Predictive Quantum Learning

Dmitry Gavinsky

NEC Labs, Princeton

Learning under Standard Setting

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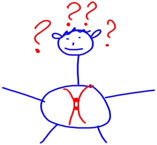

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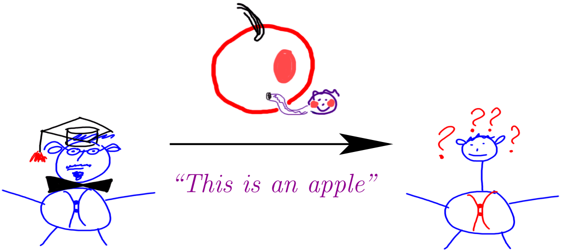

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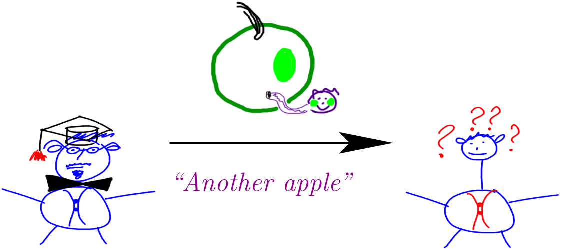

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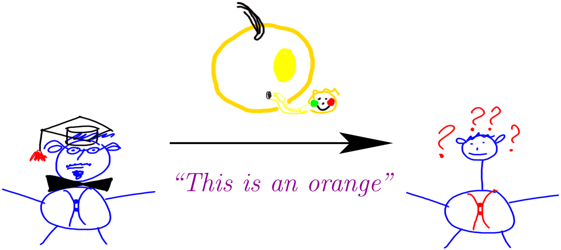

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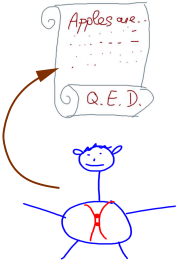

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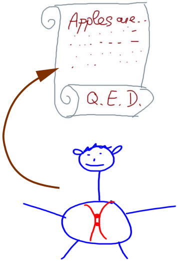

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▶ This model is called *Probably Approximately Correct (PAC)*, it has been introduced by Valiant [V84].

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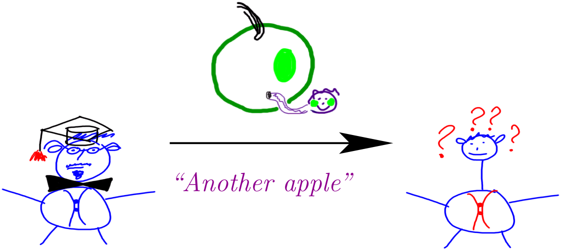

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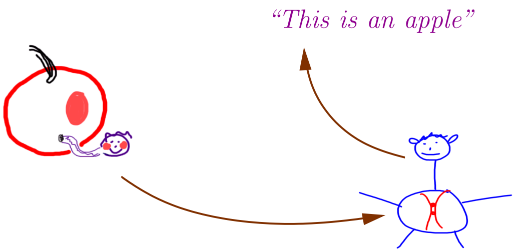

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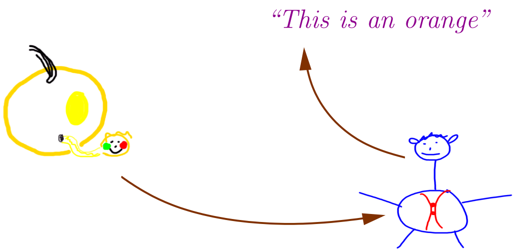

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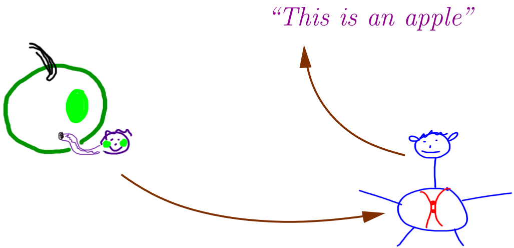

