The skeleton of information scrambling

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caltech, Inst. for quantum information & matter

NYH, *Phys. Rev. A* **95**, 012120 (2017).
NYH, B. Swingle, and J. Dressel, arXiv:1704.01971 (2017).

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KITP; Oct. 11,, 2017

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University of California, Santa Barbara
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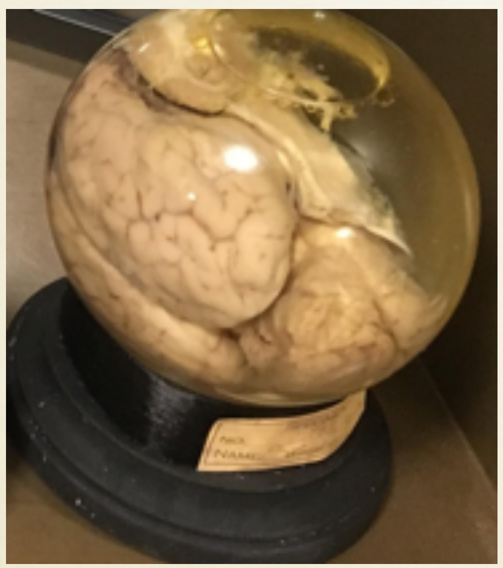
Distilled essence



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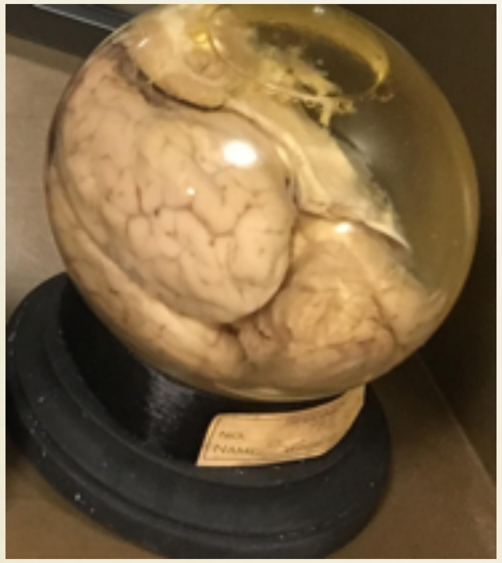

bare bones

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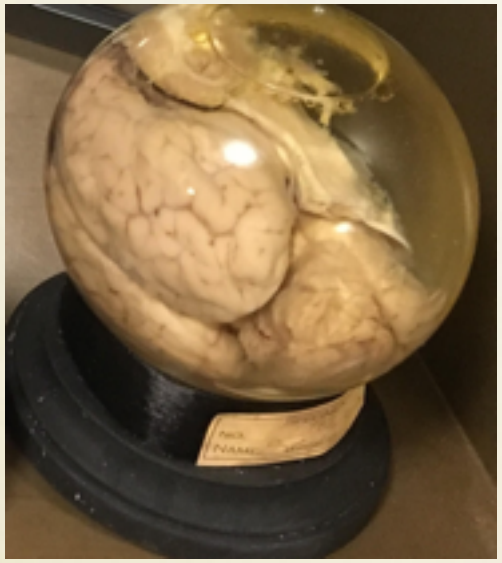

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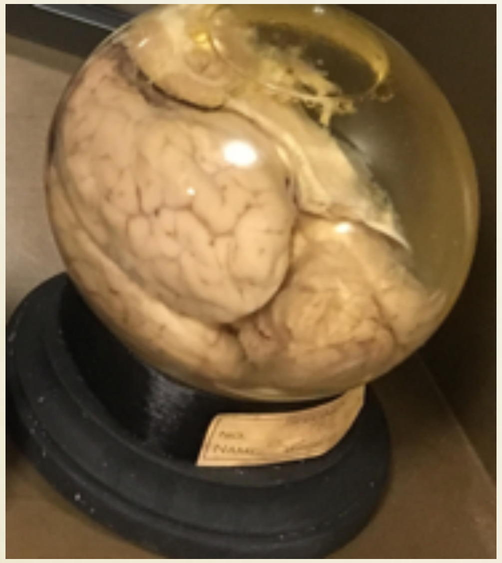

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Quantum-information scrambling

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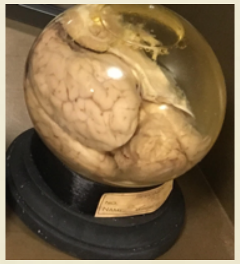

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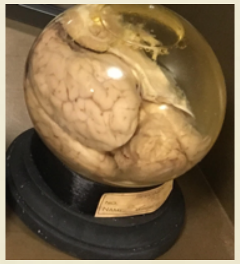
- What's its essence? ➜ 
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Outline

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- Opportunities

THE OUT-OF-TIME-ORDERED CORRELATOR (OTOC)

OTOC: Set-up

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- <u>Examples</u>

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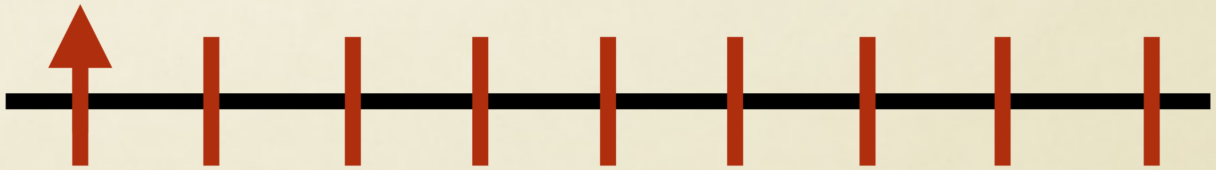
  - (2) Chain of *N* spins 

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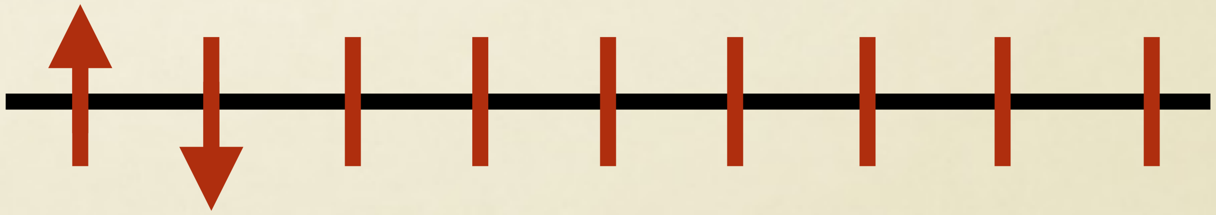
  - (2) Chain of N spins 

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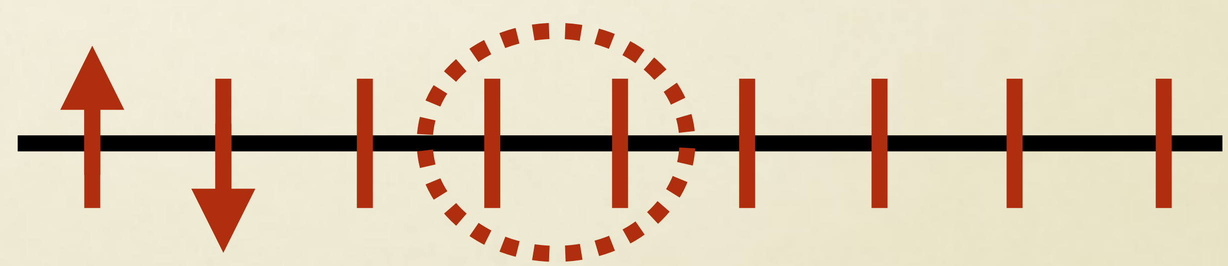

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  - $\rho = \sum_j p_j |j\rangle\langle j|$
  - Often assumed to be thermal: $e^{-H/T}/Z$

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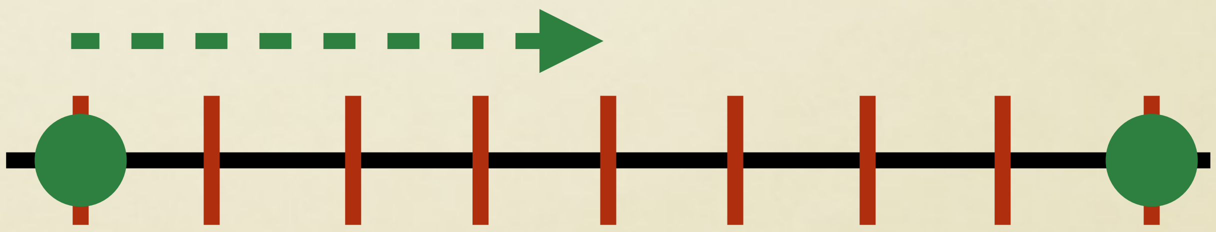

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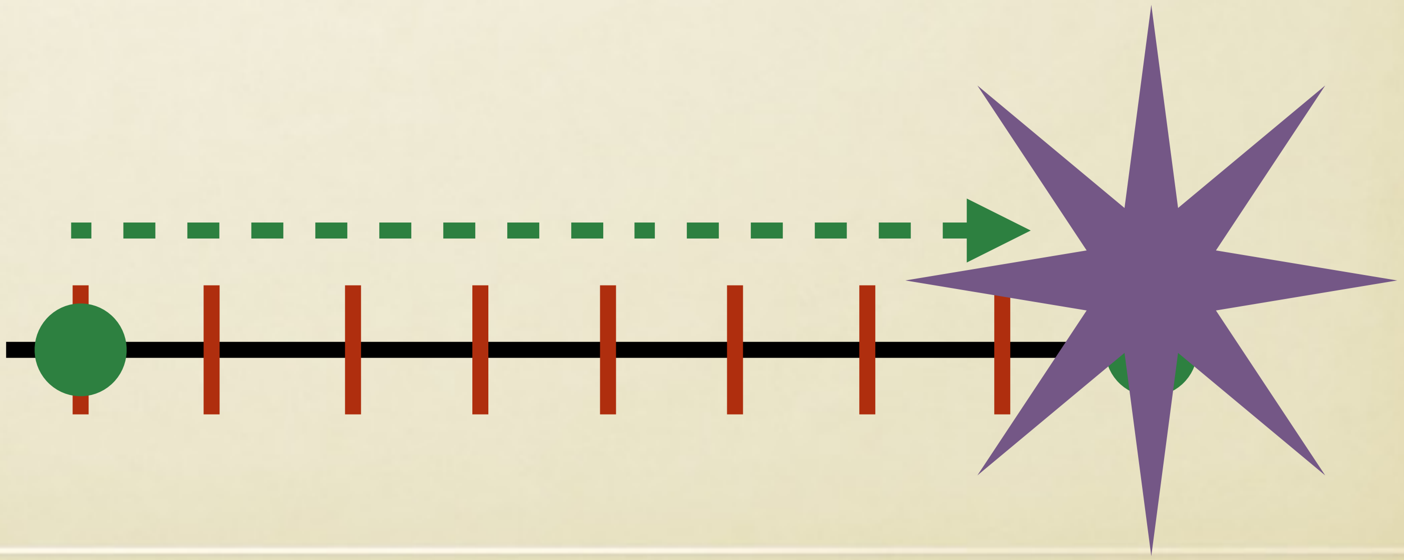

