

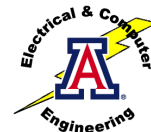
**Double Negative Metamaterial Designs, Experiments,
and Applications**

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Metamaterials

**Artificial materials that exhibit electromagnetic responses
generally not found in nature**

Metamaterials exhibit qualitatively new response functions that are not observed in the constituent materials themselves and result, for instance, from the inclusion of artificially fabricated, extrinsic, low dimensional inhomogeneities

Examples:

- Artificial dielectrics
- FSS, Electromagnetic bandgap structures
- Negative index (neg ϵ , μ) materials

Metamaterials may lead to new physics and engineering concepts

Compact DNG metamaterials having negative index of refraction have been designed, fabricated and tested experimentally

- HFSS and FDTD simulators have been used to design several DNG ($\epsilon < 0$ and $\mu < 0$) metamaterials (MTMs)
- Extraction formula have been derived to determine the MTM's effective permittivity and permeability
- Experimental results confirm the realization of DNG MTMs that are matched to free space and have a negative index of refraction
- Several potential applications have been studied:
Efficient Electrically Small Antennas (EESAs)

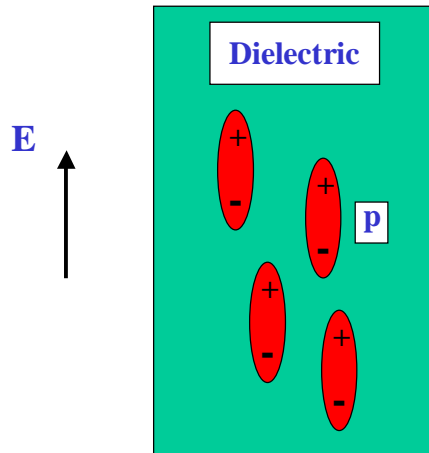


Metamaterials lead to a variety of novel electromagnetic effects

- The propagation characteristics of waves in DNG media ($\epsilon < 0, \mu < 0$) confirm the possibility of a negative index of refraction
- Negative angles of refraction exist for DNG media
- DNG Drude MTMs have been characterized with an FDTD simulator and confirm
 - ❖ paraxial beam focusing
 - ❖ negative angles of refraction for power flow
- Demonstrate phase and phase front compensators



EM Properties of aggregates of atoms / molecules are typically characterized by their electric and magnetic dipole moments



$$P = \sum_i p_i / V$$

$$= \epsilon_0 \chi E$$

$$\epsilon = \epsilon_0 (1 + \chi)$$

χ = electric susceptibility



The Debye and Lorentz linear polarization models produce well-known material responses

Debye Model

$$\partial_t P_x + \Gamma_E P_x = \epsilon_0 \Gamma_E \chi_\alpha E_x$$

Lorentz Model

$$\partial_t^2 P_x + \Gamma_E \partial_t P_x + \omega_0^2 P_x = \epsilon_0 \omega_0^2 \chi_\alpha E_x$$

$$\hat{P}_x(\omega) = \frac{\omega_0^2 \chi_\alpha}{-\omega^2 + j\omega\Gamma_E + \omega_0^2} \epsilon_0 \hat{E}_x(\omega)$$

Drude Model

$$\partial_t^2 P_x + \Gamma_E \partial_t P_x = \epsilon_0 \omega_p^2 E_x$$



Several metamaterial models have been studied. They produce a variety of novel electromagnetic responses.

Time Derivative Debye Model

$$\partial_t P_x + \Gamma_E P_x = \epsilon_0 \Gamma_E \chi_\alpha E_x + \epsilon_0 \chi_\beta \partial_t E_x$$

Time-Derivative Lorentz Model

$$\partial_t^2 P_x + \Gamma_E \partial_t P_x + \omega_0^2 P_x = \epsilon_0 \omega_0^2 \chi_\alpha E_x + \epsilon_0 \omega_0 \chi_\beta \partial_t E_x$$

Two Time-Derivative Lorentz Model

$$\partial_t^2 P_x + \Gamma_E \partial_t P_x + \omega_0^2 P_x = \epsilon_0 \omega_0^2 \chi_\alpha E_x + \epsilon_0 \omega_0 \chi_\beta \partial_t E_x + \epsilon_0 \chi_\gamma \partial_t^2 E_x$$

These models have been implemented and tested with our 1D, 2D, and 3D FDTD simulators



Material responses are incorporated into our FDTD Maxwell equation solver through equivalent polarization and magnetization models

★ Recursive Convolution Method

$$D = \epsilon * E$$

✂ Auxiliary Differential Equation Method

$$A(\partial_t) D = B(\partial_t) E$$

⊙ Polarization / Magnetization Method

$$A(\partial_t) P = B(\partial_t) E$$

P & M equations are solved self-consistently with Maxwell equations



The matched DNG medium was simulated with a lossy Drude model

Lossy Drude permittivity

$$\epsilon(\omega) = \epsilon_0 \left(1 - \frac{\omega_{pe}^2}{\omega(\omega + i\Gamma_e)} \right)$$

Lossy Drude permeability

$$\mu(\omega) = \mu_0 \left(1 - \frac{\omega_{pm}^2}{\omega(\omega + i\Gamma_m)} \right)$$

Matching conditions

$$\begin{aligned} \omega_{pe} &= \omega_{pm} \\ \Gamma_e &= \Gamma_m \\ Z(\omega) &= \sqrt{\frac{\mu(\omega)}{\epsilon(\omega)}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = Z_0 \end{aligned}$$



The 1D time domain equations solved with the FDTD simulator for the matched DNG medium were straight-forward

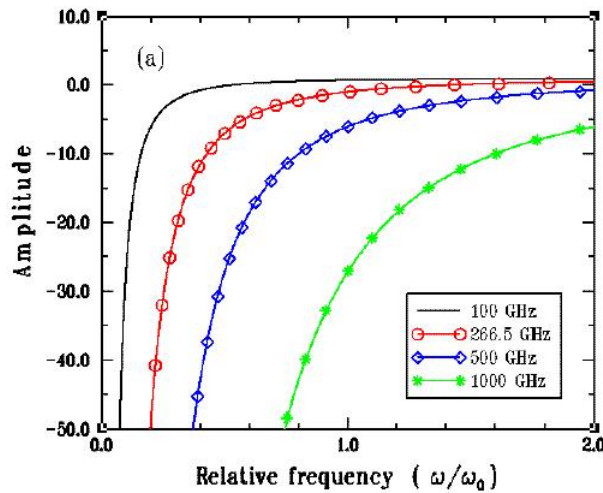
$$\begin{aligned} \partial_t E_x &= \frac{1}{\epsilon_0} (-\partial_z H_y - J_x) \\ \partial_t J_x + \Gamma J_x &= \epsilon_0 \omega_p^2 E_x \\ \partial_t H_y &= \frac{1}{\mu_0} (-\partial_z E_x - K_y) \\ \partial_t K_y + \Gamma K_y &= \mu_0 \omega_p^2 H_y \end{aligned}$$

Note: K_x has been normalized by μ_0 to make the magnetic current equation dual to the electric current definition.

Note: Matching is achieved by placing the E's, H's, J's and K's in FDTD cells in the correct manner



Real part of lossy Drude model for different plasma frequencies

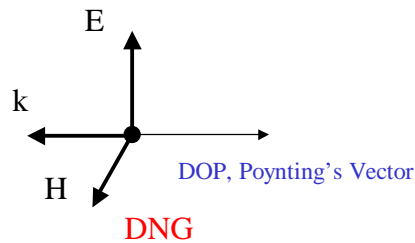
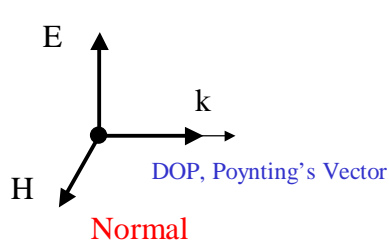


$f_0 = 30 \text{ GHz}$



Wave properties in DNG media are “unusual”

- ★ Wave propagation and power flow is causal
- ★ The medium is right-handed with respect to the direction of propagation
- ★ The medium is left-handed with respect to the direction the wave vector direction



Ziolkowski and Heyman have determined the correct, causal square root choice and confirmed it with FDTD simulations

For a slightly lossy DNG medium

$$n \simeq -\sqrt{|\epsilon_r \mu_r|} \left[1 - i \frac{1}{2} \left(\frac{\epsilon_i}{|\epsilon_r|} + \frac{\mu_i}{|\mu_r|} \right) \right]$$

$$\zeta \simeq +\sqrt{|\mu_r / \epsilon_r|} \left[1 - i \frac{1}{2} \left(\frac{\mu_i}{|\mu_r|} - \frac{\epsilon_i}{|\epsilon_r|} \right) \right]$$

