QUANTUM CATASTROPHES

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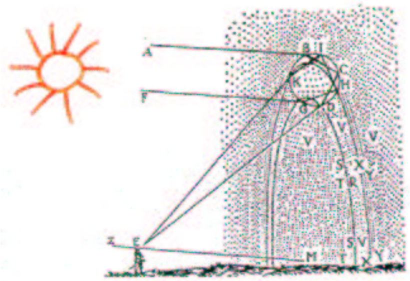
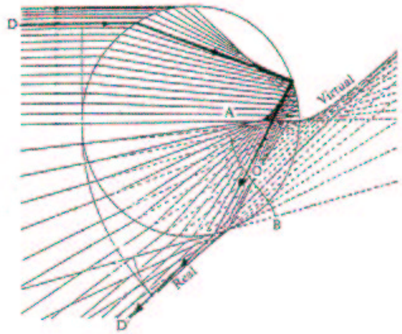
DIFFRACTION CATASTROPHES : RAINBOW





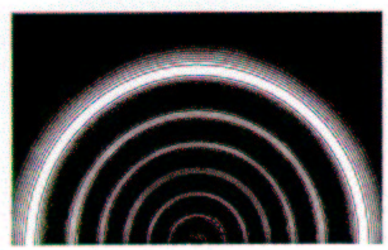
CROSS SECTION OF LIGHT RAYS BECOMES INFINITE

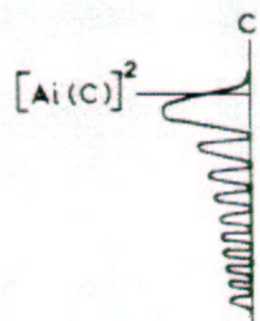
LIGHT-RAY CATASTROPHE

WAVE OPTICS RESOLVES THE CATASTROPHE

NEW PHENOMENON – INTERFERENCE

SUPERNUMERARY ARCS





PHASE SINGULARITIES OF WAVES

TIDES IN THE NORTH SEA

Dislocations in scalar wave fields

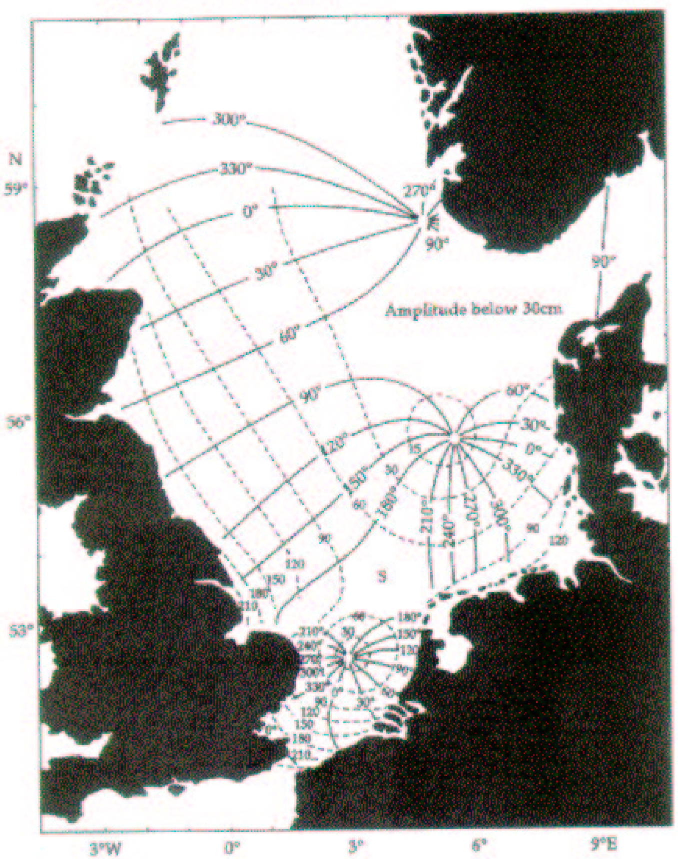
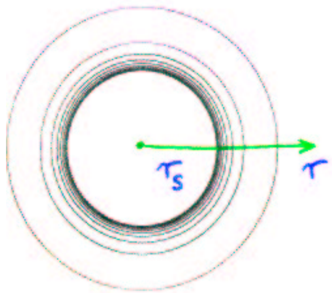


