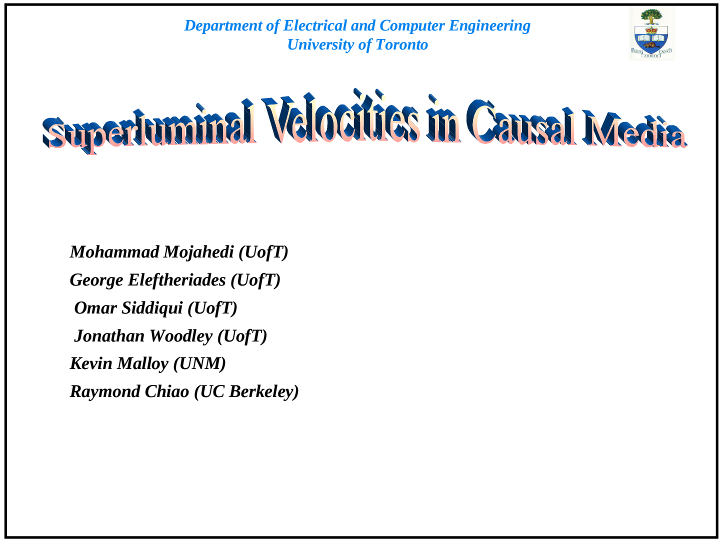
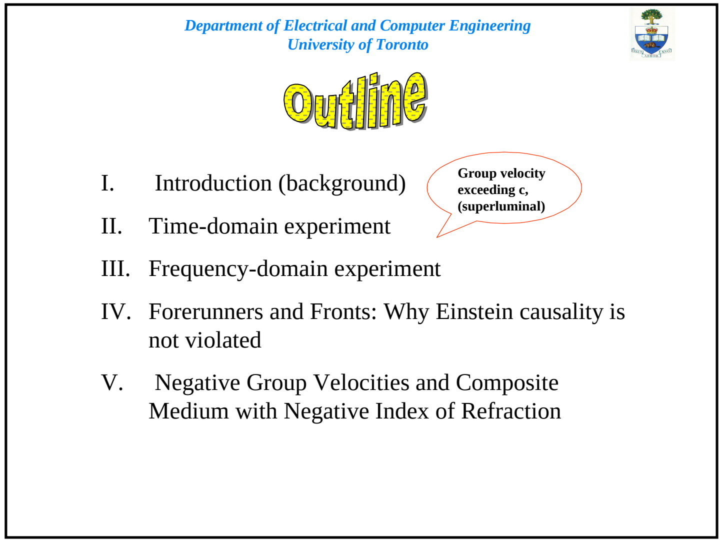


Department of Electrical and Computer Engineering
University of Toronto

Superluminal Velocities in Causal Media

Mohammad Mojahedi (UofT)
George Eleftheriades (UofT)
Omar Siddiqui (UofT)
Jonathan Woodley (UofT)
Kevin Malloy (UNM)
Raymond Chiao (UC Berkeley)



Department of Electrical and Computer Engineering
University of Toronto

Outline

I. Introduction (background)

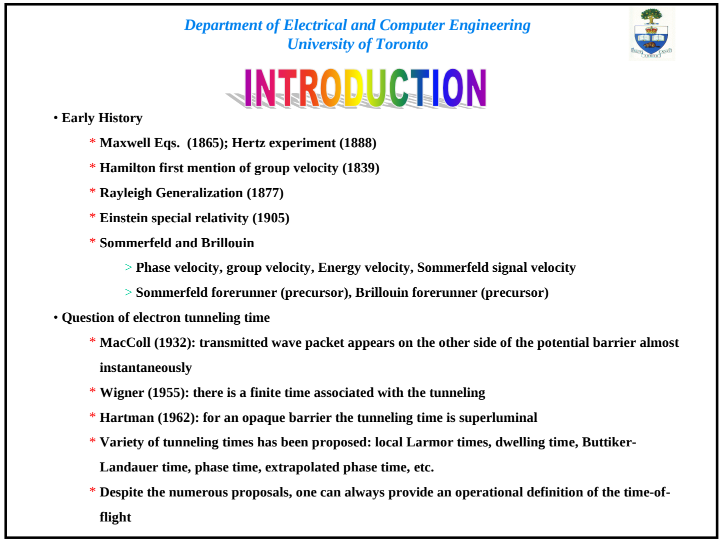
Group velocity exceeding c, (superluminal)

II. Time-domain experiment

III. Frequency-domain experiment

IV. Forerunners and Fronts: Why Einstein causality is not violated

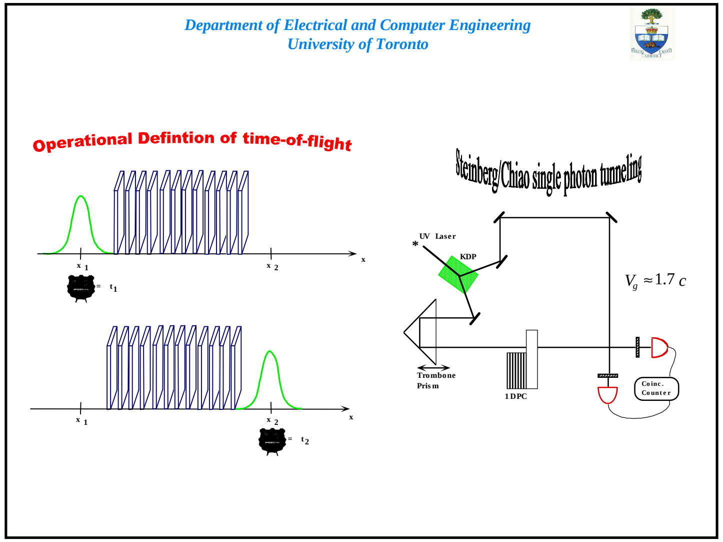
V. Negative Group Velocities and Composite Medium with Negative Index of Refraction



