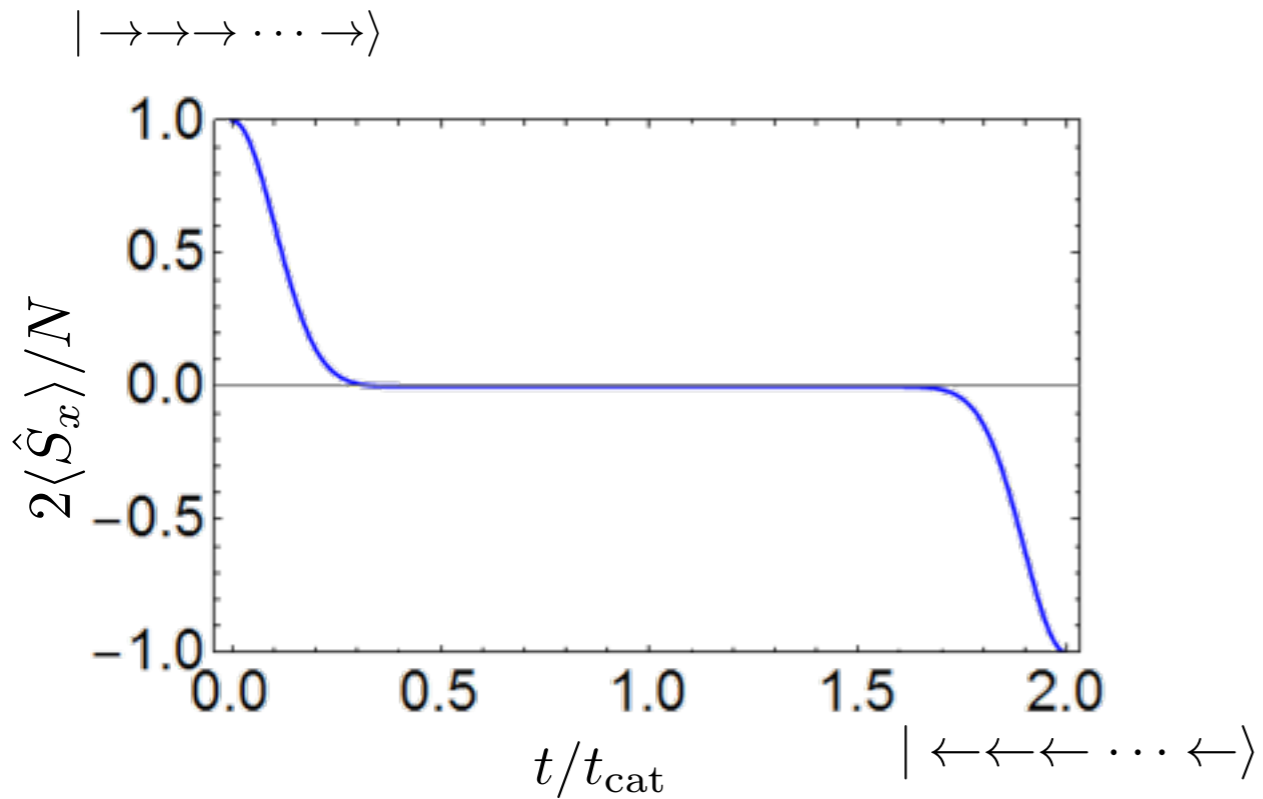
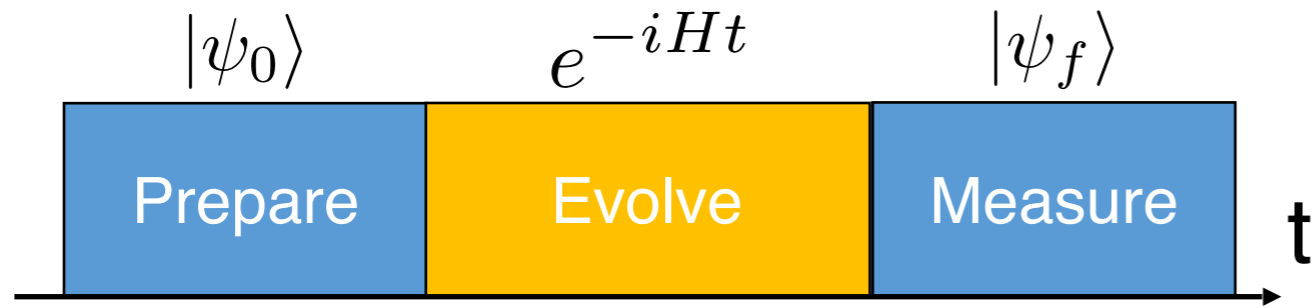
**Product state**

$$\hat{\rho} = \otimes \hat{\rho}_i$$

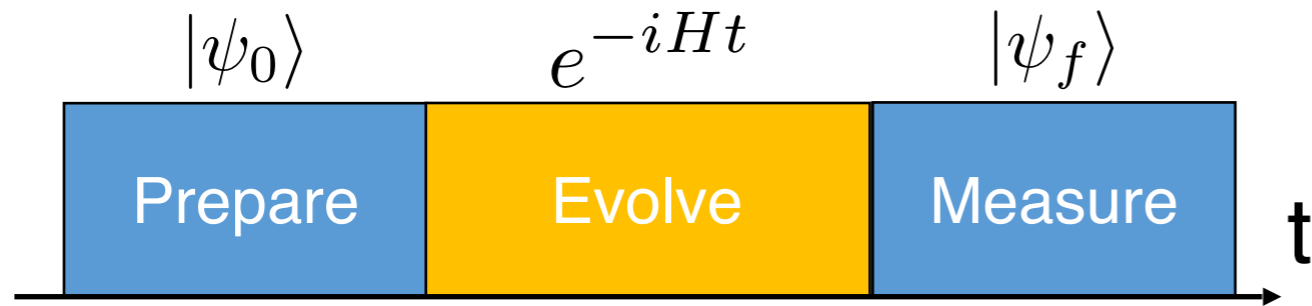
**Entangled state**

$$S_A > 0$$

# All-to-All Ising Model: One-body Observables

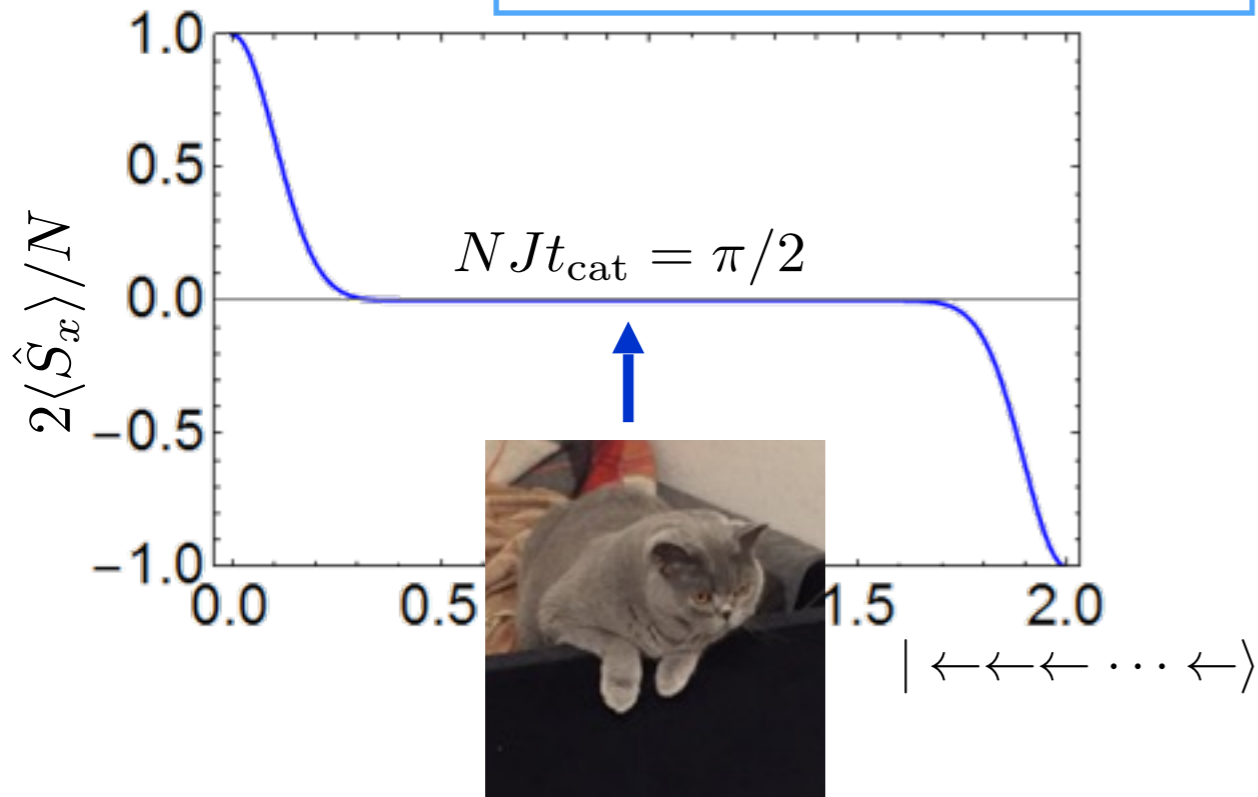


# All-to-All Ising Model: One-body Observables

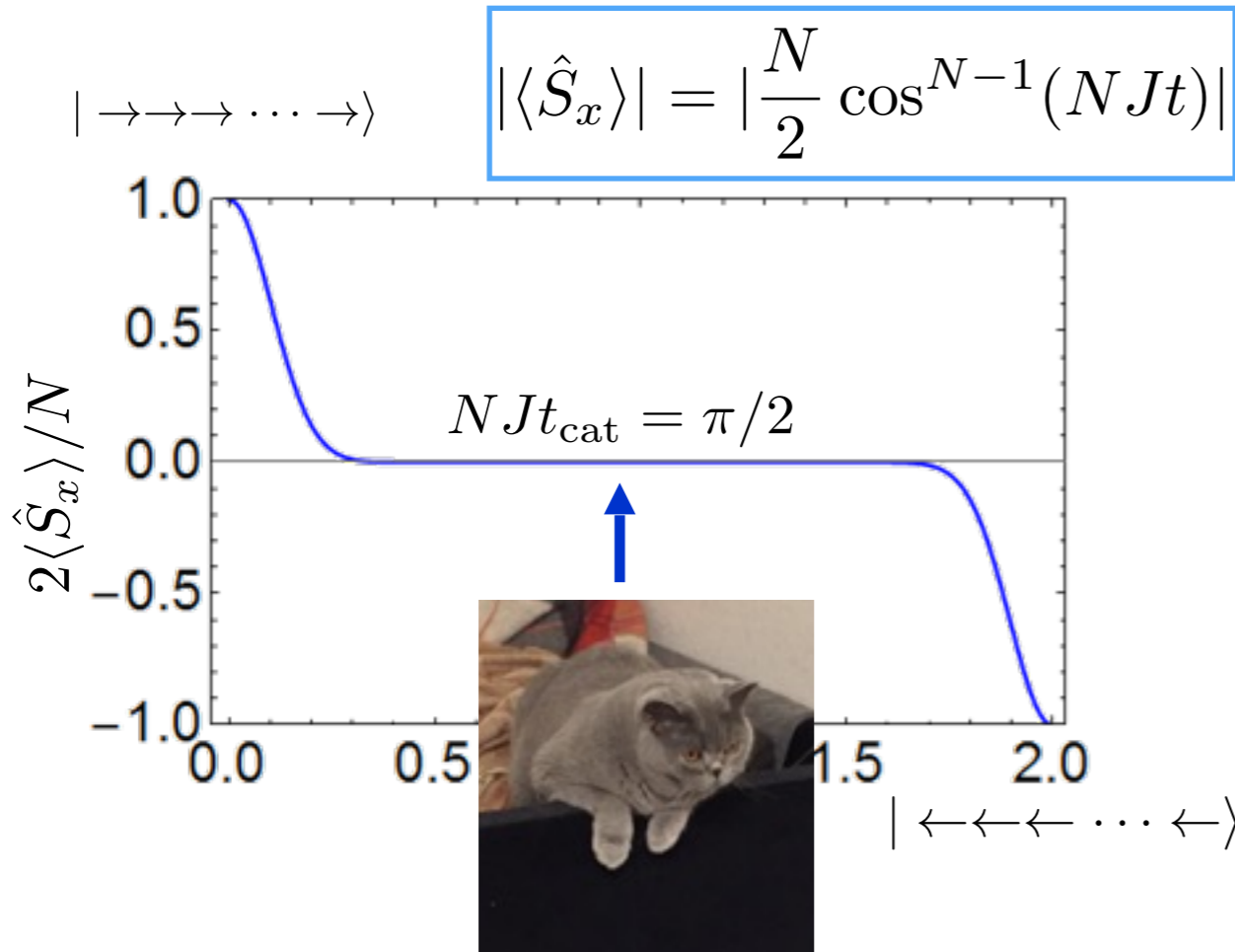
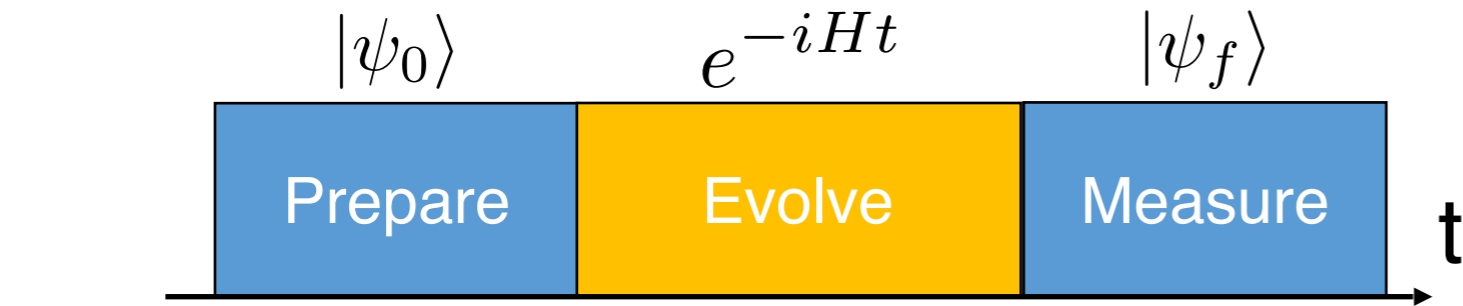


$|\rightarrow\rightarrow\rightarrow\dots\rightarrow\rangle$

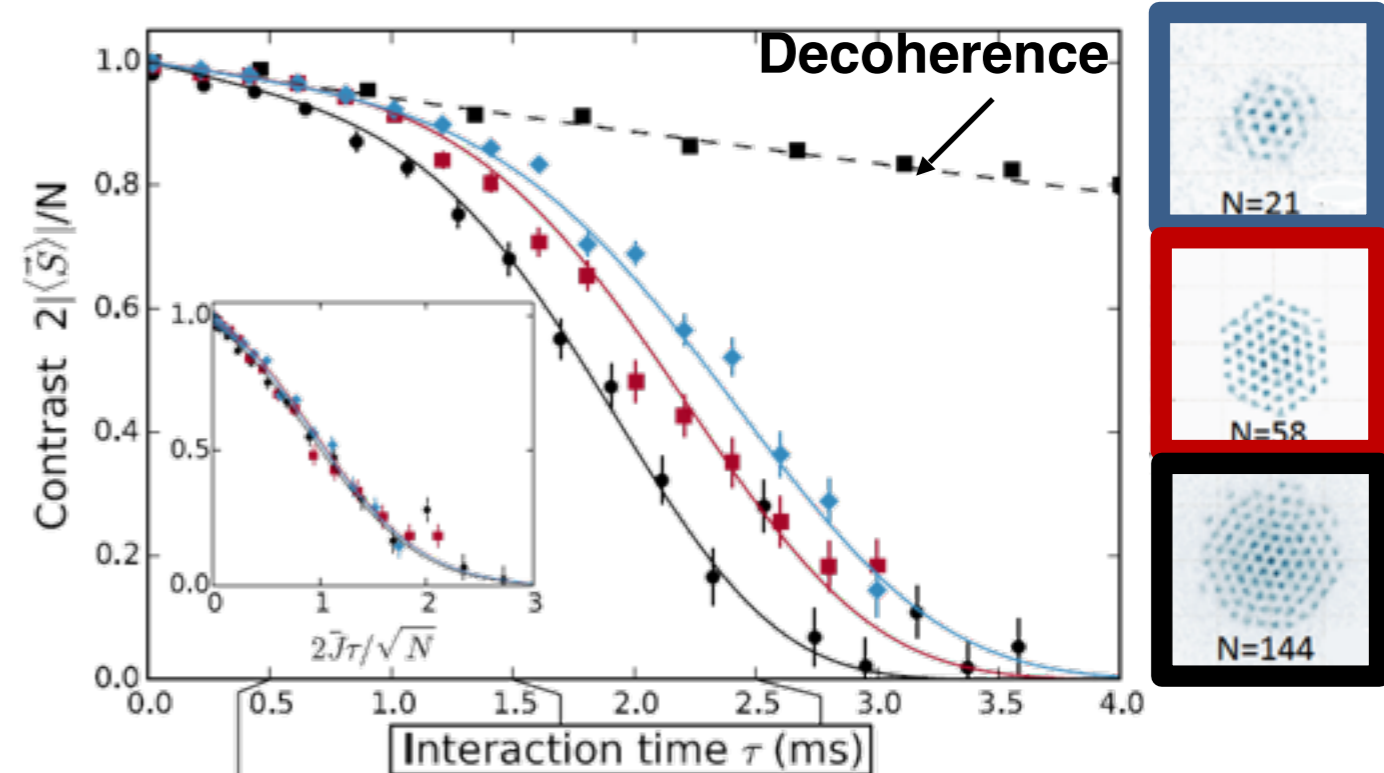
$|\langle\hat{S}_x\rangle| = \left|\frac{N}{2}\cos^{N-1}(NJt)\right|$