Figure 5.1. Co-tidal lines (in degrees) and equal amplitude lines (in centimetres) of the semi-diurnal (M_2) tide in the North Sea. S indicates a saddle point for the phase. (From Defant (1961), after Proudman & Doodson (1924).)

WAVE CATASTROPHES ? BLACK HOLES !



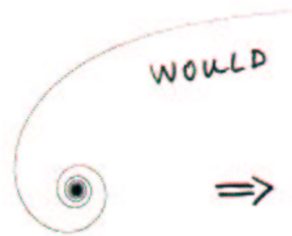
WAVES FREEZE AS

$$\varphi \sim \Theta(\zeta)\, \zeta^{i\mu}$$

$$\mu = 2 r_S \frac{\omega}{c}, \quad \zeta = r - r_S$$

$$\varphi = \Theta(\zeta)\, e^{iS}, \quad S = \mu \ln \zeta, \quad \lambda = 2\pi \left(\frac{\partial S}{\partial \zeta}\right)^{-1} = \frac{2\pi}{\mu}\zeta$$

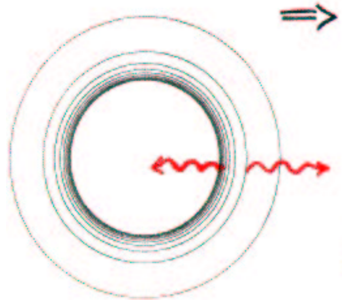
WAVE FUNCTION IS NON-ANALYTIC AT HORIZON



INWARD-FALLING OBSERVER WOULD NOT NOTICE ANYTHING UNUSUAL

$\Longrightarrow$ QUANTUM VACUUM ANALYTIC WAVE FUNCTION

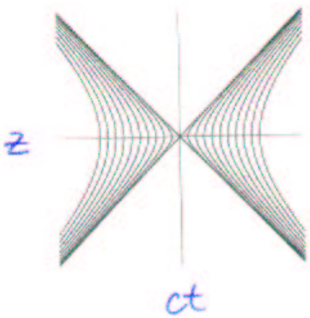
$\Longrightarrow$ $\Theta(\zeta)\,\zeta^{i\mu}$ NOT VACUUM