Department of Electrical and Computer Engineering
University of Toronto

INTRODUCTION

- Early History
  - * Maxwell Eqs. (1865); Hertz experiment (1888)
  - * Hamilton first mention of group velocity (1839)
  - * Rayleigh Generalization (1877)
  - * Einstein special relativity (1905)
  - * Sommerfeld and Brillouin
    - > Phase velocity, group velocity, Energy velocity, Sommerfeld signal velocity
    - > Sommerfeld forerunner (precursor), Brillouin forerunner (precursor)
- Question of electron tunneling time
  - * MacColl (1932): transmitted wave packet appears on the other side of the potential barrier almost instantaneously
  - * Wigner (1955): there is a finite time associated with the tunneling
  - * Hartman (1962): for an opaque barrier the tunneling time is superluminal
  - * Variety of tunneling times has been proposed: local Larmor times, dwelling time, Buttiker-Landauer time, phase time, extrapolated phase time, etc.
  - * Despite the numerous proposals, one can always provide an operational definition of the time-of-flight

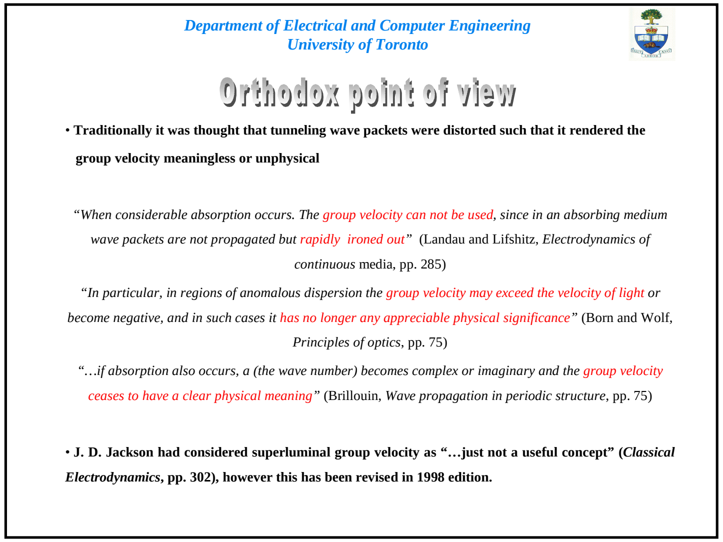


Department of Electrical and Computer Engineering
University of Toronto

Operational Defintion of time-of-flight

Steinberg/Chiao single photon tunneling

$V_g \approx 1.7\ c$



Department of Electrical and Computer Engineering
University of Toronto

Orthodox point of view

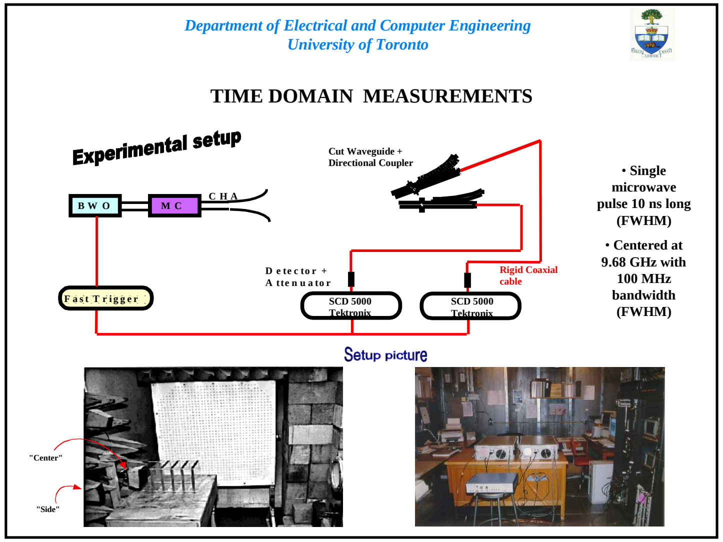
- **Traditionally it was thought that tunneling wave packets were distorted such that it rendered the group velocity meaningless or unphysical**

"*When considerable absorption occurs. The group velocity can not be used, since in an absorbing medium wave packets are not propagated but rapidly ironed out*" (Landau and Lifshitz, *Electrodynamics of continuous* media, pp. 285)

"*In particular, in regions of anomalous dispersion the group velocity may exceed the velocity of light or become negative, and in such cases it has no longer any appreciable physical significance*" (Born and Wolf, *Principles of optics*, pp. 75)

"*…if absorption also occurs, a (the wave number) becomes complex or imaginary and the group velocity ceases to have a clear physical meaning*" (Brillouin, *Wave propagation in periodic structure*, pp. 75)

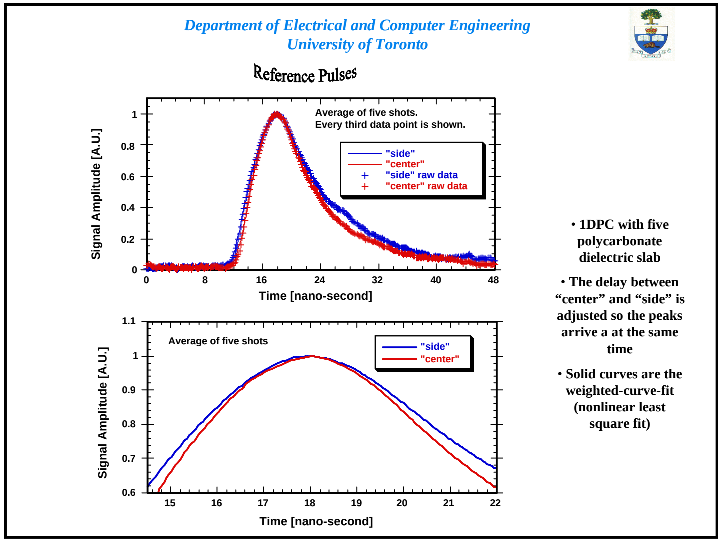
- **J. D. Jackson had considered superluminal group velocity as "…just not a useful concept" (*Classical Electrodynamics*, pp. 302), however this has been revised in 1998 edition.**