# All-to-All Ising Model: One-body Observables

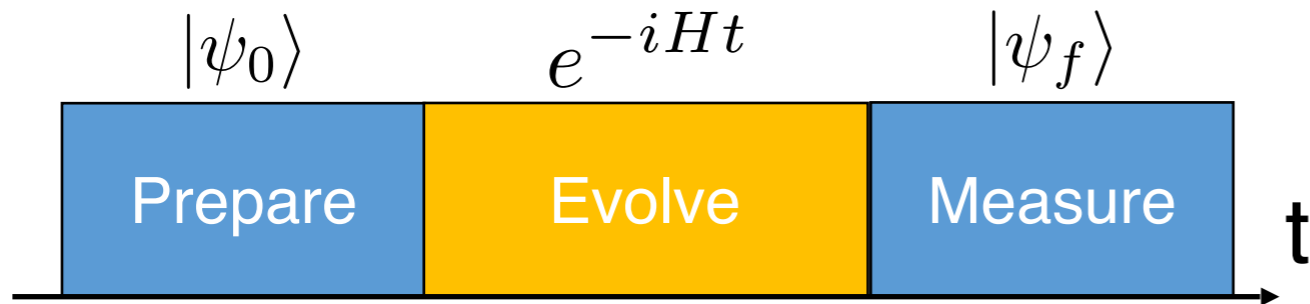


**Coherent spin demagnetization observed experimentally:** collapse of rescaled data

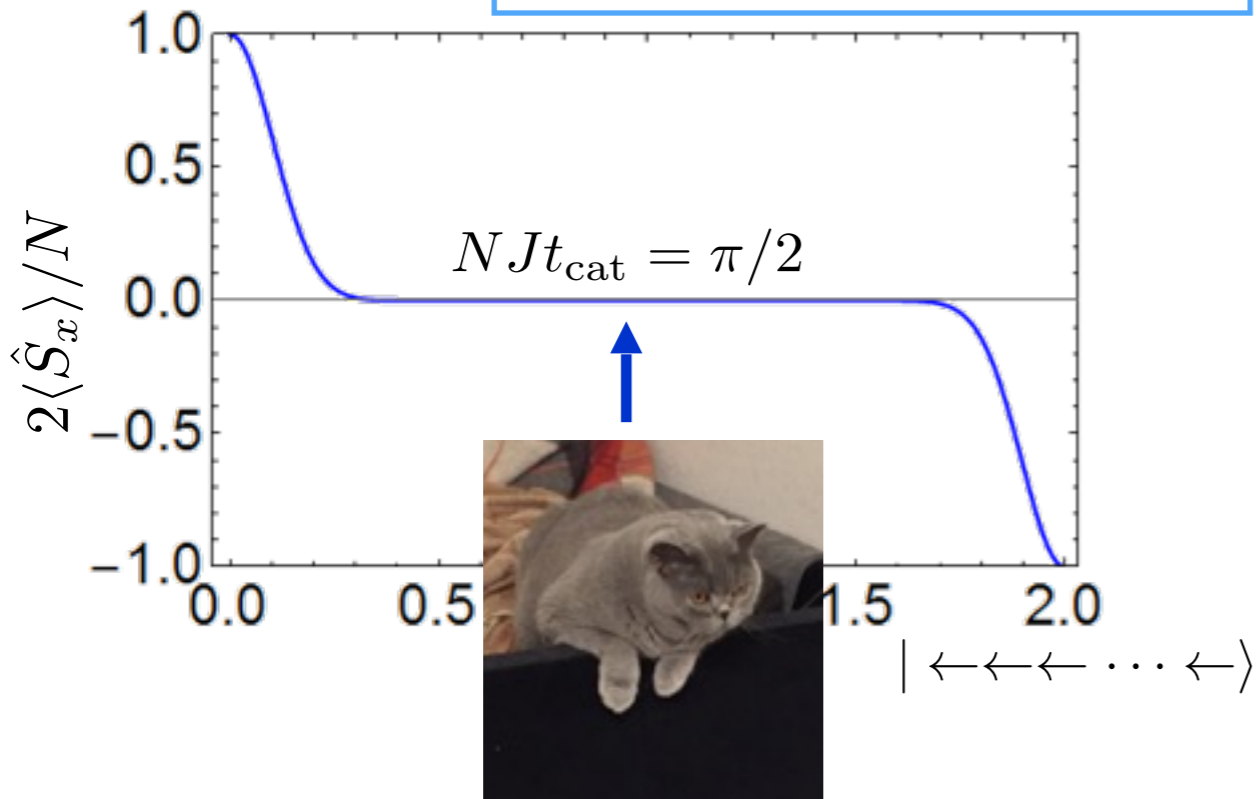


Bohnet *et al.*, Science 352, 1297(2016)

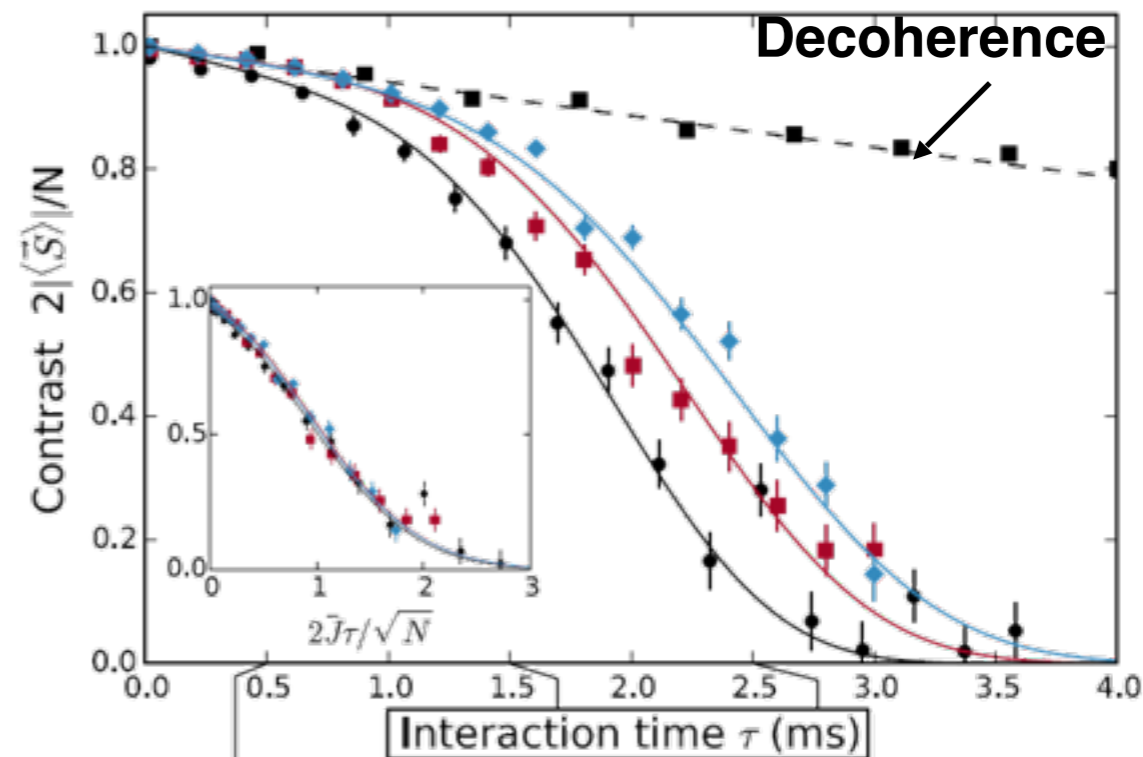
# All-to-All Ising Model: One-body Observables



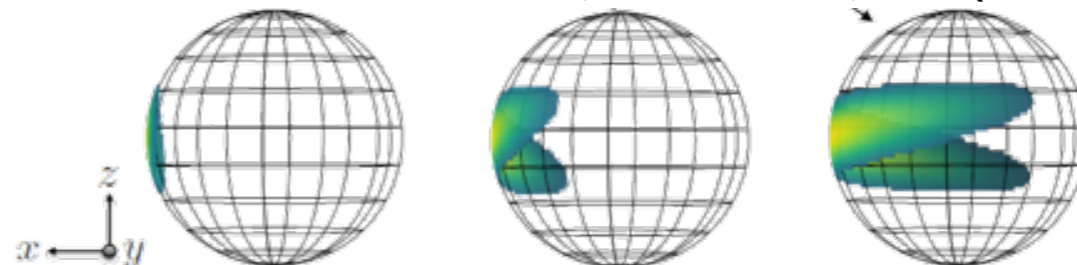
$$|\langle \hat{S}_x \rangle| = \left| \frac{N}{2} \cos^{N-1}(NJt) \right|$$



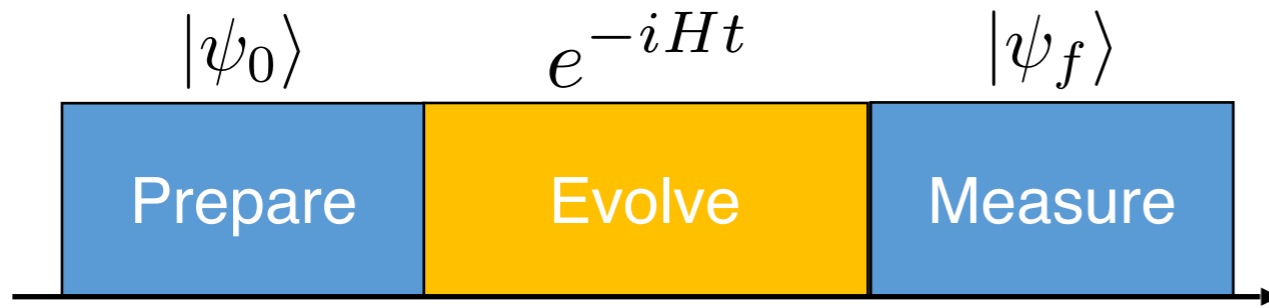
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Bohnet *et al.*, Science 352, 1297(2016)



# All-to-All Ising Model: One-body Observables



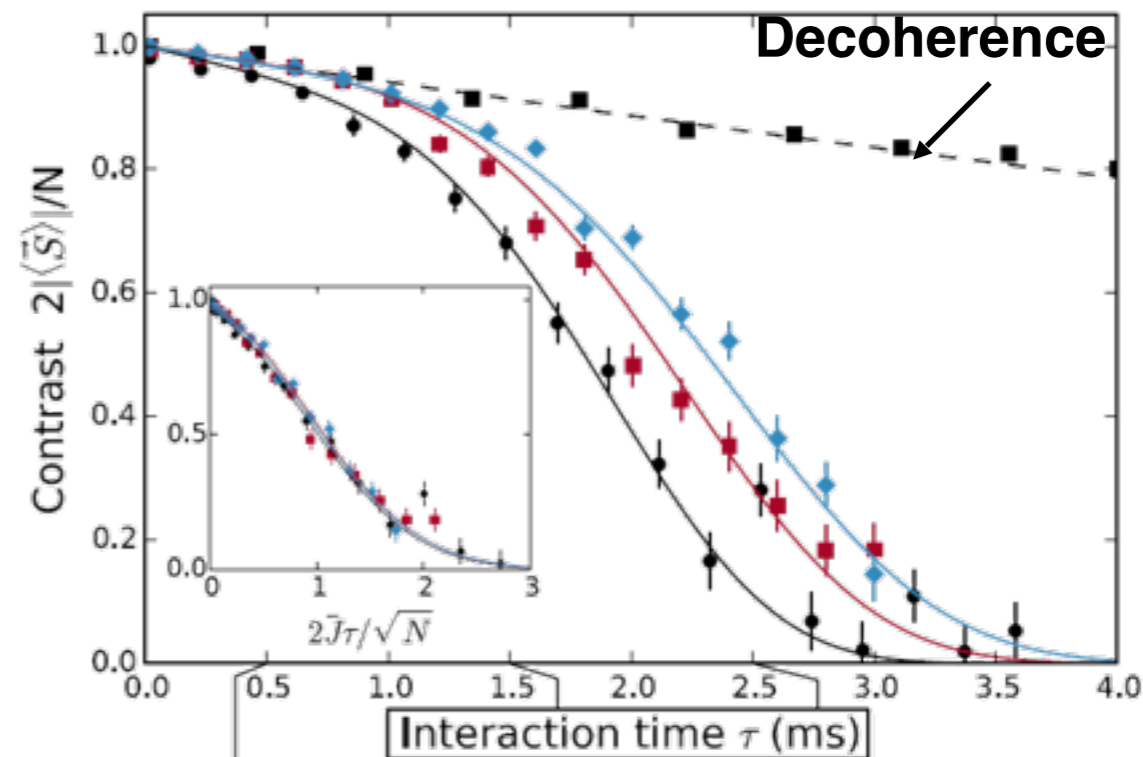
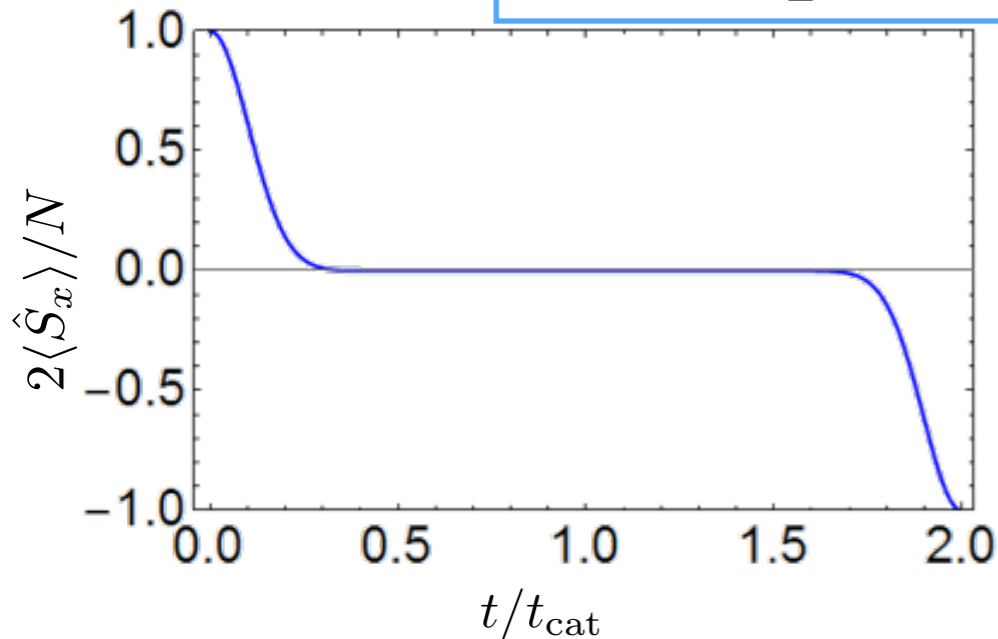
**A**



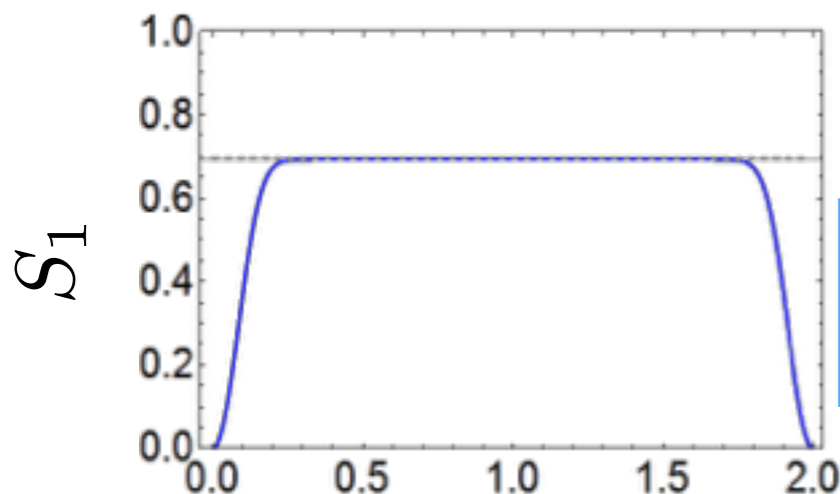
$$L_A = 1$$

**Coherent spin demagnetization observed experimentally:** collapse of rescaled data

$$|\langle \hat{S}_x \rangle| = \left| \frac{N}{2} \cos^{N-1}(NJt) \right|$$



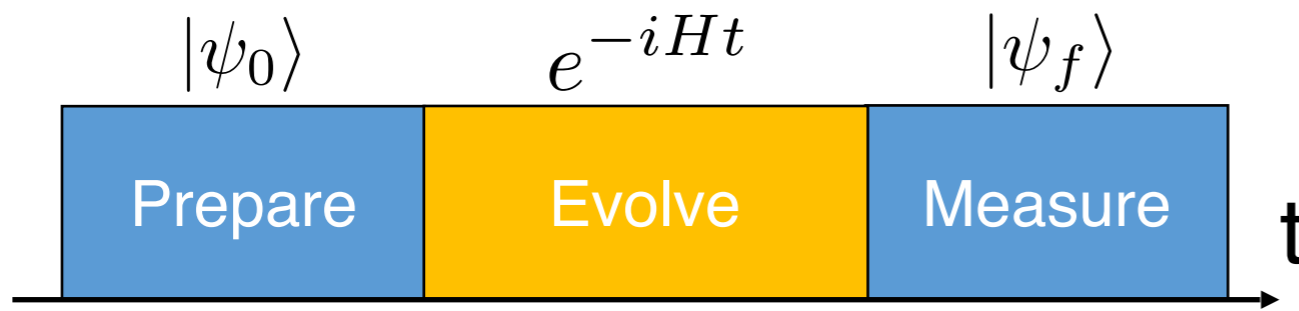
Connection to Renyi entropy of partition of length 1?



$$S_1 = -\ln \left[ \frac{1}{2} \left( 1 + \left( \frac{2}{N} \langle \hat{S}_x \rangle \right)^2 \right) \right]$$

