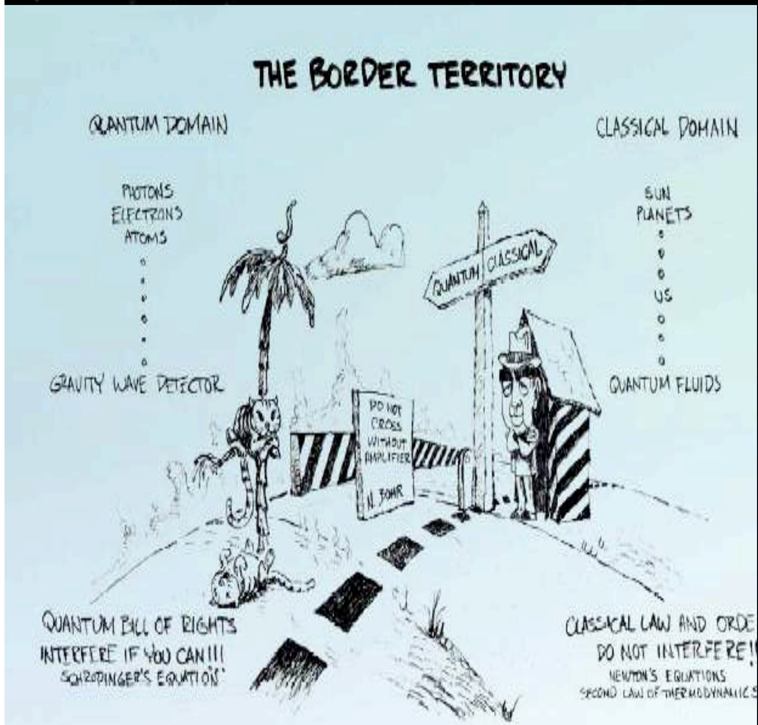
Non-Isolated Qubits:

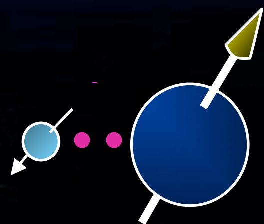
Decoherence, Entanglement Entropy and the Landau-Zener-Stückelberg-Majorana riddle

Karyn Le Hur

Yale & KITP



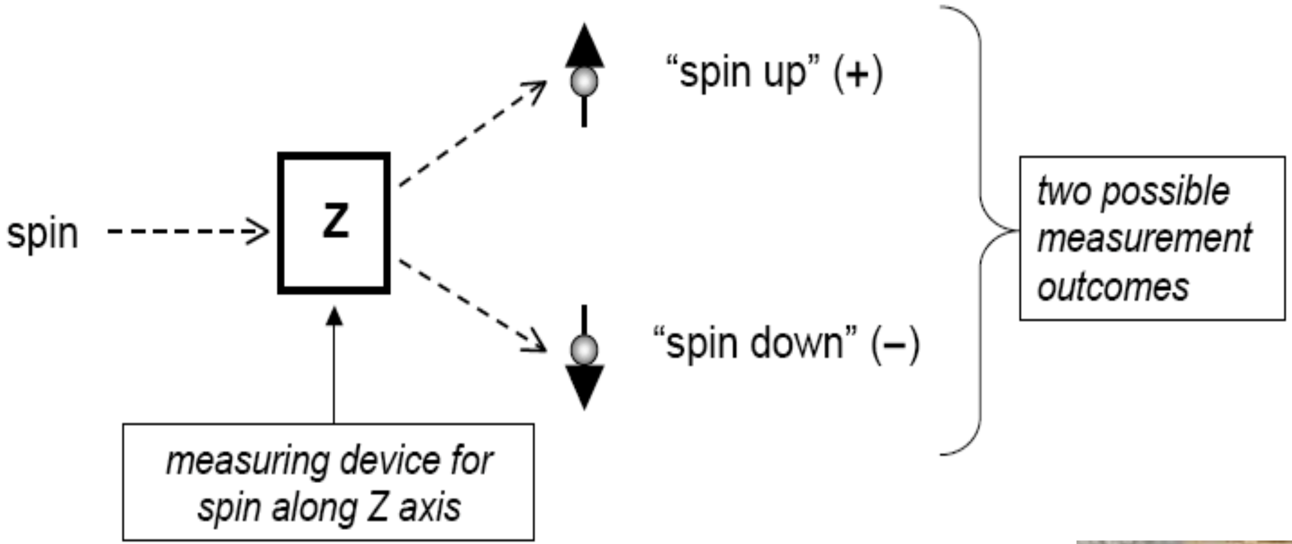
Yale Center for Quantum Information



Doucet-Beaupré Ph.
Hofstetter W.
Imambekov A.
Kopp A.
Orth P.
Roosen D.
Stanic I.

Environment?

Copenhagen interpretation: wave-function collapse
system is quantum and measuring apparatus is <u>classical</u>



Is there a quantum environment?

Sample two-state systems

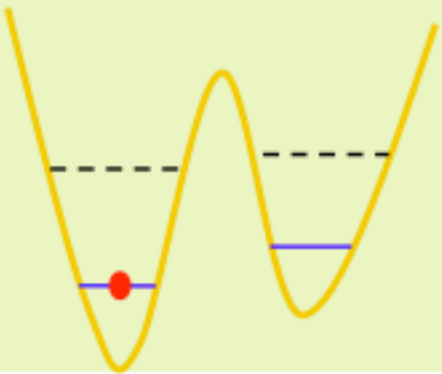
- Intrinsic two-state

Nuclei spin S=1/2   Polarization of photon (electromagnetic cavity)

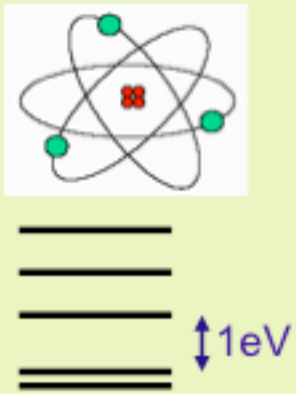
- Truncated two-state

Less than 1K

Particle in double well

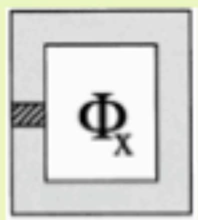


Atom

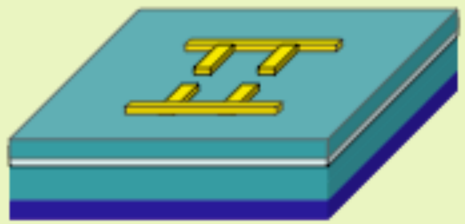


Condensed matter systems

- rf SQUID



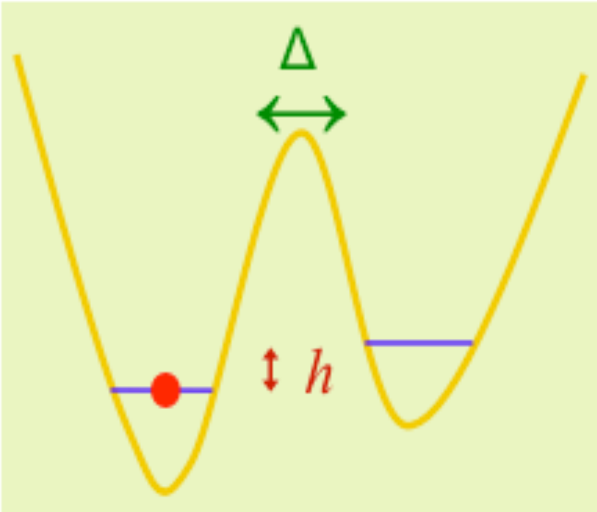
- Quantum dot



Also ultracold matter (μK~20.8kHz)

