eLOQC[a]

Produced with pdflatex and xfig

[a] Efficient Linear Optics Quantum Computation

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eLOQC Guide

Optical methods	Capabilities realized
Optical systems & ops:	Qubits and one-qubit ops
Postselection:	QC, if lucky
Teleportation:	Standard QC
Z-encoding:	Efficient QC
Loss detection, Erasure codes:	Robuster QC
Accuracy threshold:	Scalable QC

Implementation issues:

- Resource requirements
- Technological challenges

Optics for Quantum Information?

- Advantages:
  - Mostly room temperature.
  - Easy to observe interference.
  - (Coherence time/ operation time) seems long.
  - Lots of photons.
- Obstacles:
  - Need for nonlinear interactions?
  - Inefficient photodetectors.
  - Inefficient 1-photon sources.
  - Scalable mode matching.
- Optical quantum computation so far:
  - Single photon qubits, Kerr non-linearities. Milburn 1988 [1]
  - Optics and cavity QED. Turchette&*al.* 1995 [2]
  - Linear optics, many modes. Cerf&Adami&Kwiat 1998 [3]
- Problem: Capability of linear optics, 1-photon sources and photodetectors?
  - eLOQC. Knill&Laflamme&Milburn 2000 [4]
  - Linear optics with continuous variable encoding. Gottesman&Kitaev&Preskill 2000 [5]

Optical Modes

- State space of mode A:

$$|0\rangle_{\mathsf{A}}, |1\rangle_{\mathsf{A}}, \ldots, |n\rangle_{\mathsf{A}}, \ldots$$

- Operators of mode A:

$$\left(\mathbf{a}^\dagger\right)_{\mathsf{A}} : |n\rangle_{\mathsf{A}} \rightarrow \sqrt{n+1}\,|n+1\rangle_{\mathsf{A}}$$
$$\left(\mathbf{n}\right)_{\mathsf{A}} : |n\rangle_{\mathsf{A}} \rightarrow n\,|n\rangle_{\mathsf{A}}$$

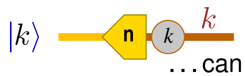
- Bosonic qubit $\mathsf{Q}(\mathsf{A},\mathsf{B})$:

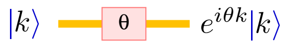
$$|\mathtt{o}\rangle_{\mathsf{Q}(\mathsf{A},\mathsf{B})} \leftrightarrow |0\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}$$
$$|\mathtt{1}\rangle_{\mathsf{Q}(\mathsf{A},\mathsf{B})} \leftrightarrow |1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}$$

Optical network notation for $\mathsf{Q}(\mathsf{A},\mathsf{B})$:

Optical Devices for LOQC

- Mode preparations: 

- Photon counter: 

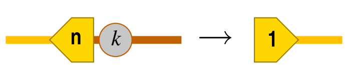
  . . . can be used for feedback.
- Phase shifter $\mathbf{P}_\theta$: 

  $e^{i\theta\mathbf{n}}$
  . . . the simplest linear optics element.
- Beam splitter $\mathbf{B}_{\theta,\phi}$: 

$$|1\rangle_A|0\rangle_B \rightarrow \cos(\theta)|1\rangle_A|0\rangle_B + e^{-i\phi}\sin(\theta)|0\rangle_A|1\rangle_B$$
$$|0\rangle_A|1\rangle_B \rightarrow -e^{i\phi}\sin(\theta)|1\rangle_A|0\rangle_B + \cos(\theta)|0\rangle_A|1\rangle_B$$

. . . another linear optics element.

On Linear Optics

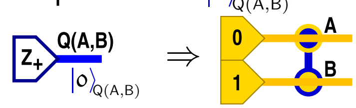
- U is *passive linear* if $U\mathbf{a}^\dagger{}_s U^\dagger = \sum_r u_{rs}\mathbf{a}^\dagger{}_r$.
  Def: $\hat{u} = \hat{u}(U)$ is the matrix with entries u_{rs}.
  - U preserves particle number.
  - $\hat{u}$ is unitary, the "second quantization" form of U.
- Example: $\hat{u}(\mathbf{B}_{45^\circ}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Def: $\mathbf{B}_{45^\circ} = \mathbf{B}_{45^\circ,0}$.
- **Theorem 1:** For every unitary $\hat{u}$, the corresponding U is implementable by a network of beam splitters and phase shifters. Reck&Zeilinger&Bernstein 1994 [6].
- U is *linear* if $U\mathbf{a}^\dagger{}_s U^\dagger = \sum_r (a_{rs}\mathbf{a}^\dagger{}_r + b_{rs}\mathbf{a}_r) + c_s$
- Active linear + 

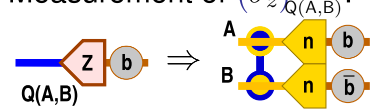
  Hong&Mandel 1986 [7]

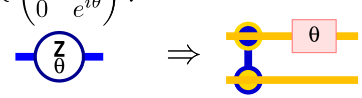
Linear Optics No-Go?

