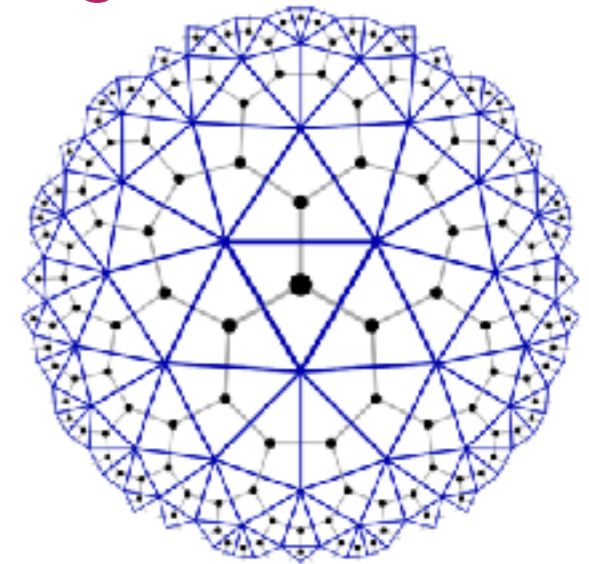
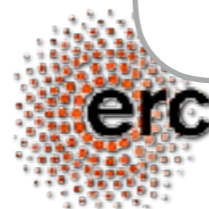


# Towards quantum simulation showing super-polynomial speedups

Holography and criticality  
in matchgate tensor

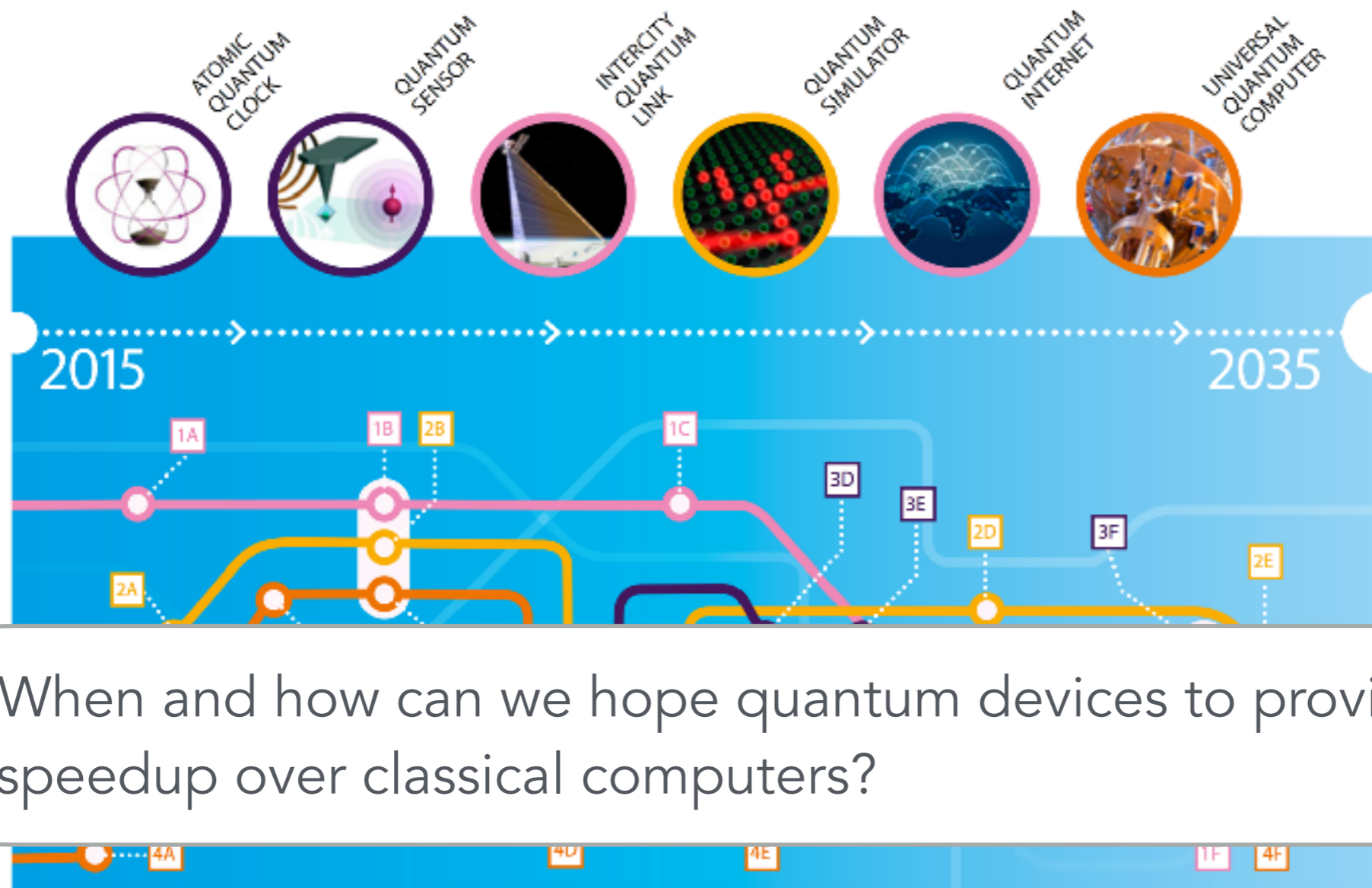


Jens Eisert, Freie Universität Berlin



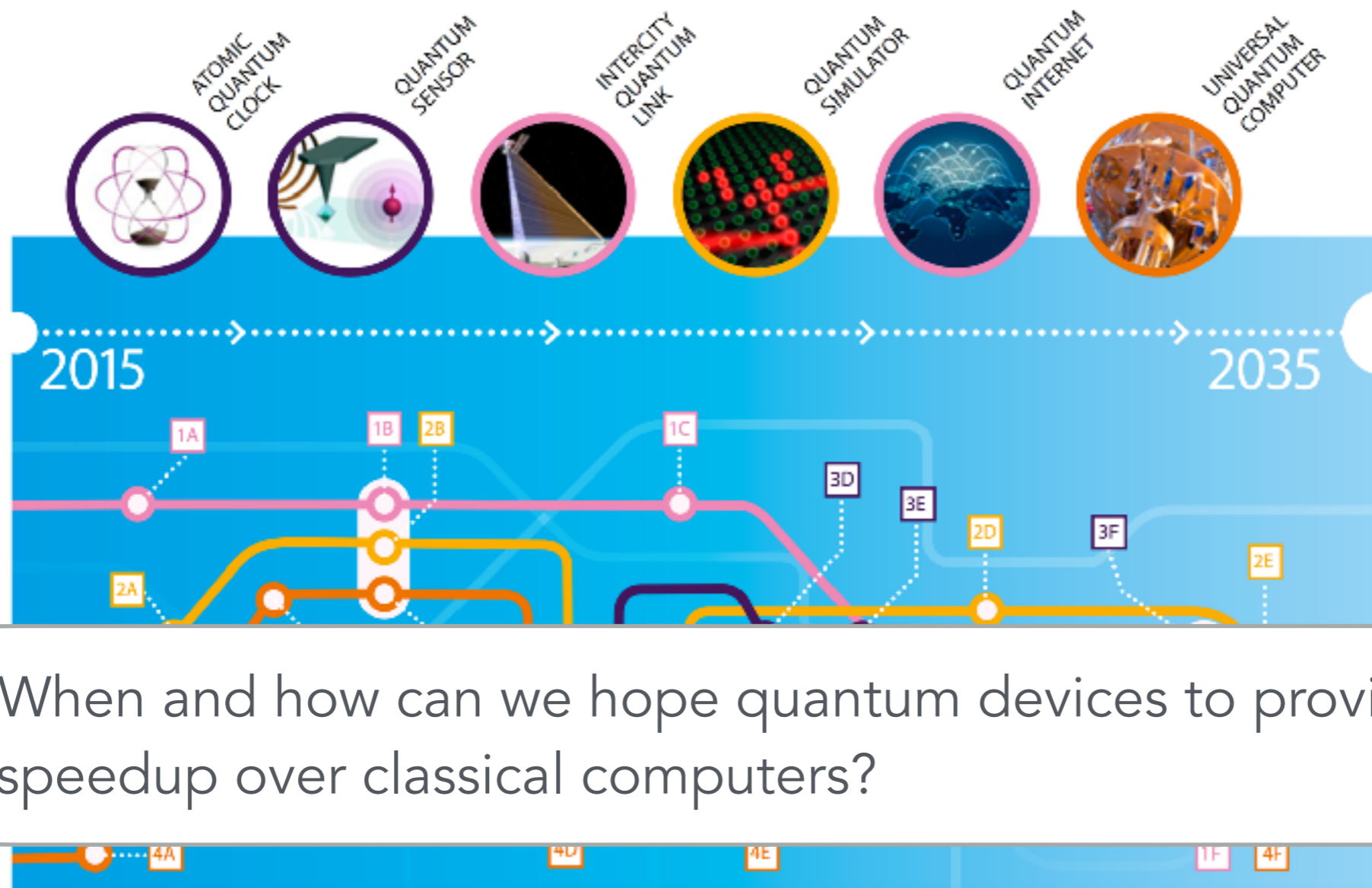
With Dom Hangleiter, Martin Schwarz, Robert Raussendorf, Juan Bermejo-Vega

# Towards quantum simulation showing super-polynomial speedups



- When and how can we hope quantum devices to provide a speedup over classical computers?

# Towards quantum simulation showing super-polynomial speedups



- When and how can we hope quantum devices to provide a speedup over classical computers?

QUANTUM  
INTERNET

UNIVERSAL  
QUANTUM  
COMPUTER



- IBM, Microsoft, Google efforts

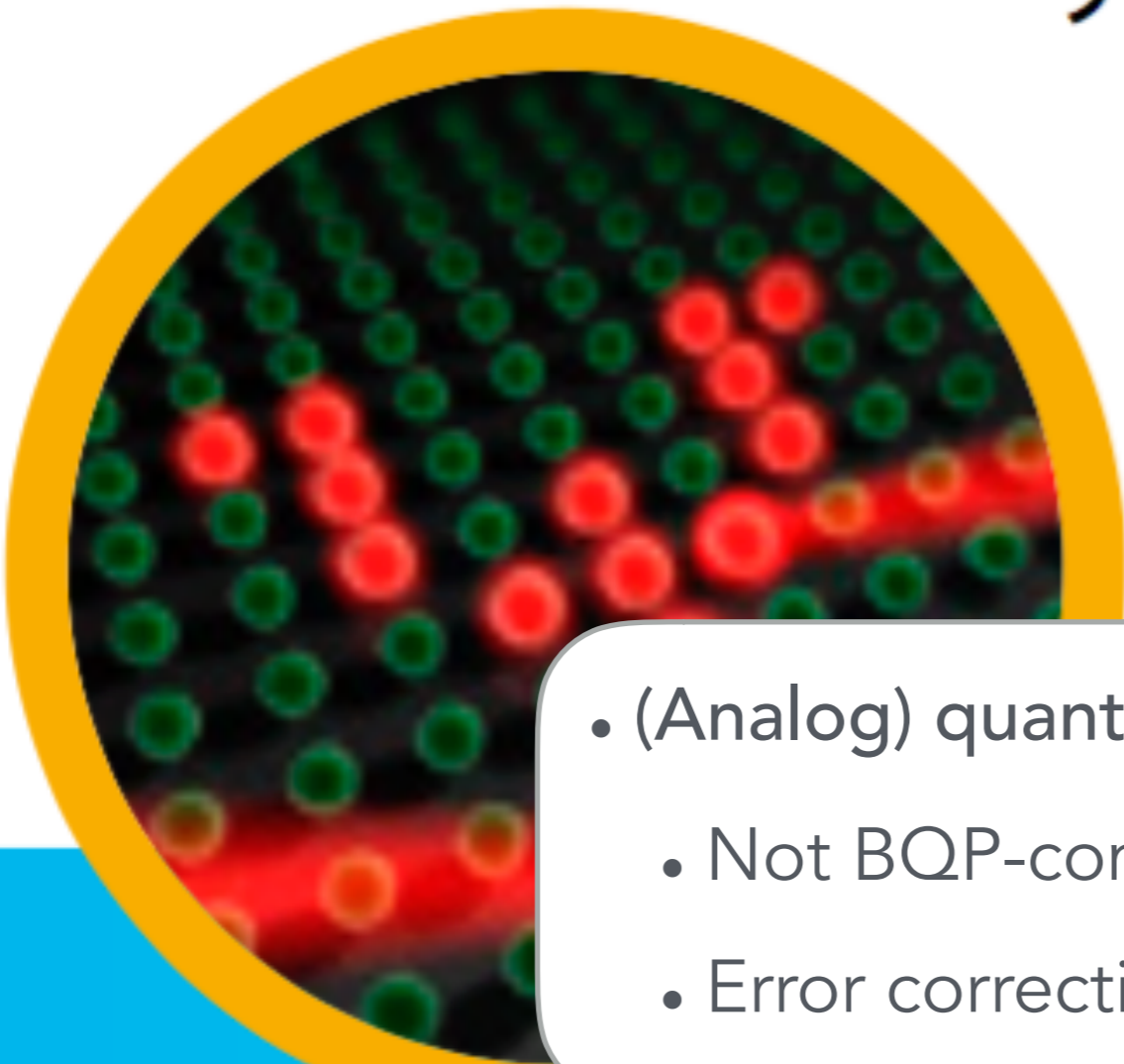
- When and how can we hope quantum devices to provide a speedup over classical computers?



QUANTUM  
LINK

QUANTUM  
SIMULATOR

QUANTUM  
LINK

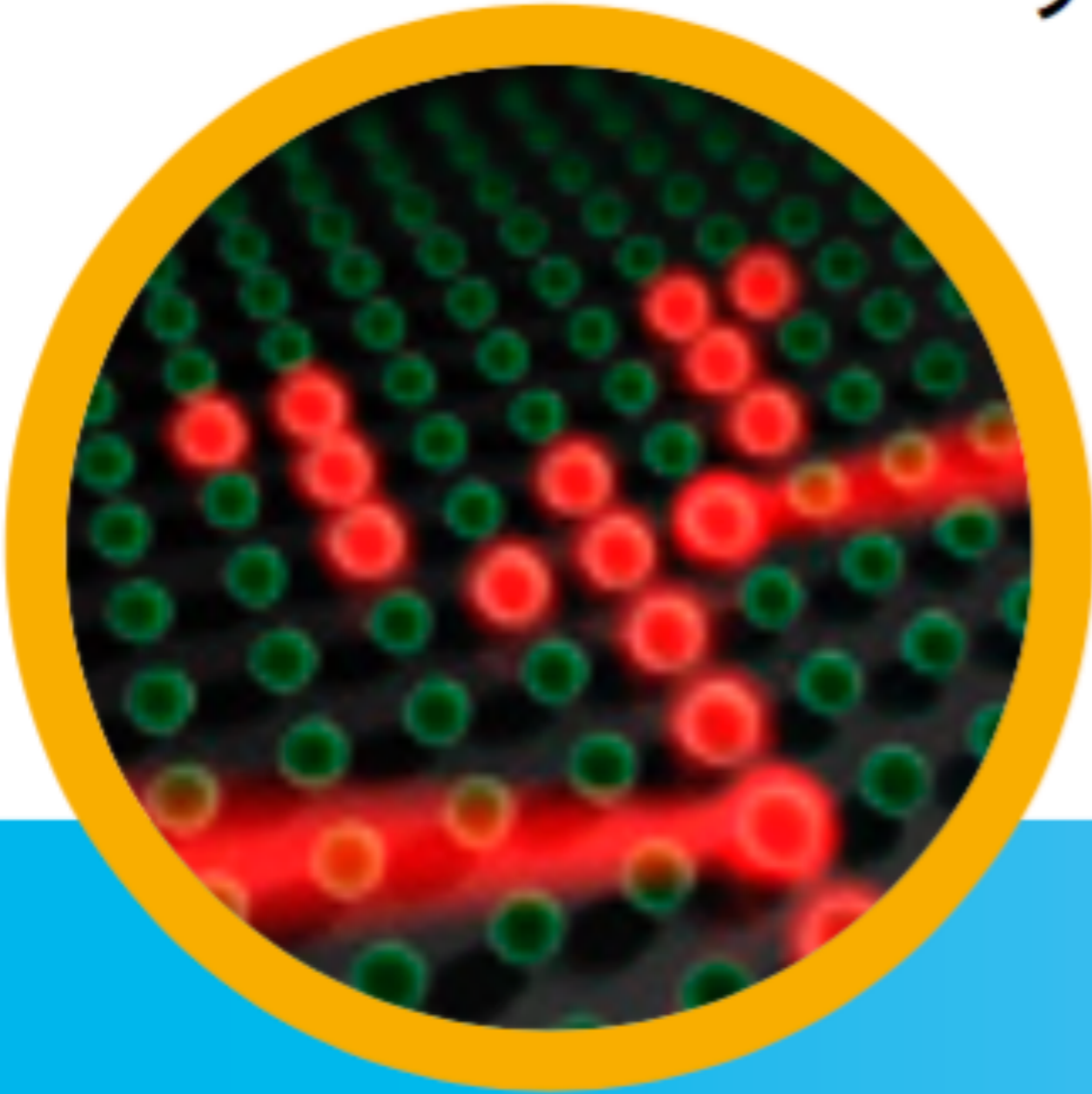


- (Analog) quantum simulators
  - Not BQP-complete, what is computational power?
  - Error correction/fault tolerance unavailable

ENERGY  
QUANTUM  
LINK

QUANTUM  
SIMULATOR

QUANTUM  
LINK

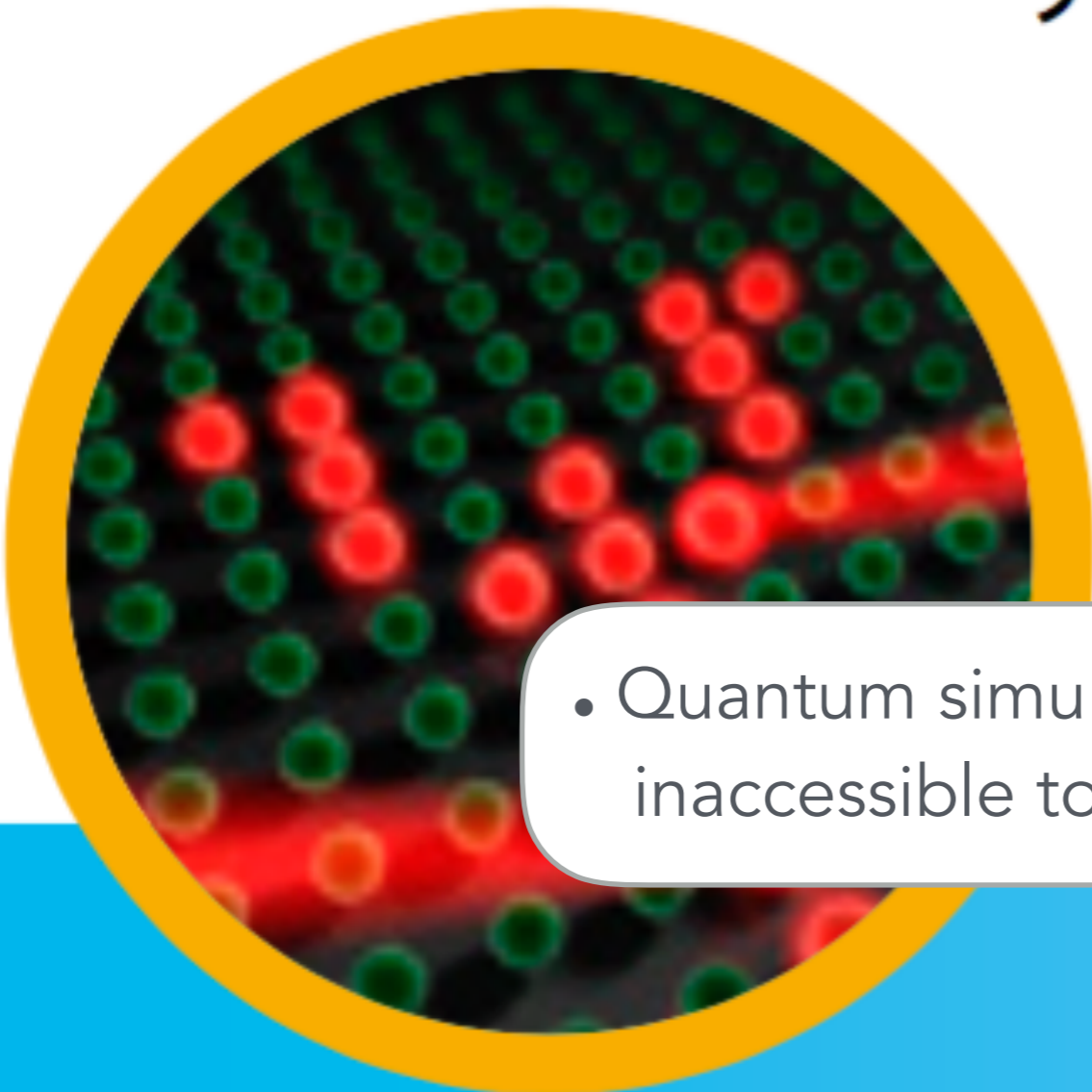




QUANTUM  
LINK

QUANTUM  
SIMULATOR

QUANTUM



- Quantum simulators should solve problems inaccessible to classical computers

ERCITY  
QUANTUM  
LINK

QUANTUM  
SIMULATOR

QUA  
IN



- How do we know we have done the right thing?