If $\epsilon < 0$ and $\mu < 0$, this means the correct causal choice gives

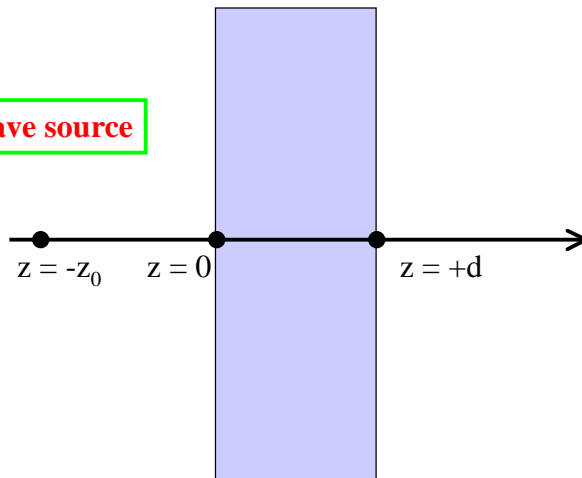
$$n_r(\omega) < 0 \quad n_i(\omega) > 0$$

$$Z_r(\omega) > 0$$



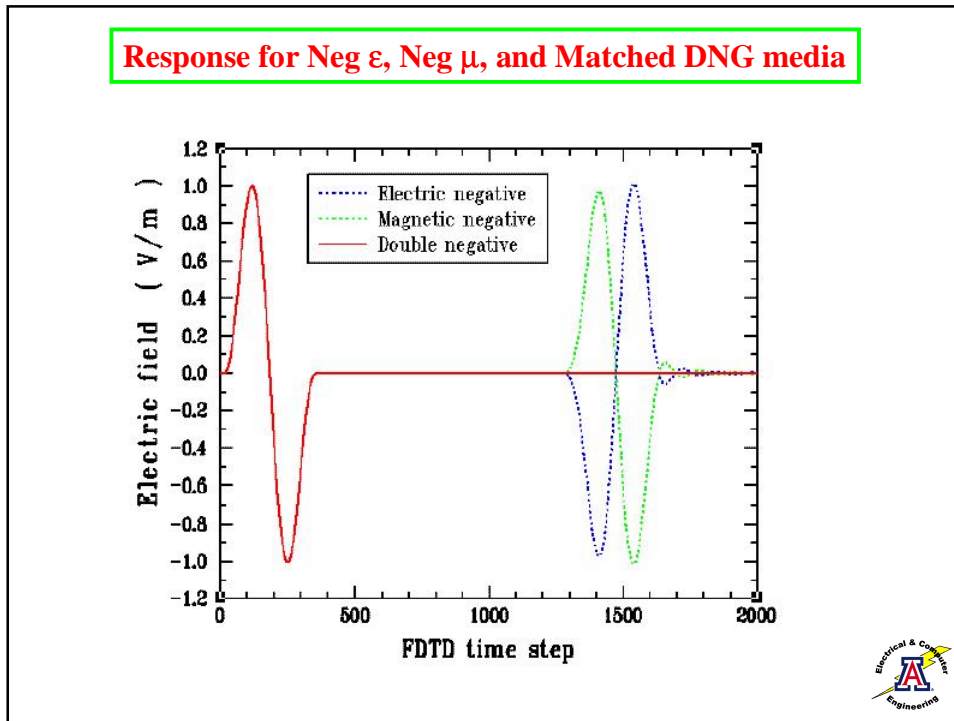
Analytical and FDTD simulation problem geometries

Plane wave source



2TDLM slab, DNG slab





Several EM quantities were monitored

Correct form of Poynting's theorem in a dispersive medium


$$-\int_{\Sigma=\partial V} \vec{S} \cdot \hat{n}_{\Sigma} d\Sigma = \int_V [\epsilon_0 \vec{E} \cdot \partial_t \vec{E} + \vec{E} \cdot \partial_t \vec{P} + \mu_0 \vec{H} \cdot \partial_t \vec{H} + \mu_0 \vec{H} \cdot \partial_t \vec{M}] dV$$

The form of Poynting's theorem in an homogenous, non-dispersive medium

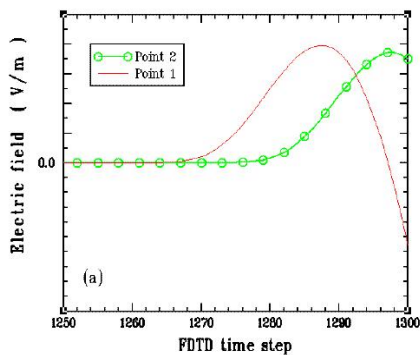
$$-\int_{\Sigma=\partial V} \vec{S} \cdot \hat{n}_{\Sigma} d\Sigma = \partial_t \int_V \left[\frac{1}{2} \epsilon |\vec{E}|^2 + \frac{1}{2} \mu |\vec{H}|^2 \right] dV = \partial_t U_{EM}$$

The index of refraction

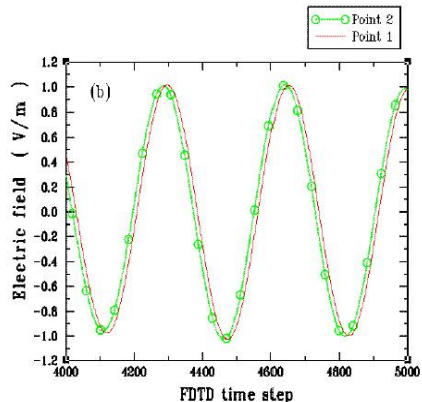
$$n_{FDTD} = \frac{1}{ik_0(z_2 - z_1)} \ln \left[\frac{\tilde{E}_x(z_2, \omega)}{\tilde{E}_x(z_1, \omega)} \right]$$



FDTD modeling of wave propagation in DNG media confirms causal behavior and negative index of refraction



Early time



Late time



The two time derivative Lorentz model produces a causal metamaterial response

Two Time-Derivative Lorentz Model

$$\partial_t^2 P_x + \Gamma_E \partial_t P_x + \omega_0^2 P_x = \epsilon_0 \omega_0^2 \chi_\alpha E_x + \epsilon_0 \omega_0 \chi_\beta \partial_t E_x + \epsilon_0 \chi_\gamma \partial_t^2 E_x$$

$$\hat{P}_x(\omega) = \frac{\omega_0^2 \chi_\alpha + j\omega \omega_0 \chi_\beta - \omega^2 \chi_\gamma}{-\omega^2 + j\omega \Gamma_E + \omega_0^2} \epsilon_0 \hat{E}_x(\omega)$$

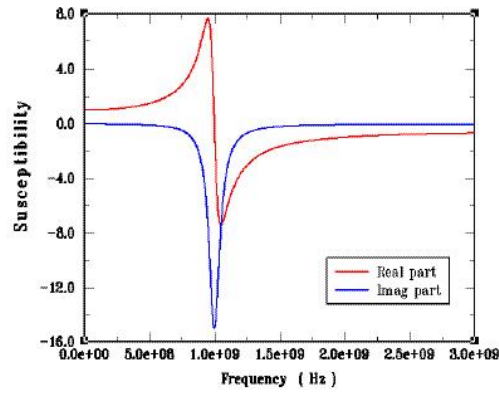
This implies the limiting properties

$$\lim_{\omega \rightarrow 0} \frac{\epsilon(\omega)}{\epsilon_0} = 1 + \chi_\alpha$$

$$\lim_{\omega \rightarrow \infty} \frac{\epsilon(\omega)}{\epsilon_0} = 1 + \chi_\gamma$$



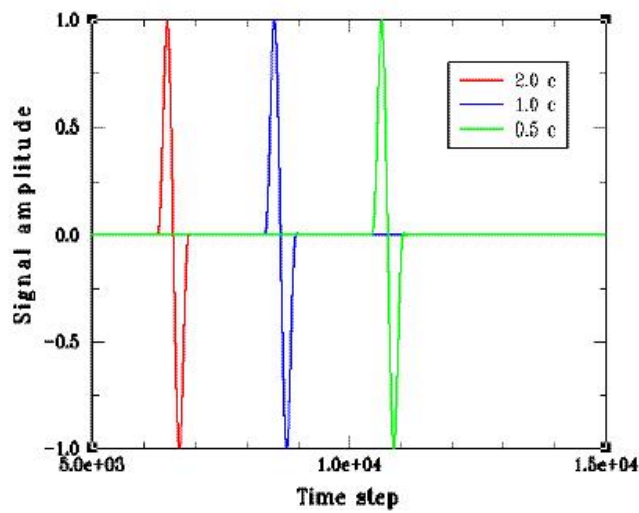
Susceptibility of 2TDLM material was designed to produce the desired superluminal medium response



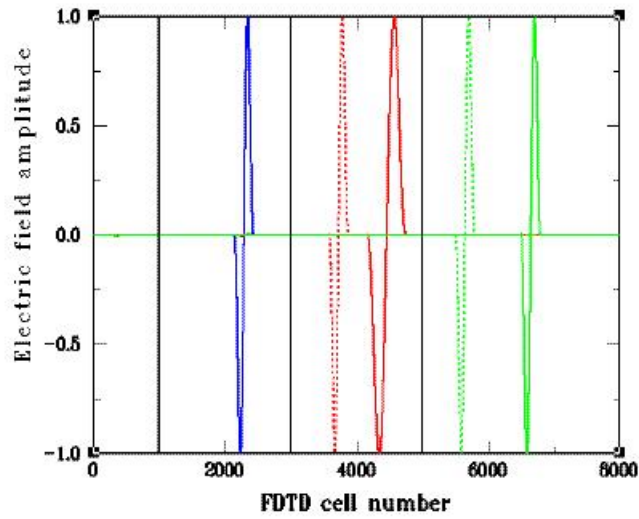
$$\chi_{\alpha}=1, \chi_{\beta}=10^{-5}, \chi_{\gamma}=-0.5, \omega_p = \omega_0 = 1 \text{ GHz}$$



