

AMPLIFICATION of SUPERLUMINALITY (?)

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THANKS → ORGANIZERS

KITP

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JEFF LINDSEEN

et al.

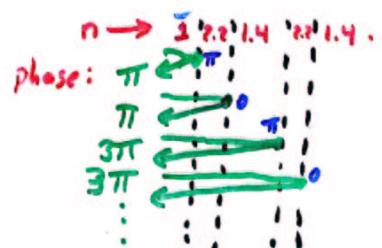
OUTLINE

- A few things I decided not to talk about (but will)
 - old MLD tunneling results
a reshaping picture to bear in mind
 - our recent 2-photon exp'ts
(nonlinear phase shifts...
slow/fast photons?
self-focussing, etc?
quantum computing? ...but phase)
 - Parallel with EIT, slow light,...
 - The Tournois effect
- Weak measurements: a way to look at tunneling
 - Can a particle be 2 places at 1 time?
- Amplification of superluminality
 - MLDs with gain

Multilayer Dielectric Mirror is precisely such a structure.

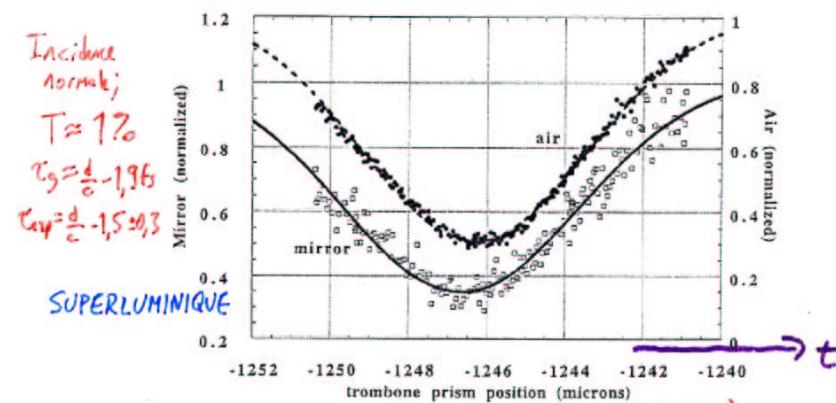
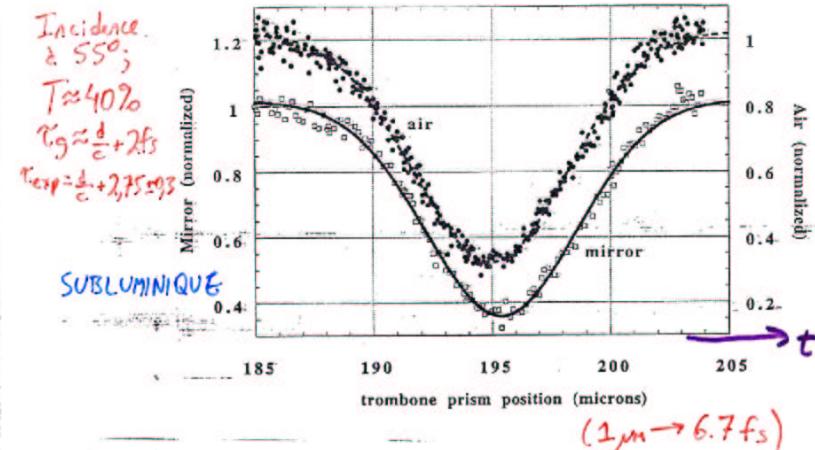


$$n_1 L_1 = n_2 L_2 = \frac{\lambda}{4}.$$



Successive reflections interfere constructively. By adding layers, even though at each interface $R \approx 5\%$, total reflectivity can be arbitrarily close to 1.

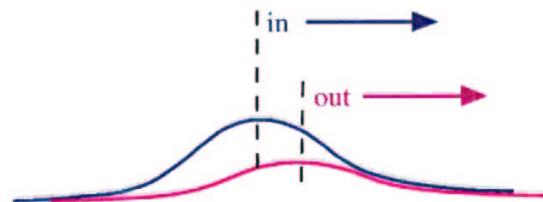
E.g., for 11 layers, $T=1.2\%$, though
 $(95\%)^{11} = 57\%.$
 (Tunnelling probability drops exponentially with d.)



[Steinberg, Kwiat, + Chiao PRL 71, 708 (1993);
 Steinberg + Chiao PRA 51, 3525 (1995).]

FIGURE 2B

FIGURE 2A

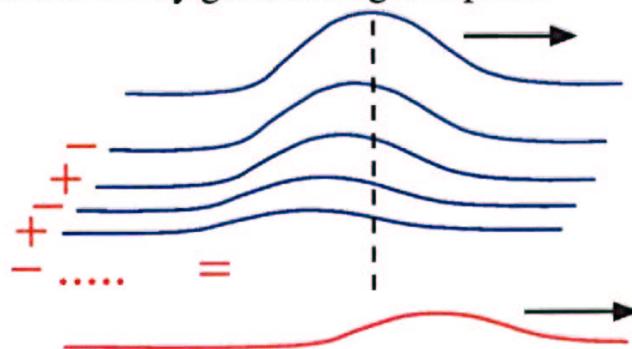


No energy need travel faster than c .

What about **information**, however?

The transmitted pulse is constructed causally out of the initial pulse. Although it *looks* as though it travelled $>c$, this is merely a result of a Taylor expansion.

A barrier is a very good analog computer.



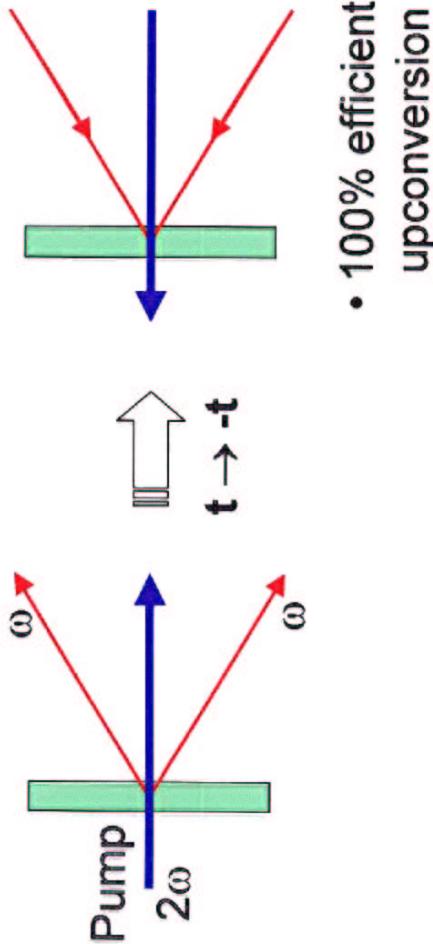
BUT: no new information propagates faster than c .



Consider...

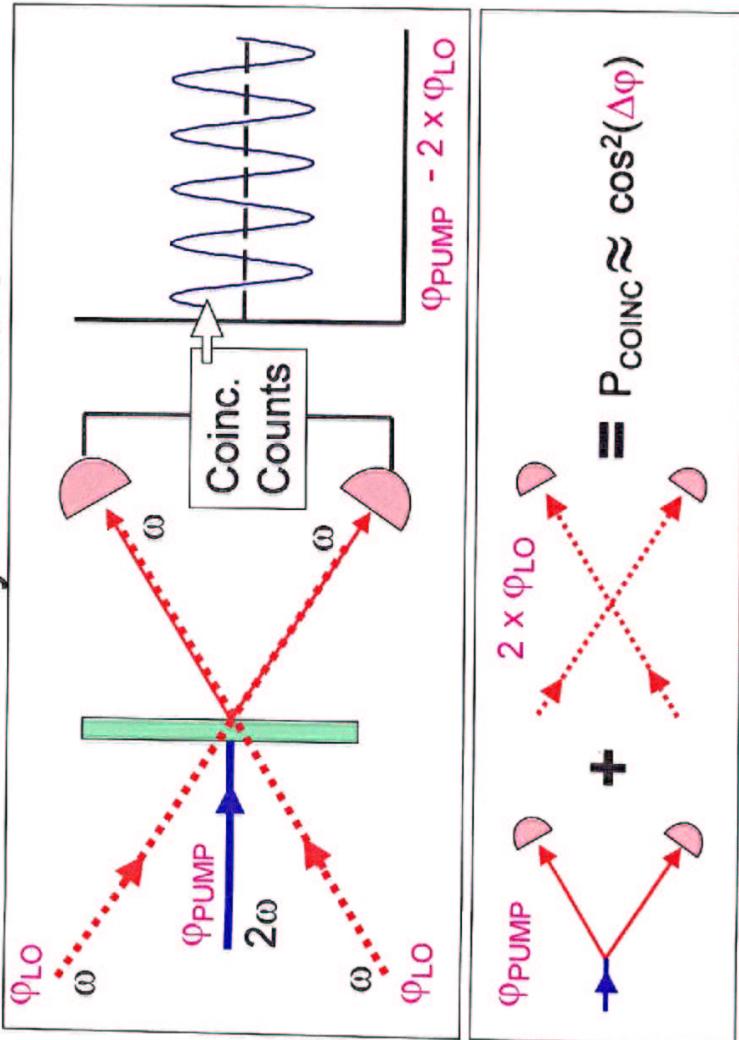
Spontaneous
Parametric Down-
conversion

Time-Reversed
Spontaneous
Parametric Down-
conversion



A "TWO-PHOTON SWITCH"?

