Reversible And Irreversible Processes In Dispersive/Dissipative Media: Electro-magnetic Free Energy And Heat Production

Dielectrics and The Riemann-Hilbert Problem

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Collaborators

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- Curtis Broadbent, Melissa Clayton (Penn. St.)
- Justin Peatross

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Goals

- General: Develop Unambiguous Notions of Free Energy & Heat in Causal **"Black-Boxes"** which are Dissipative/Non-conservative.
- Specific: Develop Algorithm to Calculate these Quantities for **"Phenomenological"** Passive, Linear Dielectrics– the Riemann Hilbert Problem.

Specific: Energy Allocation in Dielectrics—History

$$\frac{\partial u(t)}{\partial t} + c\nabla\cdot\left(\mathbf{E}\times\mathbf{H}\right)(t) = 0,$$

$$u(t) = \frac{1}{2}\left\|\mathbf{E}\right\|^2(t) + \frac{1}{2}\left\|\mathbf{H}\right\|^2(t) + \int_{-\infty}^{t}\left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau$$

Specific: Energy Allocation in Dielectrics—History

$$u(t) = \frac{1}{2}\|\mathbf{E}\|^2(t) + \frac{1}{2}\|\mathbf{H}\|^2(t) + u_{\text{int.}}(t)$$

$$u_{\text{int.}}(t) := \int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau$$

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Specific: Energy Allocation in Dielectrics—History

Electromagnetic Pulse Propagation in Causal Dielectrics, Oughstun & Sherman 1994:

$$u_{\text{int.}}(t) = \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{reactive} + \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{absorbed}$$

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Specific: Energy Allocation in Dielectrics—History

Referring to an analog of $u(t)=\frac{1}{2}\|\mathbf{E}\|^2(t)+\frac{1}{2}\|\mathbf{H}\|^2(t)+u_{\text{int.}}(t)$ O & S state:

"It is evident that one may associate the first two terms on the right-hand side of (2.2.14) with the energy density…in the field alone and the last term with the electric energy density that is interacting with the dispersive medium. A certain portion of this interaction energy is 'reactively' stored in the medium and the remainder is absorbed, giving the evolved heat in the absorptive medium.… Unfortunately, this separation is found to be intractable analytically."

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Specific: Energy Allocation in Dielectrics—History

Electromagnetic Pulse Propagation in Causal Dielectrics, Oughstun & Sherman 1994:

$$u_{\text{int.}}(t)=\left(\int_{-\infty}^{t}\left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)d\tau\right)_{reactive}+\left(\int_{-\infty}^{t}\left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)d\tau\right)_{absorbed}$$

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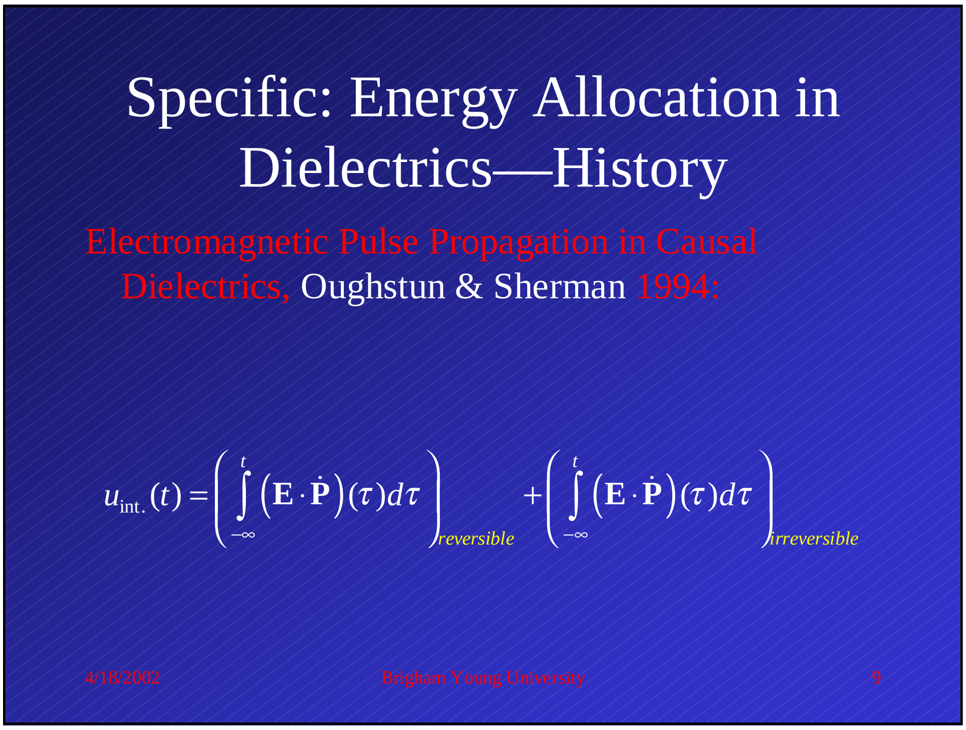
Specific: Energy Allocation in Dielectrics—History
Electromagnetic Pulse Propagation in Causal Dielectrics, Oughstun & Sherman 1994:
$u_{\text{int.}}(t)=\left(\int_{-\infty}^{t}\left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)d\tau\right)_{reversible}+\left(\int_{-\infty}^{t}\left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)d\tau\right)_{irreversible}$
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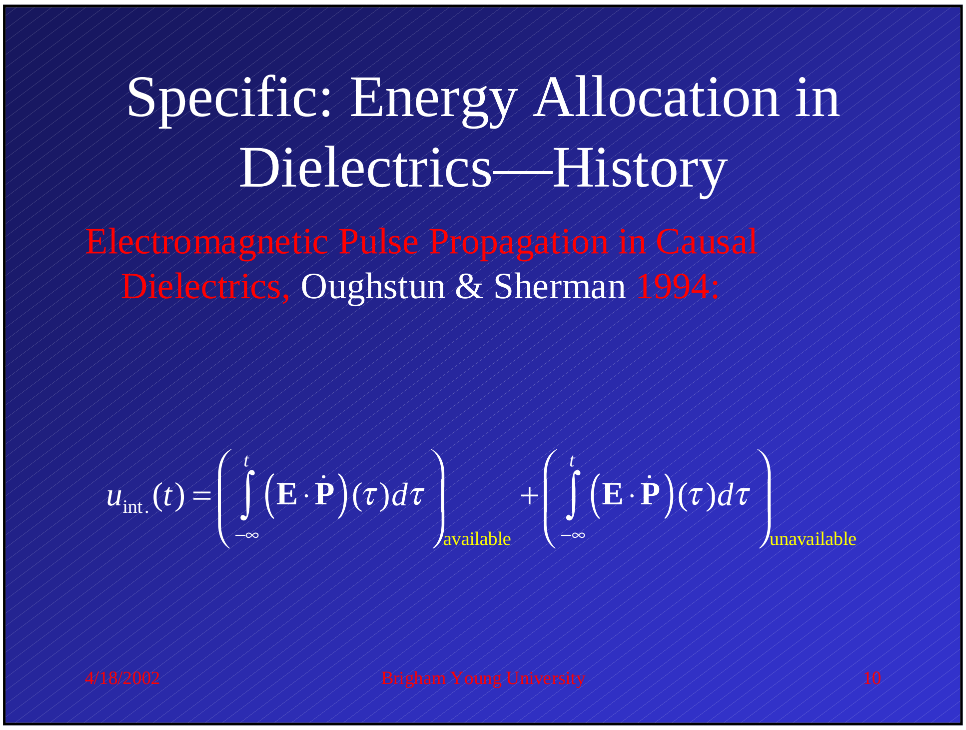
Specific: Energy Allocation in Dielectrics—History
Electromagnetic Pulse Propagation in Causal Dielectrics, Oughstun & Sherman 1994:
$u_{\text{int.}}(t)=\left(\int_{-\infty}^{t}\left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)d\tau\right)_{\text{available}}+\left(\int_{-\infty}^{t}\left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)d\tau\right)_{\text{unavailable}}$
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Specific: Energy Allocation in Dielectrics—History

Electromagnetic Pulse Propagation in Causal Dielectrics, Oughstun & Sherman 1994:

$$u_{\text{int.}}(t) = \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau \right)_{\text{available}} + \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau \right)_{\text{unavailable}}$$

To Do Work! To Do Work!

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Specific: Energy Allocation in Dielectrics—History

"In the abscence of dispersion" (i.e. with $\mathbf{D} = \varepsilon\,\mathbf{E}$, $\mathbf{B} = \mu\,\mathbf{H}$),
Landau & Lifshitz state in 1958 that

$$u := \frac{\varepsilon}{2}\|\mathbf{E}\|^2 + \frac{\mu}{2}\|\mathbf{H}\|^2$$

"has exact thermodynamic significance: it is the difference between the internal energy per unit volume with and without the field, the density and entropy remaining unchanged. In the presence of dispersion, no such simple interpretation is possible. Moreover, in the general case of arbitrary dispersion, the electromagnetic energy cannot be rationally defined as a thermodynamic quantity. This is because the presence of dispersion in general signifies a dissipation of energy, i.e. a dispersive medium is also an absorbing medium."

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Dictionary
Oughtsun & Sherman
—
Total Energy (in the Field)[D]
Heat [D]
Interaction Energy [D]
Reactive Interaction Energy [D]
Landau & Lifshitz
Internal Energy [PR]
Free Energy [PR]
Heat[PP, PR ,PT]
—
—
G. & Peatross
Total (Internal / Dynamical) Energy [P]
Total Free / Reversible Energy [P]
Heat or Irreversible Energy [P]
Interaction Energy [P]
Free / Mechanical / Reversible Energy [P]
D —Dispersive Media
P —Passive, Dispersive Media
PR —Passive, Dispersive Media;Quasi-static and/or Reversible Process
PP —Passive, Dispersive Media;Periodic Process
PT —Passive, Dispersive Media;Transient Process
D ⊃ P ⊃ PR, PP, PT
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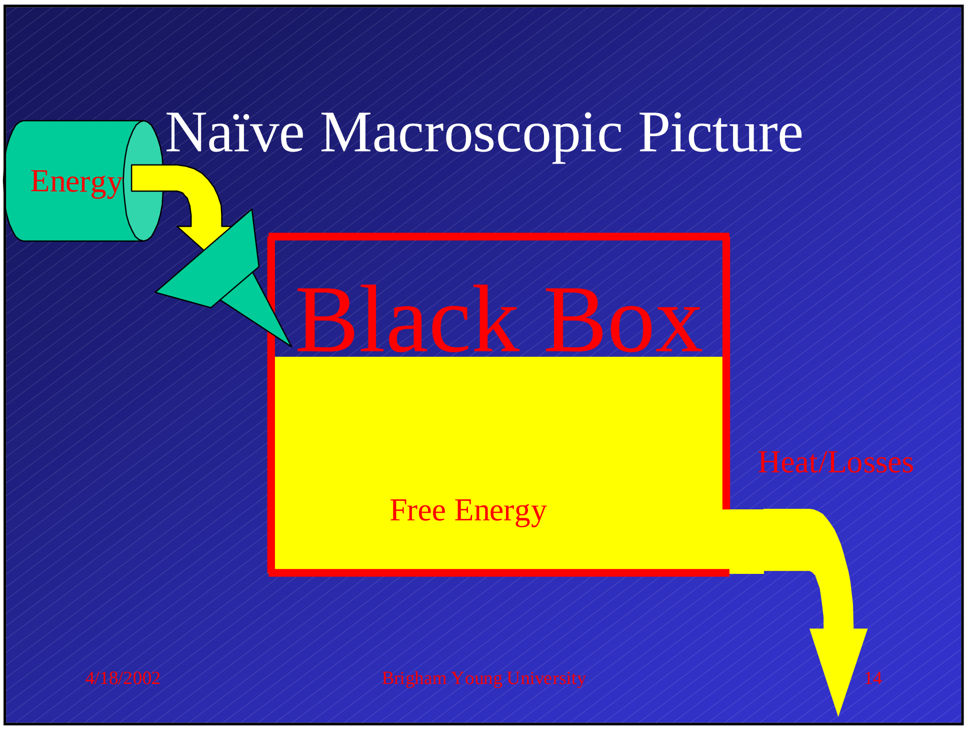
Naïve Macroscopic Picture
Energy
Black Box
Free Energy
Heat/Losses
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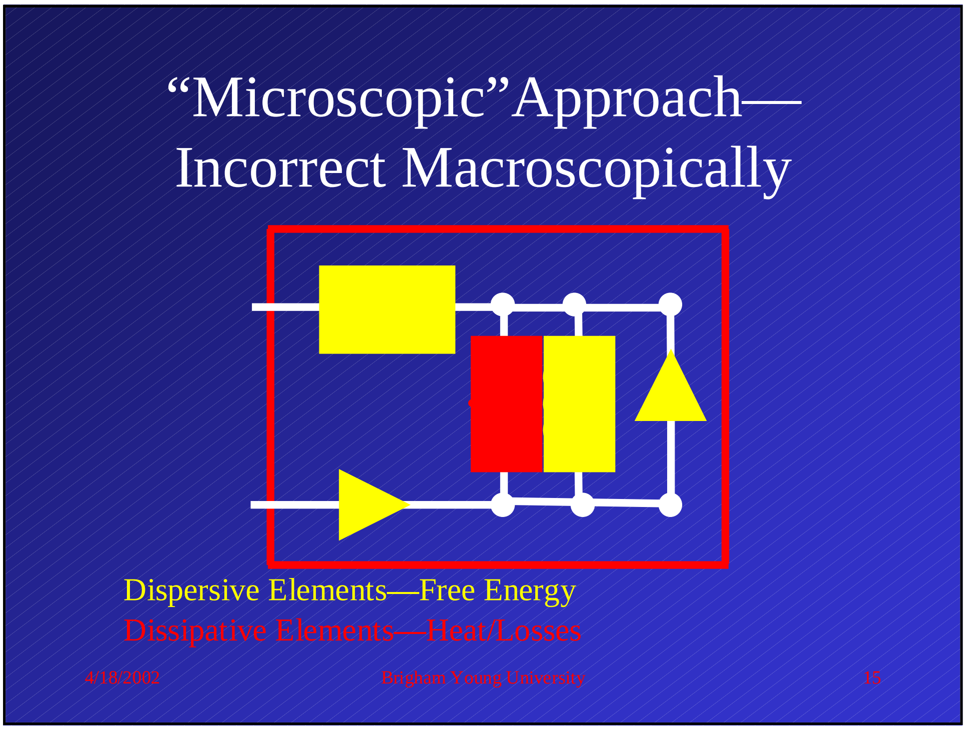
"Microscopic" Approach—
Incorrect Macroscopically
Dispersive Elements—Free Energy
Dissipative Elements—Heat/Losses
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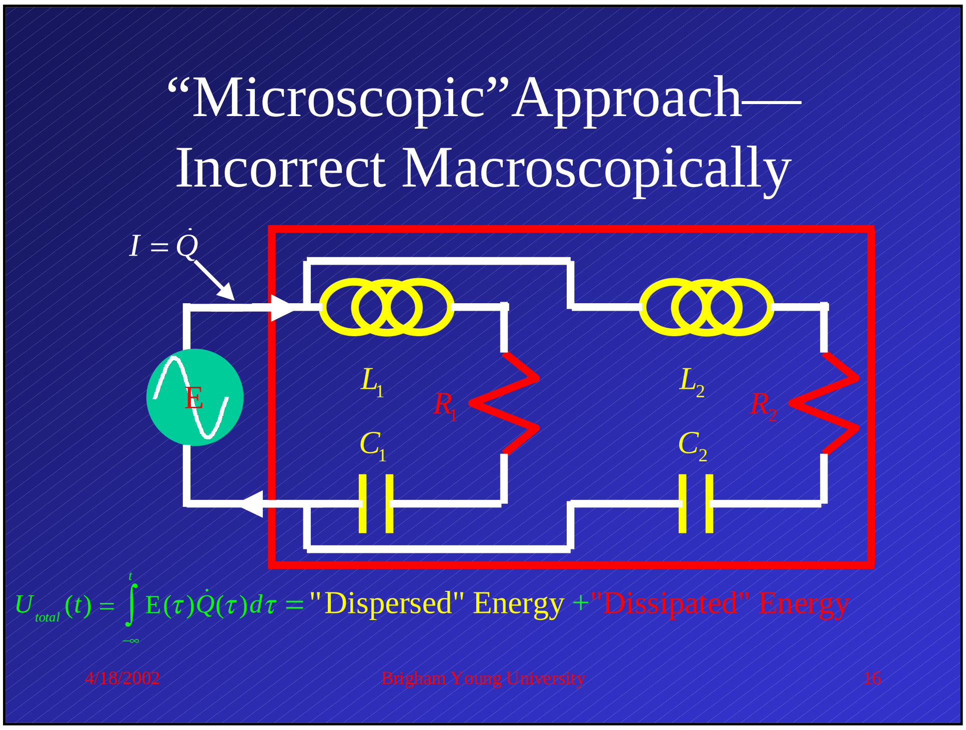
"Microscopic" Approach—
Incorrect Macroscopically
$I = \dot{Q}$
E
L_1
R_1
C_1
L_2
R_2
C_2
$U_{total}(t) = \int_{-\infty}^{t} E(\tau)\dot{Q}(\tau)d\tau =$ "Dispersed" Energy + "Dissipated" Energy
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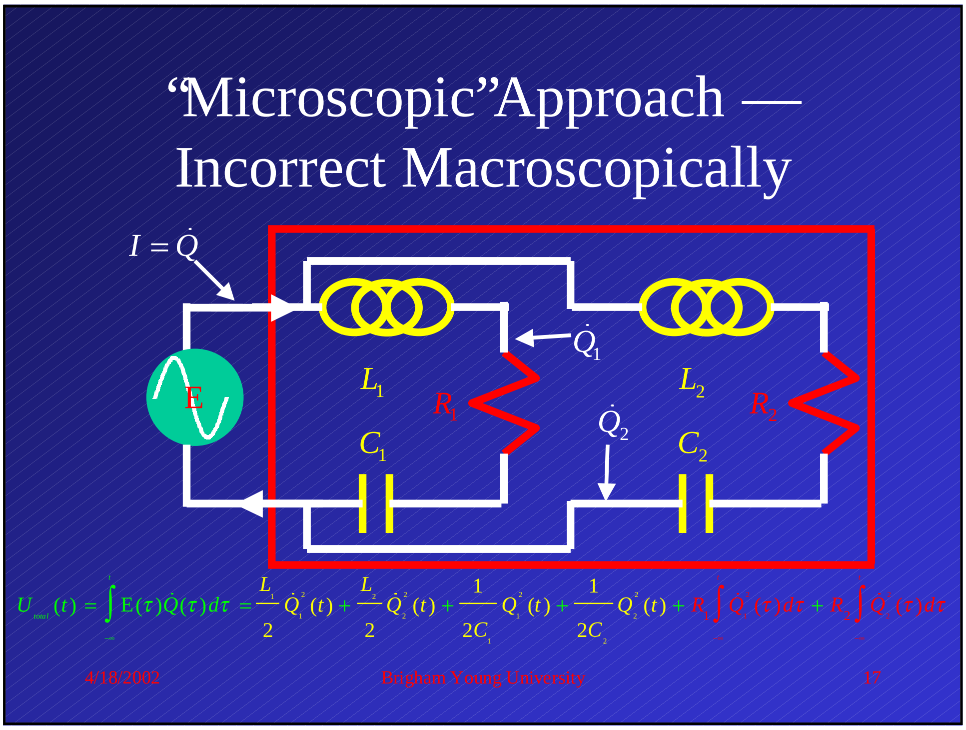

“Microscopic”Approach —
Incorrect Macroscopically
$I = \dot{Q}$
E
L_1
R_1
C_1
$\dot{Q}_1$
$\dot{Q}_2$
L_2
R_2
C_2
$U_{total}(t) = \int_{-\infty}^{t} E(\tau)\dot{Q}(\tau)d\tau = \frac{L_1}{2}\dot{Q}_1^2(t) + \frac{L_2}{2}\dot{Q}_2^2(t) + \frac{1}{2C_1}Q_1^2(t) + \frac{1}{2C_2}Q_2^2(t) + R_1\int_{-\infty}^{t}\dot{Q}_1^2(\tau)d\tau + R_2\int_{-\infty}^{t}\dot{Q}_2^2(\tau)d\tau$
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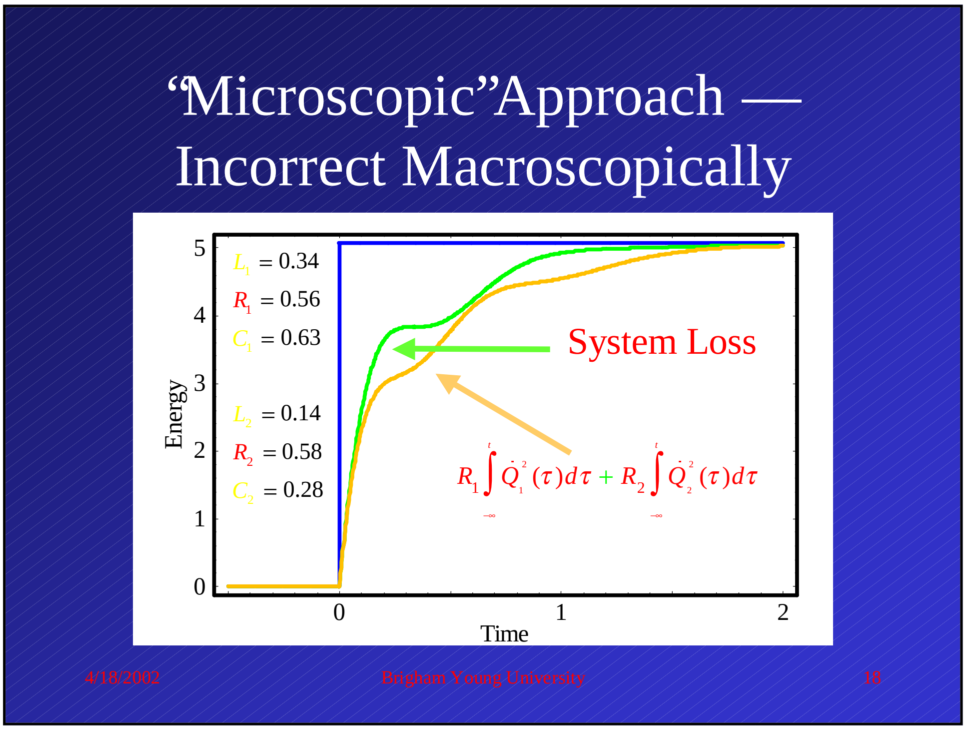