HAWKING RADIATION

$$\bar{n} = \frac{1}{e^{2\pi\mu} - 1}, \quad kT = \frac{\hbar c}{4\pi r_S}$$

ACCELERATED OBSERVER – UNRUH EFFECT

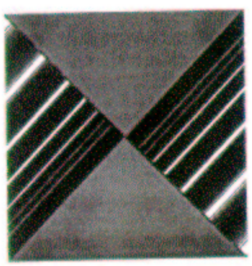


$$\frac{d}{dt}\frac{\dot z}{\sqrt{1-\dot z^2/c^2}} = a = \frac{c^2}{\zeta_0}$$

$$(ct, z) = (\zeta_0 \sinh\tau, \zeta_0 \cosh\tau)$$

$\frac{\zeta_0}{c}\tau$: PROPER TIME

- STATIONARY WAVES IN ACCELERATED FRAME



$$u_R = \Theta(\zeta)\,\zeta^{i\mu} e^{-i\mu\tau}, \quad u_L = u_R^*$$

$$\mu = \omega\frac{c}{a}$$

NON-ANALYTIC IN ζ

- MINKOWSKI-VACUUM WAVES ARE ANALYTIC

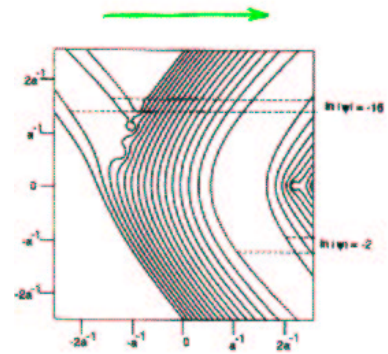
$$v_R = u_R \cosh\xi + u_L^* \sinh\xi$$
$$v_L = u_R^* \sinh\xi + u_L \cosh\xi, \quad \tanh\xi = e^{-\pi\mu}$$

- PAIR PRODUCTION – PLANCK SPECTRUM

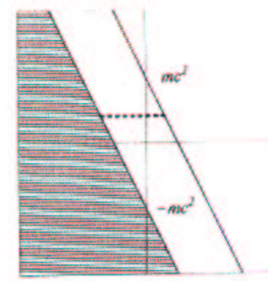
$$\bar n = \sinh^2\xi = \frac{1}{e^{2\pi\mu}-1}, \quad k_B T = \frac{\hbar a}{2\pi c}$$

ACCELERATED WAVES – SCHWINGER EFFECT

UNIFORM FORCE



POTENTIAL



$$\left((\hbar\omega + Fz)^2 + \hbar^2 c^2 \partial_z^2\right)\psi = m^2 c^4 \psi$$

$$q = \sqrt{\frac{F}{\hbar c}}\left(z + \frac{\hbar\omega}{F}\right), \quad p = -i\frac{\partial}{\partial q}$$

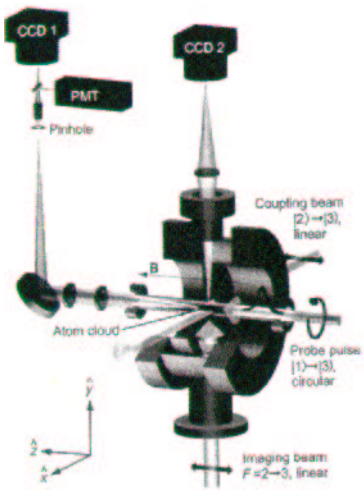
$$\hat\zeta_\pm = \frac{1}{\sqrt 2}(\hat p \pm \hat q) \Rightarrow [\hat\zeta_+, \hat\zeta_-] = i$$

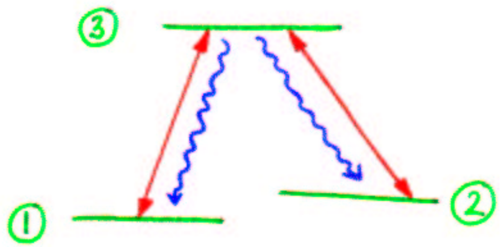
OBTAIN $\zeta_\pm \frac{\partial\psi}{\partial\zeta_\pm} = \left(\pm i\mu - \frac{1}{2}\right)\psi$

$$\psi = \zeta_\pm^{\pm i\mu - 1/2}, \quad \mu = \frac{m^2 c^3}{2\hbar F}$$

PAIR PRODUCTION $\bar n = e^{-2\pi\mu}$

SLOW LIGHT





PROBE $\quad \Omega_p = \frac{\varkappa_{13}}{\hbar} E_p \, e^{-i\omega t}$

CONTROL $\quad \Omega_c = \frac{\varkappa_{23}}{\hbar} E_c \, e^{-i\omega_c t}$

RELAXATION $\quad \gamma_{13} \, , \; \gamma_{23}$

- PREPARES ATOM IN DARK STATE

$$|a\rangle = N\left(|1\rangle - \frac{\Omega_p}{\Omega_c}|2\rangle + \frac{2(\omega-\omega_0)}{|\Omega_c|^2}\Omega_p |3\rangle\right)$$