Department of Electrical and Computer Engineering
University of Toronto

TIME DOMAIN MEASUREMENTS

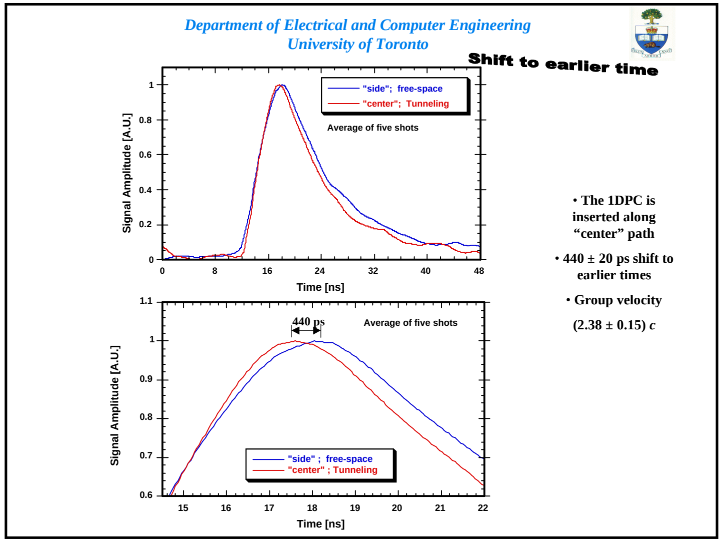
- **Single microwave pulse 10 ns long (FWHM)**
- **Centered at 9.68 GHz with 100 MHz bandwidth (FWHM)**



Department of Electrical and Computer Engineering
University of Toronto

Reference Pulses

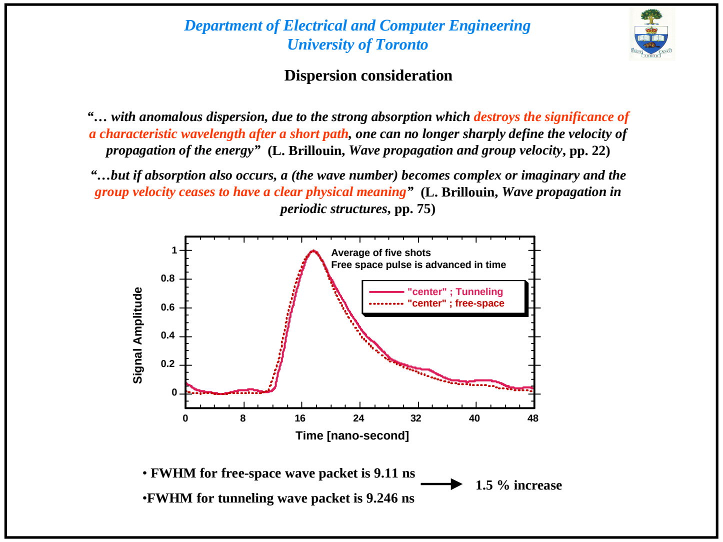
- **1DPC with five polycarbonate dielectric slab**
- **The delay between "center" and "side" is adjusted so the peaks arrive a at the same time**
- **Solid curves are the weighted-curve-fit (nonlinear least square fit)**



Department of Electrical and Computer Engineering
University of Toronto

Shift to earlier time

- **The 1DPC is inserted along "center" path**
- **440 ± 20 ps shift to earlier times**
- **Group velocity (2.38 ± 0.15) *c***



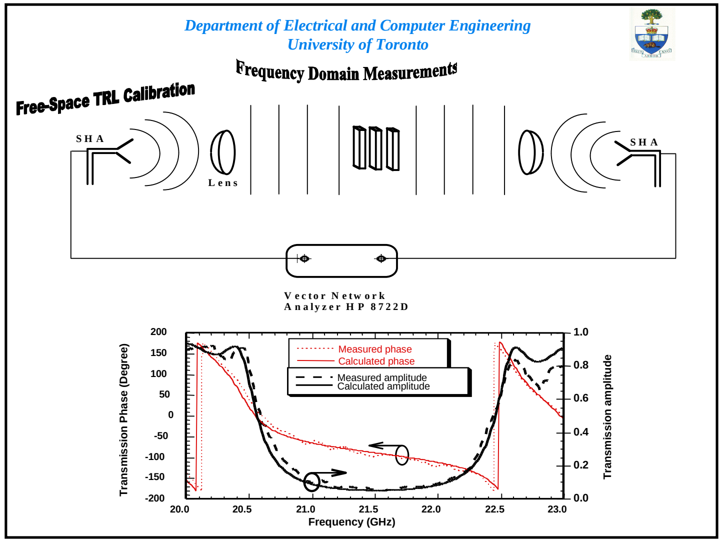
Department of Electrical and Computer Engineering
University of Toronto

Dispersion consideration

"… with anomalous dispersion, due to the strong absorption which destroys the significance of a characteristic wavelength after a short path, one can no longer sharply define the velocity of propagation of the energy" **(L. Brillouin, *Wave propagation and group velocity*, pp. 22)**

"…but if absorption also occurs, a (the wave number) becomes complex or imaginary and the group velocity ceases to have a clear physical meaning" **(L. Brillouin, *Wave propagation in periodic structures*, pp. 75)**

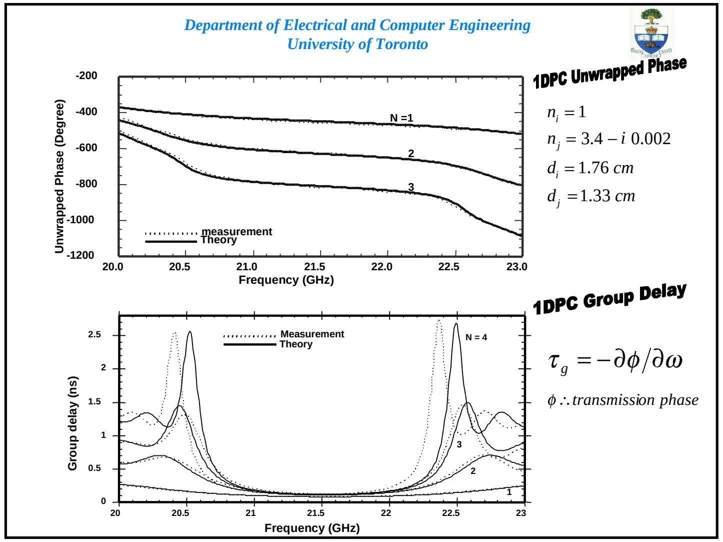
- **FWHM for free-space wave packet is 9.11 ns** → **1.5 % increase**
- **FWHM for tunneling wave packet is 9.246 ns**