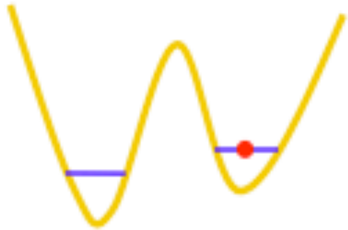
Environment = Many Particles or Fields around?

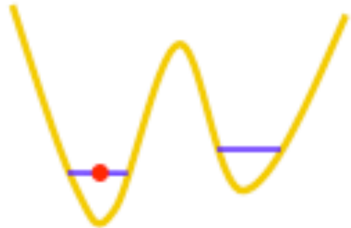
Artificial spin S=1/2



$$H_S = hS_z + \Delta(S_+ + S_-)$$

Spin S=1/2 description

$|\uparrow\rangle =$ 

$|\downarrow\rangle =$ 

particle localized ↔ spin polarized

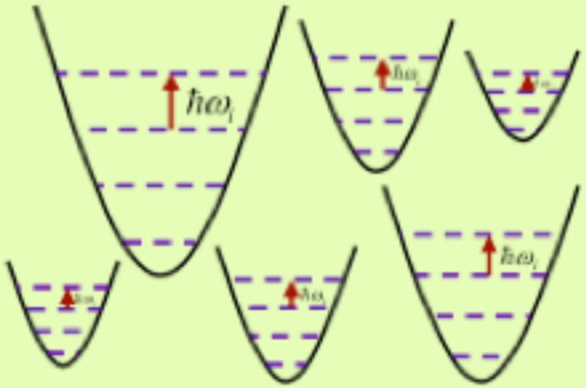
What is the effect of environment?

Understand better the quantum mechanics of non-isolated qubits

`Standard' Model of Environment

- Model the environment by quantum harmonic oscillators

$$H_B = \sum_i \left(\frac{p_i^2}{2m_i} + \frac{m_i \omega_i^2 x_i^2}{2} \right)$$



Bosonic bath

$$H_{CL} = hS_z + \Delta(S_+ + S_-) + S_z \sum_i \lambda_i x_i + H_B$$

A.J. Leggett, S. Chakravarty, A. T. Dorsey, M.P.A. Fisher, A. Garg, and W. Zwerger Rev. Mod. Phys. **59**, 1 (1987)

One important quantity is the frequency dependence of the associated coupling spectrum

$$\boxed{J(\omega) \sim \sum_i |\lambda_i|^2/(m_i\omega_i)\ \delta(\omega-\omega_i)}$$

$$J(\omega) = \alpha\omega$$

Transmission line, BEC, phonons, scalar field,…

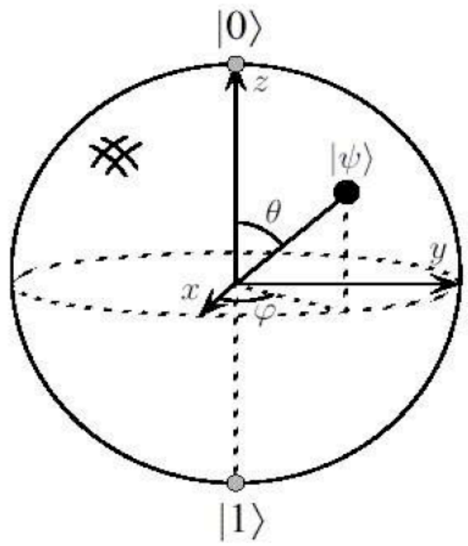
$\Delta/\omega_c \sim 0$

Separable wavefunction when $\alpha = 0$

$$|\Psi_G\rangle = |\Psi_{A=spin}\rangle \otimes |\Psi_{B=bath}\rangle$$

$$\langle S_z \rangle = -\frac{h}{\sqrt{\Delta^2 + h^2}} \qquad \langle S_x \rangle = -\frac{\Delta}{\sqrt{\Delta^2 + h^2}}$$

$$\boxed{\langle S_z \rangle^2 + \langle S_x \rangle^2 = 1}$$



Bloch sphere

Qubit lies in a pure state

Introducing the bath produces **decoherence**, quantum phase transition, entanglement entropy, `strange' Rabi oscillations, and `revisited' Landau-Zener-Stueckelberg-Majorana problem…

Many-Body Problem…

$$H\psi_G = E_G\psi_G$$

Many-body wave-function…

Trial wavefunction: BCS wavefunction for superconductivity, Jastrow wavefunctions for spin chains, quantum Hall systems,…

Sometimes Luck: **Exact solutions**

(Bethe Anstatz,mappings)

Tricky numerics

(Numerical RG)

$$\langle \Psi_G | S_x | \Psi_G \rangle = \frac{\partial E_G}{\partial \Delta}$$

$$\langle \Psi_G | S_z | \Psi_G \rangle = \frac{\partial E_G}{\partial h}$$

Spin Reduced Density Matrix

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1+\langle S_z\rangle & \langle S_x\rangle + i\cancel{\langle S_y\rangle} \\ \langle S_x\rangle - i\cancel{\langle S_y\rangle} & 1-\langle S_z\rangle \end{pmatrix}$$

Landau, 1927

$$\rho^A_{\uparrow\uparrow} = |<\Psi_G|\uparrow>|^2 = \frac{1}{2}(1+\langle\Psi_G|S_z|\Psi_G\rangle)$$

$$\rho^A_{\uparrow\downarrow} = \langle\Psi_G|\uparrow\rangle\langle\downarrow|\Psi_G\rangle$$

$|0\rangle \longleftrightarrow \begin{pmatrix}1\\0\end{pmatrix}$, $|1\rangle \longleftrightarrow \begin{pmatrix}0\\1\end{pmatrix}$

