# Finding Spacetime and Gravity from Entanglement in Conformal Field Theories


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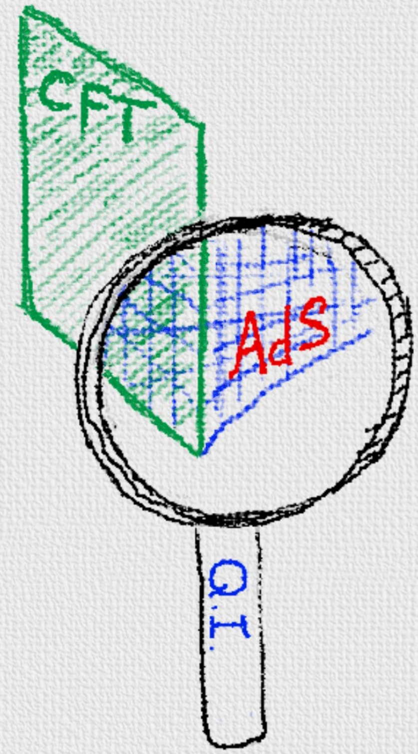
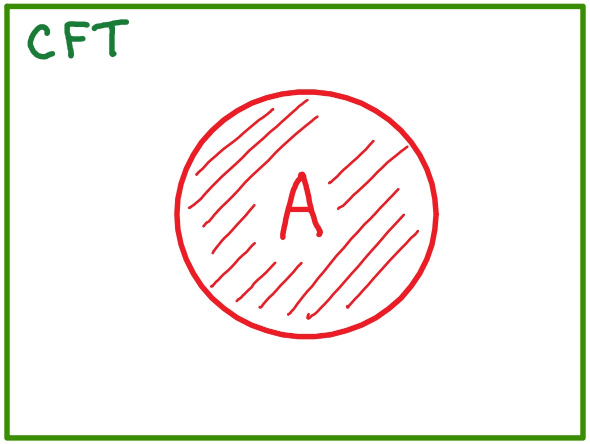
UBC


KITP, October 2017

# Entanglement entropy in CFTs




Given state $|\Psi\rangle$, region $A$, define subsystem entropy

$$S_A = -\mathrm{tr}(\rho_A \log \rho_A)$$

- quantifies entanglement of fields in $A$ with rest of system.

Divergent in continuum field theory, but have finite quantities:

- universal terms in $S_A(\varepsilon)$ UV cutoff
- combinations e.g. $S(A)+S(B)-S(A\cup B)$ Mutual Information
- vacuum subtracted: $S_A(|\Psi\rangle)-S_A(|vac\rangle)$

# PART I: AdS FROM VACUUM ENTANGLEMENT

Start with vacuum state of a 1+1d CFT


Entanglement entropy of an interval:

$$S_L = \frac{c}{3}\log\left(\frac{L}{\varepsilon}\right)$$

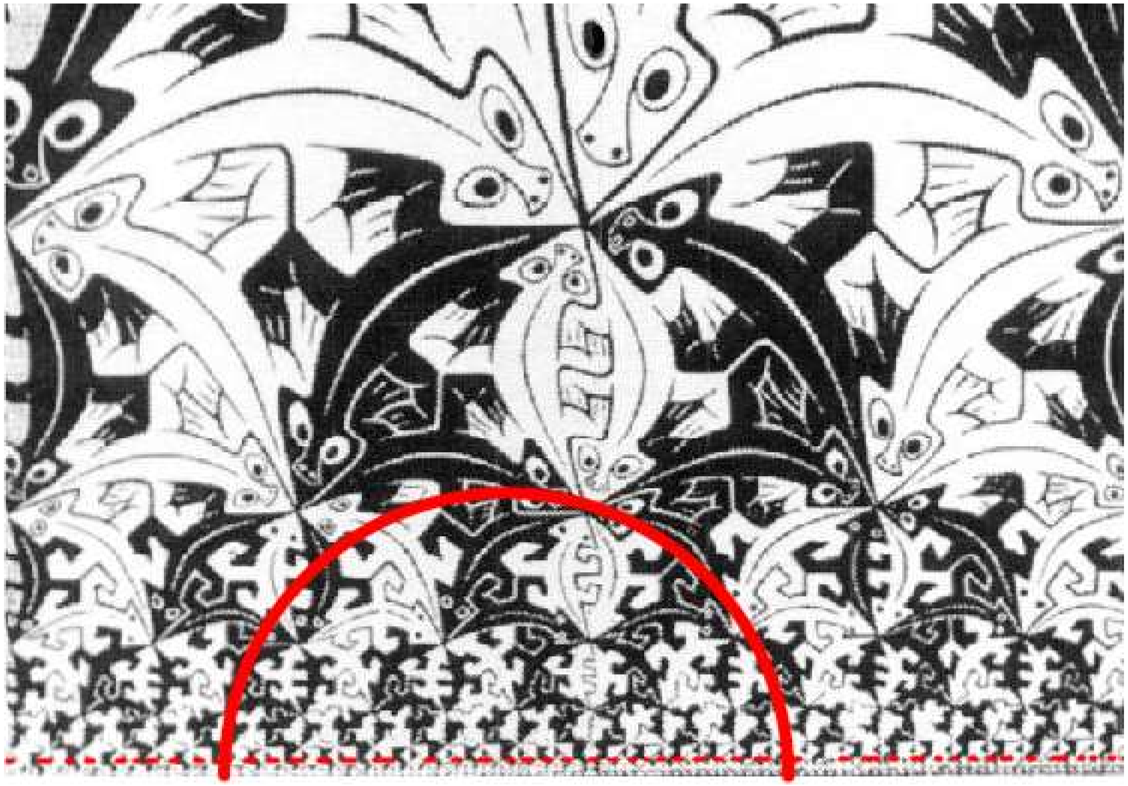
UV cutoff

central charge

This is also the answer to a geometry question...

Ryu-Takayanagi

Consider 2D negatively curved space:



Metric

$$ds^2 = \left(\frac{c}{6}\right)^2 \frac{1}{z^2}\left(dz^2 + dx^2\right)$$

Hyperbolic space

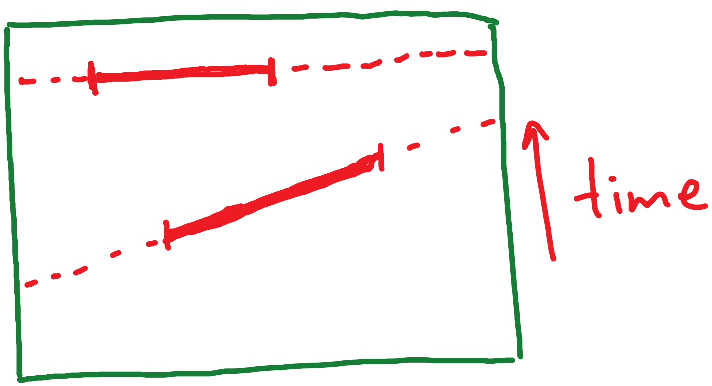
$z$

$z = \varepsilon$ ignore length below here

$x$

$L$

$$\frac{c}{3}\log\left(\frac{L}{\varepsilon}\right) =$$ Length of shortest curve through this space ending on boundary points w. $\Delta x = L$.