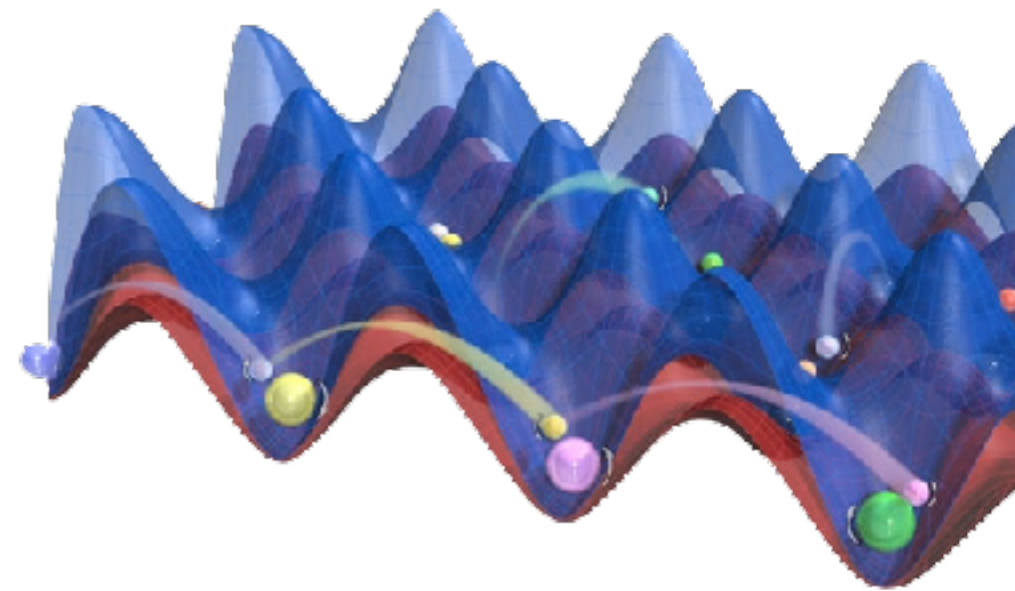
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(Analog) quantum simulators

# Analog quantum simulators



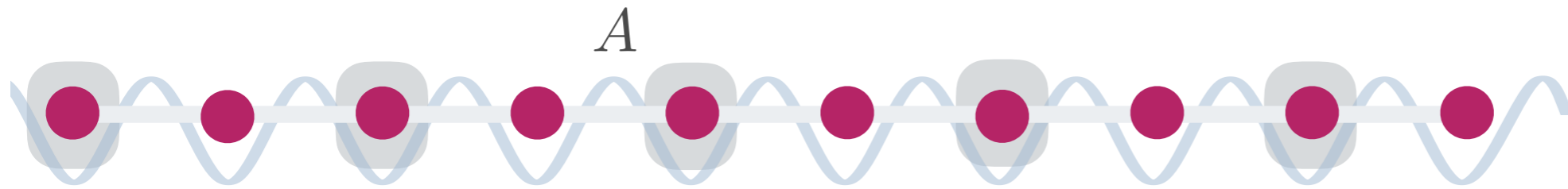
- Cold atoms in optical lattices most advanced



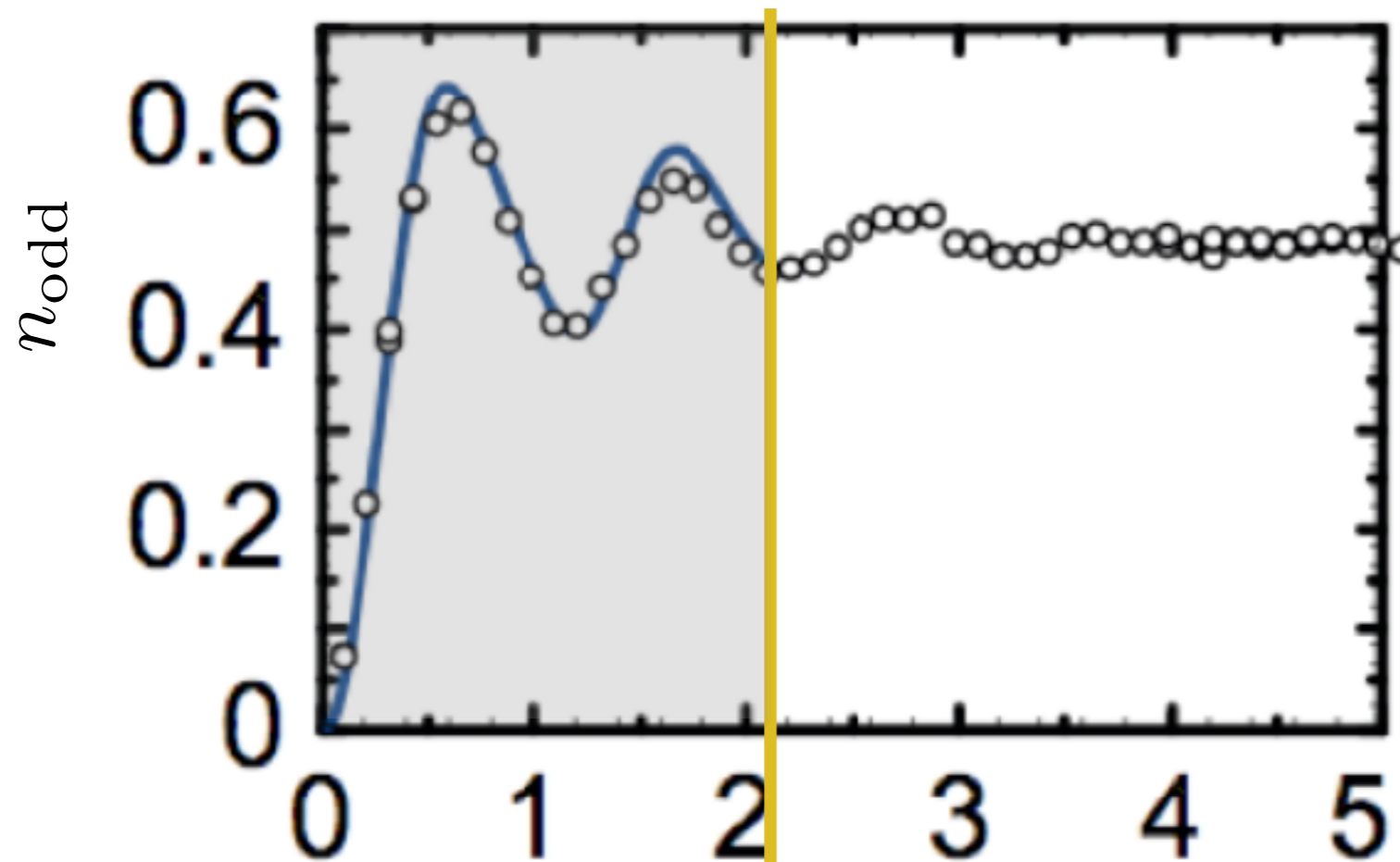
- Probe local Hamiltonians, up to  $\sim 10^5$  sites

- Ground state problems
- "Quenches"  $\rho(t) = e^{-itH} \rho e^{itH}$  (time evolution)
- Slow evolutions, reminiscent of adiabatic quantum computing

# Analog quantum simulators



- Equilibration and thermalisation of atoms in optical super-lattices (MPQ)
- Imbalance as function of time for  $|\psi(0)\rangle = |0, 1, \dots, 0, 1\rangle$  under Bose-Hubbard Hamiltonian



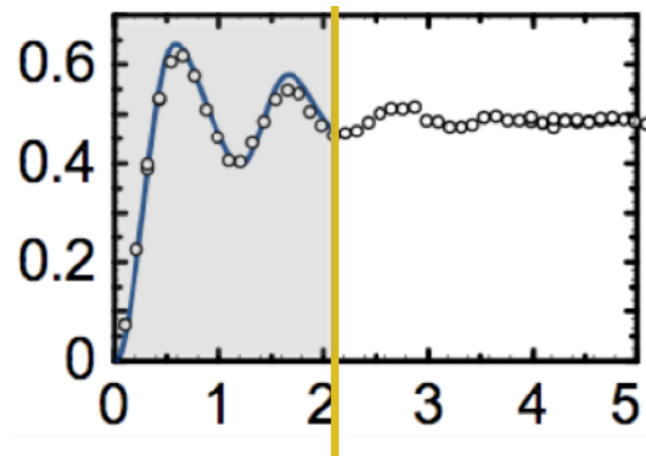
Best available classical matrix-product state simulation,  
bond dimension 5000



# Analog quantum simulators



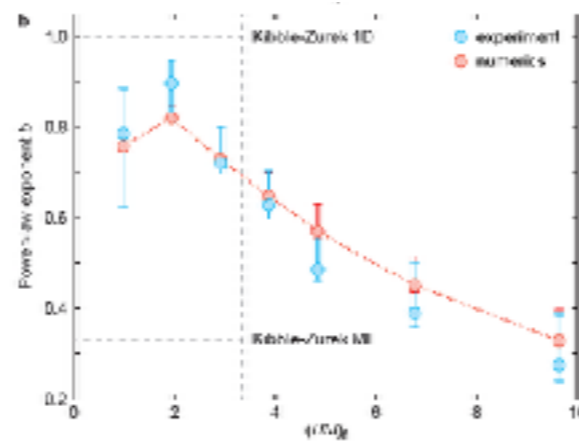
## Equilibration



Short times can be efficiently simulated

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, Nature Phys 8, 325 (2012)

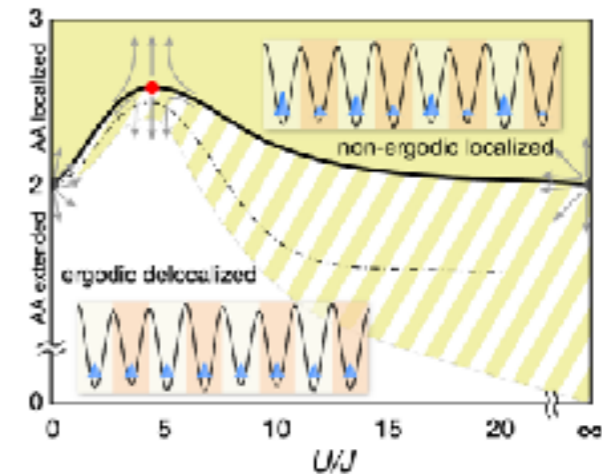
## Kibble-Zurek mechanism



1D systems can be efficiently simulated,  
2D systems not

Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, Proc Natl Acad Sci 112 3641 (2015)

## Many-body localization

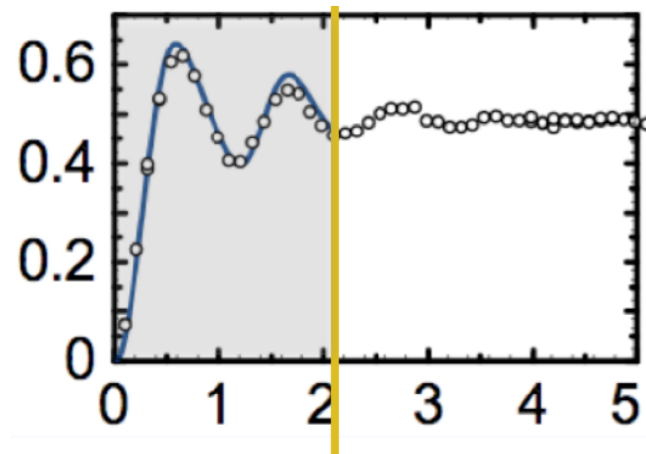


Schreiber, Hodgman, Bordia, Lüschen, Fischer, Vosk Altman, Schneider, Bloch, Science 349, 842 (2015)

# Analog quantum simulators

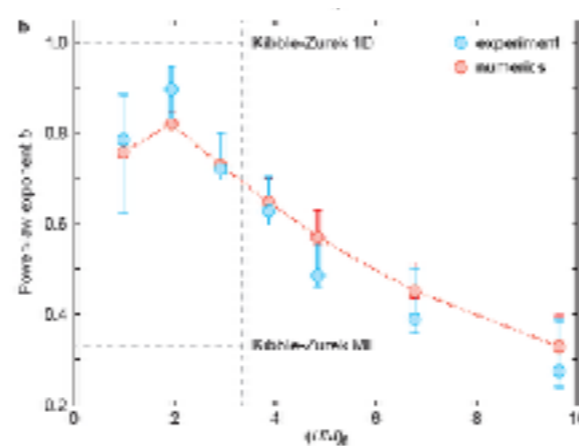


## Equilibration



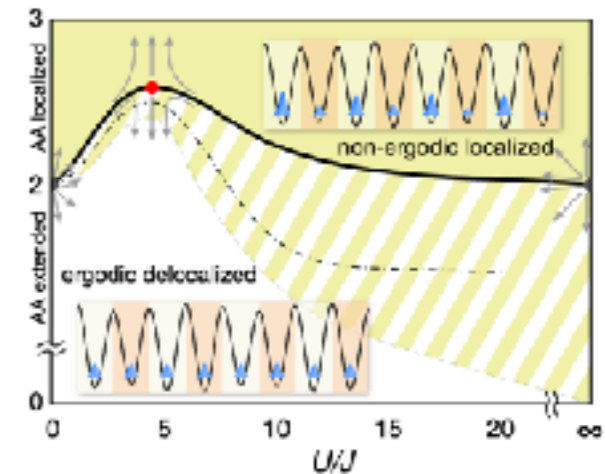
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## Many-body localization



Schreiber, Hodgman, Bordia, Lüschen, Fischer, Vosk, Altman, Schneider, Bloch, Science 349, 842 (2015)

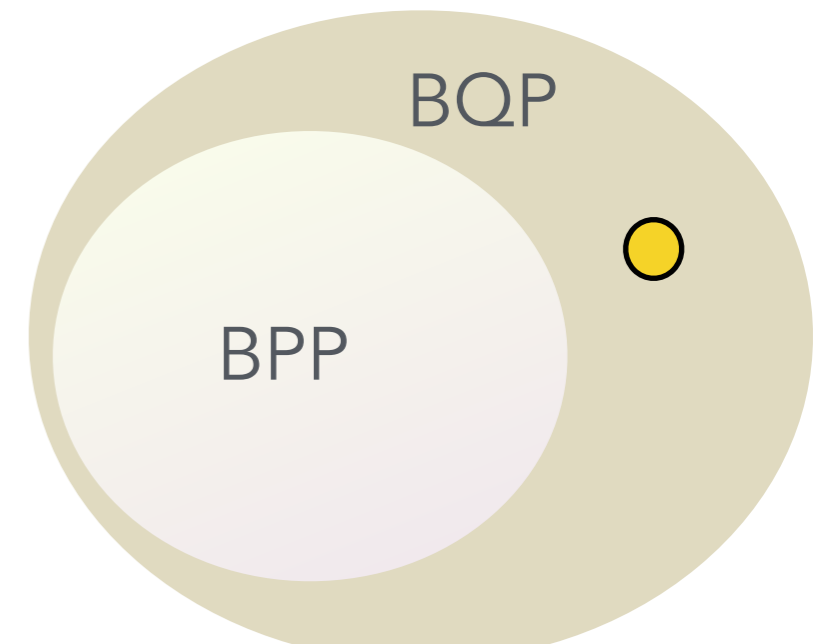
Trotz  
Eiser

- **Dynamical quantum simulators**

Existing quantum simulators outperform state-of-the-art simulations on classical supercomputers

- Cleverer simulation method?