- ADIABATIC PASSAGE $\Omega_c \to 0$ POSSIBLE

ENERGY OF INDUCED DIPOLES

$$\varepsilon_0 \chi \, |E|^2 = \frac{|\varkappa_{13}|^2}{2\hbar} \, n_A \, \frac{2(\omega-\omega_0)}{|\Omega_c|^2} \, |E|^2$$

SLOW-LIGHT POLARITONS

CLASSICAL FIELD THEORY

$$\mathcal{L} = \frac{\hbar}{2}\left((1+\alpha)(\partial_t\varphi)^2 - c^2(\partial_z\varphi)^2 - \alpha\,\omega_0^2\varphi^2\right)$$

$$\varphi = \left(\frac{\varepsilon_0}{\hbar\omega}\right)^{1/2} E_p$$

$$\alpha = \frac{|\varkappa_{13}|^2}{|\varkappa_{23}|^2}\,\frac{\hbar\omega_0 n_A}{\varepsilon_0 I_c} \; , \quad v_g = \frac{c}{1+\alpha}$$

$$\left(\partial_t (1+\alpha)\partial_t - c^2\partial_z^2 + \alpha\,\omega_0^2\right)\varphi = 0$$

QUANTUM FIELD THEORY

$$\hat{\varphi} = \int \left(\hat{a}_q u_q(t,z) + \hat{a}_q^\dagger u_q^*(t,z)\right) dq$$

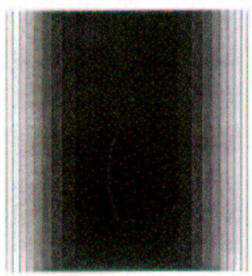
$$(u_q , u_{q'}) = \delta(q-q') \; , \quad (u_q , u_{q'}^*) = 0$$

$$(\varphi_1 , \varphi_2) \equiv i\int_{-\infty}^{+\infty}\left(\varphi_1^*\partial_t\varphi_2 - \varphi_2\partial_t\varphi_1^*\right)(1+\alpha)\,dz$$

$$[\hat{a}_q , \hat{a}_{q'}^\dagger] = \delta(q-q') \; , \quad [\hat{a}_q , \hat{a}_{q'}] = 0$$

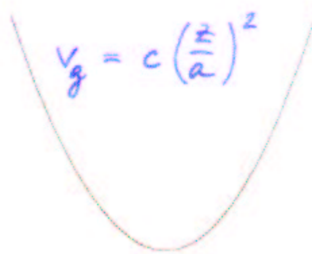
SLOW-LIGHT CATASTROPHE

I_c



$$\alpha = \frac{\varkappa}{I_c} = \left(\frac{a}{z}\right)^2$$

$$v_g \approx \frac{c}{\alpha}$$

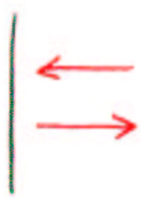


$$v_g = c\left(\frac{z}{a}\right)^2$$

$$\varphi = \sqrt{z}\, J_{\pm\nu}\!\left(\frac{\omega}{c} z\right) e^{-i\omega t}$$

$$\nu = \frac{1}{2}\sqrt{1-\frac{\delta}{\delta_0}}\,, \quad \delta = \frac{\omega-\omega_0}{\omega_0}\,, \quad \delta_0 = \frac{c^2}{8a^2\omega_0^2}$$