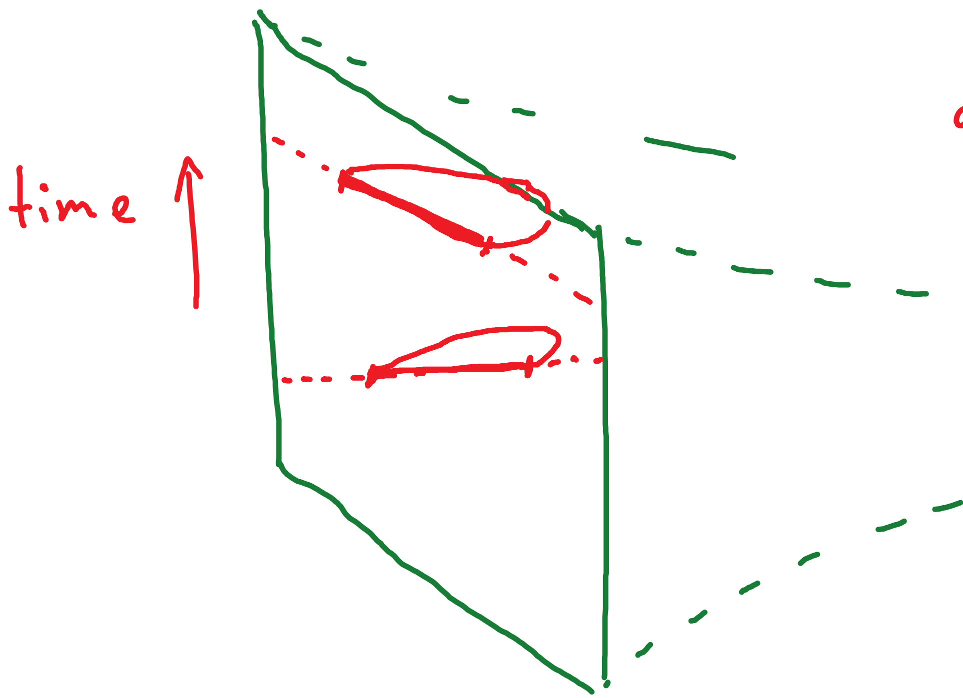

More general: can ask about regions in different frames of reference

Entropy matches lengths of curves *extremizing* area in a spacetime w. metric

$$ds^2 = \left(\frac{c}{6}\right)^2 \frac{1}{z^2}\left(dz^2 - dt^2 + dx^2\right)$$

$$\parallel$$

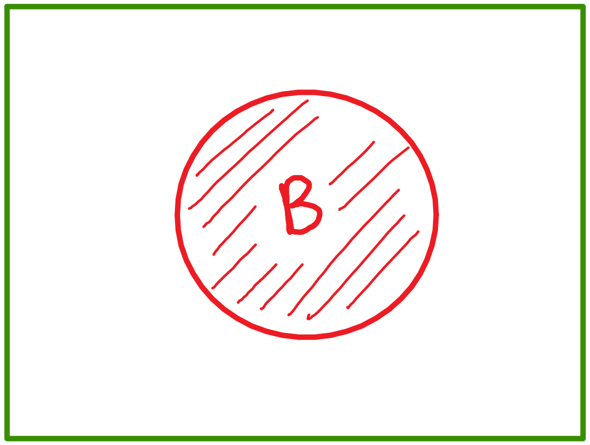
2+1D Anti de Sitter spacetime




higher central charge $\longleftrightarrow$ lower curvature

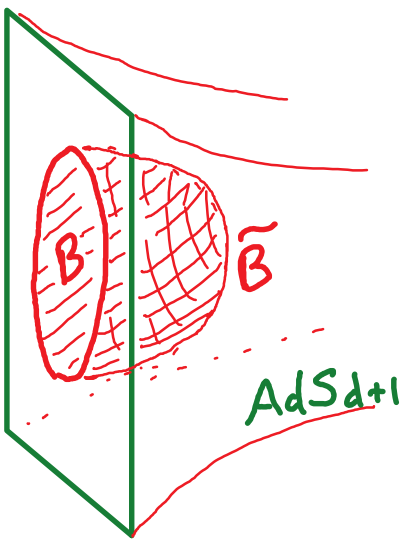
This also works in higher dimensions:

Casini, Huerta, Myers




Vacuum entanglement entropy of ball-shaped region in $CFT_d$

Regulator-indendent part same for all CFTs up to overall constant $a^*$




Matches areas of extremal surfaces in $AdS_d$

$$\boxed{S_B^{CFT} = \text{Area}(\tilde{B})}$$

extremizes area in $AdS_{d+1}$

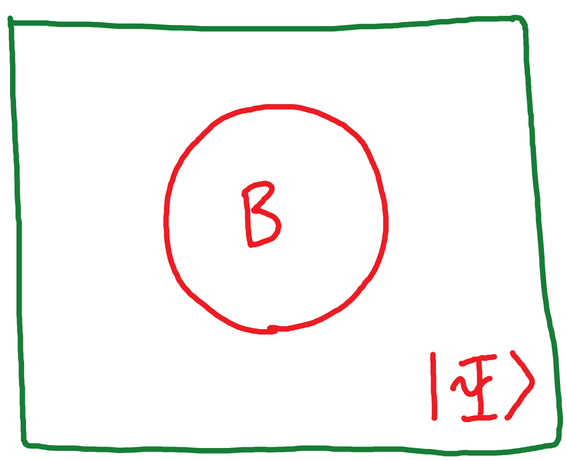
$$ds^2 = (a^*)^{\frac{2}{d-1}} \frac{1}{z^2}\left(dz^2 - dt^2 + d\vec{x}^{\,2}\right)$$

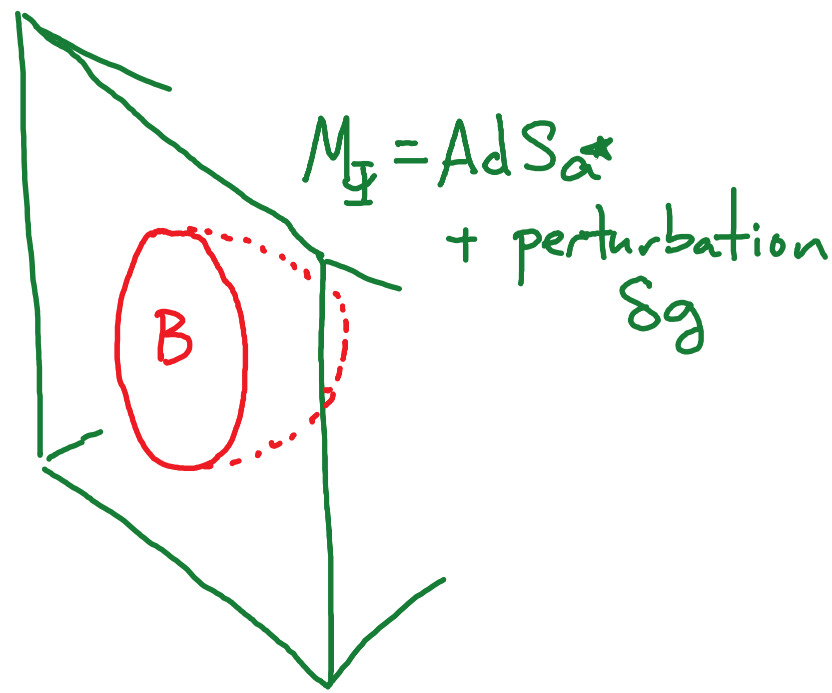
Parameter $a^*$ determines AdS curvature

# PART II: FIRST ORDER PERTURBATIONS

Take $|0\rangle \to |\Psi\rangle = |0\rangle + \delta|\Psi\rangle$

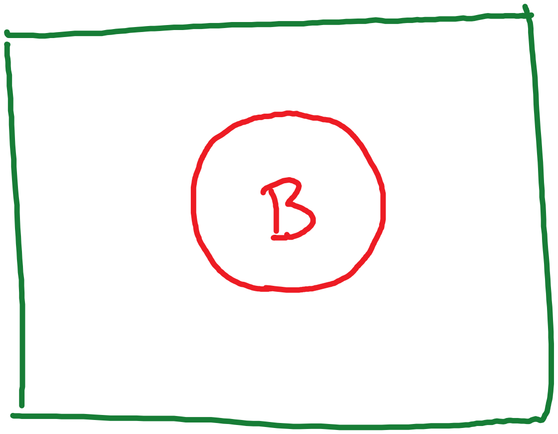
Can we still represent entanglement entropies via geometry?