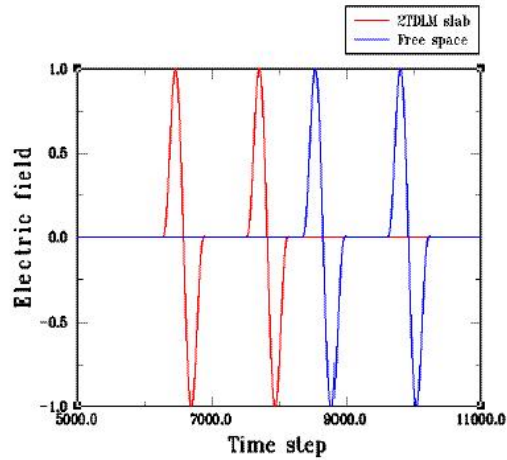
Propagation through 2TDLM slabs with $\chi_{\gamma} = +0.5, 0.0, -0.5$ demonstrate the superluminal effect



FDTD simulations of plane wave propagation through a matched 2TDLM metamaterial confirms the superluminal behavior



FDTD simulations of plane wave propagation through a matched 2TDLM metamaterial slab confirms the causal superluminal transmission of information



**Double negative (DNG) media ($\epsilon < 0$ and $\mu < 0$)
can be realized with metamaterial constructs**

- “Perfect Lens Effect”
Pendry, PRL Oct. 2000,
- Waves are not focused in general DNG medium
but rather beams are produced
Ziolkowski and Heyman, PRE, October 2001
- Direction of Power Flow
Positive: Valanju, Walser, Valanju, PRL, May 2002
Negative: Caloz, Xhang, Itoh, JAP, December 2001
Negative: Kong, Wu, Zhang, Microwave Opt. Tech.
Lett., April 2002
Negative: Ziolkowski and Heyman, PRE, October 2001



**General result:
Beams are formed in the DNG slab rather than foci**

NOTE: Slab solution is independent of square root choice

NOTE: Correct propagating and evanescent spectra

NOTE: Foci appear only for one special case:

$$\epsilon = -\epsilon_0, \mu = -\mu_0, n = -1$$

Then $\kappa = \kappa_0 = +1$ and a perfect foci appear for

$$z_{f1} = |z_0| \quad (\text{in slab}) \quad z_{f2} = 2d - |z_0| \quad (\text{beyond slab})$$



**General lossy, dispersive DNG slab:
Beams are formed rather than foci**

Paraxial result

$$g \approx \frac{1}{-2ik_0} e^{+ik_0(|z_0| - \tilde{n}z)} e^{-i\frac{k_z^2}{2k_0}(|z_0| - z/\tilde{n})} \quad 0 < z < d$$

$$g \approx \frac{1}{-2ik_0} e^{+ik_0[z + |z_0| - d(1 + \tilde{n})]} e^{-i\frac{k_z^2}{2k_0}[z + |z_0| - d(1 + 1/\tilde{n})]} \quad z > d$$

If $z_{f1} < d$ and $z_{f2} > d$, Paraxial foci appear at

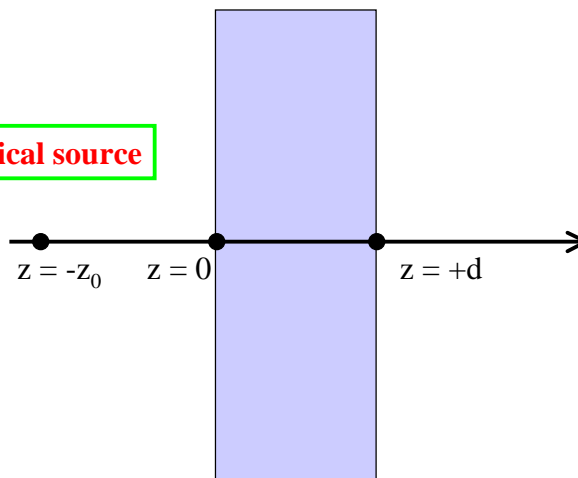
$$z_{f1} = \tilde{n}|z_0|, \quad z_{f2} = d(1 + 1/\tilde{n}) - |z_0|$$

Note: $z_{f1} = d/2$ if $z_0 = d/2\tilde{n}$ and $z_{f2} = d(1 + 1/2\tilde{n})$



Analytical and FDTD simulation problem geometries

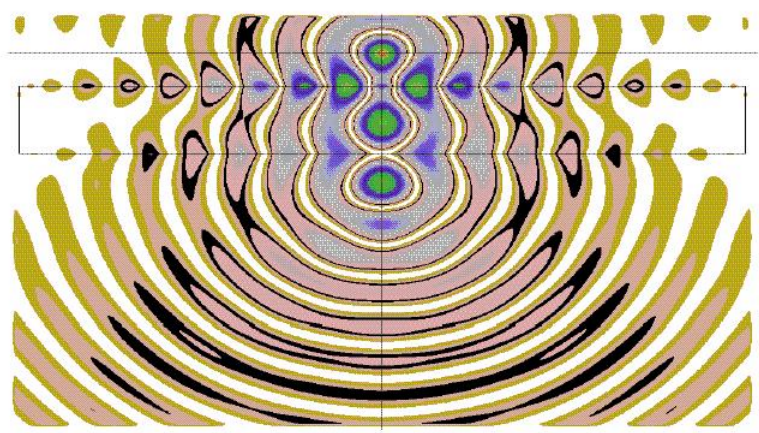
Cylindrical source



2TDLM slab, DNG slab

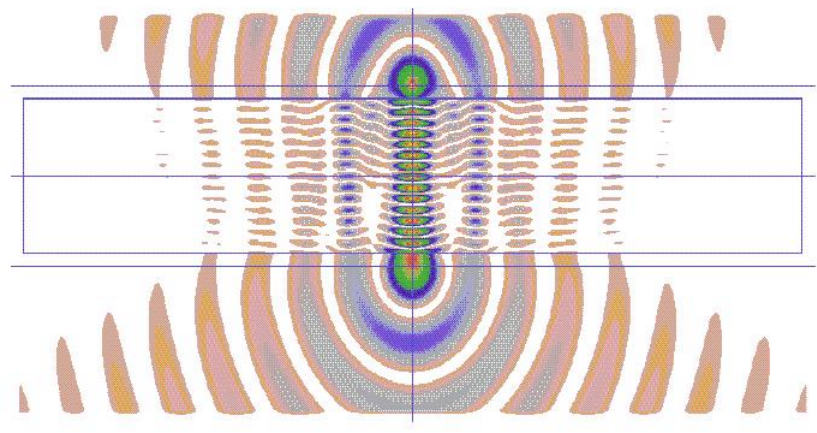


FDTD simulations of cylindrical wave interaction with lossy Drude DNG slab with $\epsilon(\omega_0)/\epsilon_0 = \mu(\omega_0)/\mu_0 = -1$ show no foci



Electric field intensity at one time over FDTD simulation space

FDTD simulations of cylindrical wave interaction with lossy Drude DNG slab with $\epsilon(\omega_0)/\epsilon_0 = \mu(\omega_0)/\mu_0 = -6$ show the predicted beam formation



Electric field intensity at one time over FDTD simulation space

DNG slab solution requires a negative angle of refraction for both phase and Poynting's vector, but phase and Poynting's directions are opposite

Regular medium

Free space

Normal interface

DNG Medium

Free space

DNG interface

Snell's Law: $\theta_{\text{trans}} = \sin^{-1}(\theta_{\text{inc}} / n)$

Negative angle of refraction for power flow follows immediately from Maxwell's equations

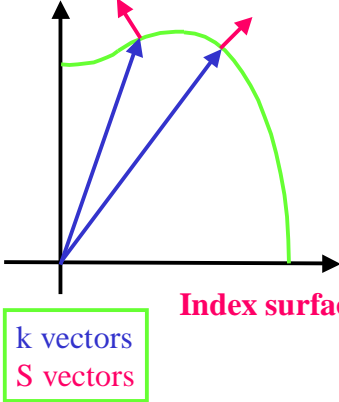
$k_z = + (\omega^2 \epsilon\mu - k_x^2)^{1/2}$

Normal DPS medium

$k_z = - (\omega^2 \epsilon\mu - k_x^2)^{1/2}$

DNG medium

Super-prism effect requires local modifications of the wave vector – index surface




Index surface

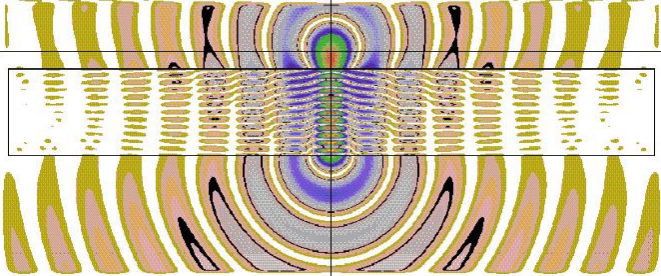
k vectors
S vectors

- EBG produces local curvature variations in the index surface
- Poynting's vector is perpendicular to index surface
- Negative angle of refraction is realized
- Are the CLS-CLL metamaterial effects the same??
- Material extraction indicates simpler DNG properties

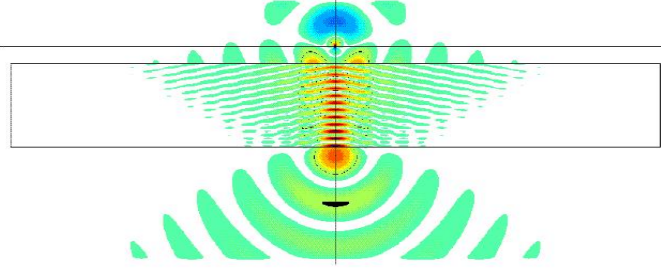
- Superprism effect (negative index of refraction) associated with EBGs (electromagnetic bandgaps) Notomi, Kosaka, ...