“Microscopic”Approach —
Incorrect Macroscopically
$L_1 = 0.34$
$R_1 = 0.56$
$C_1 = 0.63$
$L_2 = 0.14$
$R_2 = 0.58$
$C_2 = 0.28$
System Loss
$R_1\int_{-\infty}^{t}\dot{Q}_1^2(\tau)d\tau + R_2\int_{-\infty}^{t}\dot{Q}_2^2(\tau)d\tau$
Energy
Time
0 1 2 3 4 5
0 1 2
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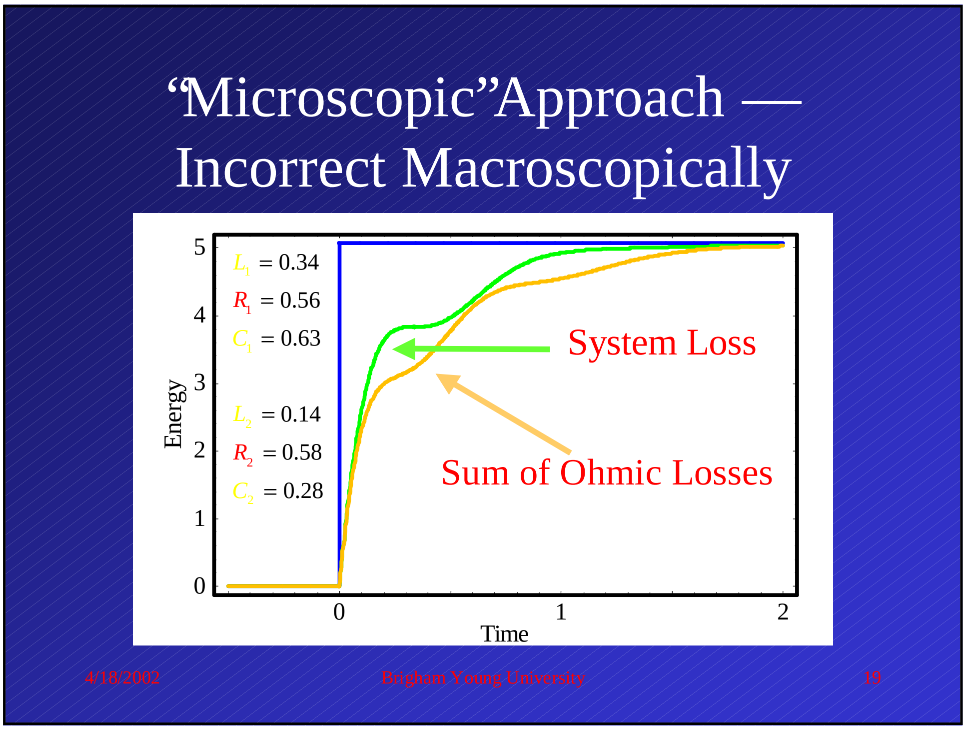
"Microscopic" Approach — Incorrect Macroscopically
L_1 = 0.34
R_1 = 0.56
C_1 = 0.63
L_2 = 0.14
R_2 = 0.58
C_2 = 0.28
Energy
Time
System Loss
Sum of Ohmic Losses
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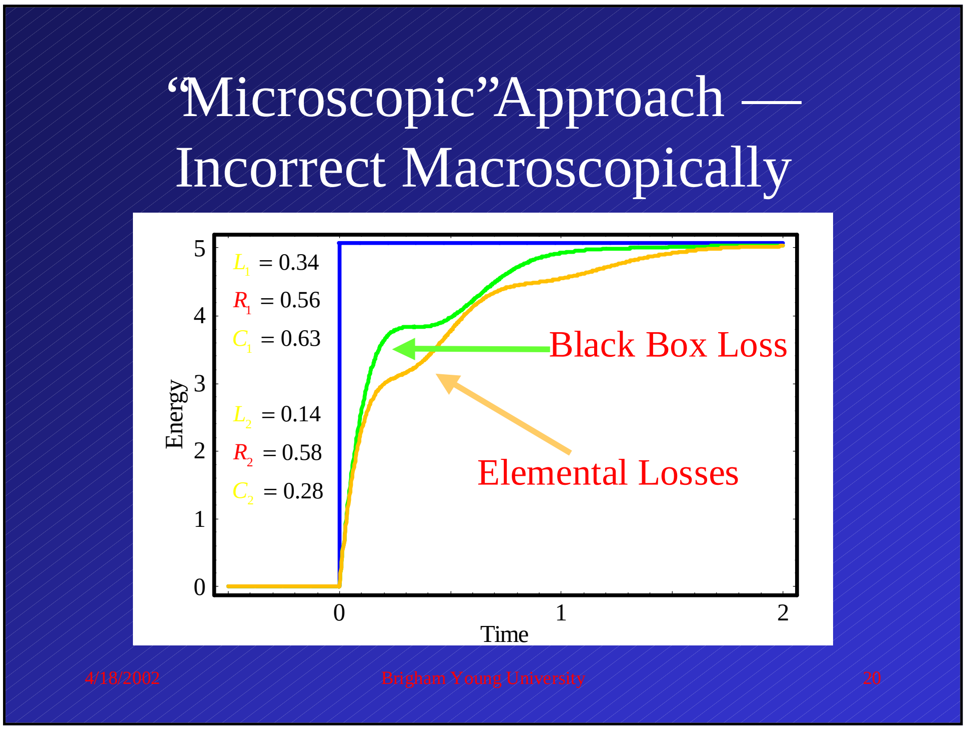
"Microscopic" Approach — Incorrect Macroscopically
L_1 = 0.34
R_1 = 0.56
C_1 = 0.63
L_2 = 0.14
R_2 = 0.58
C_2 = 0.28
Energy
Time
Black Box Loss
Elemental Losses
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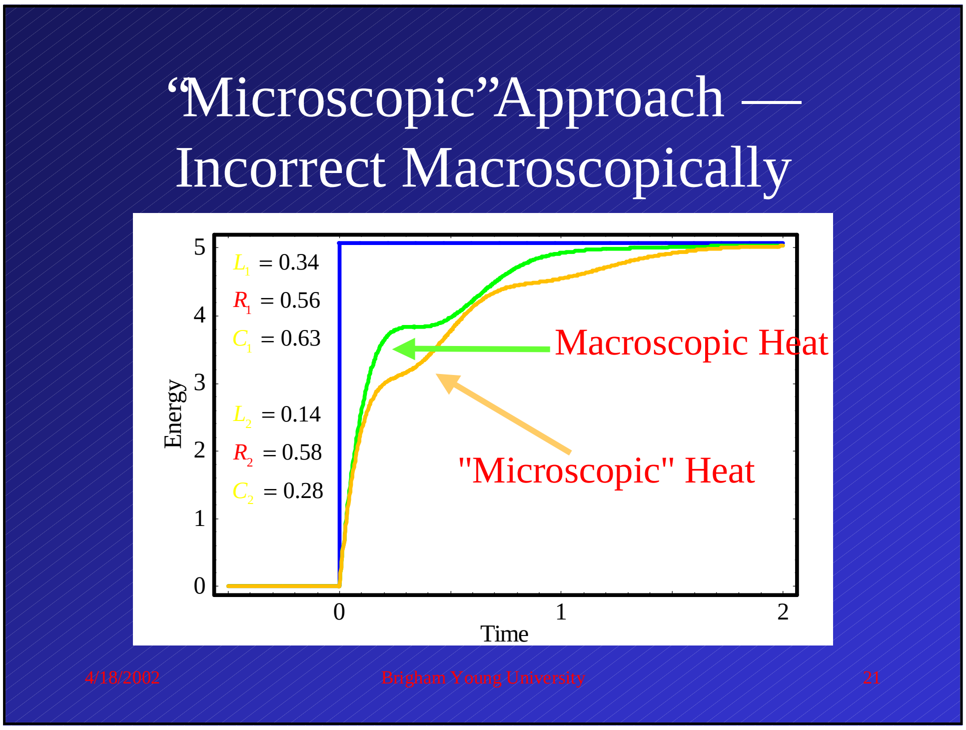

“Microscopic”Approach —
Incorrect Macroscopically
L1 = 0.34
R1 = 0.56
C1 = 0.63
L2 = 0.14
R2 = 0.58
C2 = 0.28
Macroscopic Heat
"Microscopic" Heat
Energy
Time
0 1 2 3 4 5
0 1 2
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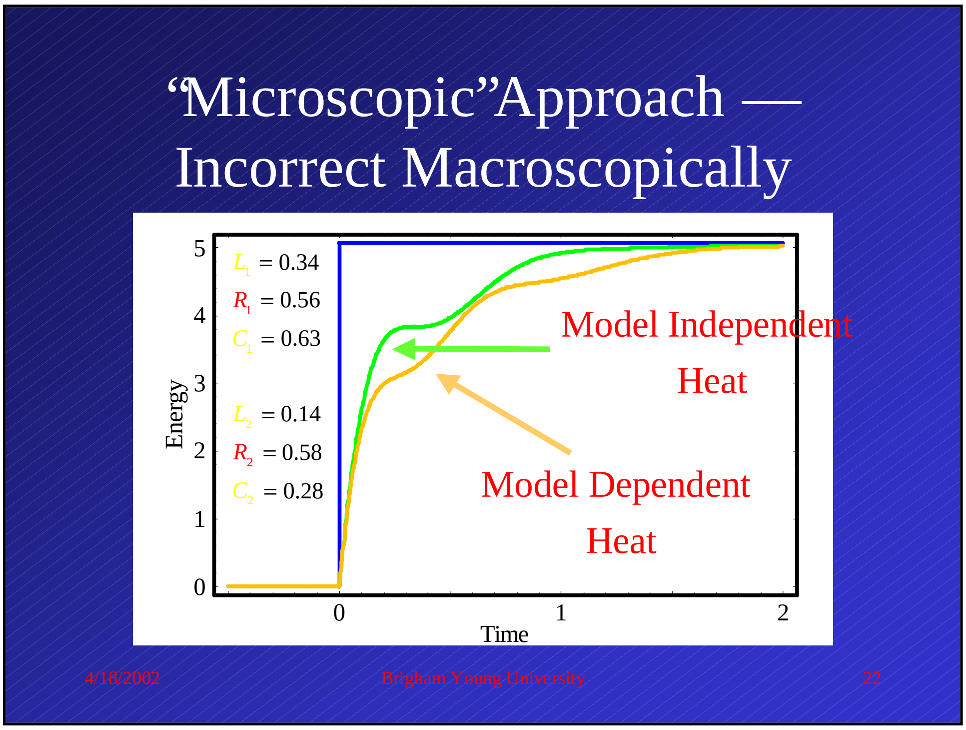

“Microscopic”Approach —
Incorrect Macroscopically
L1 = 0.34
R1 = 0.56
C1 = 0.63
L2 = 0.14
R2 = 0.58
C2 = 0.28
Model Independent Heat
Model Dependent Heat
Energy
Time
0 1 2 3 4 5
0 1 2
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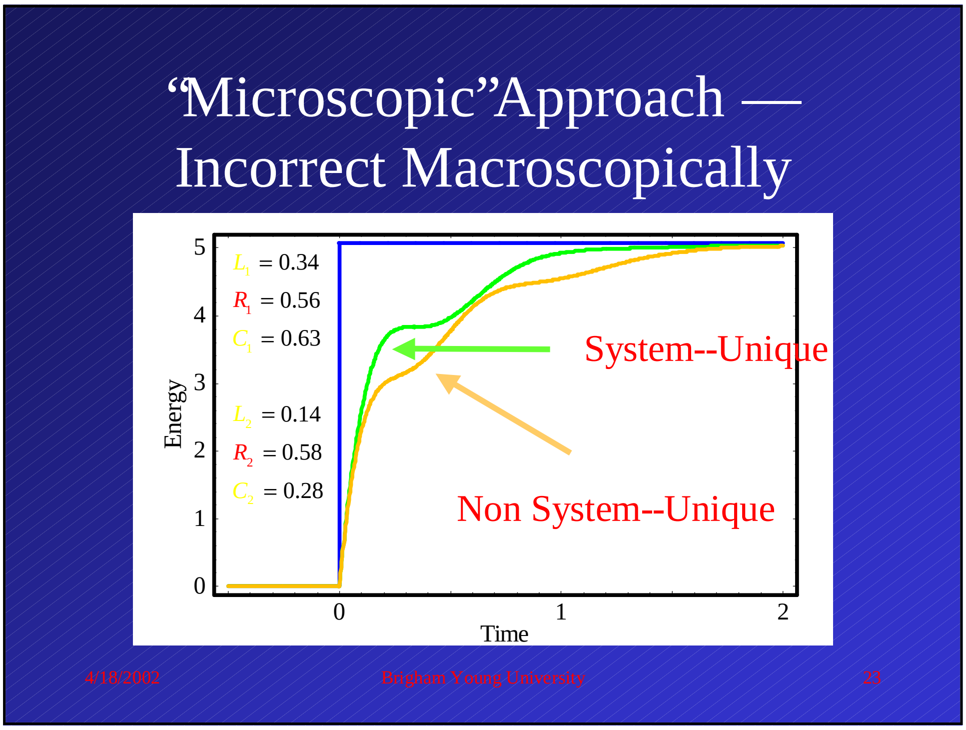

"Microscopic"Approach —
Incorrect Macroscopically
Energy
Time
L_1 = 0.34
R_1 = 0.56
C_1 = 0.63
L_2 = 0.14
R_2 = 0.58
C_2 = 0.28
System--Unique
Non System--Unique
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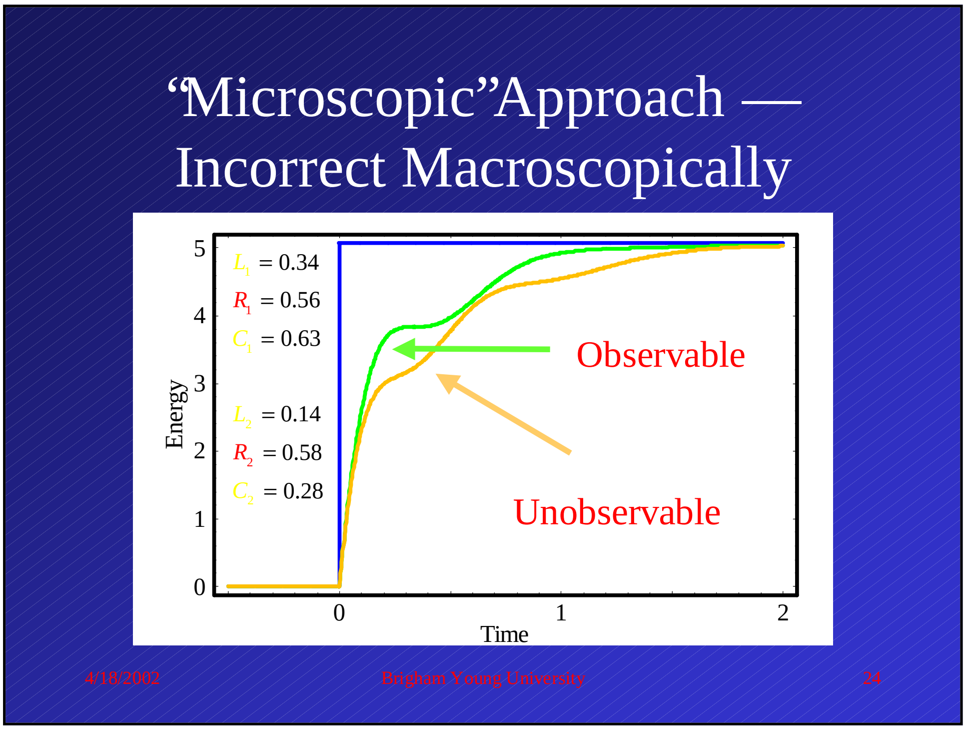

"Microscopic"Approach —
Incorrect Macroscopically
Energy
Time
L_1 = 0.34
R_1 = 0.56
C_1 = 0.63
L_2 = 0.14
R_2 = 0.58
C_2 = 0.28
Observable
Unobservable
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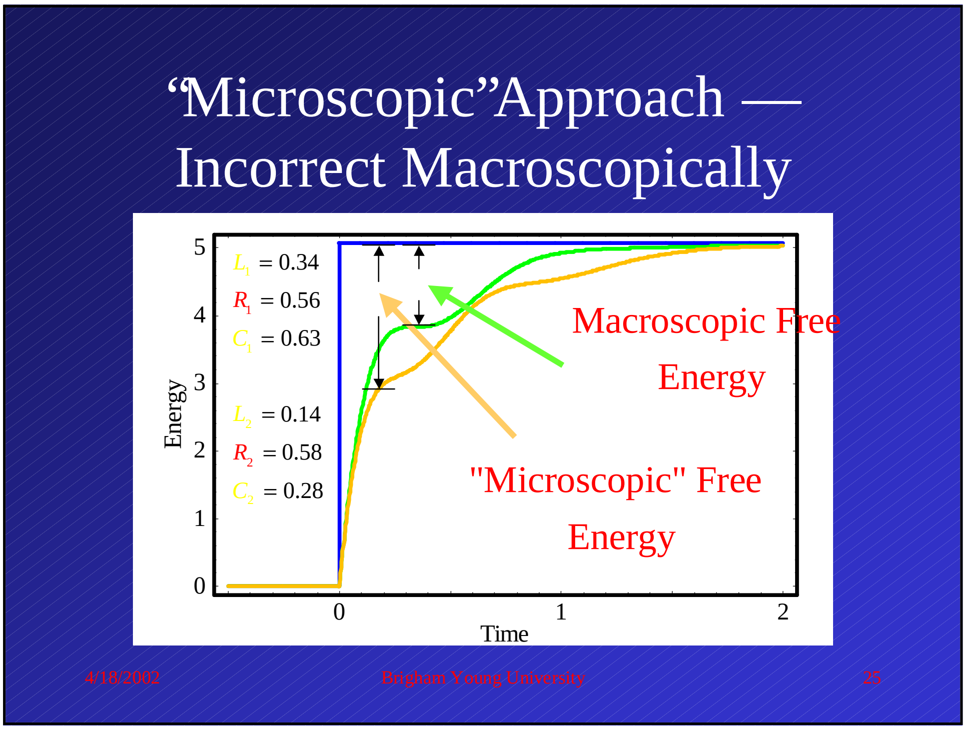

"Microscopic" Approach —
Incorrect Macroscopically
L_1 = 0.34
R_1 = 0.56
C_1 = 0.63
L_2 = 0.14
R_2 = 0.58
C_2 = 0.28
Energy
Time
Macroscopic Free Energy
"Microscopic" Free Energy
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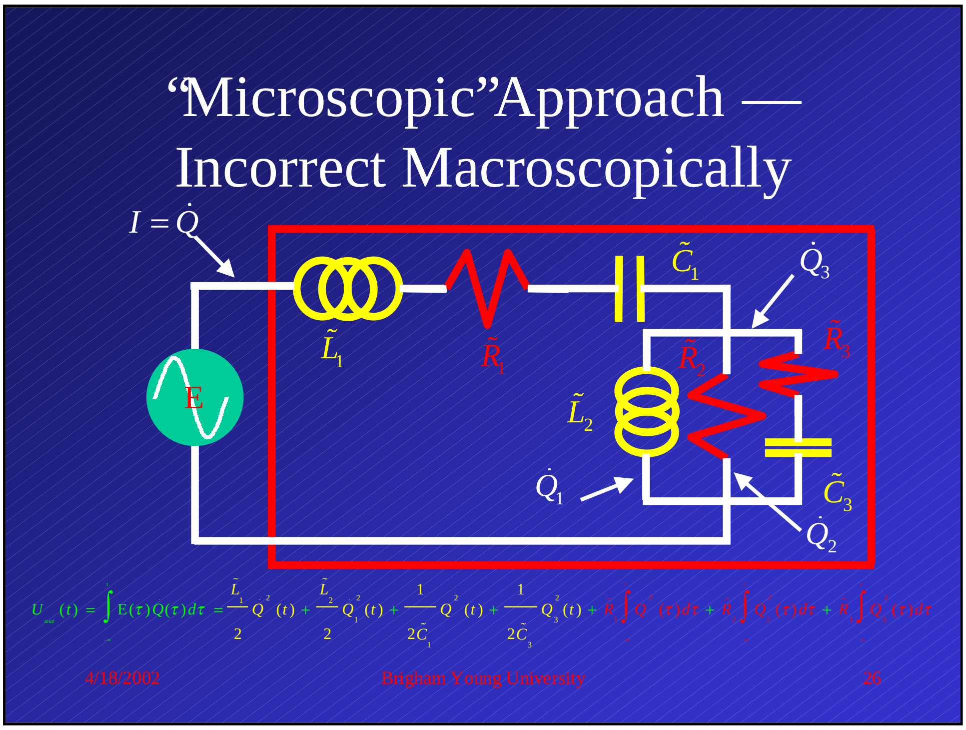

"Microscopic" Approach —
Incorrect Macroscopically
$I = \dot{Q}$
E
$\tilde{L}_1$
$\tilde{R}_1$
$\tilde{C}_1$
$\dot{Q}_3$
$\tilde{R}_3$
$\tilde{R}_2$
$\tilde{L}_2$
$\dot{Q}_1$
$\tilde{C}_3$
$\dot{Q}_2$
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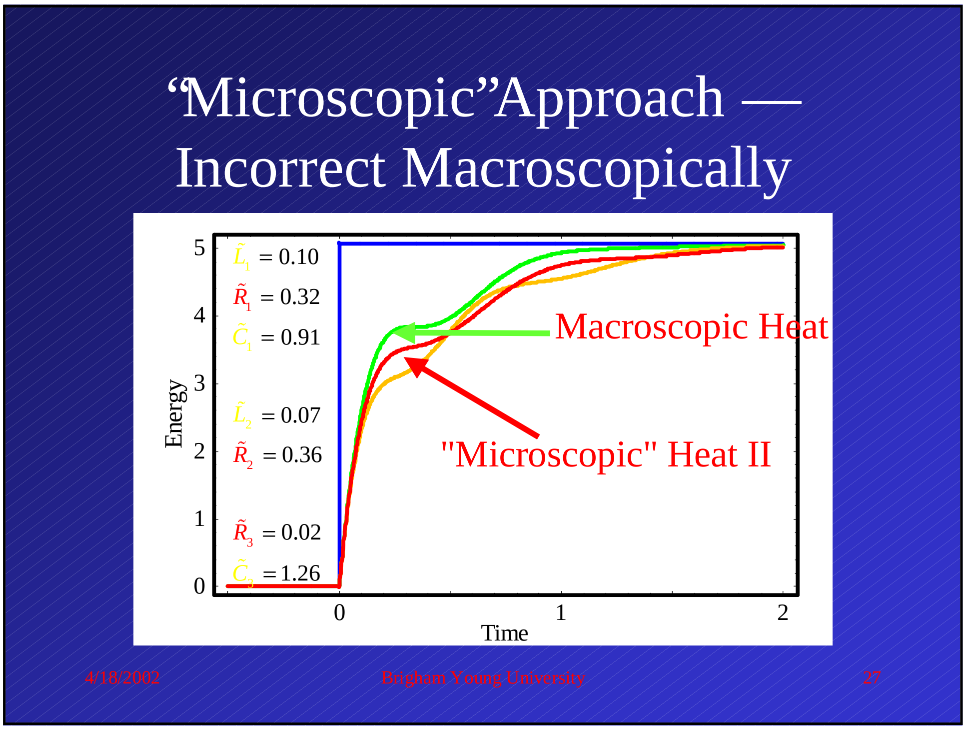
“Microscopic”Approach — Incorrect Macroscopically
$\tilde{L}_1$ = 0.10
$\tilde{R}_1$ = 0.32
$\tilde{C}_1$ = 0.91
$\tilde{L}_2$ = 0.07
$\tilde{R}_2$ = 0.36
$\tilde{R}_3$ = 0.02
$\tilde{C}_3$ = 1.26
Macroscopic Heat
"Microscopic" Heat II
Energy
Time
0
1
2
3
4
5
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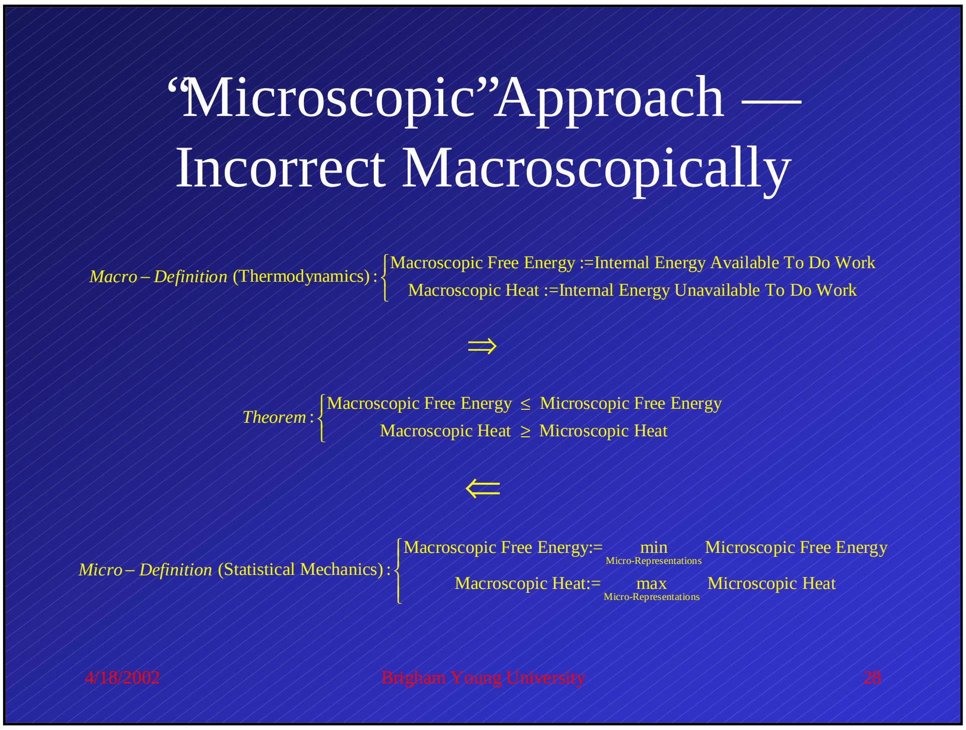
“Microscopic”Approach — Incorrect Macroscopically
Macro – Definition (Thermodynamics): Macroscopic Free Energy :=Internal Energy Available To Do Work
Macroscopic Heat :=Internal Energy Unavailable To Do Work
⇒
Theorem: Macroscopic Free Energy ≤ Microscopic Free Energy
Macroscopic Heat ≥ Microscopic Heat
⇐
Micro – Definition (Statistical Mechanics): Macroscopic Free Energy:= min Micro-Representations Microscopic Free Energy
Macroscopic Heat:= max Micro-Representations Microscopic Heat
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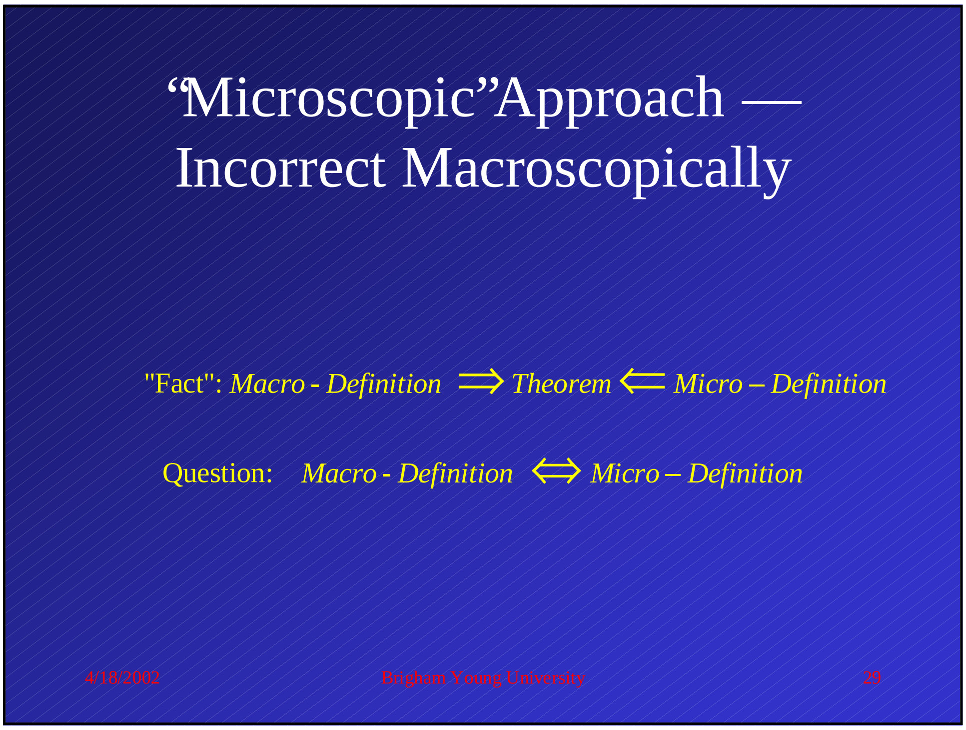
"Microscopic" Approach — Incorrect Macroscopically
"Fact": *Macro - Definition* $\Longrightarrow$ *Theorem* $\Longleftarrow$ *Micro – Definition*
Question: *Macro - Definition* $\Longleftrightarrow$ *Micro – Definition*
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"Microscopic" Approach — Incorrect Macroscopically
Question: *Macro - Definition* $\Longleftrightarrow$ *Micro – Definition*
$\Longrightarrow$
Unavailability of Internal Energy To Do Work $\Longleftrightarrow$ Maximisation of Possible Heating
$\Longleftrightarrow$
Fact: Unavailability of Internal Energy To Do Work $\Longleftrightarrow$ Maximisation of Entropy
$\Longleftrightarrow$
Fact: Unavailability of Internal Energy To Do (Ordered) Work $\Longleftrightarrow$ Possibility for Disorder
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"Microscopic" Approach — Incorrect Macroscopically

Question: *Macro-Definition* $\Longleftrightarrow$ *Micro-Definition*

$\Rightarrow$

Macro-Definition is "Thermodynamically [Stat. Mechanically] significant"
--Landau & Lifshitz

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General Principles

- An unambiguous, dynamical notion of Free Energy and Heat exists for a system irrespective of the microscopic mechanisms giving rise to its dispersive and dissipative properties whenever…
  - There is a dynamical notion of the Total (Internal) Energy deposited in the system that can be viewed as the ***effect*** of a (macroscopic) ***cause*** –i.e. there are strict distinctions between past and future. By definition, a "causal" medium should have such notions.