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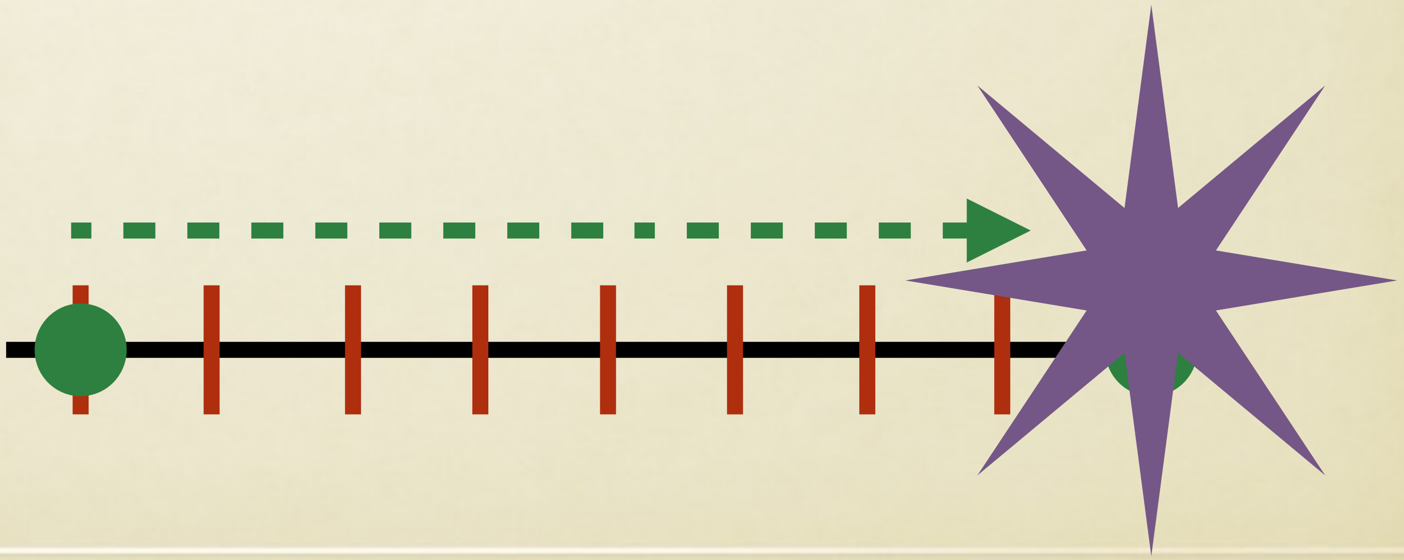

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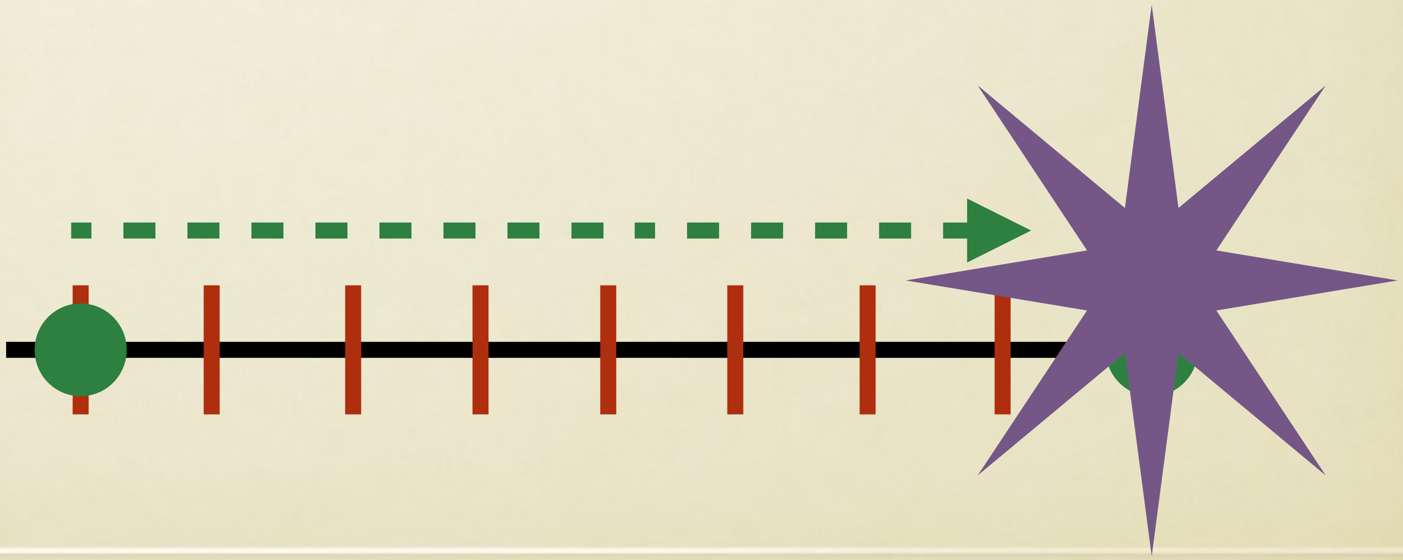

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→ 4 terms

Focus on the most important. →

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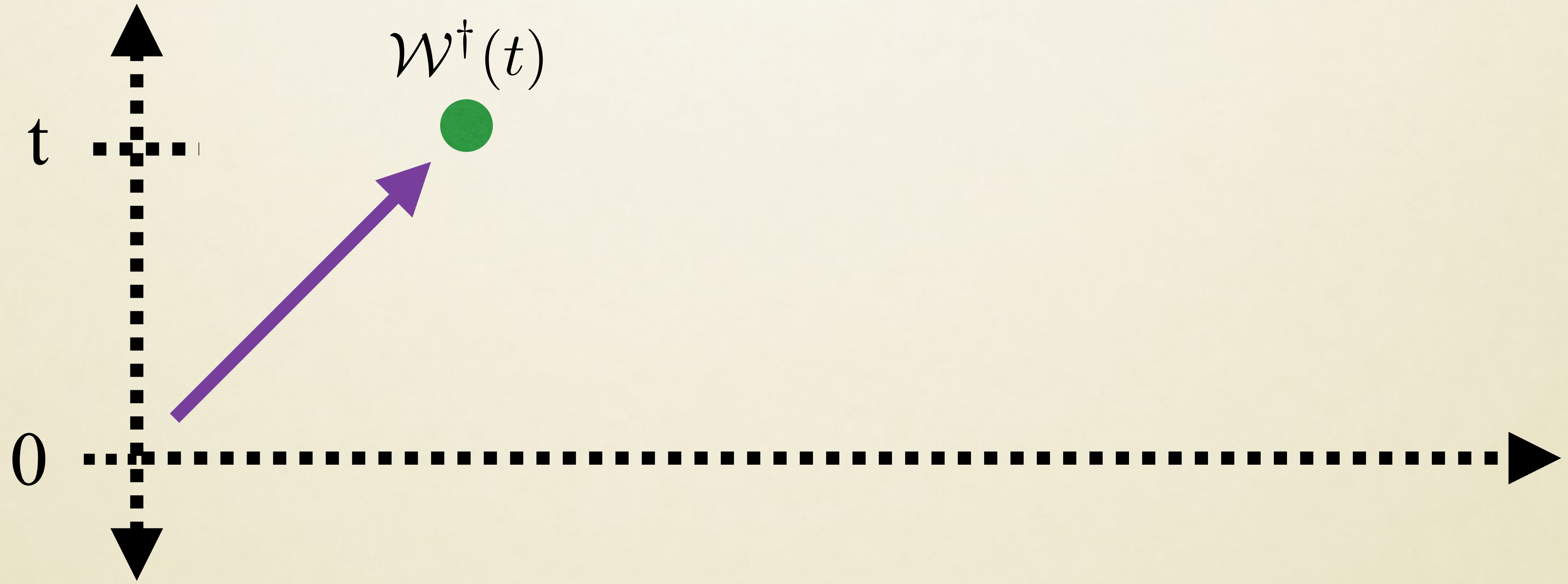