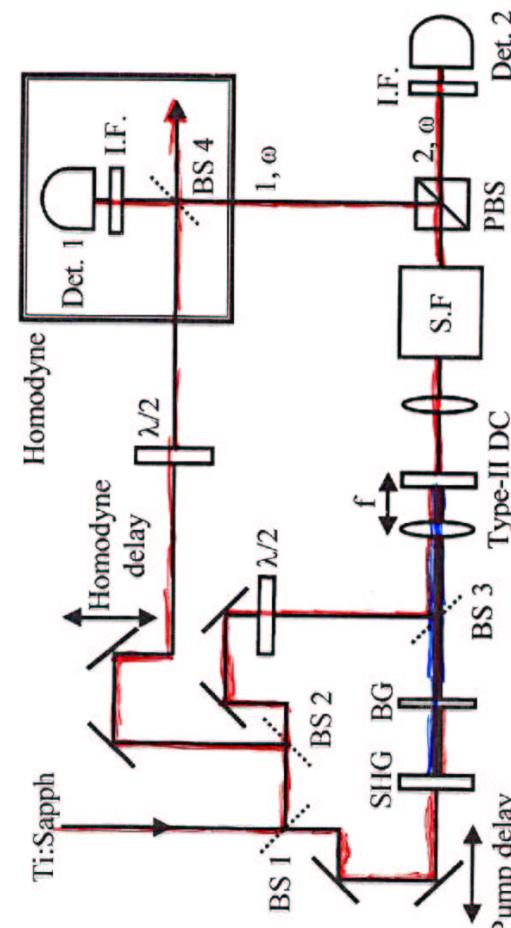
The Feynman Paths



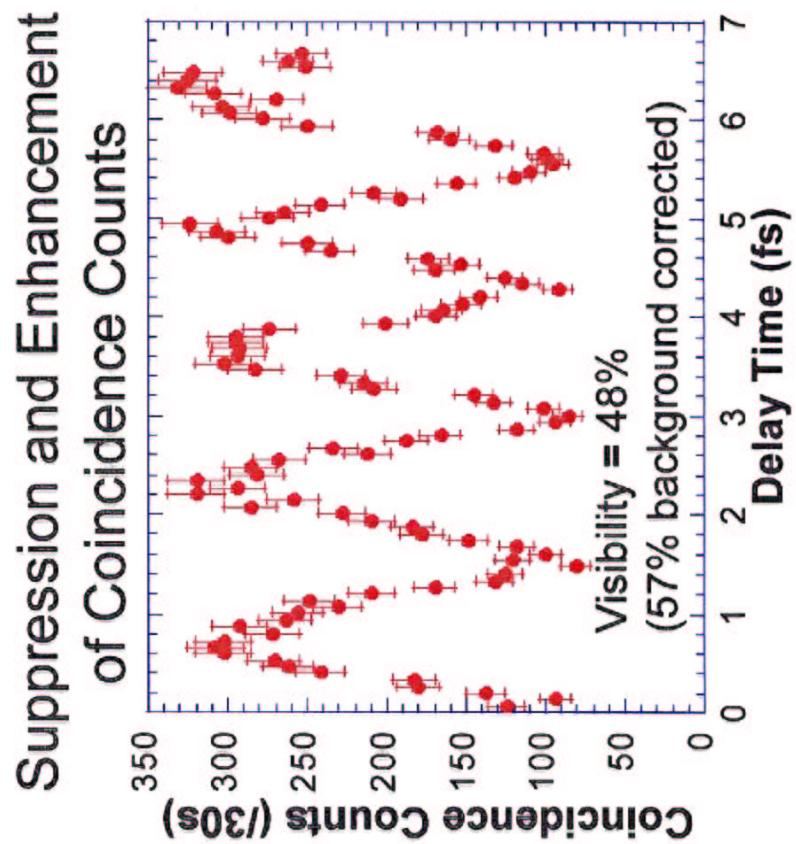
$$\phi_{\text{PUMP}} = 2 \times \phi_{\text{LO}} + \phi_{\text{LO}} = P_{\text{COINC}} \approx \cos^2(\Delta\phi)$$



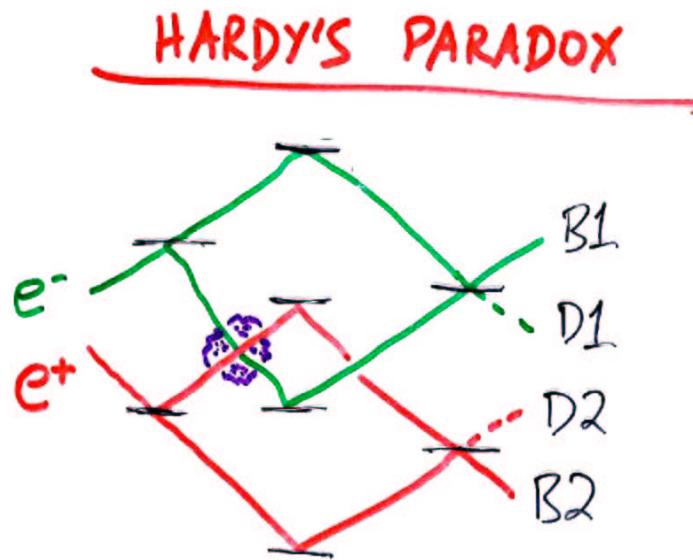
PQE XXXI



DISTORTIVE SPATIAL



PQE XXXI



D1 → e^+ was in the way

D2 → e^- was in the way

D1 & D2 → both were in the way

↳ they annihilate

↳ should never see them

BUT: QM says sometimes you do.

CONVENTIONAL CONCLUSION:

We can't reason about the past;
can't discuss what we don't measure.

OUTLANDISH CONCLUSION?

Re-examine logic, probability,...

What would weak measurements say?

$D1 \& D2$ fire $\Rightarrow e^- + e^+$ were "IN"

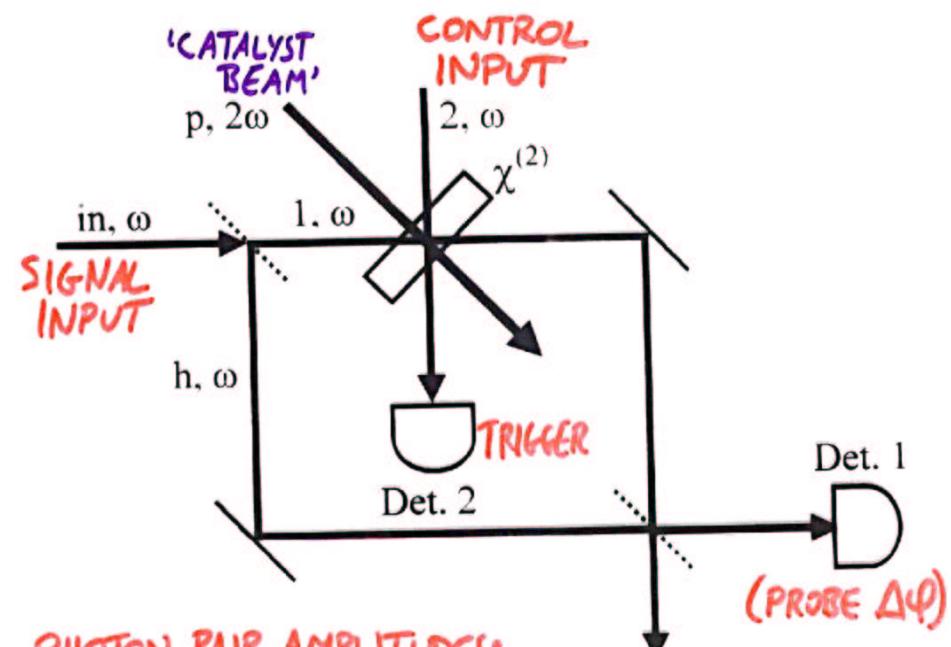
$$P(e^- \text{ IN}) = 1$$

$$P(e^+ \text{ in}) = 1$$

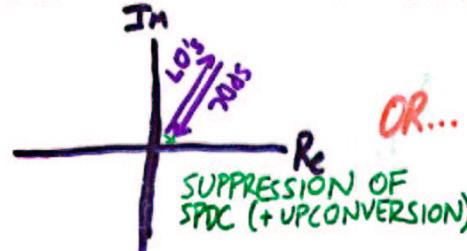
They didn't annihilate $= e^- + e^+$ weren't both "IN"

$$P(e^- \text{ in} \& e^+ \text{ in}) = 0$$

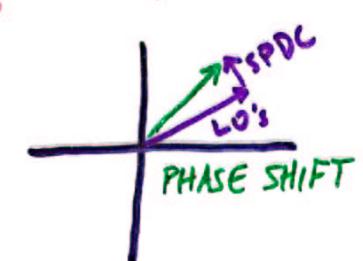
	$e^- \text{ IN}$	$e^- \text{ OUT}$	
$e^+ \text{ IN}$	0	1	$\Rightarrow 1$
$e^+ \text{ OUT}$	1	-1	$\Rightarrow 0$
	\Downarrow	\Downarrow	
	1	0	

CROSS-PHASE MODULATION
AT THE TWO-PHOTON LEVEL?