Department of Electrical and Computer Engineering
University of Toronto

Frequency Domain Measurements

Free-Space TRL Calibration



Department of Electrical and Computer Engineering
University of Toronto

1DPC Unwrapped Phase

$n_i = 1$

$n_j = 3.4 - i\ 0.002$

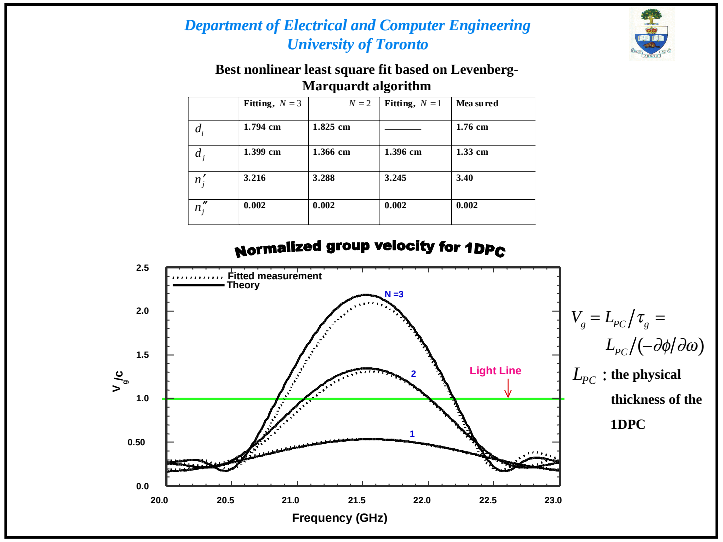
$d_i = 1.76\ cm$

$d_j = 1.33\ cm$

1DPC Group Delay

$$\tau_g = -\partial\phi/\partial\omega$$

ϕ ∴ *transmission phase*



Department of Electrical and Computer Engineering
University of Toronto

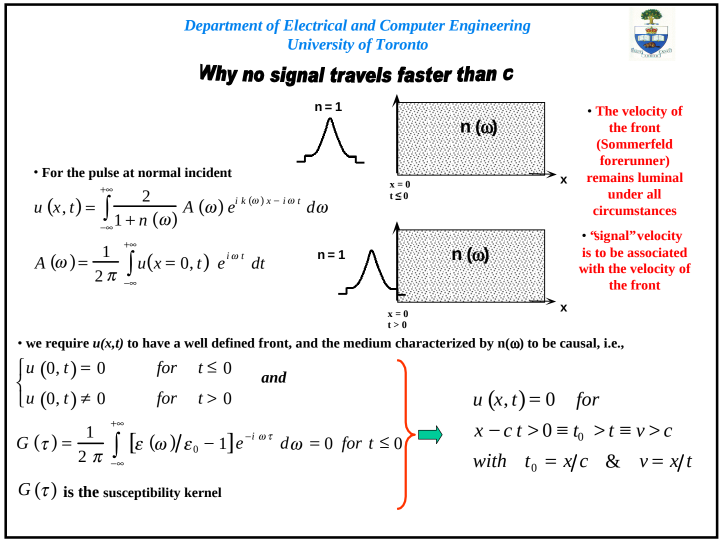
Best nonlinear least square fit based on Levenberg-Marquardt algorithm

	Fitting, $N = 3$	$N = 2$	Fitting, $N = 1$	Measured
d_i	1.794 cm	1.825 cm	———	1.76 cm
d_j	1.399 cm	1.366 cm	1.396 cm	1.33 cm
n_j'	3.216	3.288	3.245	3.40
n_j''	0.002	0.002	0.002	0.002

Normalized group velocity for 1DPC

$V_g = L_{PC}/\tau_g = L_{PC}/(-\partial\phi/\partial\omega)$

L_{PC} : **the physical thickness of the 1DPC**



Department of Electrical and Computer Engineering
University of Toronto

Why no signal travels faster than c

- For the pulse at normal incident

$$u(x,t)=\int_{-\infty}^{+\infty}\frac{2}{1+n(\omega)}A(\omega)\,e^{i\,k(\omega)\,x-i\,\omega\,t}\,d\omega$$

$$A(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}u(x=0,t)\;e^{i\,\omega\,t}\,dt$$

- **The velocity of the front (Sommerfeld forerunner) remains luminal under all circumstances**
- **"signal" velocity is to be associated with the velocity of the front**

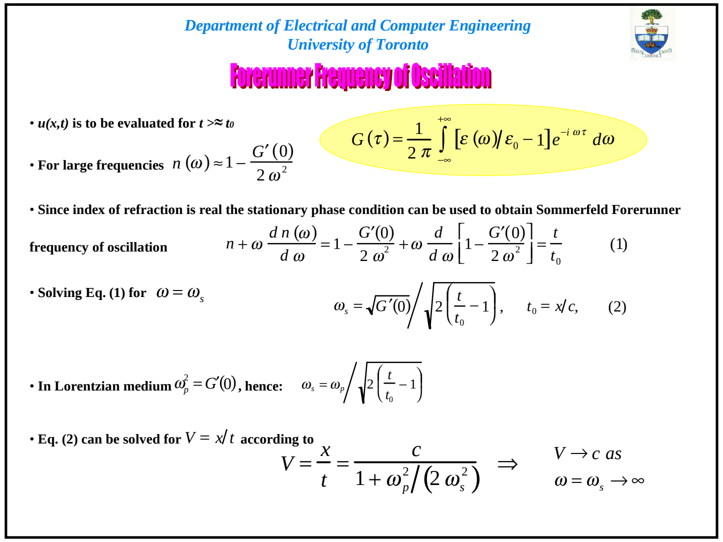
- we require *u(x,t)* to have a well defined front, and the medium characterized by n(ω) to be causal, i.e.,

$$\begin{cases}u(0,t)=0 & for\quad t\le 0\\ u(0,t)\neq 0 & for\quad t>0\end{cases}\qquad \textit{and}$$

$$G(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[\varepsilon(\omega)/\varepsilon_0-1\right]e^{-i\,\omega\,\tau}\,d\omega=0\;\; for\;\; t\le 0$$

$G(\tau)$ is the susceptibility kernel

$$\Rightarrow\quad u(x,t)=0\quad for$$
$$x-c\,t>0\equiv t_0>t\equiv v>c$$
$$with\quad t_0=x/c\quad \&\quad v=x/t$$



Department of Electrical and Computer Engineering
University of Toronto

Forerunner Frequency of Oscillation

- *u(x,t)* is to be evaluated for $t \gtrapprox t_0$
- For large frequencies $n(\omega)\approx 1-\dfrac{G'(0)}{2\,\omega^2}$

$$G(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[\varepsilon(\omega)/\varepsilon_0-1\right]e^{-i\,\omega\,\tau}\,d\omega$$