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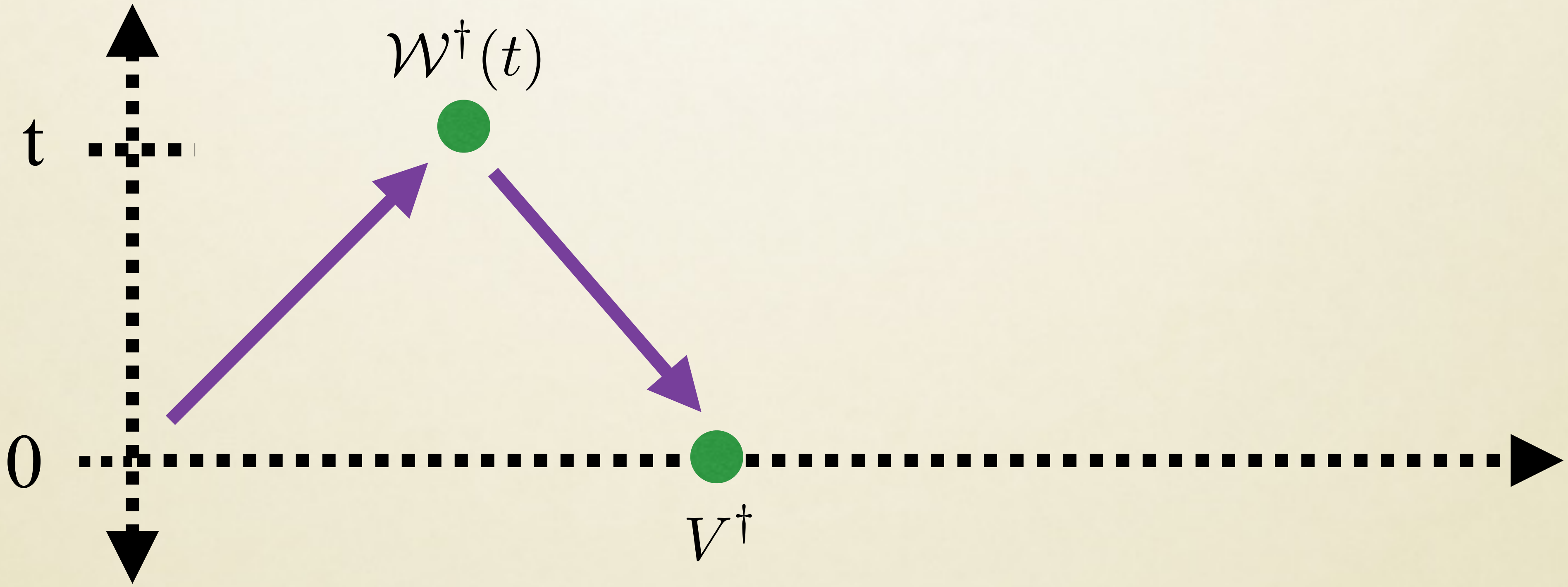

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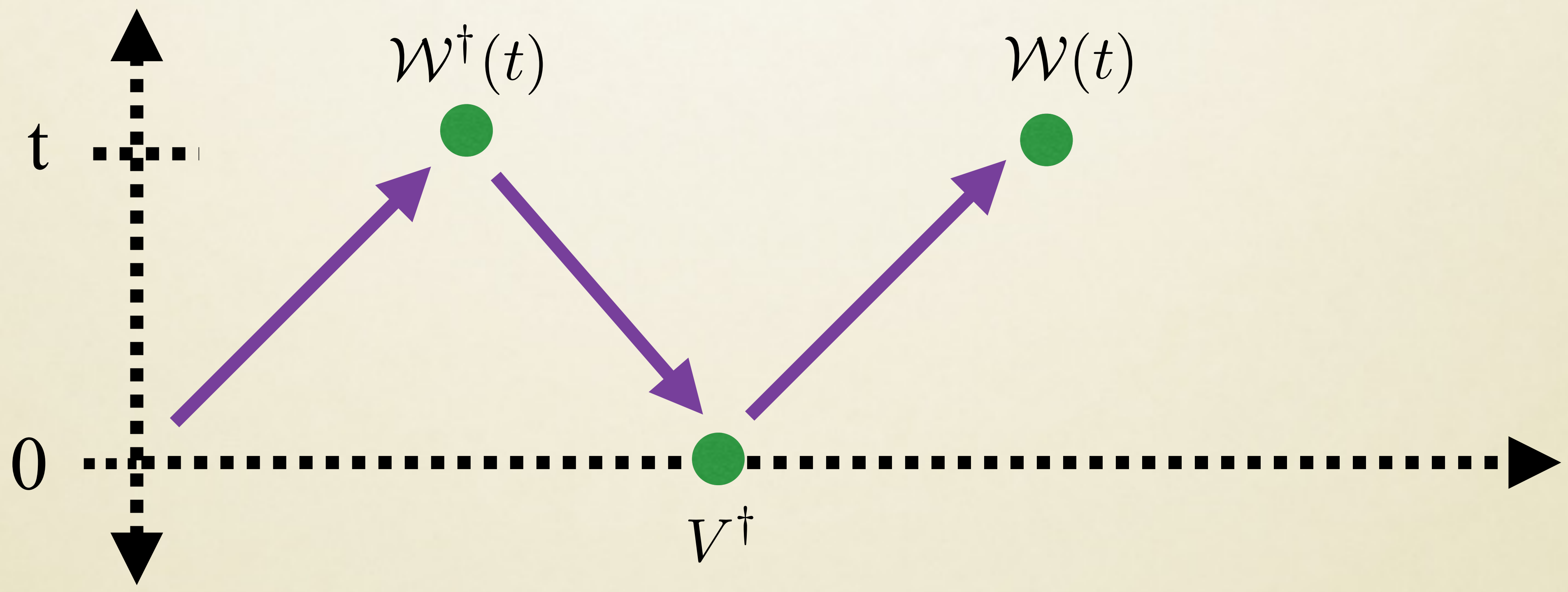

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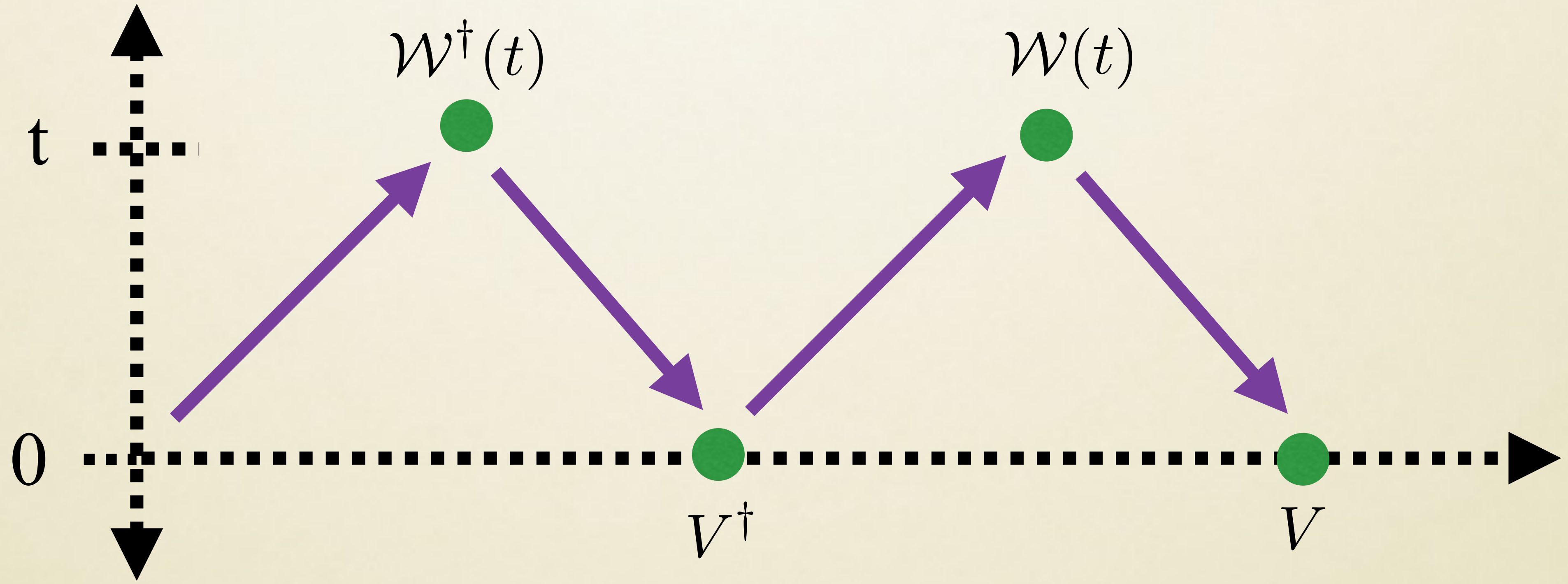

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- Previously known connections
  - Sensitivity to "perturbation in initial conditions"
  - Semiclassical
- New insight from the skeleton in the OTOC

Decomposition of the OTOC

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Doesn't decompose

Decomposition of the OTOC

← Doesn't decompose

$$\langle \mathcal{W}^\dagger(t) V^\dagger \mathcal{W}(t) V \rangle$$

← Decomposes

decomposing the OTOC

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$$F(t) := \mathrm{Tr}\left(\mathcal{W}^\dagger(t) V^\dagger \mathcal{W}(t) V \rho\right)$$

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$$\sum_{v_1} v_1 \, \Pi^V_{v_1}$$

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$$\sum_{w_3} w_3^* \, \Pi^{\mathcal{W}}_{w_3} \qquad \sum_{v_2} v_2^* \, \Pi^{V}_{v_2} \qquad \sum_{w_2} w_2 \, \Pi^{\mathcal{W}}_{w_2} \qquad \sum_{v_1} v_1 \, \Pi^{V}_{v_1}$$

$$= \sum_{v_1, w_2, v_2, w_3} w_3^* v_2^* w_2 v_1 \, \mathrm{Tr}\left(U^\dagger \Pi^{\mathcal{W}}_{w_3} U \Pi^{V}_{v_2} U^\dagger \Pi^{\mathcal{W}}_{w_2} U \Pi^{V}_{v_1} \rho\right)$$

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$$\Pi_{w_3}^{\mathcal{W}(t)} \qquad \Pi_{w_2}^{\mathcal{W}(t)}$$

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$$\mathrm{Tr}\left(\Pi_{w_3}^{\mathcal{W}(t)} \Pi_{v_2}^{V} \Pi_{w_2}^{\mathcal{W}(t)} \Pi_{v_1}^{V} \rho \right) =: \tilde{\mathcal{A}}_\rho(v_1, w_2, v_2, w_3)$$