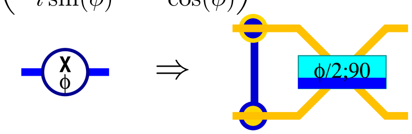
- Passive linear optics $\simeq$ classical wave mechanics?
- **Theorem 2:** Linear optics and particle detectors for fermions are efficiently classically simulatable.
  Valiant 2001 [8], Terhal&DiVincenzo 2001 [9], Knill 2001 [10]
- **Theorem 3:** Linear optics and measurement of $\hat{x} \propto \mathbf{a}^\dagger + \mathbf{a}$ is efficiently classically simulatable. Bartlett&*al.* 2001 [11]
- **Conjecture 4:** Number state preparations, linear optics and measurement of exactly one mode (no feedback) are efficiently classically simulatable.
- **Conjecture 5:** Coherent state preparations, passive linear optics and photodectors with feedback are efficiently classically simulatable. Ivan Deutsch 2001
  See also: Lütkenhaus&Calsamiglia&Suominen 1999 [12], Calsamiglia 2001 [13]

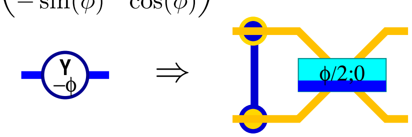
Bosonic Qubit Operations

- Preparation of $|\mathsf{o}\rangle_{Q(A,B)}$:


- Measurement of $\langle\sigma_z\rangle_{Q(A,B)}$:


- $Z = \sigma_z$ rotation by θ, $e^{-i\sigma_z\theta/2}$:
  $\propto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$:


- $X = \sigma_x$ rotation by ϕ, $e^{-i\sigma_x\phi/2}$:
  $\begin{pmatrix} \cos(\phi) & -i\sin(\phi) \\ -i\sin(\phi) & \cos(\phi) \end{pmatrix}$:


- $Y = \sigma_y$ rotation by ϕ, $e^{-i\sigma_y\phi/2}$:
  $\begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix}$:


- Still need a "nonlinear" coupling.

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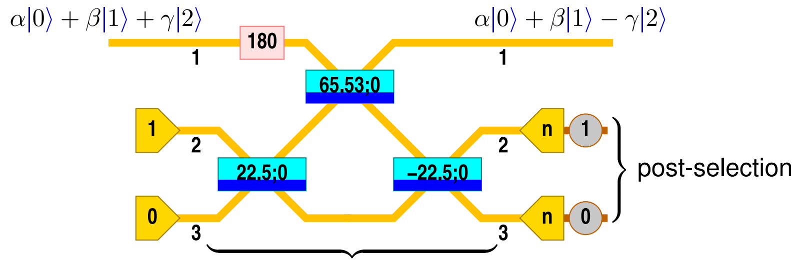
Postselected Non-linear Phaseshift

- NS(θ) : $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \rightarrow \alpha|0\rangle + \beta|1\rangle + e^{-i\theta}\gamma|2\rangle$
- Postselected NS$(180°)$, prob$_{\text{succ}} = 1/4$:

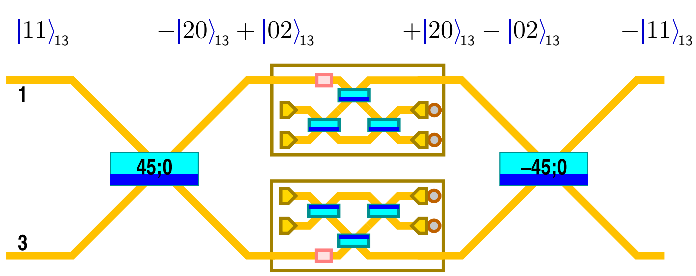


$$\hat{U} = \begin{pmatrix} 1-2^{1/2} & 2^{-1/4} & (3/2^{1/2}-2)^{1/2} \\ 2^{-1/4} & 1/2 & 1/2-1/2^{1/2} \\ (3/2^{1/2}-2)^{1/2} & 1/2-1/2^{1/2} & 2^{1/2}-1/2 \end{pmatrix}.$$

See also: Ralph&*al.* 2001 [14], Rudolph&Pan 2001 [15], Pittman&*al.* 2001 [16]

Postselected Conditional Sign Flip

- qCS(θ) for qubits:
  $\alpha|\mathtt{00}\rangle + \beta|\mathtt{01}\rangle + \gamma|\mathtt{10}\rangle + \delta|\mathtt{11}\rangle \rightarrow \alpha|\mathtt{00}\rangle + \beta|\mathtt{01}\rangle + \gamma|\mathtt{10}\rangle + e^{i\theta}\delta|\mathtt{11}\rangle$
- Postselected CS$(180°)$, CS$_{1/16}$ with prob$_{\text{succ}} = 1/16$:



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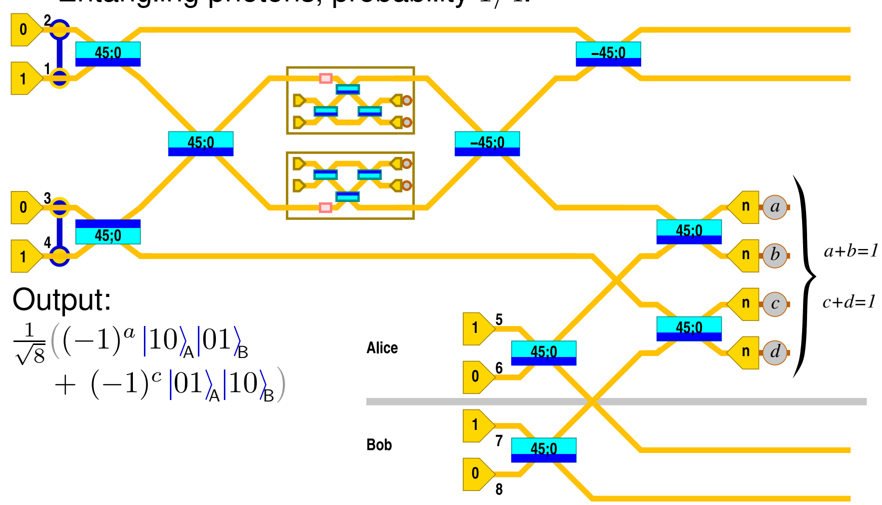
Entanglement “without” Interaction

- Entangling photons, probability $1/4$.