$|\uparrow\rangle \equiv |0\rangle$, $|\downarrow\rangle \equiv |1\rangle$

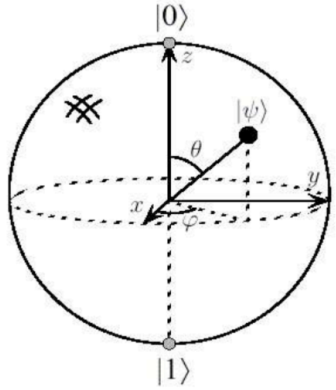
$$\rho = |\Psi_G\rangle\langle\Psi_G|$$

$$\rho_A = \mathrm{Tr}_B|\Psi_G\rangle\langle\Psi_G|$$

Measurement on Spin (A) ONLY

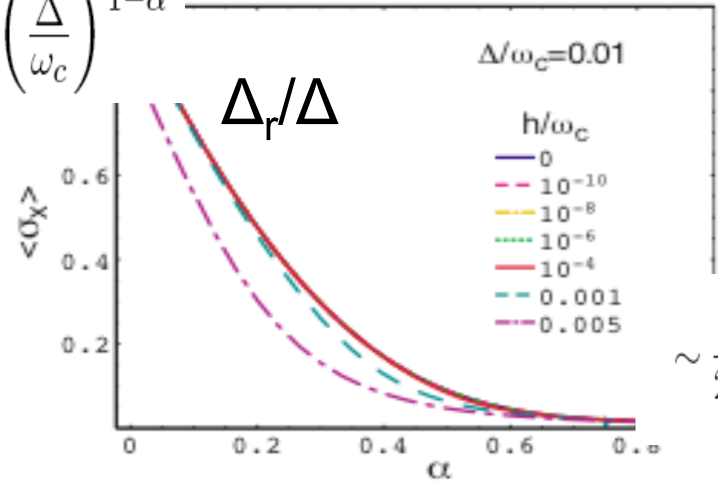
Decoherence of the Qubit…

Karyn Le Hur, arXiv: 0711.2301 (Annals of Physics, Survey 2008)
*A. Kopp and K. Le Hur, PRL **98**, 220401 (2007)*
K. Le Hur, Ph. Doucet-Beaupre, and W. Hofstetter, PRL 2008



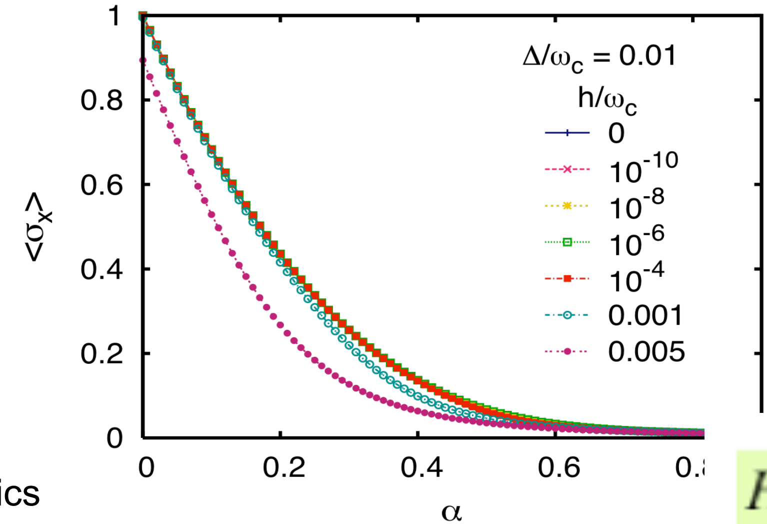
$$\Delta_r = \Delta \left(\frac{\Delta}{\omega_c}\right)^{\frac{\alpha}{1-\alpha}}$$

Exact solution



$$\sim \frac{1}{2\alpha - 1}\frac{\Delta}{\omega_c}$$

Tricky numerics



Dephasing

$$S^+ \rightarrow S^+ e^{i\Omega}$$

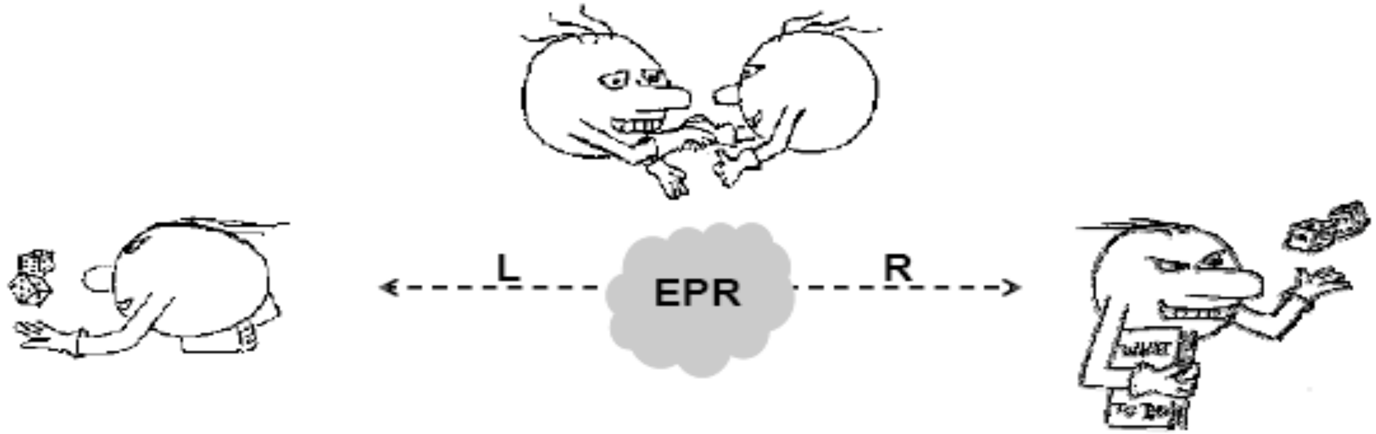
$$\Omega = \sum_\alpha \frac{\lambda_\alpha}{\hbar m_\alpha \omega_\alpha^2} p_\alpha$$

For all couplings:

$$\langle \Psi_G | S_x | \Psi_G \rangle = \frac{\partial E_G}{\partial \Delta}$$

$$H_{CL} = hS_z + \Delta(S_+ + S_-) + S_z \sum_i \lambda_i x_i + H_B$$

What is Entanglement?
Spooky action at Distance (Einstein)



Simple example: 2 Qbits forming a singlet

$$|\Psi_S\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle)$$

Wave function is NOT factorizable into individual wave functions…
Quantum states of 2 (or more) particles are linked together

$$|\Psi_G\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\chi_\uparrow\rangle \pm |\downarrow\rangle|\chi_\downarrow\rangle)$$

Entropy as a Measure of Entanglement:

A = Spin and B = environment

Measuring spin observables

Get reduced density matrix for A (*Landau, 1927*)

$$\mathbf{E = -} \mathrm{Tr}(\rho_A \log \rho_A)$$

purity

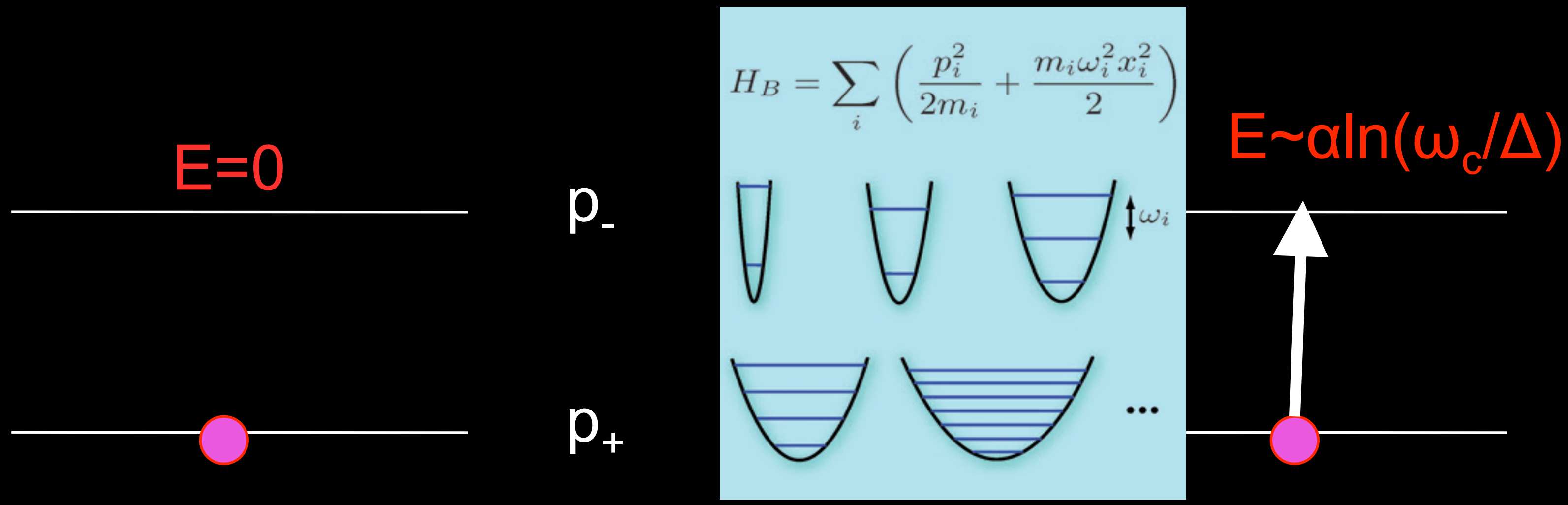
$$\mathrm{Tr}\rho_A^2$$

Resulting entropy may be non-zero if correlations between A & B

$$E = - \mathrm{Tr}\, \rho_A \log_2 \rho_A = - p_+ \log_2 p_+ - p_- \log_2 p_-$$

$$p_\pm = \left(1 \pm (\langle S_x \rangle^2 + \langle S_z \rangle^2)^{1/2}\right)/2$$

Entanglement & Mixed State for spin



$\alpha = 0$ Small $\alpha \neq 0$

Spin is in a pure state

$0 \leq E \leq 1$

$$p_\pm = \left(1 \pm (\langle S_x\rangle^2 + \langle S_z\rangle^2)^{1/2}\right)/2$$

$$E = -p_+ \log_2 p_+ - p_- \log_2 p_-$$

$\alpha = 1/2$ and $h = 0$, $E \sim 1$

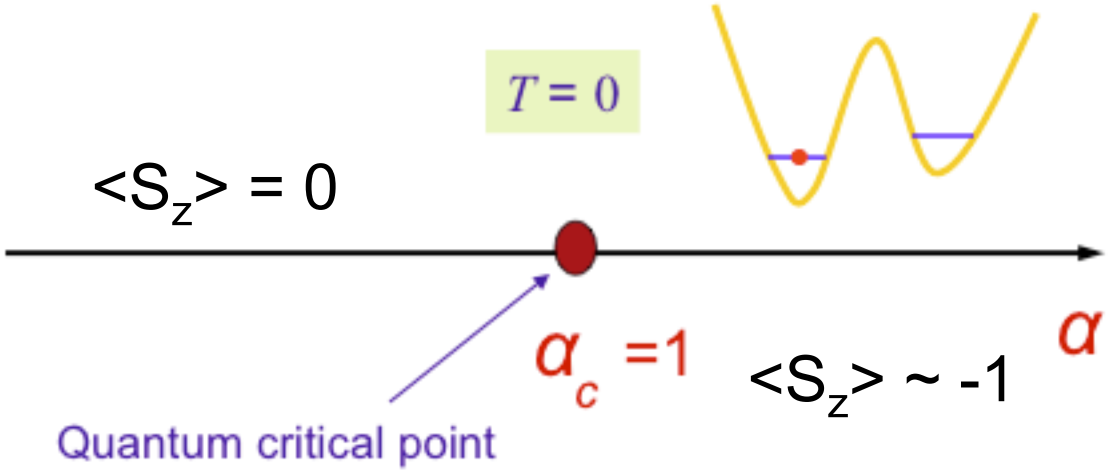
Quantum phase transition in CL model

$$\frac{1}{2}\left\langle \sum_i \lambda_i x_i(t) \cdot \sum_i \lambda_i x_i(0) \right\rangle_\omega = \hbar J(\omega) \coth(\omega / 2k_B T)$$

Ohmic dissipation

$$J(\omega) = \alpha\,\omega$$

Dissipation strength



Limit $h=0^+$

- ❑ Classical phase transition tuned by temperature

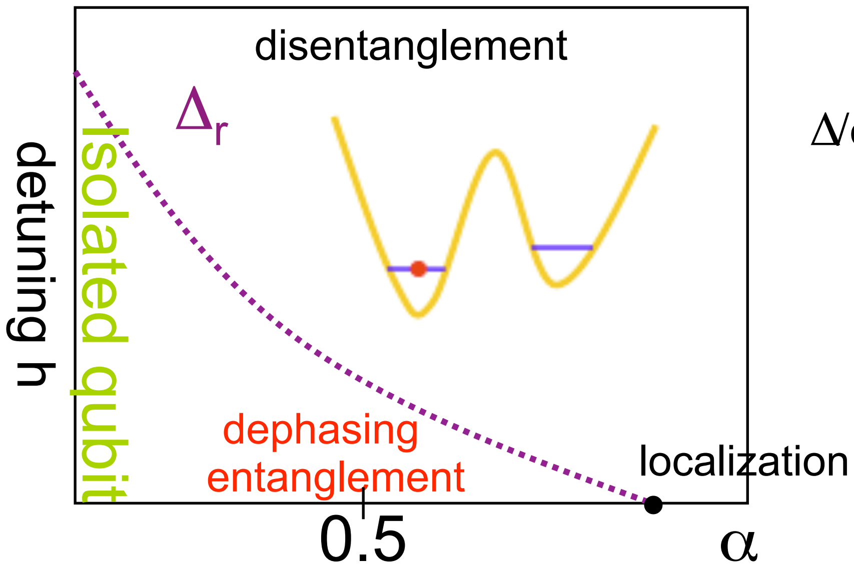
  liquid-solid
  liquid-gas
  …

- ❑ Quantum phase transition at $T = 0$ tuned by intrinsic parameters
  - *Metal-insulating* in 2DEG
  - *Insulating-superconducting* in high-temperature superconducting cuprates
  - *Spin ordered - disordered* in quantum spin models