Bohnet *et al.*, Science 352, 1297(2016)

# All-to-All Ising Model: One-body Observables



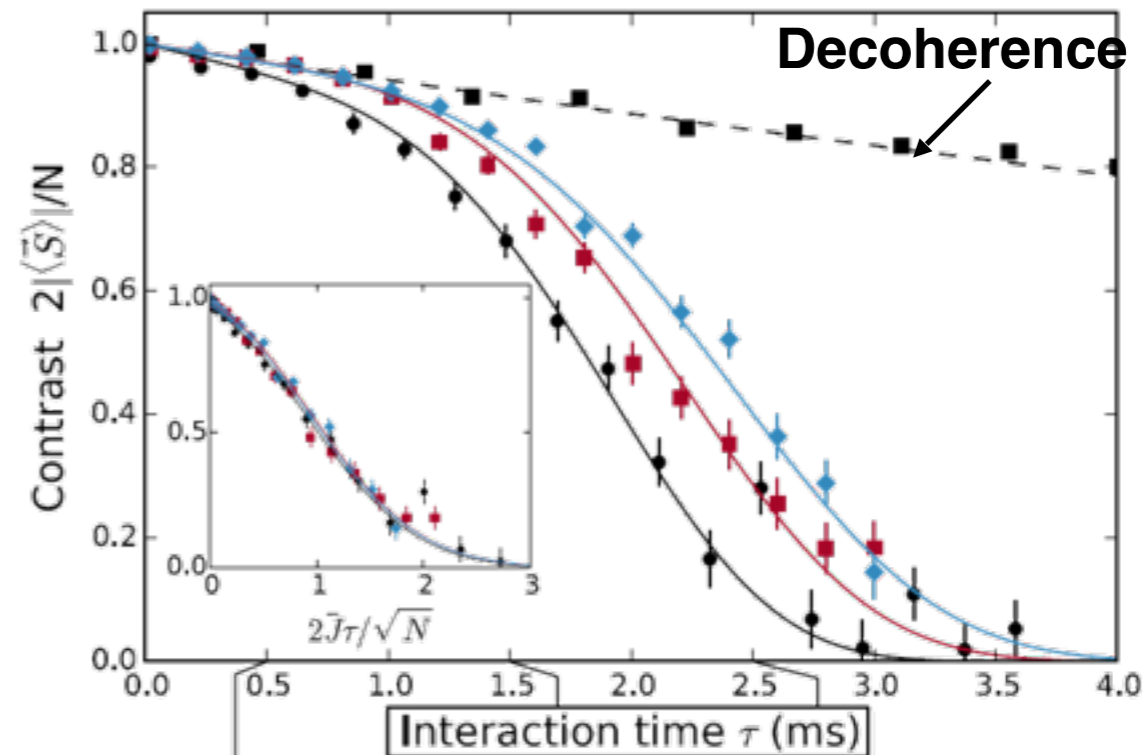
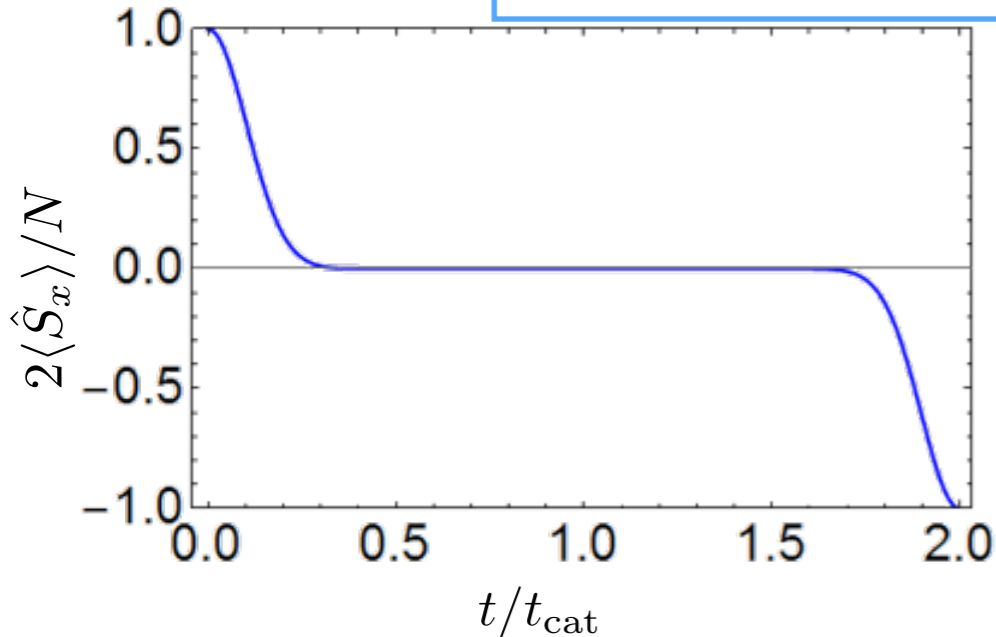
**A**



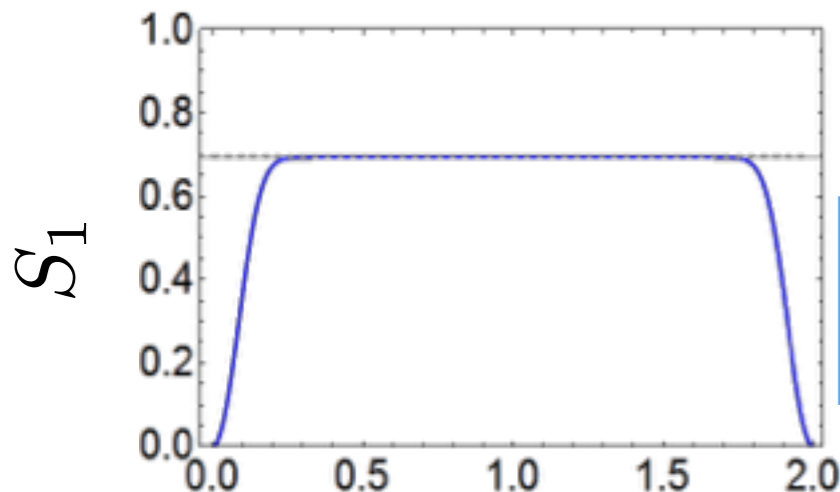
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**Coherent spin demagnetization observed experimentally:** collapse of rescaled data

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Connection to Renyi entropy of partition of length 1?

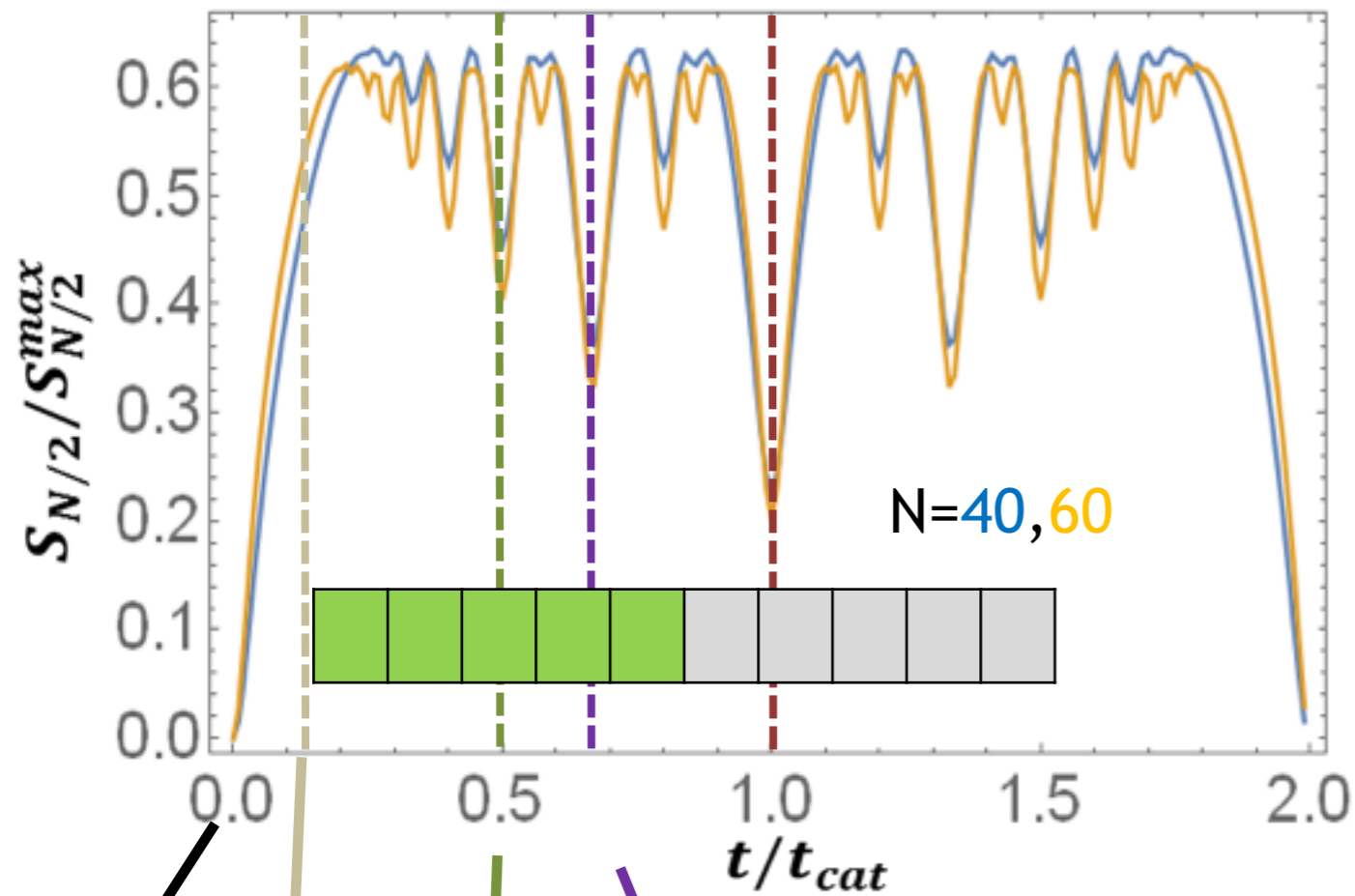


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Bohnet *et al.*, Science 352, 1297(2016)

Can we understand what the system is doing in more detail with global measurements?

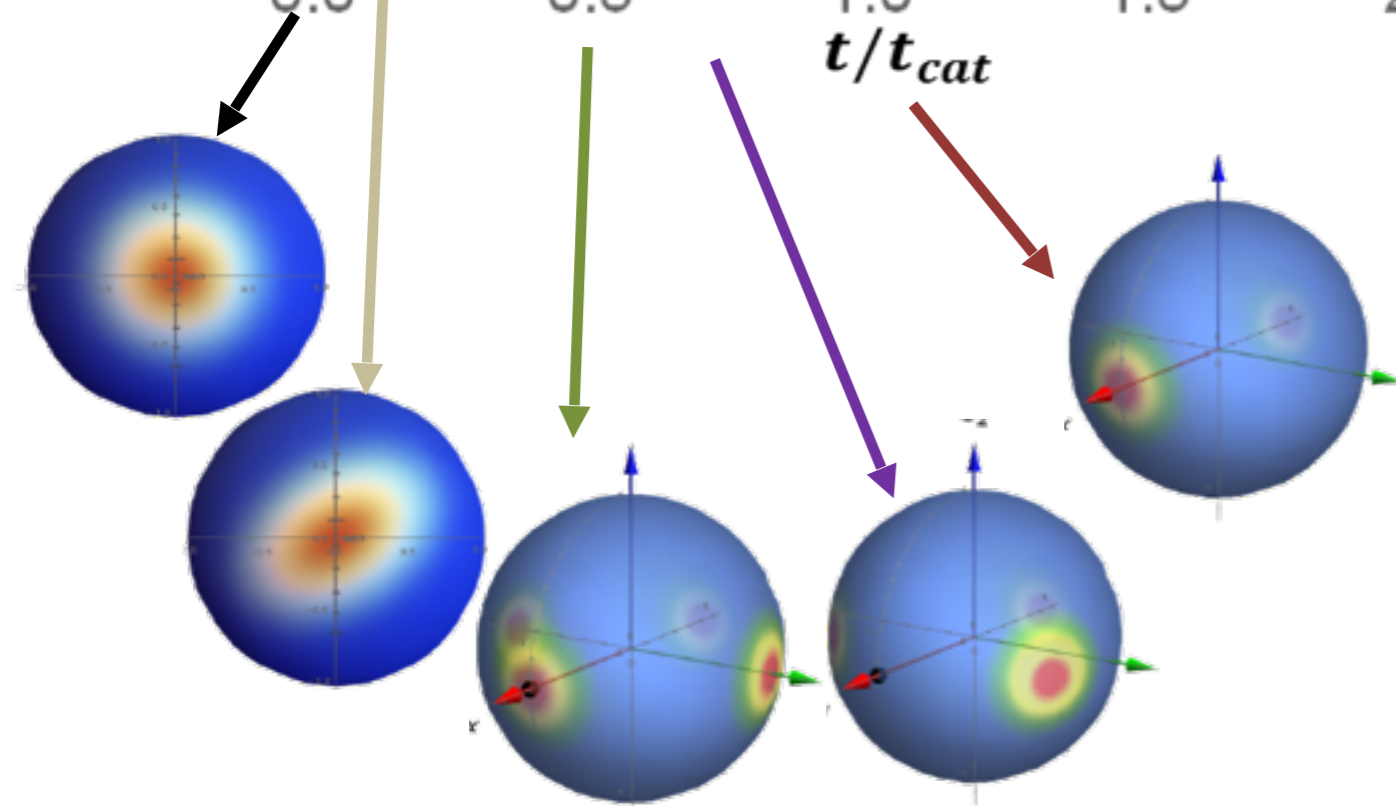
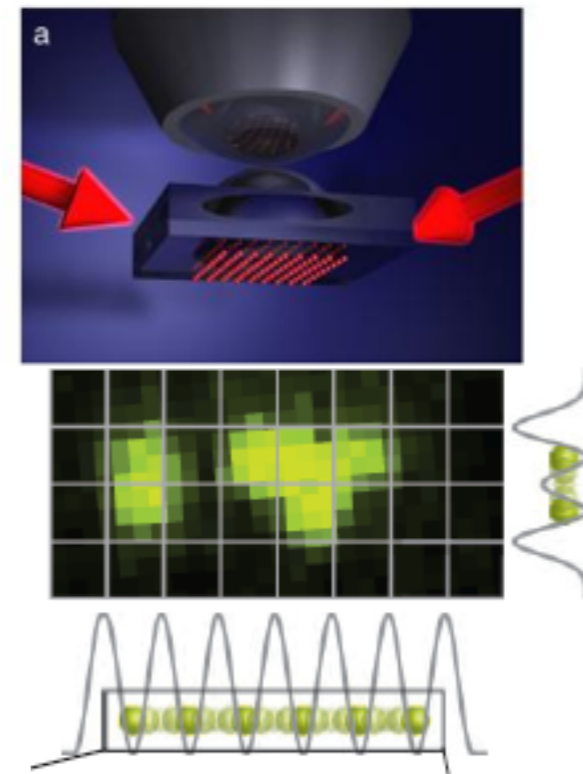
# All-to-All Ising Model: Entanglement



**Entanglement entropy for the half-chain: features instead of the plateau**

**Caveats:**

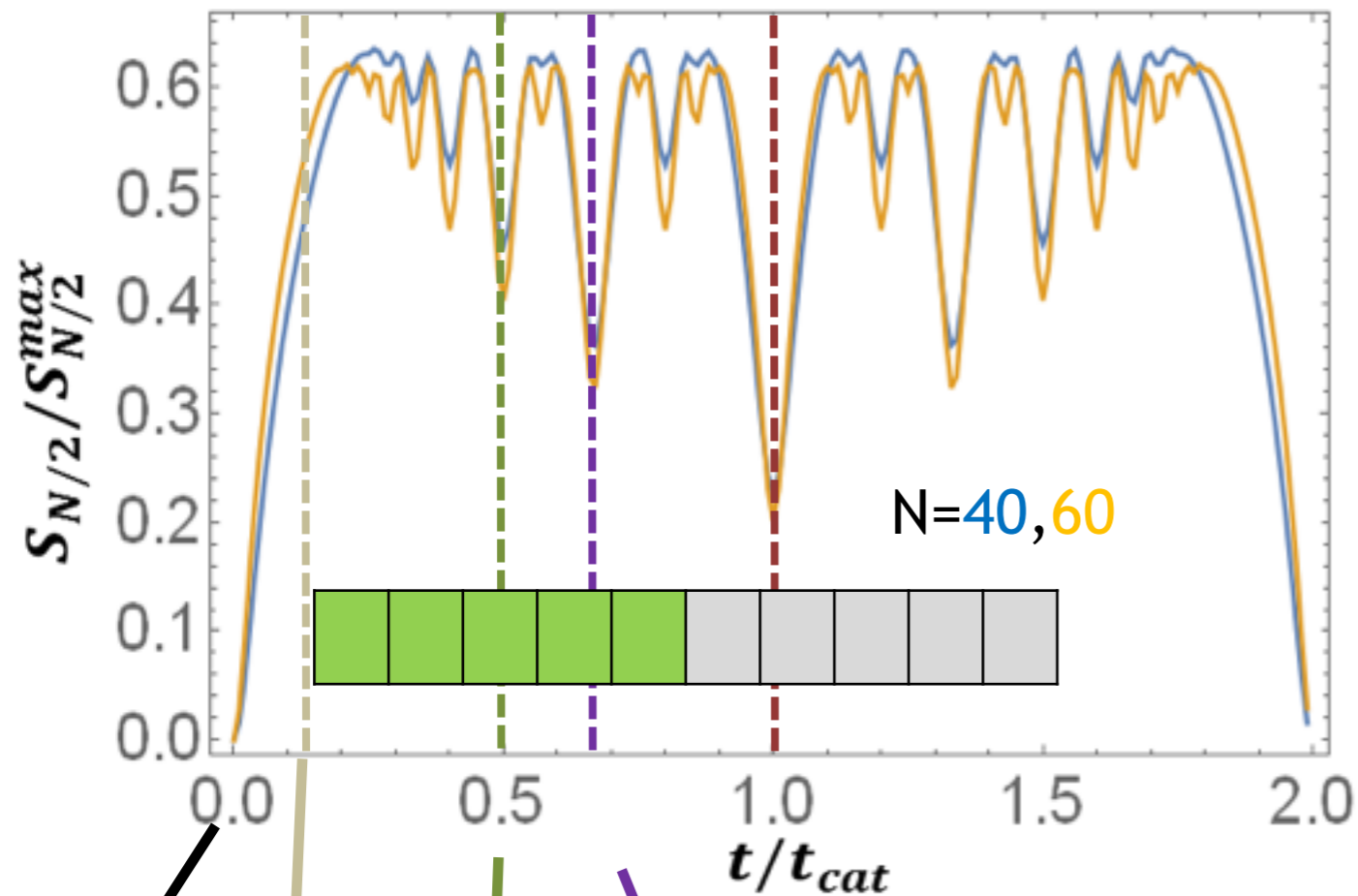
- Single site resolution (quantum gas microscope)
  - Limited to  $\sim 10$  ions
- Intractable for large systems