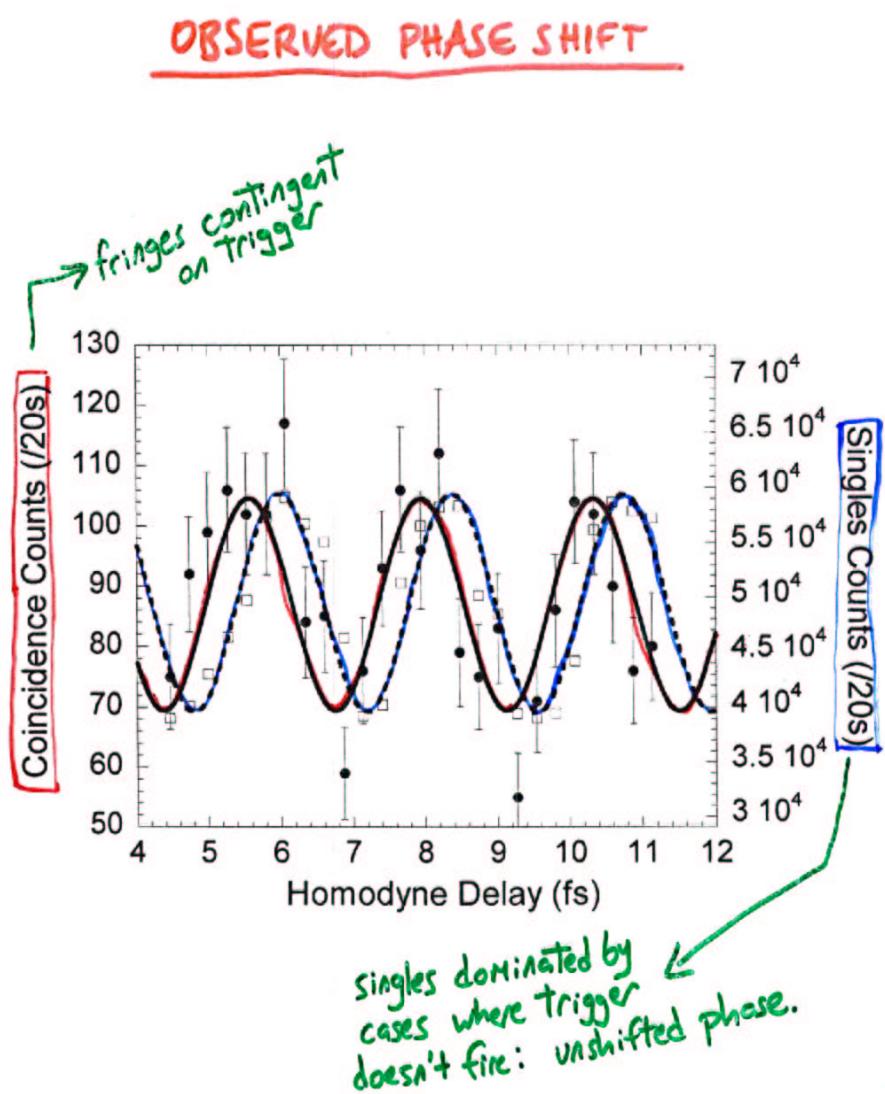
PHOTON PAIR AMPLITUDES:



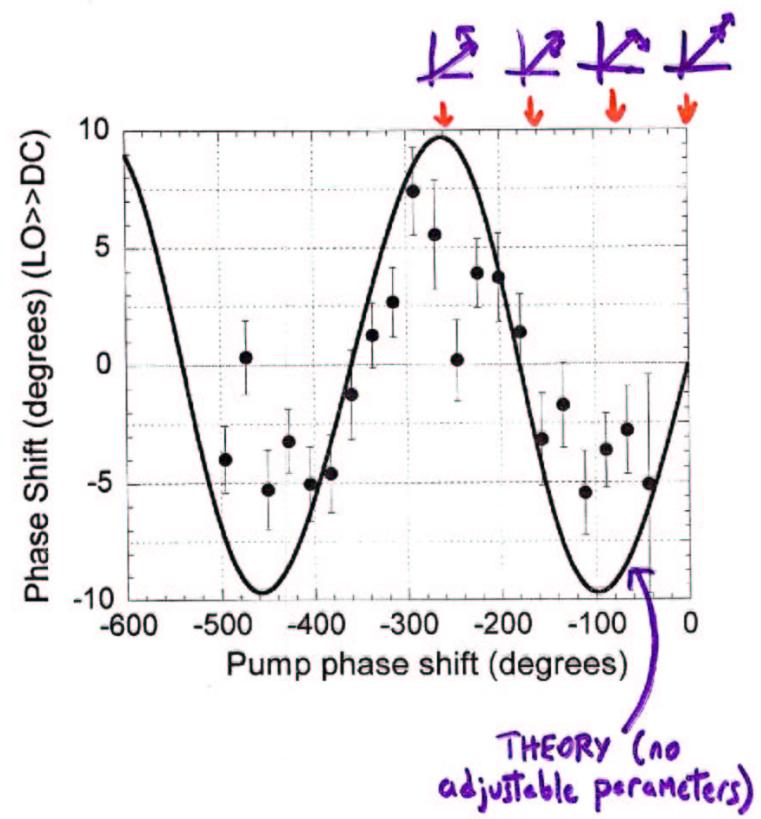
SUPPRESSION OF SPDC (+ UPCONVERSION)



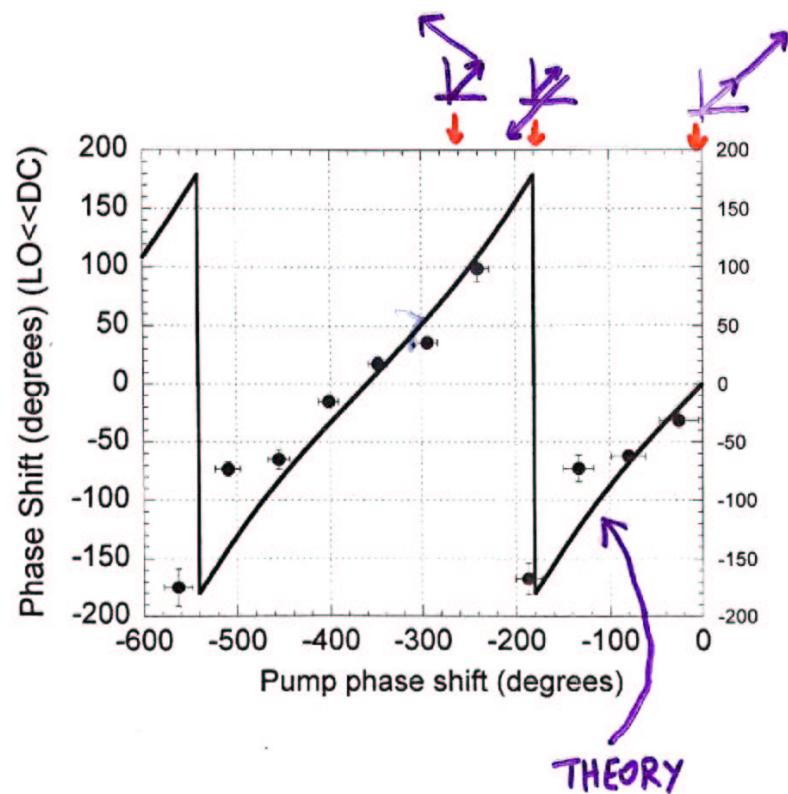
PHASE SHIFT



LOW PHASE-SHIFT REGIME (LO's \gg SPDC)



HIGH PHASE-SHIFT REGIME (SPDC \gg LO's)



A^v THEOREM (INTERESTING? RELEVANT? SURPRISING?)

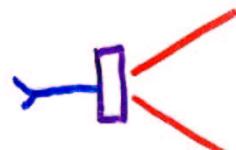
FOR A 3-LEVEL EIT SYSTEM (which one might use in a quantum logic gate) TO WORK FOR UNENTANGLED INPUTS, AND LEAVE THEM IN A PURE STATE (i.e., not entangled with the atoms), BOTH INPUTS MUST BE IN COHERENT STATES (e.g., not $|0\rangle$ and $|1\rangle$ qbit states in Fock space).

$$\begin{array}{ccc} |\alpha\rangle_1 \rightsquigarrow & - & \rightsquigarrow |\beta'\rangle_2 \\ |\beta\rangle_2 \rightsquigarrow & + & \rightsquigarrow |\alpha'\rangle_1 \end{array}$$

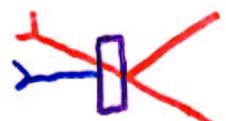
NEVER

$$\begin{array}{ccc} |m\rangle_1 \rightsquigarrow & - & \rightsquigarrow e^{imn\pi*} |n\rangle_2 \\ |n\rangle_2 \rightsquigarrow & + & \rightsquigarrow - |m\rangle_1 \end{array}$$

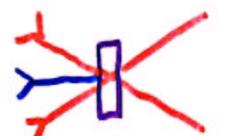
THE MENAGERIE



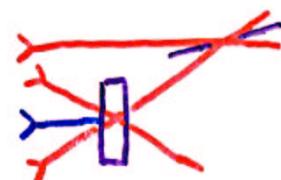
DOWN-CONVERSION
(nondet. single-photon prep)



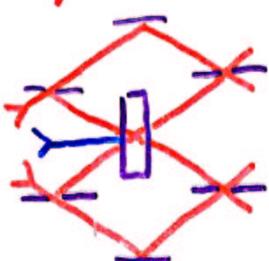
QUANTUM STATE PREP



2-PHOTON SWITCH



2-PHOTON
CROSS-PHASE MODULATION



HARDY'S PARADOX

IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. 33, NO. 4, APRIL 1997

Negative Group Delay Times in Frustrated Gires-Tournois and Fabry-Perot Interferometers

Pierre Tournais

Abstract— It is demonstrated in this paper that a Gires-Tournois interferometer illuminated with an angle of incidence greater than the critical angle for total internal reflection introduces a negative group delay time, whatever the orientation of the electric field vector of the wave with respect to the plane of incidence, when the evanescent wave in the dielectric layer is reflected by a dielectric substrate whose refractive index is between those of the incident medium and of the dielectric layer. When the evanescent wave in the dielectric layer is reflected by a nonabsorbing metal, the group delay time is negative when the electric field vector is in the plane of incidence and positive when the electric field vector is perpendicular to the plane of incidence. Similarly, a frustrated Fabry-Perot interferometer shows negative group delay times for angles of incidence greater than specific *p*-wave and *s*-wave critical angles.

