Thermalization and Entropy Production in Closed Systems of Interacting Fermi Particles

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Problems:
- Entropy production $S(t)$
- conditions for statistical approach?
- choice of the basis: small number of particles, the classical limit
- methods for description:
  - dynamical interaction...
  - structure of the LDOS and eigenstates 
  - Fermi-Dirac, Bose-Einstein, Boltzmann
- time dependence for: $n_{\xi}(t)$, $N_{\phi}(t)$, $\Delta(t)$

Isolated systems of interacting particles.

$V$ stands for two-body "elastic" interaction

$\gamma = \frac{\pi}{\sqrt{\hbar V}}$, $\eta$ - number of non-interacting particles (or quasi-particles)
In collaboration with: 

- G. Casati 
- Università di Milano, Italy 
- N. G. Bangalore, FI 
- Bangalore, FI 
- 1996

Conservation of Nature: First results in chemical, condensed matter, atomic, and in nuclear physics 

$\rho_2 = \frac{1}{2} \sum_{\alpha \beta} (\psi_{\alpha \beta})^2$, F. Izrailev, B. Parent, and G. E. Volovik

Two-body random interaction model: $\gamma = \frac{1}{2} \sum_{\alpha \beta} (\psi_{\alpha \beta})^2$, F. Izrailev, B. Parent, and G. E. Volovik
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Therefore, when $n \to \infty$, $4 \alpha \propto E^2$.

Which reads to:

$$E = \frac{Ze_{\text{e}}^2}{Z}$$

or

$$E = \frac{Ze_{\text{e}}^2}{Z}$$

However, for finite systems with few particles, a more correct definition is:

For Gaussian form of $p(E) = \frac{1}{\sqrt{2\pi} \sigma^2} e^{-\frac{E-E_0}{2\sigma^2}}$:

$$\bar{E} = \frac{\int E p(E) dE}{\int p(E) dE} = E_0$$

Thermodynamic temperature and entropy.

Different definitions of temperature.
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Two coupled non-linear oscillators

\[ H = \frac{1}{2} \sum_i \left( p_i^2 + H^2 \right) + \frac{1}{4} \sum_{i<j} \left( x_i x_j + y_i y_j \right) \]


Two interacting particles in a box

\[ \text{Particle in the rippled channel} \]

\[ H = \frac{1}{2} \left( p_x^2 + p_y^2 + \frac{1}{2} (x^2 + y^2) \right) \]


Classical analog of $f$-function (shape of $E$)

\[ E = \frac{1}{2} \sum_i \left( p_i^2 + H^2 \right) + \frac{1}{4} \sum_{i<j} \left( x_i x_j + y_i y_j \right) \]

For fixed $H = E$:

Classical analog of strength function

Classical analogs of $f$-function and eigenfunctions (or $f$-function)
functions with $\rho = 0$: (a) are the same plot in semilog scale.

overlapping over the central energy region (see Figure 3, this paper).

and classical (dashed line) for the case $L = 3.5$ (dashed line).

FIG. 3: The LDOS distribution in the energy representation.

Another representation for the localization in energy space.

Time-reversal symmetry.

Localization in energy shell.
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Many-body systems

Thermalization time in closed system

Time dependence of occupation numbers

Both to be published in Phys. Rev. E


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States in closed many-body systems

Unconventional decay law for excited

Phys. Rev. (in press)

Grant-pr/010525

Chaotic many-body systems

Determinism in the local space of closed

Entropy production and wave packet

V. Fermaglio & E. Izrailev

Grant 2001

Written quantum

\[ \Psi_{i} = \sqrt{\frac{\mu}{2\pi \hbar}} e^{i \frac{\mu}{2\pi \hbar} H} \]

Min. CD/016036

F. Borghesi, G. Cederko, F. Izrailev, G. Cascarri

(2002)

E. Cascarri - Como, Italy

F. H. I. - Pittsburgh, USA

G. Cederko - Brescia, Italy

F. Borghesi -...
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Quant-ph/0102028

(1) Fermi-Dirac, F. I. Zel'dovich

\[ \omega(\mu) \approx C. e^{-\frac{\mu}{kT}} \]

When \( \beta \) is the Gauss, for large times

Long time: \( \mu \approx \beta \)

(2) Gaussian part of the peak of SF (loos)

\[ \mu = C. e^{-\frac{\beta}{kT}} \]

(3) Short time, \( \beta \ll kT \)

\[ \mu = C. e^{-\frac{\beta}{kT}} \]

Limit case:

C. function (SF)

F. I. Zel'dovich (2000)

(1) Fermi-Dirac

\[ F(E) = \frac{e^{-\beta E}}{1 + e^{-\beta E}} \]

(2) Gaussian

\[ F(E) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(E - \mu)^2}{2\sigma^2}} \]

\[ \sigma \approx \frac{\beta}{2} \]

\[ \mu \approx 0 \]

\[ \beta \ll \frac{1}{kT} \]