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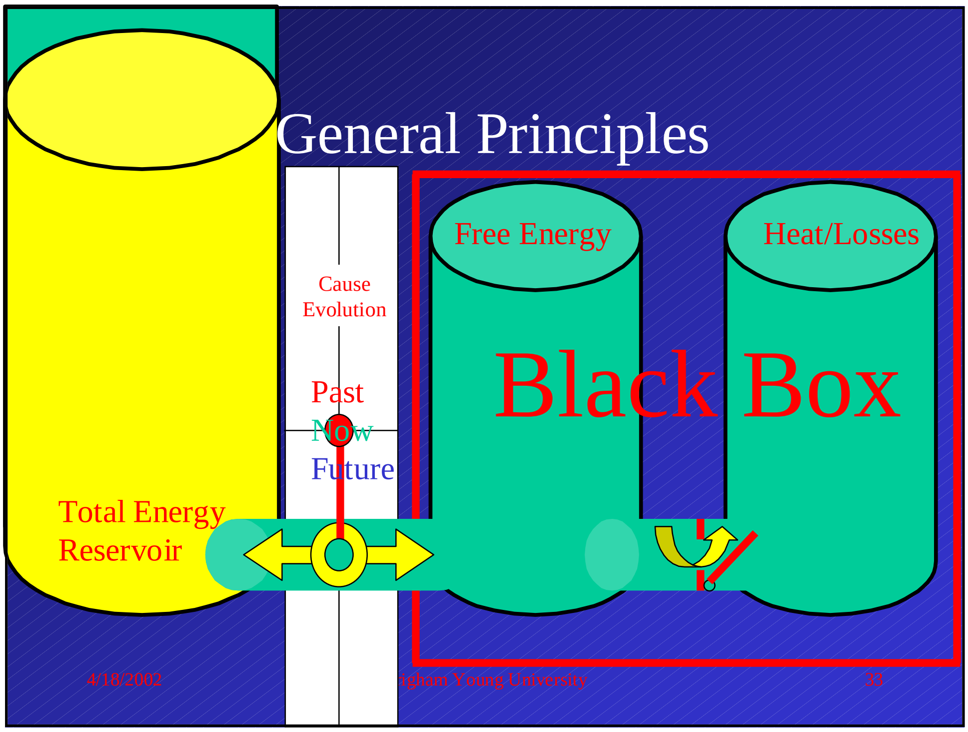
General Principles
Free Energy
Heat/Losses
Cause
Evolution
Past
Now
Future
Black Box
Total Energy
Reservoir
4/18/2002
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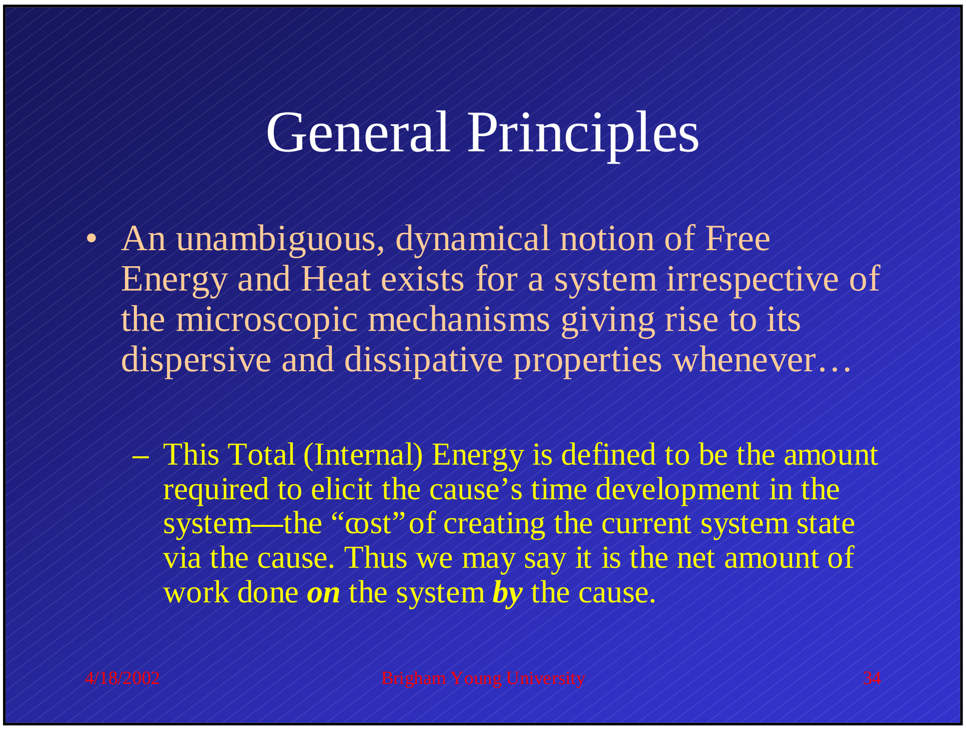
General Principles
• An unambiguous, dynamical notion of Free Energy and Heat exists for a system irrespective of the microscopic mechanisms giving rise to its dispersive and dissipative properties whenever…
– This Total (Internal) Energy is defined to be the amount required to elicit the cause's time development in the system—the "cost" of creating the current system state via the cause. Thus we may say it is the net amount of work done *on* the system *by* the cause.
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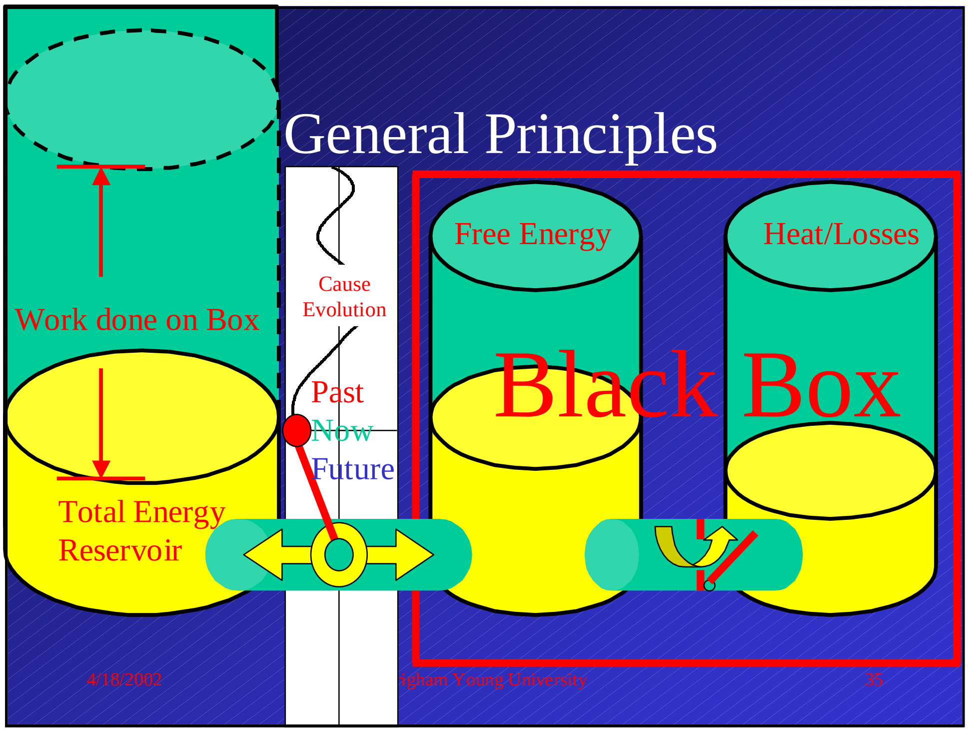
General Principles
Work done on Box
Total Energy
Reservoir
Cause
Evolution
Past
Now
Future
Free Energy
Heat/Losses
Black Box
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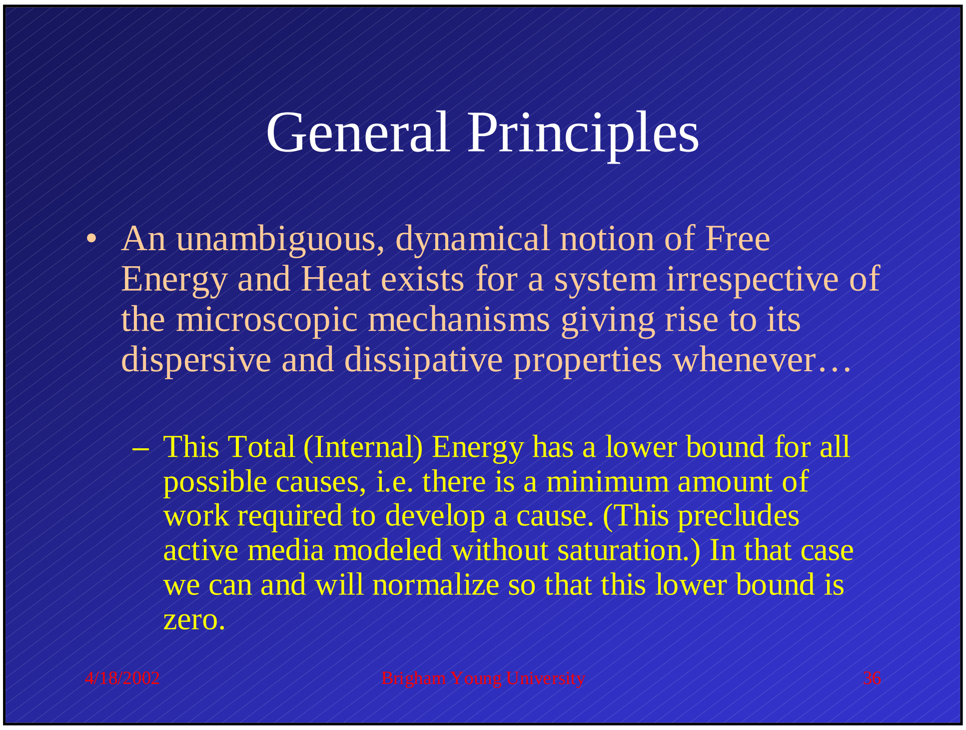
General Principles
• An unambiguous, dynamical notion of Free Energy and Heat exists for a system irrespective of the microscopic mechanisms giving rise to its dispersive and dissipative properties whenever…
– This Total (Internal) Energy has a lower bound for all possible causes, i.e. there is a minimum amount of work required to develop a cause. (This precludes active media modeled without saturation.) In that case we can and will normalize so that this lower bound is zero.
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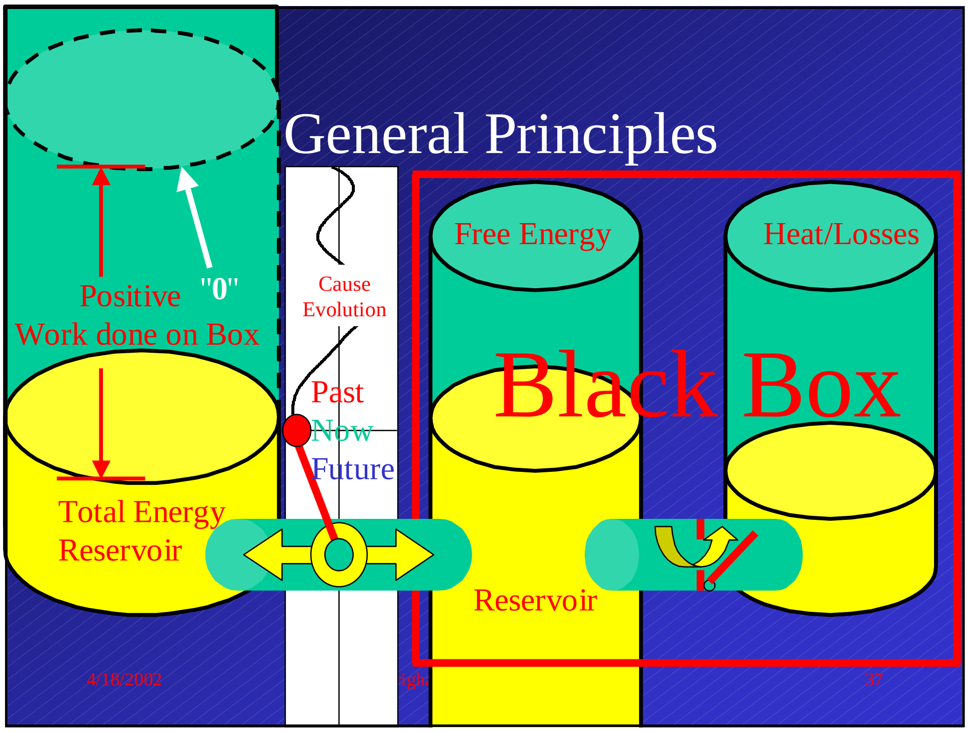

General Principles

- An unambiguous, dynamical notion of Free Energy and Heat exists for a system irrespective of the microscopic mechanisms giving rise to its dispersive and dissipative properties whenever…
  - We define the Free Energy practically: ***the current Free Energy of the system is the maximum amount of work the system can perform against the cause in the future, i.e. the maximum component of the current Total (Internal) Energy that the cause can subsequently recover from the system.***

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General Principles
• An unambiguous, dynamical notion of Free Energy and Heat exists for a system irrespective of the microscopic mechanisms giving rise to its dispersive and dissipative properties whenever…
– We define the Heat practically: *the current Heat of the system is the component of the current Total (Internal) Energy that the cause cannot, under any circumstances, subsequently recover from the medium.*
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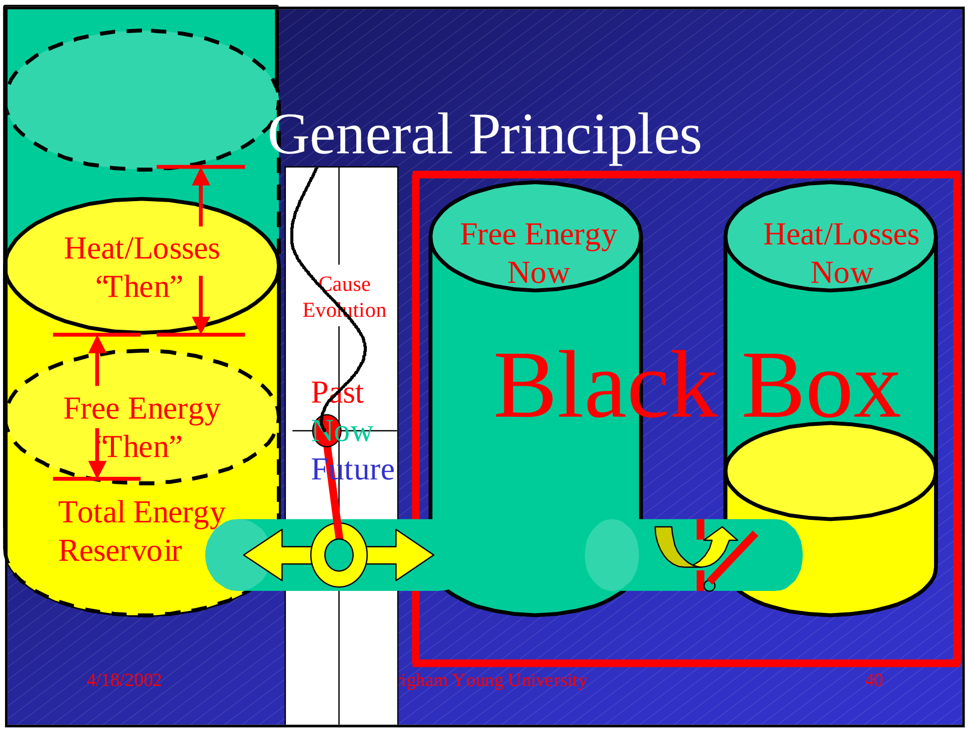
General Principles
Heat/Losses
"Then"
Free Energy
"Then"
Total Energy
Reservoir
Cause
Evolution
Past
Now
Future
Free Energy
Now
Heat/Losses
Now
Black Box
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General Principles

- An unambiguous, dynamical notion of Free Energy and Heat exists for a system irrespective of the microscopic mechanisms giving rise to its dispersive and dissipative properties whenever…
  - **Accounting:** The Heat of the system at any time is equivalent to the minimum value the Total (Internal) Energy can take at any time in the future, holding the past fixed. The Free Energy at that time is, then, equivalent to the difference between the current Total (Internal) Energy and the Heat.

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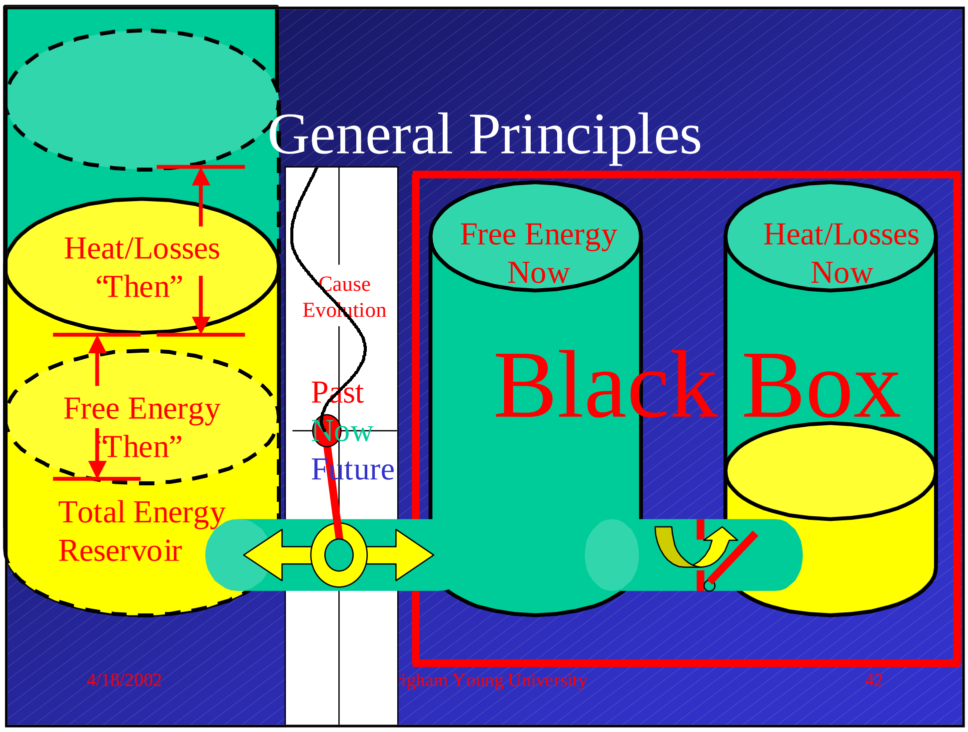

General Principles

- An unambiguous, dynamical notion of Free Energy and Heat exists for a system irrespective of the microscopic mechanisms giving rise to its dispersive and dissipative properties whenever…
  - **Consistency:** Since the Total (Internal) Energy depends only on the cause’s past, and since it has a lower bound, the minimum required in the definitions exists over all possible future causes and, so, is unique.

