Towards Quantum Computation with Trapped Ca\(^+\) Ions

- Ion traps for quantum computation
- Ion motion in linear traps
- Nonclassical states of motion, decoherence times
- Addressing individual ions
- Sideband cooling of the common motion
- Heating and cooling of an ion string
- Entanglement with trapped ions
- Cavity QED with a single ion

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What do you need for a quantum computer?

- systems with longlived quantum states
- single quantum systems which can be initialized, measured and manipulated
- controllable interaction between quantum systems
## Quantum Computer: Implementation with Trapped Ions

<table>
<thead>
<tr>
<th>HARDWARE/OPERATION</th>
<th>REQUIREMENTS</th>
<th>TRAPPED IONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-bits</td>
<td>long coherence times</td>
<td>isolated in free space (frequency standards)</td>
</tr>
<tr>
<td>Q-register</td>
<td>row of Q-bits</td>
<td>linear ion traps, ion strings</td>
</tr>
<tr>
<td>Q-gate</td>
<td>interaction between Q-bits, operations on individual Q-bits</td>
<td>Coulomb repulsion spatial separation allows one to address individual ions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>state preparation</th>
<th>Optical cooling, state preparation with laser pulses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computation</td>
<td>coherent state manipulation</td>
<td>internal, external excitations, entanglement</td>
</tr>
<tr>
<td>Output</td>
<td>state measurement (100% efficiency)</td>
<td>Quantum jump technique</td>
</tr>
</tbody>
</table>
Ion storage generics

Ion confinement requires a focusing force in 3 dimensions:

\[ \vec{F} \sim -\vec{r} \Rightarrow \vec{F} = e\vec{E} = -e\nabla \Phi \Rightarrow \Phi \sim \vec{r}^2 \]

quadrupole potential

\[ \Phi = \frac{\Phi_0}{r_0^2}(x^2 + y^2 - 2z^2) \]

Paul trap: \( \Phi_0 = U_0 + V_0 \cos \Omega t \)  

Penning trap: \( \Phi_0 = U_0 + \text{axial magn. field} \)

equation of motion in a Paul trap:

\[ a \sim U_0, q \sim V_0 \]

\[ \ddot{x} + (a - 2q \cos \Omega t) \frac{\Omega^2}{4} x = 0 \]

MATHIEU EQUATION

frequencies of secular motion: \( \omega_x, \omega_y, \omega_z \) 

superimposed is micromotion with: \( \Omega \)

\[ \omega \approx (a + \frac{1}{2} q^2) \Omega \]
Paul trap

endcap electrode

ring electrode

lens

fluorescence detection

cooling beam

z

y

x
Quantum Computer with Trapped Ions


- ions in linear trap
  - quantum bits, quantum register
    - narrow optical transitions
    - groundstate Zeeman coherences
  - 2-qubit quantum gate
  - state vector of quantum computer
  \[ \Psi = \sum_x c_x |x_{L-1} \ldots x_0 \rangle \otimes |0\rangle_{CM} \]

- state measurement with 100% efficiency, quantum jump technique
- decoherence small (!?), heating small (!?)
- quantum computation as a series of quantum gate operations (series of laser pulses)
Linear Ion Traps

Paul mass filter

Innsbruck
Los Alamos

Boulder, Mainz, Aarhus

München

Boulder
Innsbruck linear ion trap

\[ \omega_z \approx 700 \text{ kHz} \quad \omega_{x,y} \approx 1.2 - 2 \text{ MHz} \]
Innsbruck linear ion trap (2000)

$\omega_z \approx 0.7 - 2 \text{ MHz} \quad \omega_{x,y} \approx 1.5 - 4 \text{ MHz}$
String of Ca\(^+\) ions in a linear Paul trap
qubit on narrow S - D quadrupole transition
\[ \tau \approx 1 \text{s} \]
Three required steps

- Absorption spectrum: Resolve secular motion
  
- Additional cooling stage: Sideband cooling

- Addressing individual ions

200 kHz

"red" "blue" sideband

S\textsubscript{1/2} D\textsubscript{5/2}

729 nm
electrooptic deflector
Spectroscopy with quantized fluorescence (quantum jumps)

- P: Monitor absorption and emission cause fluorescence steps (digital quantum jump signal).
- D: Laser detuning.
- S: Histogram of absorption events.
State detection by quantized fluorescence

D state occupied
S state occupied

detection efficiency:

99.85%
Excitation spectrum of a single ion
Narrow Carrier Resonance

Laser Detuning at 729 nm (Hz)