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OTOC quasiprobability

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OTOC quasiprobability

Background: Quasiprobabilities

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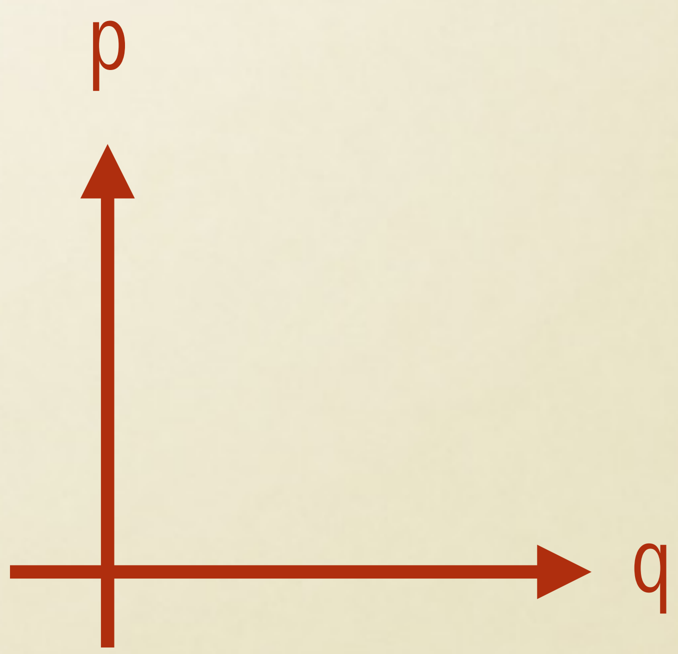
- Think: statistical-mechanics phase-space distribution, but for quantum systems



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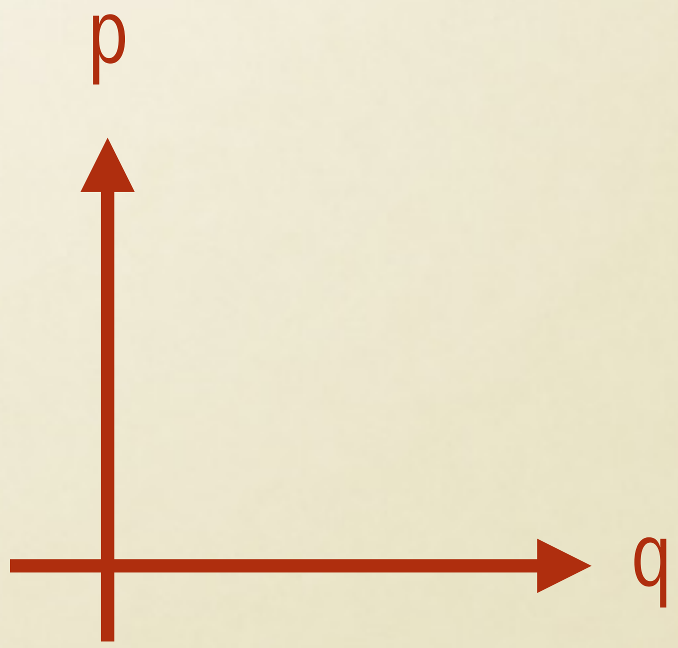

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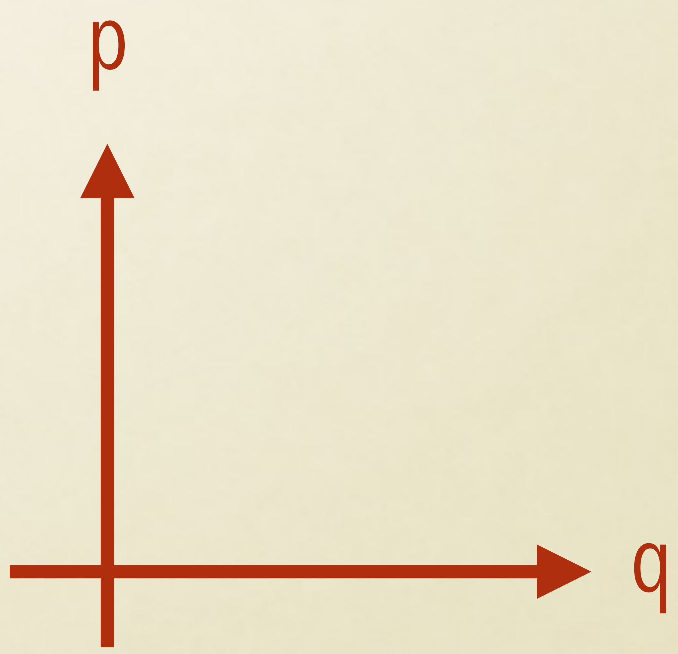
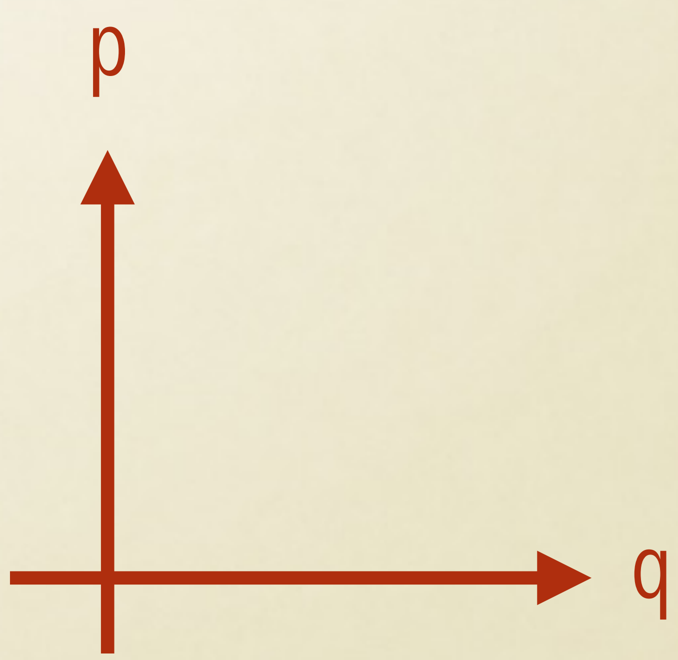
- Like a probability
- But can assume negative and nonreal values



What's a quasiprobability?

- Used mostly in quantum optics
- Think: statistical-mechanics phase-space distribution, but for quantum systems
- Like a probability
- But can assume negative and nonreal values
- Most famous example: Wigner function



Kirkwood-Dirac (KD) quasiprobability

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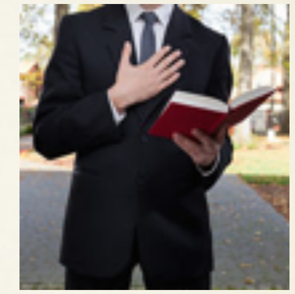

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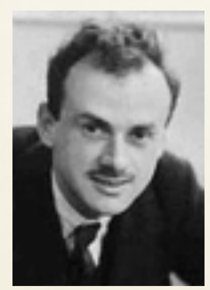

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- Can be inferred from weak measurements $\Rightarrow$ can be used to measure the OTOC

Relevance of the KD quasiprobability

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An extended KD quasiprobability is **the**  **in the OTOC.**

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The quasiprobability behind the OTOC

- NYH, *Phys. Rev. A* **95**, 012120 (2017).
- NYH, B. Swingle, and J. Dressel, arXiv:1704.01971 (2017).

Visualizing the OTOC quasiprobability

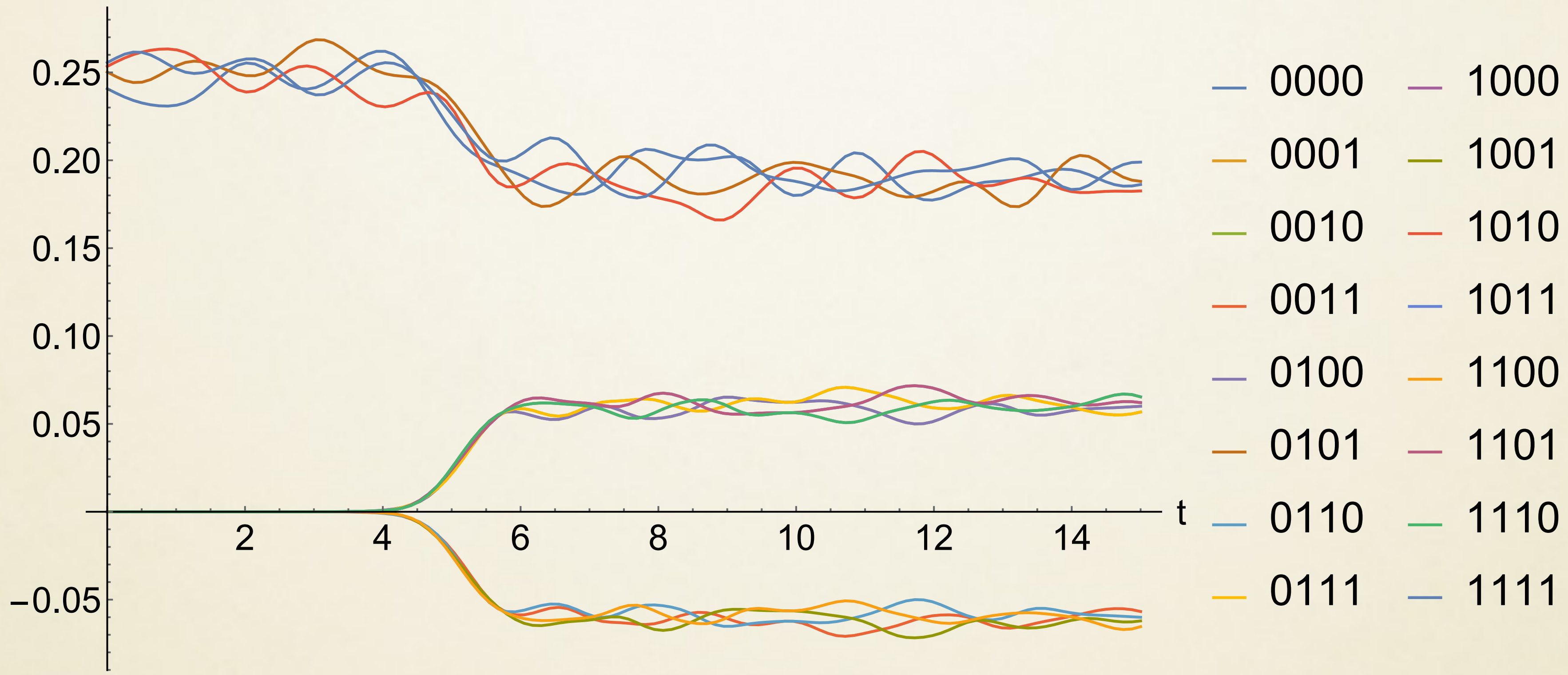

FIG. 13: Real part of $\tilde{\mathscr{A}}_\rho$ as a function of time. Random pure state. Nonintegrable parameters, $N = 10$, $\mathcal{W} = \sigma_1^z$, $V = \sigma_N^z$.

$$\tilde{A}_\rho(v_1, w_2, v_2, w_3)$$

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    - Leggett-Garg inequalities
    - Meaning of “”maximal noncommutation”

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- **Etc.** ⟶ arXiv:1704.01971 (2017).