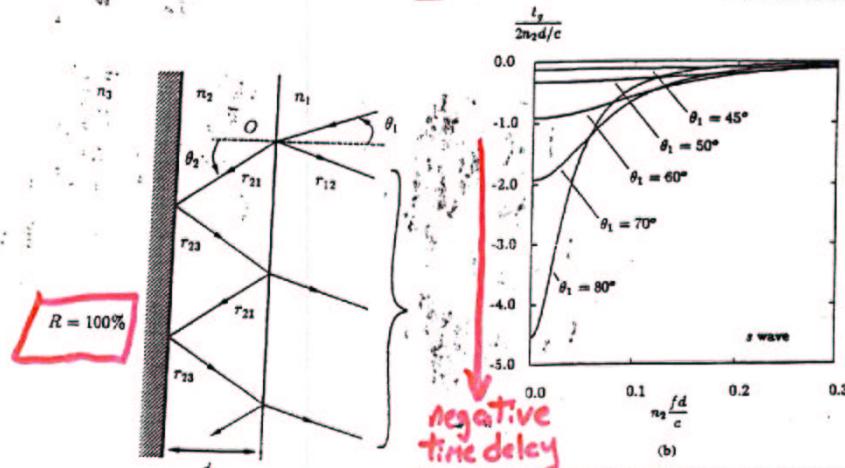


Fig. 1. The Gires-Tournois interferometer.

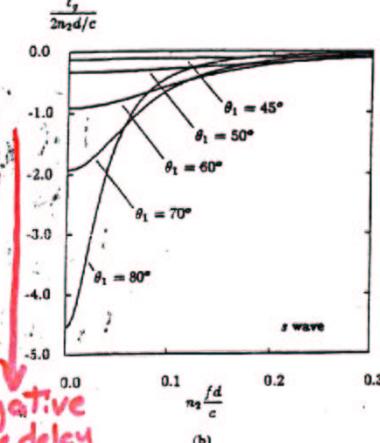
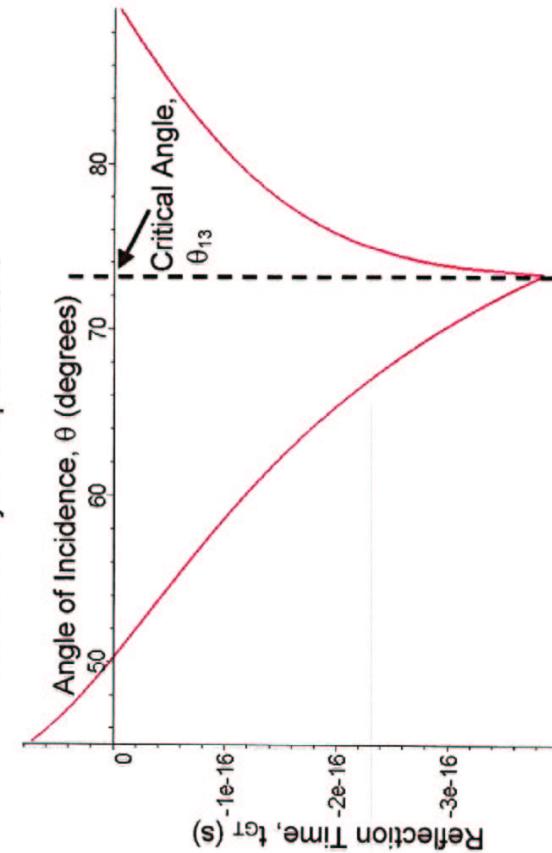


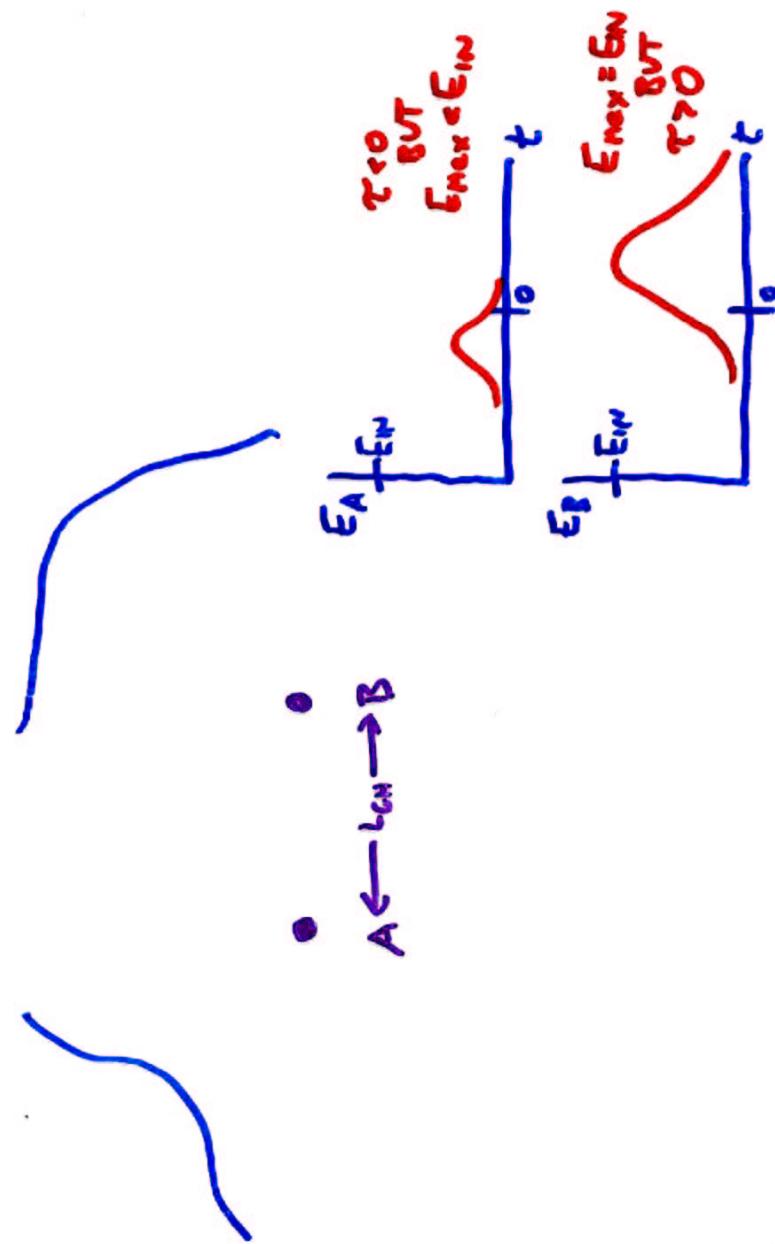
Fig. 2. Phase difference and group delay time in a frustrated Gires-Tournois interferometer, for an incident *s* wave and $n_1/n_2 = 1.5$, in the idealized case where $r_{23} = 1$.

Frustrated Gires-Tournois Interferometer

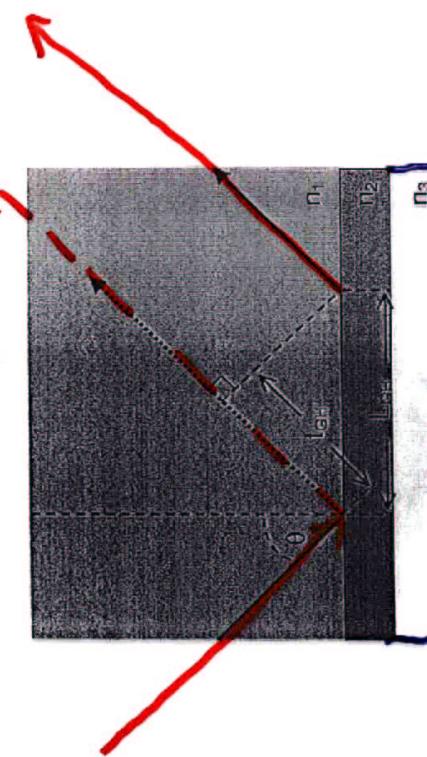
- When $n_1 > n_3 > n_2$ and $\theta > \theta_{13} = \sin^{-1}(n_3/n_1)$, where θ_{13} is the critical angle, the reflection time is negative.
 - The light undergoes total internal reflection since $\theta > \theta_{13} > \theta_{12}$ and the reflection probability is 100%.
 - The minimum negative reflection time is one optical period. $t_{GT\min} = -1/\omega$
-