① $\delta < \delta_0$



TOTAL REFLECTION

② $\delta > \delta_0$

HORIZON , $\nu = i\mu$

$$\varphi \sim z^{i\mu + 1/2}$$

BLACK HOLE : $\varphi \sim z^{i\mu}$

ANALYTIC VACUUM MODES

ADIABATIC PROCESSES CONSERVE ANALYTICITY

YET DETECTOR MODES ARE NON-ANALYTIC

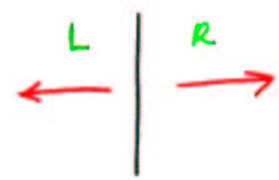
$$u_R^{\pm} = \frac{\Theta(z)}{\sqrt{2c}}\, e^{-\mu\pi/2} \sqrt{\frac{\omega_0}{c} z}\; J_{\pm i\mu}\!\left(\frac{\omega}{c}\right) e^{-i\omega t}$$

$$w_R = \frac{u_R^+ - e^{-\pi\mu} u_R^-}{\sqrt{1-e^{-2\pi\mu}}}\,, \qquad u_R = u_R^-$$

$$w_L(z) = w_R(-z)\,, \qquad u_L(z) = u_R(-z)$$

u :

w :



ANALYTIC VACUUM MODES

$$v_R = \frac{w_L}{e^{\pi\mu}+e^{-\pi\mu}} - i\, w_R - i\, e^{-2i\omega t_0} \frac{w_R^*}{e^{\pi\mu}+e^{-\pi\mu}}$$

$$v_R^+ = \frac{u_R + i\, e^{\pi\mu} u_L}{\sqrt{1+e^{2\pi\mu}}}$$

RADIATION

MODE OPERATORS $\hat{a}$: DETECTOR , $\hat{b}$: VACUUM

$$\hat{a}_R = \frac{\hat{b}_L}{e^{\pi\mu}+e^{-\pi\mu}} - i\,\hat{b}_R + i\,e^{2i\omega t_0}\frac{\hat{b}_R^+}{e^{\pi\mu}+e^{-\pi\mu}}$$

$$\hat{a}_{R\perp} = \frac{1}{\sqrt{1+e^{2\pi\mu}}}\left(\hat{b}_{R\perp} + i\,e^{\pi\mu}\,\hat{b}_{L\perp}\right)$$

$$\langle \hat{a}_R^+(q_1)\;\hat{a}_R(q_2)\rangle = \langle \hat{a}_L^+(q_1)\;\hat{a}_L(q_2)\rangle = \bar{n}\;\delta(q_1-q_2)$$

$$\bar{n} = \frac{1}{(e^{\pi\mu}+e^{-\pi\mu})^2}$$

MAXIMUM : $\bar{n} = \frac{1}{4}$, SUN LIGHT : $\bar{n} = 0.01$

PHOTON FLUX : $4\cdot 10^{14}\frac{A}{\lambda_0^2}\cdot\delta_0$, $\delta_0 = \frac{c^2}{8a^2\omega_0^2}$

PROBLEM : $\alpha = \frac{\varkappa}{I_c}$ VIOLATED AT SOME STAGE

TRANS - PLANCKIAN PROBLEM

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QUANTUM EFFECTS OF WAVE CATASTROPHES

- WAVE OPTICS RESOLVES RAY CATASTROPHES

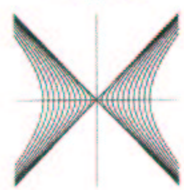
  INTERFERENCE PHENOMENA

- QUANTUM NATURE RESOLVES WAVE CATASTROPHES

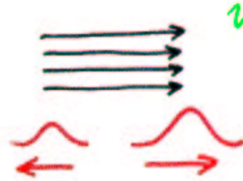
PAIR CREATION

BLACK HOLES , HAWKING 1974

$\zeta^{i\mu}$, $\bar{n} = \frac{1}{e^{2\pi\mu}-1}$

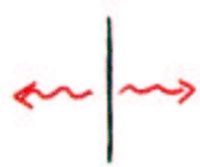


ACCELERATED FRAMES , UNRUH 1976



UNIFORM FORCE , SCHWINGER 1951

$\zeta^{i\mu-1/2}$, $\bar{n} = e^{-2\pi\mu}$



FROZEN LIGHT

$\zeta^{i\mu+1/2}$, $\bar{n} = \frac{1}{(e^{\pi\mu}+e^{-\pi\mu})^2}$