Output:
$\frac{1}{\sqrt{8}}\big((-1)^a\,|10\rangle_A|01\rangle_B$
$+\ (-1)^c\,|01\rangle_A|10\rangle_B\big)$

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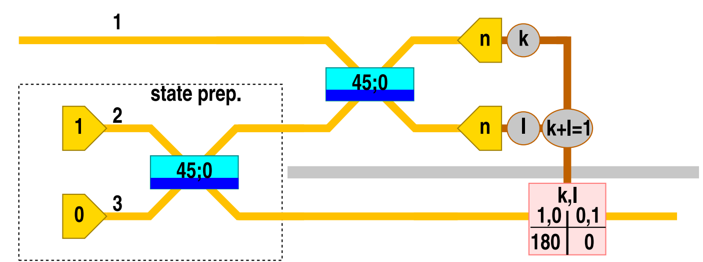
Trivial Teleportation

- Teleportation of one mode, success probability $1/2$:

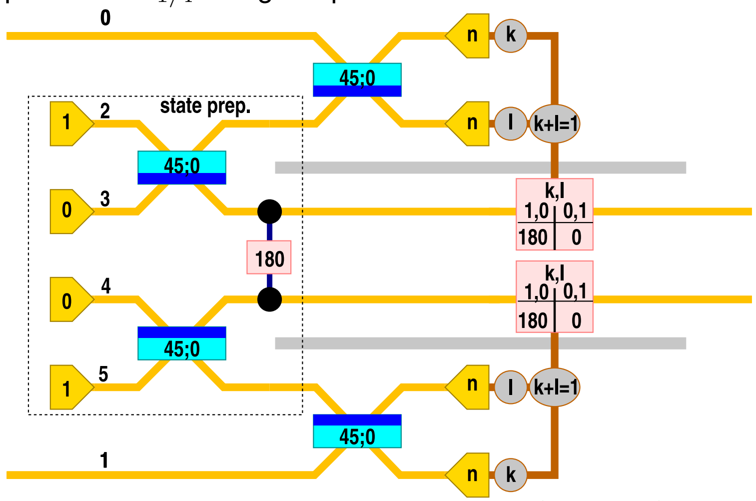


$\big(\alpha|0\rangle_1 + \beta|1\rangle_1\big) \longrightarrow$

$\frac{1}{2}\big(-\alpha|0\rangle_3 - \beta|1\rangle_3\big)$
or
$\frac{1}{2}\big(+\alpha|0\rangle_3 + \beta|1\rangle_3\big)$

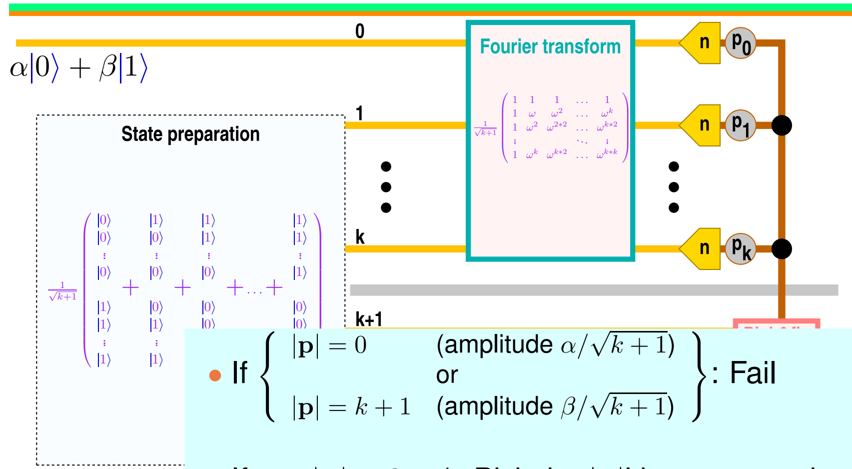
Operation $=$ State $+$ Teleportation

- Implement $\mathrm{CS}_{1/4}$ using teleportation:



Gottesman&Chuang 1999 [17]

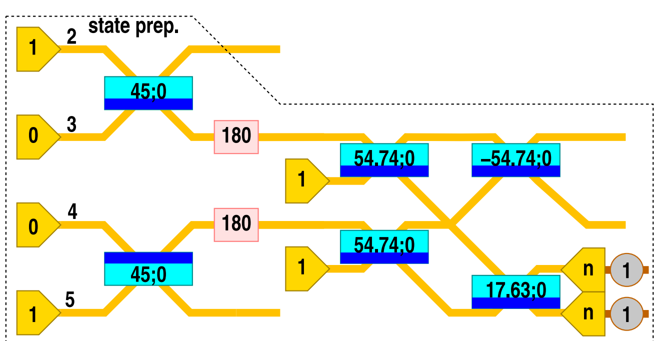
Teleportation with Probability $k/(k+1)$



- If $\left\{ \begin{array}{ll} |\mathbf{p}| = 0 & \text{(amplitude } \alpha/\sqrt{k+1}) \\ & \text{or} \\ |\mathbf{p}| = k+1 & \text{(amplitude } \beta/\sqrt{k+1}) \end{array} \right\}$: Fail
- If $1 \le |\mathbf{p}| \le k$:
  1. Pick the $|\mathbf{p}|$'th output mode.
  2. Correct the phase by $\omega^{-p.\mathbf{b}}$.

Forward to: Ts_k | $\mathrm{CS}_{4/9}$

State Preparation for $\mathrm{CS}_{1/4}$

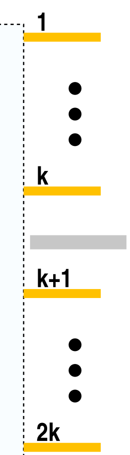


- Using: Two-mode non-linear sign shift with $\mathrm{prob}_{\mathrm{succ}} = 1/13.5$.

Go to: Details.