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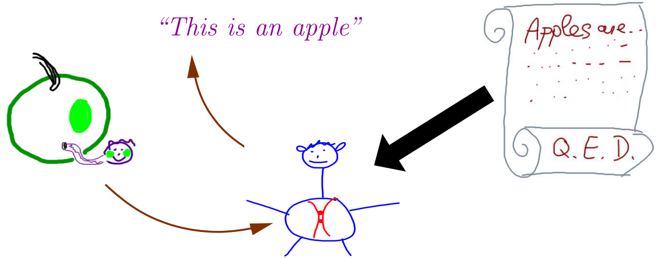

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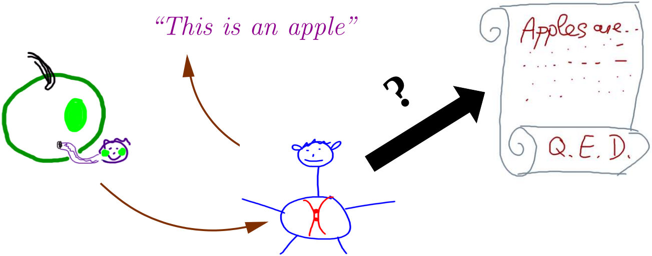
▶ Clearly, *standard learnability implies predictive learnability* (a hypothesis can be used as a distinguishing algorithm).

Does Predictive Learnability Equal Standard Learnability?



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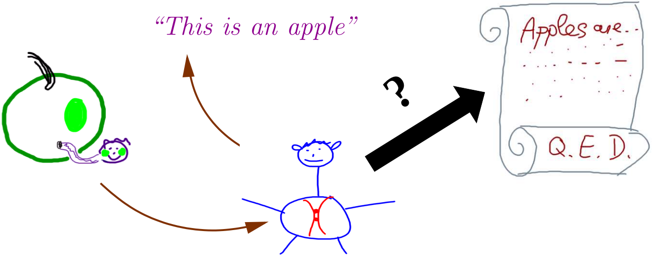

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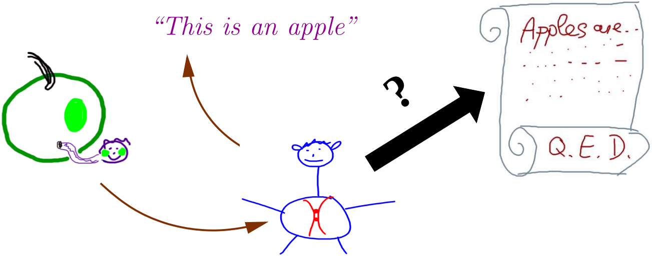

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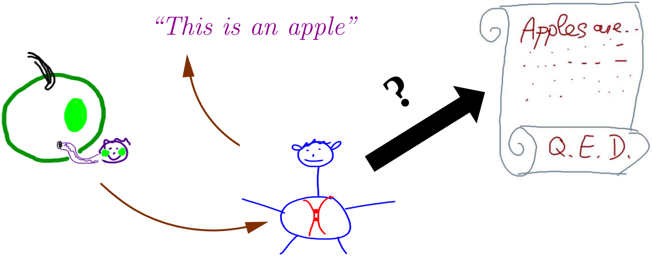

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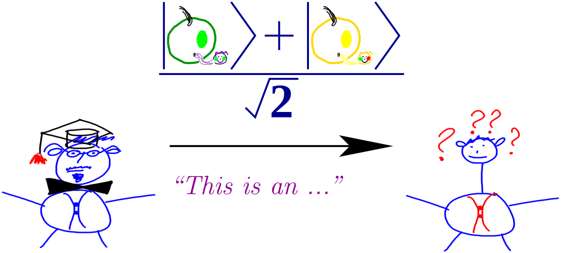
- The argument *does not translate to the case of quantum models*.
- We will see that, indeed, *quantum predictive learning is qualitatively stronger than quantum standard learning*.
- ▶ Observe that *unconditional separation between quantum and classical learning* immediately follows (we will make a more formal statement later).