## Need for intermediate problems



- **Intermediate problems**

To be sure, we must prove the hardness of the task: Identify a (feasible) task that lies outside of BPP, but is not BQP hard



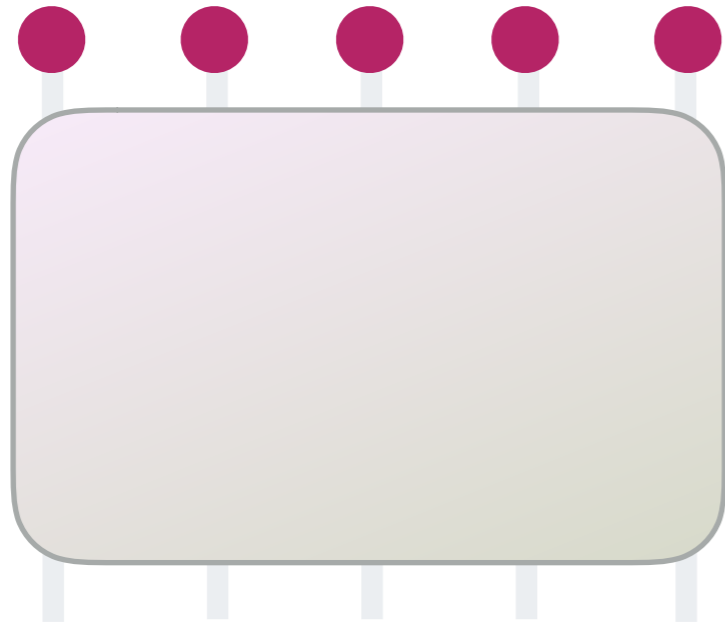
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# Super-polynomial computational speedups

# Super-polynomial speedups

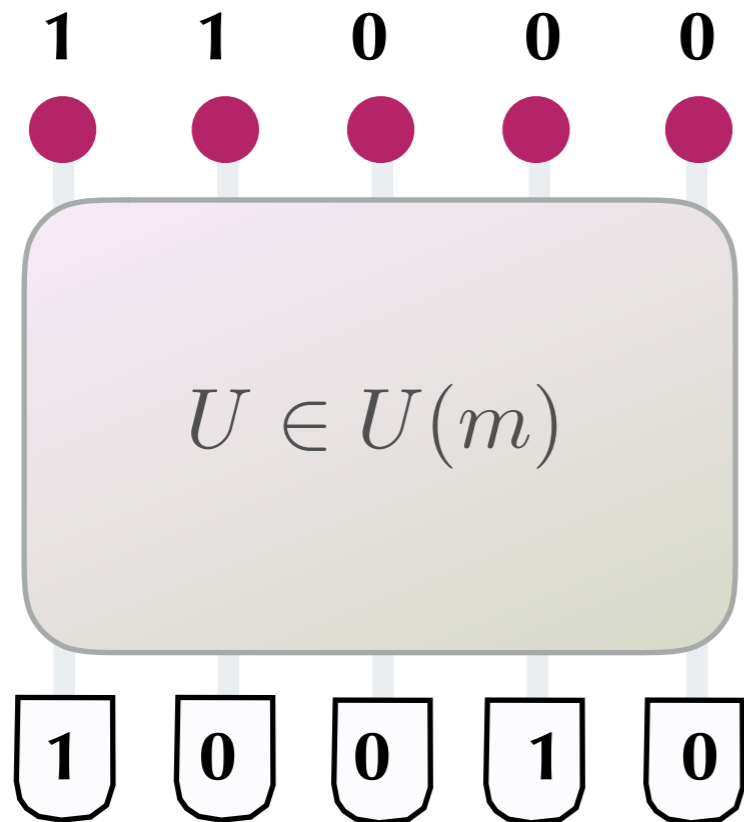
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- **Aim:** Find **some** problem with strong evidence for super-polynomial speedup
- Formerly dubbed “quantum computational supremacy”



# Super-polynomial speedups

- **Aim:** Find some problem with strong evidence for super-polynomial speedup
- Boson sampling



- $n$  bosons in  $m$  optical modes

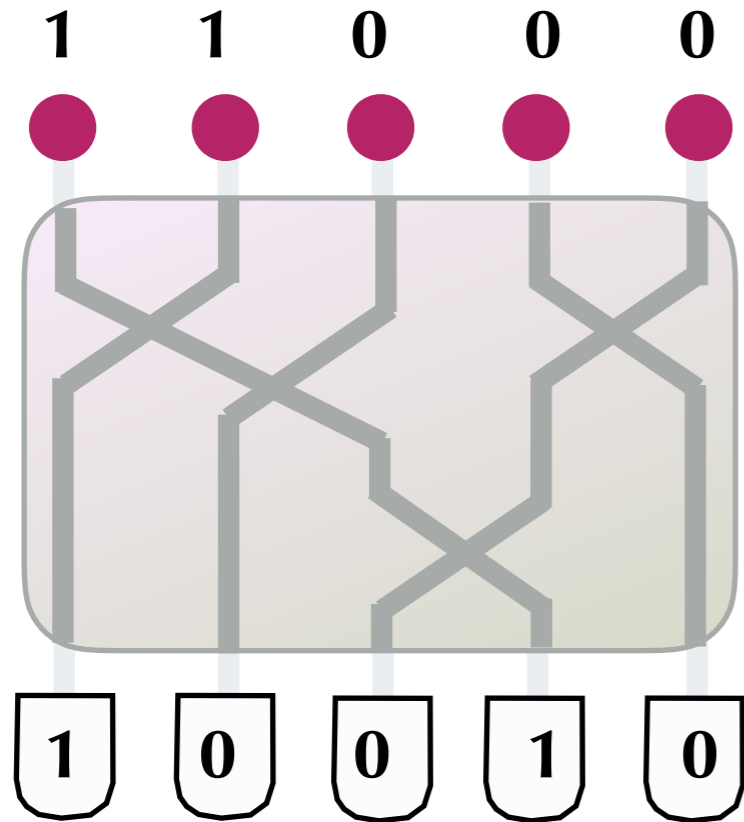
- Haar random mode transformation  $b \mapsto Ub$   
 $b = (b_1, \dots, b_m)^T$

- Photon detection



# Super-polynomial speedups

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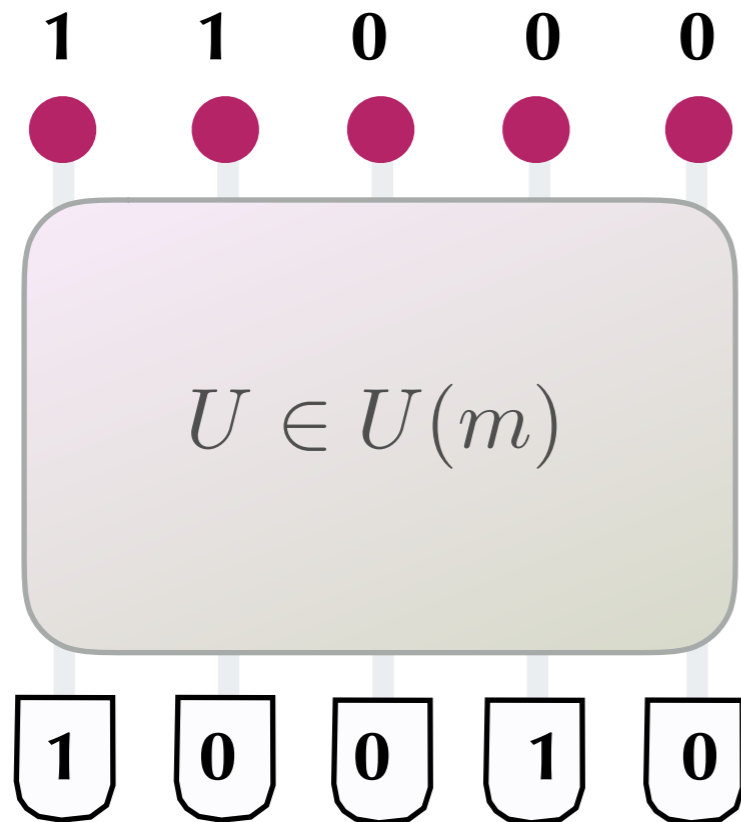
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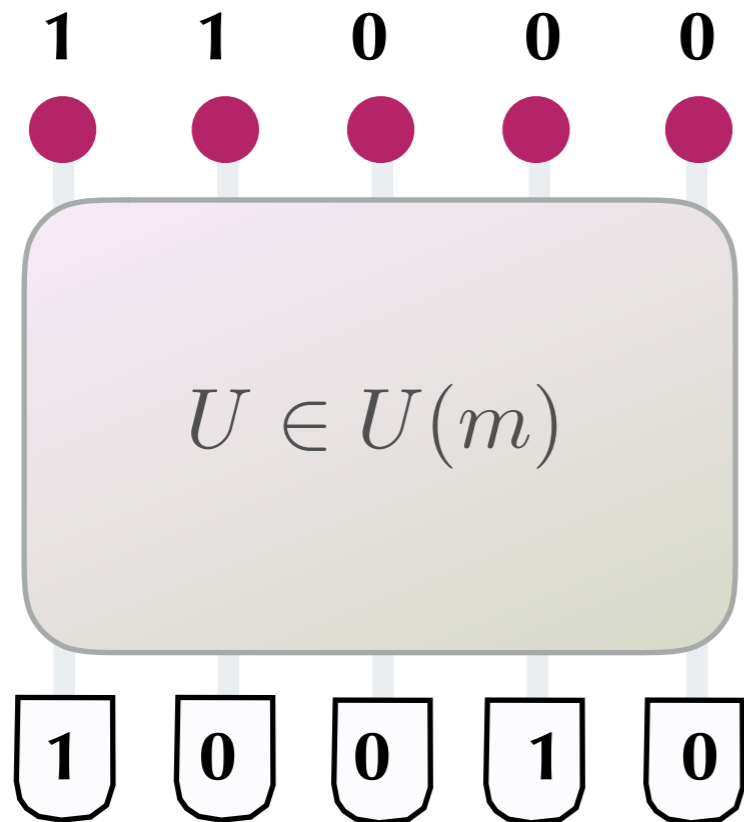
- Photon detection

## • Theorem:

Sampling from a distribution close in  $l_1$  norm to boson sampling distribution is "computationally hard" with high probability if the unitary  $U$  is chosen from Haar measure and  $m$  increases sufficiently fast with  $n$  ( $m \in \Omega(n^5)$ )

# Super-polynomial speedups

- **Aim:** Find some problem with strong evidence for super-polynomial speedup
- Boson sampling



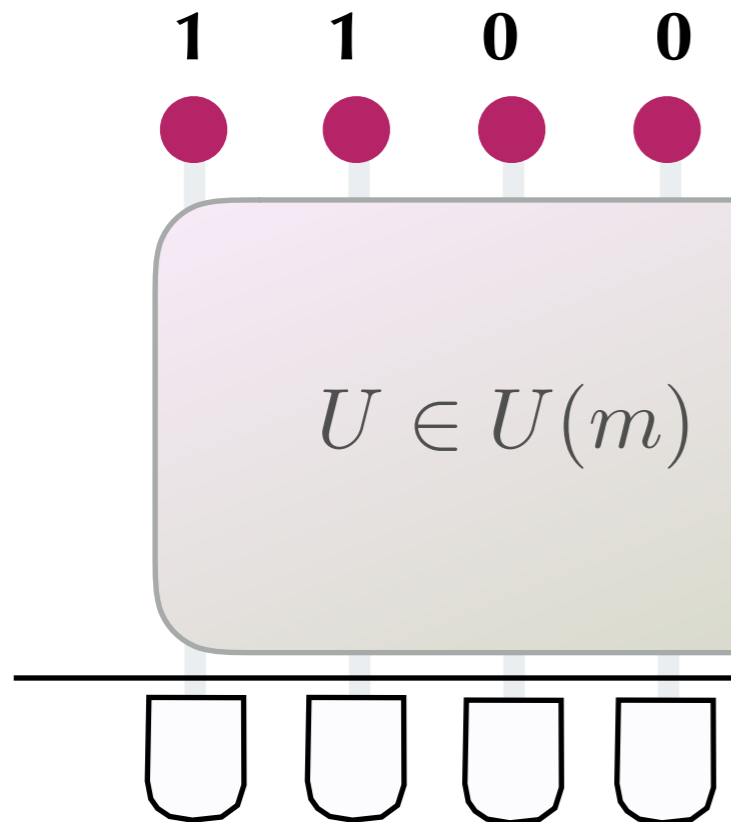
- $n$  bosons in  $m$  optical modes

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- Photon detection

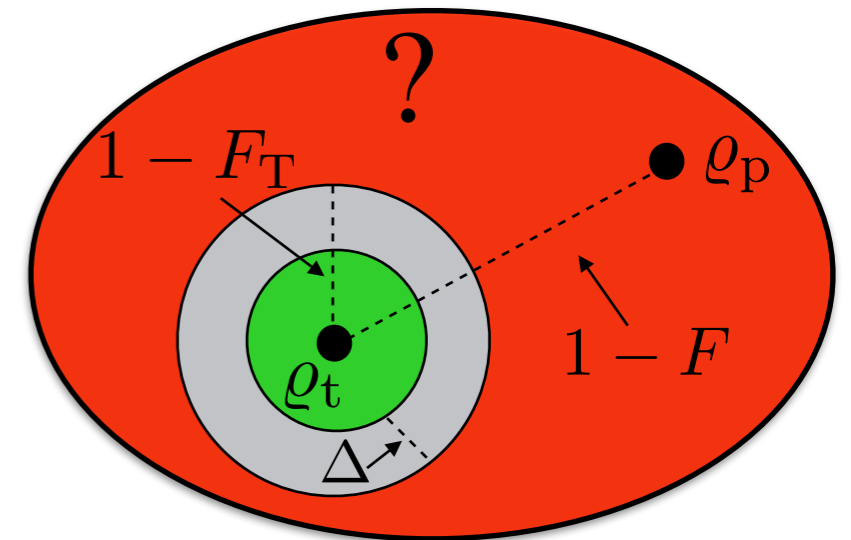
## Super-polynomial speedup

- **Aim:** Find some problem
- Boson sampling



- **Certification** using continuous-variables:

- Target fidelity  $F(\rho_t, \rho_p) \geq F_T$  with anticipated state  $\rho_t$



- Can perform robust fidelity certification, with

$$O\left(\frac{\text{poly}(m, 1/\Delta)}{\log(1/(1-\alpha))}\right)$$

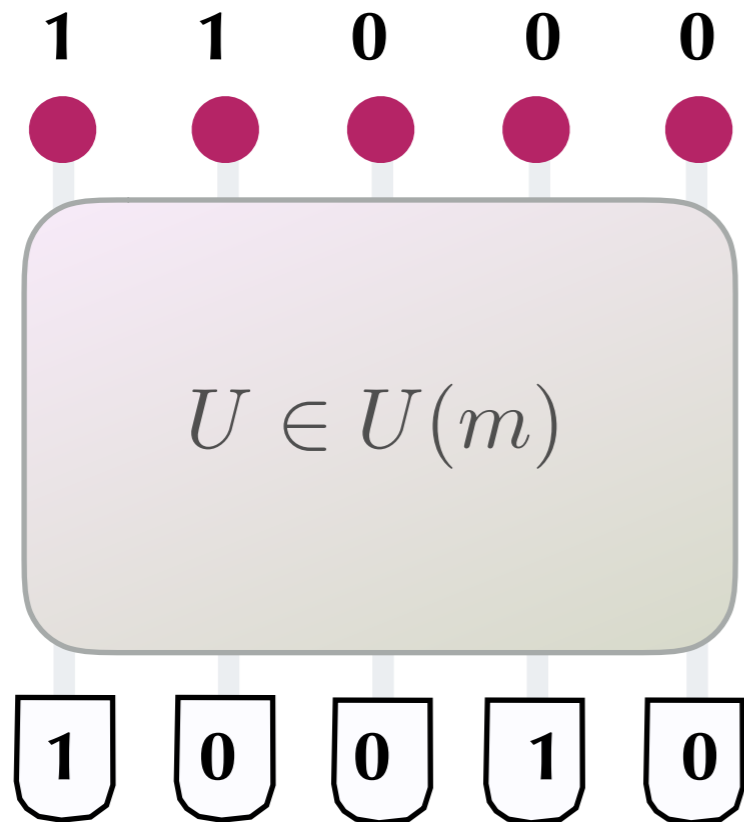
many preparations and **homodyne** measurements,  
with success probability  $\alpha$

Aolita, Gogolin, Kliesch, Eisert, Nature Communications 6, 8498 (2015)

- Not quite good enough

# Super-polynomial speedups

- **Aim:** Find some problem with strong evidence for super-polynomial speedup
- Boson sampling



- $n$  bosons in  $m$  optical modes

- Haar random mode transformation

$$b = (b_1, \dots, b_m)^T$$



# Super-polynomial speedups

- **Aim:** Find some problem with strong evidence for super-polynomial speedup
- Boson sampling

1 0 0 1 0

- Let  $m \geq n^{5.1}$  and let  $U \in U(m)$  be Haar random. Then with probability at least  $1 - \delta$ , for every  $T$  and every  $\epsilon > 0$ , there exists a circuit of size  $T \text{poly}(n, 1/\epsilon, 1/\delta)$  that samples a distribution that is  $\epsilon$ -indistinguishable from the boson sampling distribution by circuits of size at most  $T$

## Super-polynomial speedups

---

- **Aim:** Find **some** problem with strong evidence for super-polynomial speedup
- Boson sampling

- **Common prejudice:** In order to be able to verify a quantum simulation, one needs to be able to efficiently simulate it

# Super-polynomial speedups

- **Aim:** Find some problem with strong evidence for super-polynomial speedup
- **Boson sampling**  
Aaronson, Arkhipov, Th Comp 9, 143 (2013)
- **IQP circuits**  
Bremner, Montanaro, Shepherd: Phys Rev Lett 117, 080501 (2016)  
Bremner, Jozsa, Shepherd, arXiv:1005.1407
- **Random universal circuits**  
Boixo, Isakov, Smelzanski, Babbush, Ding, Jiang, Bremner, Martinis, Neven, arXiv:1608.00263 (2016)
- **Ising-type interactions**  
Gao, Wang, Duan, Phys Rev Lett, 118, 040502 (2017)

+ : Provable classical hardness with  $l_1$ -errors  
(under reasonable assumptions)

- : Very hard to implement: Either

- Hard to scale up with present technology
- Arbitrary gate choices necessary
- High (56) periodicity of Hamiltonian

---

Feasible quantum simulators showing a speedup

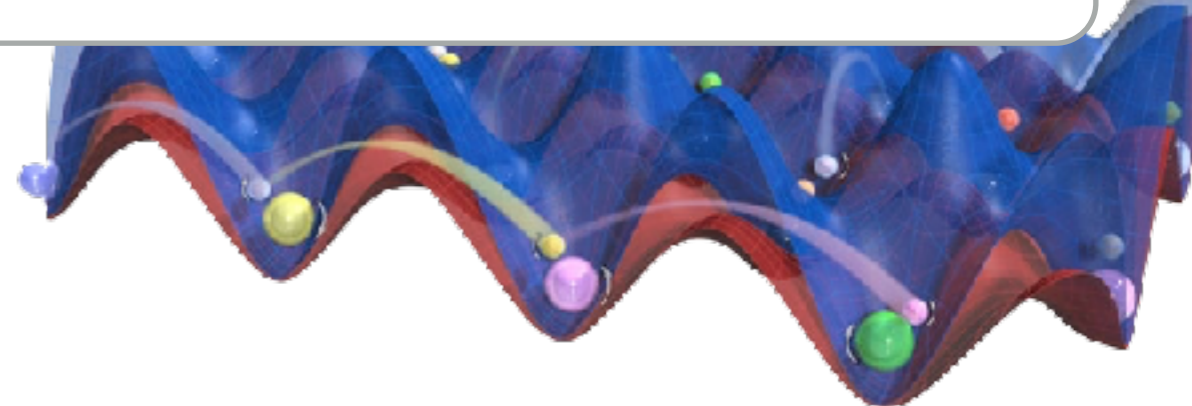
# Super-polynomial speedups

- **Aim:** Find some problem with strong evidence for super-polynomial speedup

Combine benefits of both worlds, bring speedups closer to experiment



- **Hamiltonian quench architecture**
- **Low periodicity** of the interaction Hamiltonian (NN or NNN)
- **Hardness proofs** with  $l_1$ -norm error (under some assumptions)