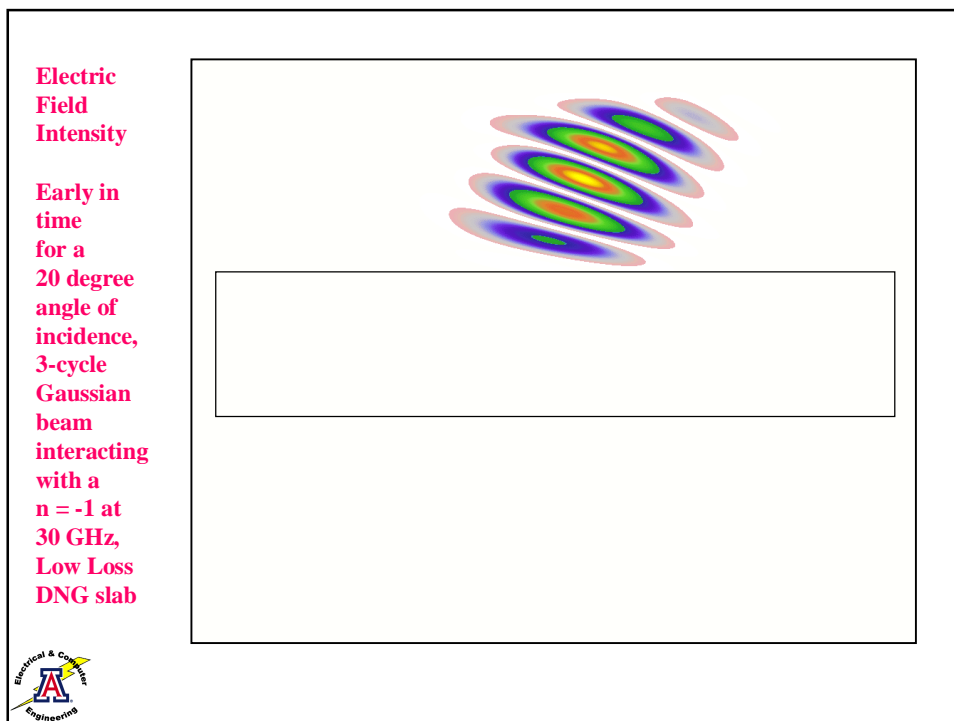
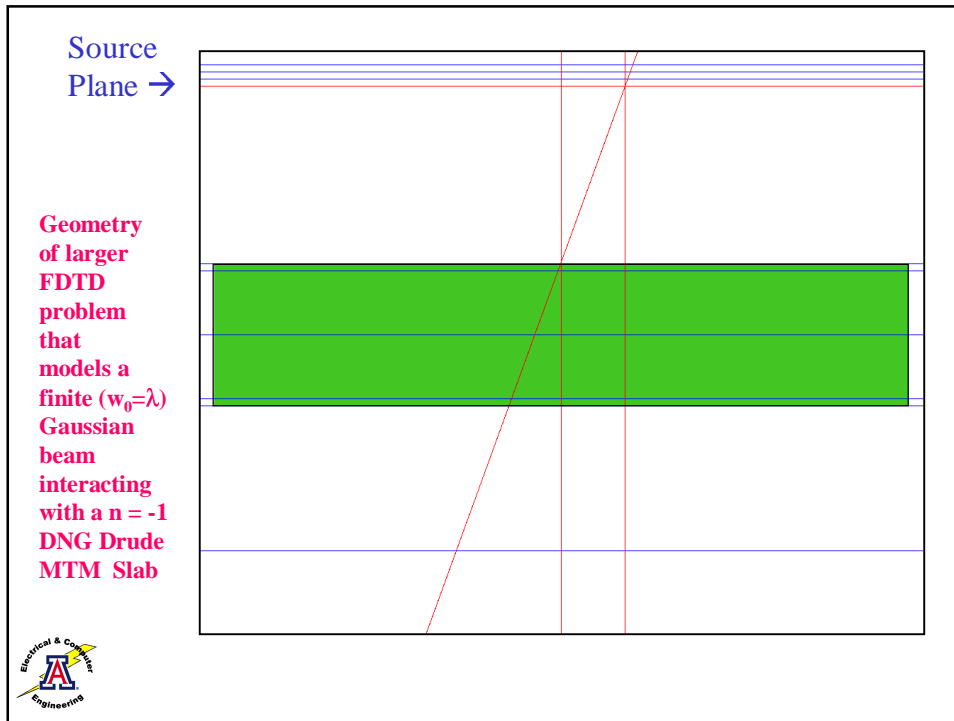
Electric field intensity at one time over FDTD simulation space

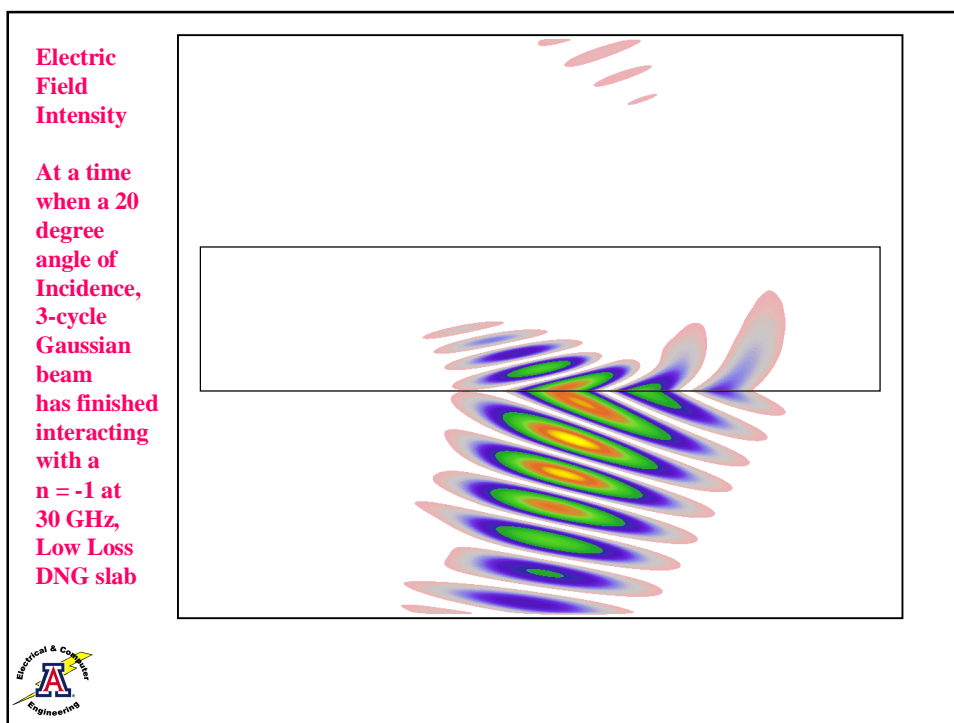
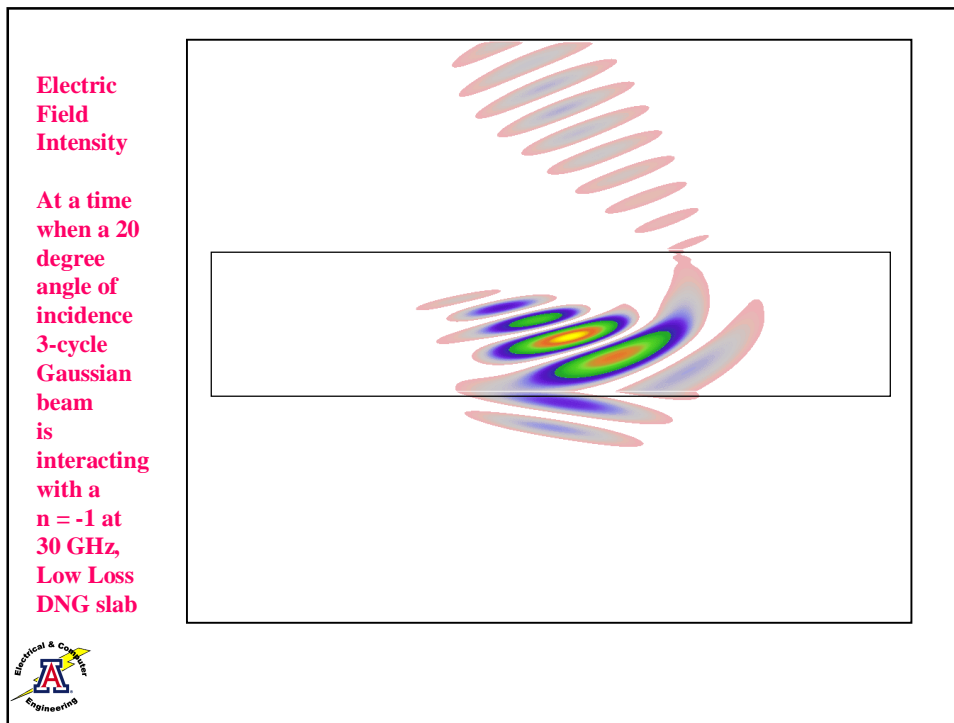