$$\text{Entropy}(B)_\Psi \overset{?}{\doteq} \text{Area}(\tilde{B})_{M_\Psi}$$

Use CFT result:




vacuum density matrix for a ball is thermal w.r.t. $H_B$ $(T=1)$

$$\rho_0 \sim e^{-H_B}$$

energy density

$$H_B = \int_{Ball} f(x)\, T_{00}(x)$$

Casini, Huerta, Myers

(Quantum) First Law of Thermodynamics:

$$\delta S_B = \delta \langle H_B \rangle$$

Gives:

$$\delta S_B = \int_{Ball} f(x)\, \delta \langle T_{00}(x) \rangle$$

entropy perturbation determined by energy density perturbation

# Infinitesimal ball:

$$\delta S_{B_\epsilon(x)} \sim \langle T_{00}(x) \rangle \qquad \text{(I)}$$

Use this to rewrite

$$\delta S_B = \int_{Ball} f(x)\, \delta S_{B_\epsilon(x)} \qquad \text{(II)}$$

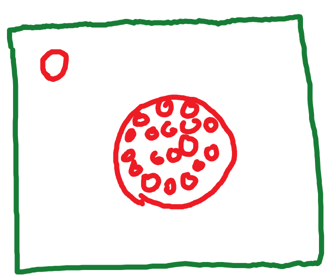
entropy for large ball

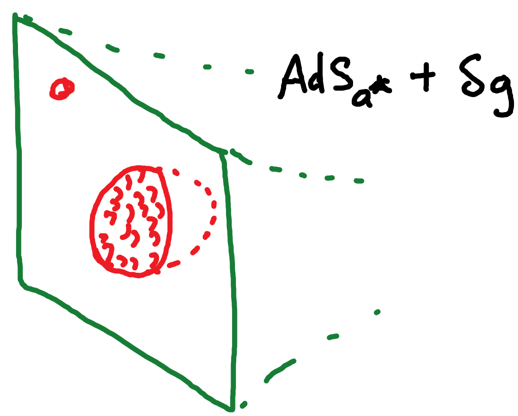
entropy for infinitesimal ball

If $AdS_{d+1} + \delta g$ calculates ball entanglements for $|0\rangle + \delta|\psi\rangle$:

(I) $\Longrightarrow$ Asymptotic $\delta g$ determined by $\langle T_{\mu\nu}\rangle$

(II) $\Longrightarrow$ $\delta g$ satisfies Einstein eqns. linearized about AdS





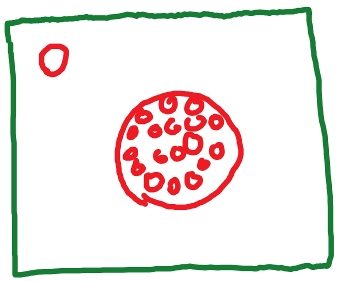

Lashkari; McDermott, MVR

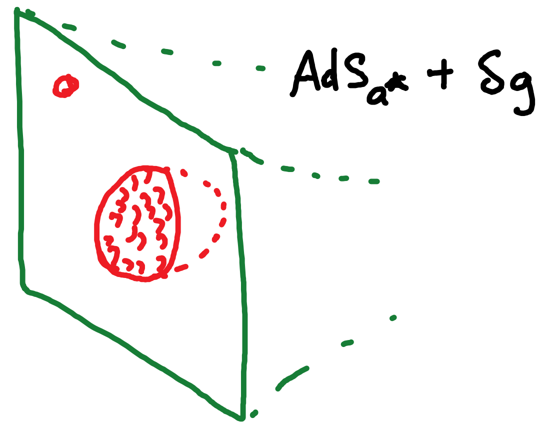
Faulkner, Guica, Hartman, Myers, MVR

If $AdS_{d^*} + \delta g$ calculates ball entanglements for $|0\rangle + \delta|\psi\rangle$:

(I) $\Longrightarrow$ Asymptotic $\delta g$ determined by $\langle T_{\mu\nu}\rangle$

(II) $\Longrightarrow$ $\delta g$ satisfies Einstein eqns. linearized about AdS






* locality is an output *

* This is enough info to construct $\delta g$ from $\langle T_{\mu\nu}\rangle$ *

For any CFT, any state $|\Psi\rangle = |0\rangle + \delta|\Psi\rangle$, $AdS_{d^*} + \delta g(\langle T\rangle)$ correctly calculates ball entanglement entropies for $|\Psi\rangle$ to first order.