- Since index of refraction is real the stationary phase condition can be used to obtain Sommerfeld Forerunner frequency of oscillation

$$n+\omega\frac{d\,n(\omega)}{d\,\omega}=1-\frac{G'(0)}{2\,\omega^2}+\omega\frac{d}{d\,\omega}\left[1-\frac{G'(0)}{2\,\omega^2}\right]=\frac{t}{t_0}\qquad(1)$$

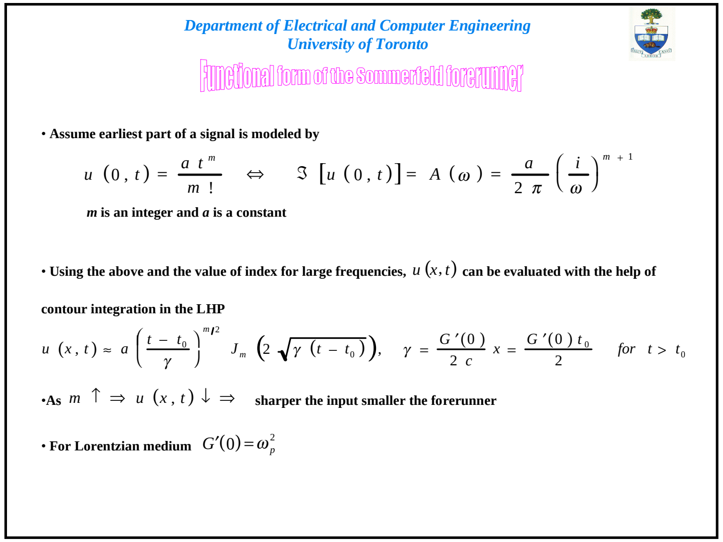
- Solving Eq. (1) for $\omega=\omega_s$

$$\omega_s=\sqrt{G'(0)}\Big/\sqrt{2\left(\frac{t}{t_0}-1\right)},\qquad t_0=x/c,\qquad(2)$$

- In Lorentzian medium $\omega_p^2=G'(0)$, hence: $\omega_s=\omega_p\Big/\sqrt{2\left(\dfrac{t}{t_0}-1\right)}$

- Eq. (2) can be solved for $V=x/t$ according to

$$V=\frac{x}{t}=\frac{c}{1+\omega_p^2/\left(2\,\omega_s^2\right)}\;\Rightarrow\;\begin{matrix}V\to c\;\;as\\ \omega=\omega_s\to\infty\end{matrix}$$



Department of Electrical and Computer Engineering
University of Toronto

Functional form of the Sommerfeld forerunner

- Assume earliest part of a signal is modeled by

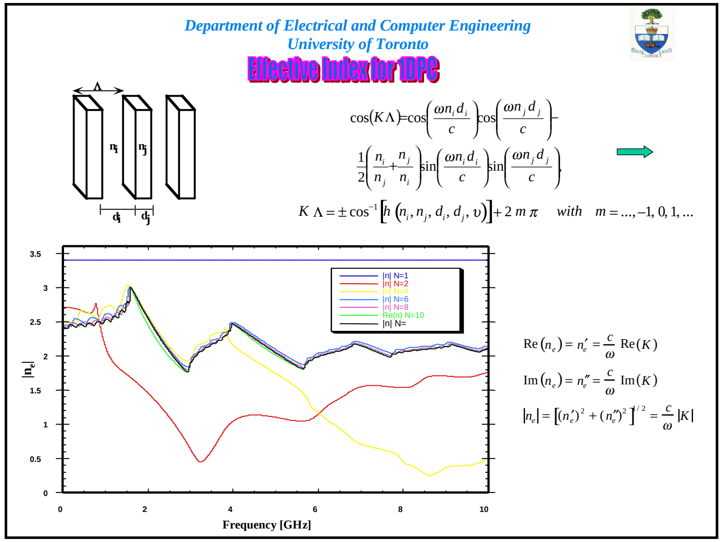
$$u(0,t)=\frac{a\,t^m}{m\,!}\quad\Leftrightarrow\quad\Im\left[u(0,t)\right]=A(\omega)=\frac{a}{2\pi}\left(\frac{i}{\omega}\right)^{m+1}$$

m is an integer and *a* is a constant

- Using the above and the value of index for large frequencies, $u(x,t)$ can be evaluated with the help of contour integration in the LHP

$$u(x,t)\approx a\left(\frac{t-t_0}{\gamma}\right)^{m/2}J_m\left(2\sqrt{\gamma\,(t-t_0)}\right),\quad \gamma=\frac{G'(0)}{2\,c}\,x=\frac{G'(0)\,t_0}{2}\qquad for\quad t>t_0$$

- As $m\uparrow\;\Rightarrow\;u(x,t)\downarrow\;\Rightarrow$ sharper the input smaller the forerunner
- For Lorentzian medium $G'(0)=\omega_p^2$



Department of Electrical and Computer Engineering
University of Toronto

Effective Index for 1DPC

$$\cos(K\Lambda)=\cos\left(\frac{\omega n_i d_i}{c}\right)\cos\left(\frac{\omega n_j d_j}{c}\right)-\frac{1}{2}\left(\frac{n_i}{n_j}+\frac{n_j}{n_i}\right)\sin\left(\frac{\omega n_i d_i}{c}\right)\sin\left(\frac{\omega n_j d_j}{c}\right),$$