Recap

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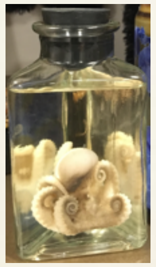
- Decomposing the OTOC → 

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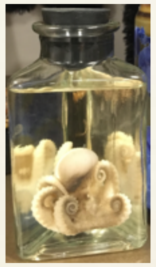
- Decomposing the OTOC → 
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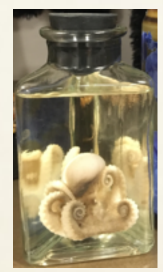

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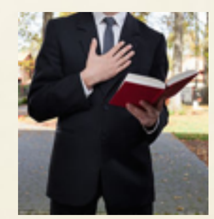

- The quasiprobability behind the OTOC → $\mathrm{Tr}\left(\Pi_{w_3}^{\mathcal{W}(t)}\Pi_{v_2}^{V}\Pi_{w_2}^{\mathcal{W}(t)}\Pi_{v_1}^{V}\rho\right)$

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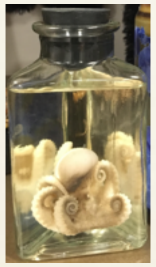

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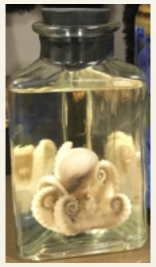

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Thanks for your time!

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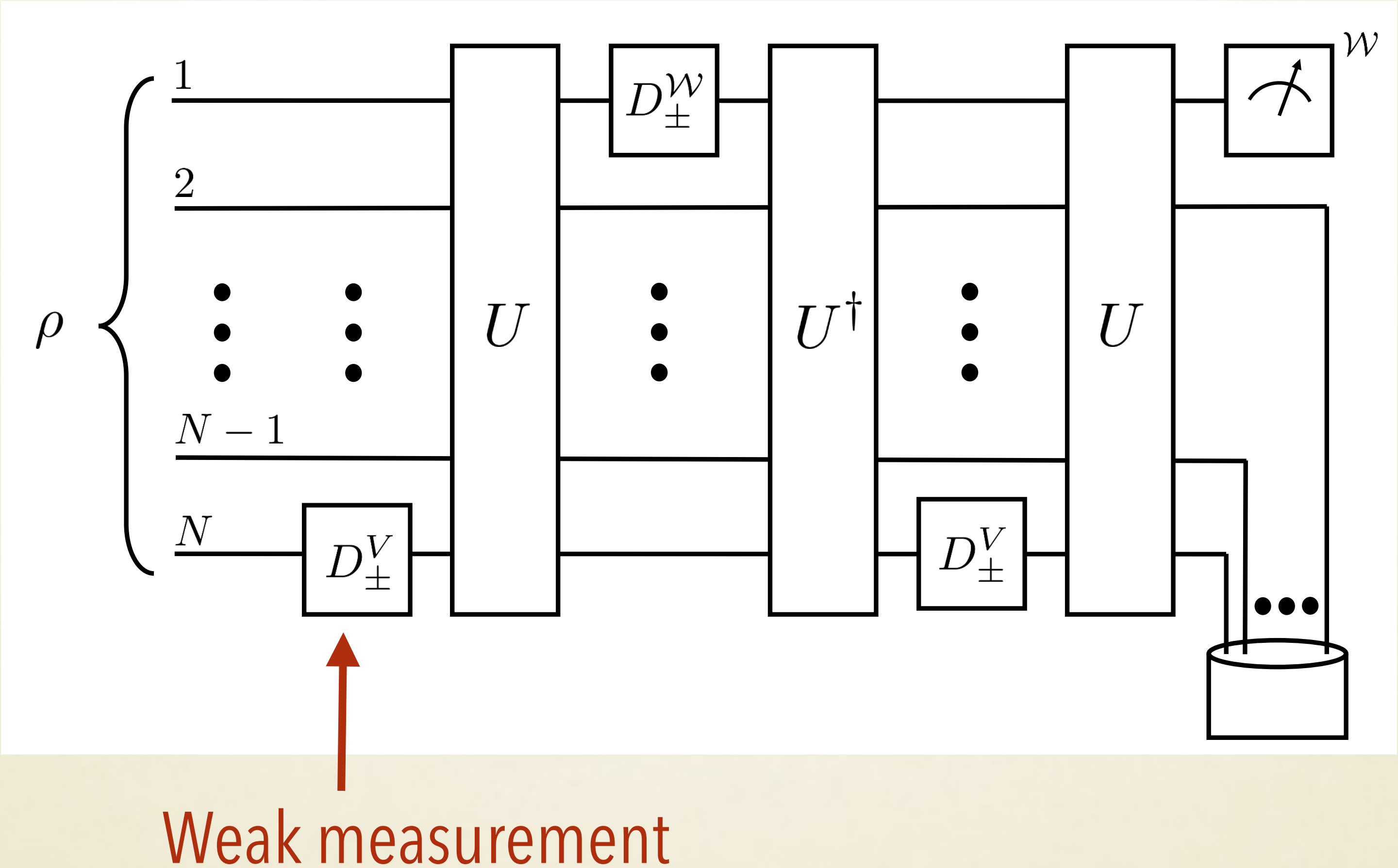
Measuring the OTOC quasiprobability with weak measurements



- $\text{Tr}\left(\Pi^{\mathcal{W}(t)}_{w_3}\Pi^{V}_{v_2}\Pi^{\mathcal{W}(t)}_{w_2}\Pi^{V}_{v_1}\rho\right)$
- No, we needn't discard data.

Comparison of OTOC-measurement schemes

	Yunger Halpern/ our weak meas.	Yunger Halpern interferometry	Swingle *et al.*	Yao *et al.*	Zhu *et al.*
Key tools	Weak measurement	Interference	Interference, Lochschmidt echo	Ramsey interfer., Rényi-entropy meas.	Quantum clock
What's inferable from the measurement?	(1) $F(t)$, $\tilde{A}_\rho$, & ρ or (2) $F(t)$ & $\tilde{\mathscr{A}}_\rho$	$F^{(\mathscr{K})}(t)$, $\tilde{A}^{\mathcal{K}}_\rho$, & ρ $\forall \mathcal{K}$	$\Re(F(t))$ or $\lvert F(t)\rvert^2$	Regulated correlator $F_{\text{reg}}(t)$	$F(t)$
Generality of ρ	Arbitrary $\rho \in \mathcal{D}(\mathcal{H})$	Arbitrary $\rho \in \mathcal{D}(\mathcal{H})$	Arbitrary $\rho \in \mathcal{D}(\mathcal{H})$	Thermal: $e^{-H/T}/Z$	Arbitrary $\rho \in \mathcal{D}(\mathcal{H})$
Ancilla needed?	Yes	Yes	Yes for $\Re(F(t))$, no for $\lvert F(t)\rvert^2$	Yes	Yes
Ancilla coupling global?	No	Yes	No	No	Yes
How long must ancilla stay coherent?	1 weak measurement	Whole protocol	Whole protocol	Whole protocol	Whole protocol
# time reversals	2	0	1	0	2
# copies of ρ needed / trial	1	1	1	2	1
Signal-to-noise ratio	To be determined [114]	To be determined [114]	Constant in N	$\sim e^{-N}$	Constant in N

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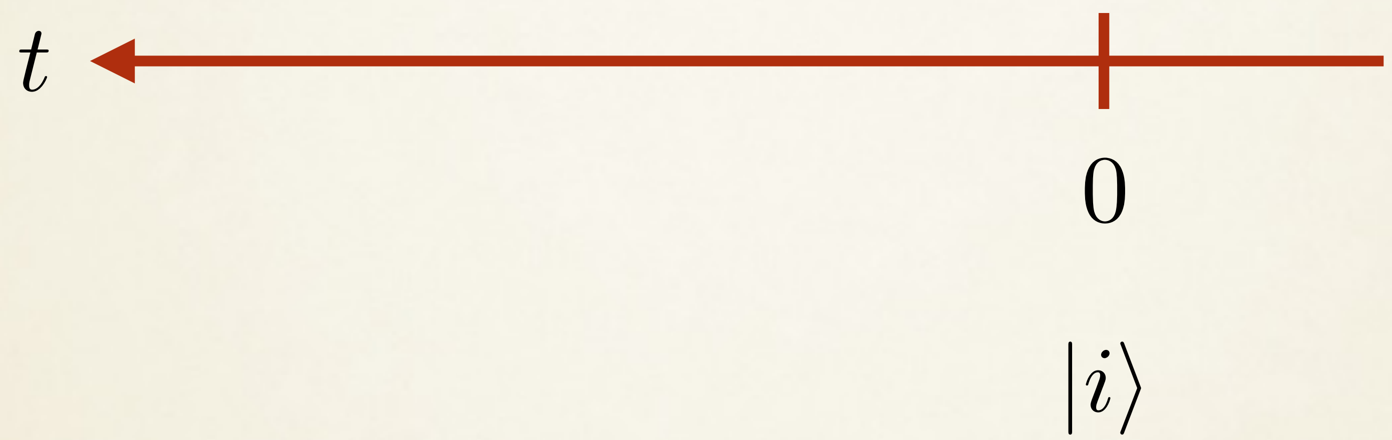