Tel_k: Initial State

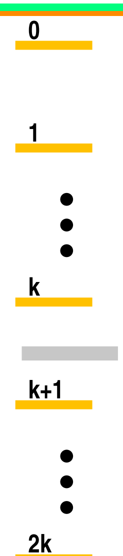
$$\mathrm{Ts}_k = \frac{1}{\sqrt{k+1}} \left(\begin{array}{c} |0\rangle \\ |0\rangle \\ \vdots \\ |0\rangle \\ \\ |1\rangle \\ |1\rangle \\ \vdots \\ |1\rangle \end{array} + \begin{array}{c} |1\rangle \\ |0\rangle \\ \vdots \\ |0\rangle \\ \\ |0\rangle \\ |1\rangle \\ \vdots \\ |1\rangle \end{array} + \begin{array}{c} |1\rangle \\ |1\rangle \\ \vdots \\ |0\rangle \\ \\ |0\rangle \\ |0\rangle \\ \vdots \\ |1\rangle \end{array} + \ldots + \begin{array}{c} |1\rangle \\ |1\rangle \\ \vdots \\ |1\rangle \\ \\ |0\rangle \\ |0\rangle \\ \vdots \\ |0\rangle \end{array} \right)$$



Tel$_k$: Adjoin Input

$$
\frac{1}{\sqrt{k+1}}\left(
\begin{matrix}\alpha|0\rangle\\|0\rangle\\|0\rangle\\|0\rangle\\\vdots\\|1\rangle\\|1\rangle\\|1\rangle\\\vdots\end{matrix}
+
\begin{matrix}\mathsf{S}\begin{pmatrix}\alpha|1\rangle\\|0\rangle\\|0\rangle\\|0\rangle\end{pmatrix}\\\vdots\\|0\rangle\\|1\rangle\\|1\rangle\\\vdots\end{matrix}
+
\begin{matrix}\mathsf{S}\begin{pmatrix}\alpha|1\rangle\\|1\rangle\\|0\rangle\\|0\rangle\end{pmatrix}\\\vdots\\|0\rangle\\|0\rangle\\|1\rangle\\\vdots\end{matrix}
+
\begin{matrix}\mathsf{S}\begin{pmatrix}\alpha|1\rangle\\|1\rangle\\|1\rangle\\|0\rangle\end{pmatrix}\\\vdots\\|0\rangle\\|0\rangle\\|0\rangle\\\vdots\end{matrix}
+\ldots\right)
$$

$$
+\frac{1}{\sqrt{k+1}}\left(
\begin{matrix}\beta|1\rangle\\|0\rangle\\|0\rangle\\|0\rangle\\\vdots\\|1\rangle\\|1\rangle\\|1\rangle\\\vdots\end{matrix}
+
\begin{matrix}\beta|1\rangle\\|1\rangle\\|0\rangle\\|0\rangle\\\vdots\\|0\rangle\\|1\rangle\\|1\rangle\\\vdots\end{matrix}
+
\begin{matrix}\beta|1\rangle\\|1\rangle\\|1\rangle\\|0\rangle\\\vdots\\|0\rangle\\|0\rangle\\|1\rangle\\\vdots\end{matrix}
+
\begin{matrix}\beta|1\rangle\\|1\rangle\\|1\rangle\\|1\rangle\\\vdots\\|0\rangle\\|0\rangle\\|0\rangle\\\vdots\end{matrix}
+\ldots\right)
$$



Back to: Tel$_k$ network

Mode Fourier Transform

- Fourier transform of modes $0\ldots k$: Define $\omega = e^{i2\pi/(k+1)}$.

$$
\hat{u}(\mathsf{F}_{k+1}) = \frac{1}{\sqrt{k+1}}\begin{pmatrix} 1 & 1 & 1 & \ldots & 1\\ 1 & \omega & \omega^2 & \ldots & \omega^k\\ 1 & \omega^2 & \omega^{2*2} & \ldots & \omega^{k*2}\\ \vdots & & & \ddots & \vdots\\ 1 & \omega^k & \omega^{k*2} & \ldots & \omega^{k*k}\end{pmatrix}
$$

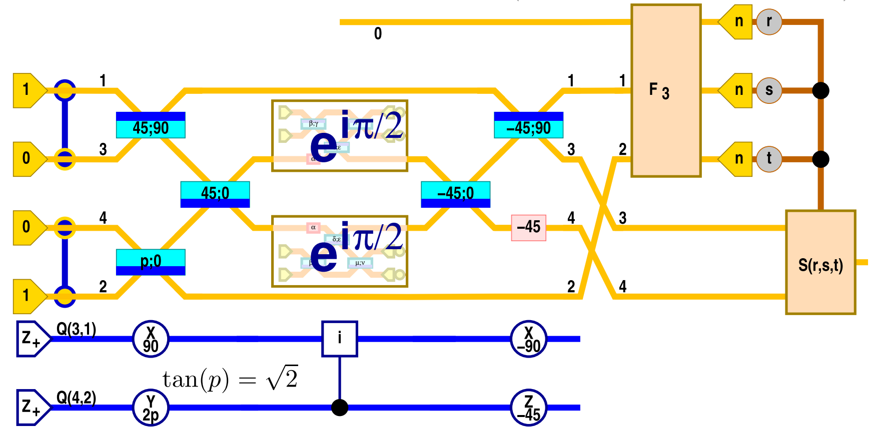
- F_{k+1} turns “shift” (S_{k+1}) into “phase” (P_{k+1}):

$$
\hat{u}(\mathsf{F}_{k+1})\begin{pmatrix} 0 & 0 & \ldots & 0 & 1\\ 1 & 0 & \ldots & 0 & 0\\ 0 & 1 & \ldots & 0 & 0\\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \ldots & 1 & 0\end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0\\ 0 & \omega & 0 & \ldots & 0\\ 0 & 0 & \omega^2 & \ldots & 0\\ \vdots & & & \ddots & \vdots\\ 0 & 0 & 0 & \ldots & \omega^k\end{pmatrix}\hat{u}(\mathsf{F}_{k+1})
$$