**Experiments: A. Kaufman *et al* Science 353, 794 (2016)**



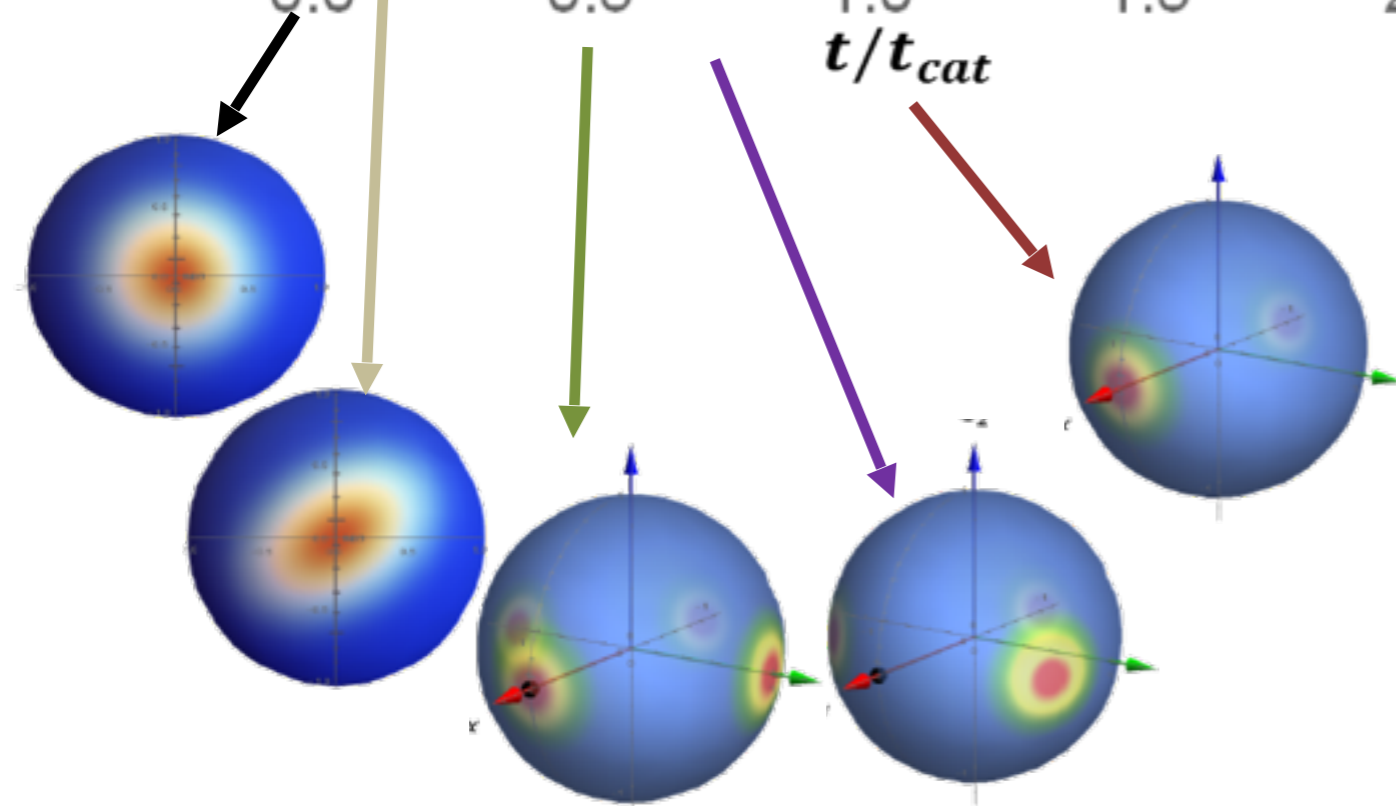
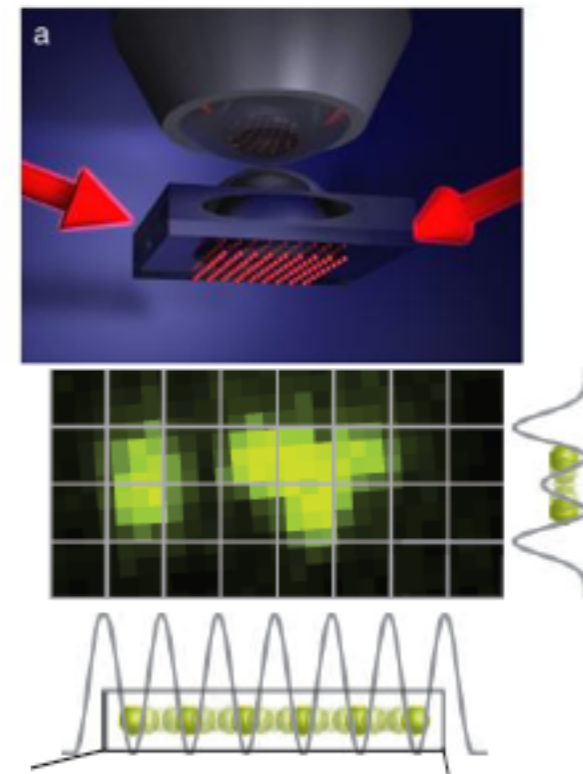
# All-to-All Ising Model: Entanglement



**Entanglement entropy for the half-chain: features instead of the plateau**

**Caveats:**

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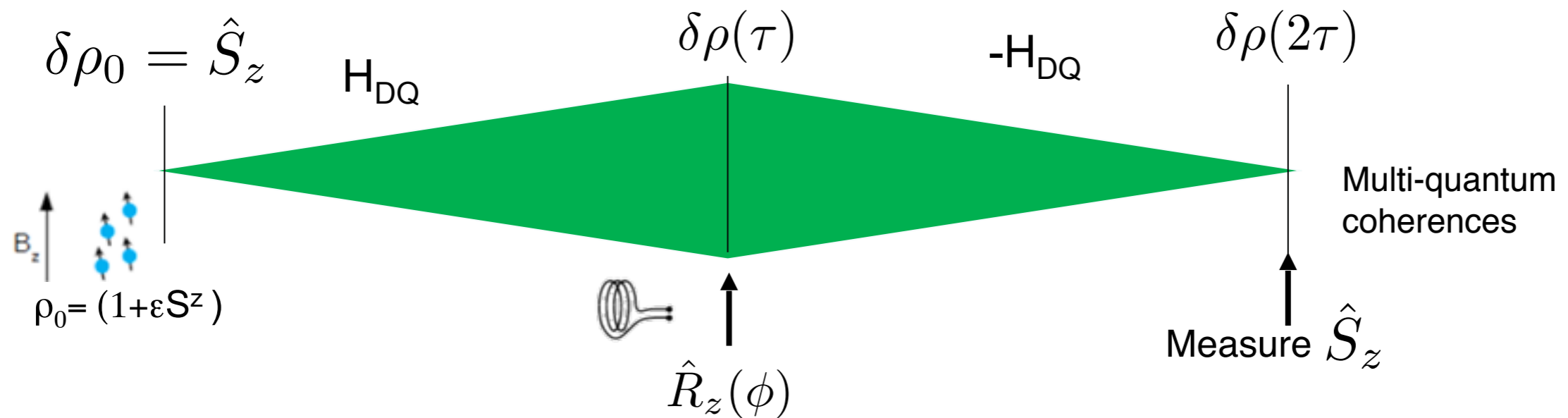
Experiments: A. Kaufman *et al* Science 353, 794 (2016)

**Large system + absence of single site resolution?**

# Multiple Quantum Coherence (MQC) Spectrum

## Multi-Quantum coherence spectrum (NMR)

$$H_{\text{DQ}} \propto \sum_{ij} J_{ij} (\hat{\sigma}_i^+ \hat{\sigma}_j^+ + \hat{\sigma}_i^- \hat{\sigma}_j^-)$$

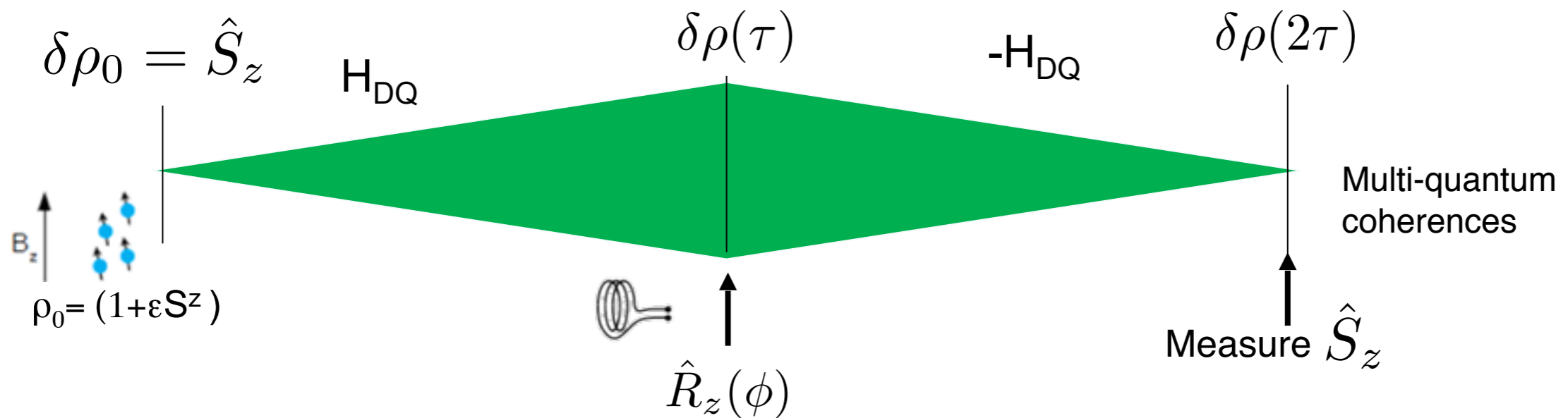


1. Start with a highly mixed state
2. Evolve
3. Rotate
4. Time-reverse
5. Measure magnetization: here fully characterizes the state

M. Munowitz and M. Mehring, Sol. St. Com., 64, 605 (1987)

# Multiple Quantum Coherence (MQC) Spectrum

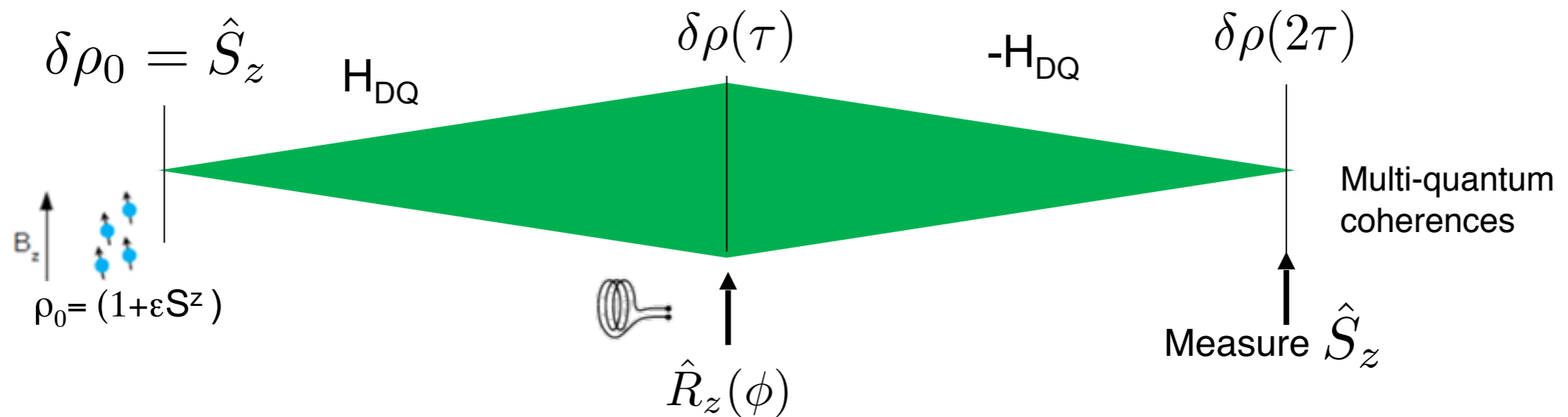
$$\langle \hat{S}_z \rangle = \text{Tr} [\delta\rho_0 \delta\rho(2\tau)] = \text{Tr} \left[ \delta\rho_0 e^{itH_{\text{DQ}}} e^{-i\phi\hat{S}_z} e^{-itH_{\text{DQ}}} \rho_0 e^{-itH_{\text{DQ}}} e^{i\phi\hat{S}_z} e^{itH_{\text{DQ}}} \right]$$



1. Start with a highly mixed state
2. Evolve
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# Multiple Quantum Coherence (MQC) Spectrum

$$\langle \hat{S}_z \rangle = \text{Tr} [\delta \rho_0 \delta \rho(2\tau)] = \text{Tr} \left[ \underbrace{\delta \rho_0}_{V^\dagger(0)} \underbrace{e^{itH_{\text{DQ}}} e^{-i\phi \hat{S}_z} e^{-itH_{\text{DQ}}}}_{W^\dagger(t)} \underbrace{\rho_0}_{V(0)} \underbrace{e^{-itH_{\text{DQ}}} e^{i\phi \hat{S}_z} e^{itH_{\text{DQ}}}}_{W(t)} \right]$$

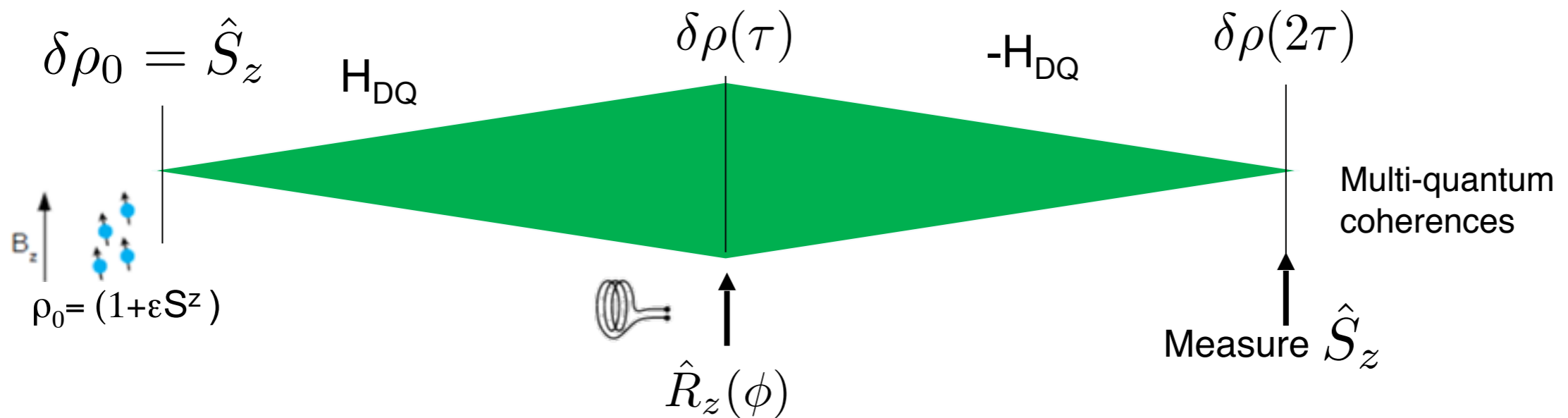


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$$F(t) = \langle W^\dagger(t) V^\dagger(0) W(t) V(0) \rangle$$

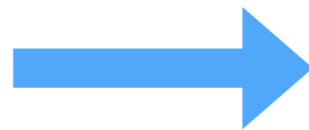


1. Start with a highly mixed state
2. Evolve
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4. Time-reverse
5. Measure magnetization: here fully characterizes the state

# Multi-quantum Coherences

How do we generalize coherences from **one** particle to **many**?

$$\hat{\rho} = \begin{bmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{bmatrix}$$



**N=3**

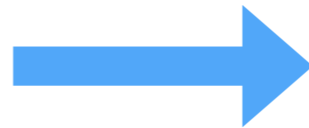
	↑↑↑	↓↑↑	↑↓↑	↑↑↓	↓↓↑	↓↑↓	↓↓↑	↓↓↓
↑↑↑	0		m=1		m=2			3
↓↑↑								
↑↓↑			m=0		m=1			
↑↑↓								
↓↓↑								
↓↑↓			m=-1		m=0			
↓↓↑								
↓↓↓	-3		m=-2		m=-1			0

$$\hat{\rho} = \sum_m \hat{\rho}_m$$

# Multi-quantum Coherences

How do we generalize coherences from **one** particle to **many**?

$$\hat{\rho} = \begin{bmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{bmatrix}$$