D-state excitation probability

Spectral resolution: $6.6 \cdot 10^{-13}$

270 (90) Hz
Ramsey spectroscopy on quadrupole transition

Vary T: measure phase coherence on superposition states

\[ |S_{1/2}\rangle + |D_{5/2}\rangle \]

1/e coherence time: 2ms
Laser linewidth: 75(10)Hz
Laser Cooling of Trapped Atoms

\[ g_n,1 - e_n,1 - e_n,2 + g_n,1 + g_n,2 \]

\[ \Gamma < \nu \] weak confinement, Doppler cooling
\[ E_D = \hbar \Gamma / 2, \langle n \rangle \gg 1 \]

\[ \nu > \Gamma \] strong confinement, sideband cooling
\[ E_S = \hbar \nu \left( \Gamma^2 / 4 \nu^2 + 1/2 \right), \langle n \rangle \ll 1 \]

Regimes:
Absorption on quadrupole transition

(with motional sidebands)

\[ P_0 \approx \frac{A_B - A_R}{A_B} \]

Absorption events

Laser detuning (MHz)
Sideband cooling on $S_{1/2} - D_{5/2}$ transition

Cooling cascade:


Effective two-level system:

\[
\Gamma_{\text{eff}} \approx \frac{\Omega_{PD}^2}{\Gamma_{SP}^2 + 4\Delta_{PD}^2} \Gamma_{SP}
\]
Sideband absorption spectrum

99.9 % ground state population

after sideband cooling

after Doppler cooling $\langle n_z \rangle = 1.7$

Generation and manipulation of Fock states

- |d,0>
- |d,1>
- |d,2>
- |s,0>
- |s,1>
- |s,2>
- |d,0>
- |d,1>
- |p,1>
- |d,2>
- |d,3>
- |s,0>
- |s,1>
- |s,2>
- |s,3>

π-pulse
Preparation of Fock states


Population of D state

Time (µs)

nr=0, nz=0

nr=0, nz=1
Cooling and heating


cooling: $0.2 \text{ ms} \text{ phonon}$

heating:
- radial: $70 \text{ ms} \text{ phonon}$
- axial: $190 \text{ ms} \text{ phonon}$
Addressing of individual ions in a linear Paul trap

Experimental setup:
- Laser beam steering with an electrooptic deflector
- Fiber output at 729nm
- Paul trap viewport
- Fiber output at 729nm
- Telescope
- Dichroic beamsplitter
- Detection at 397nm
- CCD

Intensity of addressing beam at ion position


waist:

3.7 (0.3) µm

Intensity determined by measuring the light shift of the addressing beam.
Addressing of ions in linear trap

Measurement of light shift by addressing beam

Beam diameter: $2w_0 = 2.7 \, \mu m$
Excitation spectrum of two ions

Vibrational Modes:
- A: Axial
- R: Radial
- B: Breathing

\[ \omega_R = \sqrt{3} \omega_A \]
\[ \omega_W = \sqrt{\omega_A^2 - \omega_R^2} \]
Sideband cooling of two ions

Detuning at 729 nm (MHz)

RED sidebands

- $v_z$
- $\sqrt{3}v_y$
- $v_y$

BLUE sidebands

$P_0 > 98\%$
$P_0 > 96\%$
$P_0 > 95\%$
Two ion cooling and heating

cooling of the rocking mode

\[ \omega_R = \sqrt{\omega_{axial}^2 - \omega_{radial}^2} \]

\[ \tau_{cool} = 1 \text{ ms} \]

\[ \tau_{heat} = \frac{120 \text{ ms}}{\text{phonon}} \]
Rabi oscillations of two ions, one illuminated

[Diagram showing Rabi oscillations with ground state probability over time and energy levels |e,0⟩ and |g,1⟩ indicated.]
Rabi oscillations of two ions, one illuminated

![Graph showing Rabi oscillations](qt0354, 30.5.2000)

- **Ground State Probability** vs. **Time (µs)**

- **States**: $|e,0\rangle$, $|e,1\rangle$, $|g,0\rangle$, $|g,1\rangle$

- **Note**: blue sideband of CM motion
Off-resonant carrier excitation

Problem:
- AC Stark shifts
- off-resonant (carrier) excitation (spectator modes)

Solution:
cooling of all spectator modes

\[ \omega_z = 2\pi \cdot 1.8 \text{MHz} \]
\[ \Omega_{\text{carrier}} = 2\pi \cdot 1.09 \text{MHz} \]
Ground state cooling with quantum interference


\[ |n\rangle \rightarrow |n-1\rangle \] transitions are enhanced by bright resonance

\[ |n\rangle \rightarrow |n\rangle \] transitions are suppressed by quantum interference
Levels and transitions in $^{40}\text{Ca}^+$