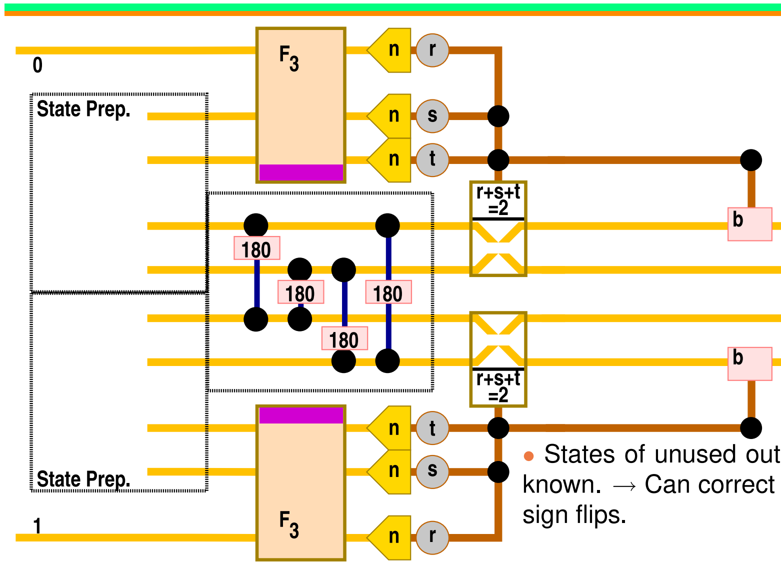
Back to: Tel$_k$ network

Tel$_k$ State Preparation Algorithm

- Ts_2 is a bosonic qubit state: $\mathsf{Ts}_2 = \frac{1}{\sqrt{3}}\left(|1\rangle_{3,1}|1\rangle_{4,2} + |0\rangle_{3,1}|1\rangle_{4,2} + |0\rangle_{3,1}|0\rangle_{4,2}\right)$.



Conditional Sign Flip with Probability $4/9$



- States of unused outputs are known. → Can correct for extra sign flips.

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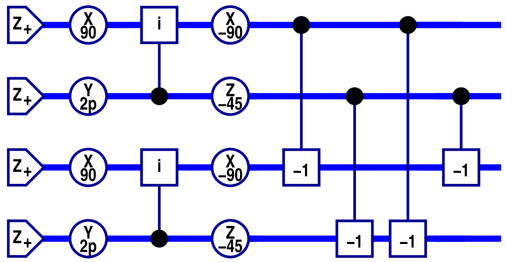
State Preparations for CS$_{k^2/(k+1)^2}$

- Design qubit networks and translate.

  Example: Prepare Cs_2.



- Requires 6 postselected controlled gates.
  - As shown: Expected number of gate attempts $\gg 600000$.
  - Can be improved but asymptotically still bad.

    Knill&Laflamme&Milburn 2000 [18]
  - Issue will be avoided by coding.

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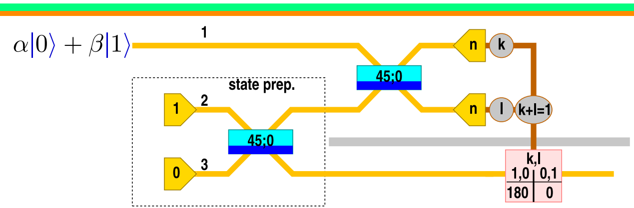
Failure Behavior of the Teleportation Schemes

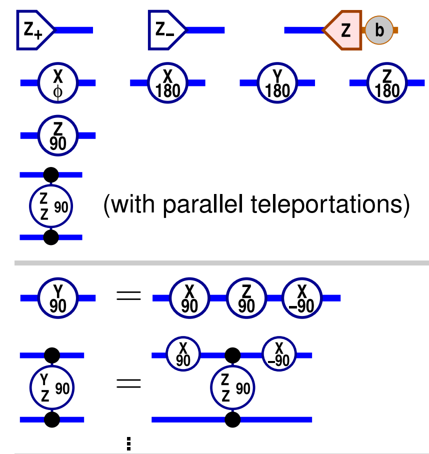
- Tel$_1$:



  - $k+l=0$: Input was $|0\rangle$.
  - $k+l=2$: Input was $|1\rangle$.
  - Failure of Tel$_1$ is measurement of $|0\rangle$ vs. $|1\rangle$.
    Probability of $\mathbf{n}$-measurement error: $1/2$.
- Generalization to Tel$_k$: Def: $p=$ number of photons detected.
  - $p=0$: Input was $|0\rangle$. $p=k+1$: Input was $|1\rangle$.
  - Failure of Tel$_k$ is measurement of $|0\rangle$ vs. $|1\rangle$.
    Probability of $\mathbf{n}$-measurement error: $1/(k+1)$.

Forward to: $\mathcal{X}_2$

Failure Model for Ideal eLOQC Gates



Detected Z-meas. error models	Physical	Encoded
	No error	No error
	No error	No error
	No error	Z, prob. f
(with parallel teleportations)	Z, prob. f; indep. Z, prob. f	Z, prob. f; indep. Z, prob. f
$=$	No error	Y, prob. f
$=$ ⋮	Y, prob. f; indep. Z, prob. f	Y, prob. f; indep. Z, prob. f

- Measurement errors can be assumed to follow the operation.