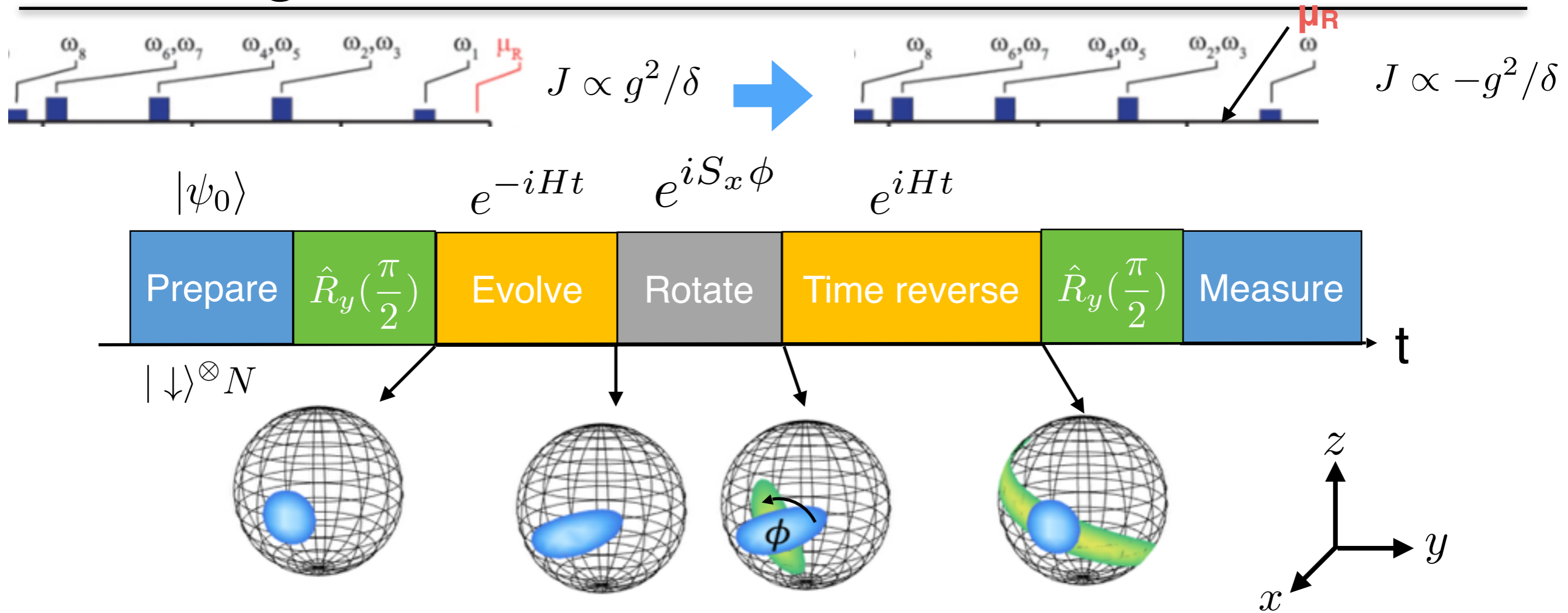
**N=3**

	↑↑↑	↓↑↑	↑↓↑	↑↑↓	↓↓↑	↓↑↓	↓↓↓
↑↑↑	0		m=1		m=2		3
↓↑↑							
↑↓↑			m=0		m=1		
↑↑↓							
↓↓↑							
↓↑↓			m=-1		m=0		
↓↓↑							
↓↓↓	-3		m=-2		m=-1		0

$$\hat{\rho} = \sum_m \hat{\rho}_m$$

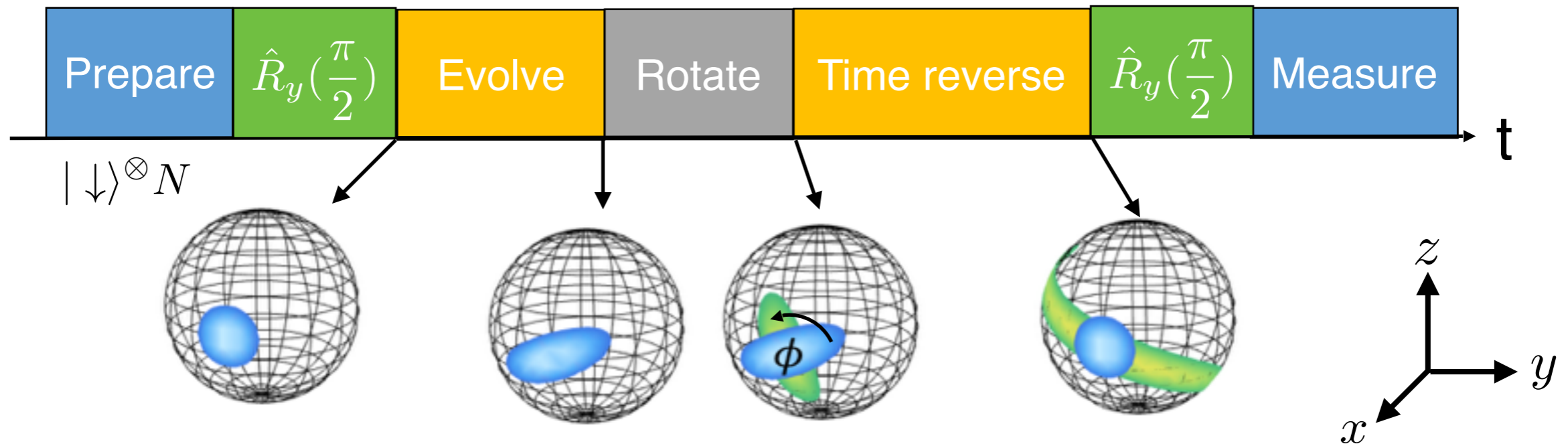
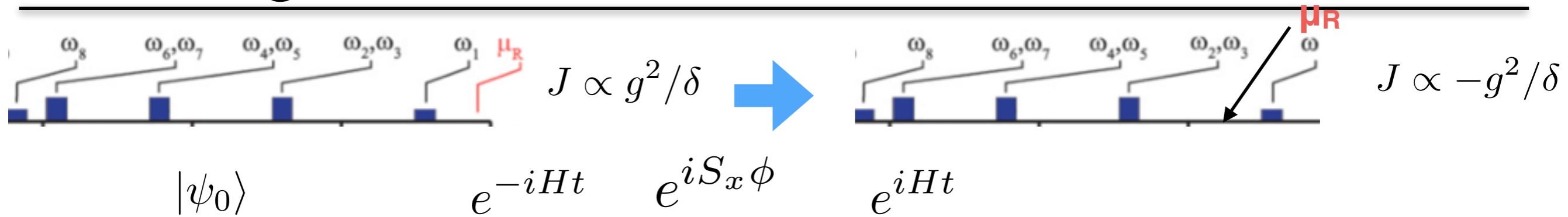
$$\hat{\rho}_m = \langle \{(n-m) \downarrow\} | \hat{\rho} | \{n \downarrow\} \rangle \neq 0$$

# Reversing Time with Ions

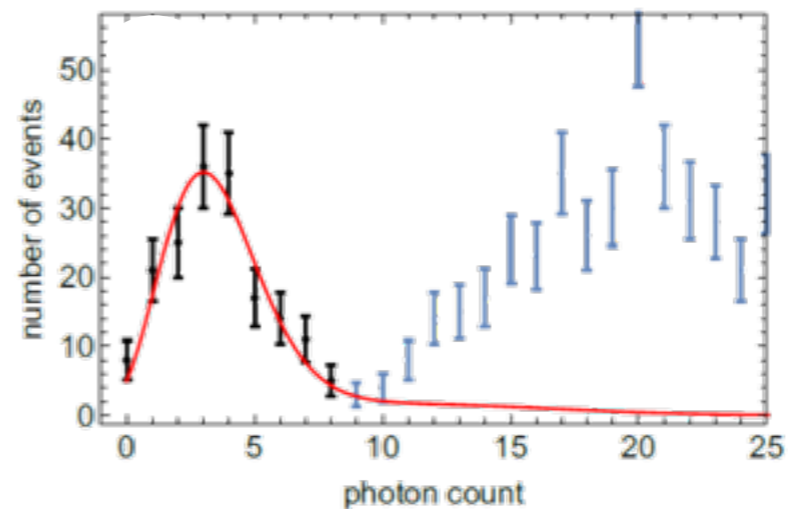




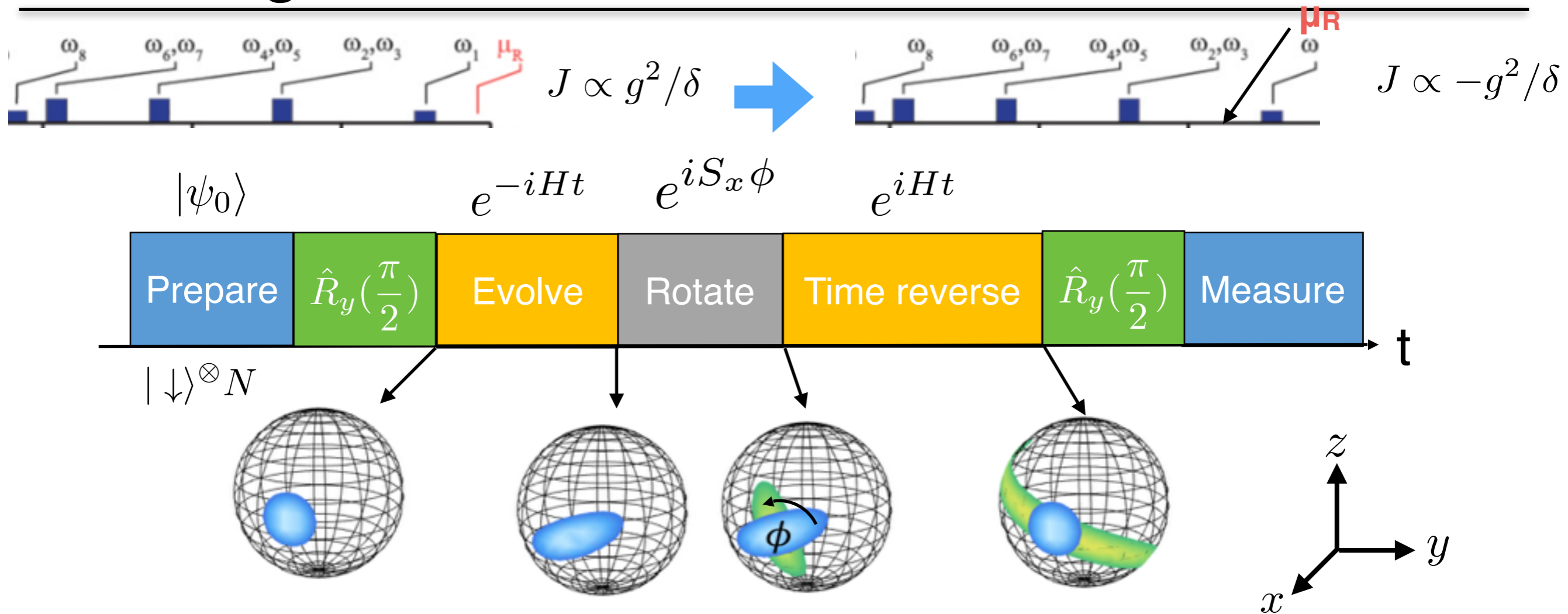
# Reversing Time with Ions



**Pure initial state  $\rightarrow$  Measure Fidelity**  
 Probability of all down



# Reversing Time with Ions



## Many-body Loschmidt echo

$$\begin{aligned}
 \langle \rho_0 \rangle &= \text{Tr}[\rho_0 \rho_f] = \text{Tr}[\rho_0 e^{itH} e^{-i\phi \hat{S}_x} e^{-itH} \rho_0 e^{itH} e^{i\phi \hat{S}_x} e^{-itH}] \\
 &= \sum_{m=-N}^N \underbrace{\text{Tr}[\rho_m \rho_m^\dagger]}_{I_m} e^{im\phi}
 \end{aligned}$$

$I_m$  = Multi-quantum coherences

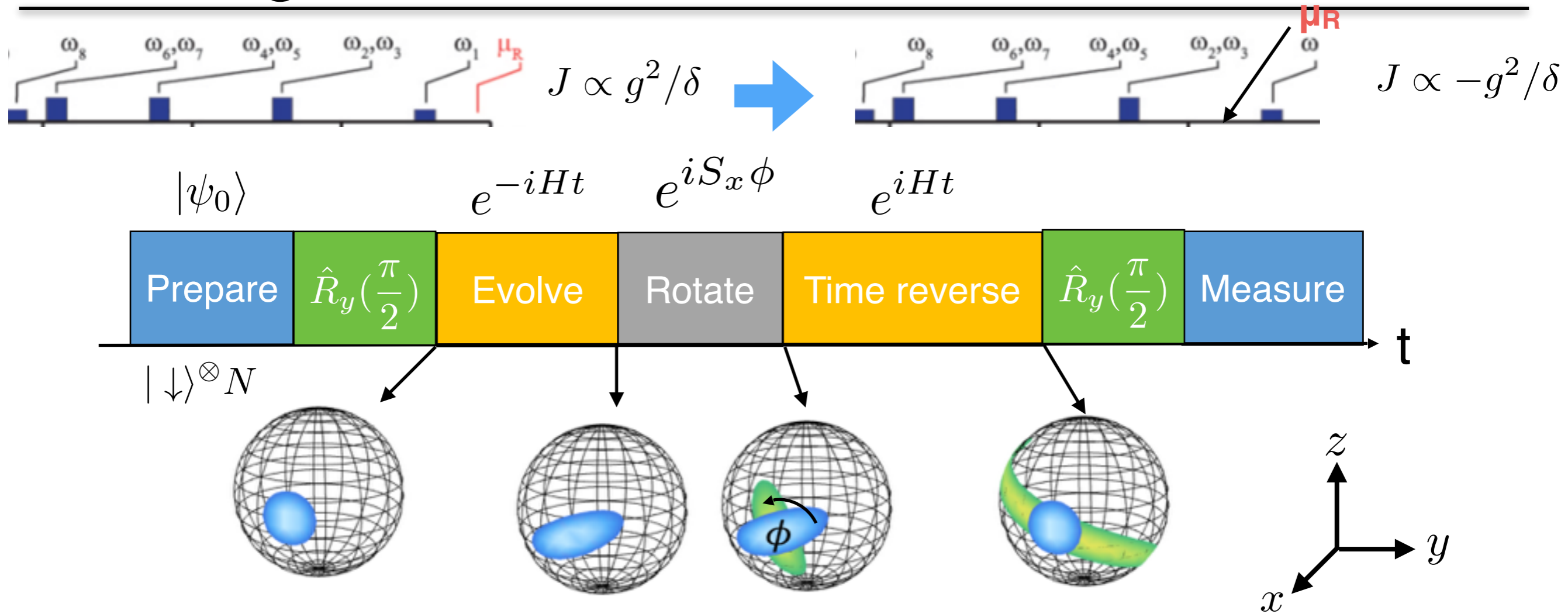
$$I_m = \text{Tr}[\rho_{-m}(t) \rho_m(t)]$$

Type of purity

$$I_0 = \text{Tr}[\rho_0^2(t)]$$

Fourier transform:  $\phi$  gives the Multi-Quantum spectrum.

# Reversing Time with Ions



## Many-body Loschmidt echo

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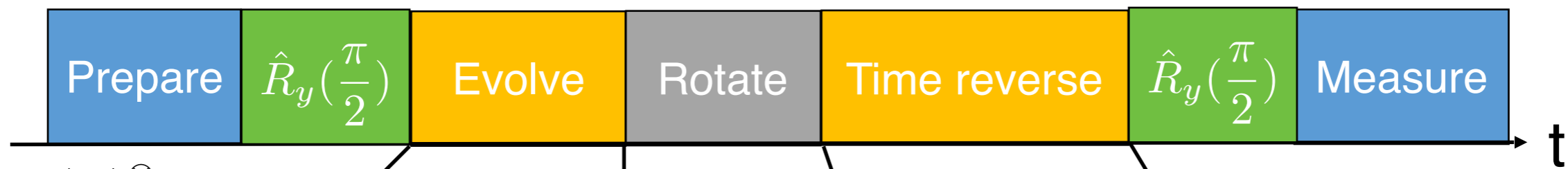
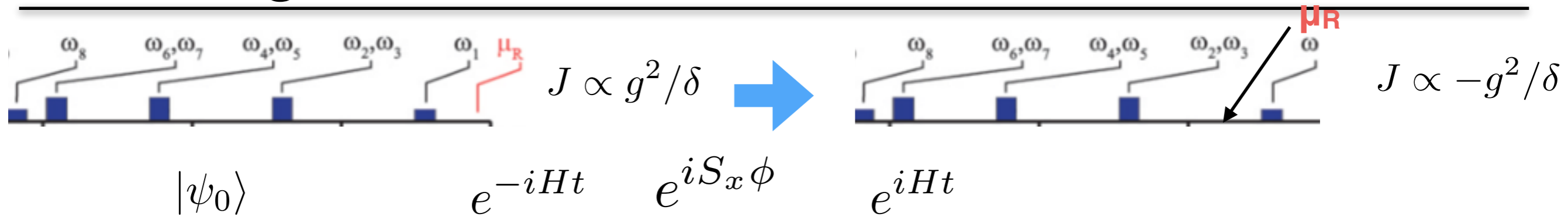
Type of purity

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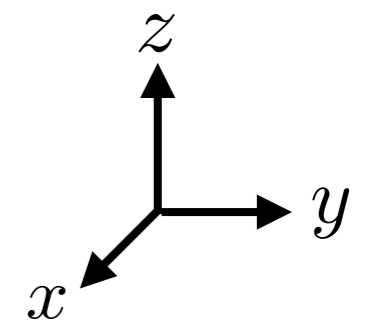
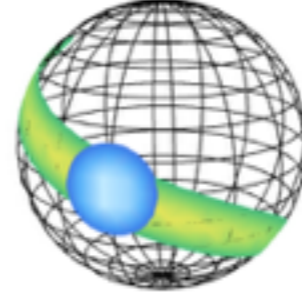
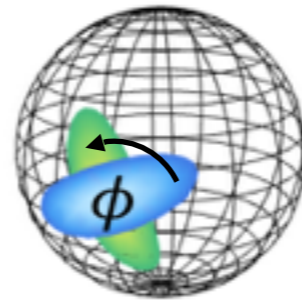
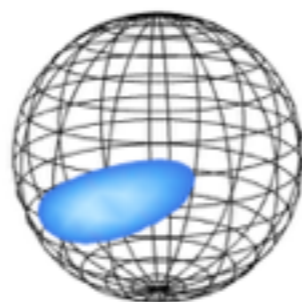
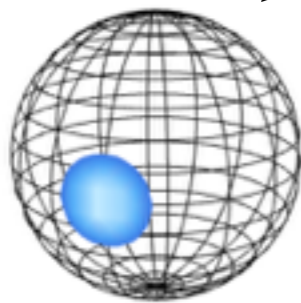
**Fourier transform:**  $\phi$  gives the Multi-Quantum spectrum.