Expected Reflection Time vs. Angle of Incidence
For Stated System Specifications





THE GIRES-TOURNOIS DECAY
OR: A Cautionary Tail



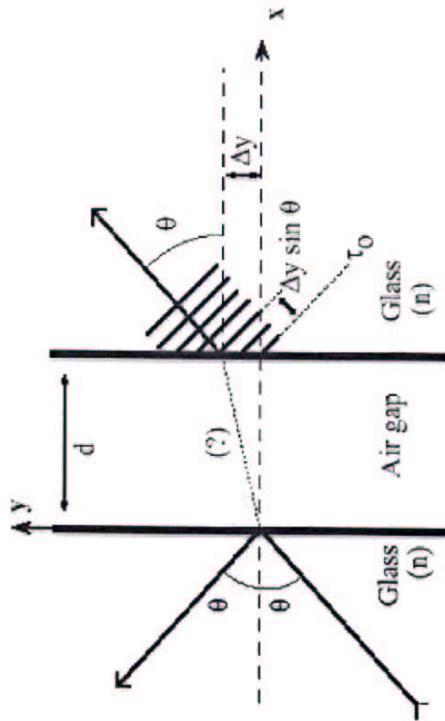


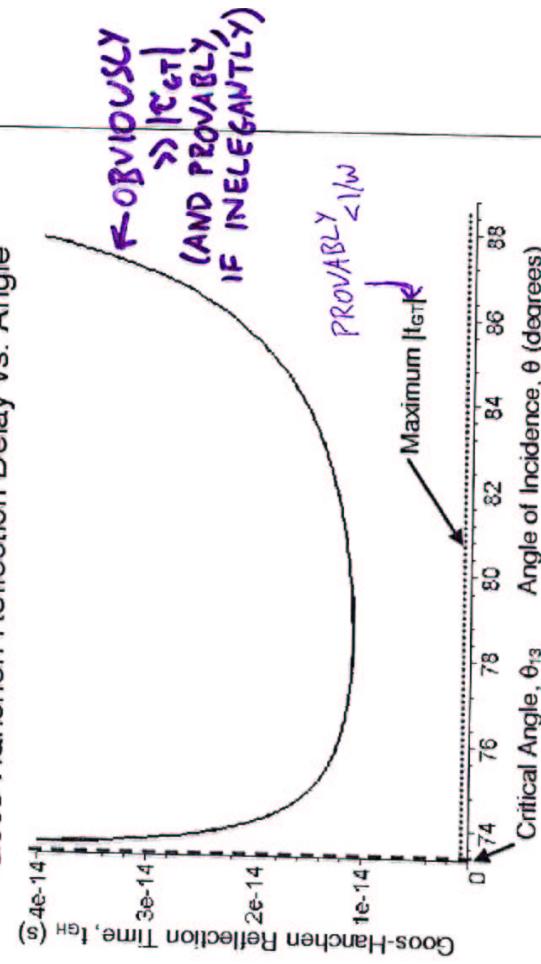
Figure 5.5: Regardless of incident angle θ , the transmitted beam reaches $y = 0$ sooner than it reaches $y = \Delta y$, by a delay of $(n/c)\Delta y \sin \theta$.

microcl 17 -juliet 2002

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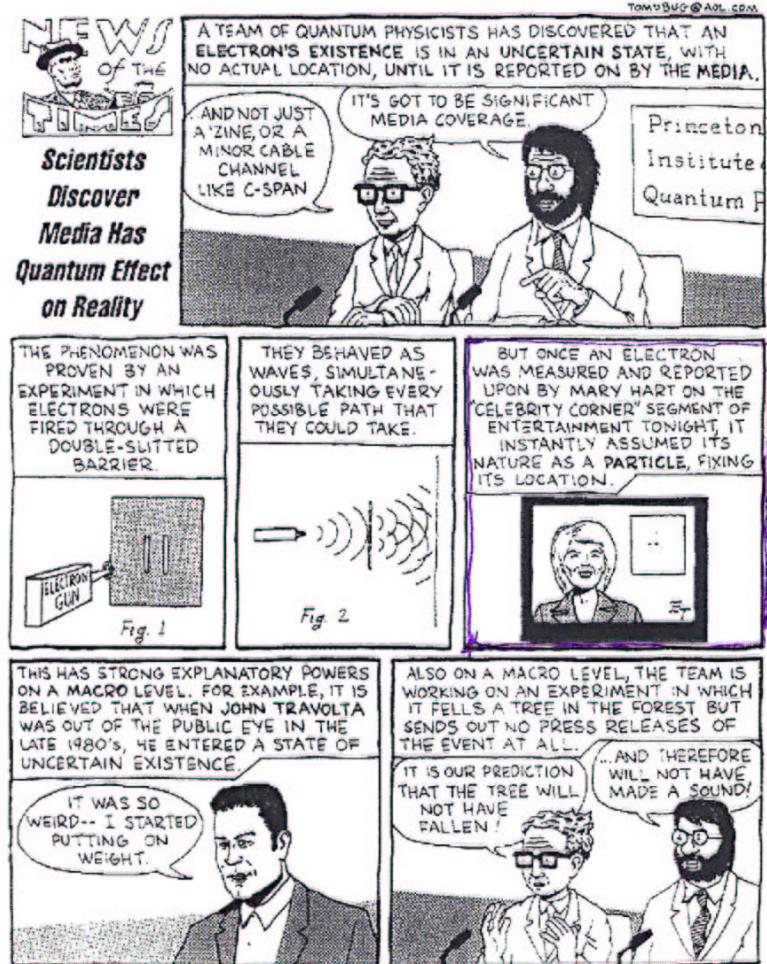
$$\tau_r = \tau_{gr} + \tau_{gh} > 0$$

Goos-Hanchen Reflection Delay vs. Angle



AND NOW FOR SOMETHING COMPLETELY DIFFERENT...

TOM the DANCING BUG



So far for an ensemble (or a classical wave), no surprises.

Center-of-Mass moves slower than light



(in fact, com is reflected)

How does a Copenhagenist understand that the subensemble which is transmitted appears early?



PROP. <C, BUT
INSTANTANEOUS
COLLAPSE

REMAINING QUESTION:

If you are not comfortable with collapse, how should you think about "that subset of the particles which is ultimately transmitted"?

'WEAK' (CONDITIONAL) MSMTS

The Schröd. eq. is t -reversible:
why should a MSMT at t_2 depend on preparation
at t_1 , but not on selection at t_3 ?

von Neumann MSMT theory:
Irreversible measurement

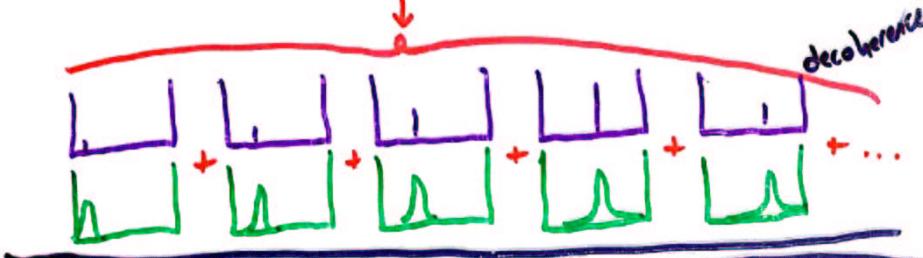
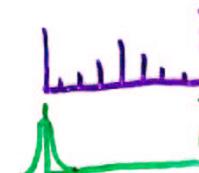
$$\tilde{\gamma}(x)$$

$$\tilde{\gamma}(t)$$

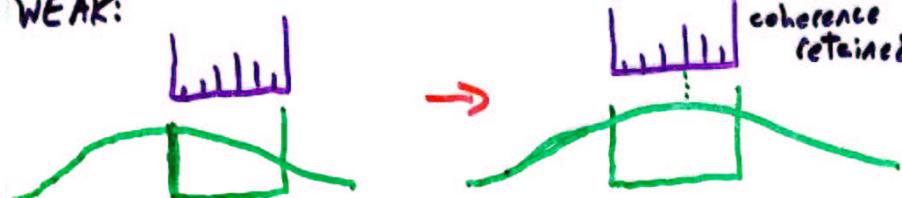
$$\tilde{\gamma}(t) \sim g(t) X \cdot P$$

pointer momentum
particle position
(or other observable)

STRONG:



WEAK:



Prepare particle in $|i\rangle$

Measure observable A weakly

Postselect particles in $|f\rangle$

Outcome of each weak MSMT very uncertain

But on average, find $\frac{\langle f|A|i\rangle}{\langle f|i\rangle} \equiv \langle A \rangle_{fi}$

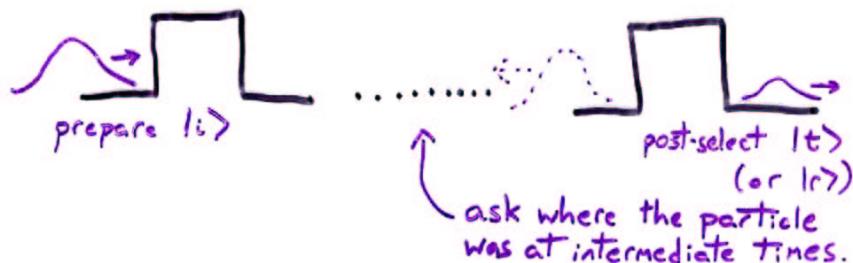
N.B. For $|f\rangle = |i\rangle$, $\langle A \rangle_{fi} = \langle A \rangle$

- For $\langle f|i\rangle \ll 1$, $\langle A \rangle_{fi}$ may be outside spectrum of A
- $\langle A \rangle_{fi}$ may be complex

$$\begin{aligned} A|i\rangle &= a|i\rangle \\ \text{OR } A|f\rangle &= a|f\rangle \end{aligned} \quad \} \rightarrow \langle A \rangle_{fi} = a$$

$$\begin{aligned} \langle A+B \rangle_{fi} &= \langle A \rangle_{fi} + \langle B \rangle_{fi} \\ \text{Even if } [A, B] &\neq 0! \\ P(A \& B | f) &= P(B | f) P(A | B) \\ \sum_j P(f_j) \langle A \rangle_{f_j i} &= \langle A \rangle \Rightarrow T/\tau_T + |R|^2 \tau_R = \tau_d \\ \downarrow \\ \sum \langle i | f_j \rangle \langle f_j | i \rangle \frac{\langle f_j | A | i \rangle}{\langle f_j | i \rangle} &= \sum \langle i | f_j \rangle \langle f_j | A | i \rangle = \langle i | A | i \rangle \end{aligned}$$

How does this apply to tunneling?



$$P(x) = |\psi(x)|^2 = \underbrace{\langle \psi | \psi \rangle}_{\text{Proj}(x)} \langle \psi | x \rangle \langle x | \psi \rangle$$