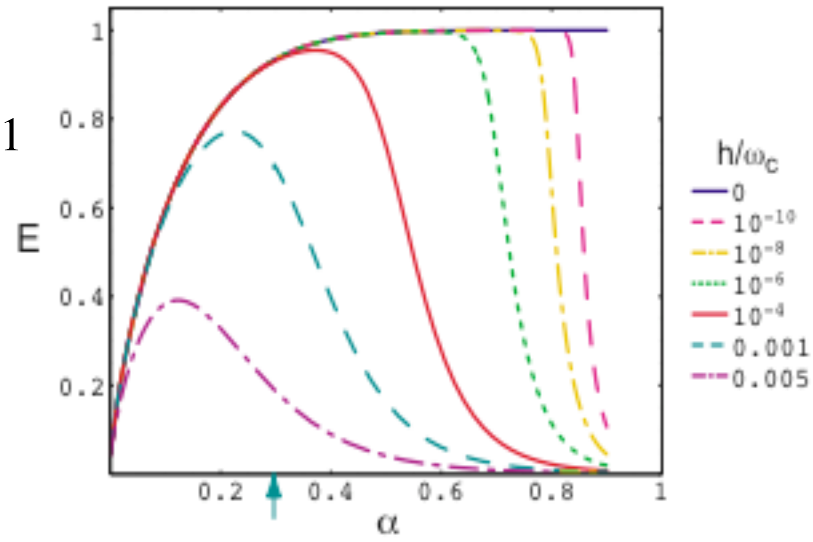
Ising spin chains: Dyson, Thouless, Fisher
Kondo model: Anderson, Hamman, Yuval

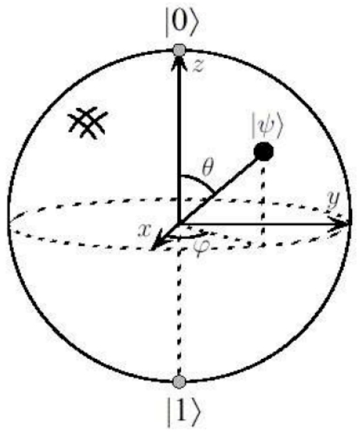
Quantum Entanglement

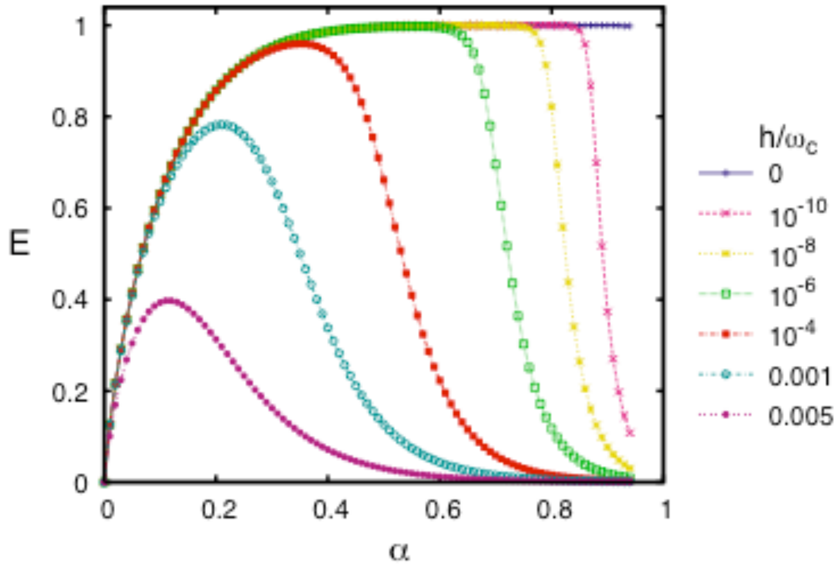
$$\Delta_r = \Delta(\Delta/\omega_c)^{\alpha/1-\alpha}$$



$\Delta/\omega_c = 0.01$







For $\alpha<<1/2$, geometric phase is robust

What we have learnt so far

Decoherence: Maximal Entanglement with the environment

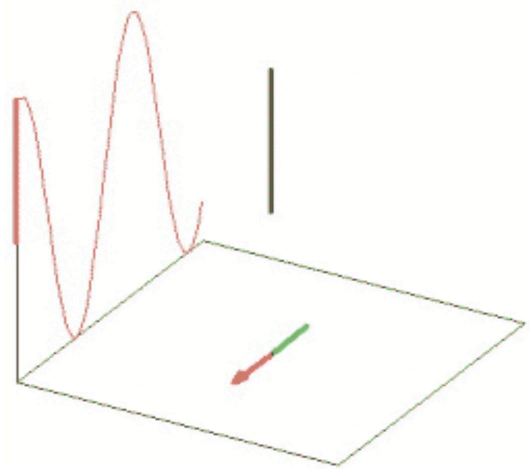
Leads to the destruction of quantum superposition

Also the reason why quantum computers become classical…

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1 & \sim \Delta/\omega_c \to 0 \\ \sim \Delta/\omega_c \to 0 & 1 \end{pmatrix}$$

Quantum to Classical

Rabi oscillations



The red segment visualizes the longitudinal component of the Bloch vector (<$\mathbf{S_z}$>)

Apply a magnetic field in the plane (red/green arrow): Rabi oscillations…

Nobel price, 1944

Many-Body Generalization

$$H_{CL} = \cancel{hS_z} + \Delta(S_+ + S_-) + S_z + H_B$$

A weak-coupling Expansion:
Feynman-Vernon Path Integral Approach (NIBA)…

$$\boxed{\langle \dot{S}_z \rangle + \int_{-\infty}^{t} F(t-t')\langle S_z(t')\rangle dt' = 0}$$

F(t) decays as a power law at T=0: Non-Markovian evolution

- $\alpha=0$ $\quad$ $<S_z(t)> = \cos(\Delta t)$
- $0<\alpha<1/2$ $\quad$ $<S_z(t)> = \cos(\Omega t)\exp-(\Upsilon t)$
- $\alpha=1/2$ $\quad$ $<S_z(t)> = \exp(-\Upsilon t)$