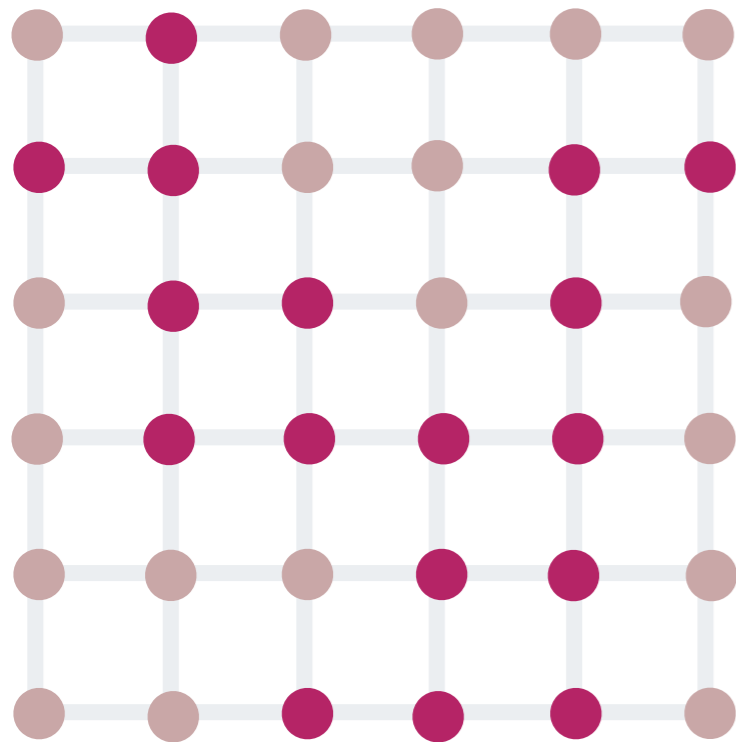




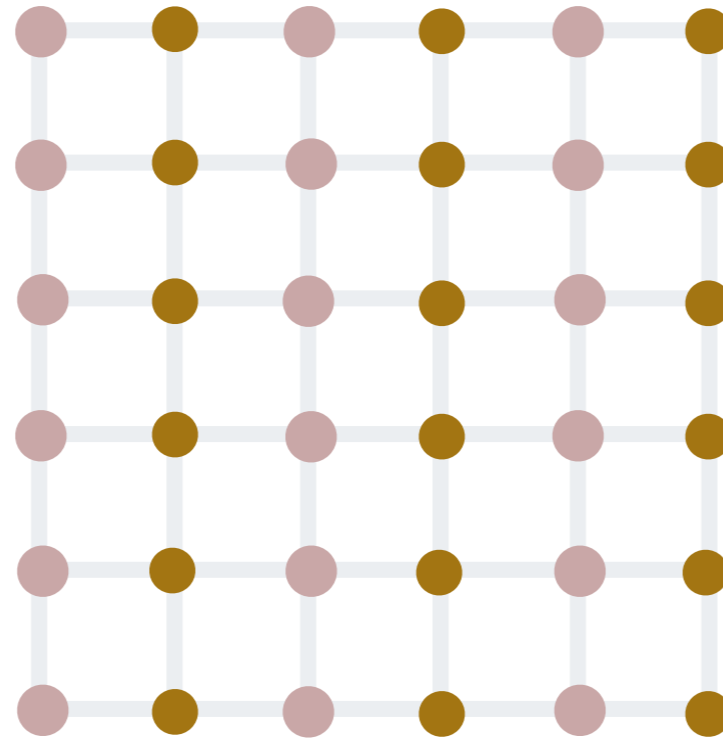
# Simple Ising models

- **Aim:** Find some problem with strong evidence for super-polynomial speedup

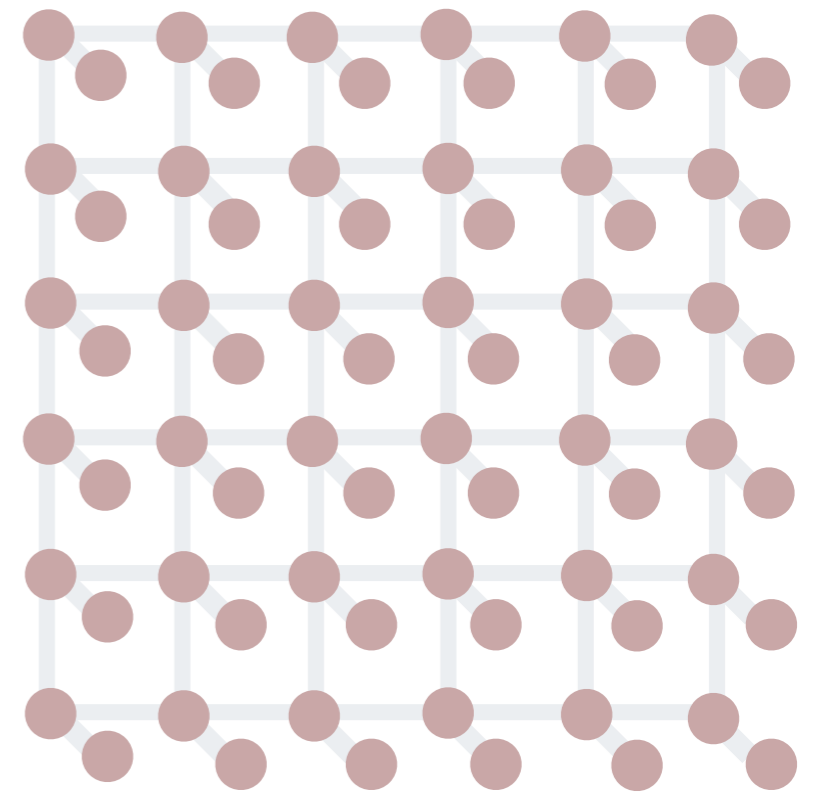
Combine benefits of both worlds, bring speedups closer to experiment



Random



Periodic



Translationally invariant

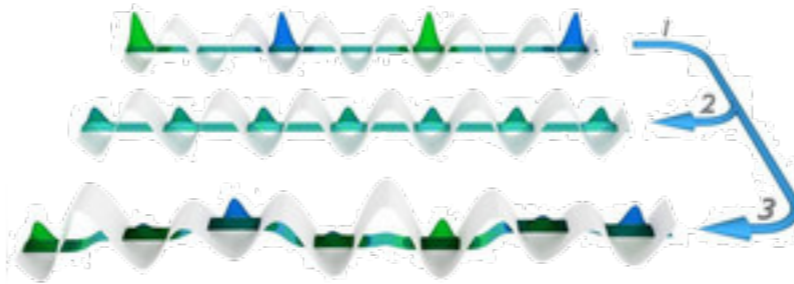
# Simple Ising models

- Prepare  $N$  qubits in  $n \times m$  square lattice in product

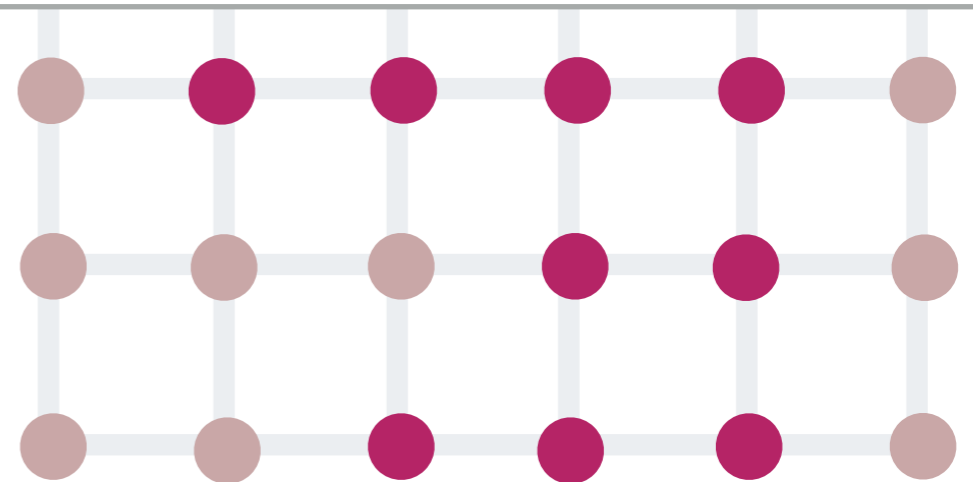
$$|\psi_\beta\rangle = \bigotimes_{i,j=1}^{n,m} (|0\rangle + e^{i\beta_{i,j}} |1\rangle)$$

with  $\beta_{i,j} \in \{0, \pi/4\}$ ,  $\{\bullet, \bullet\}$  i.i.d. randomly

- Reminiscent of disordered optical lattices



Schreiber, Hodgman, Bordia, Lüschen, Fischer, Vosk, Altman, Schneider, Bloch, Science 349, 842 (2015)



# Simple Ising models

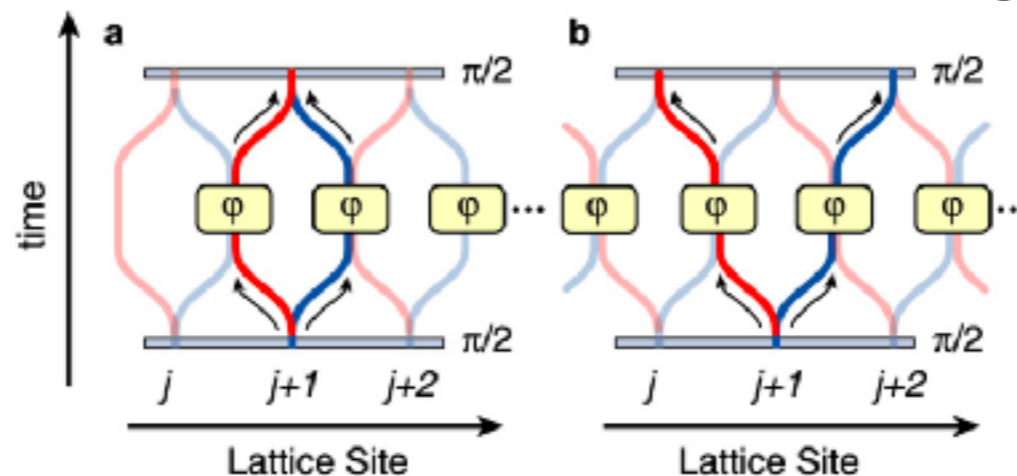
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- Quench to  $H = \sum_{(i,j) \in E} Z_i Z_j + \frac{\pi}{4} \sum_{i \in V} Z_i$  and evolve under  $U = e^{iH}$

- Controlled coherent collisions long realized



Mandel, Greiner, Widera, Rom, Hänsch, Bloch, Nature, 425, 937 (2003)

# Simple Ising models

- Prepare  $N$  qubits in  $n \times m$  square lattice in product

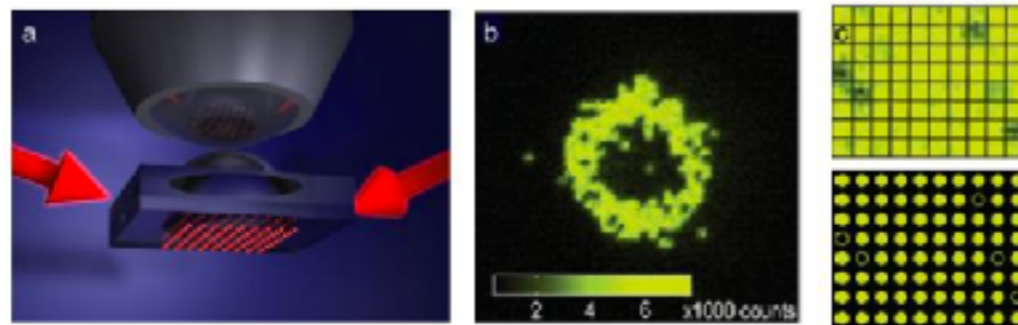
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- Measure all qubits in  $X$ -basis

- Single-site addressing possible (within limits)



Bakr, Gillen, Peng, Foelling, Greiner, Nature 462, 74–77 (2009)

Weitenberg, Endres, Sherson, Cheneau, Schauß, Fukuhara, Bloch, Kuhr, Nature 471, 319 (2011)

# Simple Ising models

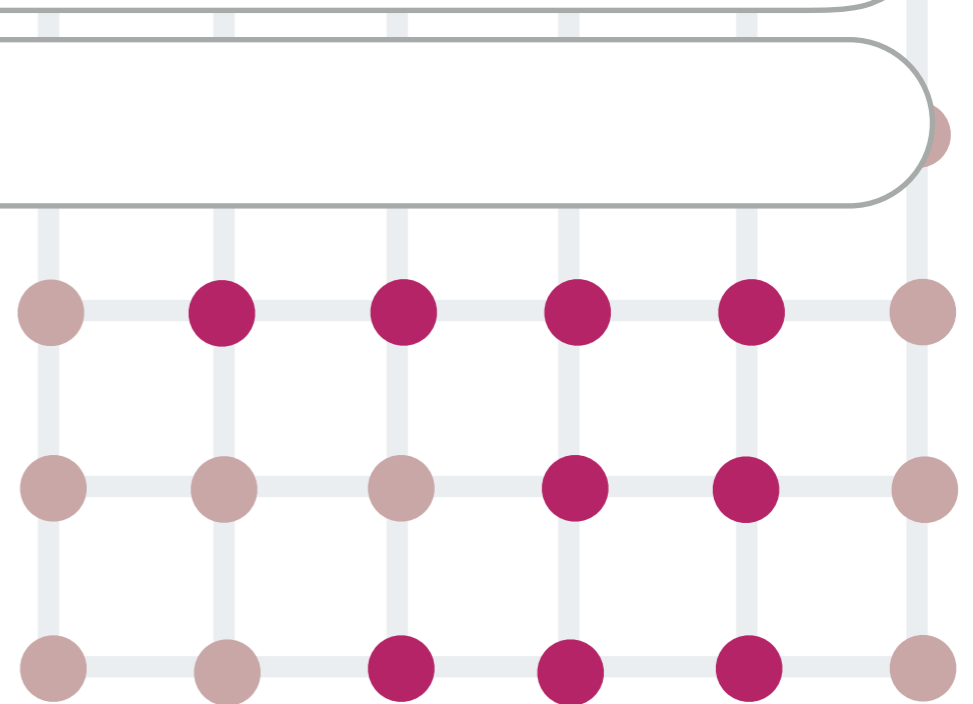
- Prepare  $N$  qubits in  $n \times m$  square lattice in product

$$|\psi_\beta\rangle = \bigotimes_{i,j=1}^{n,m} (|0\rangle + e^{i\beta_{i,j}} |1\rangle)$$

with  $\beta_{i,j} \in \{0, \pi/4\}$ ,  $\{\bullet, \bullet\}$  i.i.d. randomly

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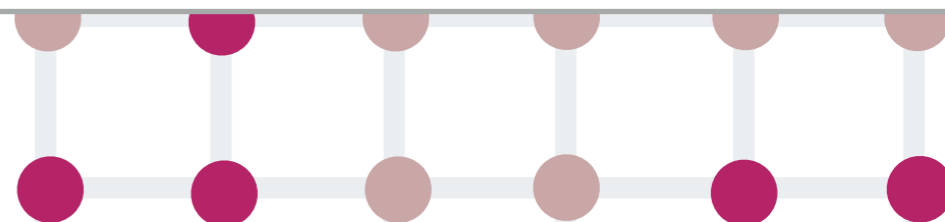




## Main result

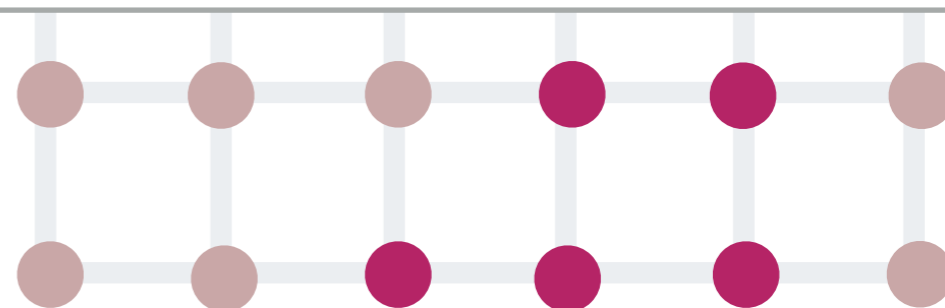
- **Theorem** (Hardness of classical sampling):

Assuming three highly plausible complexity-theoretic conjectures are true a classical computer cannot efficiently sample from the outcome distribution of our scheme up to constant error in  $l_1$  distance



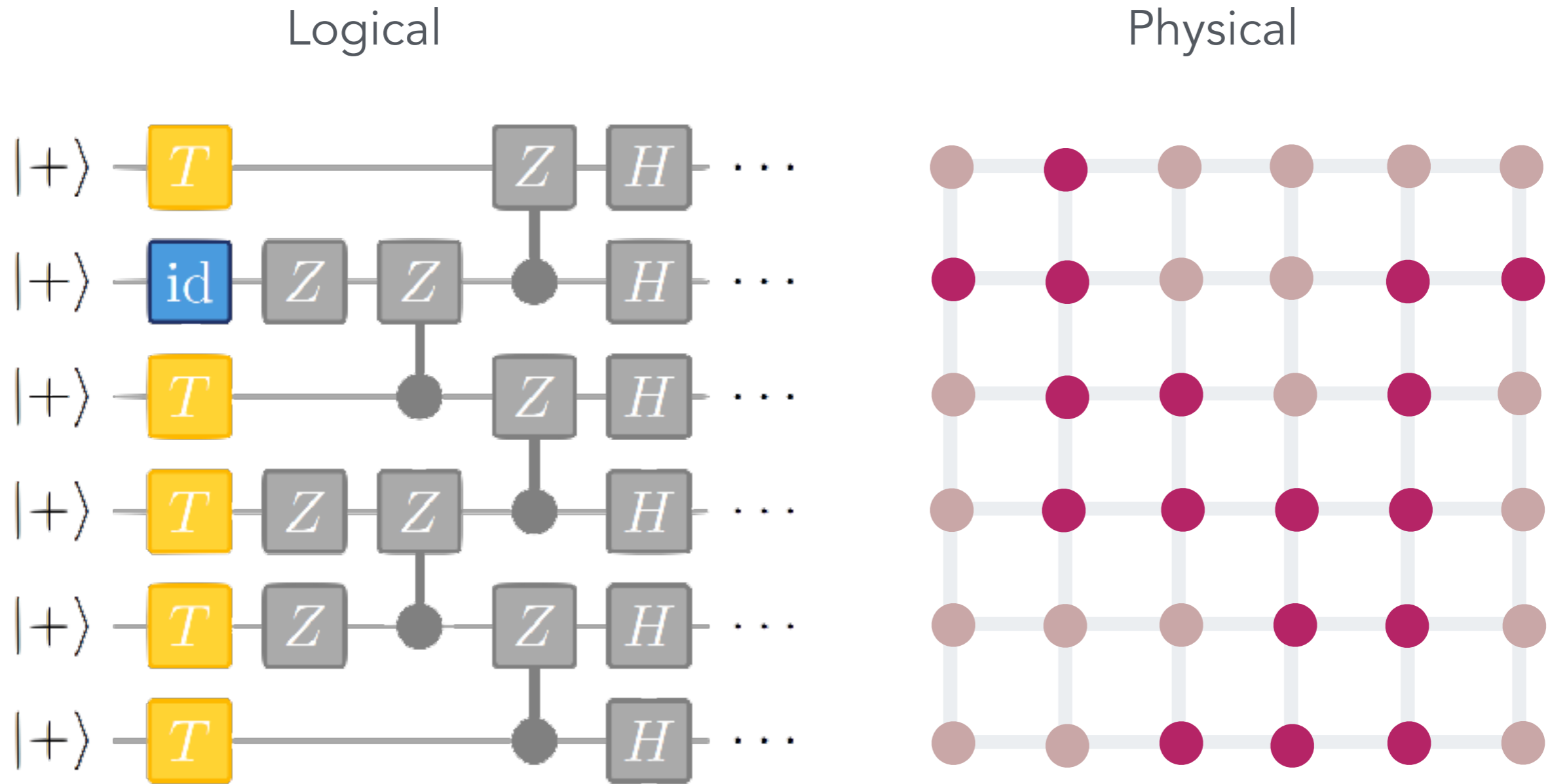
- **Lemma** (Approximation of outcome distribution in worst case):

It is #P-hard to approximate the outcome distribution of an all- $X$  measurement  $U|\psi_\beta\rangle$  to constant relative error