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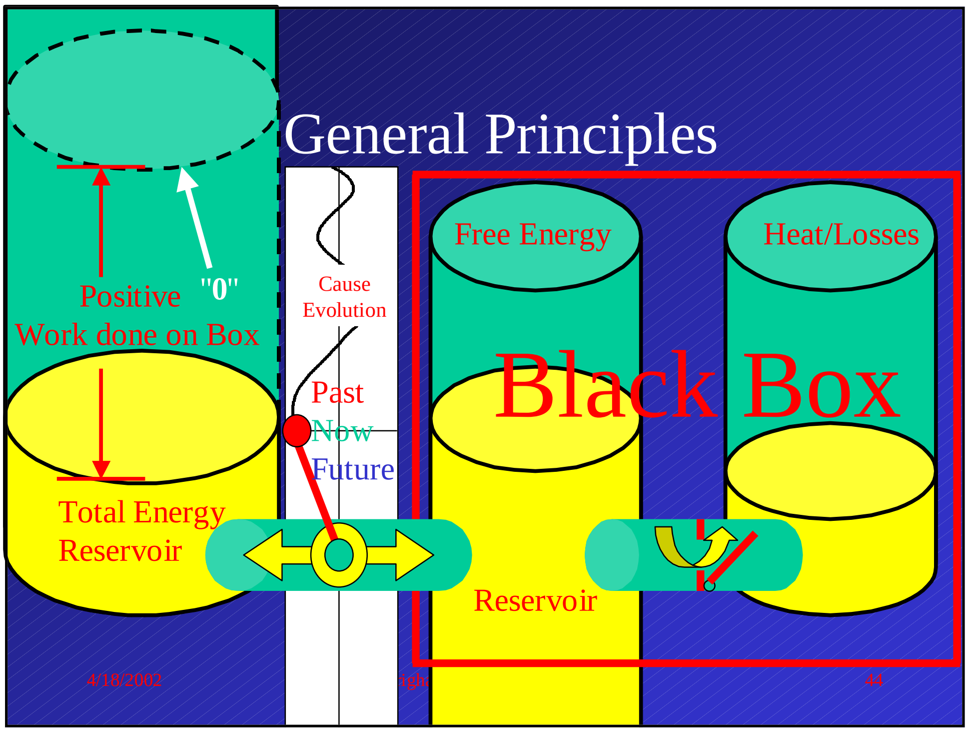

Specific: Energy Allocation in Dielectrics—Principles

$$u_{\text{int.}}(t) := \int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau$$

$$= \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{reactive} + \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{absorbed}$$

$$= \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{reversible} + \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{irreversible}$$

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Specific: Energy Allocation in Dielectrics—Principles

$$u_{\text{int.}}(t) = \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{reversible} + \left(\int_{-\infty}^{t} \left(\mathbf{E}\cdot\dot{\mathbf{P}}\right)(\tau)\,d\tau\right)_{irreversible}$$

$$= \left(u_{\text{int.}}(t)\right)_{reversible} + \left(u_{\text{int.}}(t)\right)_{irreversible}$$

$$\left(u_{\text{int.}}(t)\right)_{irreversible} \text{ "} := \text{" } \inf_{t' \in t' \text{ s future}} u_{\text{int.}}(t') = \inf_{t'>t} u_{\text{int.}}(t')$$

$$\left(u_{\text{int.}}(t)\right)_{reversible} := u_{\text{int.}}(t) - \left(u_{\text{int.}}(t)\right)_{irreversible} \text{ "} = \text{" } \sup_{t'>t} u_{\text{int.}}(t) - u_{\text{int.}}(t')$$

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Specific: Energy Allocation in Dielectrics—Principles

$$u_{\text{int.}}(t) = u_{\text{int.}}[\mathrm{E};\chi](t) = \int_{-\infty}^{t} \left(\mathrm{E}\dot{\mathrm{P}}\right)(\tau)d\tau = \int_{-\infty}^{t} \left(\mathrm{E}\dot{\mathrm{P}}[\mathrm{E};\chi]\right)(\tau)d\tau$$

$$\hat{\mathrm{P}}(\omega) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{P}(\tau)e^{i\omega t}d\tau = \hat{\mathrm{P}}[\mathrm{E};\chi](\omega) = \chi(\omega)\hat{\mathrm{E}}(\omega)$$

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Specific: Energy Allocation in Dielectrics—Principles

$$\left(u_{\text{int.}}(t)\right)_{irreversible} \text{"}:=\text{"} \inf_{t' \in t'\text{ s future}} u_{\text{int.}}(t') := \inf_{\mathrm{E}} u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E}_{\mathrm{f}} H_t^+;\chi\right](+\infty)$$

$$= \inf_{\mathrm{E}_{\mathrm{f}}} \int_{-\infty}^{+\infty} \left(\left(\mathrm{E}H_t^- + \mathrm{E}_{\mathrm{f}} H_t^+\right)\dot{\mathrm{P}}\left[\mathrm{E}H_t^- + \mathrm{E}_{\mathrm{f}} H_t^+;\chi\right]\right)(\tau)d\tau$$

$$\left(\mathrm{E}H_t^- + \mathrm{E}_{\mathrm{f}} H_t^+\right)(\tau) = \mathrm{E}(\tau)H_t^-(\tau) + \mathrm{E}_{\mathrm{f}}(\tau)H_t^+(\tau) = \mathrm{E}(\tau)H(t-\tau) + \mathrm{E}_{\mathrm{f}}(\tau)H(\tau - t)$$

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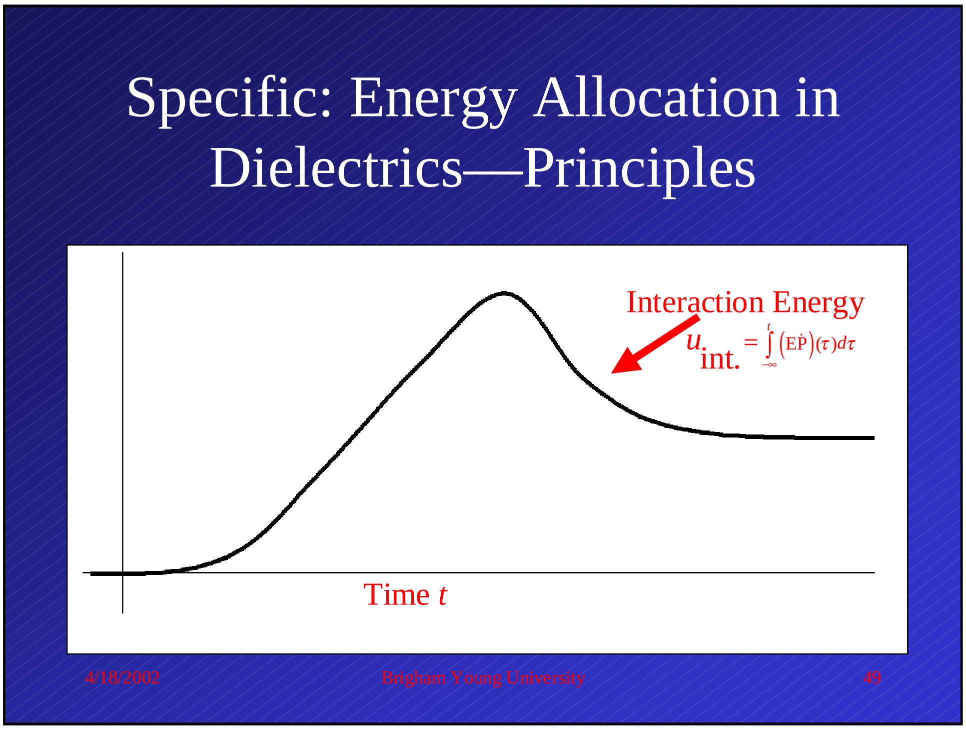
Specific: Energy Allocation in Dielectrics—Principles
Interaction Energy
$u_{int.} = \int_{-\infty}^{t} (E\dot{P})(\tau)\,d\tau$
Time t
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Brigham Young University
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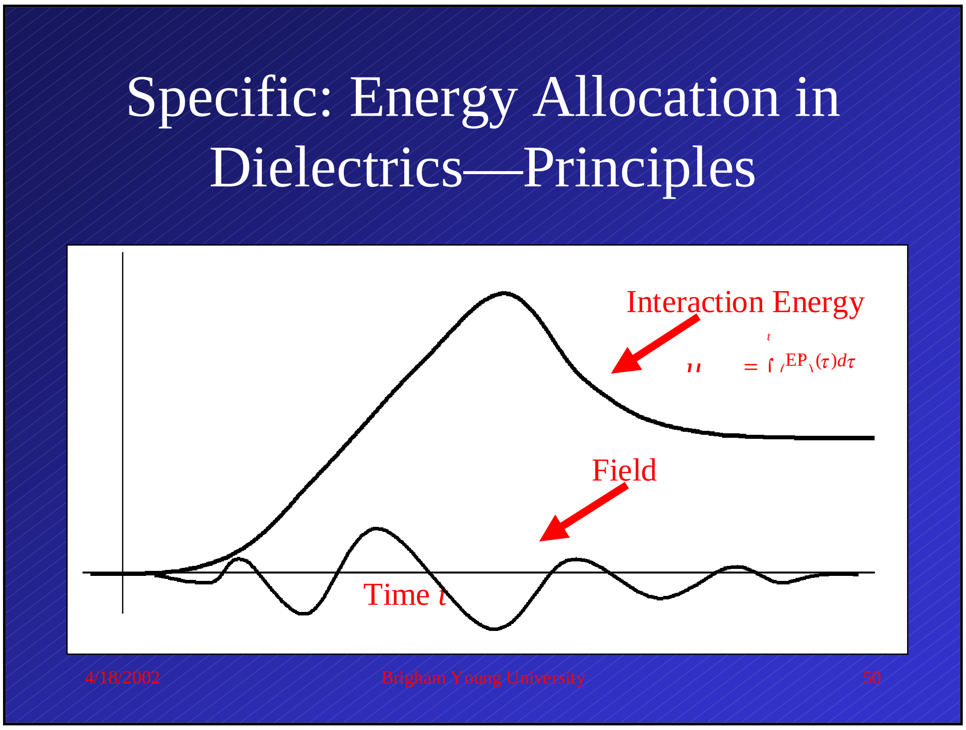
Specific: Energy Allocation in Dielectrics—Principles
Interaction Energy
Field
Time t
4/18/2002
Brigham Young University
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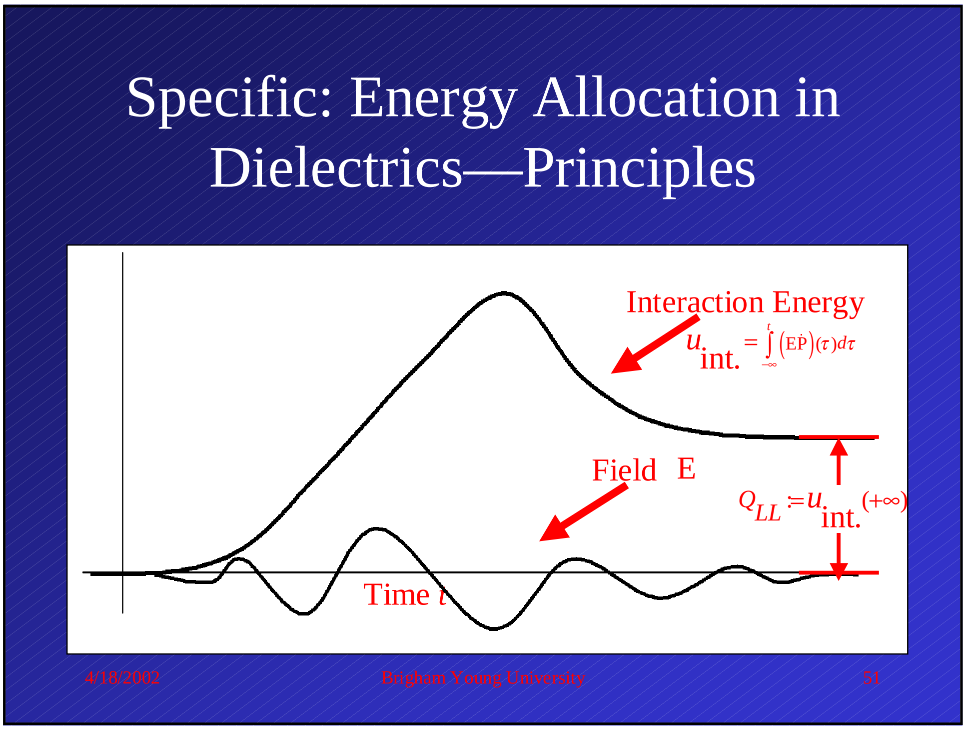
Specific: Energy Allocation in Dielectrics—Principles
Interaction Energy
$u_{\text{int.}} = \int_{-\infty}^{t} (E\dot{P})(\tau)d\tau$
Field E
$Q_{LL} := u_{\text{int.}}(+\infty)$
Time t
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Brigham Young University
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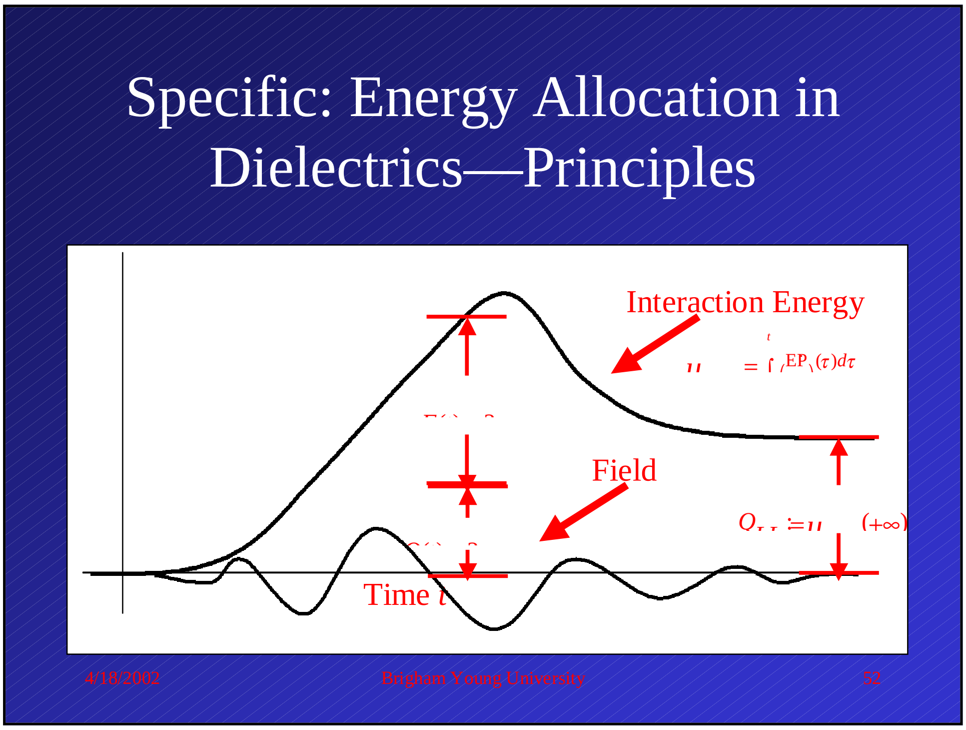
Specific: Energy Allocation in Dielectrics—Principles
Interaction Energy
Field
(+∞)
Time t
4/18/2002
Brigham Young University
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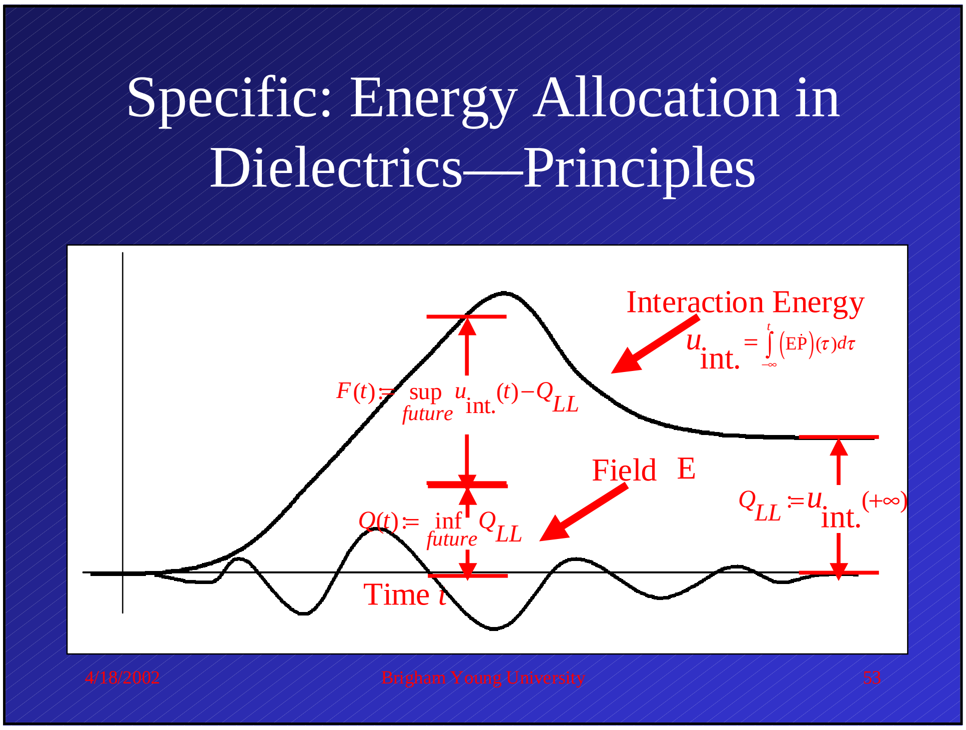
Specific: Energy Allocation in Dielectrics—Principles
Interaction Energy
$u_{\text{int.}} = \int_{-\infty}^{t} (E\dot{P})(\tau)d\tau$
$F(t) := \sup_{future} u_{\text{int.}}(t) - Q_{LL}$
Field E
$Q(t) := \inf_{future} Q_{LL}$
$Q_{LL} := u_{\text{int.}}(+\infty)$
Time t
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Brigham Young University
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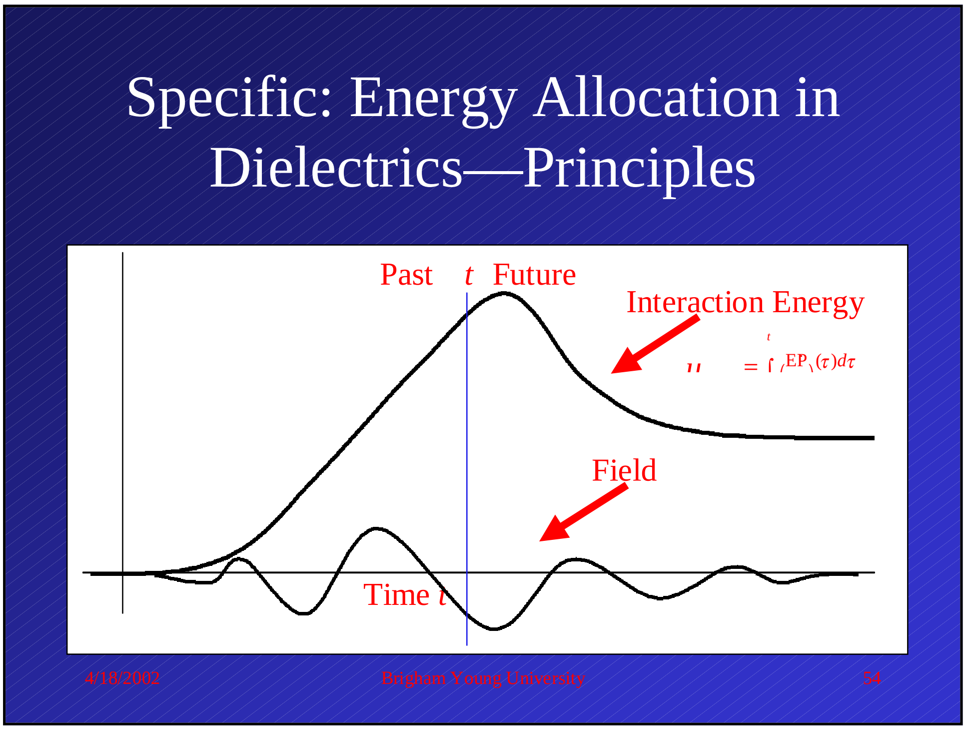
Specific: Energy Allocation in Dielectrics—Principles
Past t Future
Interaction Energy
Field
Time t
4/18/2002
Brigham Young University
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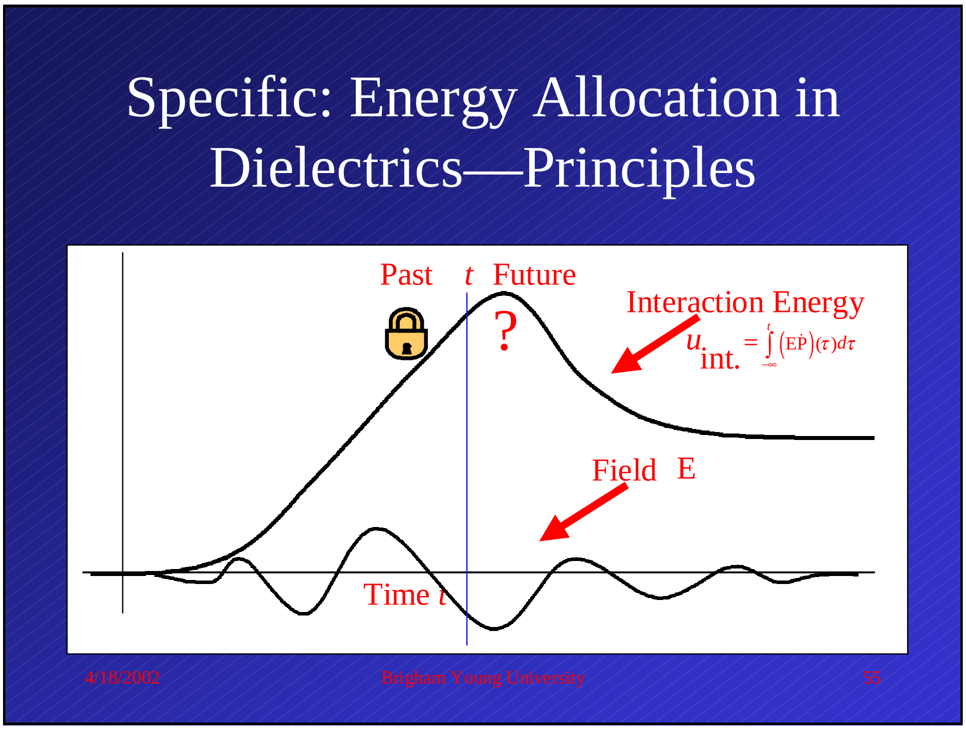

Specific: Energy Allocation in Dielectrics—Principles
Past
t
Future
?
Interaction Energy
$u_{\text{int.}} = \int_{-\infty}^{t} (\mathrm{E}\dot{\mathrm{P}})(\tau)\,d\tau$
Field E
Time t
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Brigham Young University
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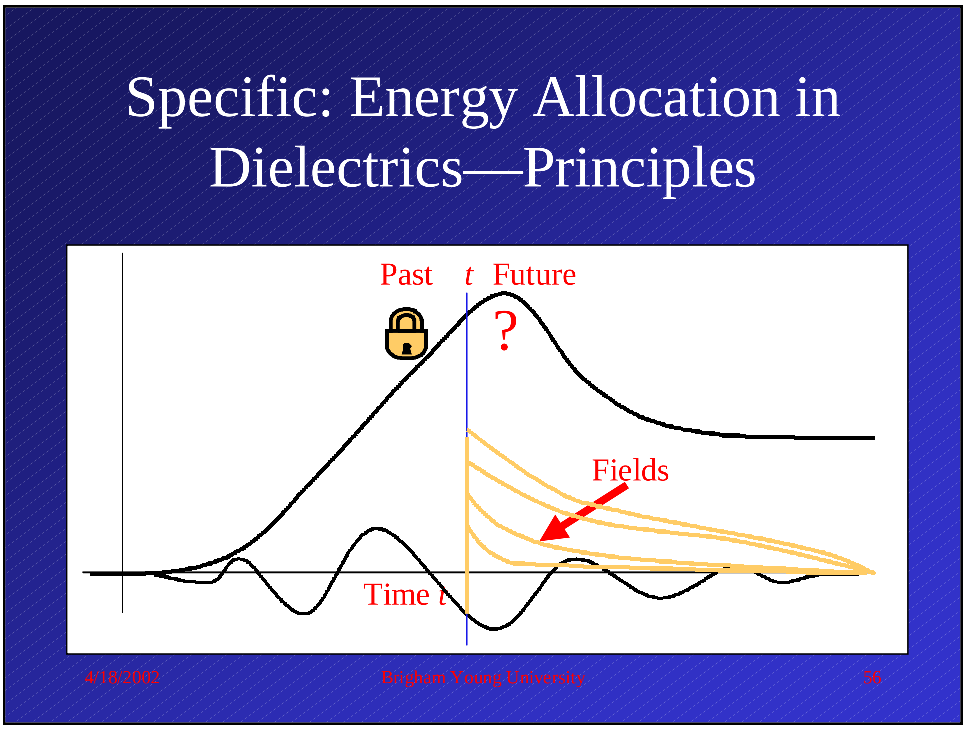