**Connection to Renyi entropy?**

# Reversing Time with Ions



$|\downarrow\rangle^{\otimes N}$



## Many-body Loschmidt echo

$$\langle \rho_0 \rangle = \text{Tr}[\rho_0 \rho_f] = \text{Tr}[\rho_0 \underbrace{e^{itH}}_{V^\dagger(0)} \underbrace{e^{-i\phi \hat{S}_x}}_{W^\dagger(t)} \underbrace{e^{-itH}}_{V(0)} \rho_0 \underbrace{e^{itH}}_{W(t)} \underbrace{e^{i\phi \hat{S}_x}}_{V^\dagger(0)} e^{-itH}] = \sum_{m=-N}^N \text{Tr}[\rho_m \rho_m^\dagger] e^{im\phi}$$

$V^\dagger(0)$   $W^\dagger(t)$   $V(0)$   $W(t)$

$$I_m = \text{Tr}[\rho_{-m}(t) \rho_m(t)]$$

Type of purity

## Fidelity OTOC

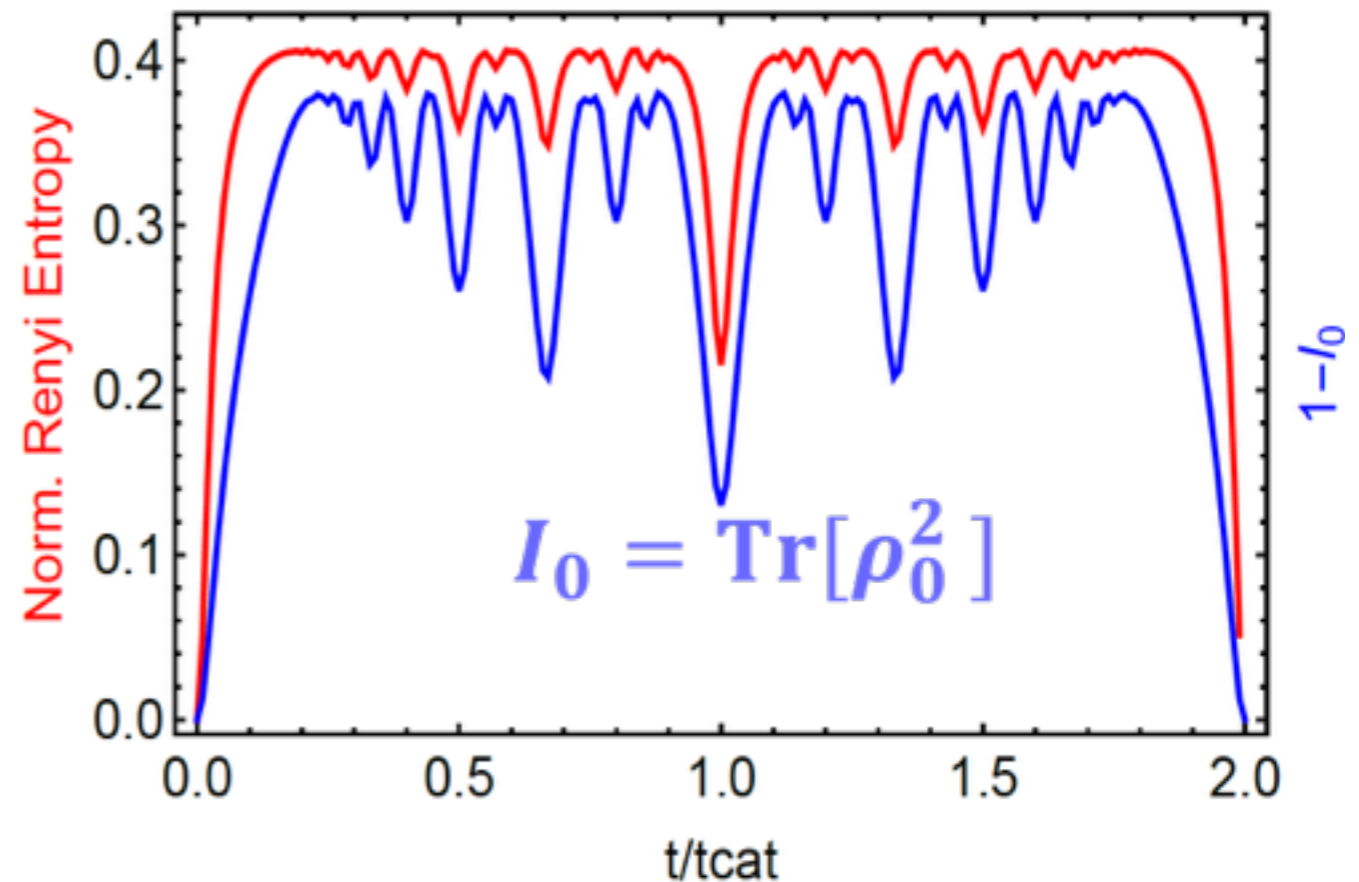
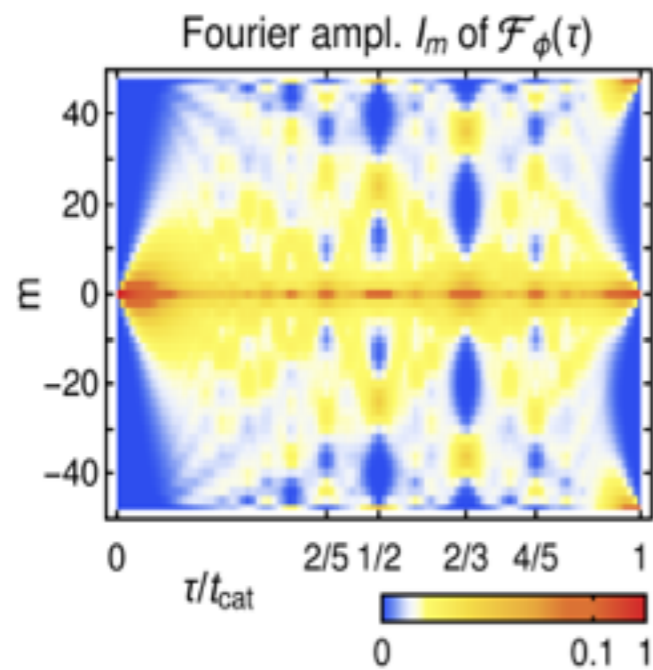
$$I_0 = \text{Tr}[\rho_0^2(t)]$$

Fourier transform:  $\phi$  gives the Multi-Quantum spectrum.

Connection to Renyi entropy?

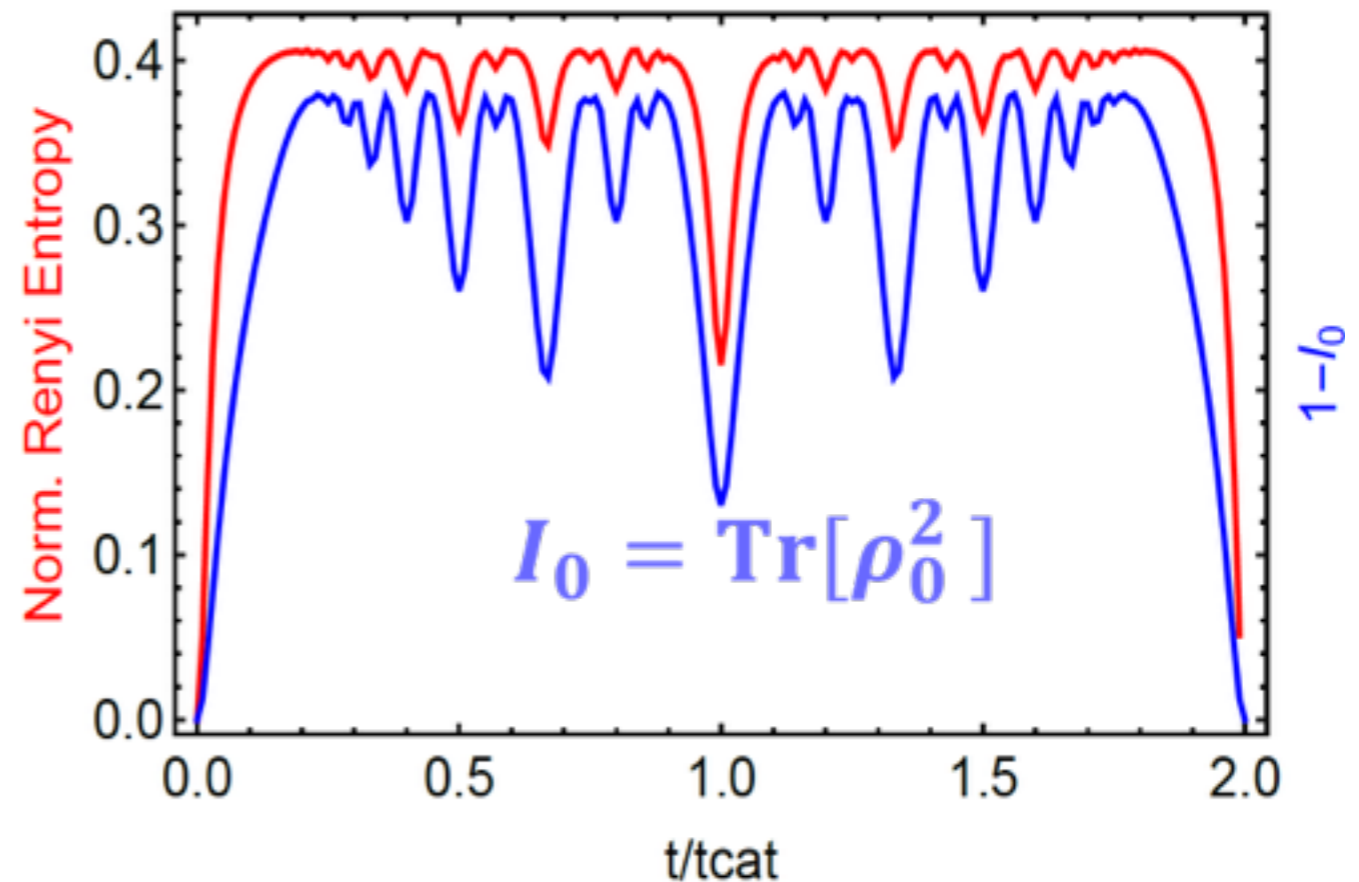
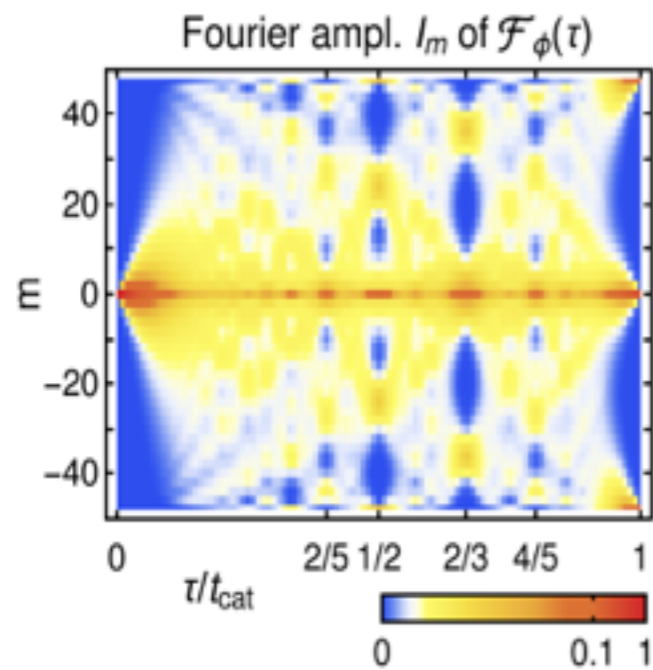
# MQC in All-to-all Ising Model

Information stored in the initial (local) state is distributed, through the interactions, over many-body degrees of freedom of the system.

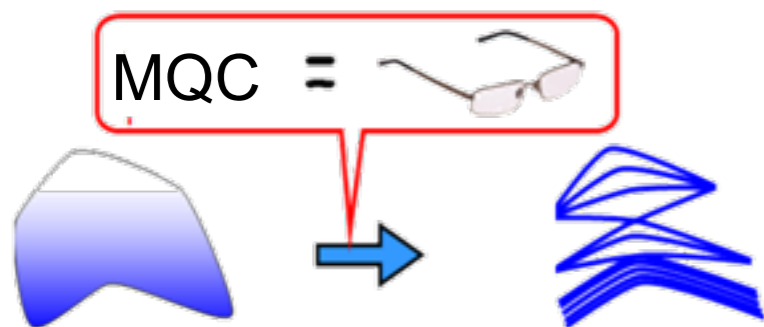


# MQC in All-to-all Ising Model

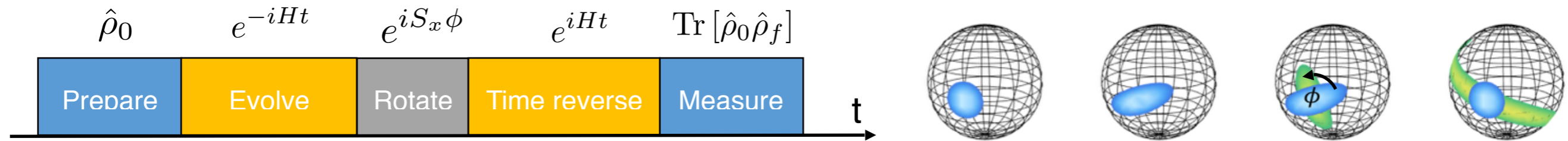
Information stored in the initial (local) state is distributed, through the interactions, over many-body degrees of freedom of the system.



Detailed structure of the state without single-site resolution



# Fidelity Measurement



## Many-body Loschmidt echo

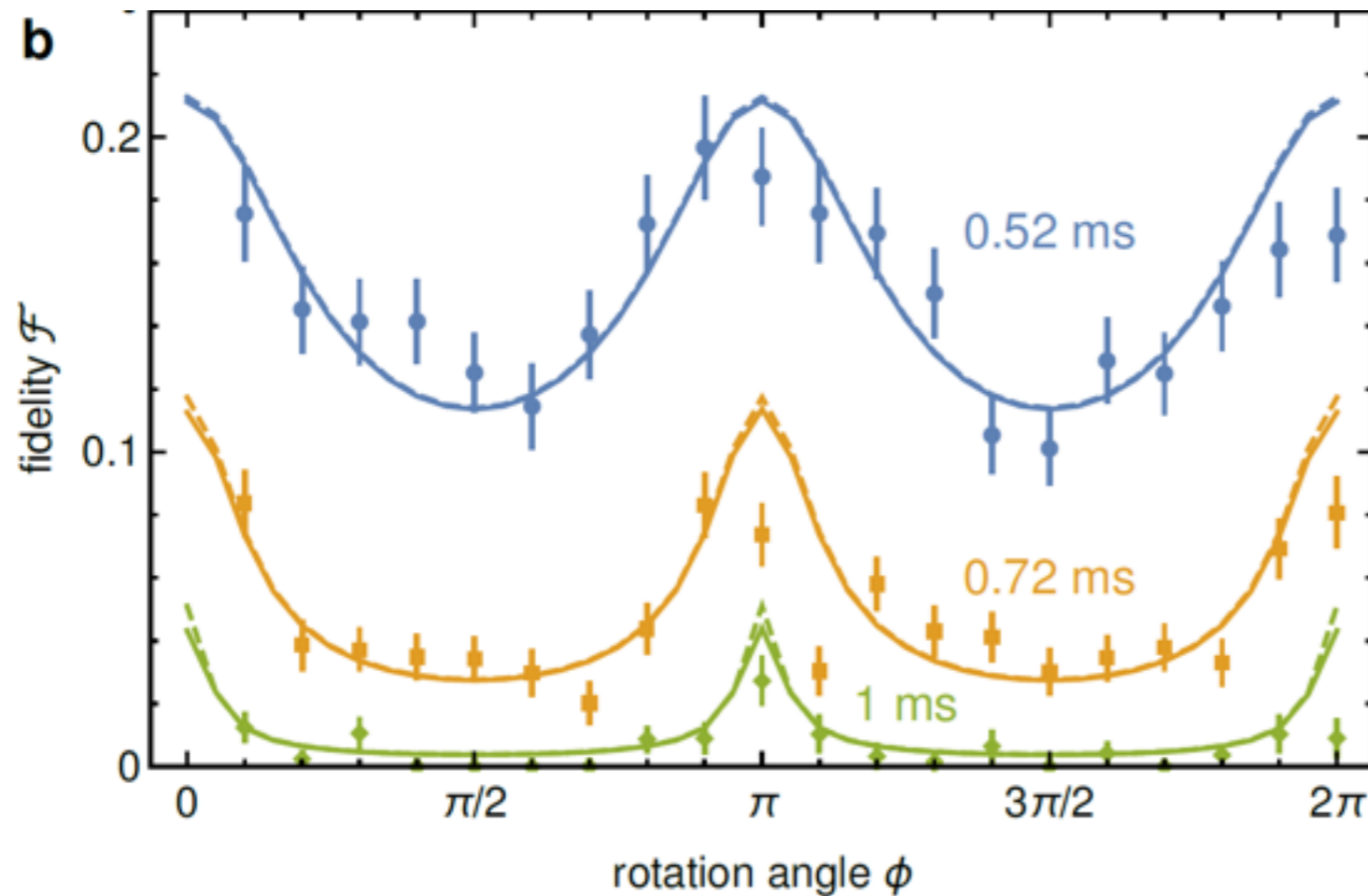
$$\begin{aligned}
 \langle \rho_0 \rangle &= \text{Tr}[\rho_0 \rho_f] = \text{Tr}[\rho_0 e^{itH} e^{-i\phi \hat{S}_x} e^{-itH} \rho_0 e^{itH} e^{i\phi \hat{S}_x} e^{-itH}] \\
 &= \sum_{m=-N}^N \underbrace{\text{Tr}[\rho_m \rho_m^\dagger]}_{I_m} e^{im\phi}
 \end{aligned}$$