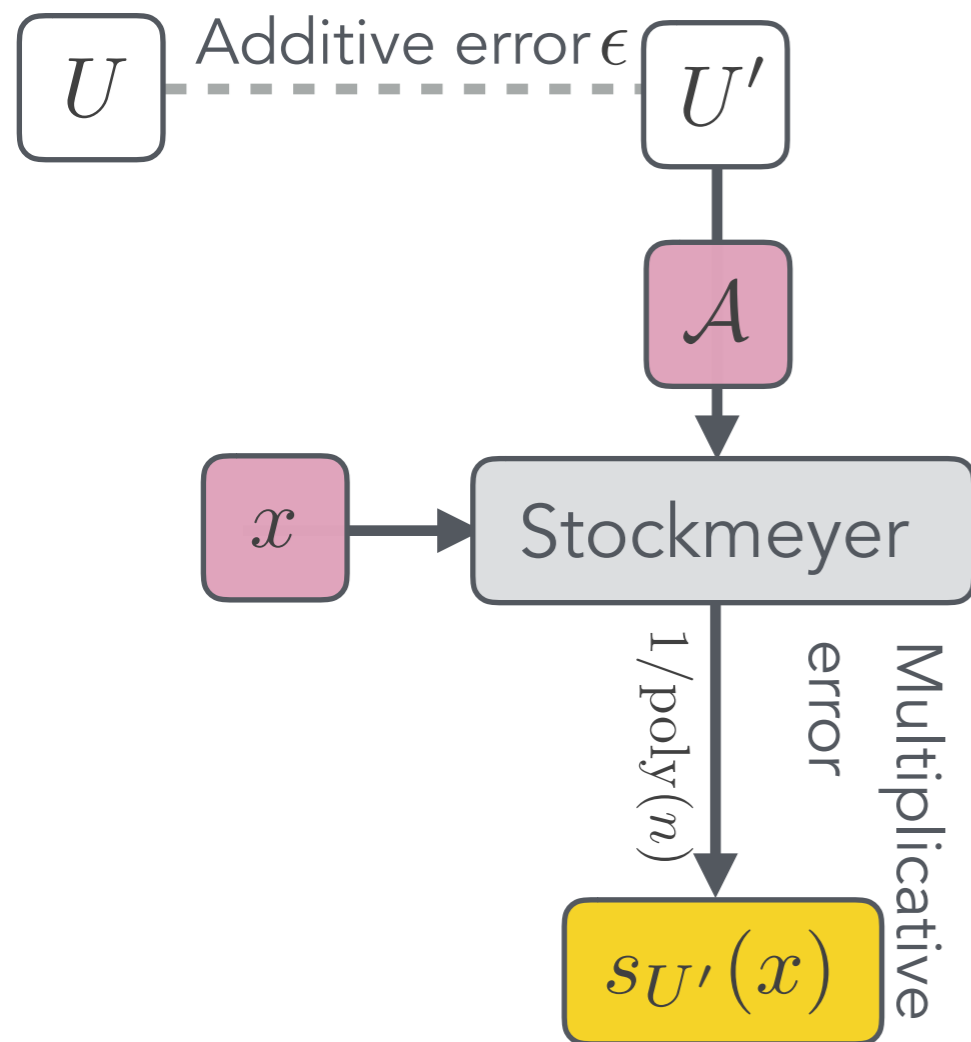
# Equivalence to MBQC

- A hint at the argument: Universal architecture for postBQP



## • Ingredients

- A classical algorithm  $\mathcal{A}$  that samples from the output distribution of  $U'$  such that  $\|p_{U'} - p_U\|_{l_1} \leq \epsilon$
- A binary string  $x$



- Approximate sampling  $\|p_U - p_{U'}\|_{l_1} < \epsilon$

- Stockmeyer error  $|s_{U'}(x) - p_{U'}(x)| \leq \frac{p_{U'}(x)}{\text{poly}(n)}$

$$|s_{U'}(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}} + \frac{\epsilon}{2^n \delta} (1 + o(1))$$

- **Polynomial hierarchy:** The polynomial hierarchy is infinite (i.p.  $P \neq NP$ )
- **Average case complexity:** For a constant fraction of the instances it is as hard to sample from the outcomes of measurements as in worst case
- **Anti-concentration:** The output probabilities of  $U|\psi_\beta\rangle$  anticongentrate, i.e.,

$$\text{prob}_U \left( |\langle a|U|\psi_\beta\rangle|^2 \geq \frac{1}{2^N} \right) \geq \frac{1}{e}$$

# Complexity theoretic conjectures

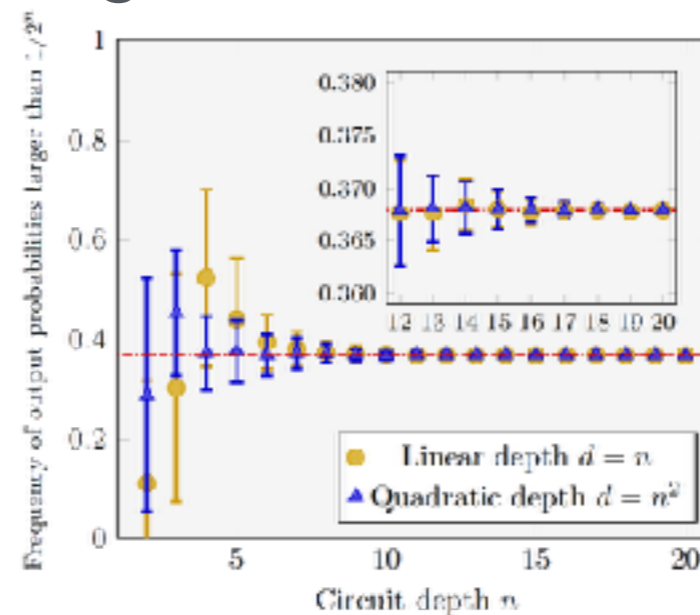
- **Polynomial hierarchy:** The polynomial hierarchy is infinite (i.p.  $P \neq NP$ )

- **Average case complexity:** It is as hard to sample from the distribution of outputs of a function as it is to compute the function value. (i.e., it is as hard to sample from the distribution of outputs of a function as it is to compute the function value.)

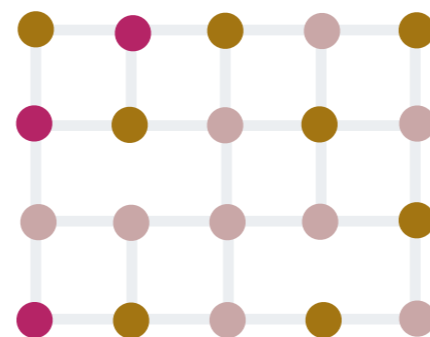
- **Anti-concentration:** The distribution of output probabilities is not concentrated, i.e.,

$$\text{prob}_U \left( |\langle a | U | \psi_\beta \rangle|^2 \geq \frac{1}{2^n} \right) > \frac{1}{2}$$

- Strong numerical evidence



- Rigorous proofs for random circuits



Hangleiter, Bermejo-Vega, Schwarz, Eisert, arXiv:1706.03786  
 Mann, Bremner, arXiv:1711.00686



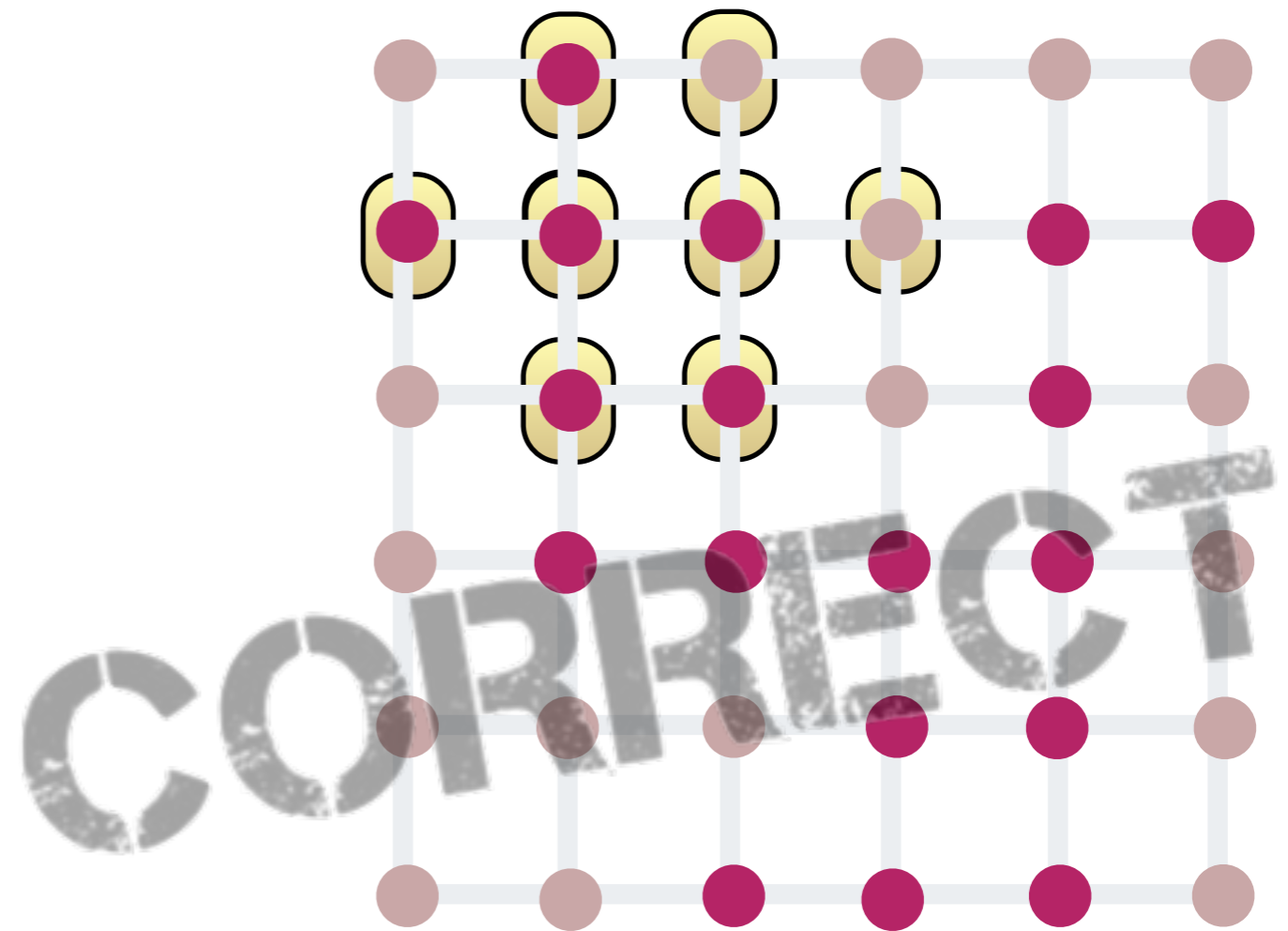
## Complexity theoretic conjectures

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- This quantum simulation is intractable for classical computers

# Certification of trustworthy simulators

- One can with  $\theta(N)$  many measurements detect closeness in  $l_1$ -norm!



Bermejo-Vega, Hangleiter, Schwarz, Raussendorf, Eisert, arXiv:1703.00466

Hangleiter, M. Kliesch, M. Schwarz, J. Eisert, Quantum Sci Technol 2, 015004 (2017)

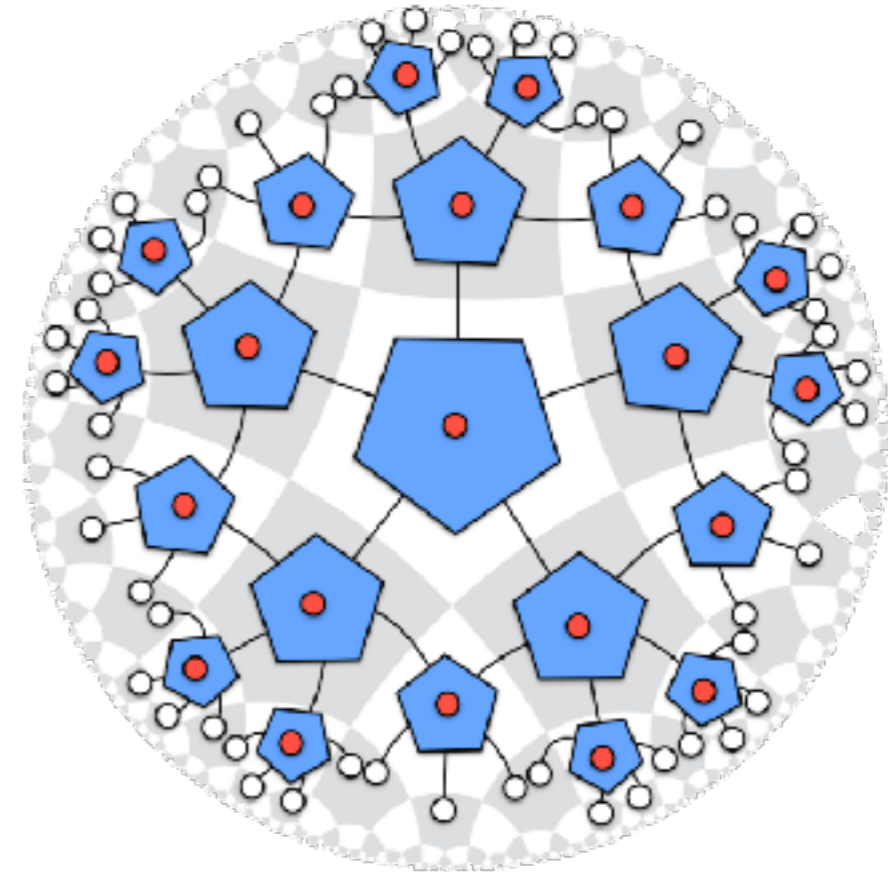
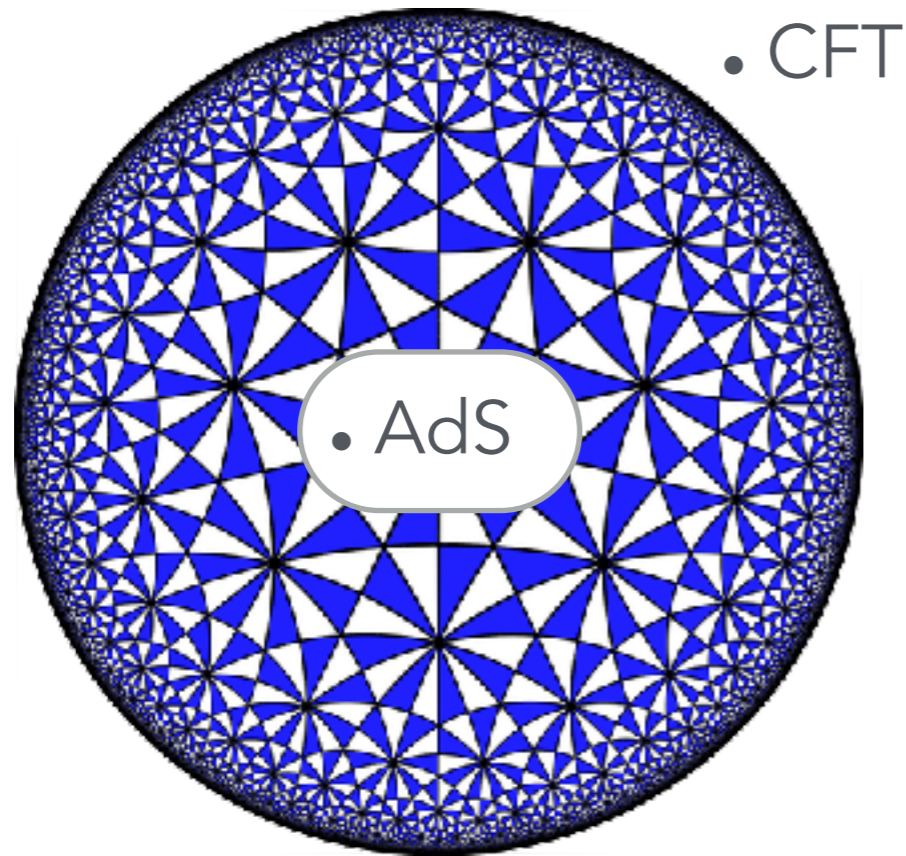
Cramer et al, Nature Comm 1, 149 (2010)

- **Common prejudice:** In order to be able to verify a quantum simulation, one needs to be able to efficiently simulate it

---

Teaser: Holography and criticality  
in matchgate tensor networks

# Toy models for the AdS/CFT correspondence



- Duality between Einstein gravity in  $D + 2$  Anti de Sitter spacetime and
- Conformal field theory in  $D + 1$  dimensions

- Quantum error correction: Holographic pentagon code

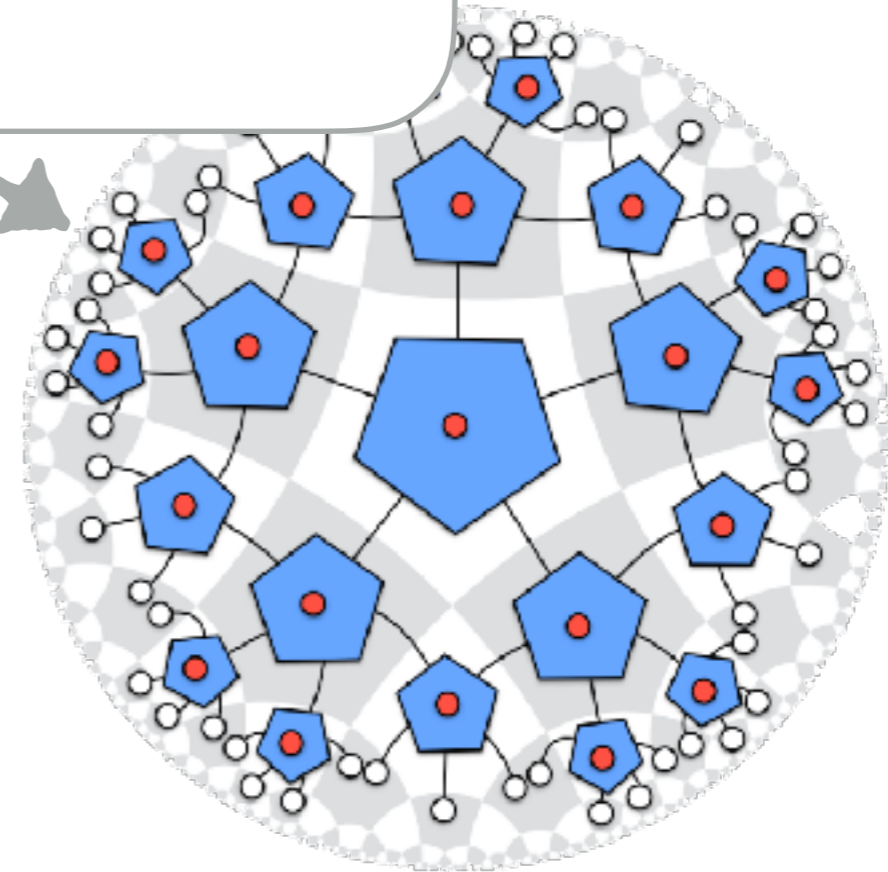
- Isometric tensor from bulk to boundary

- Realizes Ryu-Takayanagi formula

Almheiri, Dong, Harlow, JHEP 1504, 163 (2015)

Pastawski, Yoshida, Harlow, Preskill, JHEP 2015, 149 (2015)

Ryu, Takayanagi, Phys Rev Lett 96, 181602 (2006)