A Two-Qubit Code: $\mathcal{X}_2$

- $\mathcal{X}_2$'s logical states:

  $|\mathtt{0}\rangle_{\mathsf{L}(1,2)} = \frac{1}{\sqrt{2}}\left(|\mathtt{00}\rangle_{12} + |\mathtt{11}\rangle_{12}\right), \quad |\mathtt{1}\rangle_{\mathsf{L}(1,2)} = \frac{1}{\sqrt{2}}\left(|\mathtt{01}\rangle_{12} + |\mathtt{10}\rangle_{12}\right)$
- Effects on states, by example:

$$\begin{matrix}\alpha|\mathtt{0}\rangle_{\mathsf{L}} \\ + \\ \beta|\mathtt{1}\rangle_{\mathsf{L}}\end{matrix} = \left.\begin{matrix}\frac{\alpha}{\sqrt{2}}\left(|\mathtt{00}\rangle + |\mathtt{11}\rangle\right) \\ + \\ \frac{\beta}{\sqrt{2}}\left(|\mathtt{01}\rangle + |\mathtt{10}\rangle\right)\end{matrix}\right\} \xrightarrow{P_{\mathtt{0}*}} \left\{\begin{matrix}\alpha|\mathtt{00}\rangle \\ + \\ \beta|\mathtt{01}\rangle\end{matrix}\right\} \xrightarrow{U_{\mathtt{0}*}} \left\{\begin{matrix}\alpha|\mathtt{0}\rangle_{\mathsf{L}} \\ + \\ \beta|\mathtt{1}\rangle_{\mathsf{L}}\end{matrix}\right.$$

$$\xrightarrow{P_{*\mathtt{1}}} \left\{\begin{matrix}\alpha|\mathtt{11}\rangle \\ + \\ \beta|\mathtt{01}\rangle\end{matrix}\right\} \xrightarrow{U_{*\mathtt{1}}} \left\{\begin{matrix}\alpha|\mathtt{0}\rangle_{\mathsf{L}} \\ + \\ \beta|\mathtt{1}\rangle_{\mathsf{L}}\end{matrix}\right.$$

- Z-measurement errors can be corrected.

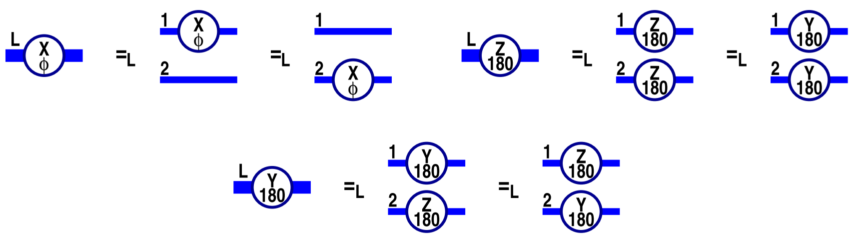
Forward to: $\mathcal{X}_2$ threshold

Encoded Operations for $\mathcal{X}_2$

- $\mathcal{X}_2$: $|0\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big), |1\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)$
- The following hold when restricted to $\mathcal{X}_2$:

$$\begin{array}{lllll} (I)_{\mathsf{L}} & =_{\mathsf{L}} II & =_{\mathsf{L}} XX & (Z)_{\mathsf{L}} =_{\mathsf{L}} ZZ & =_{\mathsf{L}} -YY \\ (X)_{\mathsf{L}} & =_{\mathsf{L}} XI & =_{\mathsf{L}} IX & (Y)_{\mathsf{L}} =_{\mathsf{L}} YZ & =_{\mathsf{L}} ZY \end{array}$$

- Error-free encoded rotations:



Forward to: $\mathcal{X}_2$'s recovery

Encoded Operations for $\mathcal{X}_2$

- $\mathcal{X}_2$: $|0\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big), |1\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)$
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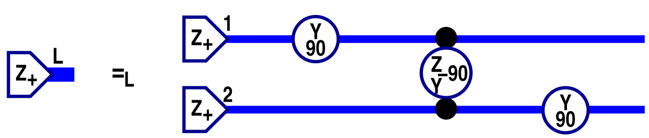
- Encoded state preparation, error-free if successful:



Forward to: $\mathcal{X}_2$'s recovery

Encoded Operations for $\mathcal{X}_2$

- $\mathcal{X}_2$: $|0\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big), |1\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)$
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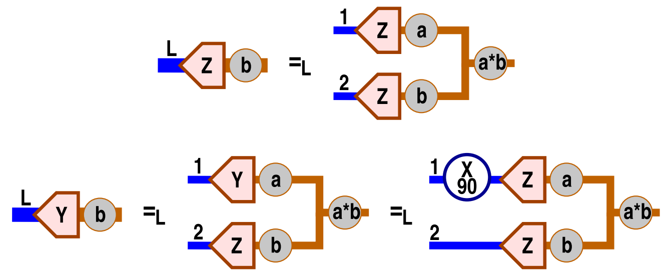
- Encoded measurements, error free:



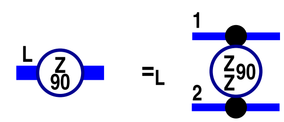
Forward to: $\mathcal{X}_2$'s recovery

Encoded Operations for $\mathcal{X}_2$

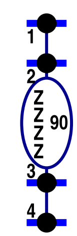
- $\mathcal{X}_2$: $|0\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big), |1\rangle_{\mathsf{L}} = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)$
- The following hold when restricted to $\mathcal{X}_2$:

$$\begin{array}{lllll} (I)_{\mathsf{L}} & =_{\mathsf{L}} II & =_{\mathsf{L}} XX & (Z)_{\mathsf{L}} =_{\mathsf{L}} ZZ & =_{\mathsf{L}} -YY \\ (X)_{\mathsf{L}} & =_{\mathsf{L}} XI & =_{\mathsf{L}} IX & (Y)_{\mathsf{L}} =_{\mathsf{L}} YZ & =_{\mathsf{L}} ZY \end{array}$$