OTOC measurement but also connected to a **multi-partite entanglement witness**:

## Quantum Fisher Information (QFI)

- How much that state changes with respect to a rotation
- Can show  $F_Q(\rho, A) \geq -2 \frac{d^2}{d\phi^2} \text{Tr}[\hat{\rho}_0 \hat{\rho}_f] |_{\phi=0}$
- MQC has even more information since not limited to small angles

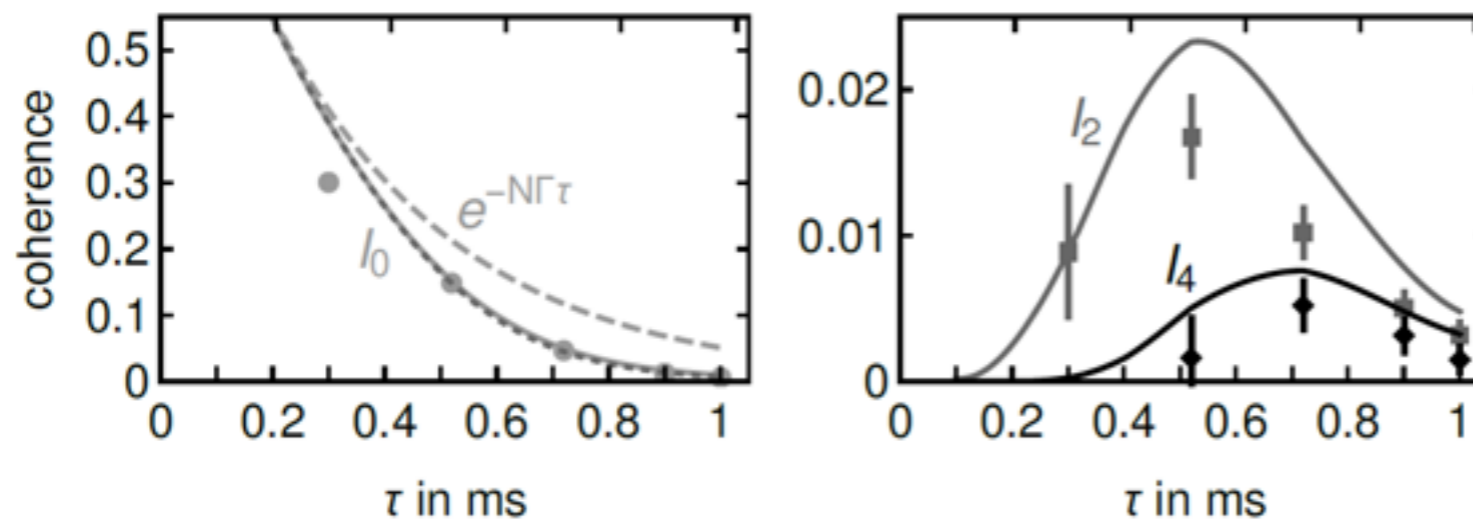
# Fidelity Measurement: Experimental Result



- OTOC witnesses multi-particle entanglement!
- Experimentally measured the OTOC
- Fully benchmarked experimental system: **spins + phonons + decoherence**

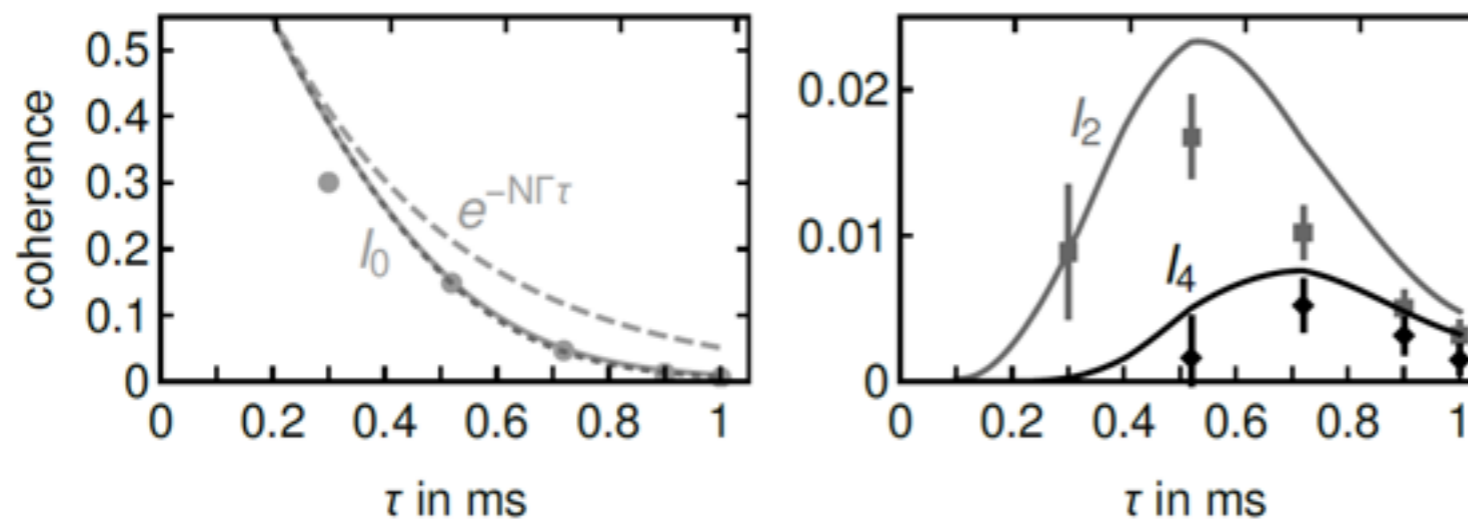
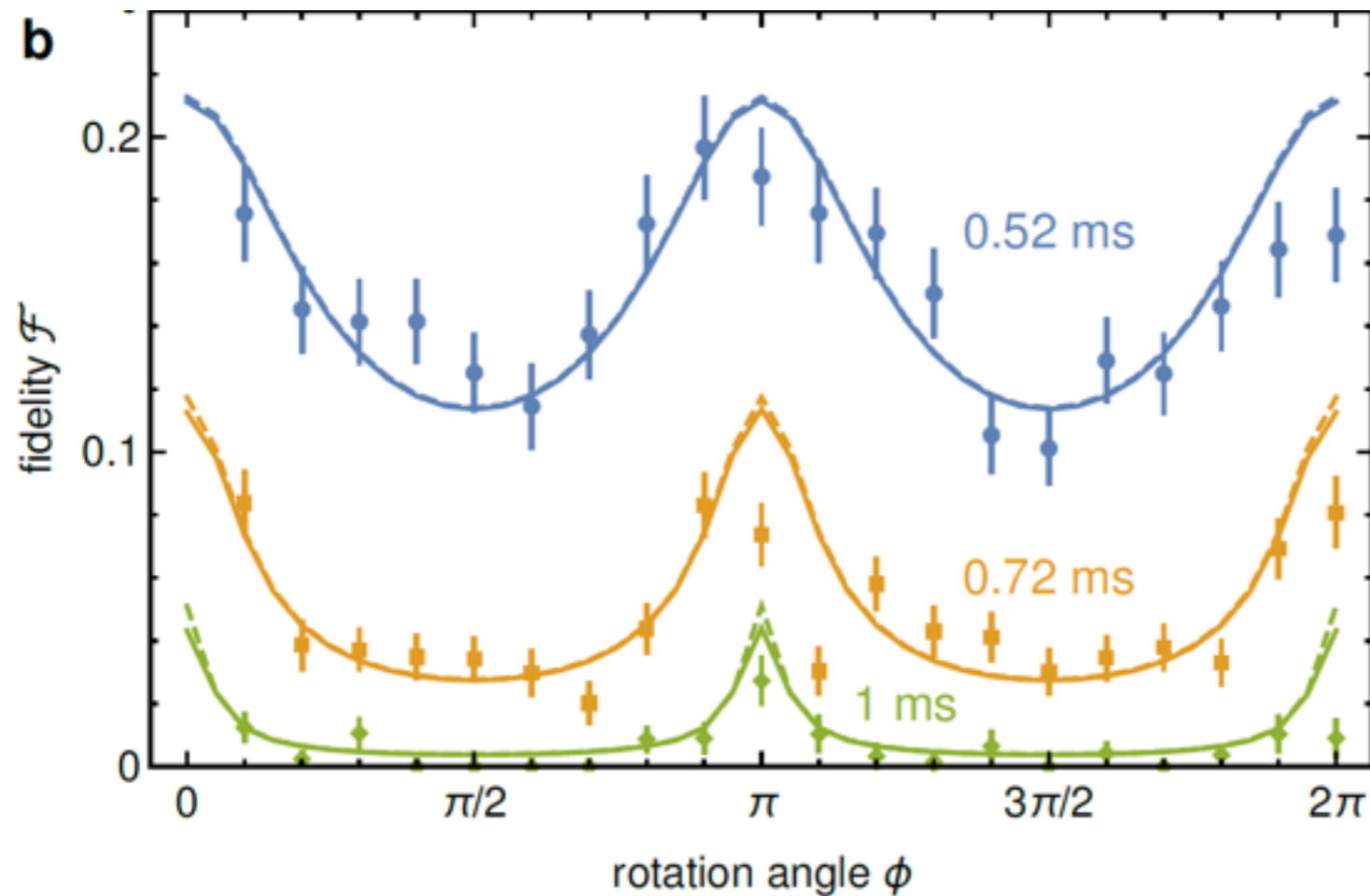
$$I_0^{pure}(\tau) = (1 + J^2\tau^2)^{-1}$$

Not a fast scrambler





# Fidelity Measurement: Experimental Result



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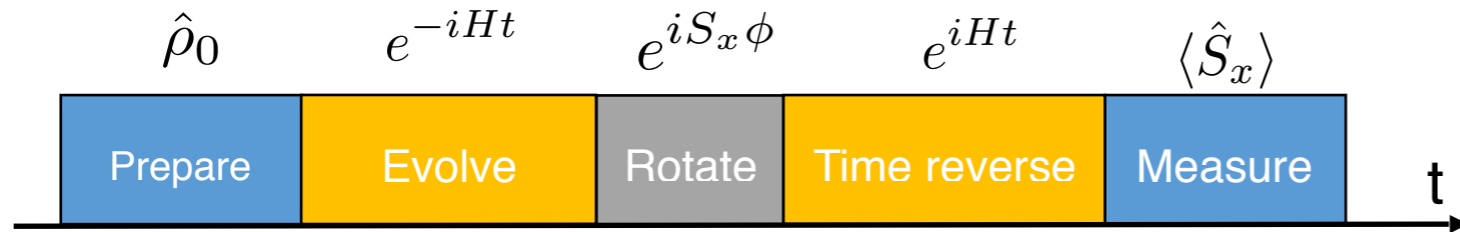
**Obstacle:**

Light scattering

$$I_0(\tau) = e^{-\Gamma N\tau} I_0^{pure}$$

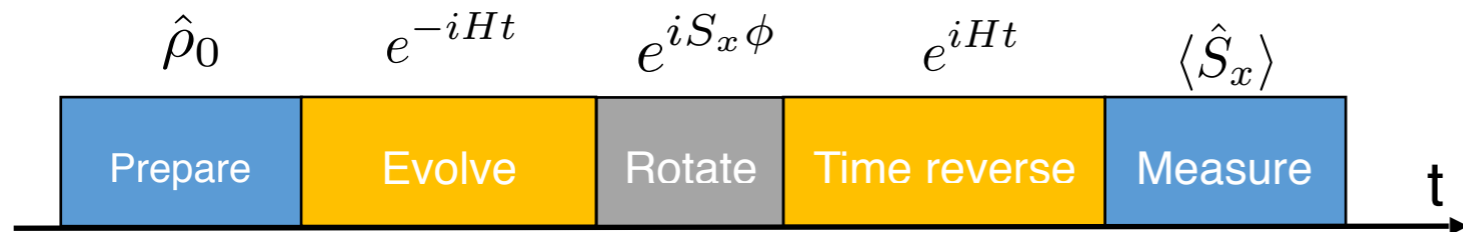
limited to 6.5% cat time

# Magnetization



$$\begin{aligned}
 \langle S_x \rangle &= \langle \Psi_0 | e^{itH} e^{i\phi S_x} e^{-itH} S_x e^{itH} e^{-i\phi S_x} e^{-itH} | \Psi_0 \rangle \\
 &= \frac{2}{N} \langle \Psi_0 | \underbrace{e^{itH} W^\dagger}_{W^\dagger(t)} \underbrace{e^{-itH} V^\dagger}_{V^\dagger(0)} \underbrace{e^{itH} W}_{W(t)} \underbrace{e^{-itH} V}_{V(0)} | \Psi_0 \rangle
 \end{aligned}$$

# Magnetization



$$\langle S_x \rangle = \langle \Psi_0 | e^{itH} e^{i\phi S_x} e^{-itH} S_x e^{itH} e^{-i\phi S_x} e^{-itH} | \Psi_0 \rangle$$

$$= \sum_m \langle \Psi | C_m | \Psi \rangle e^{i\phi m}$$

$$C_m = \sum \underbrace{\sigma_1^z \sigma_4^y \dots \sigma_k^z}$$

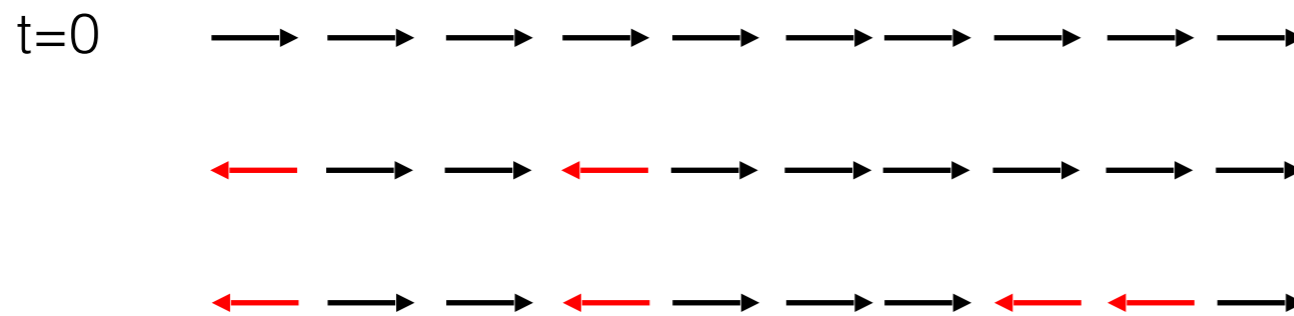
At least m terms

Fourier components of magnetization OTOC

How do the correlations propagate?

$$\sigma_i^z \sigma_j^z = \sigma_i^+ \sigma_j^+ + \sigma_i^- \sigma_j^- + \sigma_i^- \sigma_j^+ + \sigma_i^+ \sigma_j^-$$