EIT cooling
Zeeman splitting with $B = 4$ G

$|P, -\rangle$  $|P, +\rangle$

$|S, -\rangle$  $|S, +\rangle$

Doppler

75 MHz

$\sigma_+$

$\pi$
Simultaneous ground state cooling


Simultaneous ground state cooling of axial and radial motion

axial: P(0)=73%
radial: P(0)=58%

S_{1/2} to D_{1/2} excitation probability

<table>
<thead>
<tr>
<th>Sidebands</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial lower</td>
<td>-3.2 MHz</td>
</tr>
<tr>
<td>Radial lower</td>
<td>-1.6 MHz</td>
</tr>
<tr>
<td>Radial upper</td>
<td>+1.6 MHz</td>
</tr>
<tr>
<td>Axial upper</td>
<td>+3.2 MHz</td>
</tr>
</tbody>
</table>

lower sidebands  upper sidebands
Ground state cooling with quantum interference

measured cooling limit vs. light shift

Best radial (1.6 MHz) cooling:

\[ P(0) = 80\% \]

Best axial (3.2 MHz) cooling:

\[ P(0) = 90\% \]

from Doppler limit:

\[ \langle n \rangle = 17 \text{ (radial)}, \]
\[ \langle n \rangle = 8 \text{ (axial)} \]

EIT cooling: Ideal scheme for ion strings

Calculated EIT cooling for a string of 10 ions

\[ z/2 = 0.7 \text{MHz} \]
Cirac-Zoller: Steps of controlled-NOT operation

Atom #1

\[ |e\rangle_1 |0\rangle \quad \rightarrow \quad |g\rangle_1 |1\rangle \quad \xrightarrow{\pi} \quad |e\rangle_1 |0\rangle \]

Atom #2

\[ |e^{\prime}\rangle_2 |0\rangle \quad \rightarrow \quad |g\rangle_2 |1\rangle \quad \xrightarrow{2\pi} \quad |e^{\prime}\rangle_2 |0\rangle \]

Atom #1

\[ |e\rangle_1 |0\rangle \quad \rightarrow \quad |g\rangle_1 |1\rangle \quad \xrightarrow{\pi} \quad |e\rangle_1 |0\rangle \]

Qubits

<table>
<thead>
<tr>
<th>State</th>
<th>00 \rangle</th>
<th>10 \rangle</th>
<th>01 \rangle</th>
<th>11 \rangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>\langle gg</td>
<td>\langle eg</td>
<td>\langle ge</td>
<td>\langle ee</td>
</tr>
<tr>
<td>10</td>
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<td>\langle ee</td>
<td>\langle ee</td>
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Initialize after 1st \( \pi \)-pulse lower sideband

After 1st \( \pi \)-pulse lower sideband

After 2\( \pi \)-pulse to auxiliary state

After 2nd \( \pi \)-pulse lower sideband
Cirac-Zoller gate with experimental imperfections

$\pi$, $2\pi$ pulses: 2% imperfection, addressing error: 10% error in Rabi frequency,
5 % error in state preparation

π pulse 1st ion

2π rotation 2nd ion

$-\pi$ pulse 1st ion

Calculations by H. Häffner, 2001
heating is NOT the dominant problem in the near future

decoherence will allow for 10 - 50 CNOT equivalent operations with fidelity above 0.5
Towards Quantum Computation with Trapped Ca\textsuperscript{+} Ions

- Ion strings as qubits and quantum registers in linear traps
- Innsbruck Ca\textsuperscript{+} experiments
  - Spherical trap \((v_z = 4.5 \text{ MHz}, v_{x,y} = 2 \text{ MHz})\)
  - Linear trap \((v_z = 0.7 \text{ MHz}, v_{x,y} = 2 \text{ MHz}, v_z = 1.2 \text{ MHz}, v_{x,y} = 4 \text{ MHz})\)
- Spectroscopy of the S – D transition: resolution \(7 \times 10^{-13}\)
- Sideband cooling
  - Using coupled transitions, Raman cooling, EIT cooling, sympathetic cooling
- Relevant time scales
  - Coherence time: several ms
  - Heating times: > 100 ms
- Addressing of individual ions

Next:
- Preparation of Bell states, Bell measurements
- Realization of the Cirac-Zoller gate
  \(10 – 50\) CNOT gate operations currently possible
- CQED with trapped ions, interface to photonic qubits
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