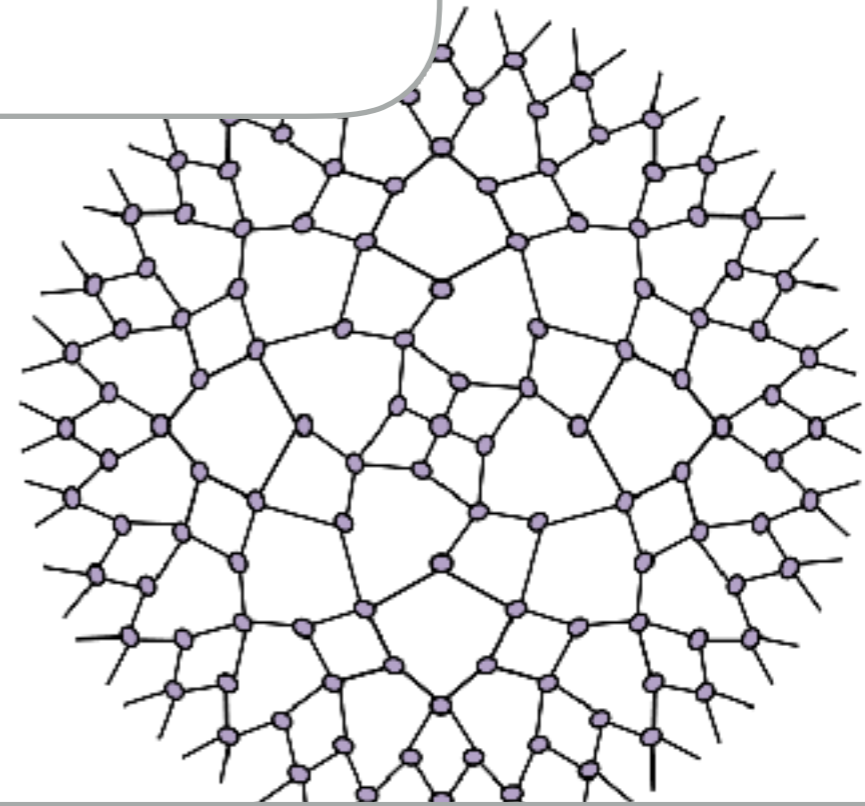
- **Quantum error correction: Holographic pentagon code**

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Almheiri, Dong, Harlow, JHEP 1504, 163 (2015)

Pastawski, Yoshida, Harlow, Preskill, JHEP 2015, 149 (2015)

Ryu, Takayanagi, Phys Rev Lett 96, 181602 (2006)



- **Multi-scale entanglement renormalization approach**

- Realise critical Ising theory on boundary

Vidal, Phys Rev Lett 101, 110501 (2008)

Evenbly, Vidal, Phys Rev B, 79, 144108 (2009)

Swingle, Phys Rev D 86, 065007 (2012)

Haegeman, Swingle, Walter, Cotler, Evenbly, Scholz, arXiv:1707.06243



- **Quantum error correction: Holographic pentagon code**

- Isometric tensor from bulk to boundary
- Realizes Ryu-Takayanagi formula

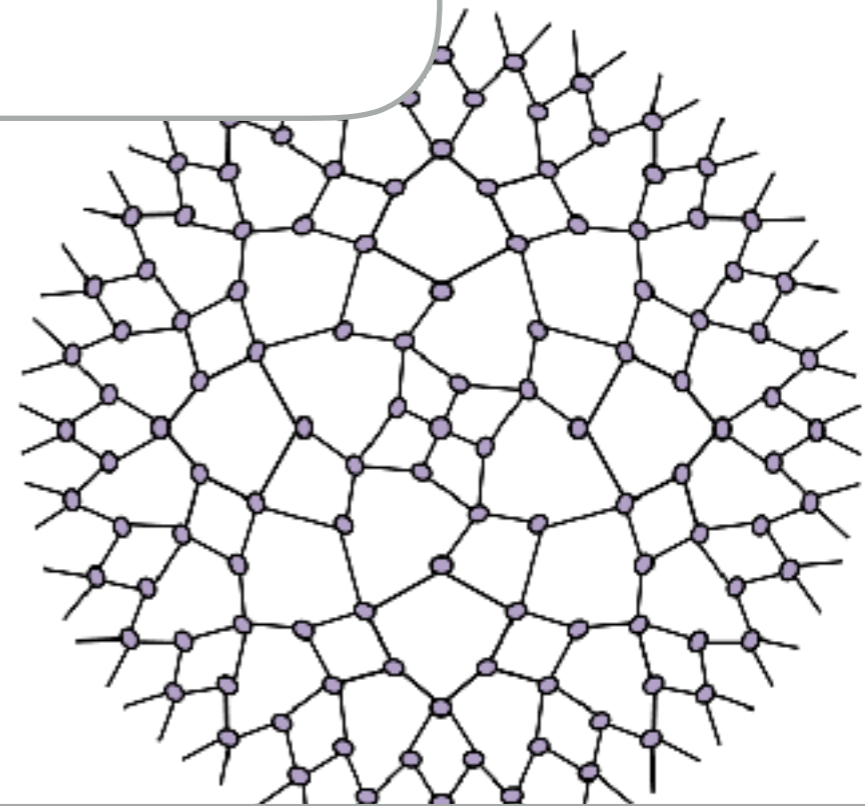
Almheiri, Dong, Harlow, JHEP 1504, 163 (2015)

Pastawski, Yoshida, Harlow, Preskill, JHEP 2015, 149 (2015)

Ryu, Takayanagi, Phys Rev Lett 96, 181602 (2006)

- **Random tensors**

Hayden, Nezami, Qi, Thomas, Walter, Yang,  
JHEP 2016, 9 (2016)



- **Multi-scale entanglement renormalization approach**

- Realise critical Ising theory on boundary

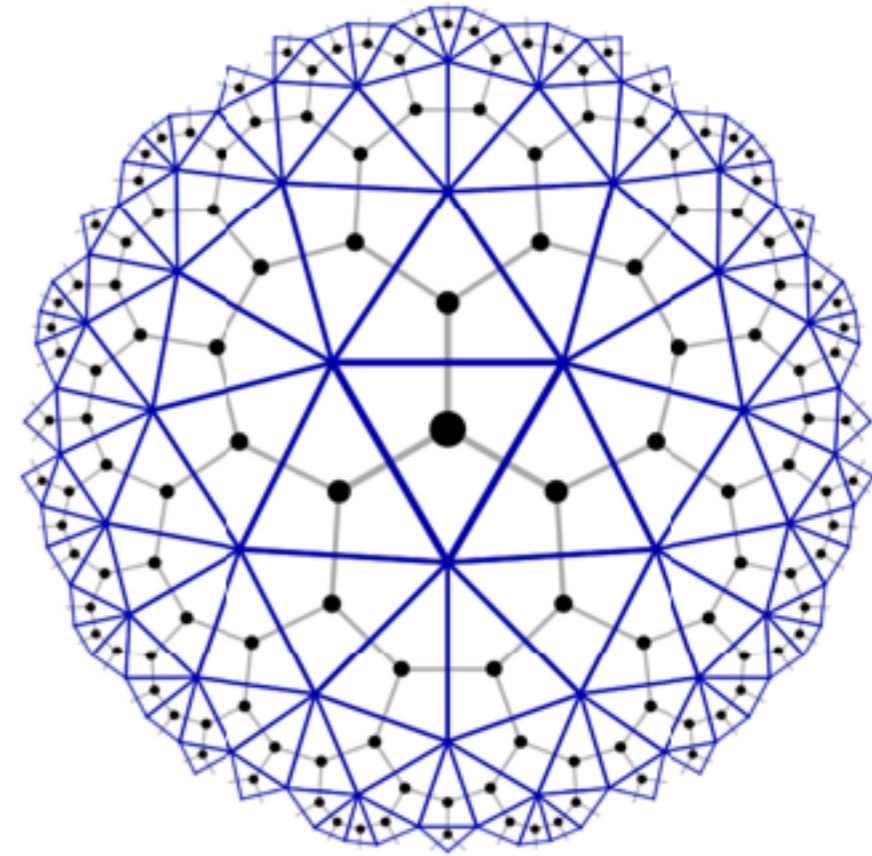
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# Matchgate tensor networks



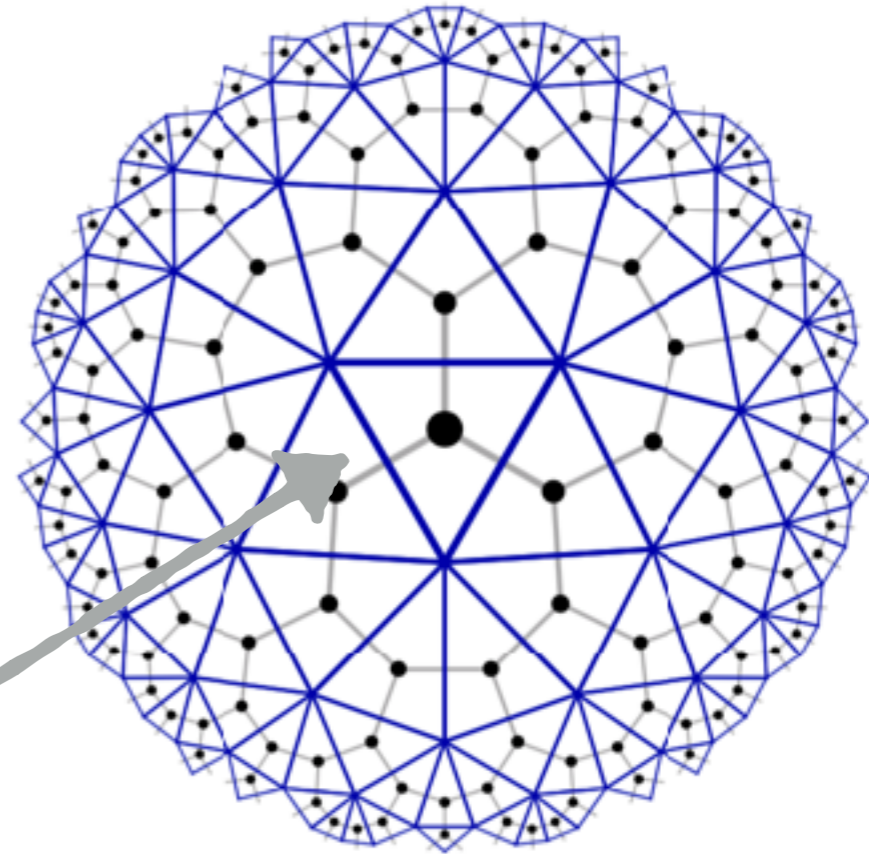
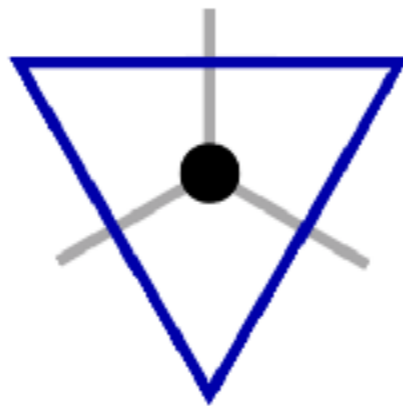
- Can one embody them in a larger framework?

# Matchgate tensor networks

- Matchgate tensor

$$T_v : \{0, 1\}^{\times r} \rightarrow \mathbb{C}$$

per vertex  $v \in V$



- Boundary state obtained by tensor contraction  $|\psi\rangle = \sum_{j \in \{0,1\}^{\times L}} \mathcal{T}(j) |j\rangle$

# Matchgate tensor networks

- Grassmann-variate characteristic function

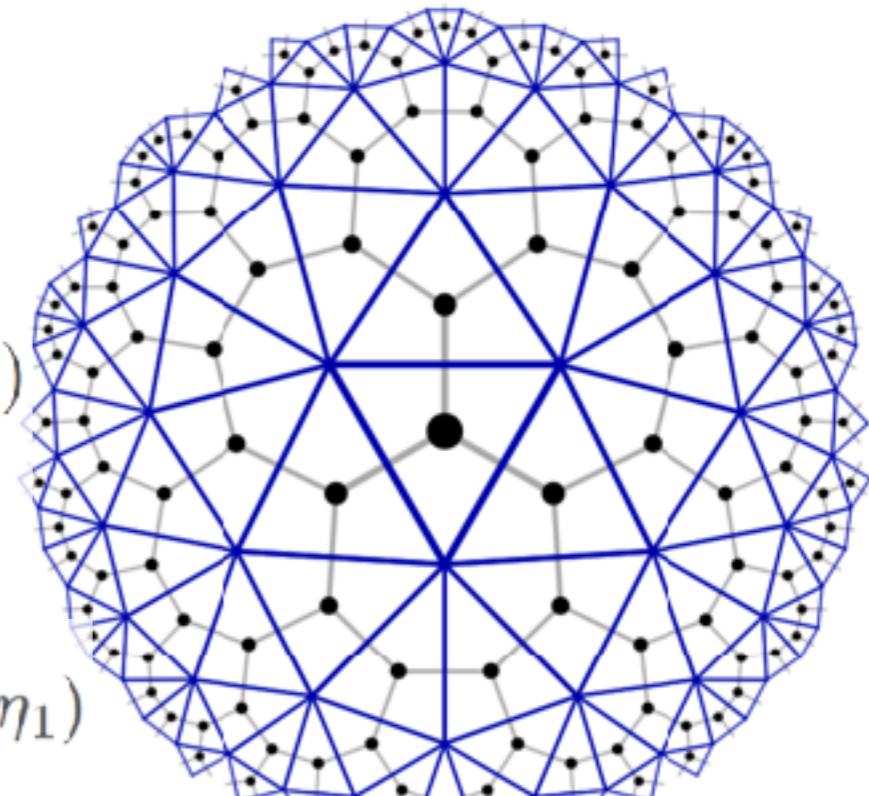
$$\Phi_T(\theta) = \sum_{j \in \{0,1\}^{\times r}} T(j) \theta_1^{j_1} \theta_2^{j_2} \dots \theta_r^{j_r}$$

$$\theta_k \theta_{k'} + \theta_{k'} \theta_k = 0$$

- Contraction  $T_{1 \star 2}(x, y) = \sum_{z \in \{0,1\}} T_1(x, z) T_2(z, y)$

gives

$$\Phi_{T_{1 \star 2}}(\xi) = \int d\eta_1 \int d\theta_{r_1} \Phi_{T_1}(\theta) \Phi_{T_2}(\eta) \exp(\theta_{r_1} \eta_1)$$



- Matchgate tensors:** Consider a rank- $r$  tensor  $T(x)$  with inputs  $x \in \{0, 1\}^{\times r}$ ,  $T(x)$  is a matchgate if there exists an antisymmetric matrix  $A \in \mathbb{C}^{r \times r}$  so that

$$T(x) = \text{Pf}(A_{|x \text{ XOR } z}) T(z)$$

where  $\text{Pf}(A)$  is the Pfaffian of  $A$  and  $A_{|x}$  is the submatrix of  $A$  acting on the subspace supported by  $x$

# Contraction

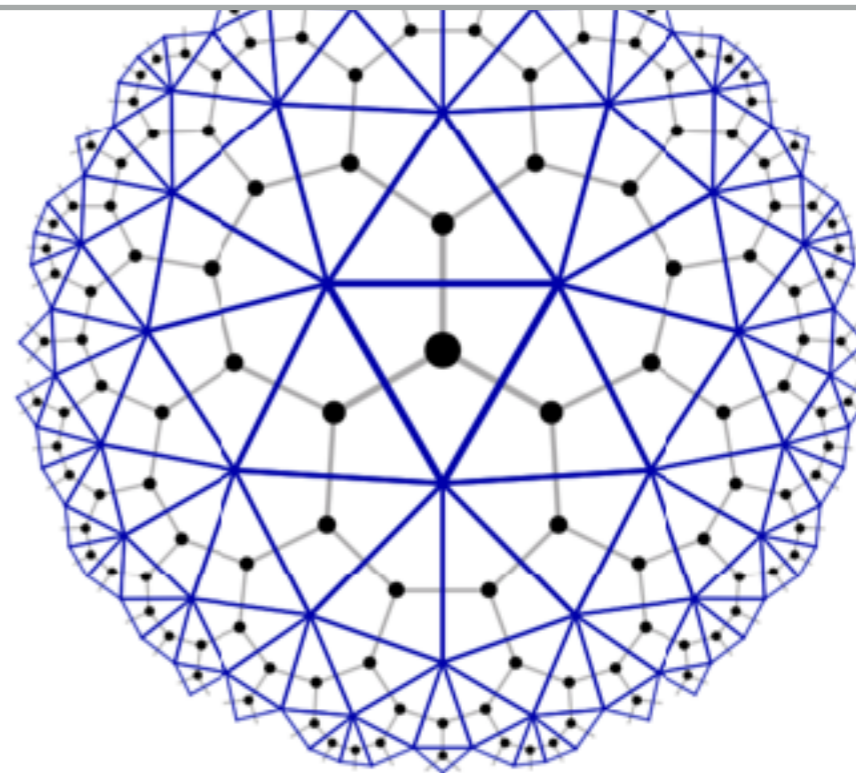
- **Observation:** The contraction requires  $O(L^2 N)$  steps for  $L$  boundary sites and  $N$  contracted tensors

- Generic even matchgate with  $\bar{z} = 0$  has

$$\Phi_T(\theta) = T(\bar{0}) \exp\left(\frac{1}{2} \sum_{j,k=1}^r A_{j,k} \theta_j \theta_k\right)$$

with generating matrix  $A$

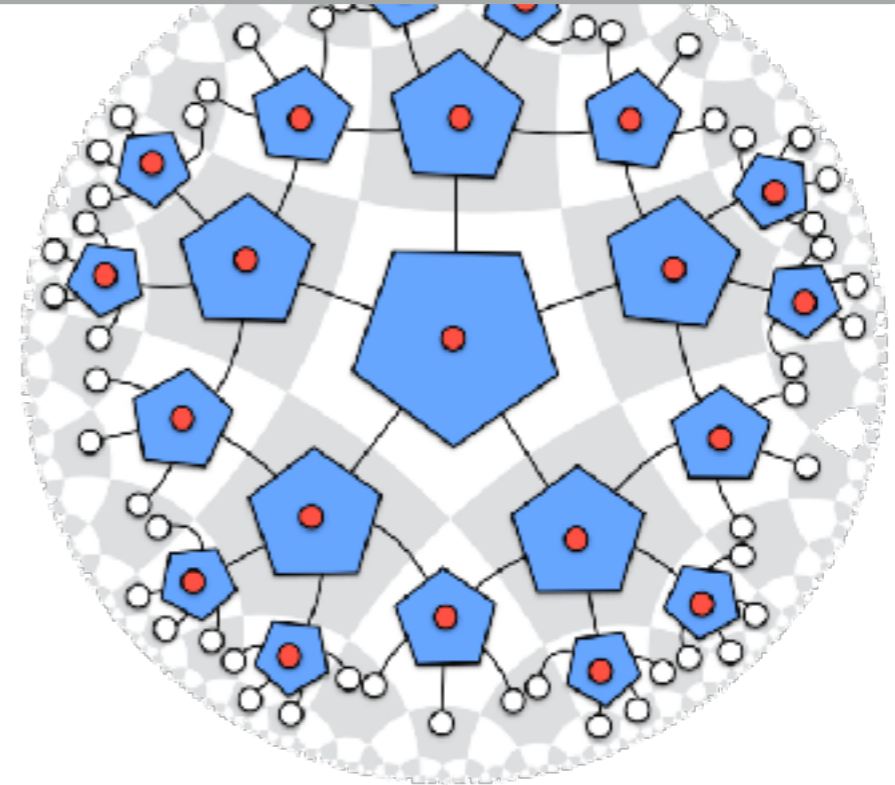
- Gives rise to various tilings, models, etc