Quality Factor $\Omega/\gamma = \cot[\pi\alpha/2(1-\alpha)]$

A. Leggett et al. Rev. Mod. Phys. **59**, 1 (1987)
U. Weiss, Quantum Dissipative Systems, 2nd edition

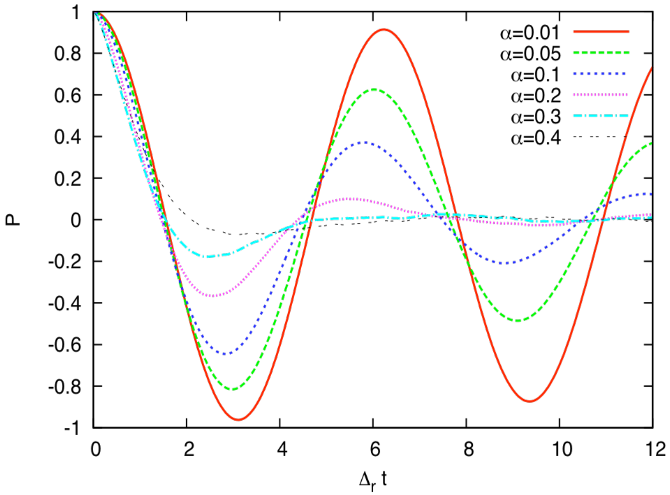
2009: New analytical Approach & tricky numerics

P. Orth, A. Imambekov, K. Le Hur, in preparation

D. Roosen, K. Le Hur, W. Hofstetter, in preparation

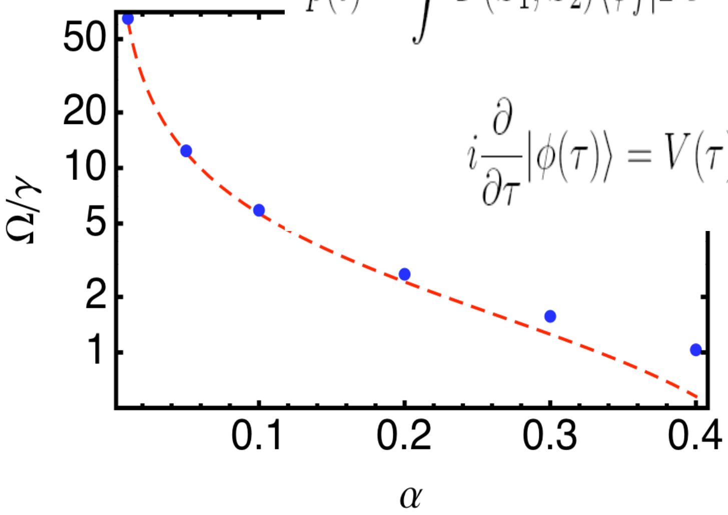
Gaussian distributed Random Noise Approach

$p = \langle \sigma_z(t) \rangle$

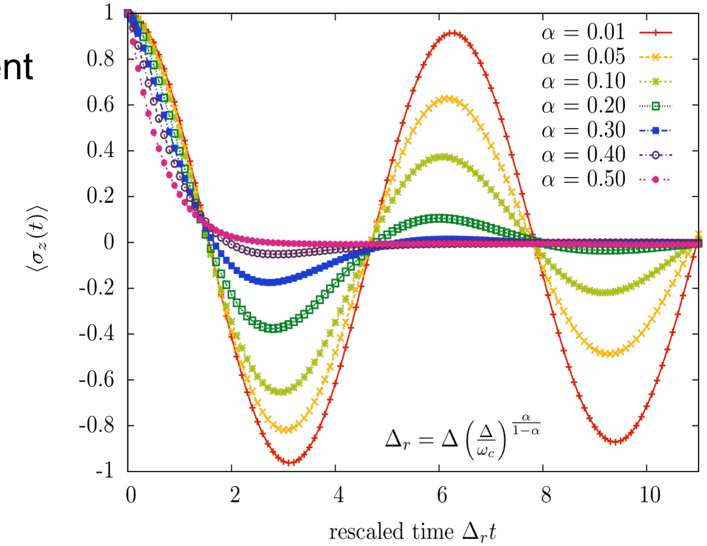


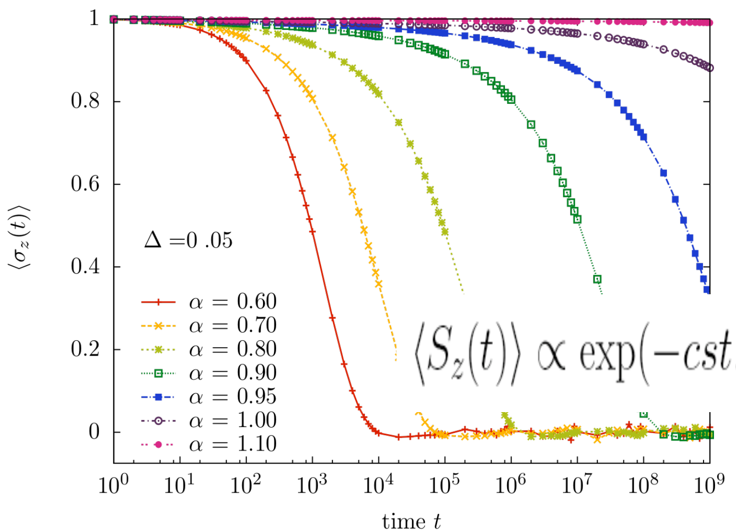
$$p(t) = \int \mathcal{D}(S_1, S_2) \langle \phi_f | T e^{-i \int_0^t dt' V(t') dt'} | \phi_i \rangle$$

$$i \frac{\partial}{\partial \tau} |\phi(\tau)\rangle = V(\tau) |\phi(\tau)\rangle$$



Time-dependent NRG





$$\langle S_z(t) \rangle \propto \exp(-cst.(t\Delta_r)^{a_\alpha})$$

A few details on the “Random Noise Approach”

Starting Point: Feynman-Vernon Path Integral Approach

$$\rho_S(x_f, x_f; t) = \int \mathcal{D}x(\tau) \int \mathcal{D}y(\tau') A[x(\tau)] A[y(\tau')] F[x(\tau), y(\tau')]$$

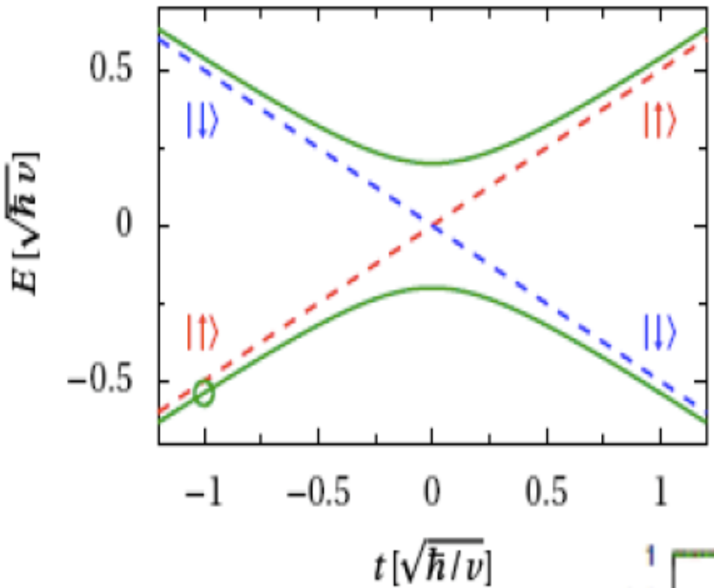
F[ξ(τ),χ(τ’)]: influence functional for the bath