Specific: Energy Allocation in Dielectrics—Principles
Past
t
Future
?
Fields
Time t
4/18/2002
Brigham Young University
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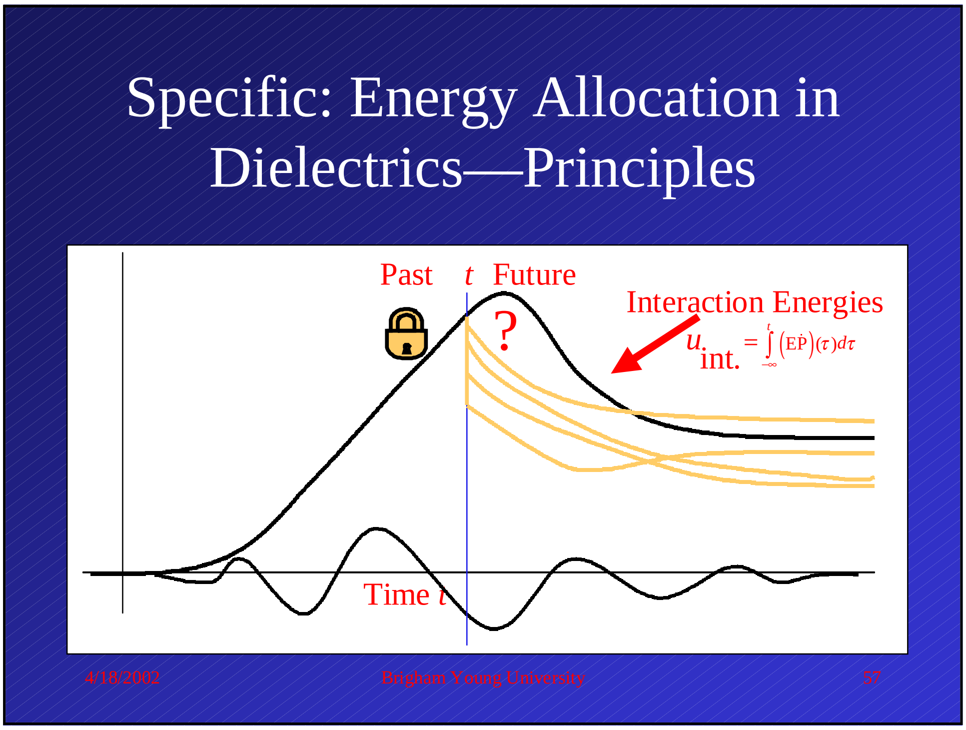
Specific: Energy Allocation in Dielectrics—Principles
Past
t
Future
?
Interaction Energies
$u_{\text{int.}} = \int_{-\infty}^{t} (E\dot{P})(\tau)\,d\tau$
Time t
4/18/2002
Brigham Young University
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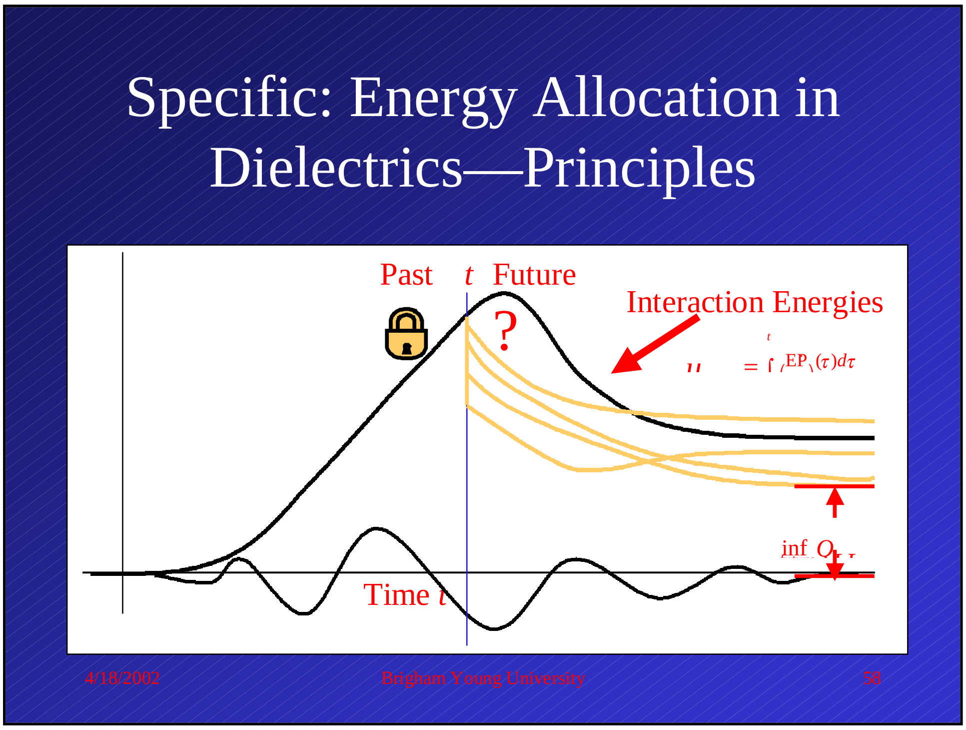
Specific: Energy Allocation in Dielectrics—Principles
Past
t
Future
?
Interaction Energies
inf
Time t
4/18/2002
Brigham Young University
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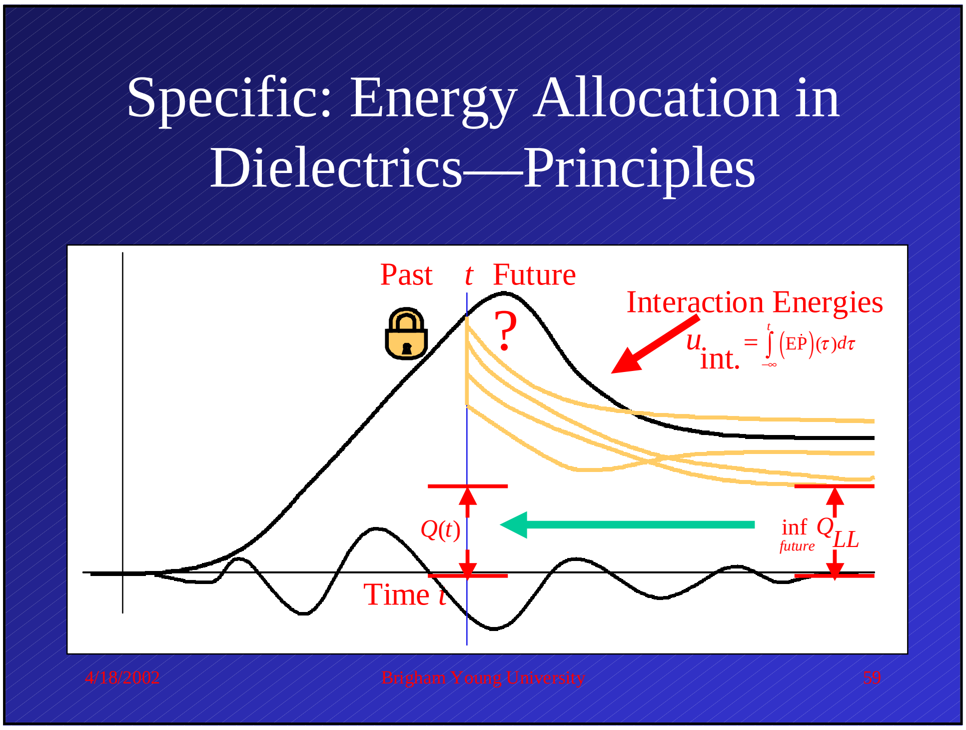
Specific: Energy Allocation in Dielectrics—Principles
Past
t
Future
?
Interaction Energies
$u_{int.} = \int_{-\infty}^{t} (E\dot{P})(\tau)d\tau$
Q(t)
inf Q_LL
future
Time t
4/18/2002
Brigham Young University
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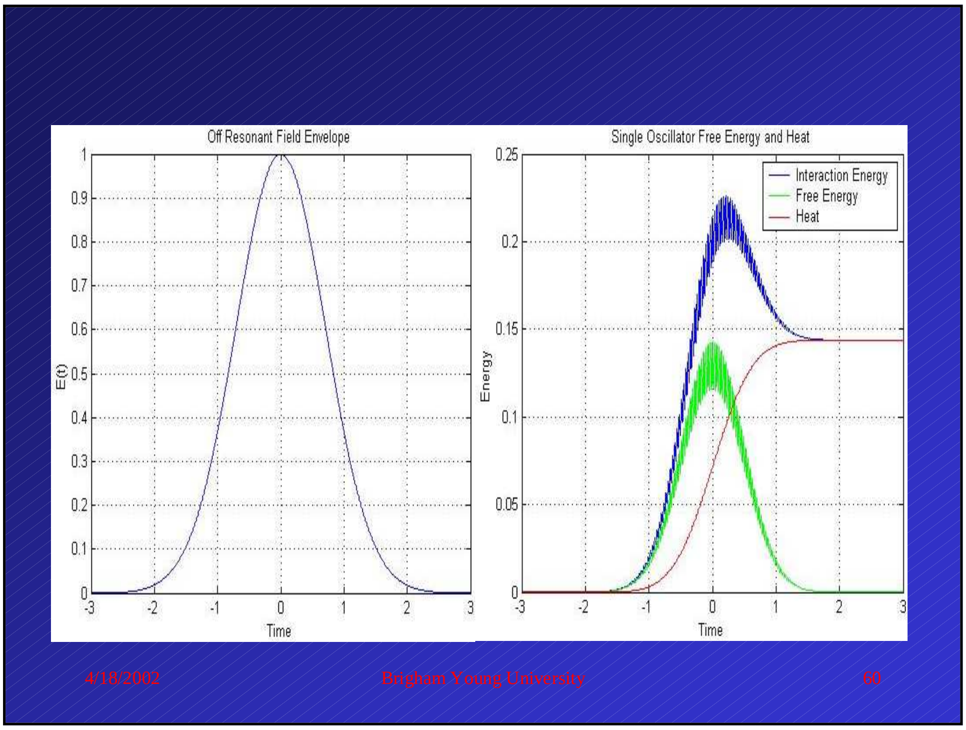
Off Resonant Field Envelope
E(t)
Time
Single Oscillator Free Energy and Heat
Interaction Energy
Free Energy
Heat
Energy
Time
4/18/2002
Brigham Young University
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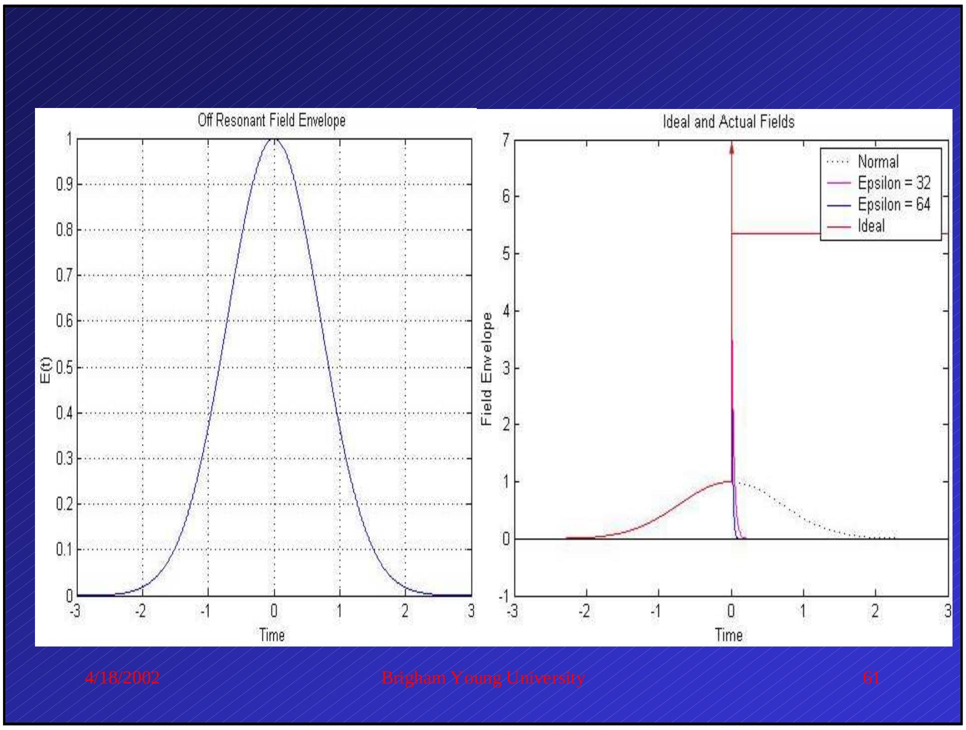
Off Resonant Field Envelope
E(t)
Time
Ideal and Actual Fields
Field Envelope
Time
Normal
Epsilon = 32
Epsilon = 64
Ideal
4/18/2002
Brigham Young University
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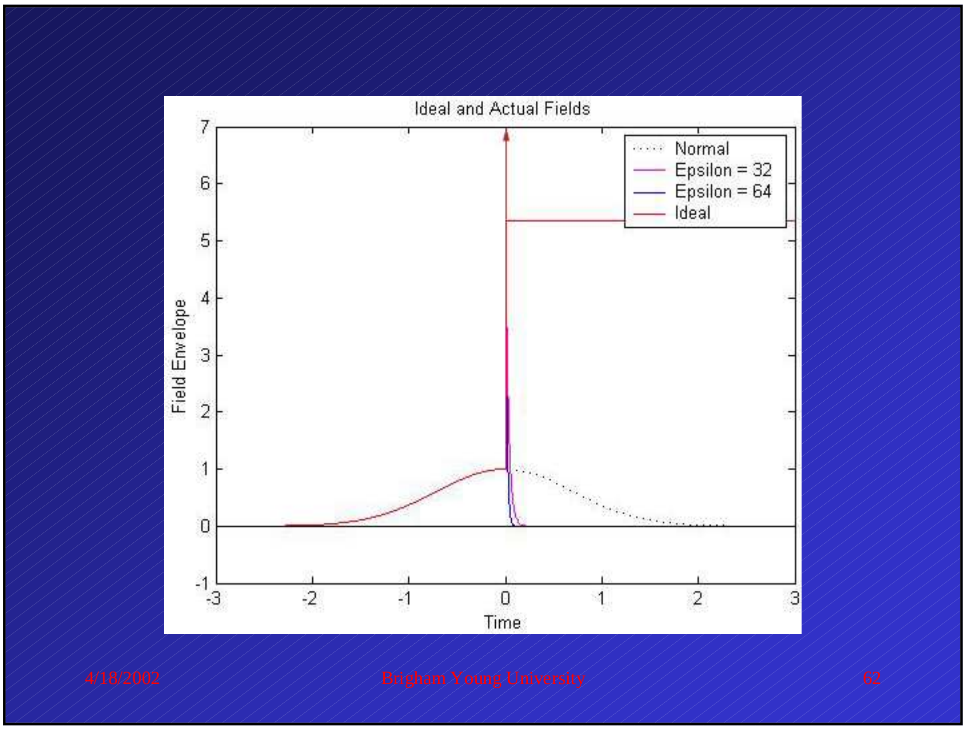
Ideal and Actual Fields
Field Envelope
Time
Normal
Epsilon = 32
Epsilon = 64
Ideal
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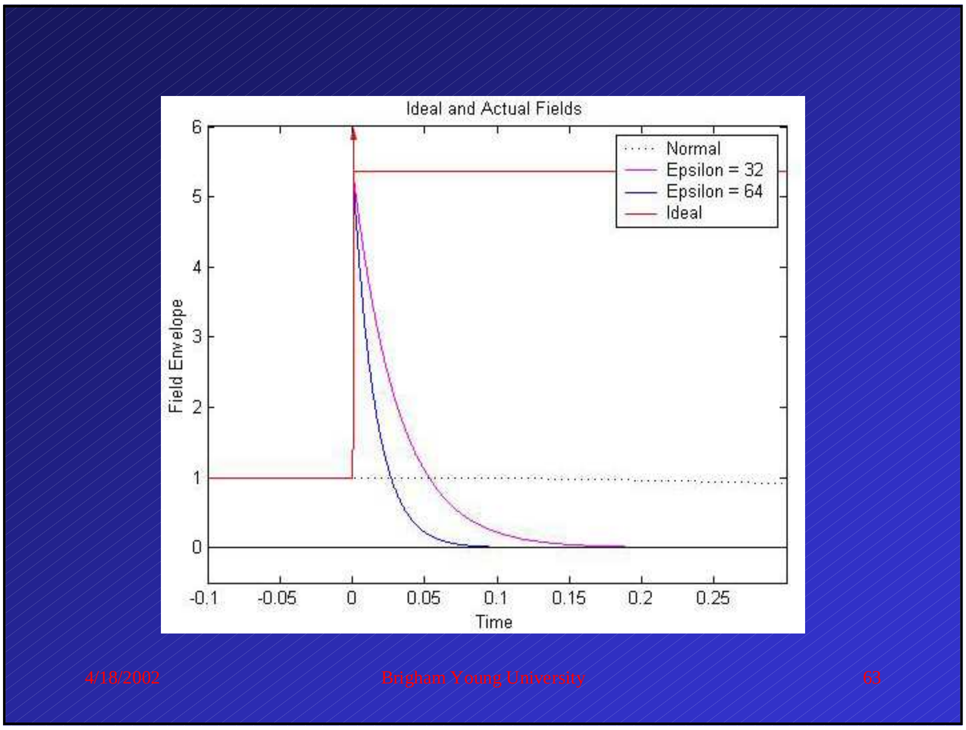
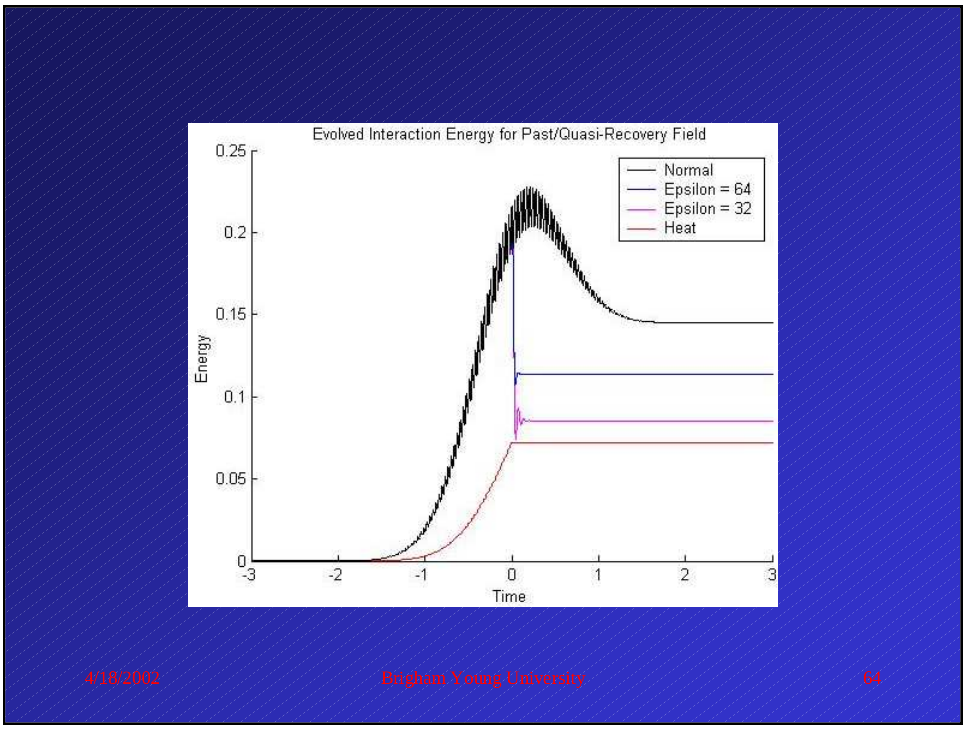
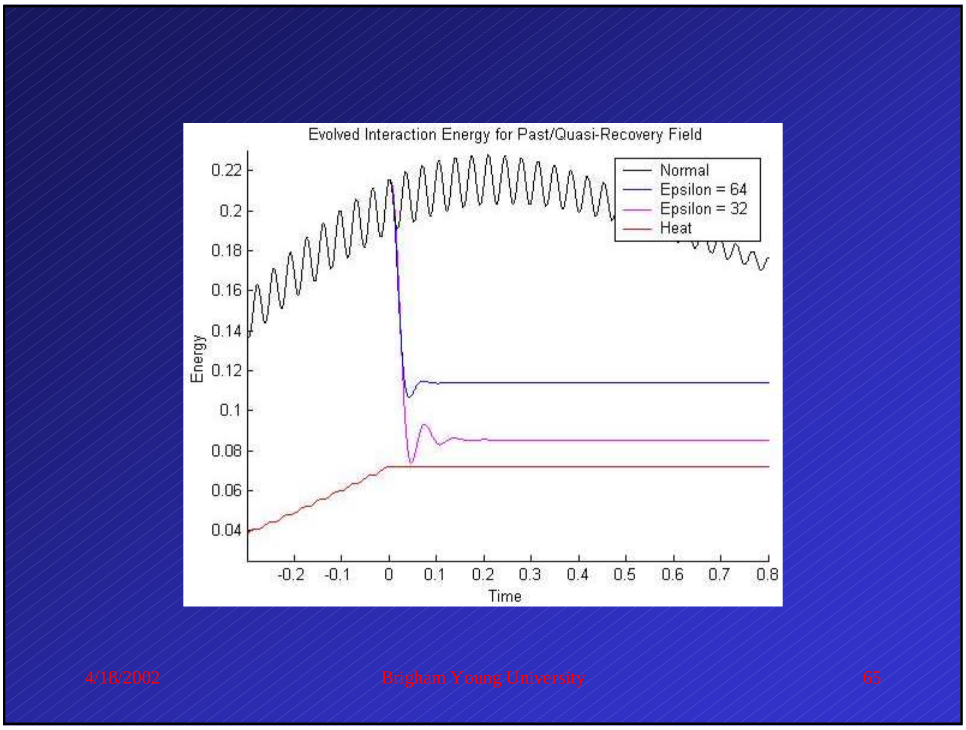
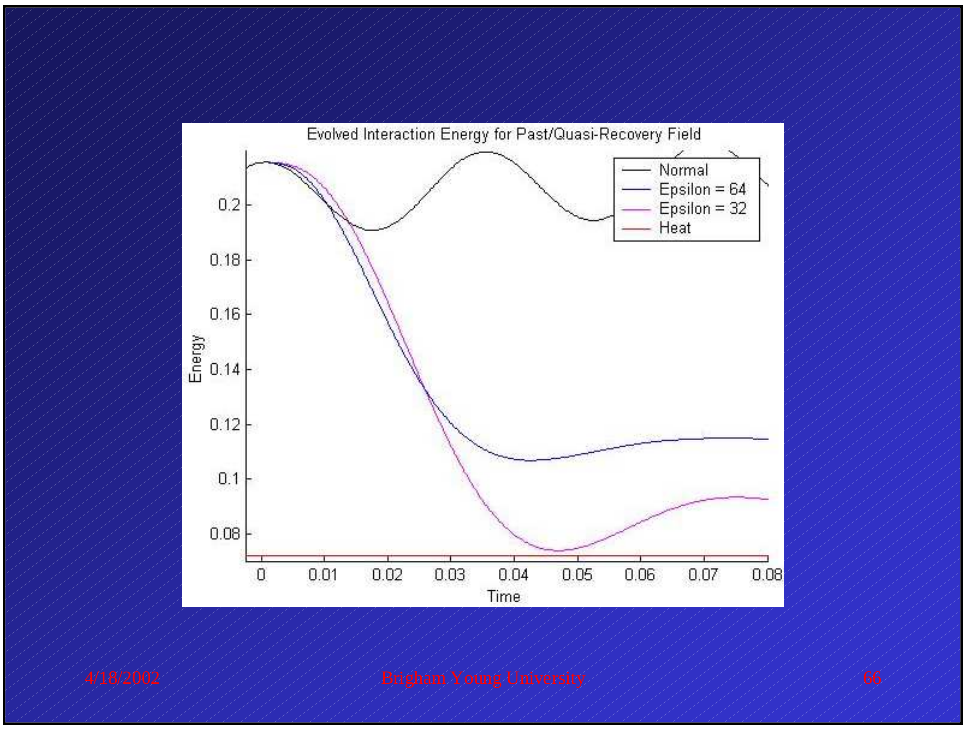
Reversible and Irreversible Processes in Dispersive/Dissipative Media: Electro-Magnetic Free Energy and Heat Production

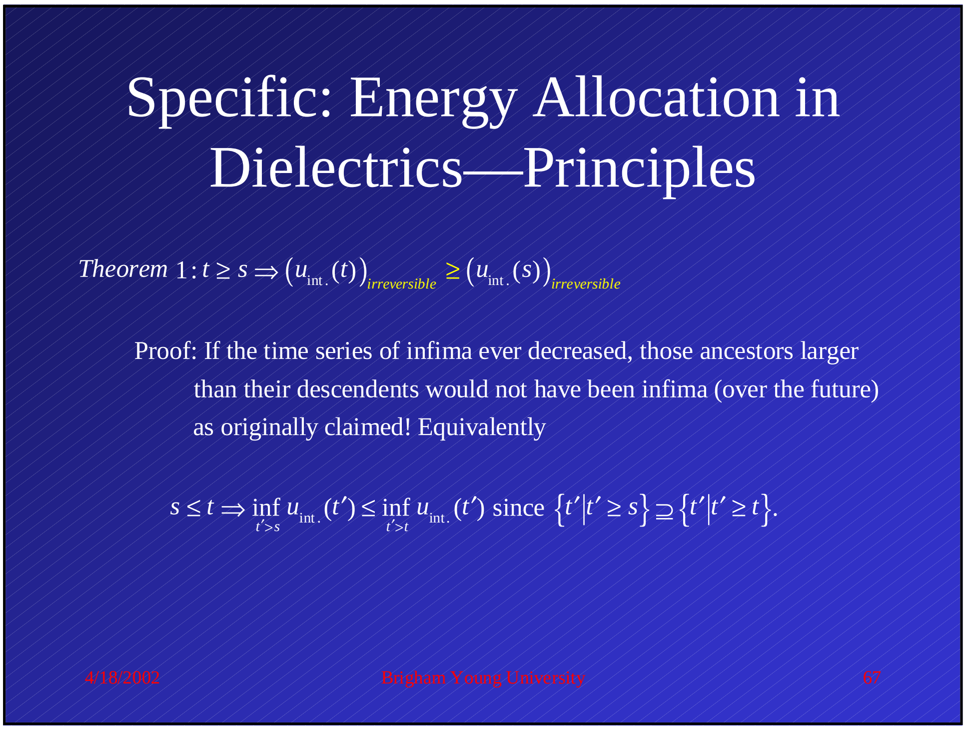




Reversible and Irreversible Processes in Dispersive/Dissipative Media: Electro-Magnetic Free Energy and Heat Production