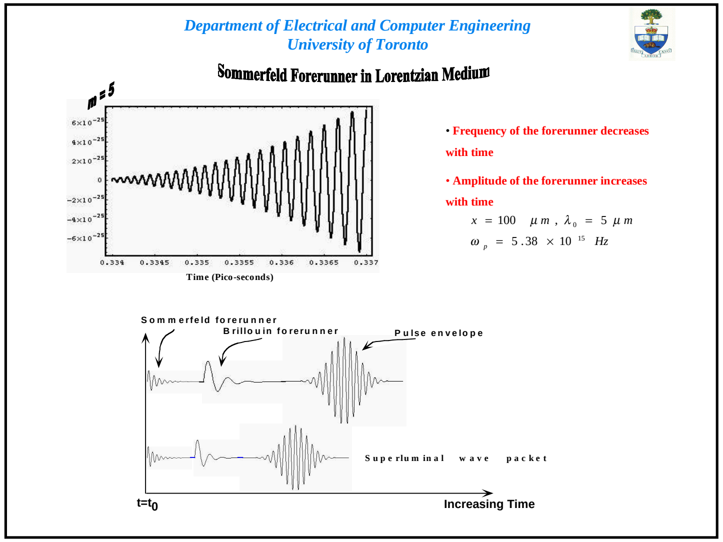
$$\Rightarrow$$

$$K\,\Lambda=\pm\cos^{-1}\left[h\left(n_i,n_j,d_i,d_j,\upsilon\right)\right]+2\,m\,\pi\qquad with\quad m=\ldots,-1,0,1,\ldots$$

$$\mathrm{Re}(n_e)=n_e'=\frac{c}{\omega}\,\mathrm{Re}(K)$$

$$\mathrm{Im}(n_e)=n_e''=\frac{c}{\omega}\,\mathrm{Im}(K)$$

$$|n_e|=\left[(n_e')^2+(n_e'')^2\right]^{1/2}=\frac{c}{\omega}|K|$$



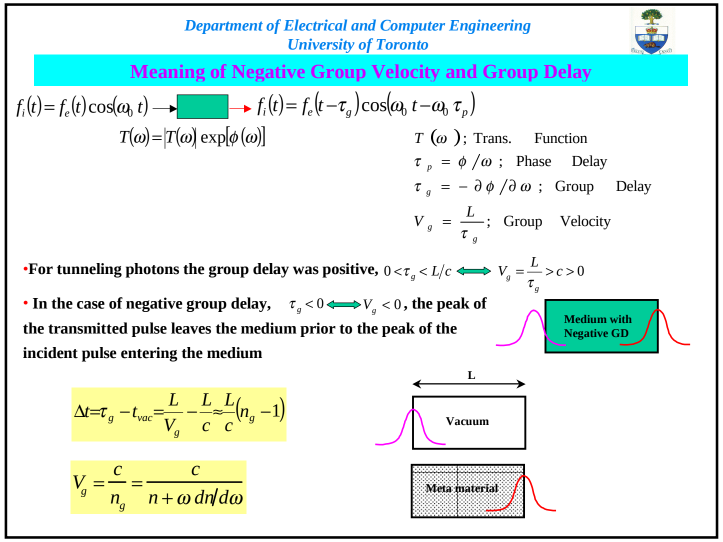
Department of Electrical and Computer Engineering
University of Toronto

Sommerfeld Forerunner in Lorentzian Medium

- **Frequency of the forerunner decreases with time**
- **Amplitude of the forerunner increases with time**

$x = 100\ \mu m,\ \lambda_0 = 5\ \mu m$

$\omega_p = 5.38 \times 10^{15}\ Hz$



Department of Electrical and Computer Engineering
University of Toronto

Meaning of Negative Group Velocity and Group Delay

$$f_i(t) = f_e(t)\cos(\omega_0 t) \longrightarrow \boxed{} \longrightarrow f_i(t) = f_e(t-\tau_g)\cos(\omega_0 t - \omega_0 \tau_p)$$

$$T(\omega) = |T(\omega)| \exp[\phi(\omega)]$$

$T(\omega)$; Trans. Function

$\tau_p = \phi/\omega$; Phase Delay

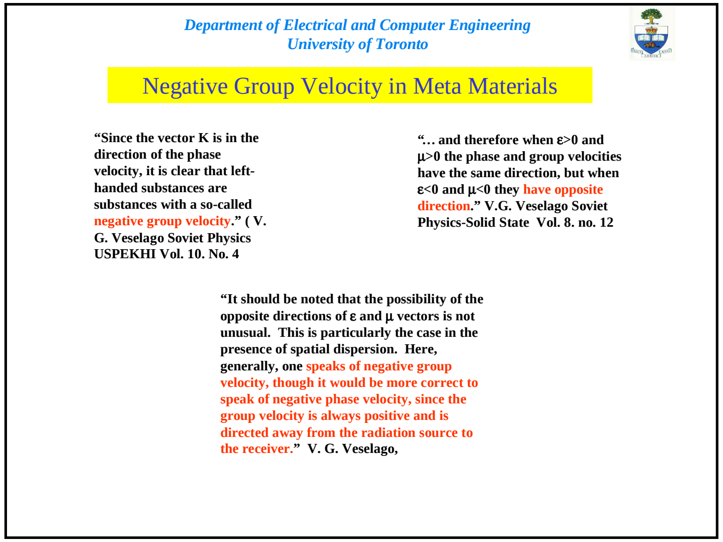
$\tau_g = -\partial\phi/\partial\omega$; Group Delay

$V_g = \frac{L}{\tau_g}$; Group Velocity

- **For tunneling photons the group delay was positive,** $0 < \tau_g < L/c \Longleftrightarrow V_g = \frac{L}{\tau_g} > c > 0$
- **In the case of negative group delay,** $\tau_g < 0 \Longleftrightarrow V_g < 0$**, the peak of the transmitted pulse leaves the medium prior to the peak of the incident pulse entering the medium**

$$\Delta t = \tau_g - t_{vac} = \frac{L}{V_g} - \frac{L}{c} \approx \frac{L}{c}(n_g - 1)$$

$$V_g = \frac{c}{n_g} = \frac{c}{n + \omega\, dn/d\omega}$$



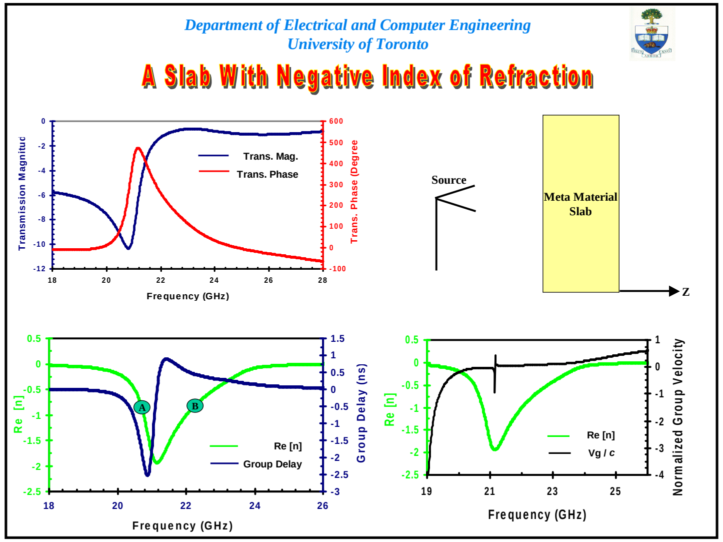
Department of Electrical and Computer Engineering
University of Toronto