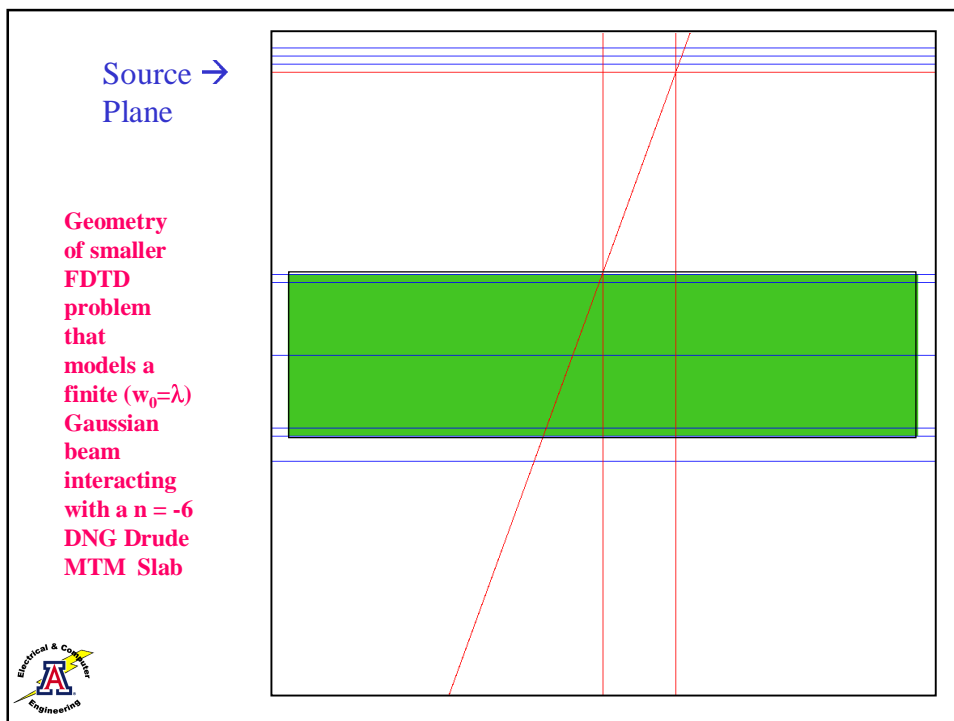
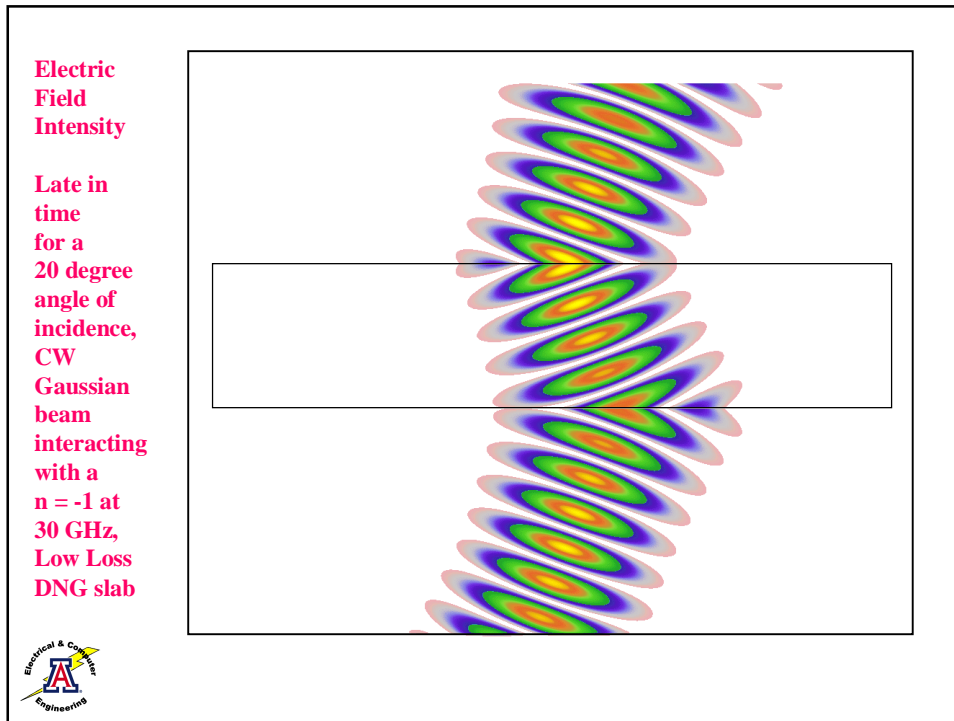
Poynting's vector amplitude at one time over FDTD simulation space