Specific: Energy Allocation in Dielectrics—Principles
$Theorem\ 1: t \geq s \Rightarrow (u_{int.}(t))_{irreversible} \geq (u_{int.}(s))_{irreversible}$
Proof: If the time series of infima ever decreased, those ancestors larger than their descendents would not have been infima (over the future) as originally claimed! Equivalently
$s \leq t \Rightarrow \inf_{t'>s} u_{int.}(t') \leq \inf_{t'>t} u_{int.}(t')$ since $\{t' | t' \geq s\} \supseteq \{t' | t' \geq t\}$.
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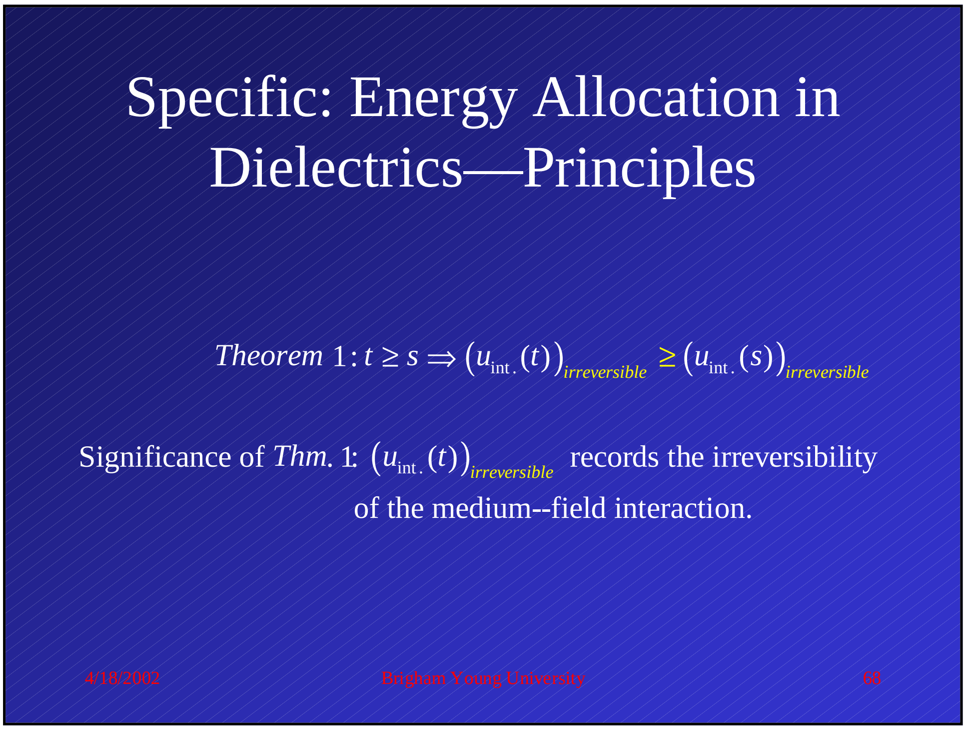
Specific: Energy Allocation in Dielectrics—Principles
$Theorem\ 1: t \geq s \Rightarrow (u_{int.}(t))_{irreversible} \geq (u_{int.}(s))_{irreversible}$
Significance of *Thm.* 1: $(u_{int.}(t))_{irreversible}$ records the irreversibility of the medium--field interaction.
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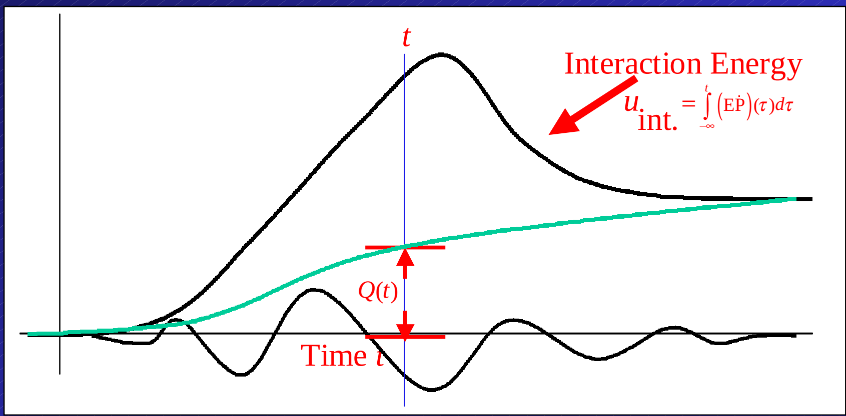
Specific: Energy Allocation in Dielectrics—Principles



Specific: Energy Allocation in Dielectrics—Principles

Theorem 2 : $(u_{int.}[\mathrm{E};\chi_1+\chi_2](t))_{irreversible} \geq (u_{int.}[\mathrm{E};\chi_1](t))_{irreversible} + (u_{int.}[\mathrm{E};\chi_2](t))_{irreversible}$

Proof Sketch: a) $u_{int.}[\mathrm{E};\chi](t)$ is linear in its second argument χ, but
b) $(u_{int.}[\mathrm{E};\chi](t))_{irreversible}$ is defined via an infemum of $u_{int.}[\mathrm{E};\chi](t)$ over its first argument E with the second argument χ fixed!

Specific: Energy Allocation in Dielectrics—Principles

Proof: $\left(u_{\text{int.}}[\mathrm{E};\chi_1+\chi_2](t)\right)_{irreversible} := \inf_{\mathrm{E_f}} u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}H_t^+;\chi_1+\chi_2\right](+\infty)$

$$= \inf_{\mathrm{E_f}}\left(u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}H_t^+;\chi_1\right](+\infty) + u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}H_t^+;\chi_2\right](+\infty)\right)$$

$$\geq \inf_{\mathrm{E_f}}\left(\inf_{\mathrm{E_f}'} u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}'H_t^+;\chi_1\right](+\infty) + \inf_{\mathrm{E_f}''} u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}''H_t^+;\chi_2\right](+\infty)\right)$$

$$= \inf_{\mathrm{E_f}'} u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}'H_t^+;\chi_1\right](+\infty) + \inf_{\mathrm{E_f}''} u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}''H_t^+;\chi_2\right](+\infty)$$

$$=: \left(u_{\text{int.}}[\mathrm{E};\chi_1](t)\right)_{irreversible} + \left(u_{\text{int.}}[\mathrm{E};\chi_2](t)\right)_{irreversible}.$$

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Specific: Energy Allocation in Dielectrics—Principles

Corollary 2: $\left(u_{\text{int.}}[\mathrm{E};\chi_1+\chi_2](t)\right)_{reversible} \leq \left(u_{\text{int.}}[\mathrm{E};\chi_1](t)\right)_{reversible} + \left(u_{\text{int.}}[\mathrm{E};\chi_2](t)\right)_{reversible}$

Significance of *C*.2 : Energy available to do work within individual constituents of composite media can never combine constructively.

Significance of *Thm.* 2: Creating composites generates irreversibility / heat / entropy.

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Specific: Energy Allocation in Dielectrics—Principles

$$Kramers-Kronig: \chi(\omega) = \lim_{\gamma\to 0} \int_{\omega_0 \in \Re} \chi_{Lorentz}[\omega_0, \gamma, \omega_p](\omega) \text{ where}$$

$$\chi_{Lorentz}[\omega_0, \gamma, \omega_p](\omega) = \frac{\omega_p^2(\omega_0)}{-\omega^2 - 2i\gamma\omega + \omega_0^2} \text{ and}$$

$$\omega_p^2(\omega_0) = \omega_0 \operatorname{Im}[\chi(\omega_0)] d\omega_0 / \pi.$$

$$\left(u_{\text{int.}}[\text{E};\chi](t)\right)_{irreversible} = \left(u_{\text{int.}}[\text{E}; \lim_{\gamma\to 0} \int_{\omega_0 \in \Re} \chi_{Lorentz}[\omega_0, \gamma, \omega_p]](t)\right)_{irreversible}$$
$$\geq \lim_{\gamma\to 0} \int_{\omega_0 \in \Re} \left(u_{\text{int.}}[\text{E}; \chi_{Lorentz}[\omega_0, \gamma, \omega_p]](t)\right)_{irreversible} = 0.$$

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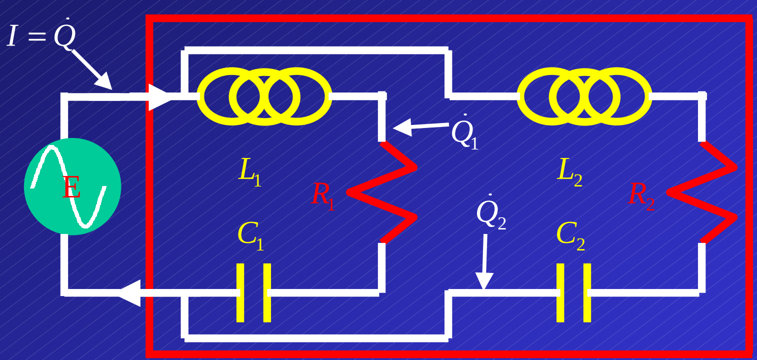
"Microscopic" Approach — Incorrect Macroscopically



$$U_{total}(t) = \int_{-\infty}^{t} \text{E}(\tau)\dot{Q}(\tau)d\tau = \frac{L_1}{2}\dot{Q}_1^2(t) + \frac{L_2}{2}\dot{Q}_2^2(t) + \frac{1}{2C_1}Q_1^2(t) + \frac{1}{2C_2}Q_2^2(t) + R_1\int_{-\infty}^{t}\dot{Q}_1^2(\tau)d\tau + R_2\int_{-\infty}^{t}\dot{Q}_2^2(\tau)d\tau$$

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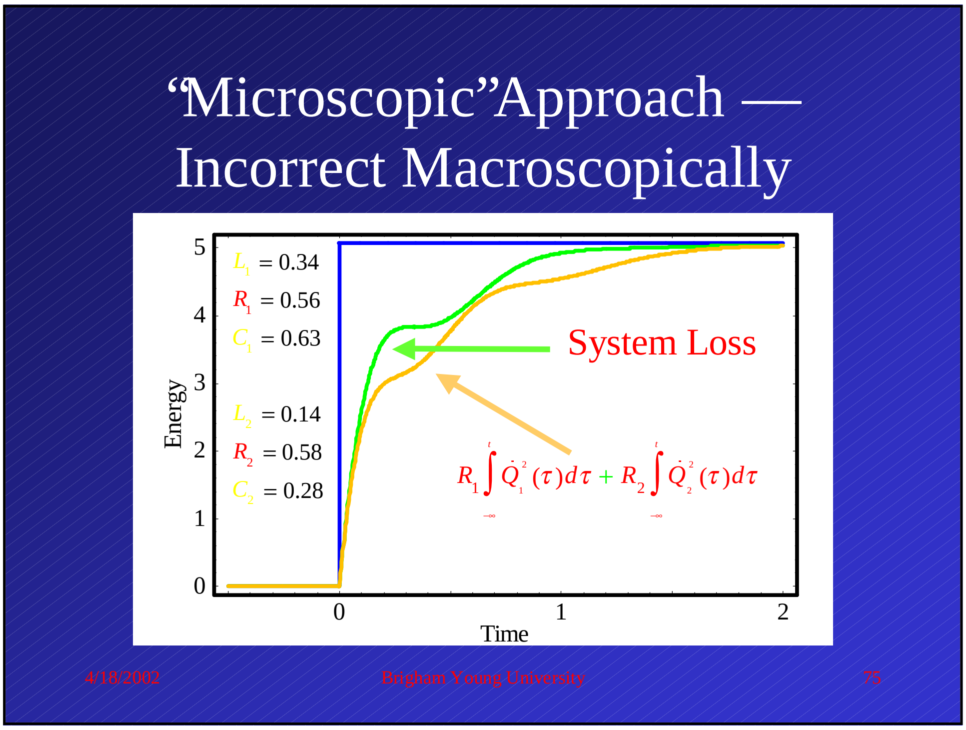

“Microscopic”Approach —
Incorrect Macroscopically
$L_1 = 0.34$
$R_1 = 0.56$
$C_1 = 0.63$
$L_2 = 0.14$
$R_2 = 0.58$
$C_2 = 0.28$
System Loss
$R_1\int_{-\infty}^{t}\dot{Q}_1^2(\tau)d\tau + R_2\int_{-\infty}^{t}\dot{Q}_2^2(\tau)d\tau$
Energy
Time
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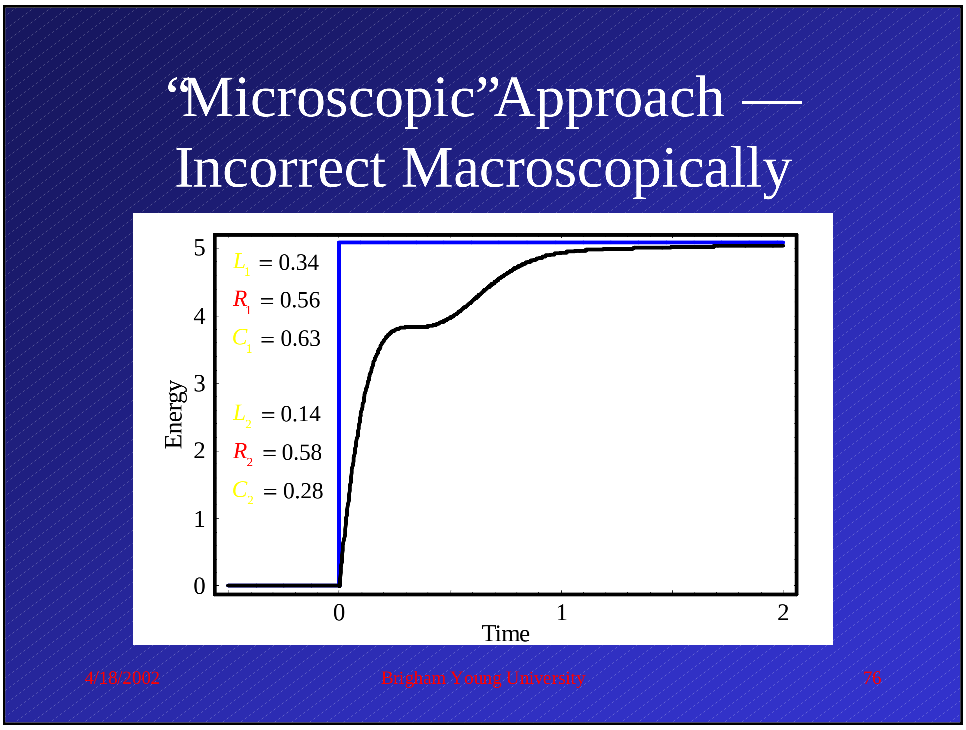

“Microscopic”Approach —
Incorrect Macroscopically
$L_1 = 0.34$
$R_1 = 0.56$
$C_1 = 0.63$
$L_2 = 0.14$
$R_2 = 0.58$
$C_2 = 0.28$
Energy
Time
4/18/2002
Brigham Young University
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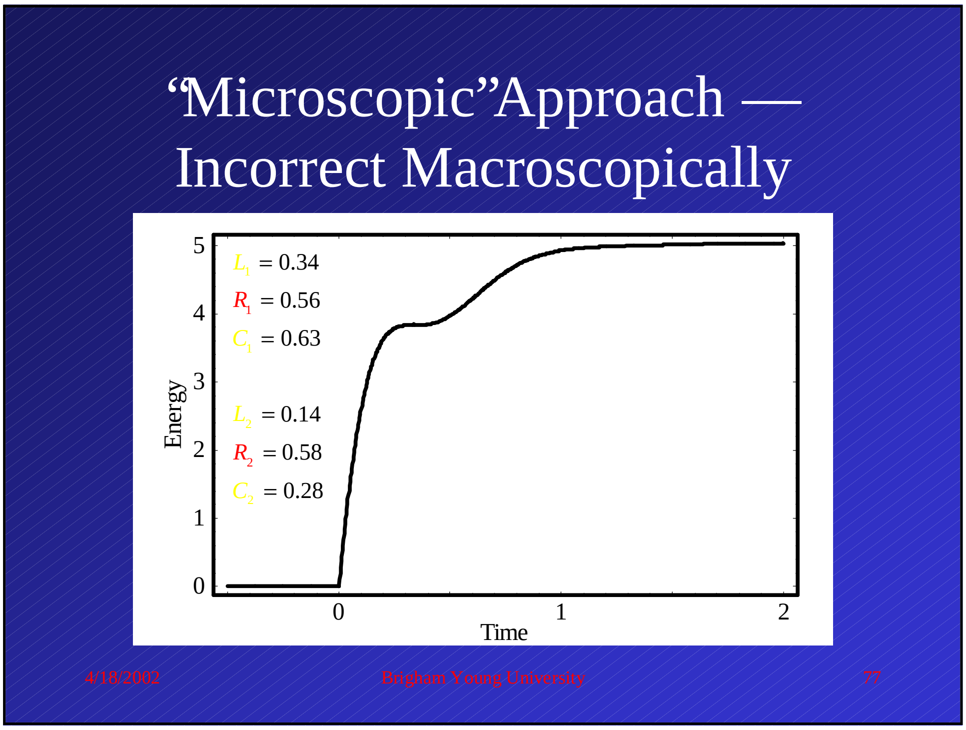

“Microscopic”Approach —
Incorrect Macroscopically
L_1 = 0.34
R_1 = 0.56
C_1 = 0.63
L_2 = 0.14
R_2 = 0.58
C_2 = 0.28
Energy
0
1
2
3
4
5
Time
0
1
2
4/18/2002
Brigham Young University
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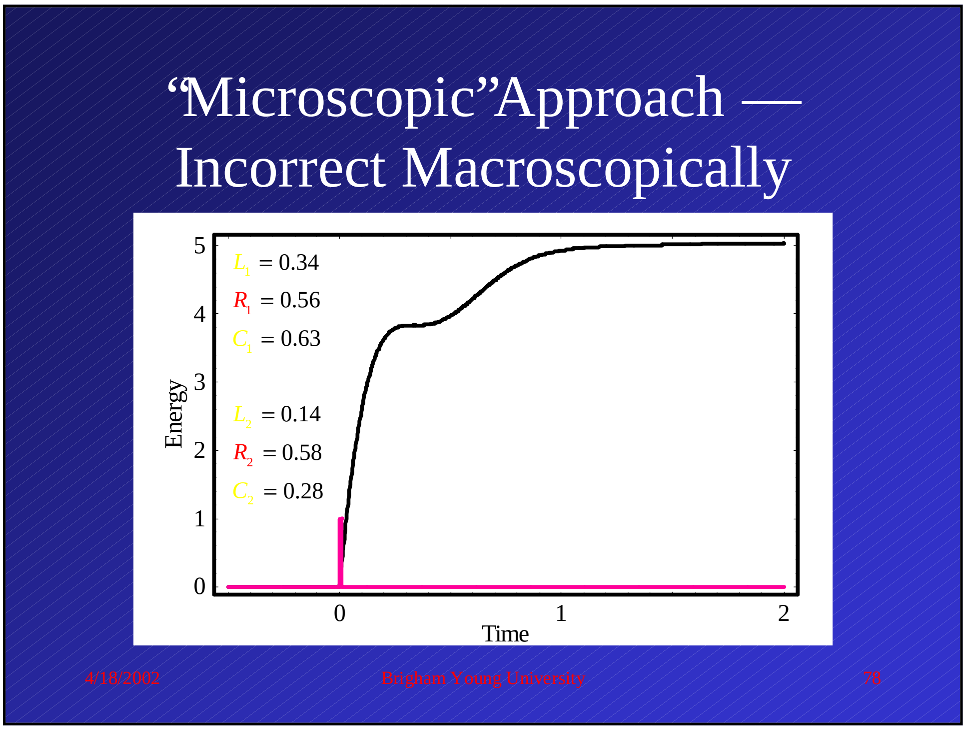