Negative Group Velocity in Meta Materials

"Since the vector K is in the direction of the phase velocity, it is clear that left-handed substances are substances with a so-called negative group velocity." (V. G. Veselago Soviet Physics USPEKHI Vol. 10. No. 4

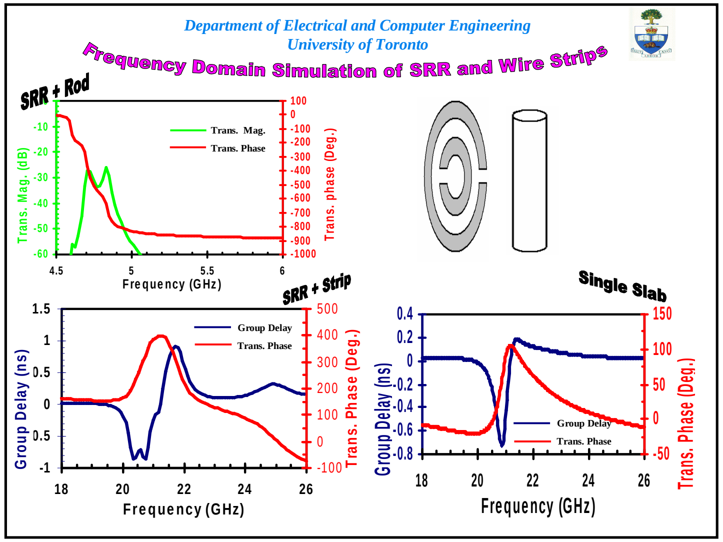
"… and therefore when ε>0 and μ>0 the phase and group velocities have the same direction, but when ε<0 and μ<0 they have opposite direction." V.G. Veselago Soviet Physics-Solid State Vol. 8. no. 12

"It should be noted that the possibility of the opposite directions of ε and μ vectors is not unusual. This is particularly the case in the presence of spatial dispersion. Here, generally, one speaks of negative group velocity, though it would be more correct to speak of negative phase velocity, since the group velocity is always positive and is directed away from the radiation source to the receiver." V. G. Veselago,



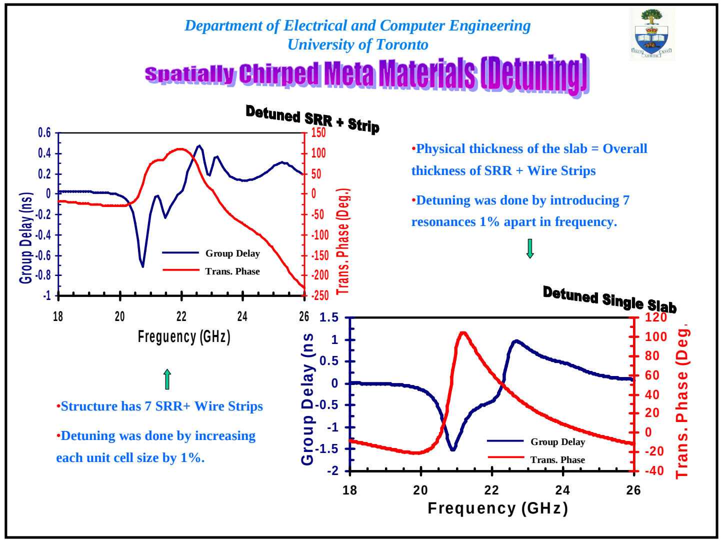
Department of Electrical and Computer Engineering
University of Toronto

A Slab With Negative Index of Refraction



Department of Electrical and Computer Engineering
University of Toronto

Frequency Domain Simulation of SRR and Wire Strips

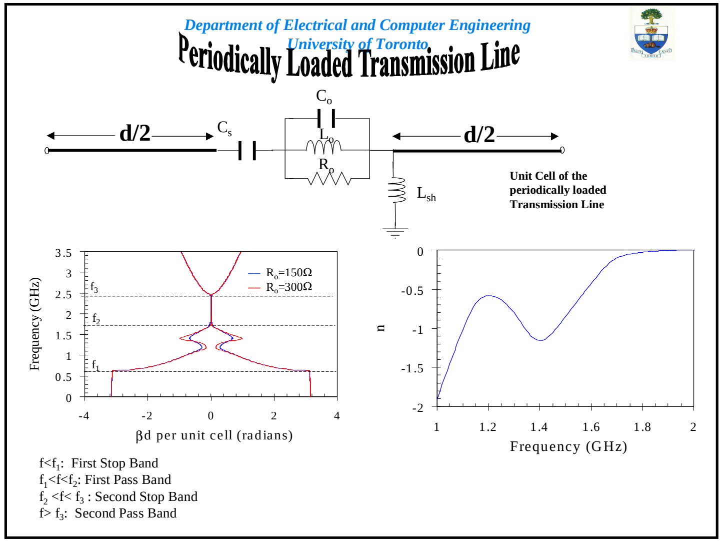


Department of Electrical and Computer Engineering
University of Toronto

Spatially Chirped Meta Materials (Detuning)

- Physical thickness of the slab = Overall thickness of SRR + Wire Strips
- Detuning was done by introducing 7 resonances 1% apart in frequency.

- Structure has 7 SRR+ Wire Strips
- Detuning was done by increasing each unit cell size by 1%.