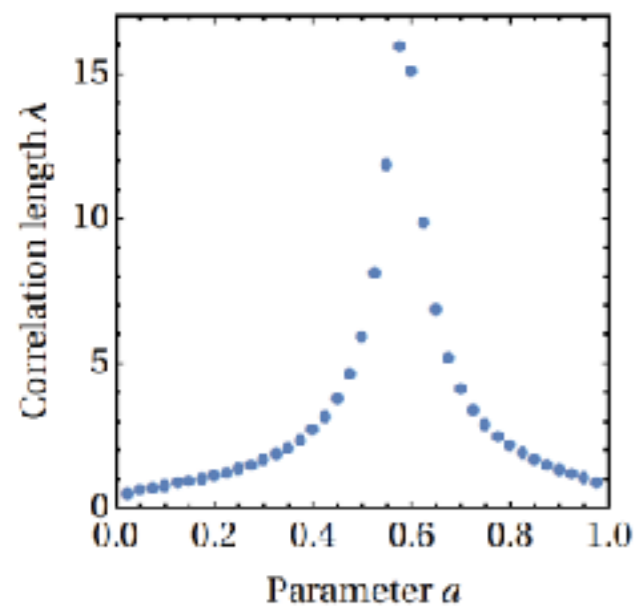
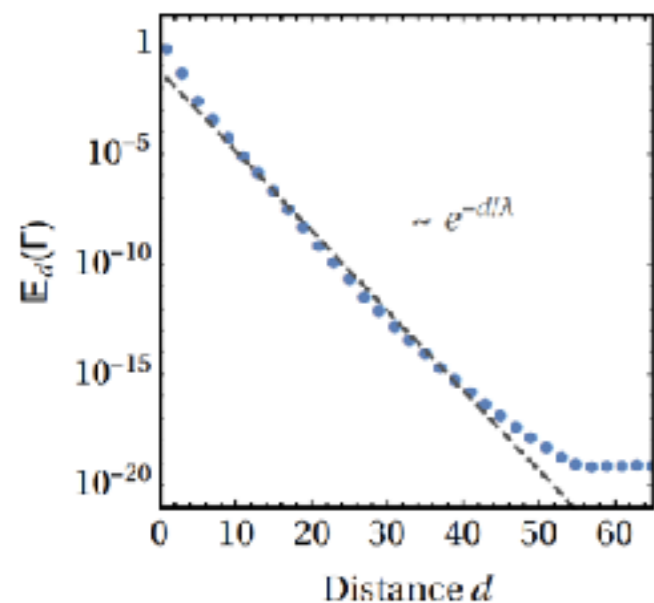


## Relationship to Pentagon code

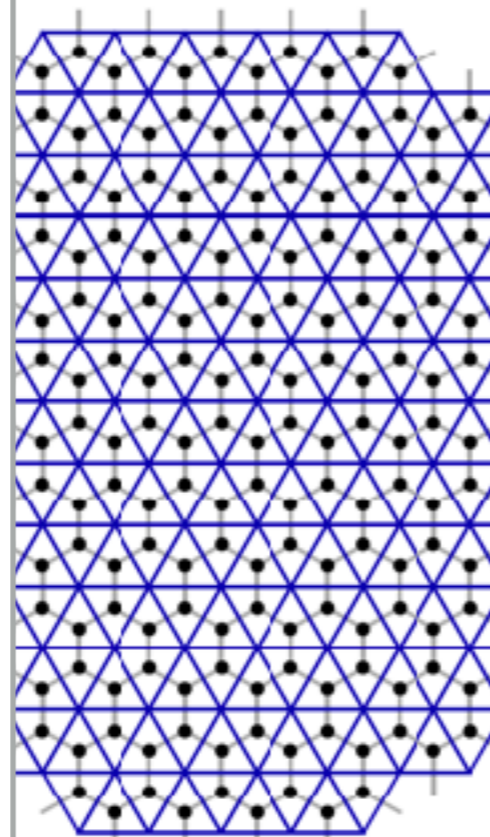
- **Observation:** The holographic pentagon code with computational basis input in the bulk yields a matchgate tensor on the boundary



- $\{3, 6\}$  gapped

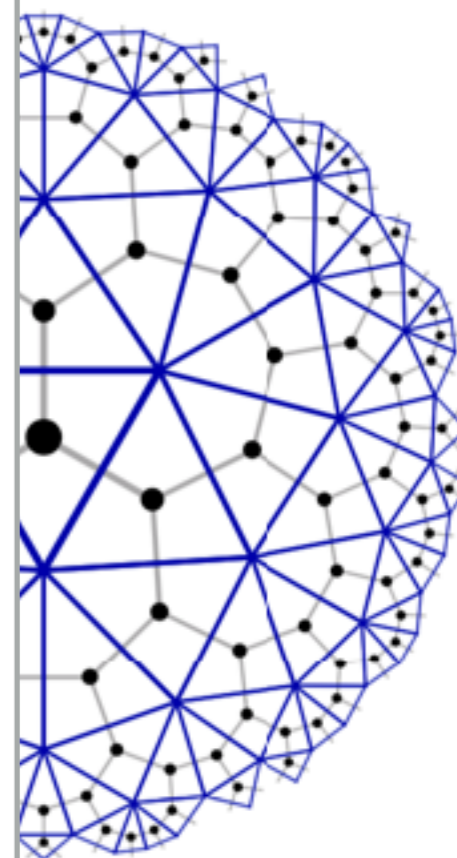
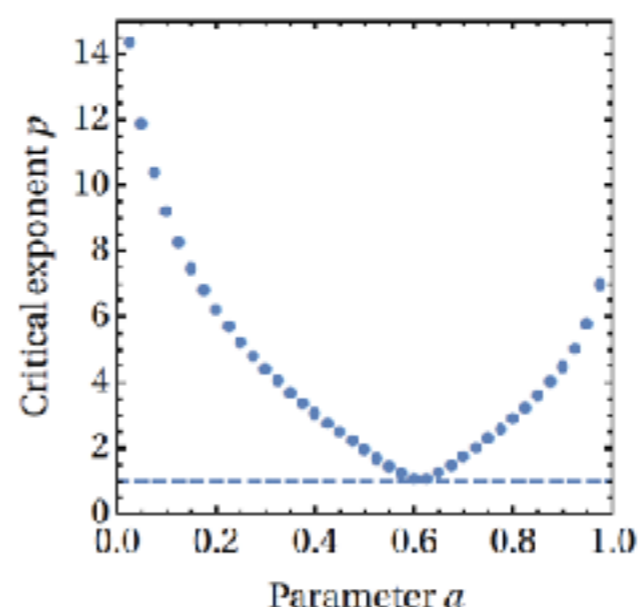
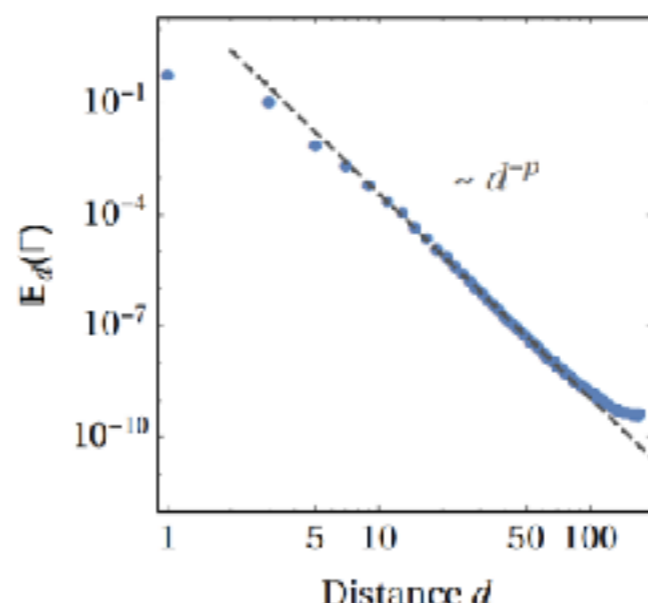


bulk curvature



$\{3, 6\}$  flat tiling

- $\{3, 7\}$  critical

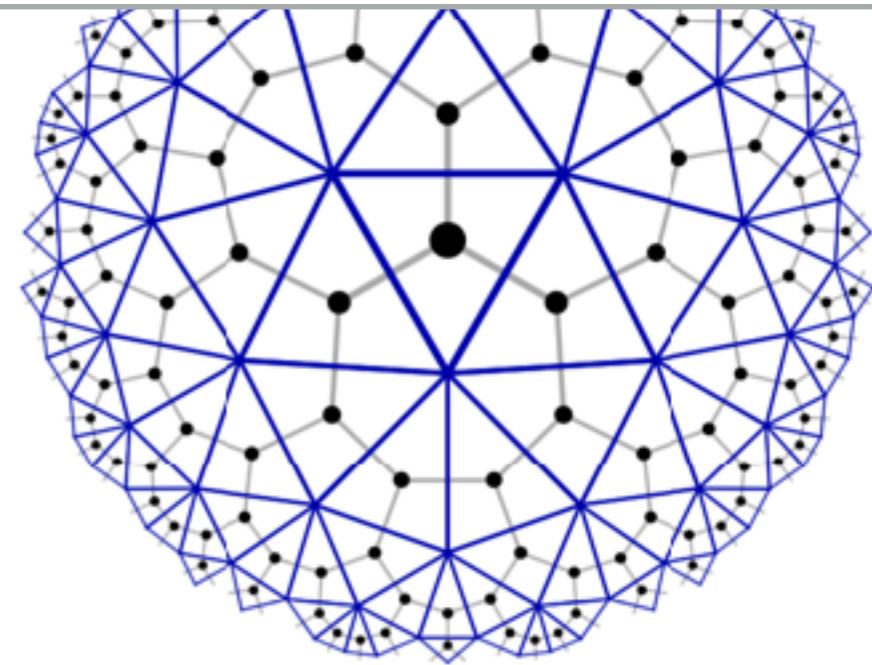
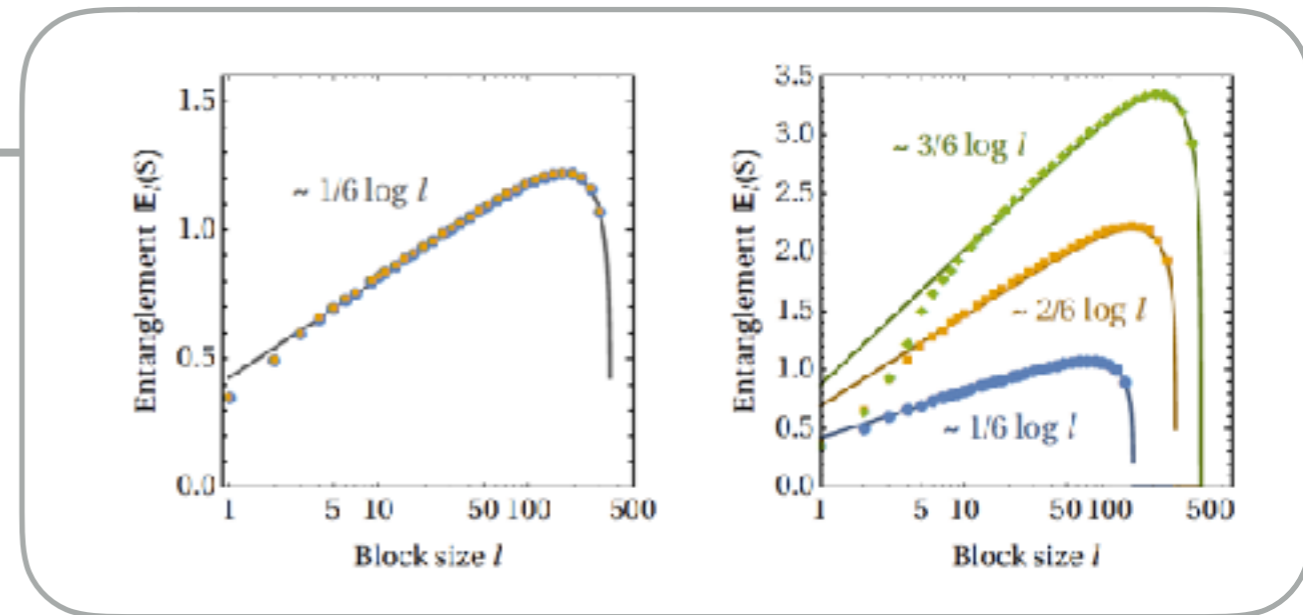


hyperbolic tiling

- Specified by anti-symmetric  $3 \times 3$ -matrix  $A$ , governed by parameter  $a$



# Entanglement entropy of CFTs



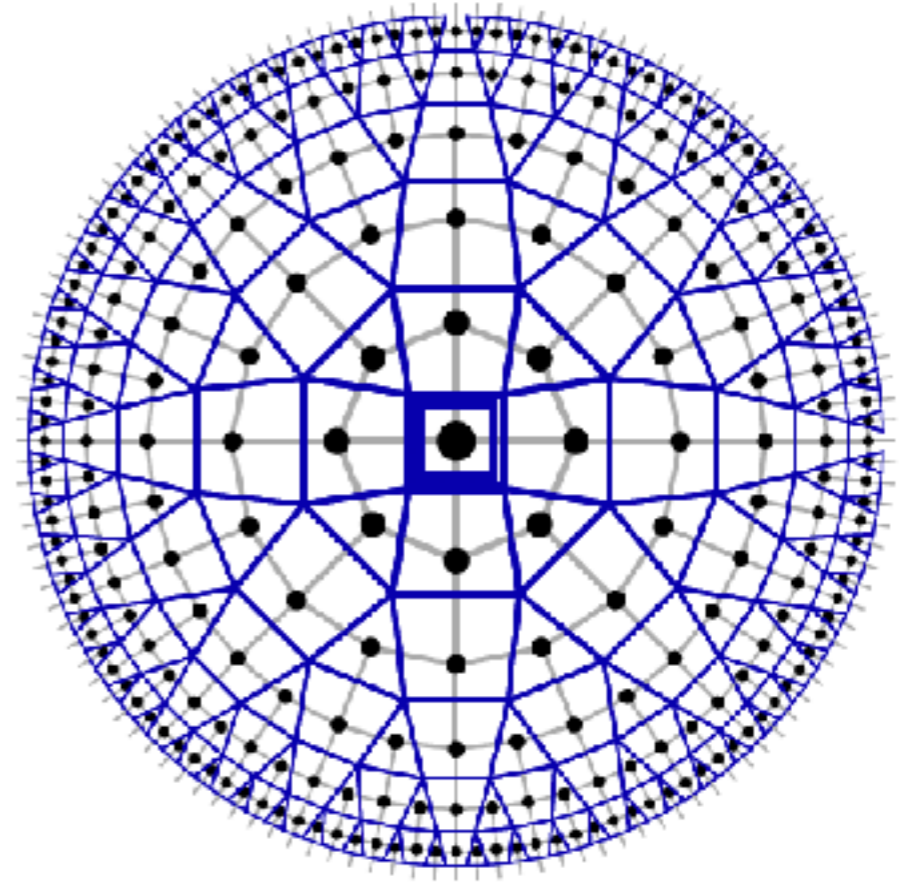
- $\{3, k\}$ ,  $k > 6$ , hyperbolic tiling

- CFT entanglement entropy: for  $c = 1/2$ ,  $\epsilon \approx 0.08$ , critical Ising theory

$$S_\ell = \frac{c}{3} \ln \left( \frac{L}{\pi \epsilon} \sin \frac{\pi \ell}{L} \right) \simeq \frac{c}{3} \ln \frac{\ell}{\epsilon} + O((\ell/L)^2)$$

and copies thereof

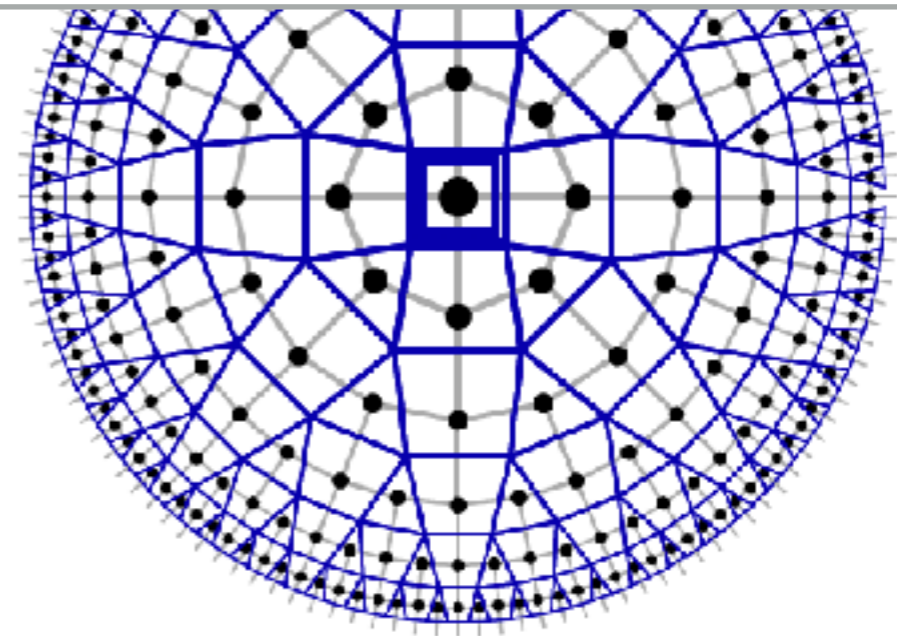
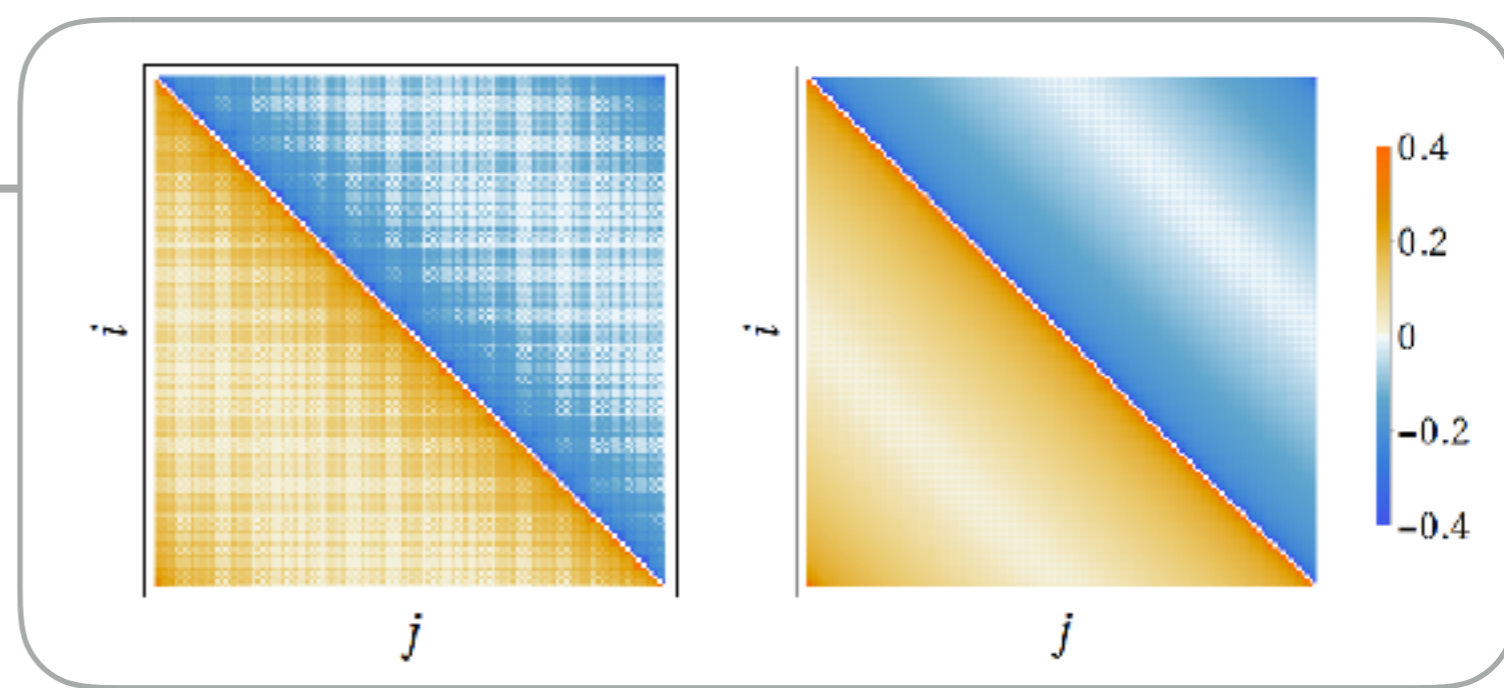
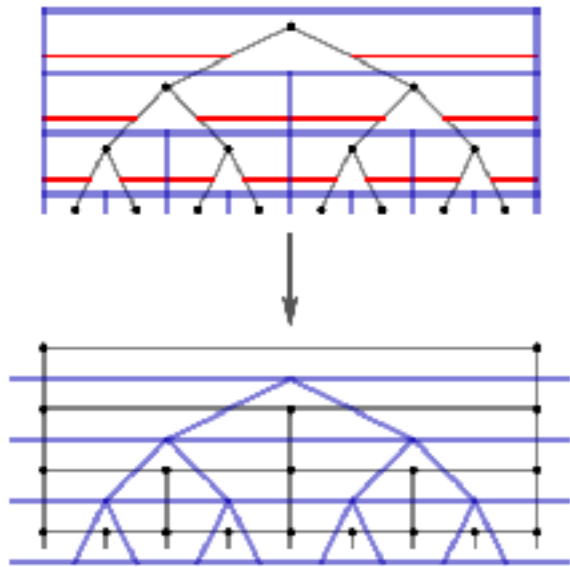
# Translationally invariant MERA