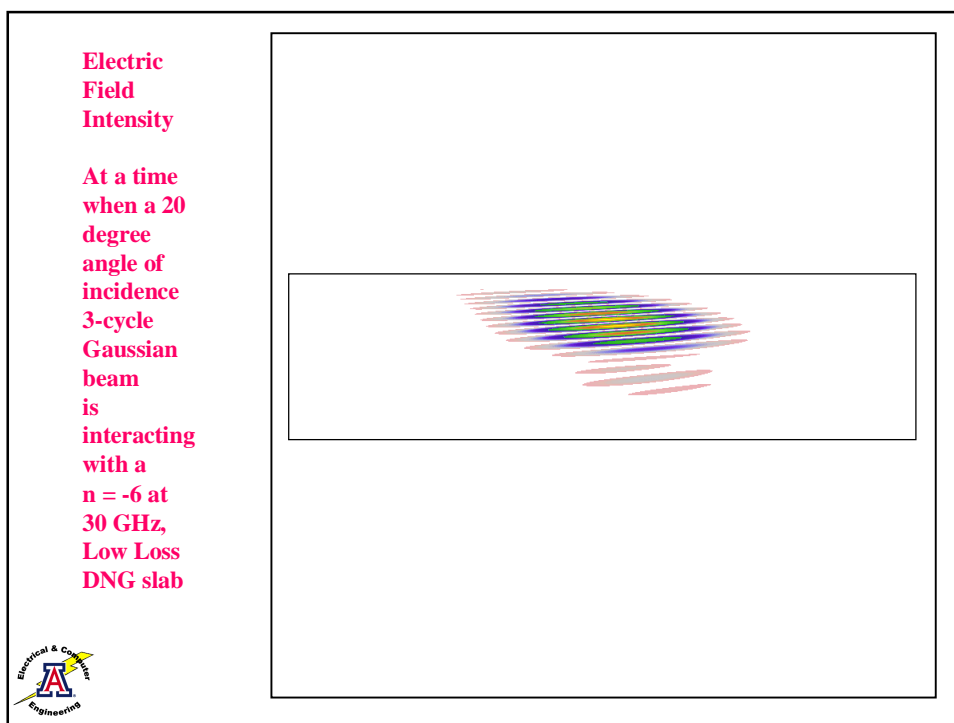


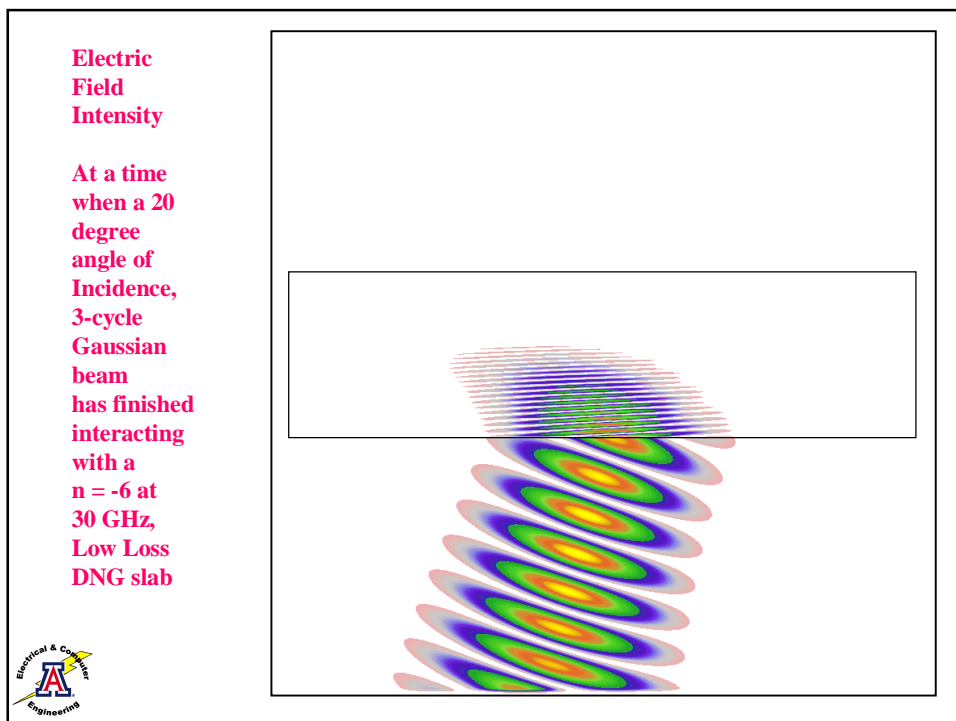
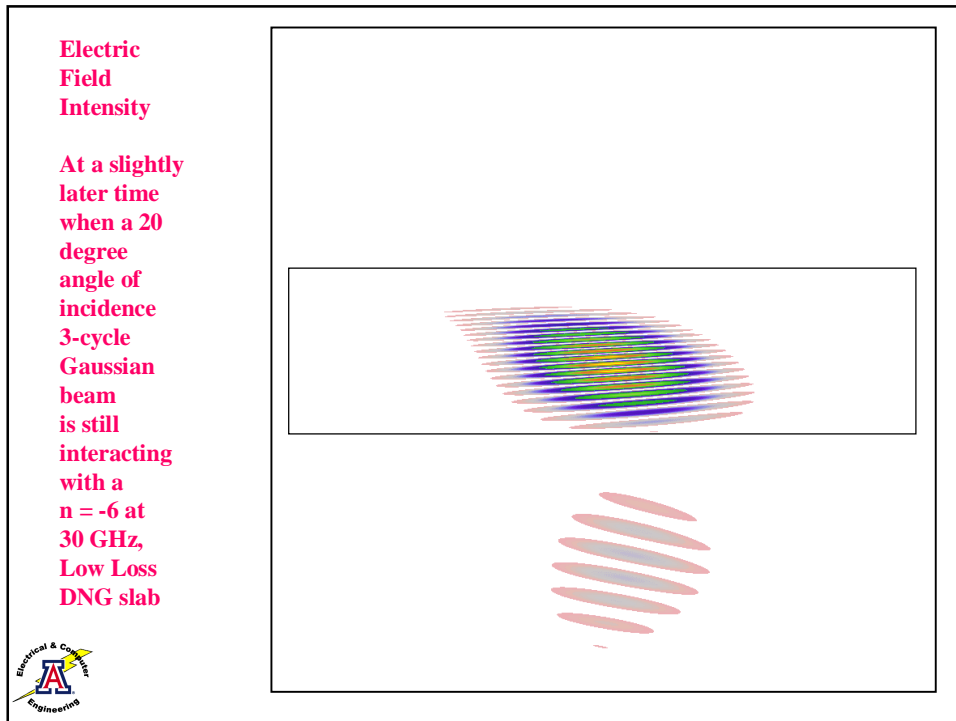


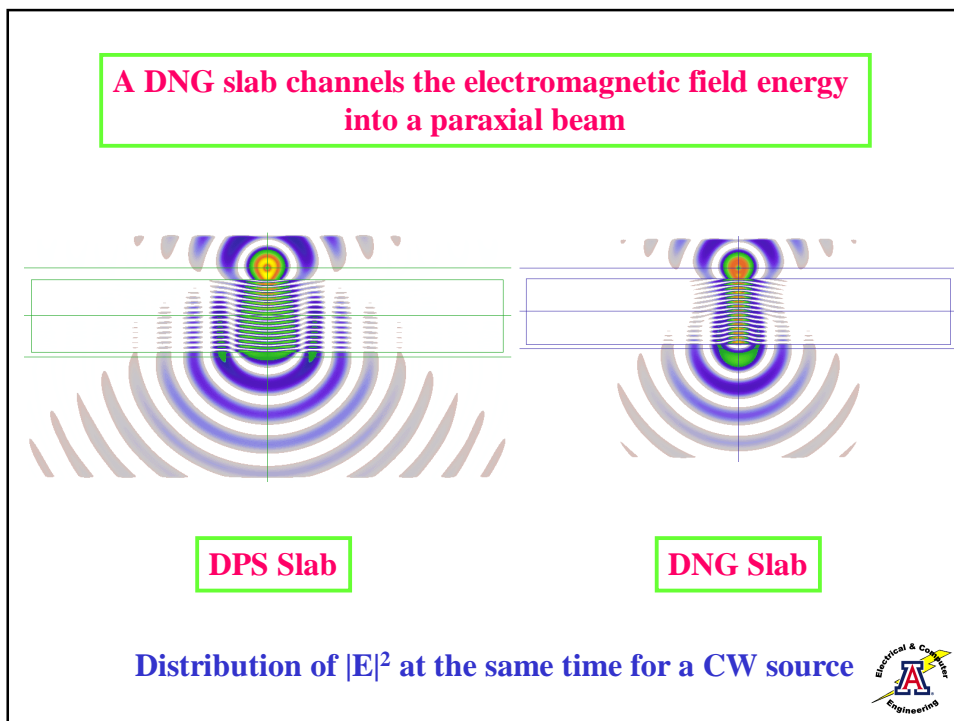
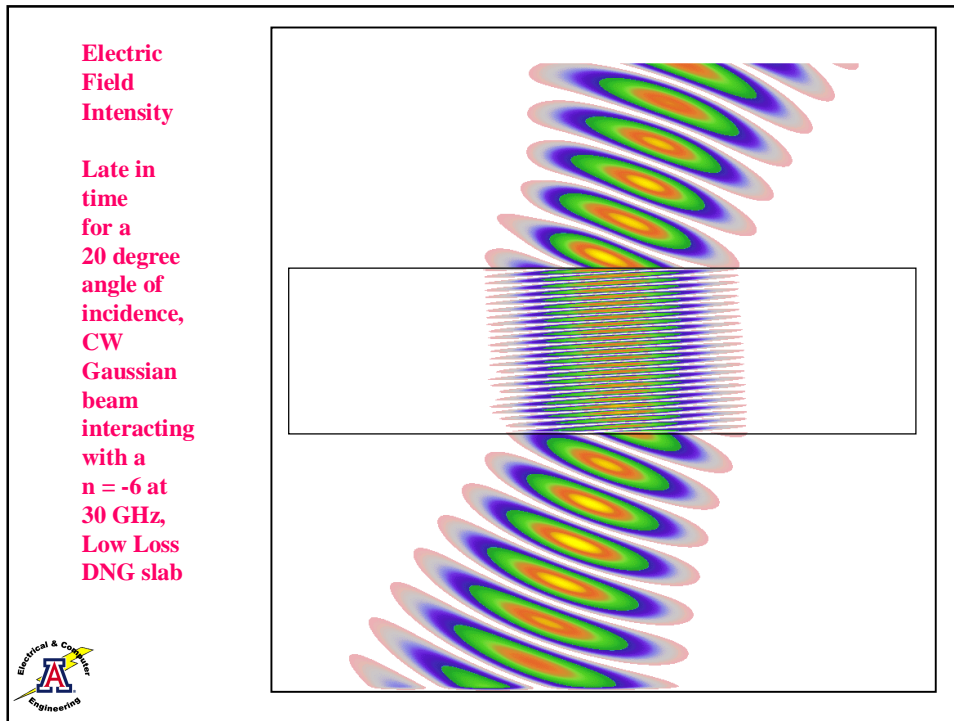












Matched DNG medium could lead to phase compensation techniques and devices

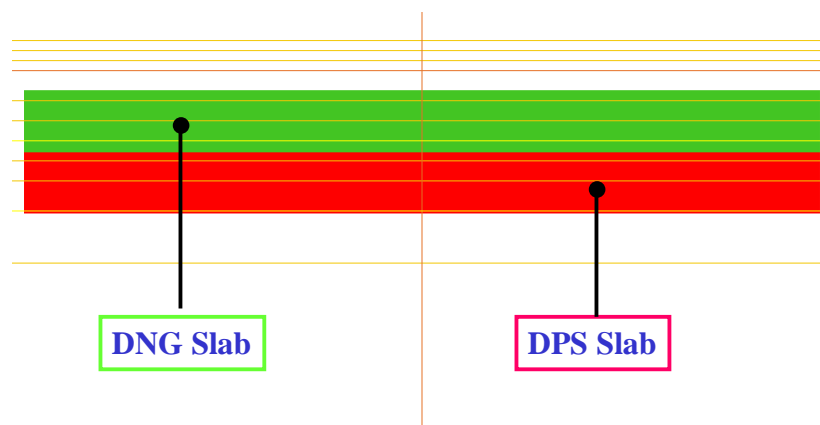
Matched media: $Z = Z_0$ so there are no reflections