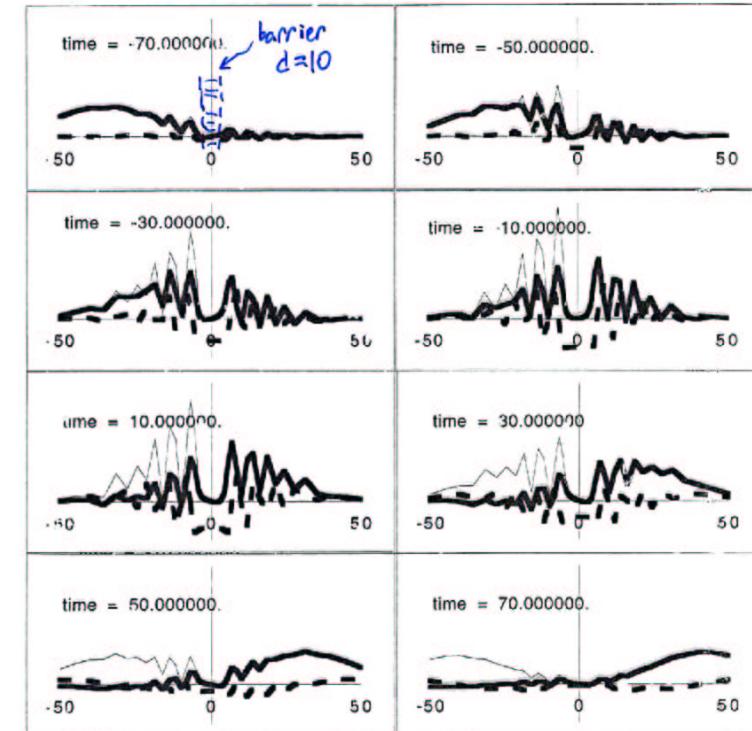
The prob. of being at x is just the expectation value of the projector onto x .

$$\begin{aligned} \text{Bayes's thm.} \Rightarrow P(x \mid \text{trans}) &= \frac{P(x \& \text{trans})}{P(\text{trans})} \\ &= \frac{\langle \psi | t \rangle \langle t | \psi \rangle \langle \psi | x \rangle \langle x | \psi \rangle}{\langle \psi | t \rangle \langle t | \psi \rangle} \\ &= \frac{\langle \psi | t \rangle \langle t | x \rangle \langle x | t \rangle}{\langle \psi | t \rangle \langle t | \psi \rangle} = \frac{\langle t | x \rangle \langle x | t \rangle}{\langle t | t \rangle} \end{aligned}$$

Precisely AhV's result. $P(x|\text{trans}) = \frac{1}{T} \int_T \psi^*(x) \psi(x)$.

- We can write the prob. distrib. of either trans. or refl. particles, as a function of time.
- We can integrate over time + over the barrier to obtain a total "conditional dwell time."
- BUT: these results are complex.

35 52M
52.15



— Prob. distrib. of tunneling particle
— " " " reflected "

---- "Back-action" on quantum measurement device

N.B. ① very little 'real time' spent at $x=0$
② no preferential transmission of leading edge

THE MEANING OF 'WEAK MEASUREMENTS' WITH COMPLEX VALUES

$$\tau_T \rightarrow \tau_d - i \tau_{BL}$$

"Has anyone ever seen a stopwatch with complex numbers on the dial?"



But consider a quantum-mechanical stopwatch.

$$\Psi(x) \sim e^{-(x-t)^2/4\sigma^2}$$

some inevitable uncertainty

$$t \text{ complex} \Rightarrow \Psi \sim e^{-(x - \operatorname{Re} t)^2/4\sigma^2} e^{i \operatorname{Im} t / 2\sigma^2} \dots$$

hand shifts by $\operatorname{Re} t$ picks up momentum of $\hbar \operatorname{Im} t / 2\sigma^2$ normalization

This is precisely the meaning of weak (or conditional) measurements.

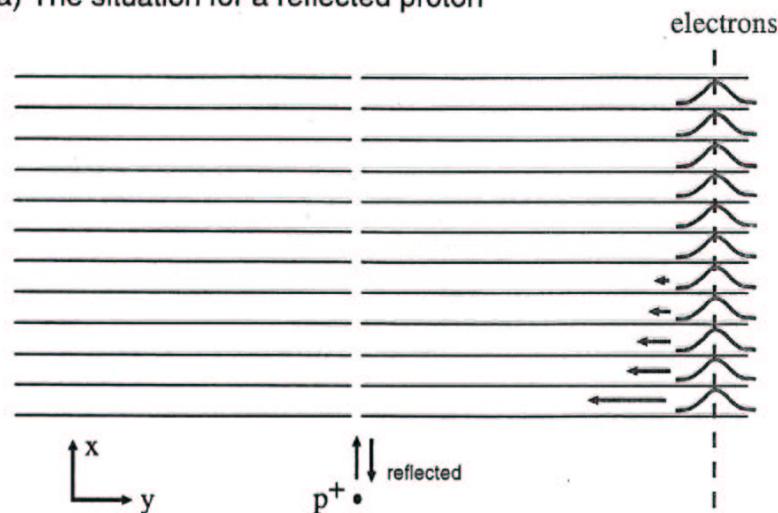
$\operatorname{Re} \tau_T$ describes clock hand's position shift (e.g., Larmor precession).

$i \operatorname{Im} \tau_T$ describes back-action (e.g., spin aligning with \vec{B} .)

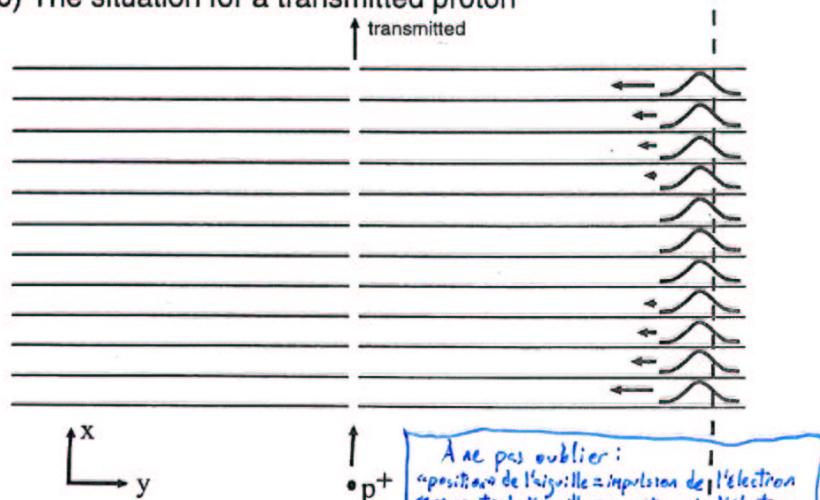
For large r , back-action vanishes, but position shift of hand remains constant.

A la recherche du temps perdu.

(a) The situation for a reflected proton

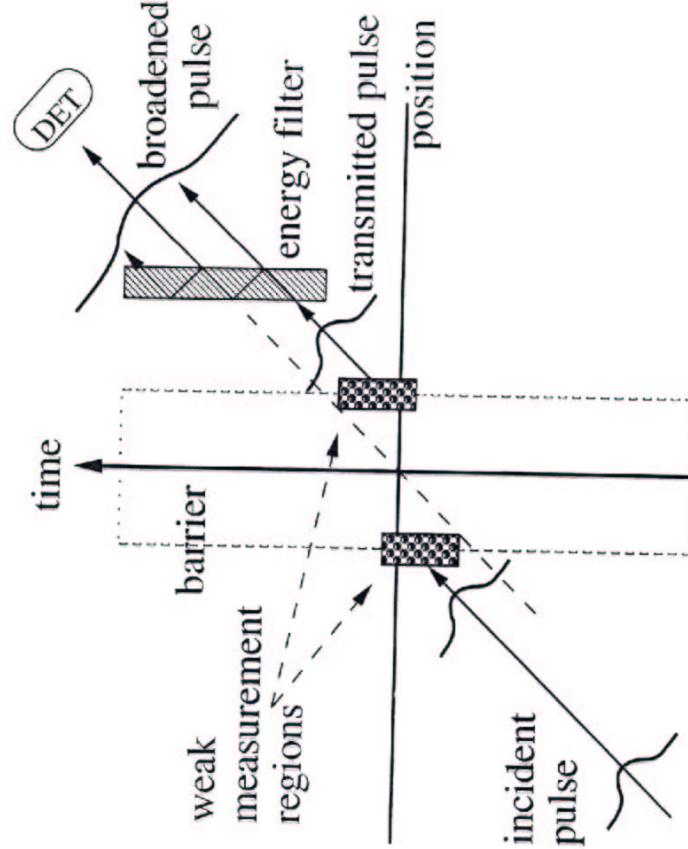


(b) The situation for a transmitted proton

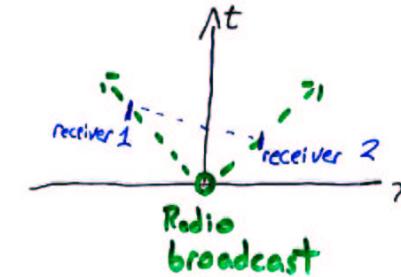


A ne pas oublier:
 - position de l'ajouille = impulsion de l'électron
 - moment de l'ajouille = position de l'électron

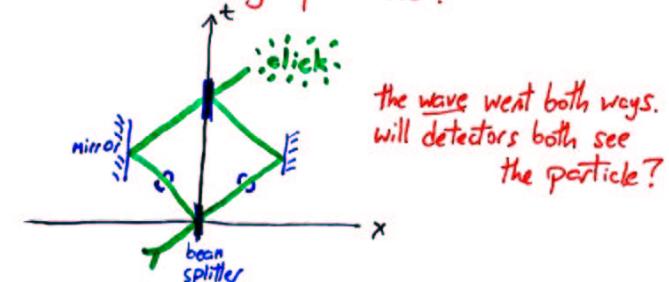
Figure 1



A cause may have two spacelike-separated effects.