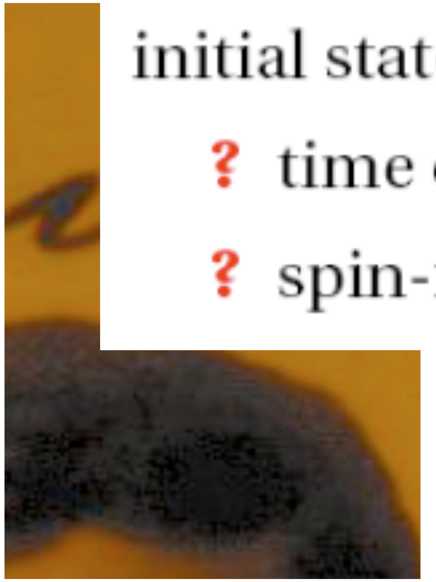
$$F[\xi(\tau), \chi(\tau')] = \exp\left[-\frac{q_0}{\pi\hbar} \int_{t_0}^{t} d\tau \int_{t_0}^{\tau} ds \left\{ -iL_1(\tau - s)\xi(\tau)\chi(s) + L_2(\tau - s)\xi(\tau)\xi(s) \right\} \right]$$

$$p(t) = \int \mathcal{D}(S_1, S_2) \langle \phi_f | T e^{-i \int_0^t dt' V(t') dt'} | \phi_i \rangle$$

$$i \frac{\partial}{\partial \tau} |\phi(\tau)\rangle = V(\tau) |\phi(\tau)\rangle$$

for N=5000 realizations of the Gaussian distributed variables…



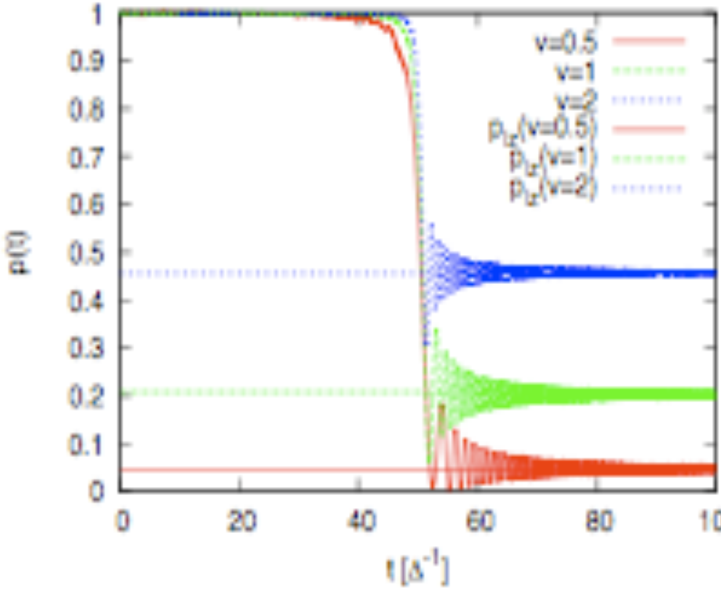


initial state: $|\psi(t=-\infty)\rangle = |\uparrow\rangle$

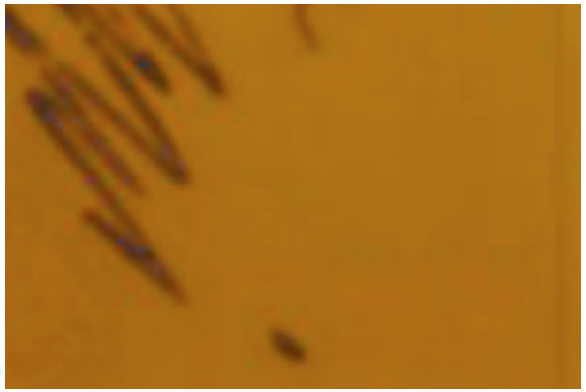
? time evolution

? spin-flip probability $P_{\uparrow\to\downarrow}$

$$H = \frac{vt}{2}\sigma^z + \frac{\Delta}{2}\sigma^x$$



$$|\Psi(t)\rangle = C_1(t)|\uparrow\rangle + C_2(t)|\downarrow\rangle$$



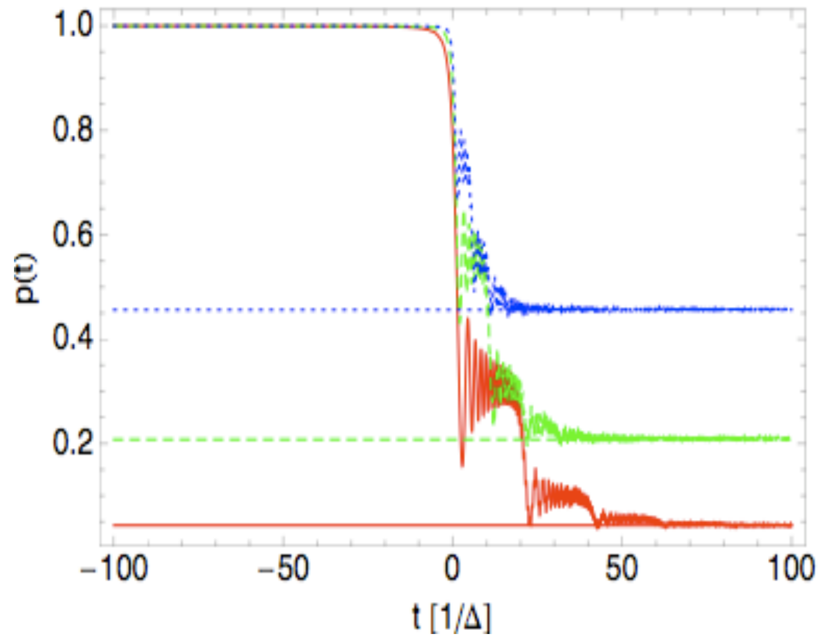
$$P_{\uparrow\to\uparrow} \overset{t\sim+\infty}{=} \exp(-\pi\Delta^2/2v)$$