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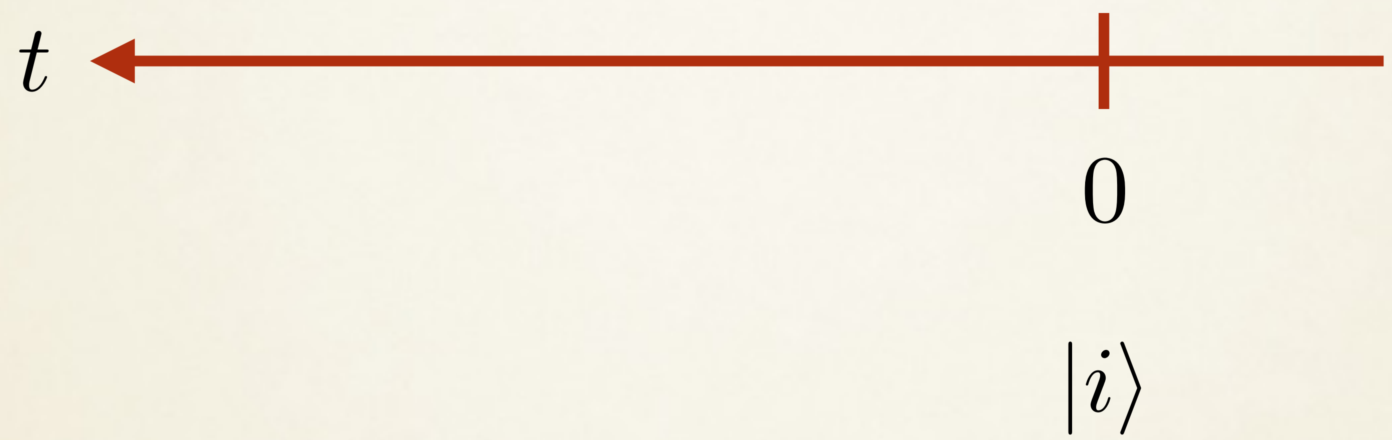

- $F = \sum_f f |f\rangle\langle f|$

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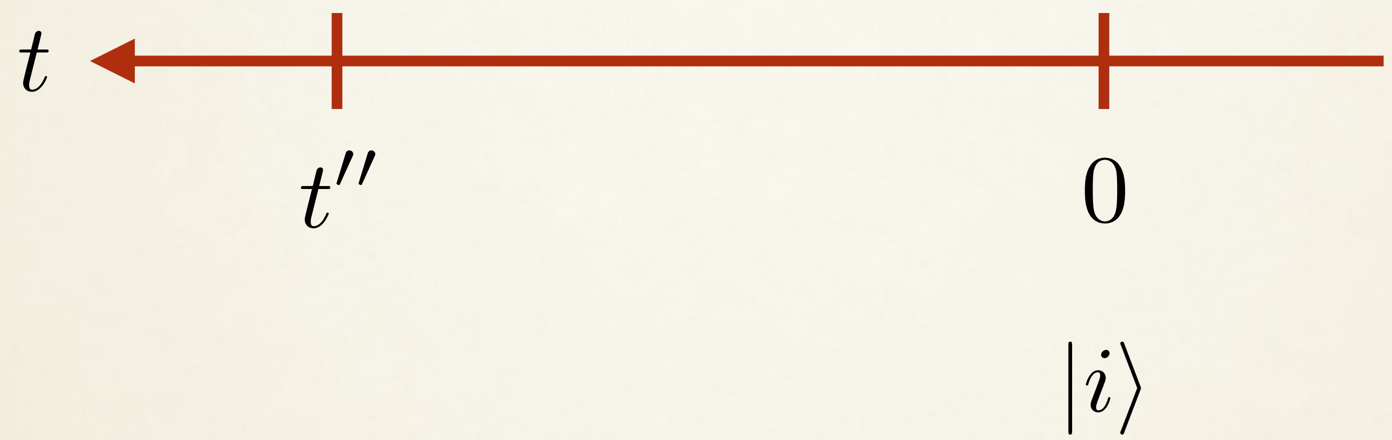

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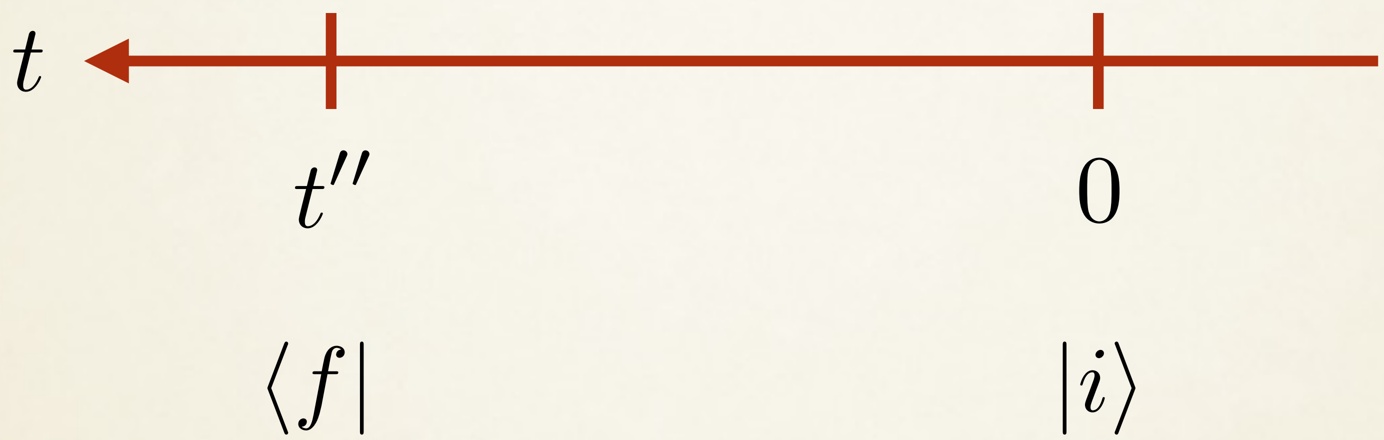

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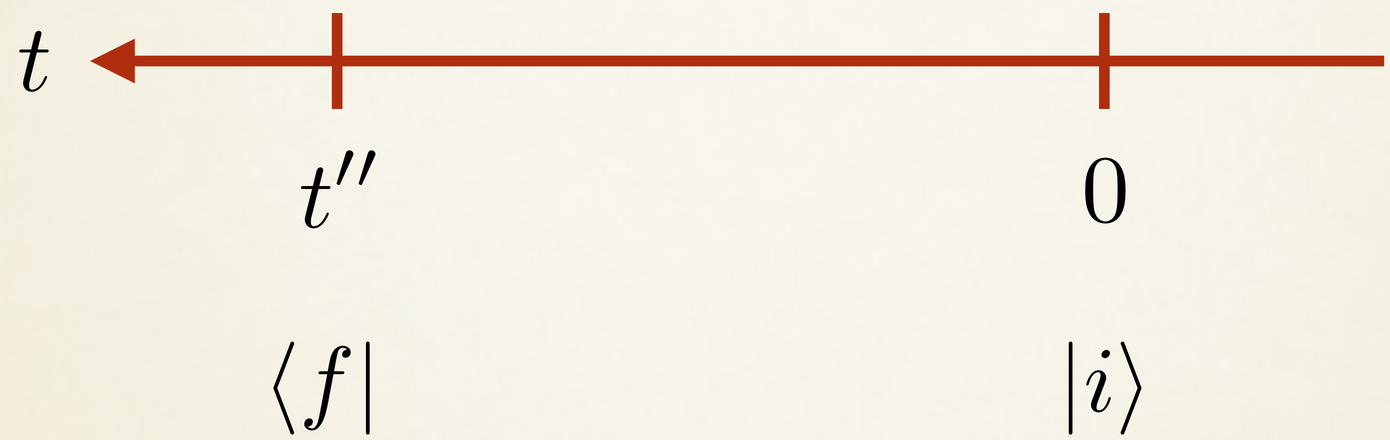

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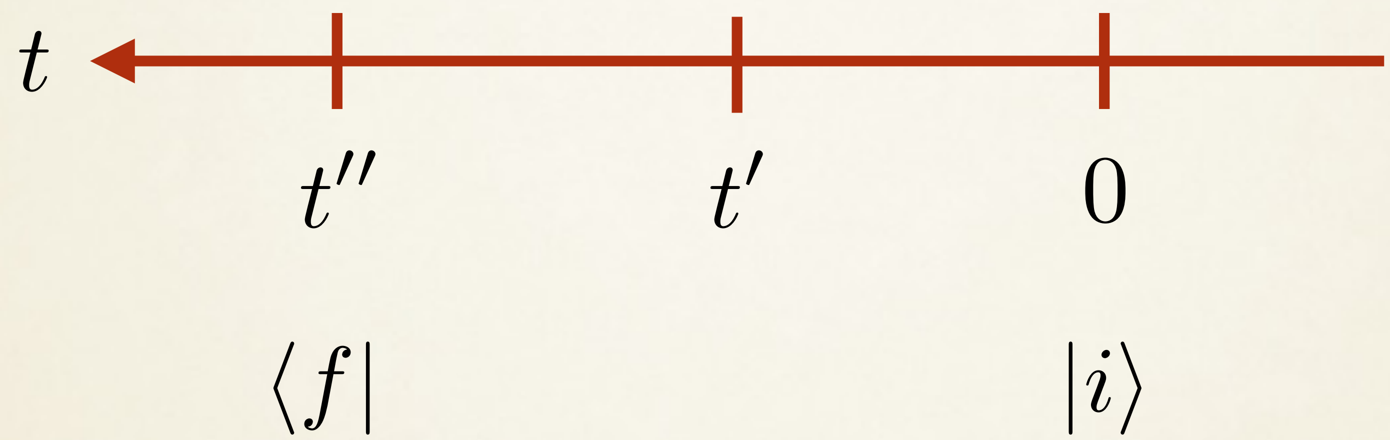