Quantum Learning



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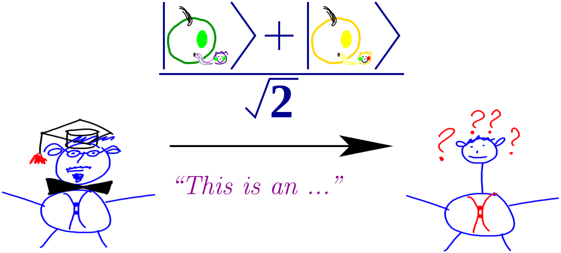

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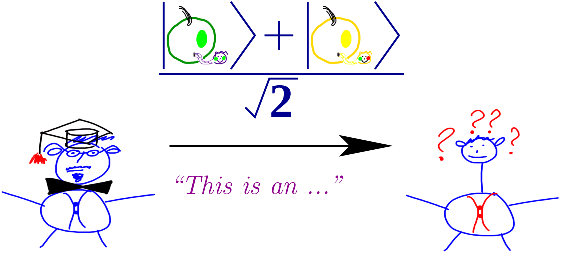

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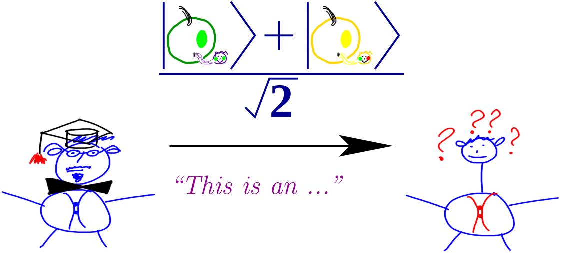

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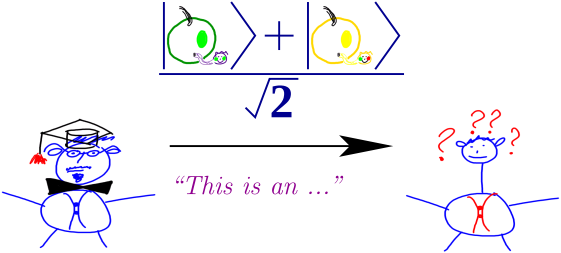

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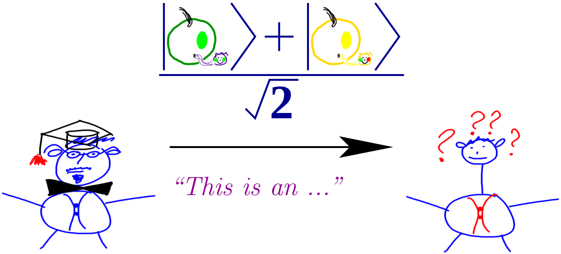
- Are quantum models stronger than classical?

Earlier Work



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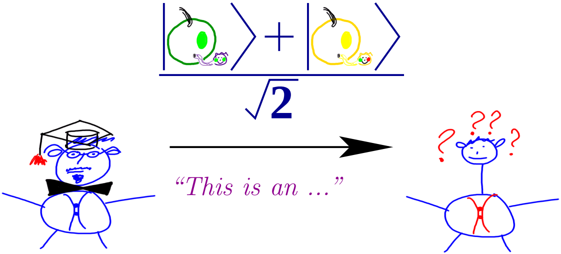

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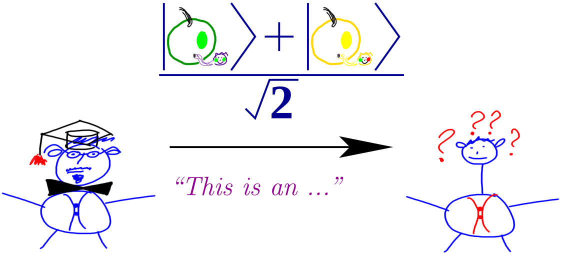

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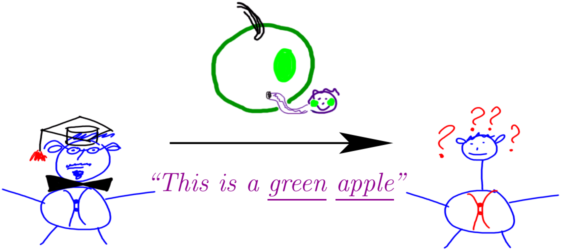

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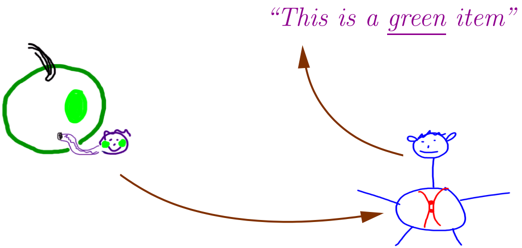