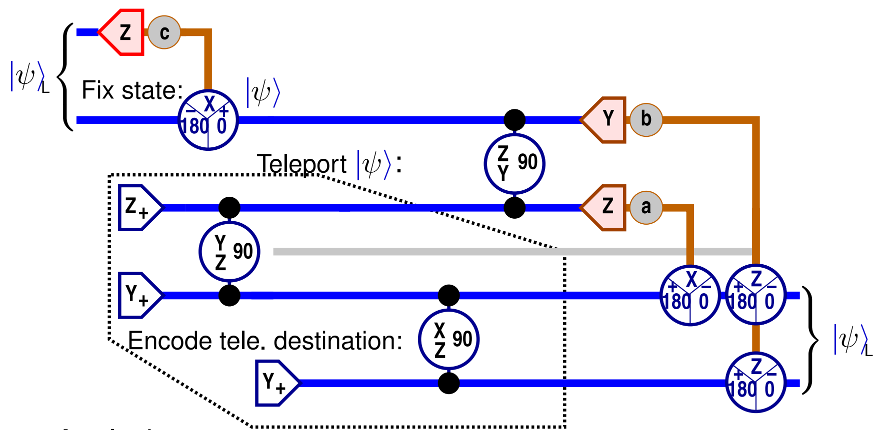
- Other encoded operations:
  - Logical Z_{90°:



    - Measurement errors require recovery attempts.
  - Logical $(ZZ)_{90^\circ}$: By teleportation. In principle:



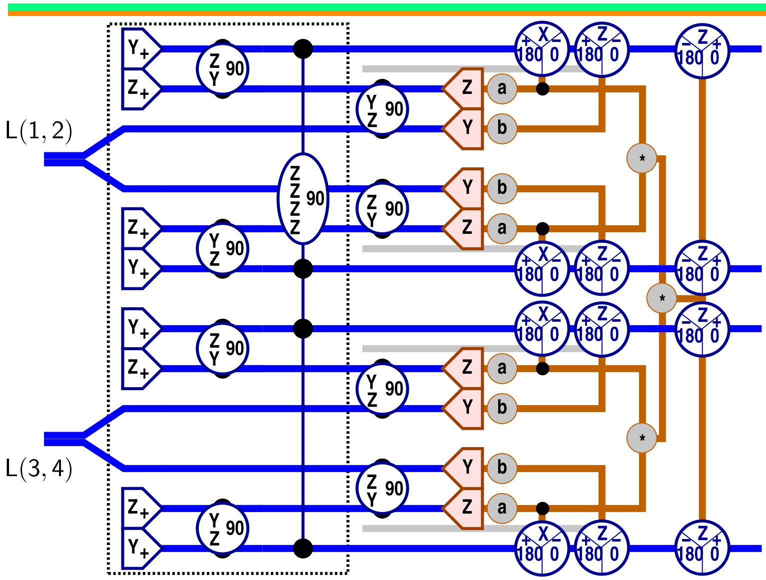
Recovery from Z-measurement Error



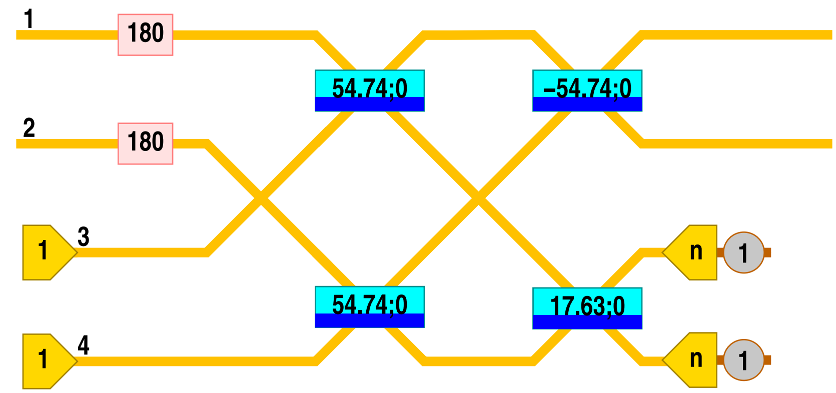
- Analysis:
  - Failure: Probability f, by $(Z)_L$ measurement.

Forward to: $\mathcal{X}_2$ threshold

Encoded $(ZZ)_{90°}$



Zooming in on eLOQC



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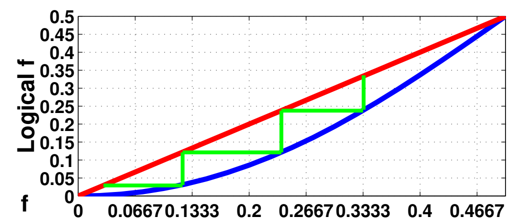
Encoded Failure Analysis

- For $(ZZ)_{90°}$, independently on each encoded qubit:
  - prob.$=f^2$: Encoded Z-measurement error.
  - prob.$=(1-f)^2$: Success.
  - prob.$=2(1-f)f$: One Z-measurement error.
    - prob.$=(1-f)$: Re-encoding succeeds.
    - prob.$=f$: Encoded Z-measurement error.
- Failure probability/encoded qubit: $f^2+2f^2(1-f)=f^2(3-2f)$.



- Z-measurement accuracy threshold $\geq .5$.