“Microscopic”Approach —
Incorrect Macroscopically
L_1 = 0.34
R_1 = 0.56
C_1 = 0.63
L_2 = 0.14
R_2 = 0.58
C_2 = 0.28
Energy
0
1
2
3
4
5
Time
0
1
2
4/18/2002
Brigham Young University
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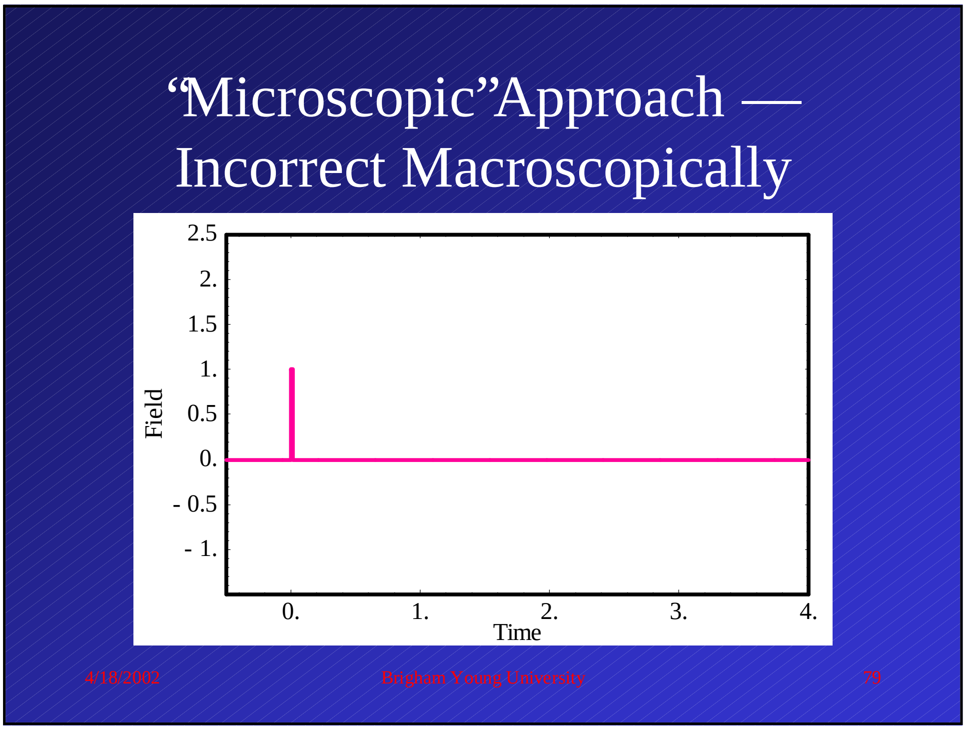
"Microscopic" Approach —
Incorrect Macroscopically
Field
Time
2.5
2.
1.5
1.
0.5
0.
- 0.5
- 1.
0.
1.
2.
3.
4.
4/18/2002
Brigham Young University
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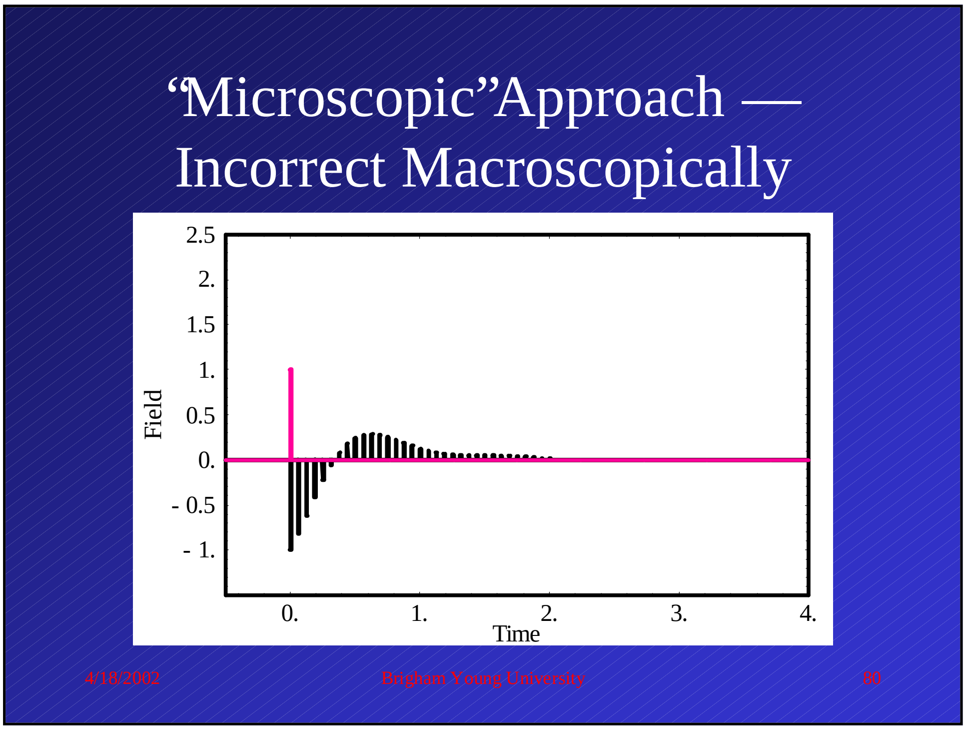
"Microscopic" Approach —
Incorrect Macroscopically
Field
Time
2.5
2.
1.5
1.
0.5
0.
- 0.5
- 1.
0.
1.
2.
3.
4.
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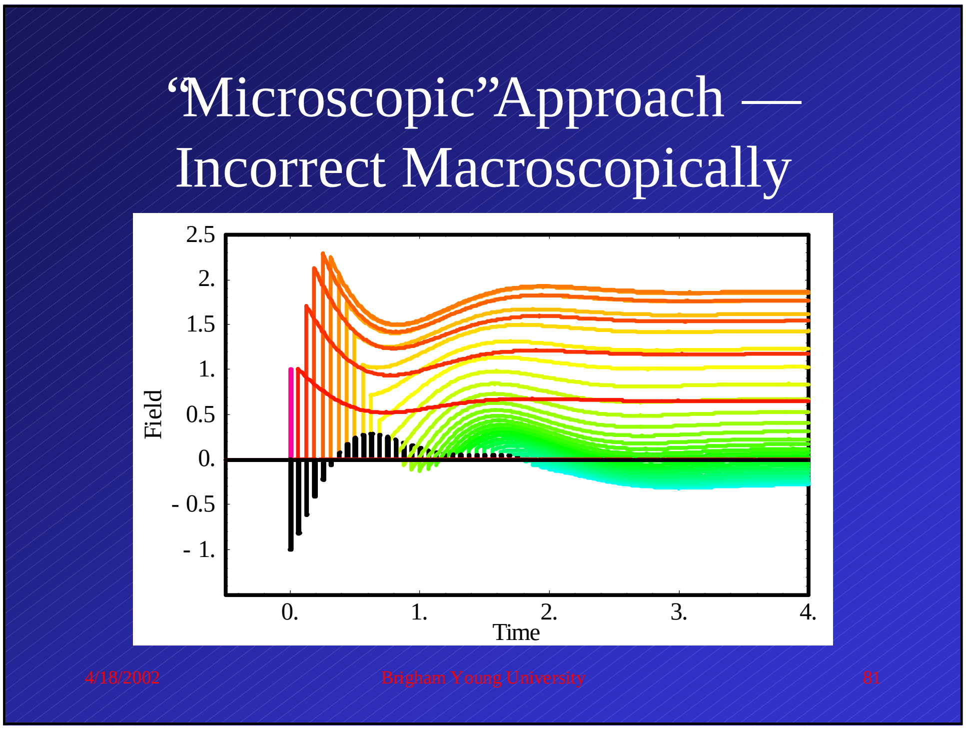
“Microscopic”Approach —
Incorrect Macroscopically
Field
2.5
2.
1.5
1.
0.5
0.
- 0.5
- 1.
0.
1.
2.
3.
4.
Time
4/18/2002
Brigham Young University
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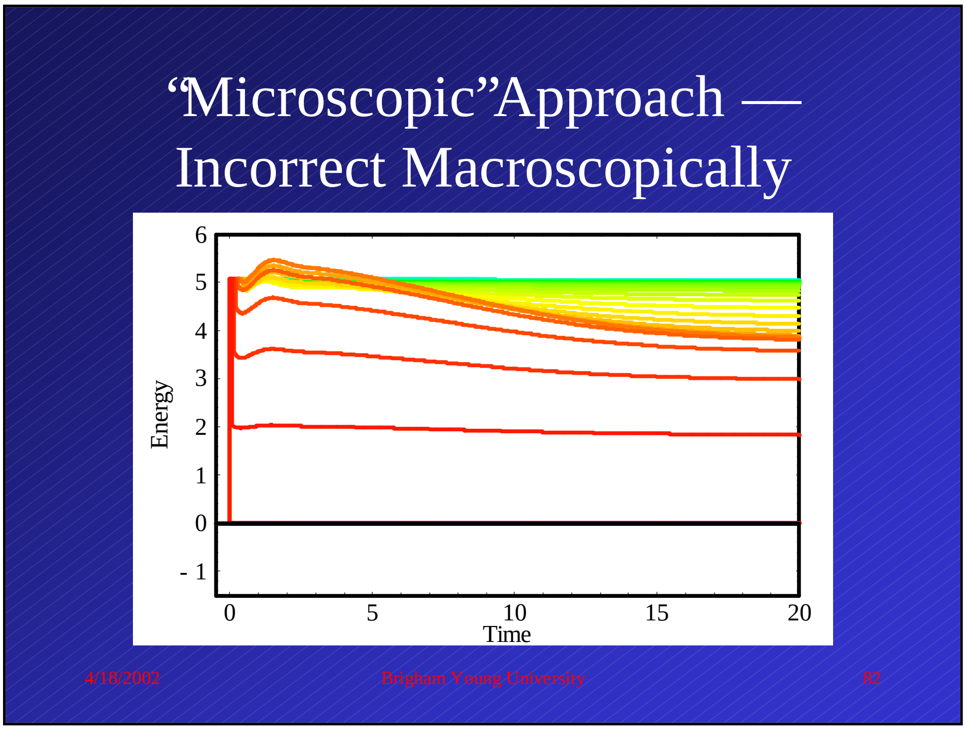
“Microscopic”Approach —
Incorrect Macroscopically
Energy
6
5
4
3
2
1
0
- 1
0
5
10
15
20
Time
4/18/2002
Brigham Young University
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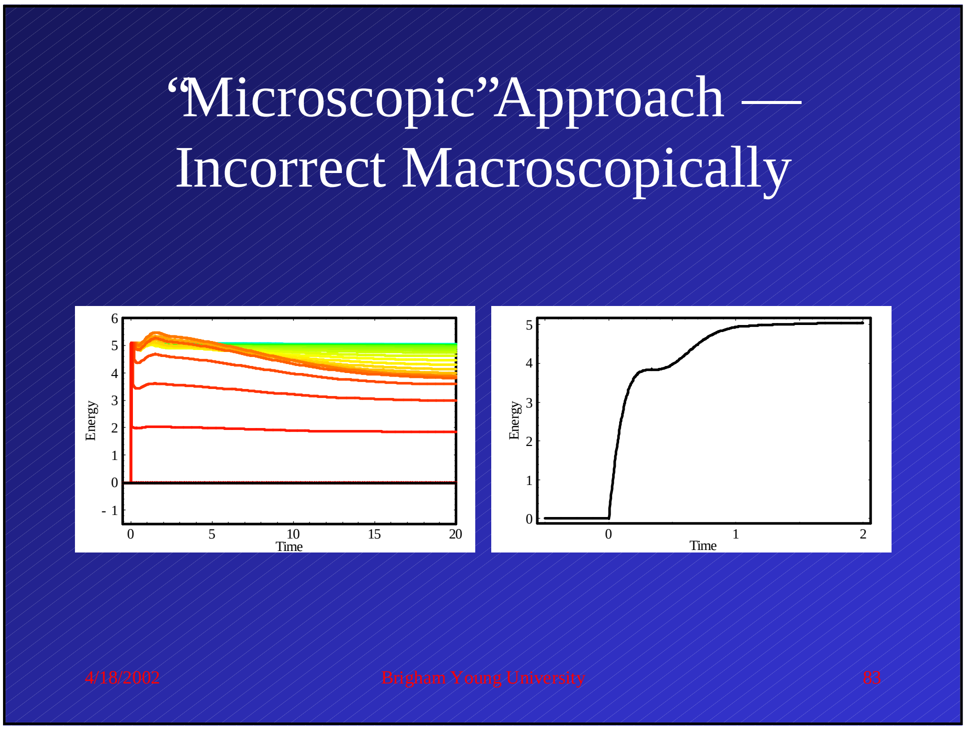
"Microscopic"Approach —
Incorrect Macroscopically
Energy
Time
Energy
Time
4/18/2002
Brigham Young University
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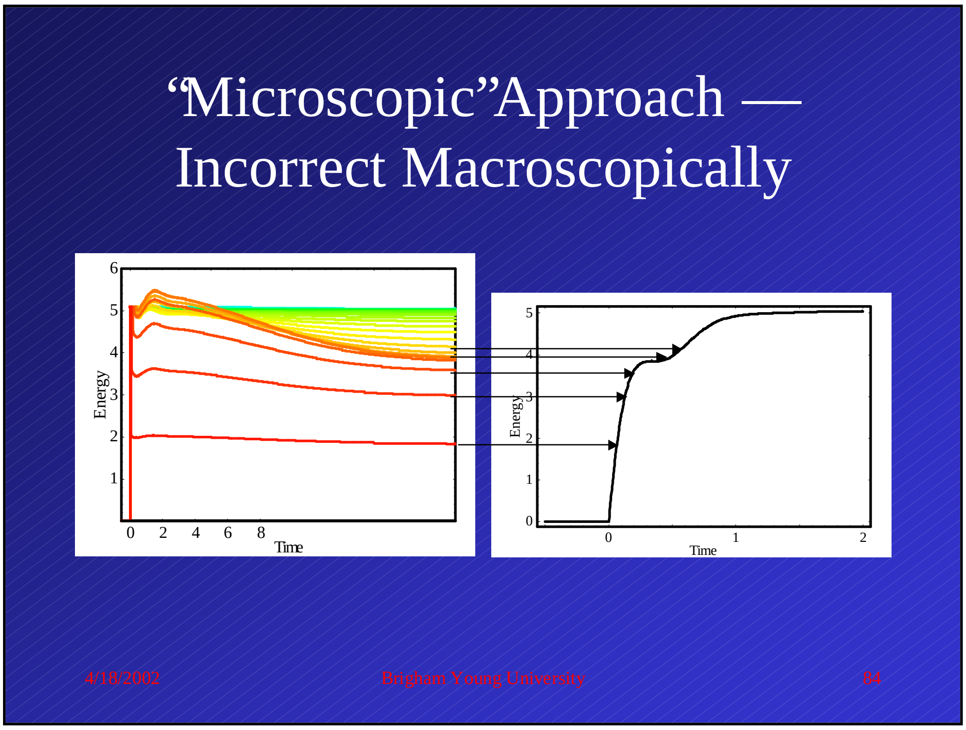
"Microscopic"Approach —
Incorrect Macroscopically
Energy
Time
Energy
Time
4/18/2002
Brigham Young University
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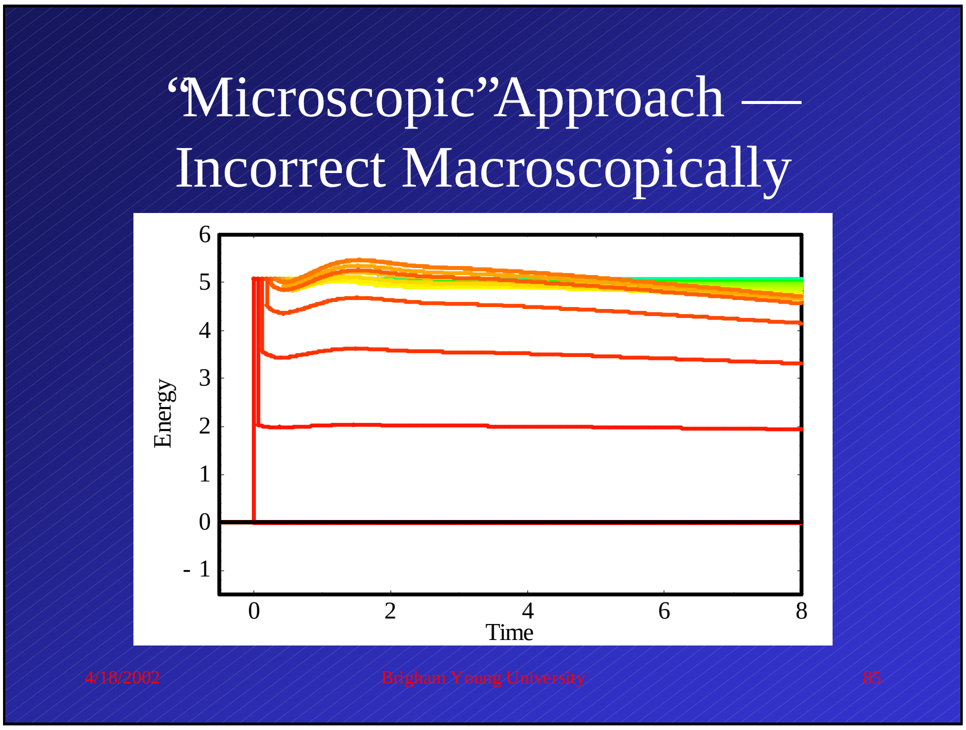
“Microscopic”Approach —
Incorrect Macroscopically
Energy
Time
6
5
4
3
2
1
0
- 1
0
2
4
6
8
4/18/2002
Brigham Young University
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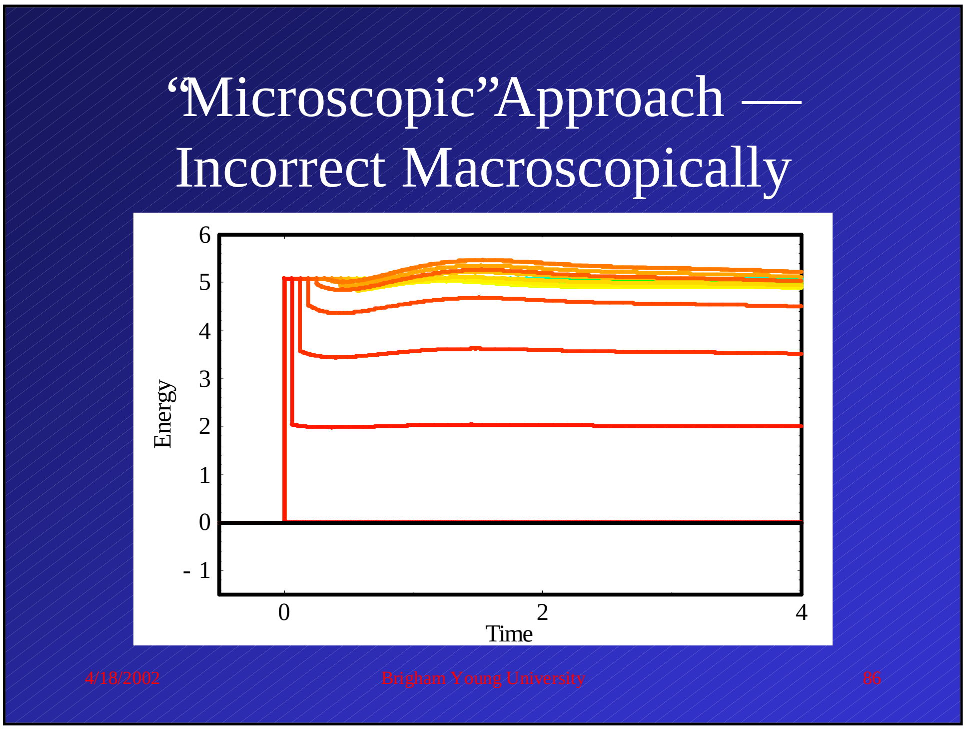
“Microscopic”Approach —
Incorrect Macroscopically
Energy
Time
6
5
4
3
2
1
0
- 1
0
2
4
4/18/2002
Brigham Young University
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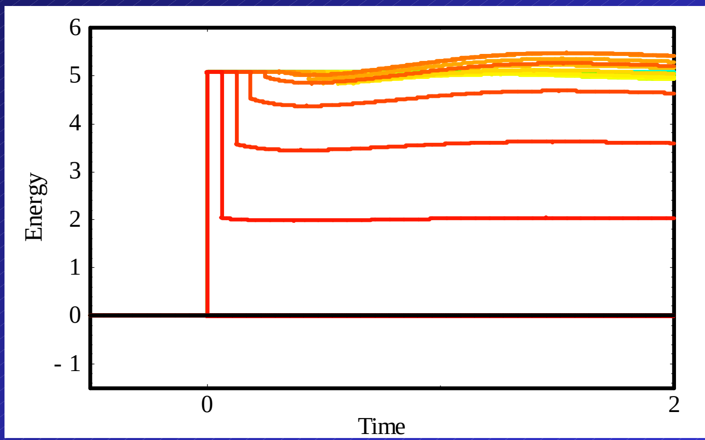
"Microscopic" Approach — Incorrect Macroscopically



Specific: Energy Allocation in Dielectrics—Mathematics

$$u_{\text{int.}}[E;\chi](t) = \int_{-\infty}^{t} \left(E\dot{P}[E;\chi]\right)(\tau)\,d\tau$$

$\hat{P}[E;\chi](\omega) = \chi(\omega)\hat{E}(\omega)$, χ causal, passive.

Specific: Energy Allocation in Dielectrics—Mathematics

Theorem (10/01/01 PRE): $u_{\text{int.}}[\mathrm{E};\chi](t) = \int_{-\infty}^{+\infty} \omega \operatorname{Im}[\chi(\omega)] \left|\hat{\mathrm{E}}_t^-(\omega)\right|^2 d\omega,$ where

the lower analytic spectrum of E at time t, $\hat{\mathrm{E}}_t^-(\omega),$ is obtained by

$$\hat{\mathrm{E}}_t^-(\omega) := e^{-i\omega t}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t} \mathrm{E}(\tau)e^{i\omega\tau}d\tau = e^{-i\omega t}\,\mathrm{F}\left[\mathrm{E}H_t^-\right](\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} \mathrm{E}(\tau+t)e^{i\omega\tau}d\tau.$$

Note that $\hat{\mathrm{E}}_t^-(\omega) \in \mathrm{A}^-$:= the set of complex-valued functions analytic and rapidly vanishing in the lower-half ω-plane. Then $\hat{\mathrm{E}}_t^+(\omega) := e^{-i\omega t}\,\mathrm{F}\left[\mathrm{E}H_t^+\right](\omega) \in \mathrm{A}^+$.

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Theorem (10/01/01 PRE) $\Rightarrow$

$$\left(u_{\text{int.}}(t)\right)_{irreversible} := \inf_{\mathrm{E_f}} u_{\text{int.}}\left[\mathrm{E}H_t^- + \mathrm{E_f}H_t^+;\chi\right](+\infty)$$

$$= \inf_{\hat{\mathrm{E}}_t^+ \in \mathrm{A}^+} \int_{-\infty}^{+\infty} \omega \operatorname{Im}[\chi(\omega)] \left|\hat{\mathrm{E}}_t^-(\omega) + \hat{\mathrm{E}}_t^+(\omega)\right|^2 d\omega.$$