- Dual MERA of the hyperbolic plane

# Translationally invariant MERA

- Mapping between standard and block-dual MERA



- Dual MERA of the hyperbolic plane
- 3 and 4-leg tensors specified by  $3 \times 3$  matrix  $A$  and  $4 \times 4$  matrix  $B$
- Gives translationally invariant state to very good approximation

## More to play

- Lesson: Matchgate tensor networks provide versatile framework
  - Random tensors?
  - Interacting theories, connection to string-nets
  - Further connections to quantum error correction?
  - Etc



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# Summary



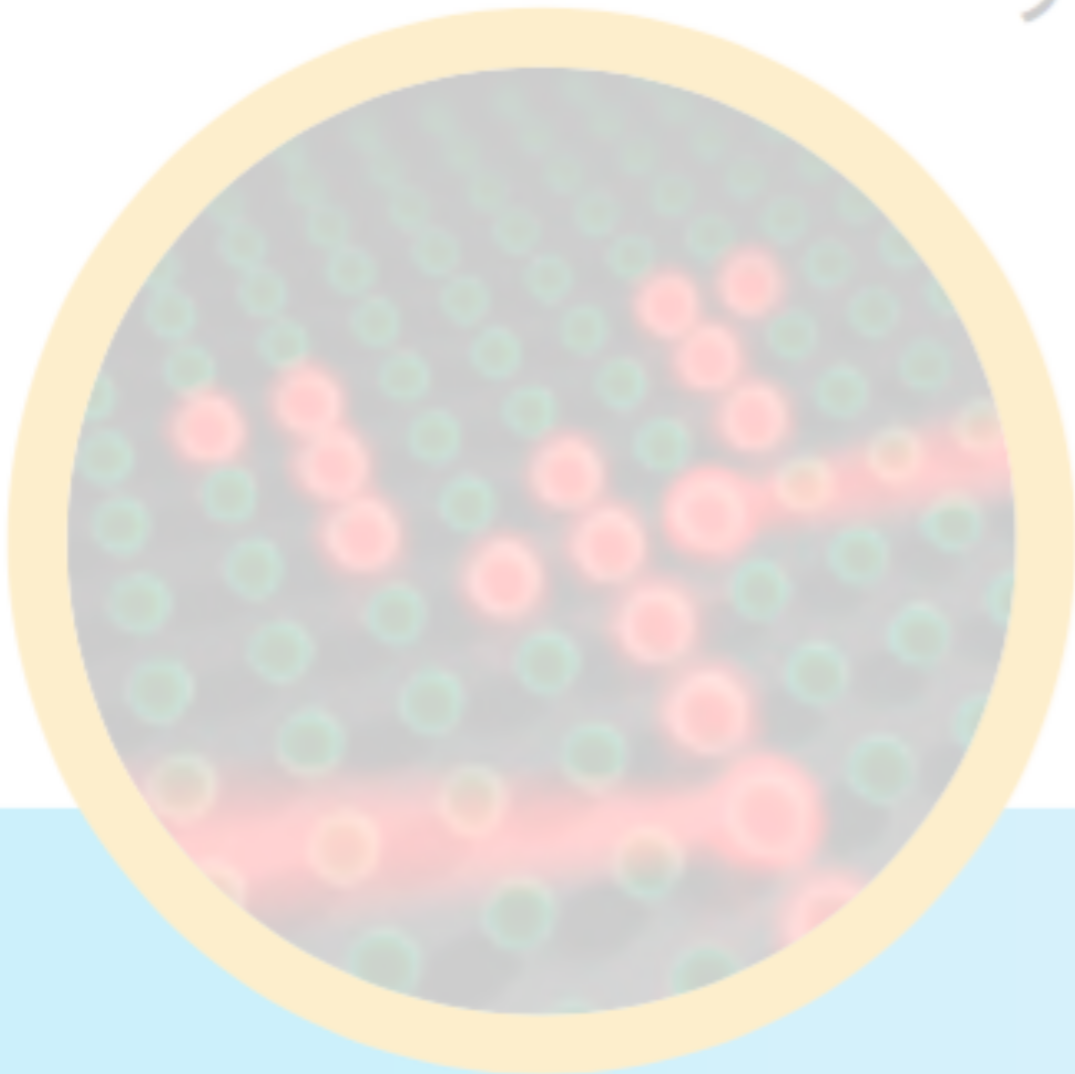
## Summary

- Hope for **feasible** quantum simulators with superpolynomial speedup

QUANTUM  
LINK

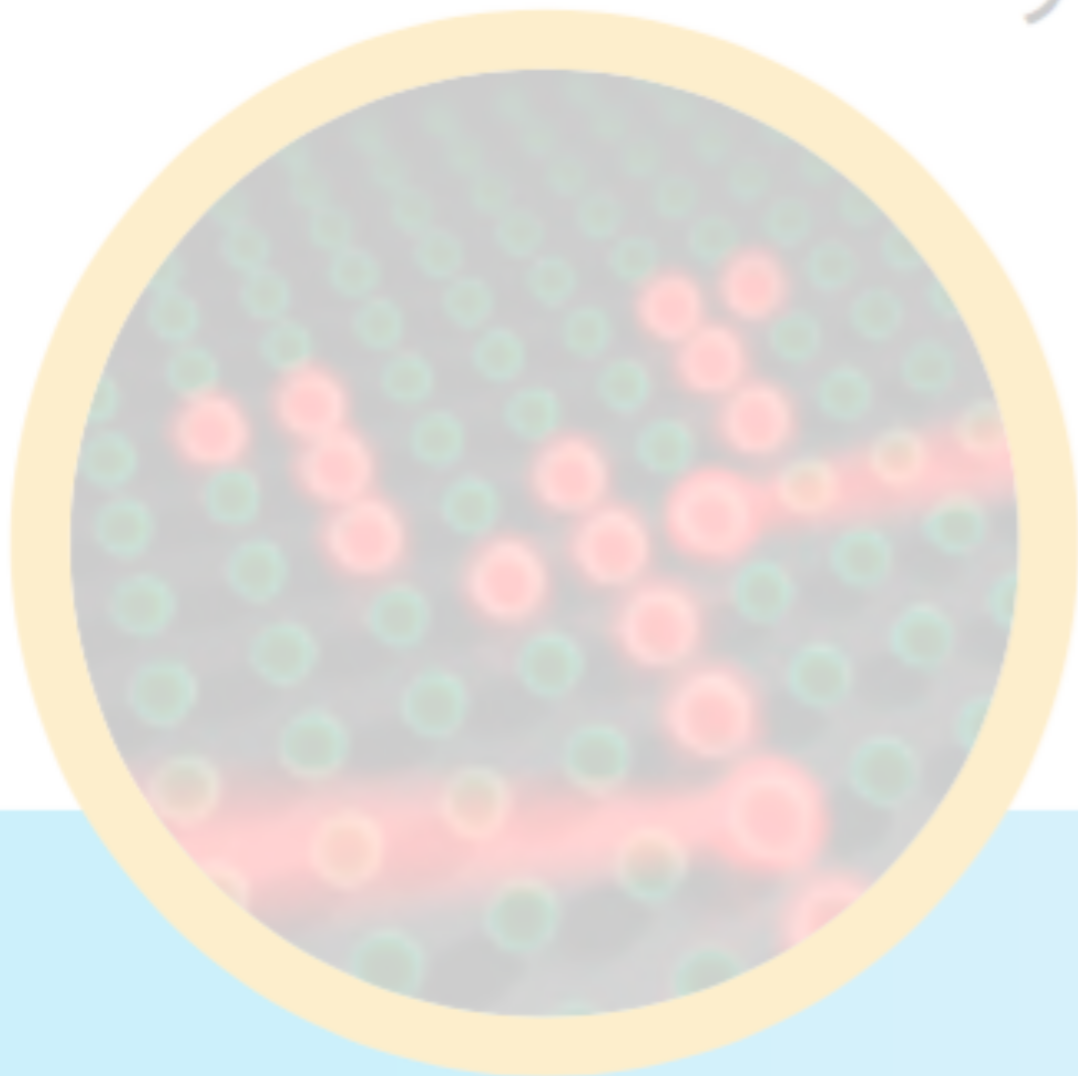
QUANTUM  
SIMULATOR

QUANTUM  
INTELLIGENCE



## Summary

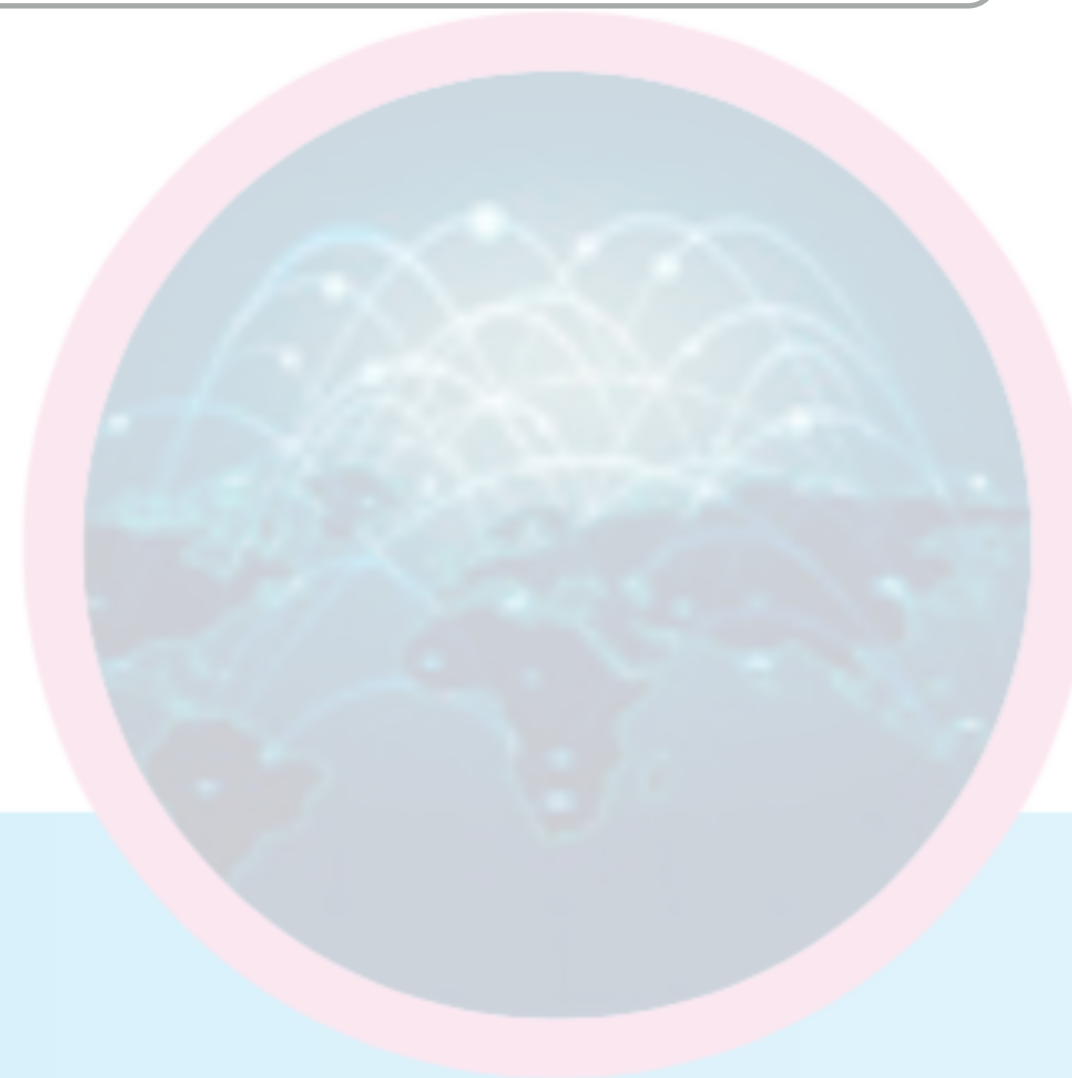
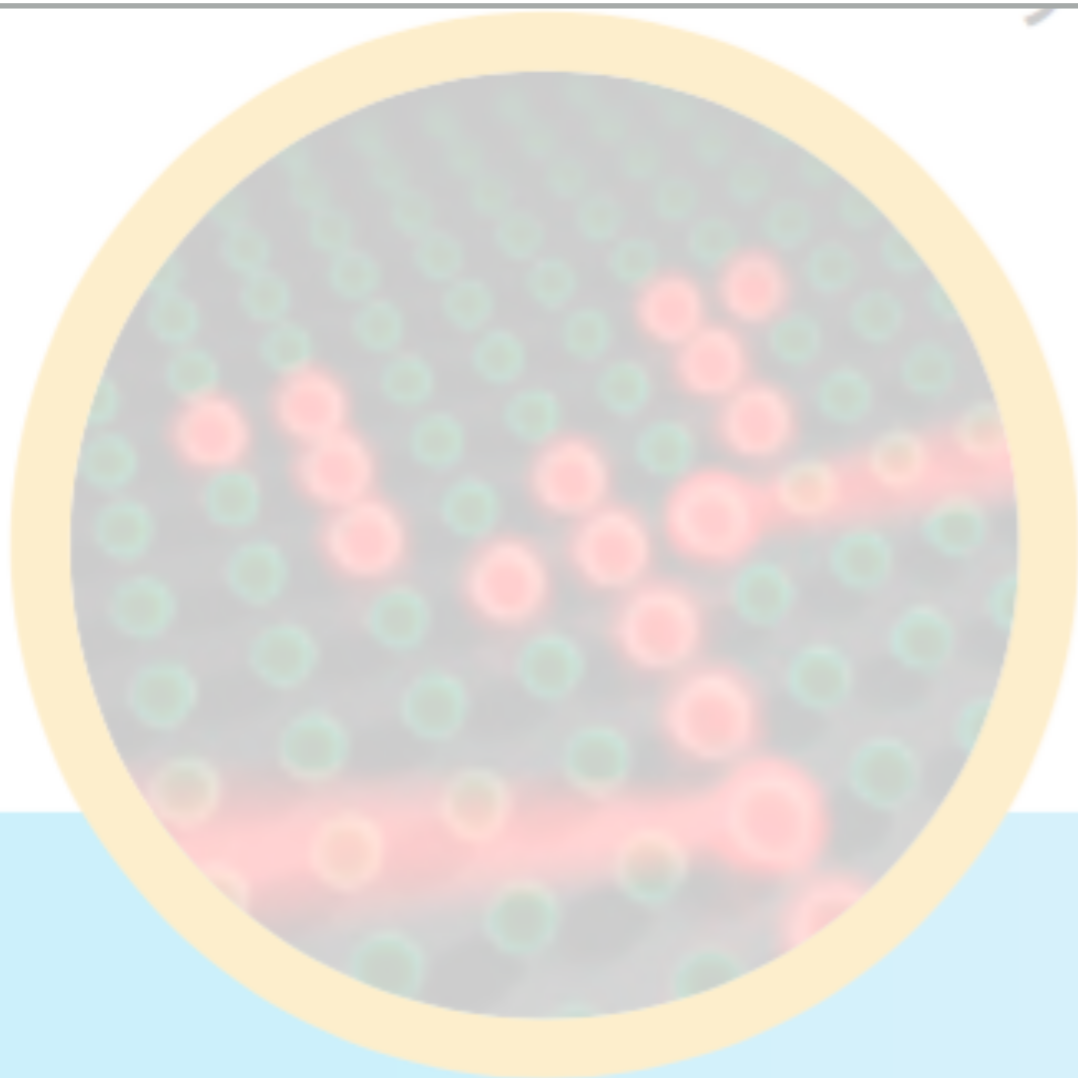
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- Is not fault tolerant, but can be **certified**





## Summary

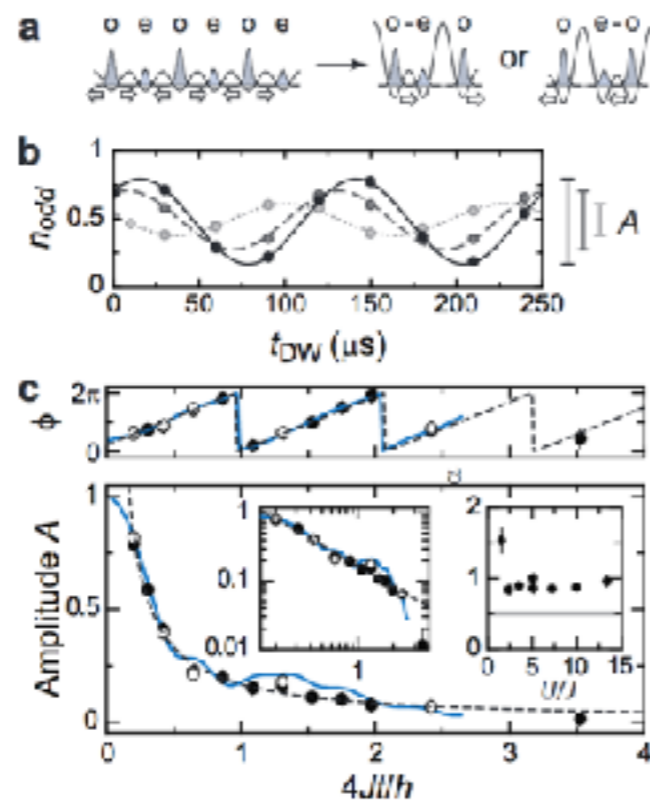
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- One can efficiently assess **correctness** - even if simulators exhibit quantum computational speedups



## Summary

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- Closer to physically more interesting schemes?



## Summary

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  - Is not fault tolerant, but can be **certified**
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- Closer to physically more interesting schemes?
  - Robustness of quantum simulators?

## Summary

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- Is not fault tolerant, but can be **certified**
- One can efficiently assess **correctness** - even if simulators exhibit quantum computational speedups

- Closer to physically more interesting schemes?
- Robustness of quantum simulators?
- Features of basic quantum error correction?



## Summary

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**Thanks for your attention!**