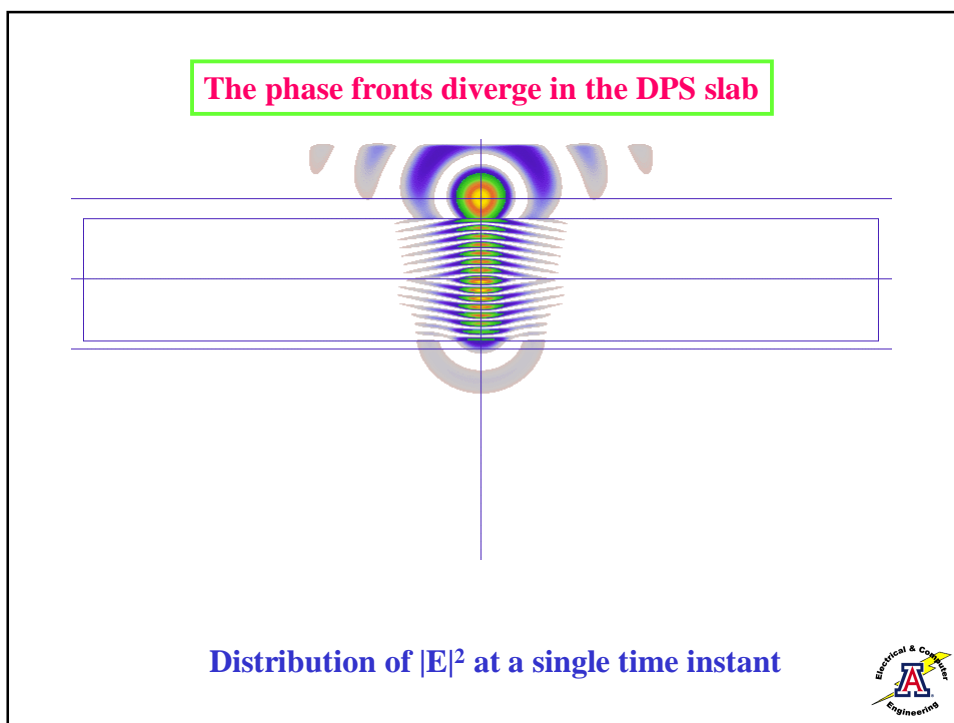
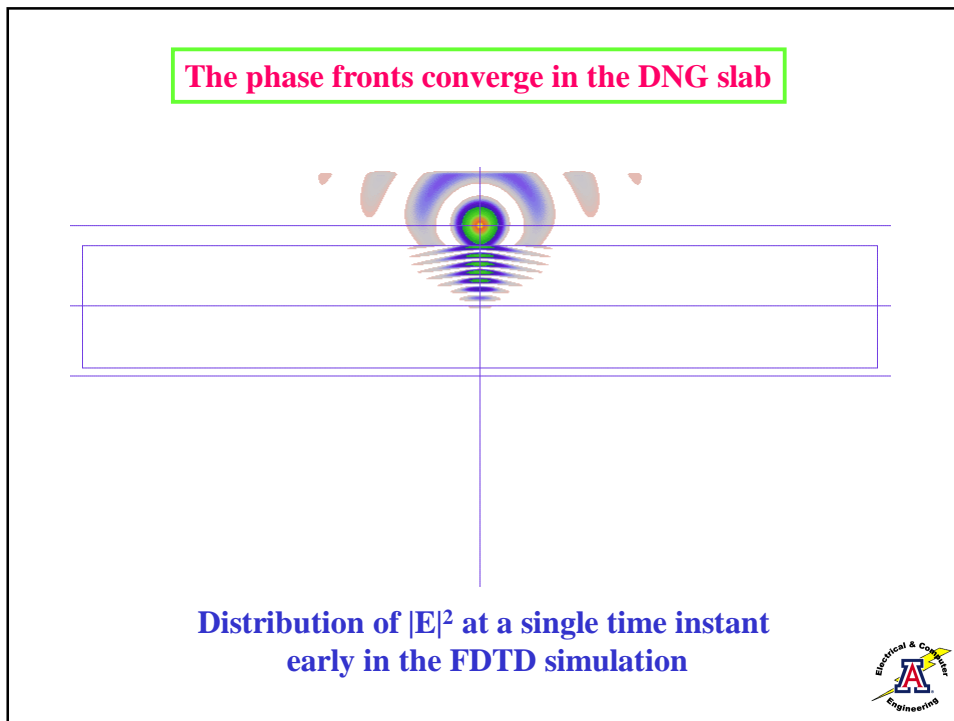
$$R = (Z - Z_0) / (Z + Z_0) = 0$$

Negative Phase: A DNG slab combined with a device that produces a positive phase shift, could lead to a zero phase point at the output of the combined device-slab

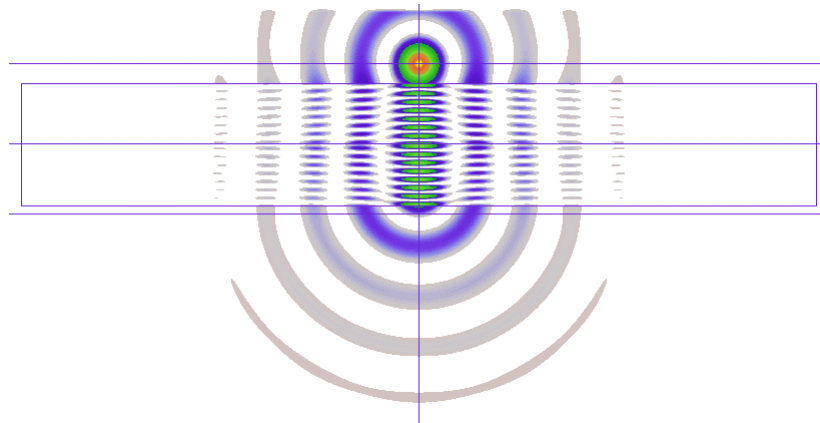


DNG slabs can be used to achieve phase front compensation





The phase fronts are planar near their exit from the DNG-DPS slab combination => 0° phase-shift delay line



**Distribution of $|E|^2$ at a single time instant
after steady state is achieved**



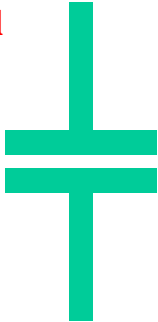
**Compact metamaterials having negative index of refraction
have been designed, fabricated and tested experimentally**

- **All structures constructed with Rogers Corporation
5880 Duroid ($\epsilon_r = 2.2$, $\mu_r = 1.0$, $\tan d = 0.0009$)
31 mil (100 mil = 2.54 mm) thick,
125 mil polyethylene spacers**
- **S-parameters measured with a free space measurement
system at X-band frequencies**
- **Experimental results confirm the realization of DNG MTMs
that are matched to free space**
- **Very good agreement between numerical and
experimental results**

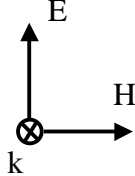


Electric dipole metamaterial

5 layers
31 x 24 elements

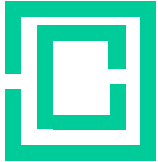


Unit element


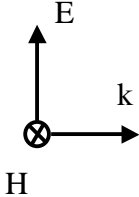


Magnetic dipole metamaterial

34 x 4 elements

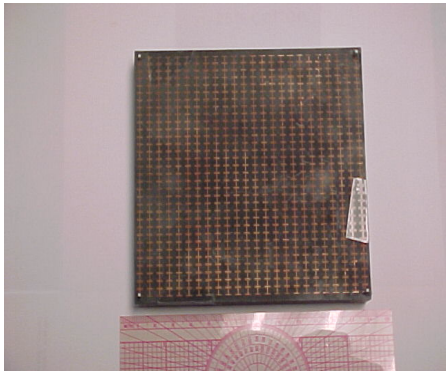


Unit element


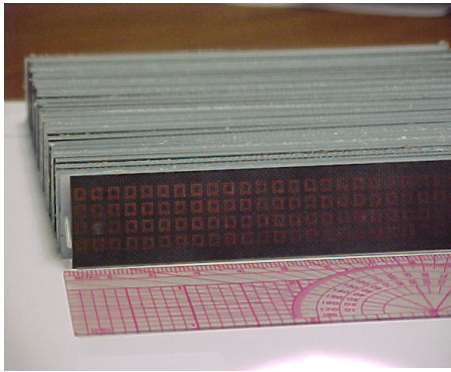


The first experiments only measured the orthogonal structure's components separately

Dipoles Only



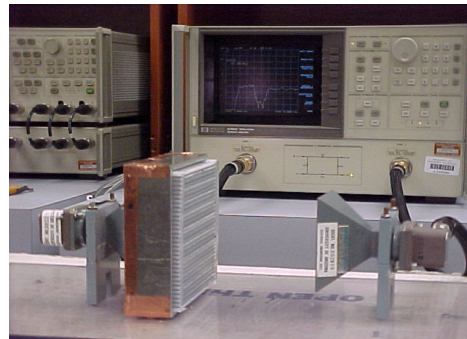
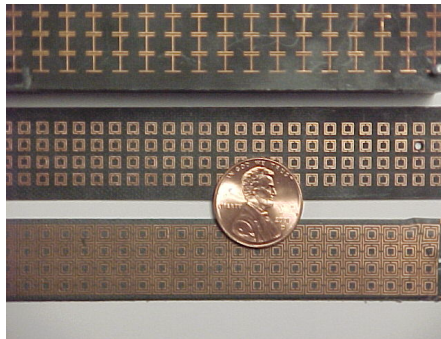
Split rings only



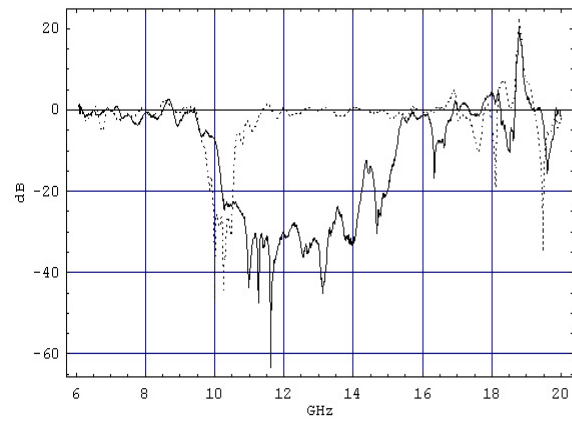
Second series of measurements were performed to confirm the HFSS simulation results showing the DNG medium effects for the planar structure

