Accuracy Thresholds: Can we beat $10^{-4}$?

The History

Accuracy Thresholds

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[Work on various topics]

- Threshold Results
- Error Correcting Codes
  - BK's algorithms
  - Quantum Algorithms

Ongoing Work

Daniel Gottesman

Can we beat $10^{-4}$?
Accuracy thresholds: Can we beat 10^{-4}?  

But note importantly:  

\[ \frac{300}{\sqrt{3}} \approx 1 \]  

Pretty simple derivation  

New observation  

Putting together all tracers we know  

Used to think: \( \sim 10^4 \) (no proof)  

What is the limit of the  

Concatenation method?  

Main observation:  

Large codes, error grows exponentially  

Small codes, error grows polynomially  

Zoom in  

Zoom in
CSS codes:

\[
\sum_{\omega \in \Omega} \omega|x\rangle
\]

\[
FT
\]

In fact:

\[
H_2H_1 = \chi
\]

Correct only:

\[
I = (0, 0), x = (0, 0), y = (0, 0), z = (0, 0)
\]

How to correct errors?

CSS codes over \(F_2\)

Again:

Correct bit flips, correct bit flips.

Again:

\[
H_1 = \chi \rightarrow \chi^y \rightarrow \chi^x \rightarrow \chi^y \rightarrow \chi^x \rightarrow \chi^y \rightarrow \chi^x \rightarrow \chi^y
\]

Correcting errors over \(F_2\).

\[
x, y, z
\]

\[
\forall \theta \in F_2
\]
3. Fault Tolerant Gates

All gates can be applied

2. Polynomial codes [A. Ben-Or '80]
Repeat for phase flips in $H$ basis.

Connect

Find error classifier

Measure

Correct

Error correction

Use teleportation:

Degree reduction:
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corrected

1. Correct from 15.7 to 15.7

2. Error connection A |A, A

3. Error connection A |A, A

4. A |A, A

5. Correct to 15.7

6. Creating reliable ancillas

7. Creating reliable ancillas

8. Calculate thresholds

9. Constructing recipe for thresholds
Accuracy thresholds: Can we beat $10^{-4}$?

Where $P_r (\text{error in one quarter}) = 3 \times 10^{-4}$.

For large codes: Drops exponentially.

For small codes, drops slowly above expectation. Main observation.

\[ \frac{m}{n} = \frac{3}{4} \]
\[ M = \frac{t}{2} \]
\[ M = \frac{3r}{4} \]

\[ \frac{1}{e} \leq \frac{1}{4} \]
\[ \frac{1}{r} \]

Therefor: $\geq \frac{1}{n}$ gates per quarter (3n)

Proof: For $\epsilon > 0$, $\sqrt{n} > 3\epsilon n$.

How to calculate $A$, threshold?
Higher Dimensions

\[ p = 2, \quad E^2 = y = 10^{-4} \]

Thresholds

Solutions

Recall that E fields are

\[ \mathbf{F} \rightarrow (\mathbf{F}, \ldots, \mathbf{F}) \]

\[ \mathbf{F} \rightarrow \mathbf{F} \]

Each coordinate is

\[ \Rightarrow \quad |1| \text{ large.} \]

Fail. Large \( \Rightarrow \) actual program.

Problem.
Accuracy thresholds: Can we beat 10^{-4}?