Lorentz: \( \beta \gg kT \)

\[ \sqrt{\frac{A}{2\pi \sigma^2}} e^{-\frac{(E - \mu)^2}{2\sigma^2}} \]

\[ L(E) = \frac{2A}{2\pi \sigma^2} e^{-\frac{(E - \mu)^2}{2\sigma^2}} \]

\[ \mu \approx 0 \]

\[ \sigma \approx \frac{\beta}{2} \]

\[ \beta \ll \frac{1}{kT} \]

Gaussian like, with an excess

\[ \rho(E) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(E - \mu)^2}{2\sigma^2}} \]

\[ \mu \approx 0 \]

\[ \sigma \approx \frac{\beta}{2} \]

\[ \beta \ll \frac{1}{kT} \]

The form changes from the Lorentzian

\[ C. function (SF) \]
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\[ \frac{df}{dt} = f - L \cdot \frac{df}{dt} \]

\[ L = \frac{G}{2} \]

\[ W = \frac{1}{2} \frac{G}{2} \]

\[ \text{Stochastic functions: } p(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) \]

\[ A_0 \leq A_1, A_2 \leq \ldots \]

\[ \mu_0 = (\lambda_1^2 - |\nu|^2) \]

\[ \psi_n(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) \]

\[ f(x) = \psi_n(x) \]

\[ \text{Furthermore, the solution is: } \]

\[ \text{Moreover, the ground state } 10 \text{ is excited} \]

\[ \text{Then, } \]

\[ \text{11} = \frac{1}{2} \frac{\psi_n}{\psi_n} \]

\[ \text{11} - \text{exact states (of } H) \]

\[ \text{12} - \text{ground states (of } H_0) \]

\[ \mu = \mu_0 + \Delta \]
For thermalization of gases, set
\[ N_{\text{fp}} = \frac{N_{\text{fp}}}{N_{\text{fp}}} \]

For infinite number of classes,
\[ N_{\text{fp}} = \frac{N_{\text{fp}}}{N_{\text{fp}}} \]

From IPR

Solution

in the $k$-th class,
\[ N = \text{number of basis states} \]
\[ k = \text{Index of the } k \text{-th class} \]
\[ W = \text{Number of states, directly} \],

neglecting the return flow,
\[ N_{\text{fp}} = N_{\text{fp}} \]

Compare with the Cloge's tree model
\[ M_0 = \frac{M_0}{M_0} \]
\[
S(t) = \frac{N}{h} \left[ \frac{\epsilon_{\text{max}}}{\epsilon_{\text{min}}} \right] - \int_{-\infty}^{\infty} g(\epsilon) \ln g(\epsilon) \, d\epsilon
\]

Happo expression:
\[
S(t) = \frac{2^{\frac{3}{2}}} {\gamma^2} \left\{ \frac{3}{2} \right\} \ln \left[ \frac{1}{3} \right] \]

For small times,\[ \gamma \to \text{the classical limit} \]
\[ \frac{1}{t} \approx \text{constant} \]

\[ S(t) \approx \frac{1}{t} \ln \frac{1}{t} \]

Result:
\[
S(t) = - 2 \ln \epsilon_{\text{max}} - 2 \ln \epsilon_{\text{min}} - 2 \ln \left( \frac{\epsilon_{\text{max}}}{\epsilon_{\text{min}}} \right)
\]

Entropy
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Gaussian form of the strength function

\[ n(\omega) = \frac{1}{\sqrt{2\pi \hbar^2}} e^{-\frac{(\omega - \omega_0)^2}{2\hbar^2}} \]

where: \( \omega_0 \) is the center of energy spectrum

\[ W(\omega) = \frac{1}{W(\omega)} \] - return propagate to stable

[Graph showing Gaussian distribution]

\[ n(\omega) = \frac{1}{N} \sum_{\omega_n} \delta(\omega - \omega_n) \]

Then:

If \( n(\omega) \) is at the center of energy spectrum, then:

\[ \Delta E = 0 \]

Perturbation level

\[ n_E = \frac{1}{N} \sum_{\omega_n} \delta(\omega - \omega_n) \]

\[ n(\omega) = \frac{1}{N} \sum_{\omega_n} \delta(\omega - \omega_n) \]

\[ n(\omega) = \frac{1}{N} \sum_{\omega_n} \delta(\omega - \omega_n) \]

Using the cascade model, one can obtain

Time dependence of occupation numbers

Numerical data are obtained for \( m = 1, 2 \)

Standard s-d shell model

Number of particles (Fermi-participants)

Number of single-particle levels

of composite (heavy) nuclei

\[ n(\omega) = \frac{1}{N} \sum_{\omega_n} \delta(\omega - \omega_n) \]