L.D. Landau, 1932
C. Zener, 1932,
E. Majorana, 1932
E.C.G. Stückelberg, 1932

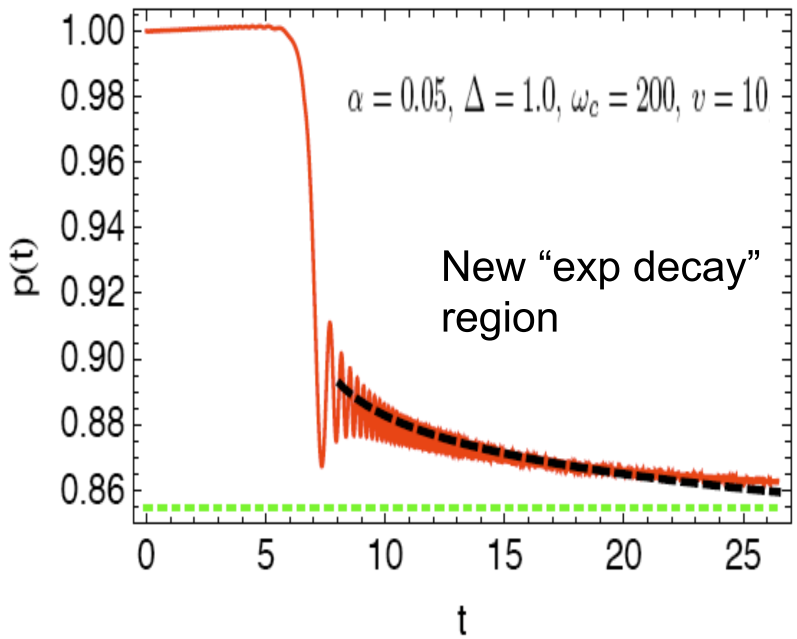
Effect of the environment…

In the case of a longitudinal coupling with the environment, the long-time probability remains unchanged ($t \gg t_c = \omega_c/v$)

No go-up Theorem (Hanggi, Kayanuma et al, 2008)

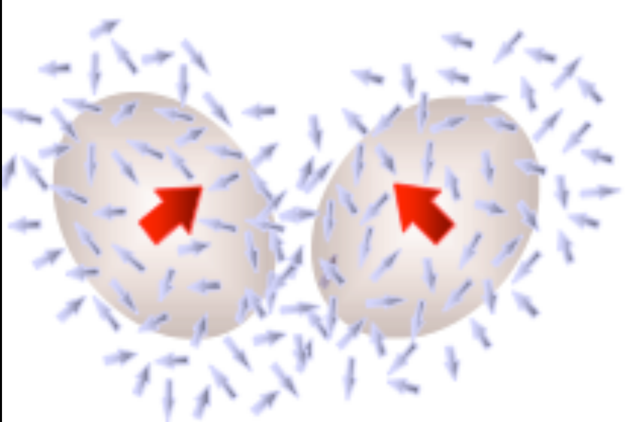


1 oscillator



bath of oscillators

More than one…



A common bath induces effective interactions between qubits

RKKY interaction for fermion baths

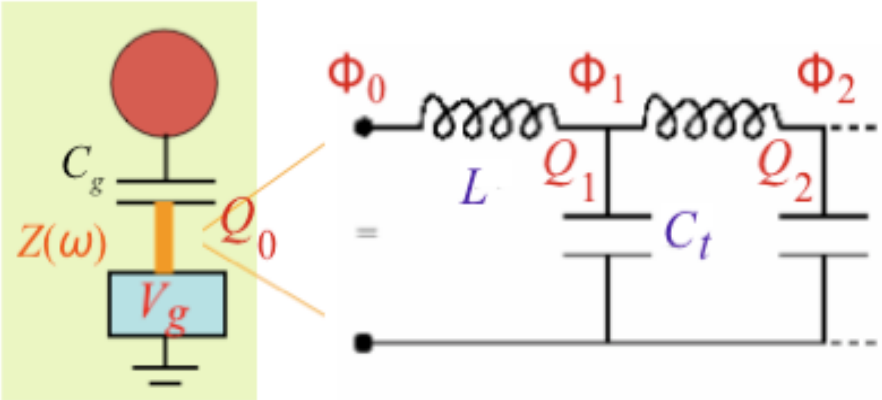
boson baths (…, Privman et al; P. Orth, I. Stanic, K. Le Hur, 2008;…)
QED cavity systems (R. Schoelkopf, S. Girvin,..),…
Good for computation (2-qubit gates)!

Possibility of maintaining “coherence” of the noisy qubit chain?

Case Δ=0: for some entangled input states no decoherence occurs
L.-M. Duan and G. C. Guo, PRA 57, 737 (1998)

Many-Body spin dynamics and Entanglement

Quantum Control…



Transmission line

$J(\omega) \propto (R/R_K)\omega$

($\alpha = R/R_K$ and R_K~6.5kΩs)

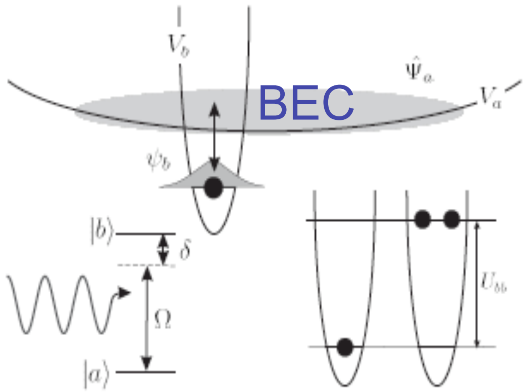
Possible "ohmic" realizations:

R. Schoelkopf et al, (2002)

Makhlin et al. Rev. Mod. Physics 73, 357 (2001),…

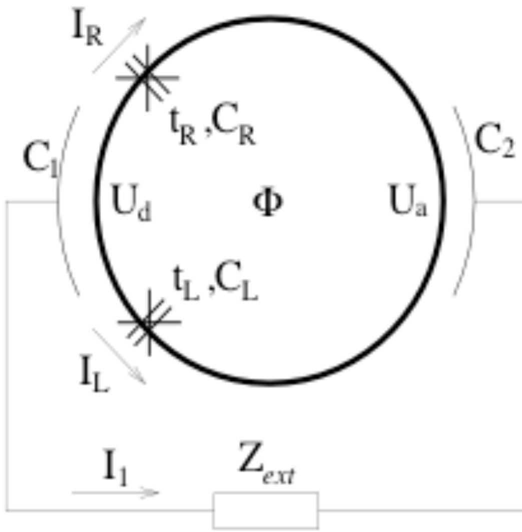
P. Cedraschi and M. Büttiker

Annals of Physics ***289****, 1-23 (2001)*



Persistent current

$$I(\alpha) \propto \langle S_x \rangle$$



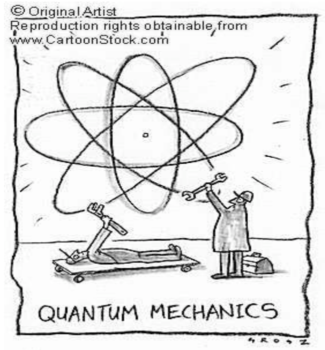
P. Zoller et al. PRL **94**, 040404 (2005)

Peter Orth, Ivan Stanic, Karyn Le Hur, PRA (2008)

Single Atom in a Trap: Ph. Gangier et al. Science **309**, 454 (2005)

Another proposal: D. Porras, F. Marquardt, J. von Delft and J. I. Cirac (2007),…

Noisy Qubits



Computing requires decoherence control…

Noise NOT always bad: coupling between qubits

"Non-Isolated" Quantum Mechanics

Thank you for your attention!

Thanks to Students, Postdocs & Collaborators!!