Go to: It is $=1$.
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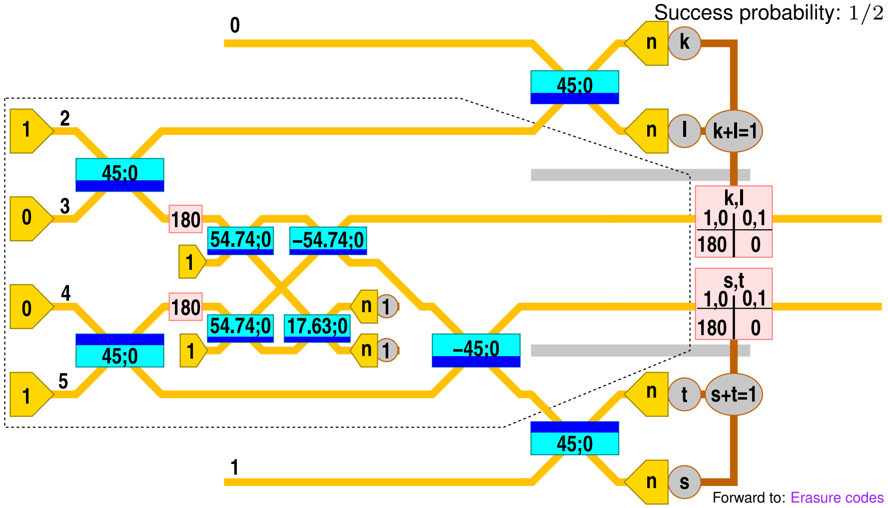
Detecting Qubit Loss

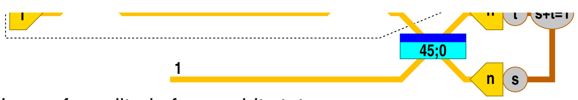
Robust teleport (rTel$_1$) with LOQC state preparation:



Forward to: Erasure codes

rTel for Loss and Leakage[a]

- Behavior for photon loss: • Includes detector inefficiency
  Every loss error is:
  - Detected, or
  - Loss in output induced by loss in preparation.
- Behavior for general leakage:
  Any input state, only photon loss in prep:
  - Output state has at most 1 photon.
- For teleportation in general:
  - Leakage in output requires state prep. error.
  - $\longrightarrow$ Leakage errors cannot accumulate.



[a] Leakage: Loss of amplitude from qubit state space.

Erasure Codes and Thresholds

- With rTel: Photon loss $=$ detected erasure error.
- Thresholds for erasure errors are $> 1\%$.

Knill&Laflamme&Milburn 2000 [19]

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Priority Based Error Control

- Errors and some methods for their control in eLOQC:

1. Z-measurement errors (failed postselected gates).
   - Use Z-detecting stabilizer codes. See eLOQC IV, V.

1.a. Phase noise.
   - Use Z-detecting codes based on good classical codes and syndrome based recovery methods.

1.b. Photon loss, detector inefficiency.
   - Use loss detection and erasure codes. See eLOQC VI, VII.

2.a. Mode matching and beam-splitter angle errors.
   - Use better calibration of the classical devices.

2.b. Detector dark counts.
   - Reduce them by engineering.

3. Remaining residual errors.
   - Use generic quantum fault-tolerance methods.

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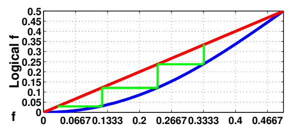
Preliminary resource analysis

- Example, assuming ideal LOQC gates.
  Design:
  - Goal: $< 5\%$ Z-measurement probability.
  - Use Tel_3 so that $f_0 = .25$.
  - Two levels of concatenation: $f_2 = 0.038$.



  Analysis:
  - Conditional resources: Knill&Laflamme&Milburn 2000 [19]
    Encoded $(ZZ)_{90^\circ}$ depends on < 240 $\mathrm{CS}_{9/16}$ gates.
  - Average total resources?
- Improvements possible:
  - Use better Z-measurement codes with Tel_1.
  - Block coding.
  - Improve state preparation networks.

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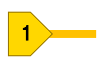
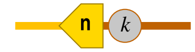
Technological challenges I



$|1\rangle$

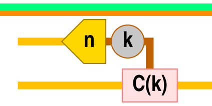
- Single photon sources:
  - Parametric down conversion conditional on detection of second photon (room temperature). Hong&Mandel 1986[7]
  - Acousto-electric (SAW devices, cryogenic).
    Kim&*al* 1999[20], Foden&*al* 2000[21]
  - Cycling single atoms or molecules (room temperature).
    Louis&Moerner 2000[22]
- SAW devices may have error rates $< 10^{-4}$.
  Janssen&Hartland 2000 [23]

Technological challenges II

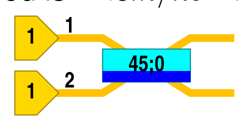


- Photodetectors:
  - Routine efficiencies $> 70\%$ (EG&G, silicon photodiodes).
  - Research level efficiency $\approx 90\%$ (with counting).
    Estimated internal efficiency $94.7 \pm 5\%$ (cryogenic operation). Takeuchi&Yamamoto&Hogue 1999[24]
  - Up to 97% with trap arrangements and silicon photodiodes? Hariharan 1999[25]
  - Efficiencies above 99.9% in "absolute radiometric measurements"? Zalewski&Duda 1983[26]
- Photon counters:
  - In principle, with beamsplitters and photodetectors.
  - Or use "avalanche multiplication effect" for visible light photon counters. Takeuchi&Yamamoto&Hogue 1999[24]

Technological challenges III



- Feedback.
  - Required to select accepted states.
  - Control mechanisms available?
  - Note: Photon speed is $\approx .3m/ns$—need for delay lines?



$|02\rangle_{12} - |20\rangle_{12}$

- Mode matching for independent photons.
  - Achievable visibilities?
  - Advantage of single (transversal) mode wave guides.
  - Lock single photon pulses with reference frequency.
  - Emission time jitter?

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