# Part III: Nonlinear Perturbations

Does this continue to work at higher orders?

Not likely to work for general states

$|\Psi_1\rangle \to M_{\Psi_1}$ $\quad |\Psi_2\rangle \to M_{\Psi_2}$ $\quad \alpha|\Psi_1\rangle + \beta|\Psi_2\rangle \longrightarrow ???$

Key result at first order: $\delta S_B$ from $\langle T \rangle$

$\downarrow$

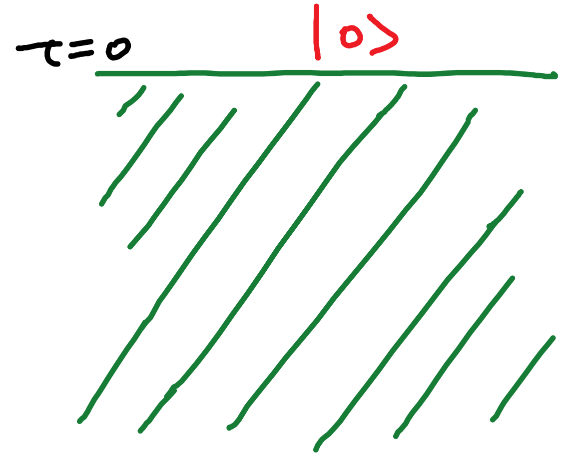
$\delta g$ from $\delta g_{\text{boundary}}$

Is there a natural family of states where $S_B$ is determined by local data $\langle \mathcal{O}_\alpha \rangle$?

# PATH INTEGRAL STATES

Recall: define vacuum by

$$|0\rangle = e^{-\varepsilon H} e^{-\varepsilon H} \cdots e^{-\varepsilon H} |\Phi_0\rangle$$



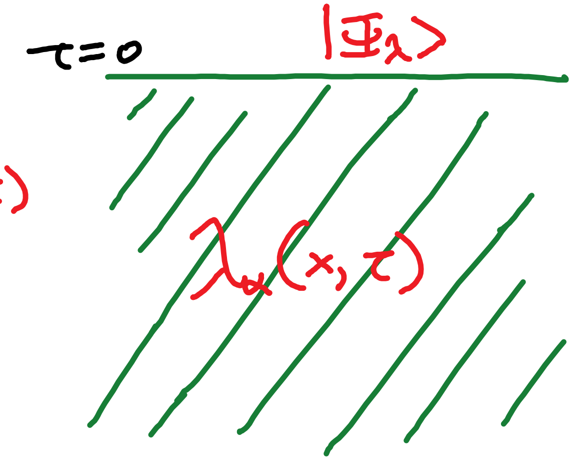

$$\downarrow$$

$$\langle \phi_0 | 0 \rangle = \int_{\phi(\tau=0)=\phi_0} [d\phi]\, e^{-S_{Euc}}$$

Add sources for local operators:

$$\langle \phi_0 | \Phi_\lambda \rangle = \int_{\phi(\tau=0)=\phi_0} [d\phi]\, e^{-S_{Euc} - \int \lambda_\alpha(x,\tau)\, \mathcal{O}_\alpha(x,\tau)}$$

turn off as
$\tau \to 0$



For $|\Psi_\lambda\rangle$ with $\lambda(\varepsilon) = \varepsilon\lambda^{(1)} + \varepsilon^2\lambda^{(2)} + \cdots$ can calculate

$$S_B = a^* S_B^{(0)} + \int_B f(x)\langle T_{00}(x)\rangle + \frac{1}{C_T}\int K_B^T(x_1,x_2)\langle T(x_1)\rangle\langle T(x_2)\rangle$$

$$+ \int K_B^\alpha(x_1,x_2)\langle \mathcal{O}_\alpha(x_1)\rangle\langle \mathcal{O}_\alpha(x_2)\rangle$$

$$+ \cdots$$

central charge defined from vacuum entanglement

central charge defined from $\langle T(x)T(0)\rangle \sim \frac{C_T}{x^{2d}}$

For $a^* = C_T$ (more general case later):