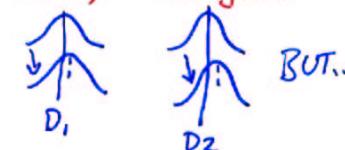


But if the cause is a single particle?



NO — a good detector collapses the state, s.t. only one detector fires.

BUT: a "weak" detector does no such thing.
Of course, it also gives no event-by-event information.



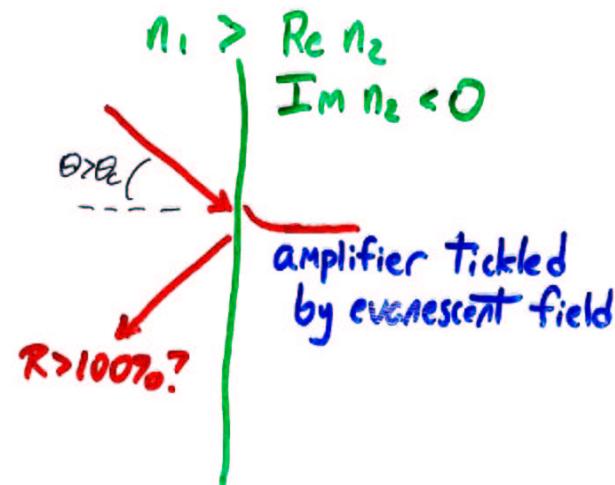
weak measurements
May (CONJECTURE) predict
no broadening. If experiment
confirms this, and a quantitative
inequality derived, we can show that
some particles are in 2 places at once.

$D_1 - D_2$
a naive corpuscular
model would say $D_1 - D_2$
undergoes a random walk.

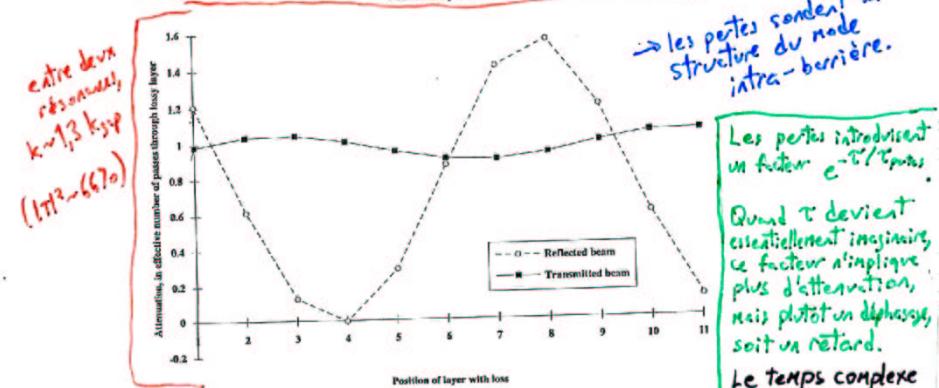
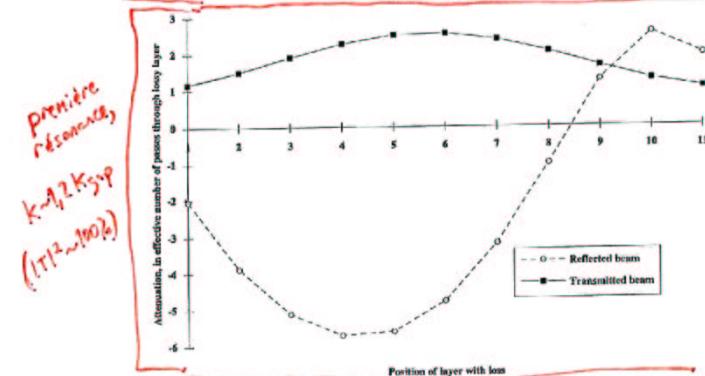
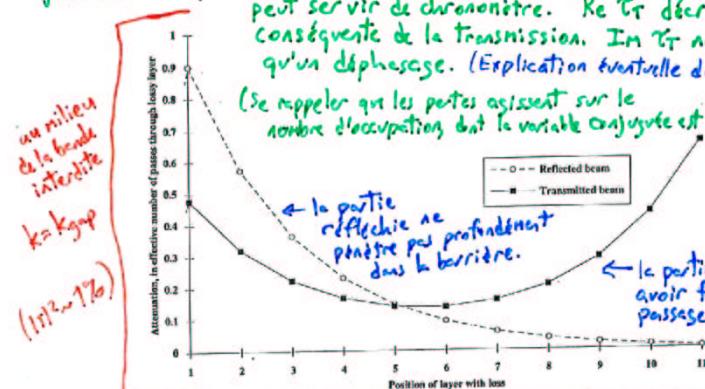
SIEGMAN'S QUESTION

What is the effect of an amplifier in a forbidden region?

For example...



Ajouter de faibles pertes dans une des couches diélectriques du miroir, ça peut servir de chronomètre. $\text{Re } T\tau$ décrit l'atténuation conséquente de la transmission. $\text{Im } T\tau$ ne décrit qu'un déphasage. (Explication éventuelle du désaccord expérimental?)
 (Se rappeler que les pertes agissent sur le nombre d'occupations, dont la variable conjuguée est la phase.)



Multilayer with gain
(DBR laser, below threshold)

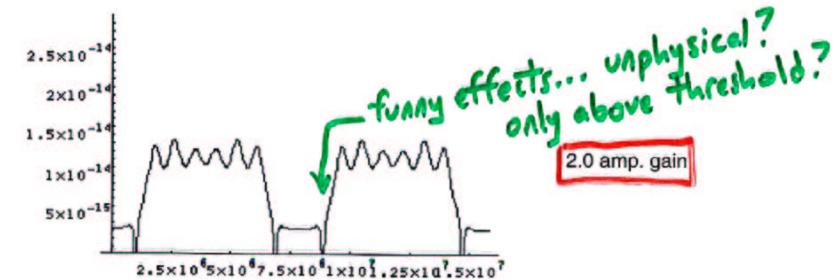
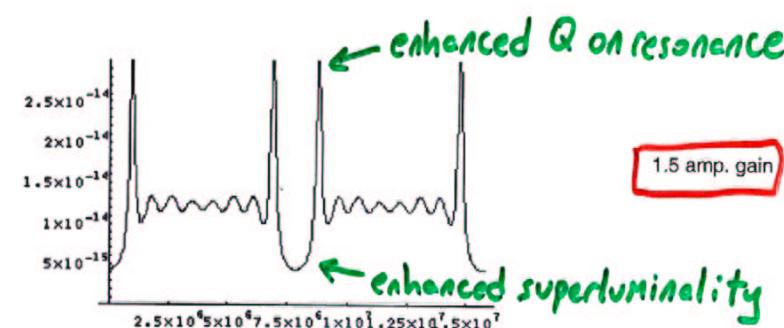
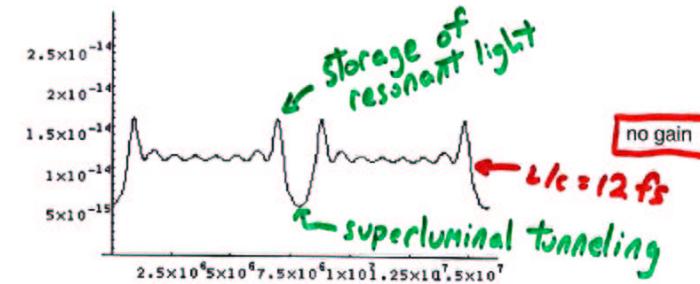


probe off
cavity resonance

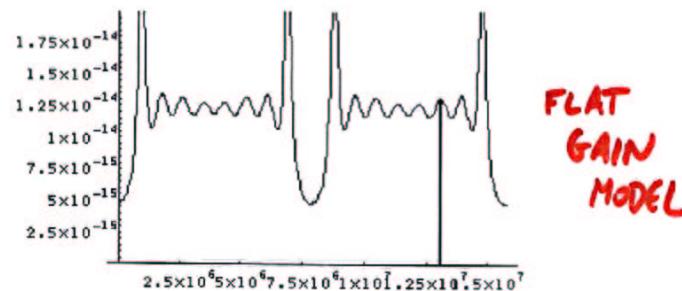
gain layer "sees"
little of probe... but
what of imaginary value
of weak dwell time?

DELAY TIMES FOR A 9-LAYER STRUCTURE

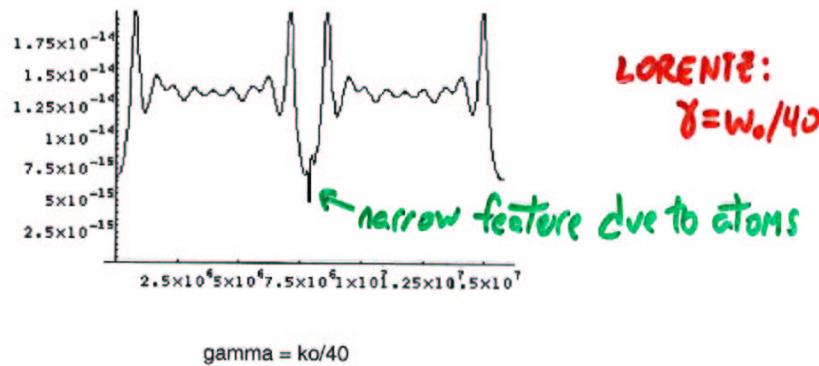
(4% reflectivity per layer makes 45% per end)