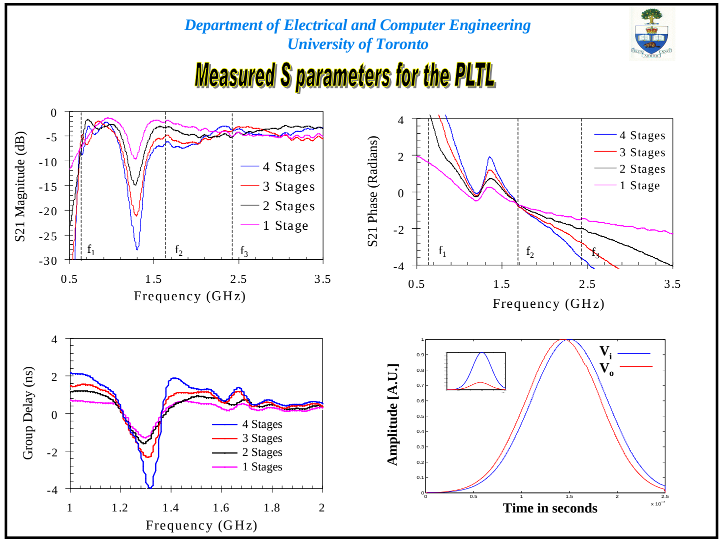


Department of Electrical and Computer Engineering
University of Toronto

Periodically Loaded Transmission Line

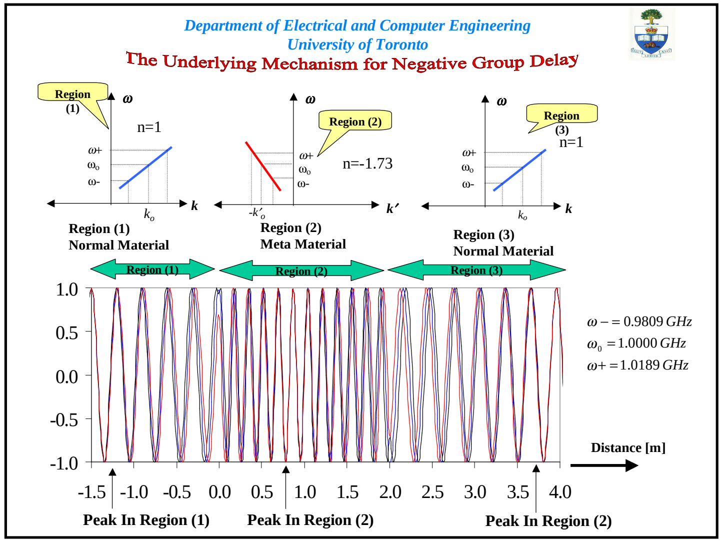
Unit Cell of the periodically loaded Transmission Line

$f<f_1$: First Stop Band
$f_1<f<f_2$: First Pass Band
$f_2 <f< f_3$: Second Stop Band
$f> f_3$: Second Pass Band



Department of Electrical and Computer Engineering
University of Toronto

Measured S parameters for the PLTL



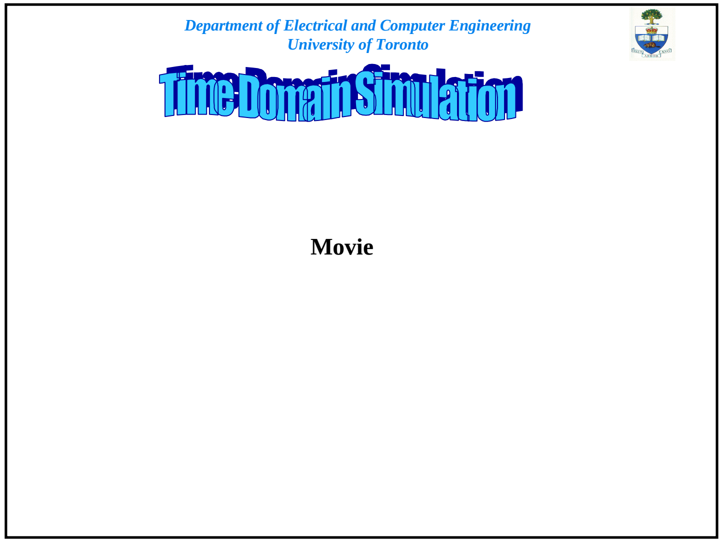
Department of Electrical and Computer Engineering
University of Toronto

The Underlying Mechanism for Negative Group Delay

$\omega - = 0.9809\,GHz$

$\omega_0 = 1.0000\,GHz$

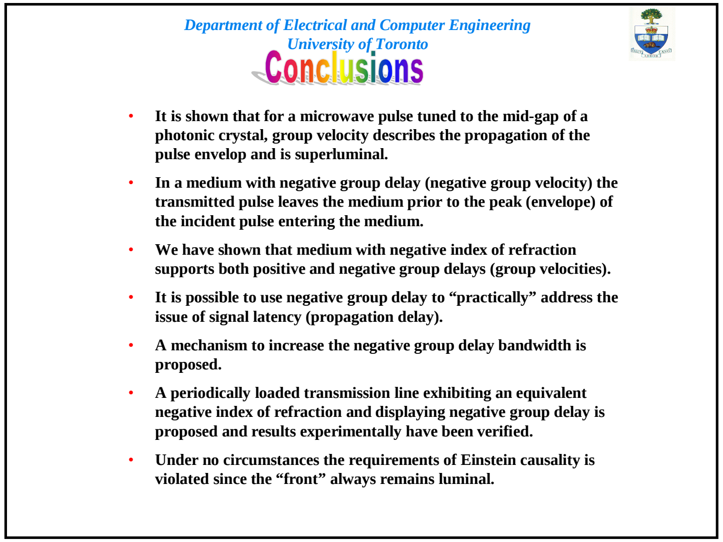
$\omega + = 1.0189\,GHz$



Department of Electrical and Computer Engineering
University of Toronto

Time Domain Simulation

Movie



Department of Electrical and Computer Engineering
University of Toronto

Conclusions

- **It is shown that for a microwave pulse tuned to the mid-gap of a photonic crystal, group velocity describes the propagation of the pulse envelop and is superluminal.**
- **In a medium with negative group delay (negative group velocity) the transmitted pulse leaves the medium prior to the peak (envelope) of the incident pulse entering the medium.**
- **We have shown that medium with negative index of refraction supports both positive and negative group delays (group velocities).**
- **It is possible to use negative group delay to “practically” address the issue of signal latency (propagation delay).**
- **A mechanism to increase the negative group delay bandwidth is proposed.**
- **A periodically loaded transmission line exhibiting an equivalent negative index of refraction and displaying negative group delay is proposed and results experimentally have been verified.**
- **Under no circumstances the requirements of Einstein causality is violated since the “front” always remains luminal.**