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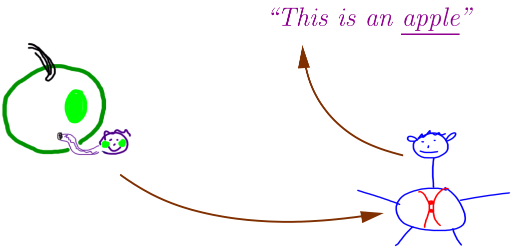

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► *$\tilde{\mathcal{C}}$ approximates $\mathcal{C}$* if $\underline{\forall c \in \mathcal{C}\ \exists \tilde{c} \in \tilde{\mathcal{C}} \text{ s.t. } \tilde{c} \approx c}$.

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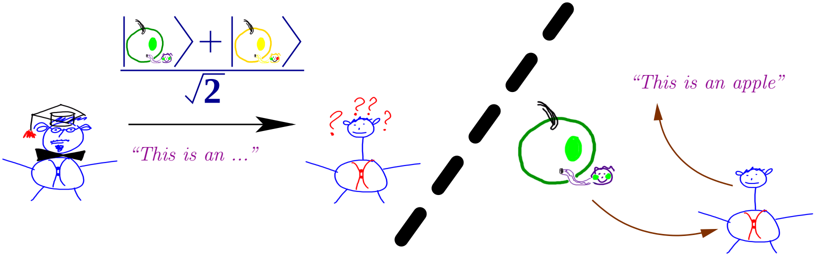
  - *unspeakable classes of functions are not learnable efficiently (either quantumly or classically)*.

► Therefore, *willing to learn unspeakable concepts*, we can only hope to do so for a *relational* class, in a *quantum predictive* model.

Our Results



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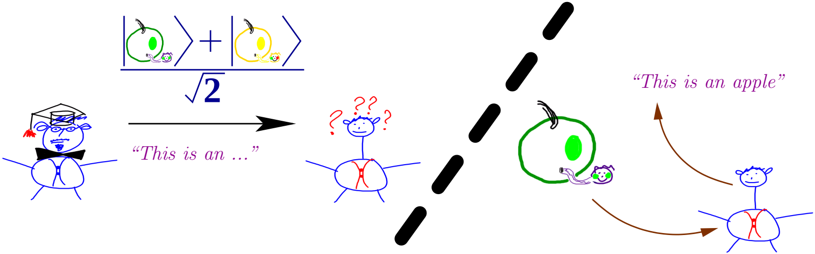

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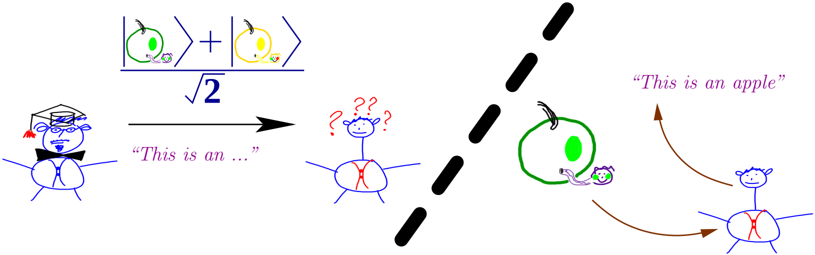

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- This construction has been inspired by a communication problem defined in [KW03] and [BJK04].

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- Given a test question $q_0 \in [N-1]$, the student performs projective measurement of $|\alpha_{x_0}\rangle$ onto $(N-1)/2$ subspaces, each spanned by a pair of vectors from

$$\left\{\left(|x_0+(2i+1)q_0\rangle, |x_0+(2i+2)q_0\rangle\right) \;\middle|\; 0\le i\le \frac{N-3}{2}\right\}.$$

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The student is ready for the testing phase.

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Efficient PQ-Learning of $\mathcal{C}$

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▶ The student responds with $(a, c_a\oplus c_{a+q_0})$, as required.

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► Therefore, *$\mathcal{C}$ is unspeakable*.

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► *Assuming $BQP \not\subseteq P/poly$, the answer is trivial* (let $\mathcal{C} \stackrel{\text{def}}{=} \{f_L\}$, for any $L \in BQP \setminus P/poly$) – the goal is to *weaken the assumptions*.


Thank you!