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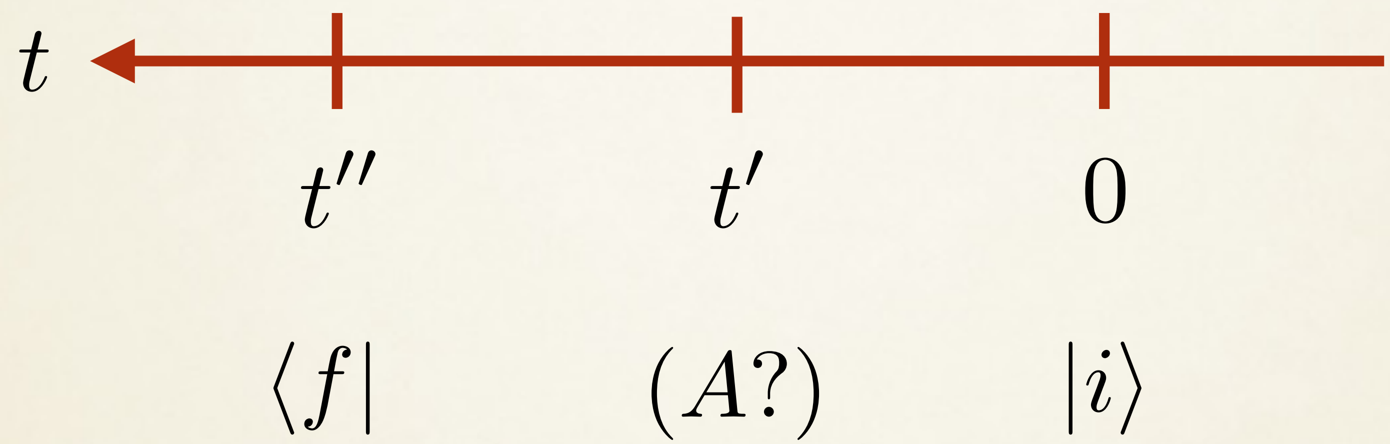

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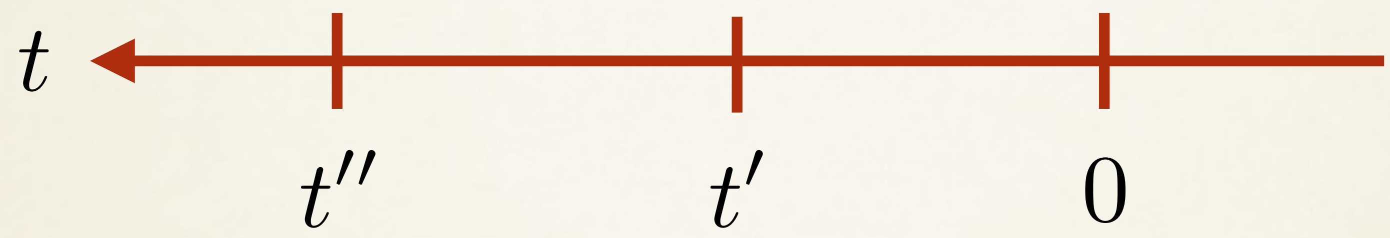

$$\langle f| \, U^{\dagger}_{t'',t'} \, (A?) \, U_{t',0} \, |i\rangle$$

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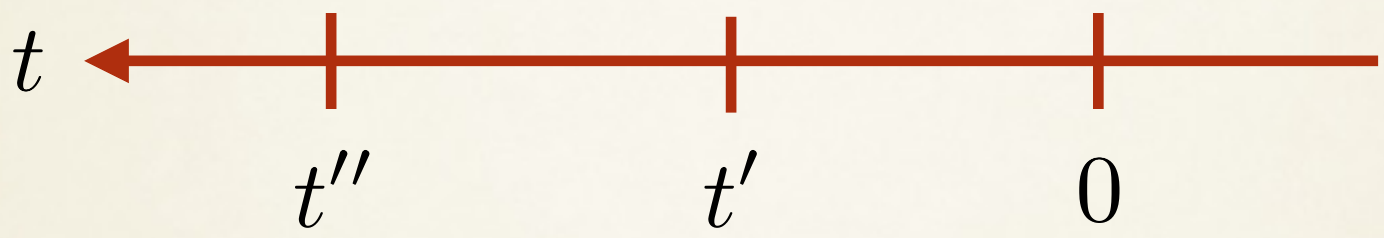

$$\langle f|\, U^\dagger_{t'',t'}\, (A?)\, U_{t',0}\, |i\rangle$$

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- What value is most reasonably attributable to *A* retrodictively, given that we prepared $|i\rangle$ and that our *F* measurement outcome yielded *f*?

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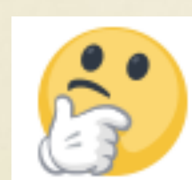

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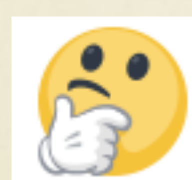

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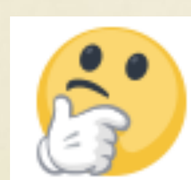

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OTOC quasiprobability

The OTOC as a signature of chaos

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- Chaos $\longleftrightarrow$ sensitivity to initial conditions

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- Compare 2 protocols that differ by an initial perturbation.

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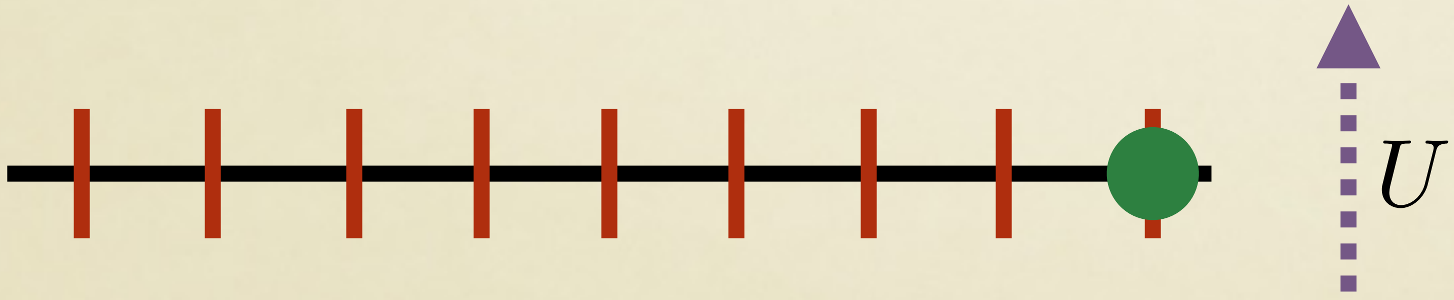

The OTOC as a signature of chaos

- Chaos $\longleftrightarrow$ sensitivity to initial conditions
- Compare 2 protocols that differ by an initial perturbation.

(1) $|\psi\rangle \mapsto \quad \mathcal{W} \; U \; V \; |\psi\rangle$

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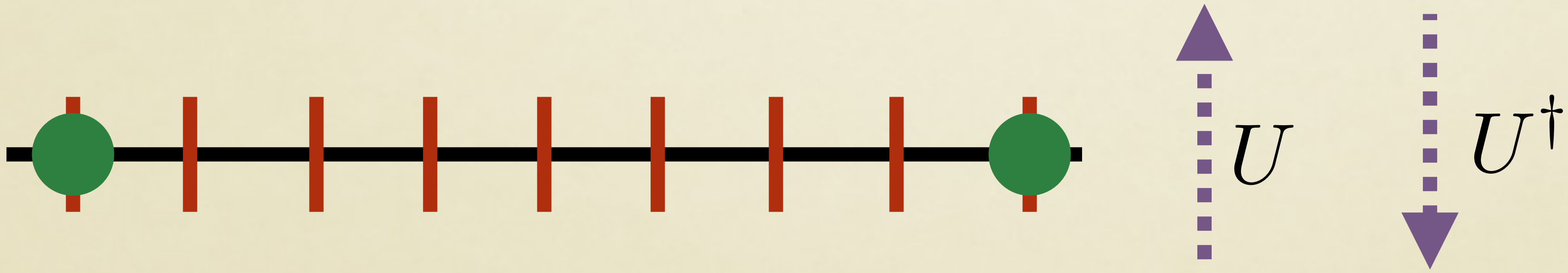

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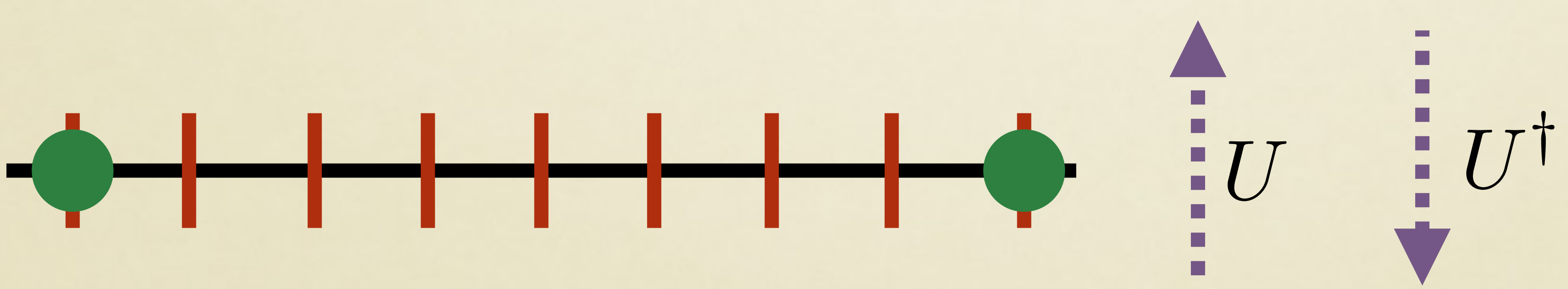

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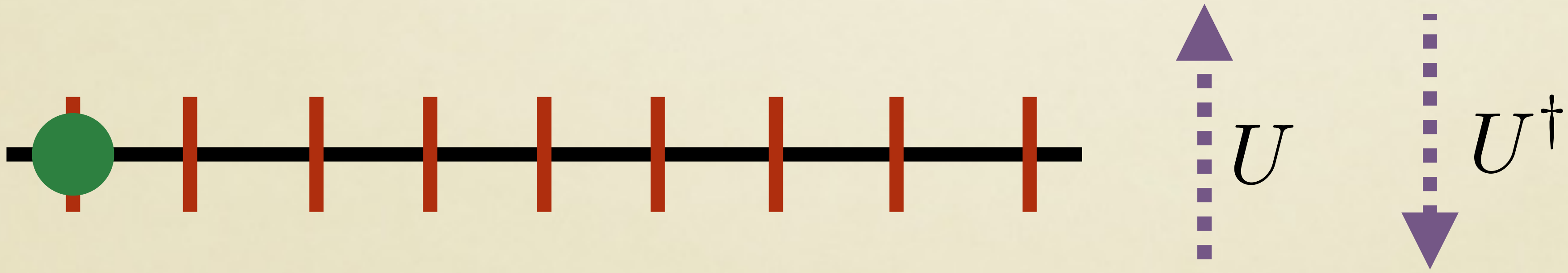