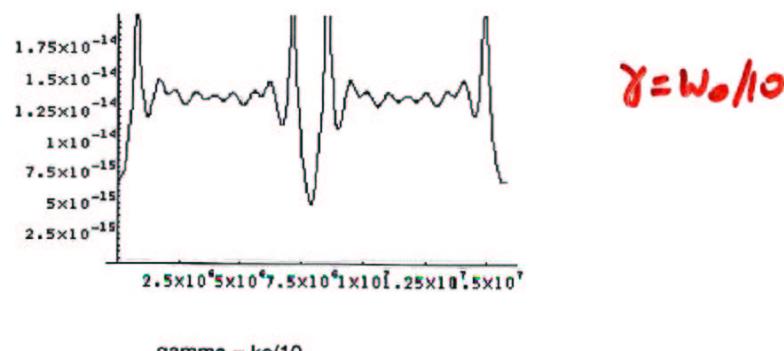
JUST DUE TO NEGLECTING GAIN DISPERSION?



amplitude gain of 1.37 (88% in intensity, single-pass)

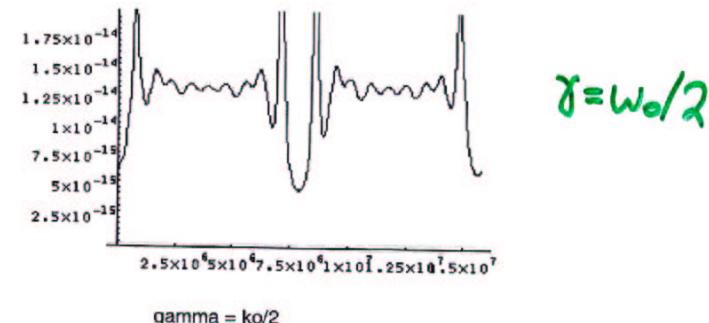


$\gamma = \omega_0/40$

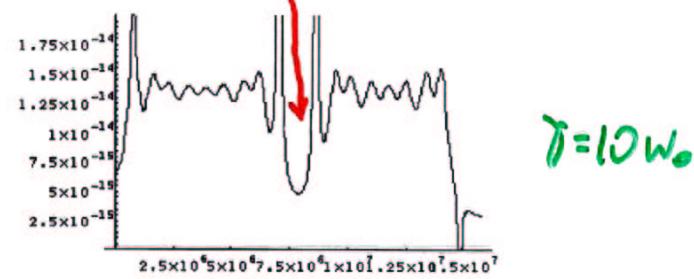


$\gamma = \omega_0/10$

FOR BIG γ , APPROACHES NAÏVE MODEL

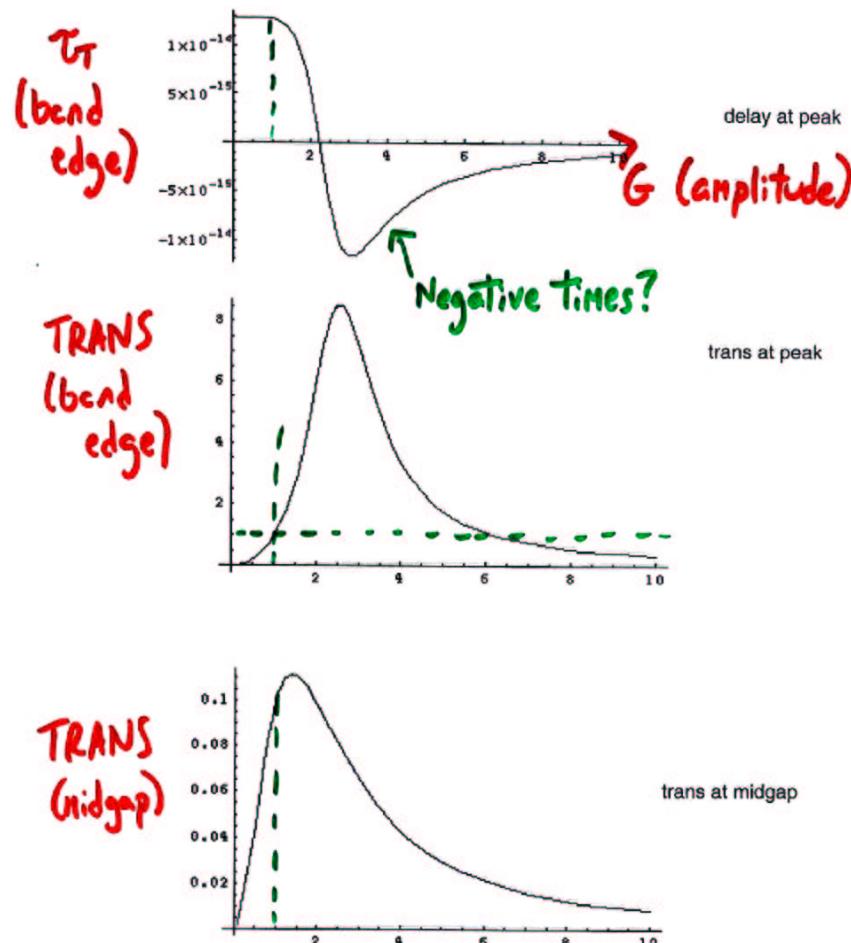


RÉTRIEVE BROAD
FLAT MINIMUM

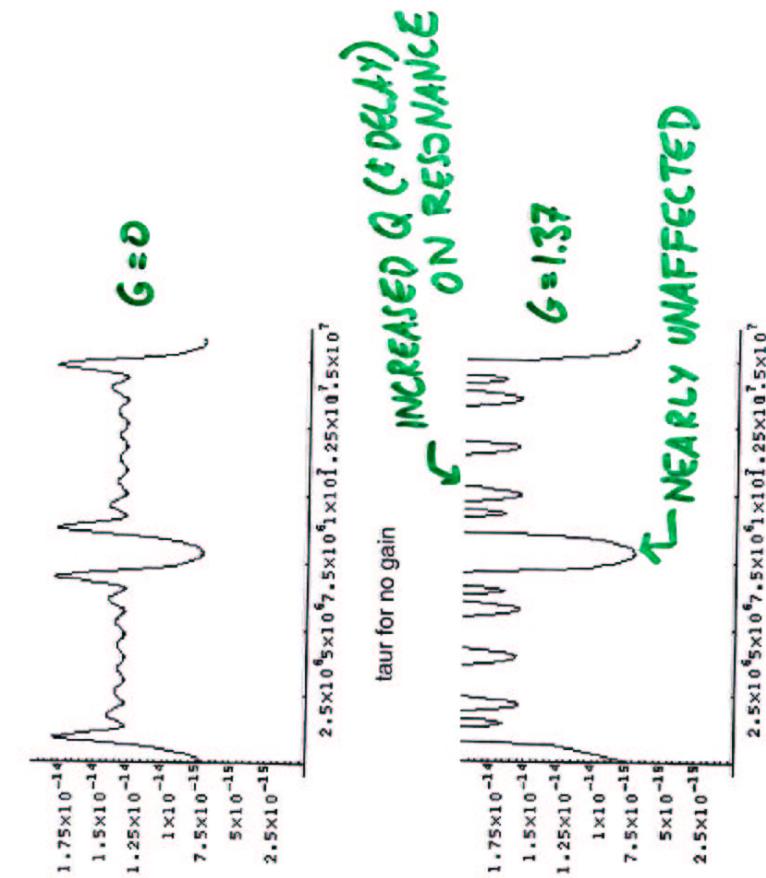


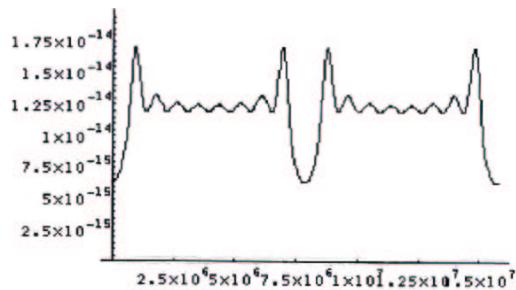
$\gamma = 10\omega_0$

MIDGAP SPEEDED UP FROM
1.9c TO 2.5c, AS
TRANSMISSION CHANGES
FROM 10% TO 11%.
(FOR 1-PASS INTENSITY GAIN OF 88%)

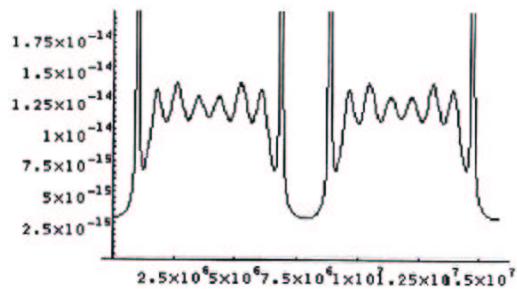


REFLECTION TIMES ARE
INSENSITIVE TO GAIN
FOR ω IN GAP.

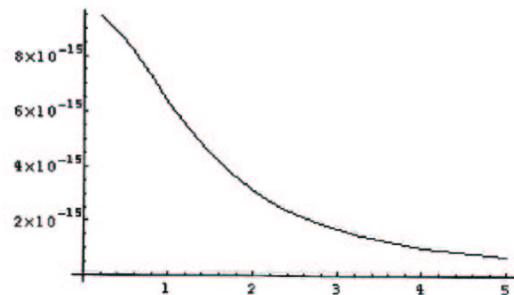




$(9L/c = 1.2 \text{ E} -14 \text{ sec})$



88% peak gain; 4% reflection per layer; 4 layers + gain + 4 layers



midgap delay versus gain

SUMMARY

- ① NLO can be done at 1-photon level with help of spectator fields... new effects should be explored
 - ② Using coherence effects for QI may be subtle since $\Delta t \Delta \phi \gtrsim 1/2$
 - ③ 2D effects introduce subtleties for defining delay times.
Passive media do not transmit/reflect light with negative delays and 100% efficiency.
 - ④ Weak measurements often describe what we really do in the lab, and may be useful for guiding the intuition.
 - ⑤ Adding gain to MLD structures may enhance superluminality without hurting transmission much (cf. longer or higher-Q structures)... perhaps less prone to instability than other gain-assisted superluminality?
- WHY?