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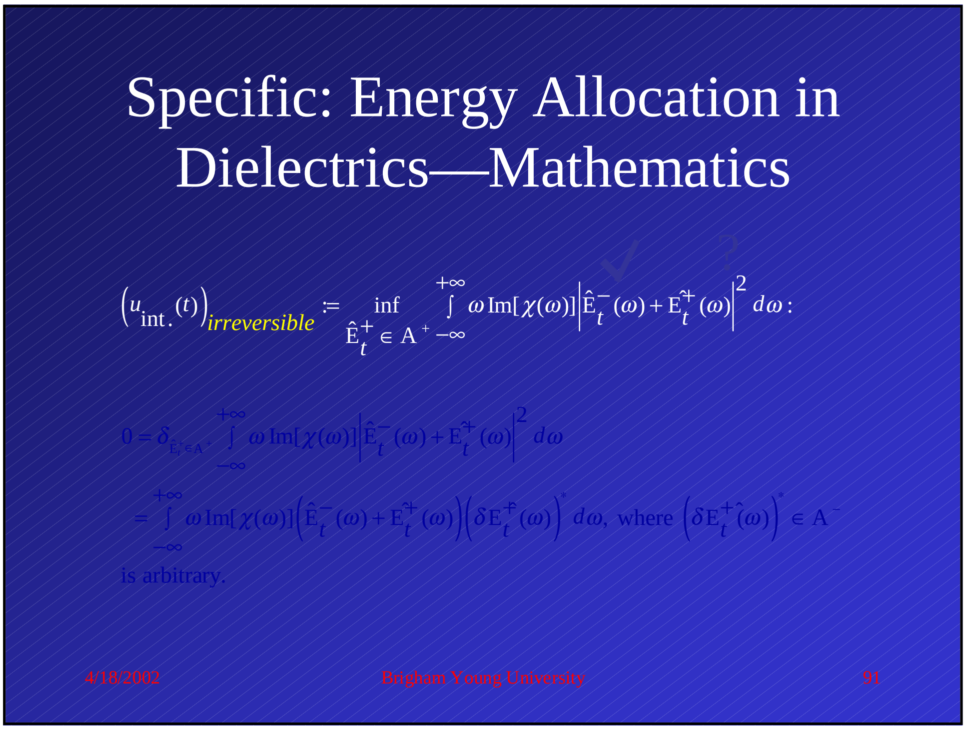
Specific: Energy Allocation in Dielectrics—Mathematics
$\left(u_{\mathrm{int.}}(t)\right)_{irreversible} := \inf_{\hat{E}_t^{+} \in A^{+}} \int_{-\infty}^{+\infty} \omega \,\mathrm{Im}[\chi(\omega)] \left| \hat{E}_t^{-}(\omega) + \hat{E}_t^{+}(\omega) \right|^2 d\omega :$
$0 = \delta_{\hat{E}_t^{+} \in A^{+}} \int_{-\infty}^{+\infty} \omega \,\mathrm{Im}[\chi(\omega)] \left| \hat{E}_t^{-}(\omega) + \hat{E}_t^{+}(\omega) \right|^2 d\omega$
$= \int_{-\infty}^{+\infty} \omega \,\mathrm{Im}[\chi(\omega)] \left(\hat{E}_t^{-}(\omega) + \hat{E}_t^{+}(\omega) \right) \left(\delta \hat{E}_t^{+}(\omega) \right)^* d\omega$, where $\left(\delta \hat{E}_t^{+}(\omega) \right)^* \in A^{-}$
is arbitrary.
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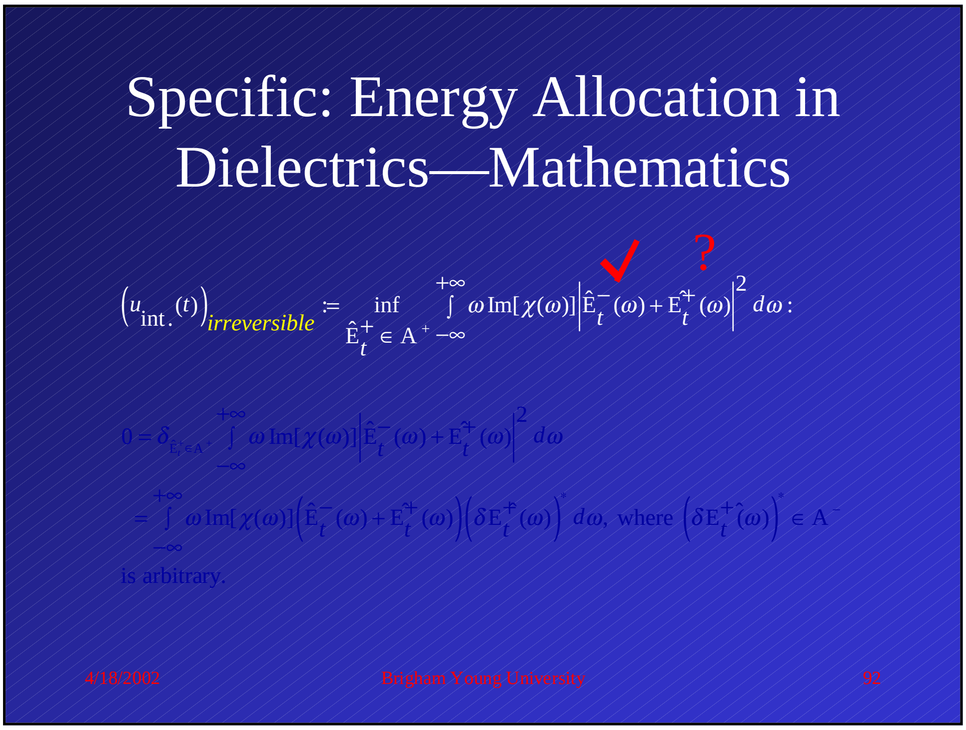
Specific: Energy Allocation in Dielectrics—Mathematics
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$\left(u_{\mathrm{int.}}(t)\right)_{irreversible} := \inf_{\hat{E}_t^{+} \in A^{+}} \int_{-\infty}^{+\infty} \omega \,\mathrm{Im}[\chi(\omega)] \left| \hat{E}_t^{-}(\omega) + \hat{E}_t^{+}(\omega) \right|^2 d\omega :$
$0 = \delta_{\hat{E}_t^{+} \in A^{+}} \int_{-\infty}^{+\infty} \omega \,\mathrm{Im}[\chi(\omega)] \left| \hat{E}_t^{-}(\omega) + \hat{E}_t^{+}(\omega) \right|^2 d\omega$
$= \int_{-\infty}^{+\infty} \omega \,\mathrm{Im}[\chi(\omega)] \left(\hat{E}_t^{-}(\omega) + \hat{E}_t^{+}(\omega) \right) \left(\delta \hat{E}_t^{+}(\omega) \right)^* d\omega$, where $\left(\delta \hat{E}_t^{+}(\omega) \right)^* \in A^{-}$
is arbitrary.
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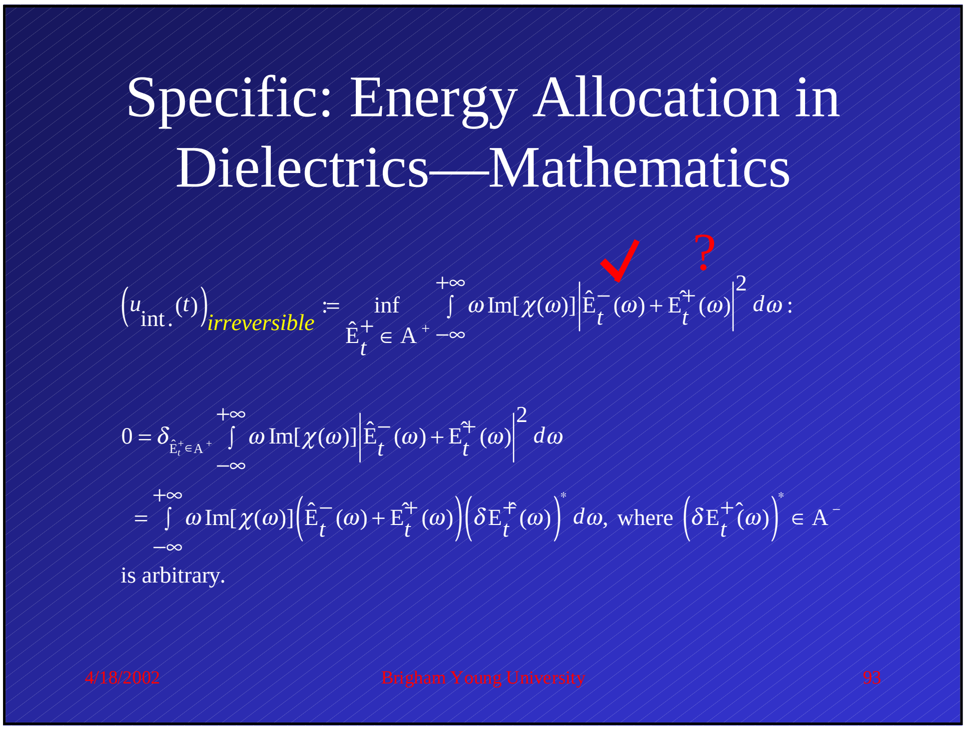
Specific: Energy Allocation in Dielectrics—Mathematics
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$\left(u_{\text{int.}}(t)\right)_{irreversible} := \inf_{\hat{E}_t^+ \in A^+} \int_{-\infty}^{+\infty} \omega \,\text{Im}[\chi(\omega)] \left|\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right|^2 d\omega :$
$0 = \delta_{\hat{E}_t^+ \in A^+} \int_{-\infty}^{+\infty} \omega \,\text{Im}[\chi(\omega)] \left|\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right|^2 d\omega$
$= \int_{-\infty}^{+\infty} \omega \,\text{Im}[\chi(\omega)] \left(\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right)\left(\delta \hat{E}_t^+(\omega)\right)^* d\omega,$ where $\left(\delta \hat{E}_t^+(\omega)\right)^* \in A^-$
is arbitrary.
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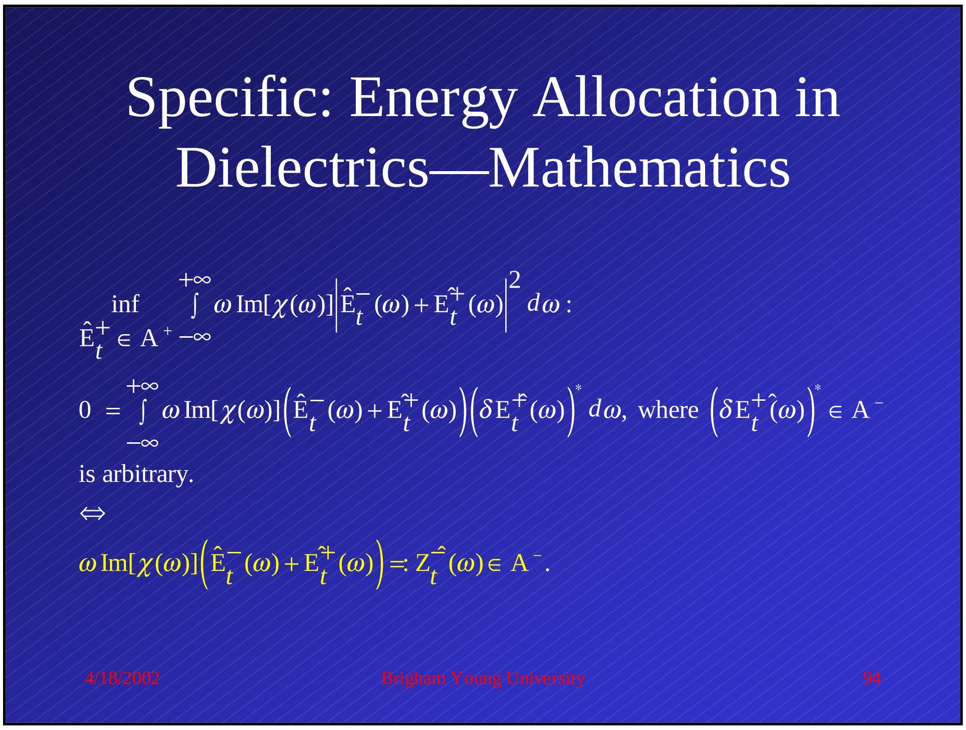
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$\inf_{\hat{E}_t^+ \in A^+} \int_{-\infty}^{+\infty} \omega \,\text{Im}[\chi(\omega)] \left|\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right|^2 d\omega :$
$0 = \int_{-\infty}^{+\infty} \omega \,\text{Im}[\chi(\omega)] \left(\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right)\left(\delta \hat{E}_t^+(\omega)\right)^* d\omega,$ where $\left(\delta \hat{E}_t^+(\omega)\right)^* \in A^-$
is arbitrary.
$\Leftrightarrow$
$\omega \,\text{Im}[\chi(\omega)] \left(\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right) =: \hat{Z}_t^-(\omega) \in A^-.$
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$$\left(u_{int.}(t)\right)_{irreversible} = \int_{-\infty}^{+\infty} \omega\, \mathrm{Im}[\chi(\omega)] \left|\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right|^2 d\omega$$

where $\hat{E}_t^+(\omega) \in A^-$ is the unique solution to the equation

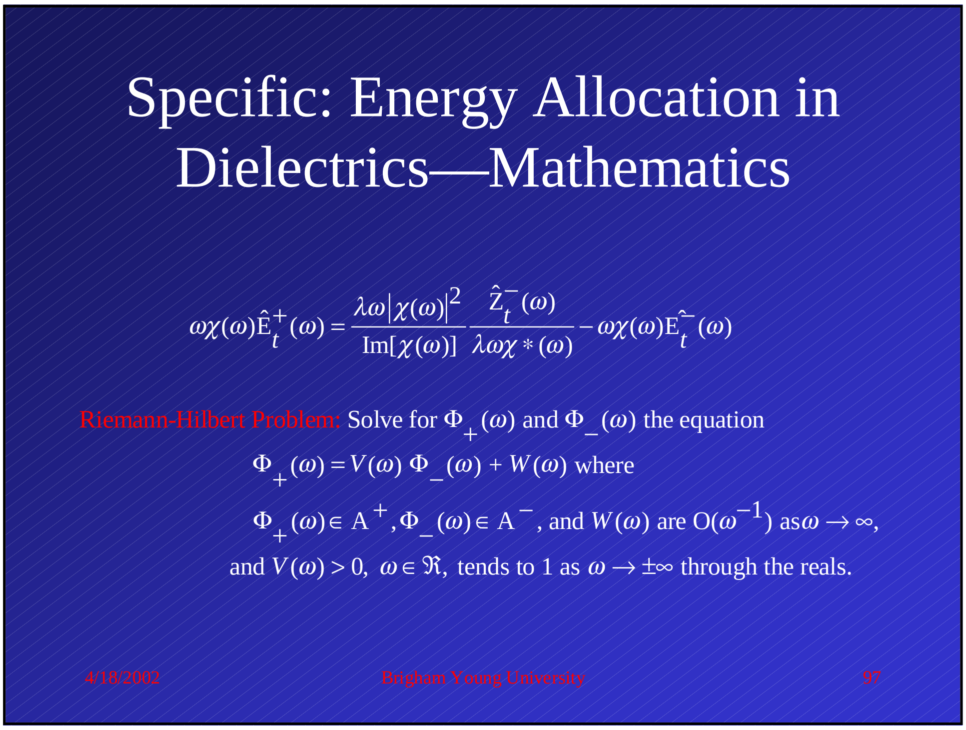
$$A^- \ni \hat{Z}_t^-(\omega) = \omega\, \mathrm{Im}[\chi(\omega)]\left(\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right).$$

Specific: Energy Allocation in Dielectrics—Mathematics

$$\hat{Z}_t^-(\omega) = \omega\, \mathrm{Im}[\chi(\omega)]\left(\hat{E}_t^-(\omega) + \hat{E}_t^+(\omega)\right)$$

$$\omega\chi(\omega)\hat{E}_t^+(\omega) = \frac{\lambda\omega|\chi(\omega)|^2}{\mathrm{Im}[\chi(\omega)]}\,\frac{\hat{Z}_t^-(\omega)}{\lambda\omega\chi^*(\omega)} - \omega\chi(\omega)\hat{E}_t^-(\omega)$$

$$\lambda := \lim_{\omega\to\infty} \frac{\mathrm{Im}[\chi(\omega)]}{\omega|\chi(\omega)|^2} > 0.$$


Specific: Energy Allocation in Dielectrics—Mathematics
$\omega\chi(\omega)\hat{E}_t^+(\omega) = \frac{\lambda\omega|\chi(\omega)|^2}{\mathrm{Im}[\chi(\omega)]}\frac{\hat{Z}_t^-(\omega)}{\lambda\omega\chi^*(\omega)} - \omega\chi(\omega)\hat{E}_t^-(\omega)$
Riemann-Hilbert Problem: Solve for $\Phi_+(\omega)$ and $\Phi_-(\omega)$ the equation
$\Phi_+(\omega) = V(\omega)\,\Phi_-(\omega) + W(\omega)$ where
$\Phi_+(\omega) \in A^+, \Phi_-(\omega) \in A^-$, and $W(\omega)$ are $O(\omega^{-1})$ as $\omega \to \infty$,
and $V(\omega) > 0$, $\omega \in \Re$, tends to 1 as $\omega \to \pm\infty$ through the reals.
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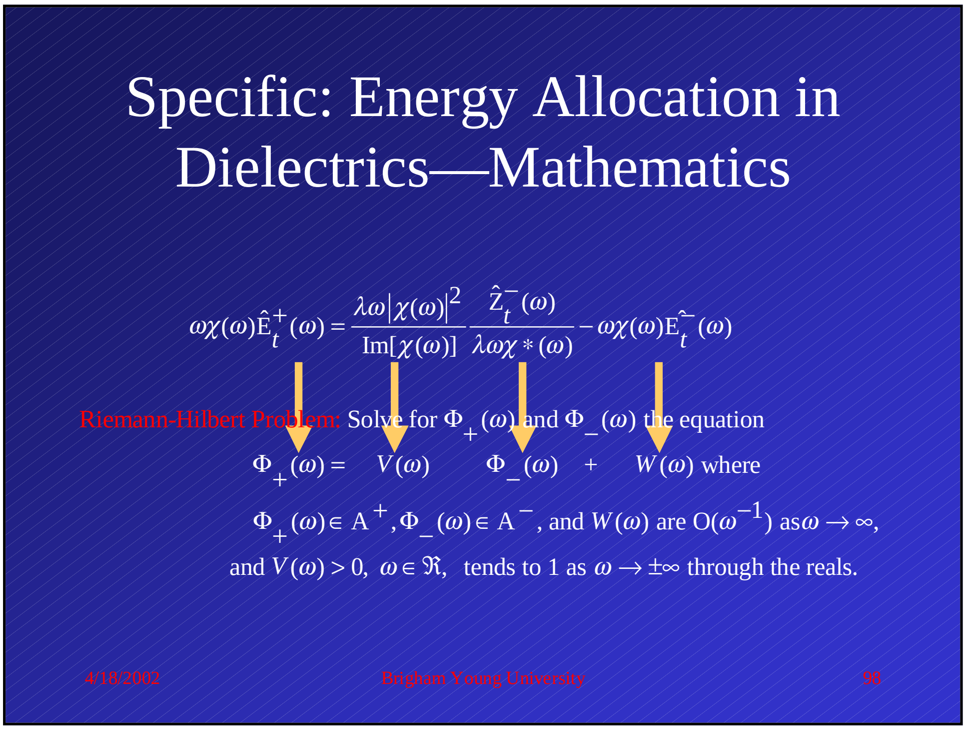
Specific: Energy Allocation in Dielectrics—Mathematics
$\omega\chi(\omega)\hat{E}_t^+(\omega) = \frac{\lambda\omega|\chi(\omega)|^2}{\mathrm{Im}[\chi(\omega)]}\frac{\hat{Z}_t^-(\omega)}{\lambda\omega\chi^*(\omega)} - \omega\chi(\omega)\hat{E}_t^-(\omega)$
Riemann-Hilbert Problem: Solve for $\Phi_+(\omega)$ and $\Phi_-(\omega)$ the equation
$\Phi_+(\omega) = V(\omega)\quad \Phi_-(\omega)\quad + \quad W(\omega)$ where
$\Phi_+(\omega) \in A^+, \Phi_-(\omega) \in A^-$, and $W(\omega)$ are $O(\omega^{-1})$ as $\omega \to \infty$,
and $V(\omega) > 0$, $\omega \in \Re$, tends to 1 as $\omega \to \pm\infty$ through the reals.
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Riemann-Hilbert Problem: $\Phi_+(\omega) = V(\omega)\,\Phi_-(\omega) + W(\omega)$

Solution: Define recursively $\phi(z \in \mathbb{C} \setminus \Re) := \exp\left(\frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{\log V(\omega)}{\omega\text{-}z} d\omega \right)$,

$\phi_+(\omega \in \Re) := \lim_{\varepsilon \searrow 0} \phi(\omega + i\varepsilon)$, $\Phi(z \in \mathbb{C} \setminus \Re) := \frac{\phi(z)}{2\pi i} \int_{-\infty}^{+\infty} \frac{W(\omega)}{\phi_+(\omega)(\omega\text{-}z)} d\omega.$

Then the solutions are $\Phi_\pm(\omega \in \Re) = \lim_{\varepsilon \searrow 0} \Phi(\omega \pm i\varepsilon)$.

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$$\left(u_{\text{int.}}(t)\right)_{irreversible} = \frac{\lambda}{2\pi} \int_{-\infty}^{t} \left| \int_{-\infty}^{+\infty} \frac{-i\omega\chi(\omega)}{\phi_+(\omega)} \hat{\mathrm{E}}_\tau(\omega) e^{-i\omega\tau}\, d\omega \right|^2 d\tau \geq 0$$

$$\left(u_{\text{int.}}(t)\right)_{reversible} = \frac{\lambda}{2\pi} \int_{t}^{+\infty} \left| \int_{-\infty}^{+\infty} \frac{-i\omega\chi(\omega)}{\phi_+(\omega)} \hat{\mathrm{E}}_t(\omega) e^{-i\omega\tau}\, d\omega \right|^2 d\tau \geq 0,$$

where $\lambda := \lim_{\omega \to \infty} \frac{\mathrm{Im}[\chi(\omega)]}{\omega|\chi(\omega)|^2} > 0$, $\phi_+(\omega) := \lim_{\varepsilon \searrow 0} \exp\left(\frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{\log \frac{\lambda\omega' |\chi(\omega')|^2}{\mathrm{Im}[\chi(\omega')]}}{\omega'\text{-}(\omega + i\varepsilon)} d\omega' \right)$,

$\hat{\mathrm{E}}_t(\omega) := \mathrm{F}\left[\mathrm{E}H_t^-\right](\omega) = e^{+i\omega t}\hat{\mathrm{E}}_t^-(\omega)$, $H_t^-(\tau) := H(t - \tau)$.

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Note the irreversibility:

$$\frac{d}{dt}\left(u_{\text{int.}}(t)\right)_{irreversible} = \frac{d}{dt}\frac{\lambda}{2\pi}\int_{-\infty}^{t}\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{E}_\tau(\omega)e^{-i\omega\tau}\,d\omega\right|^2 d\tau$$

$$=\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{E}_t(\omega)e^{-i\omega t}\,d\omega\right|^2 \geq 0,$$ as advertised.

Specific: Energy Allocation in Dielectrics—Mathematics

Note the loss of oportunity to do useful, macroscopic work after E-subsidy is over:

$$\left(u_{\text{int.}}(t)\right)_{reversible} = \frac{\lambda}{2\pi}\int_{t}^{+\infty}\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{E}_t(\omega)e^{-i\omega\tau}\,d\omega\right|^2 d\tau \geq 0,$$ so that

if $E \equiv 0$ for all $t > t_0$, then $\hat{E}_t(\omega) \equiv \hat{E}_{t_0}(\omega)$ for such t, and then

$$\frac{d}{dt}\left(u_{\text{int.}}(t)\right)_{reversible} = \frac{d}{dt}\frac{\lambda}{2\pi}\int_{t}^{+\infty}\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{E}_{t_0}(\omega)e^{-i\omega\tau}\,d\omega\right|^2 d\tau$$

$$= -\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{E}_{t_0}(\omega)e^{-i\omega t}\,d\omega\right| \leq 0.$$

Specific: Energy Allocation in Dielectrics—Mathematics

Note the reversibility:

$$\left(u_{\text{int.}}(-\infty)\right)_{reversible} = \frac{\lambda}{2\pi}\int_{-\infty}^{+\infty}\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{\mathrm{E}}_{-\infty}(\omega)e^{-i\omega\tau}\,d\omega\right|^2 d\tau = 0$$

since $\hat{\mathrm{E}}_{-\infty}(\omega) := \mathrm{F}\left[\mathrm{E}H_{-\infty}^{-}\right](\omega) = \mathrm{F}\left[\mathrm{E}\times 0\right](\omega) = \mathrm{F}\left[0\right](\omega) \equiv 0.$

Also

$$\left(u_{\text{int.}}(+\infty)\right)_{reversible} = \frac{\lambda}{2\pi}\int_{+\infty}^{+\infty}\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{\mathrm{E}}_{+\infty}(\omega)e^{-i\omega\tau}\,d\omega\right|^2 d\tau = 0.\ \text{Thus}$$

$$\left(u_{\text{int.}}(-\infty)\right)_{reversible} = \left(u_{\text{int.}}(+\infty)\right)_{reversible} = 0,\ \text{as expected.}$$

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Specific: Energy Allocation in Dielectrics—Mathematics

Note the reversibility:

0

$$\left(u_{\text{int.}}(-\infty)\right)_{reversible} = \frac{\lambda}{2\pi}\int_{-\infty}^{+\infty}\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{\mathrm{E}}_{-\infty}(\omega)e^{-i\omega\tau}\,d\omega\right|^2 d\tau = 0$$

since $\hat{\mathrm{E}}_{-\infty}(\omega) := \mathrm{F}\left[\mathrm{E}H_{-\infty}^{-}\right](\omega) = \mathrm{F}\left[\mathrm{E}\times 0\right](\omega) = \mathrm{F}\left[0\right](\omega) \equiv 0.$

Also

0

$$\left(u_{\text{int.}}(+\infty)\right)_{reversible} = \frac{\lambda}{2\pi}\int_{+\infty}^{+\infty}\left|\int_{-\infty}^{+\infty}\frac{-i\omega\chi(\omega)}{\phi_+(\omega)}\hat{\mathrm{E}}_{+\infty}(\omega)e^{-i\omega\tau}\,d\omega\right|^2 d\tau = 0.\ \text{Thus}$$

$$\left(u_{\text{int.}}(-\infty)\right)_{reversible} = \left(u_{\text{int.}}(+\infty)\right)_{reversible} = 0,\ \text{as expected.}$$

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General Principles

- Any dynamical notion of Free Energy and Heat depending solely on distinctions between dispersive and dissipative processes, either micro- or macroscopically defined, is intrinsically ambiguous/unobservable globally.
  - The notion that the dispersive elements of the system contain solely the Free Energy of the system and the dissipative elements contain solely the Heat/losses ignores the dynamic interplay between these elements.

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General Principles

- Any dynamical notion of Free Energy and Heat depending solely on distinctions between dispersive and dissipative processes, either micro- or macroscopically determined, is intrinsically ambiguous/unobservable globally.
  - The interplay between the elements does not in general allow these individual allocations to be measured "externally", i.e. to be measured without physical decomposition of the system into those elements—in case these elements are microscopic, this requires a "Maxwell Demon."

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General Principles

- Any dynamical notion of Free Energy and Heat depending solely on distinctions between dispersive and dissipative processes, either micro- or macroscopically determined, is intrinsically ambiguous/unobservable globally.
  - Consequently allocations based on individual dispersive and dissipative elements is artificial: these allocations are irrelevant to the energetics of the composite system, at least without the use of a "Maxwell Demon."

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General Principles

- Any dynamical notion of Free Energy and Heat depending solely on distinctions between dispersive and dissipative processes, either micro- or macroscopically determined, is intrinsically ambiguous/unobservable globally.
  - Two, "internally distinct" systems can have precisely the same external energetics. Because they are built with different dispersive/dissipative elements, the artificial, externally unmeasurable allocations mentioned may differ dynamically.

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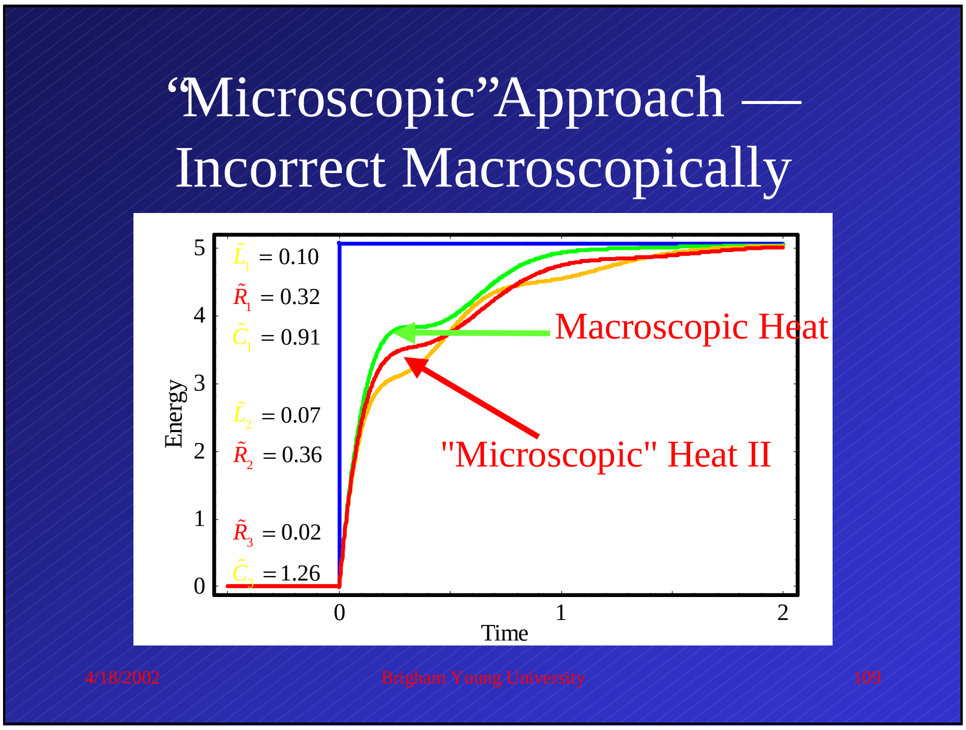
"Microscopic" Approach —
Incorrect Macroscopically
$\tilde{L}_1$ = 0.10
$\tilde{R}_1$ = 0.32
$\tilde{C}_1$ = 0.91
$\tilde{L}_2$ = 0.07
$\tilde{R}_2$ = 0.36
$\tilde{R}_3$ = 0.02
$\tilde{C}_3$ = 1.26
Macroscopic Heat
"Microscopic" Heat II
Energy
Time
0
1
2
3
4
5
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