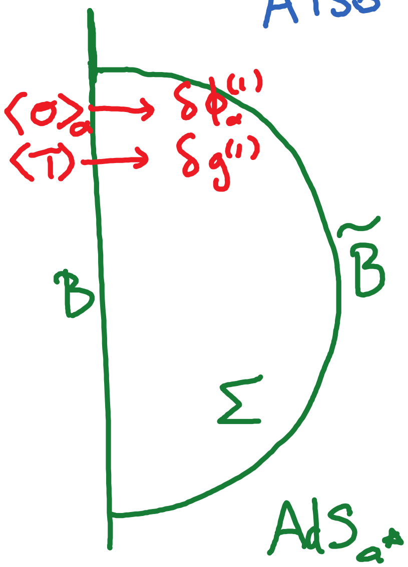
* Can rewrite 2nd order terms in terms of our auxiliary AdS. *

Start with $AdS_{a^*}$

Define $\delta g^{(1)}$ by:

$$\langle T \rangle \longrightarrow \delta g^{(1)}_{\text{boundary}} \xrightarrow{\text{1st order Einst. eqns.}} \delta g^{(1)}$$

Also introduce auxiliary scalars $\phi_\alpha$:




$$\langle O_\alpha \rangle \longrightarrow \delta\phi^{(1)}_{\alpha\,\text{boundary}} \longrightarrow \delta\phi^{(1)}_\alpha$$

solve scalar equation
w. $m^2 = (a^*)^{-\frac{1}{d-1}} \cdot d(d-\Delta)$

Then: $g^{AdS}_{a^*} + \delta g^{(1)} + \delta g^{(2)}$ calculates ball entanglement entropies for $|\Psi_\lambda\rangle$ via $S(B) = \text{Area}(\tilde{B})$ if and only if this geometry satisfies $G_{ab} = 8\pi T_{ab}$ to second order in $\lambda^2$ with $T_{ab}$ the stress tensor of the matter $\delta\phi$.

(assumes $a^* = C_T$)

## Summary:

e.g. all 2D CFTs

Euclidean path integral sources for $T_{\mu\nu}$, scalar primary $\mathcal{O}_\alpha$

Given any CFT w. $a^* = C_T$, any $|\Psi\rangle_\lambda$

① There exists $M_\psi = AdS_{a^*} + \delta g^{(1)} + \delta g^{(2)}$ s.t. $M_\psi$ calculates all ball entanglement entropies via $S(A) = \text{Area}(\tilde{A})$ up to 2nd order.

② Any such $M_\psi$ satisfies Einstein's eqns to 2nd order with $T_{\mu\nu}$ coming from matter fields determined by $\langle \mathcal{O}_\alpha \rangle$

Recent work: can extend to ANY CFT if we generalize to add 1-parameter family of higher curvature terms to gravity equations + corresponding terms to area functional

Haehl, Hijano, Parrikar, Rabideau

Note: we expect that $M_\psi$ only provides a legitimate AdS/CFT dual if the CFT satisfies additional constraints.

→ these will show up at higher order in perturbation theory

Extra: spacetimes emerging in this way have additional constraints.

CFT entanglement entropies are bound by fundamental quantum constraints.

e.g. strong subadditivity, positivity & monotonicity of relative entropy.

These translate to constraints on geometries

e.g. positivity of relative entropy $\Longrightarrow$ positive energy theorem for gravity subsystems.

Lashkari, Lin, Ooguri, Stoica, MVR.

AdS/CFT: any consistent quantum theory of gravity for asymptotically AdS spacetimes should be associated with a CFT & satisfy these constraints.

Q.I. constraints in CFT give fundamental constraints on gravitational systems.

Future directions: understand gravitational physics of more general CFT states.

→ path integral states we considered are like coherent states in the gravity language

→ to understand entanglement physics of more general states, expect that we need quantum effects in our gravitational system.

→ systematic study possible using more general path integral states (work in progress)