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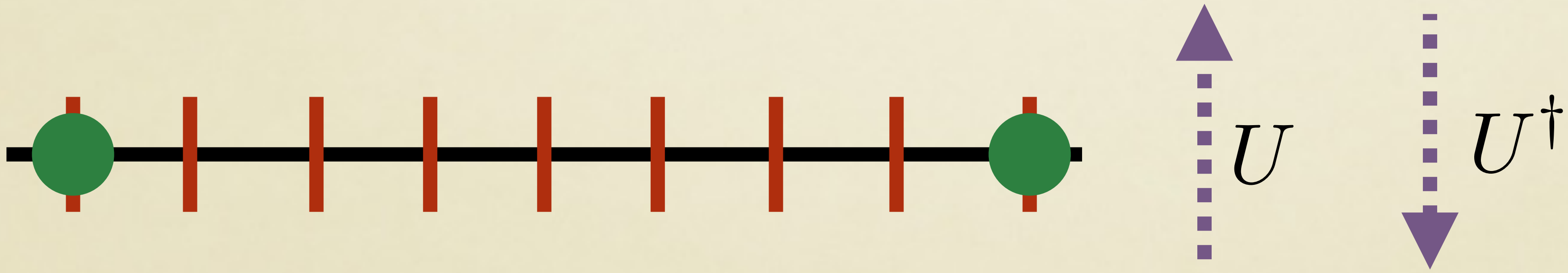

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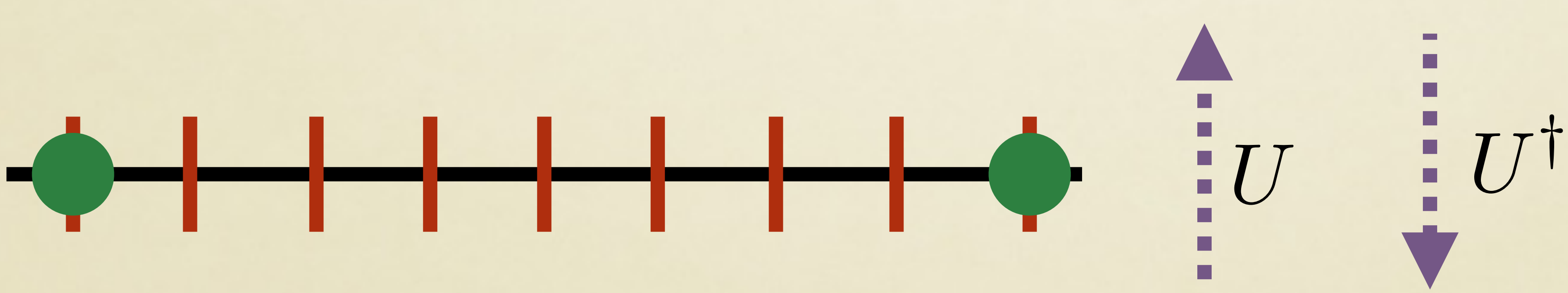

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- How much does an initial perturbation change the final state?

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  - <u>Overlap</u>: $|\langle\psi''|\psi'\rangle|$

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Lyapunov-type exponent

Fluctuation relations 

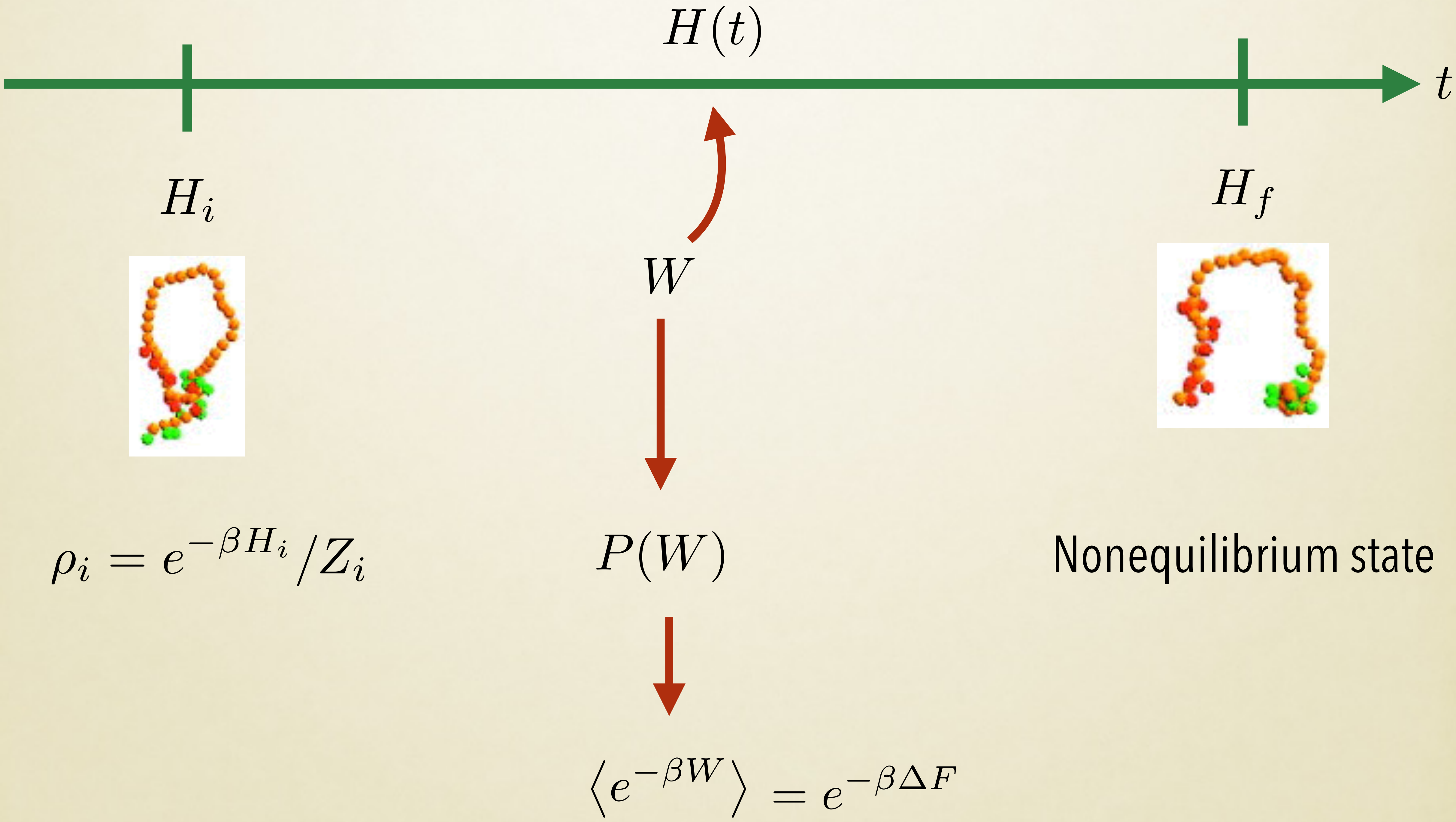
- <u>Field of physics</u>: nonequilibrium statistical mechanics
- <u>Broad strokes</u>
  - Describe systems arbitrarily far from equilibrium
  - Relate to irreversibility, Second Law, loss of information
  - Tested experimentally $\longrightarrow$ DNA, single-electron boxes, ion traps, …
  - Useful $\longrightarrow$ used to infer a free-energy difference ΔF

Jarzynski's Equality

Jarzynski, *Phys. Rev. Lett.* **78**, 2690 (1997).

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

Jarzynski's equality: $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$



Free-energy difference ΔF

$H_i,\ \beta$

$H_f,\ \beta$

$$F_i := -\frac{1}{\beta}\ln Z_i$$

$$F_f := -\frac{1}{\beta}\ln Z_f$$

$$\Delta F := F_f - F_i$$

- Applied in pharmacology, biology, and chemistry
- Difficult to measure –– idealized equilibrium quantity
- Inferred from nonequilibrium trials, via Jarzynski's Equality

Jarzynski's equality and the OTOC

- "Useful" form of Jarzynski's Equality: $\Delta F = -\frac{1}{\beta} \ln \langle e^{-\beta W} \rangle$
- Jarzynski-like equality for the OTOC: $F(t) = \frac{\partial^2}{\partial\beta \, \partial\beta'} \left\langle e^{-(\beta W + \beta' W')} \right\rangle \Big|_{\beta, \beta' = 0}$

- From different fields of physics
- Both related to time reversal, loss of information…
- They *must* be combinable!

Jarzynski-like equality for the OTOC

NYH, *Phys. Rev. A* **95**, 012120 (2017).

NYH, B. Swingle, and J. Dressel,
arXiv:1704.01971 (2017).

Strategy

- Start with a paper that casts Jarzynski's Eq. in terms of a correlation function.
  - Talkner *et al.*, *Phys. Rev. E* **75**, 050102(R) (2007).
  - 2-point, time-ordered correlator
- Deform the proof such that the OTOC pops out.
  - Build definitions by analogy.
  - Interpret physically. (Construct measurement protocols.)
  - Discover: probabilities $\longmapsto$ quasiprobabilities

Definitions

$$F(t) = \frac{\partial^2}{\partial\beta\,\partial\beta'} \left\langle e^{-(\beta W + \beta' W')} \right\rangle \Big|_{\beta,\beta'=0}$$

- *W, W′* → measurable random variables analogous to thermodynamic work
- < . > → average w.r.t. complex distribution
    - Constructed from quasiprobability
- β, β′ → real parameters

The quasiprobability behind the OTOC

- Jarzynski's Equality casts ΔF in terms of the characteristic function of a probability distribution.

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

- The Jarzynski-like equality casts the OTOC in terms of the characteristic function of a summed **quasi**probability distribution.

Signals nonclassical behavior

Signals noncomutation