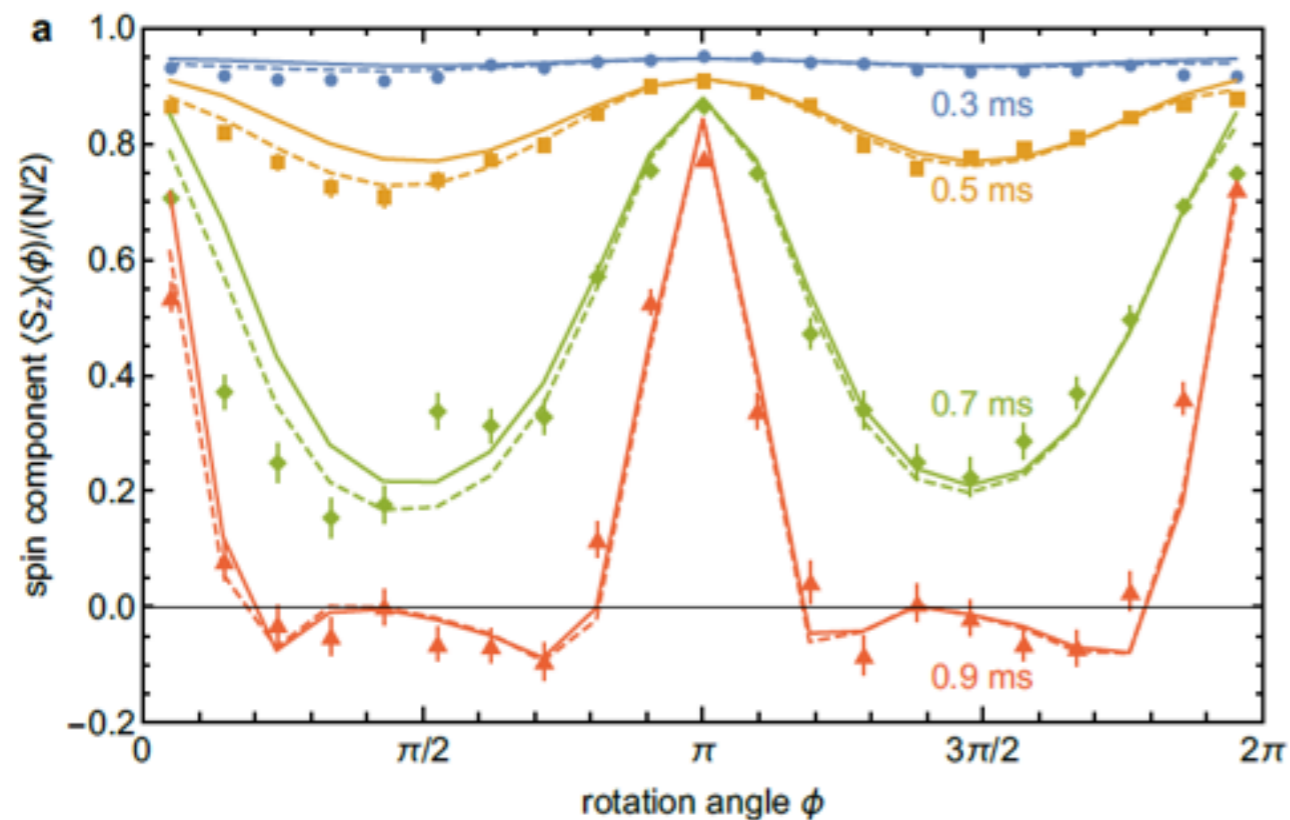
- **Signals the buildup of at least m-body correlations**
- **Far less sensitive to decoherence**



$$C_0(\tau) \sim e^{-\Gamma\tau} C_0^{\text{pure}}$$

time

# Magnetization OTOC: Experimental Results

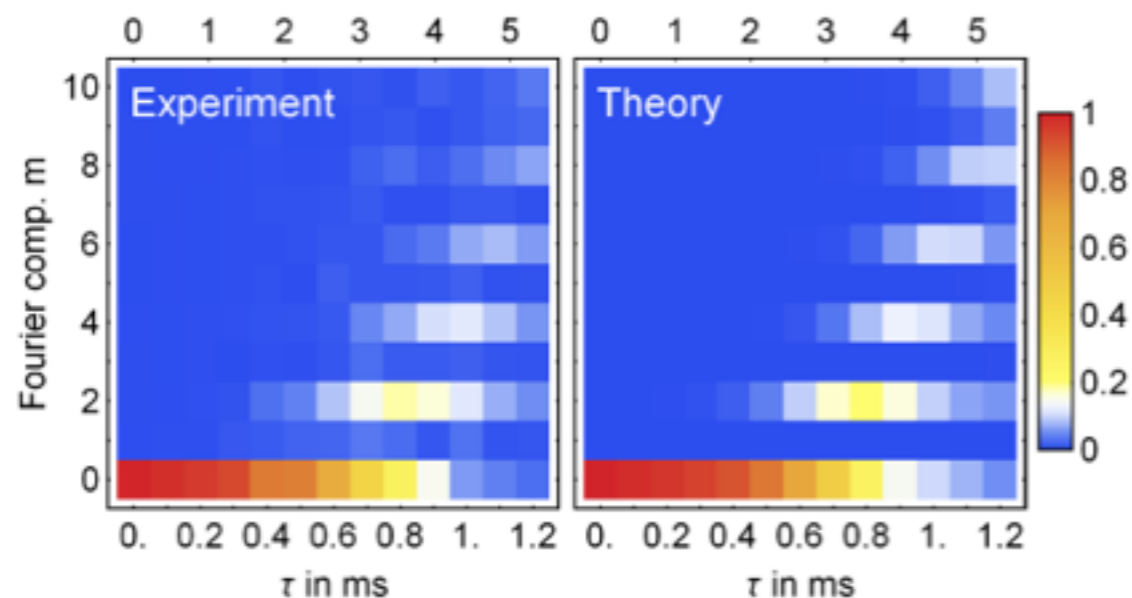


$$H_{SS} = \frac{J}{N} \hat{S}_z^2$$

$J \sim 5\text{kHz}$   
 $N = 111$   
 $\Gamma = 93\text{Hz}$

## Global measurements + Time reversal

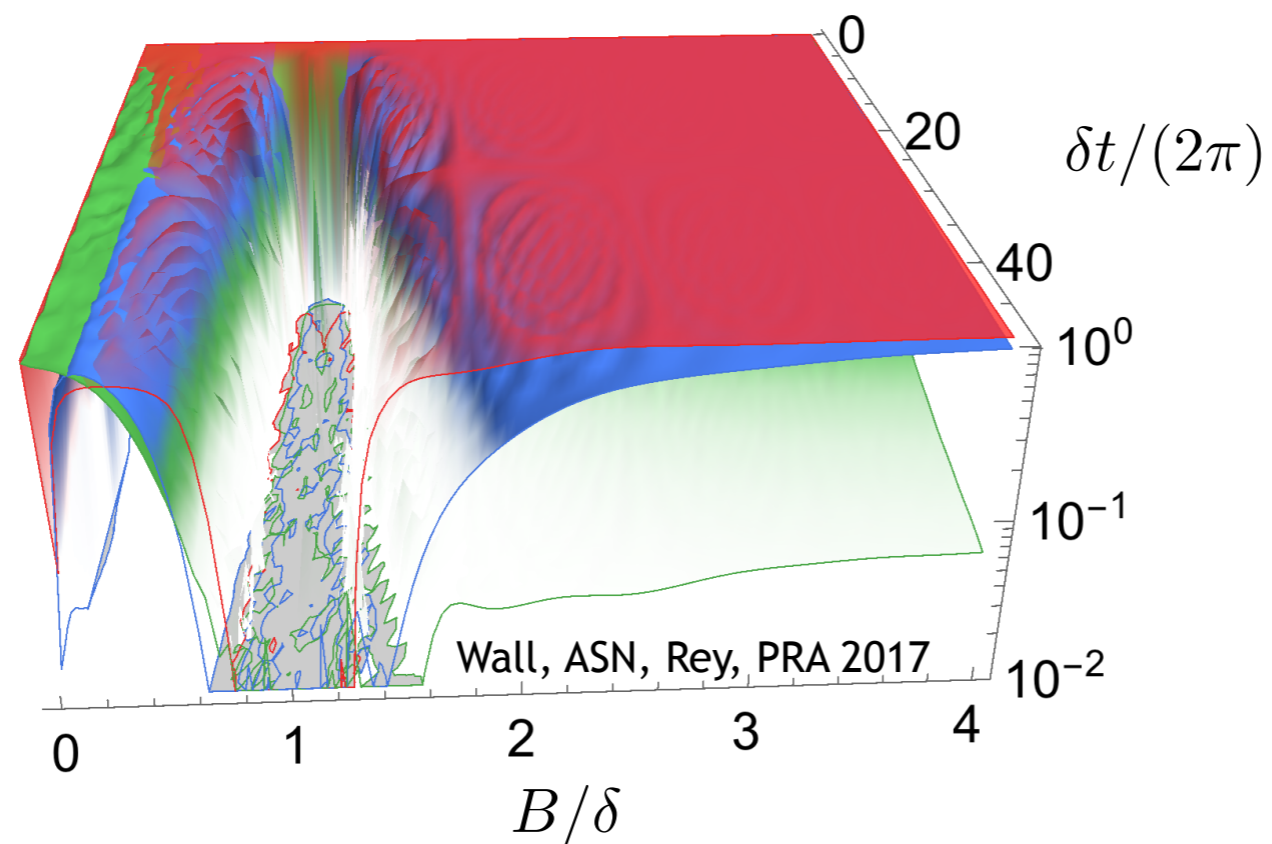
- OTOC
- Benchmark the simulator
- Decoherence and Hamiltonian
- Characterization of the final state
- Dynamics of correlations



# Transverse Magnetic Field

- The most straight-forward next step experimentally:
  - Add a transverse field: non-commuting term

$$\hat{H} = -\delta\hat{a}^\dagger\hat{a} - g(\hat{a} + \hat{a}^\dagger)\hat{S}_z + B(t)\hat{S}_x$$



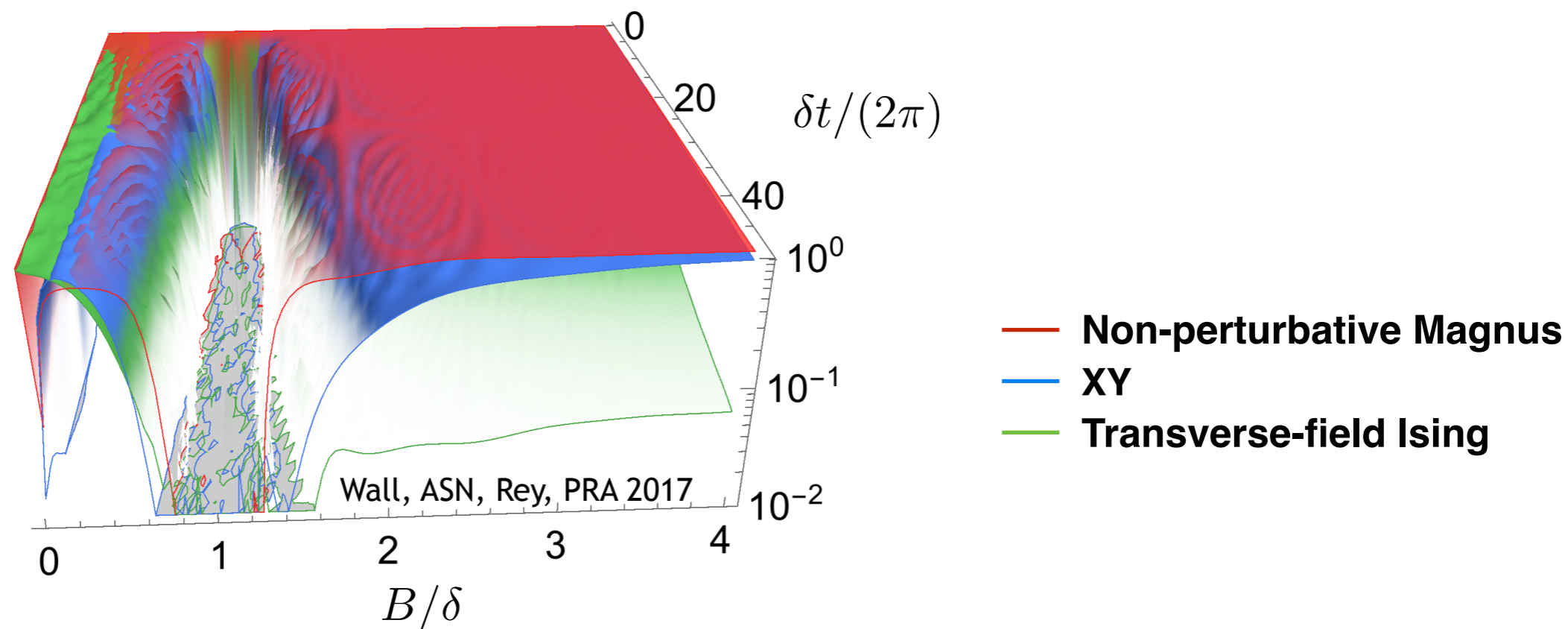
- Non-perturbative Magnus
- XY
- Transverse-field Ising

# Transverse Magnetic Field

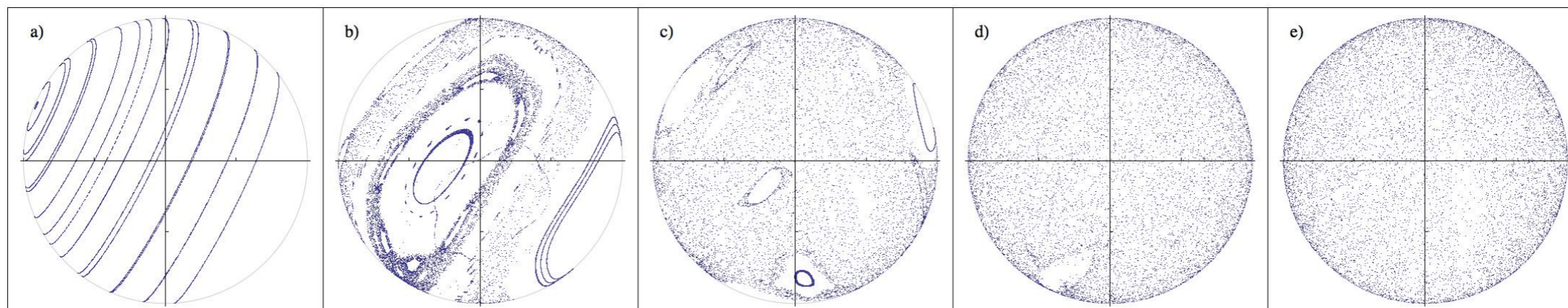
- The most straight-forward next step experimentally:
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$$\hat{H} = -\delta \hat{a}^\dagger \hat{a} - g(\hat{a} + \hat{a}^\dagger) \hat{S}_z + B(t) \hat{S}_x$$

**Dicke model**

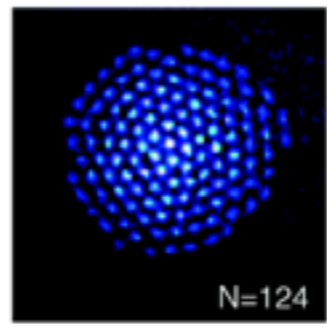


**classical chaos with increasing g**

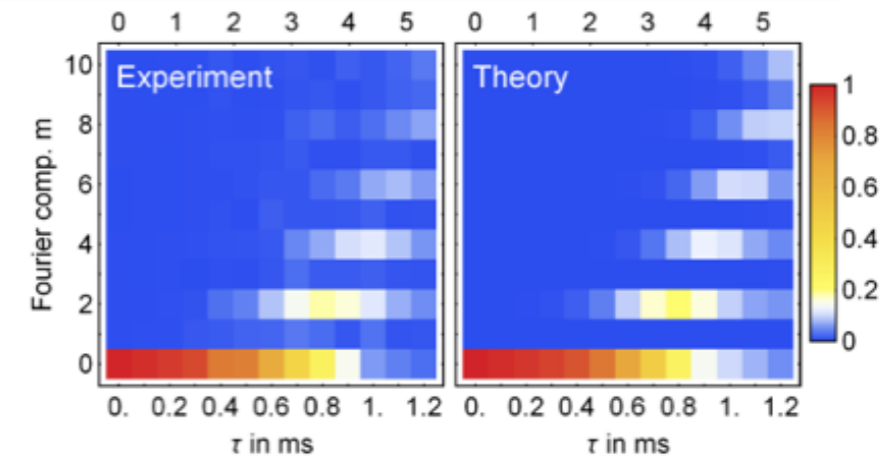


# Conclusions and Outlook

Benchmarking of a trapped-ion simulator of **Ising** and **Dicke** models of  $\sim 100$  ions



Observation of the dynamics of quantum coherences and correlations



But what about our relatively simple model?

- Fine grain information about the state only from global measurements
- Characterize spread of coherences (Fidelity) and correlations (Magnetization)
- Connection to entanglement witnesses (Fidelity)

## Future Directions:

- Scrambling in the Dicke model
- States outside of the symmetric manifold
- Dynamical phase transition in the Dicke model
- Preparation of the spin-phonon cat state



## References

- Garttner, Bohnet, **ASN**, Wall, Gilmore, Bollinger, Rey '17
- Wall, **ASN**, Rey, '16, '17
- Garttner, Hauke, Rey '17
- **ASN**, Lewis-Swann, Garttner, Gilmore, Jordan, Rey, Bollinger, In preparation.

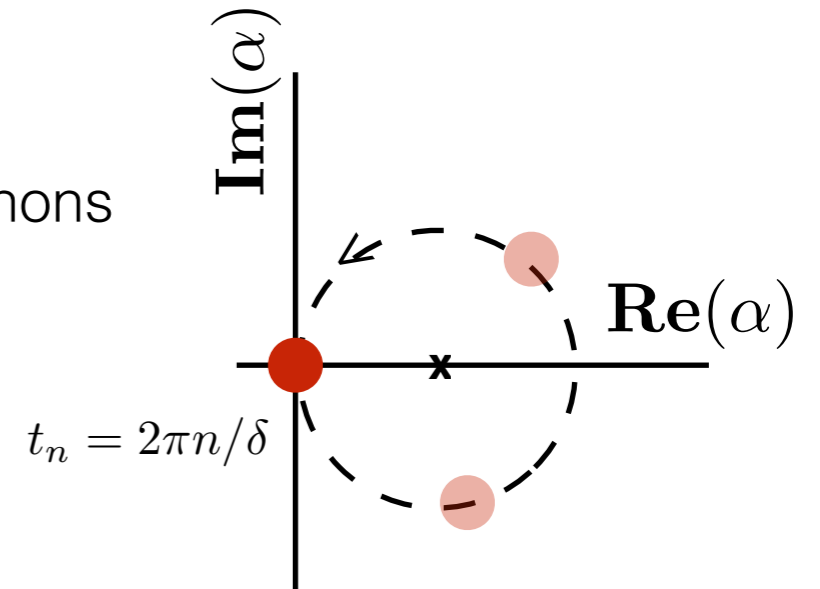
# Stroboscopic Spin Dynamics

Dynamics generated by:  $U(t) = U_{\text{SP}}(t)U_{\text{SS}}(t)$

$$H_{\text{SP}} = \hat{D}(\alpha\hat{S}_z) \quad \text{spin-dependent displacement of phonons}$$

- Spin and motion decouple at specific times  $t = 2\pi n/\delta$

$$U_{\text{SP}}(t_n) = I$$



Stroboscopically measure the same dynamics as the Ising model:

$$H_{\text{SS}} = \frac{J}{N} \hat{S}_z^2 \quad J \sim 1/\delta \quad \text{Uniform couplings when coupled only to the COM}$$