CLSs, SRRs, & composite planar structure

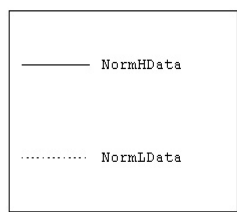
Free space measurement system

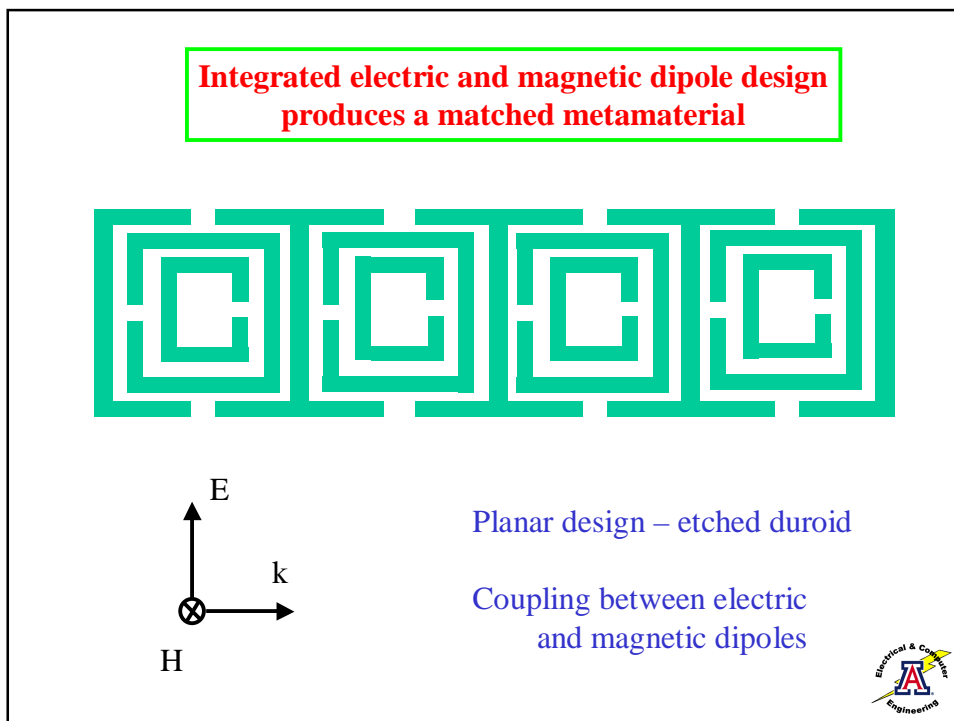
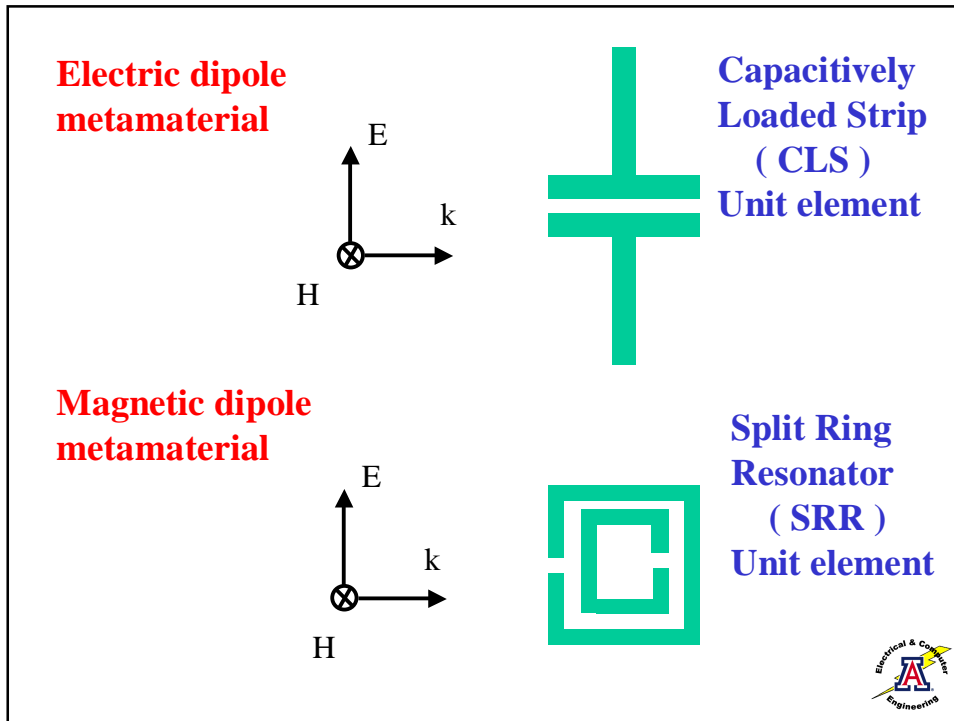


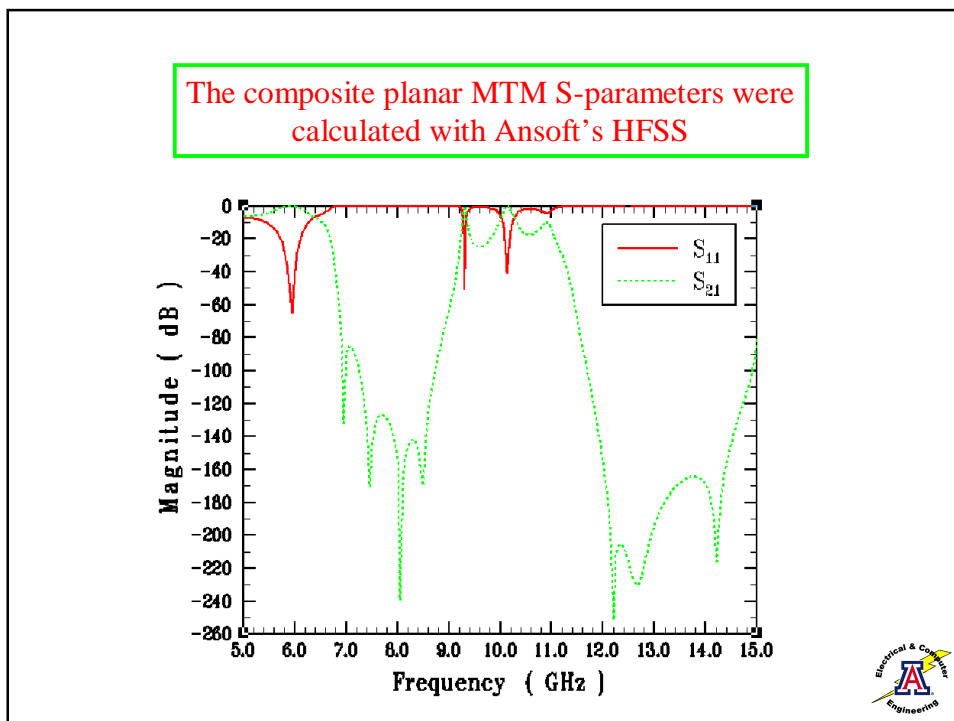
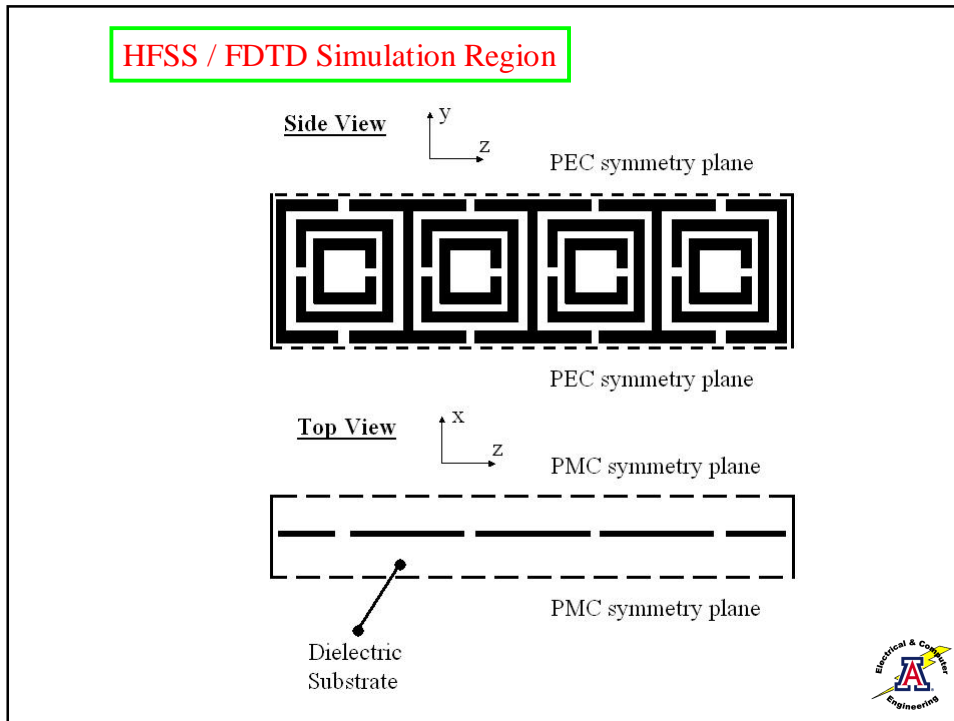
The S_{21} experimental data shows the predicted reflection bands for the electric and magnetic dipole metamaterials



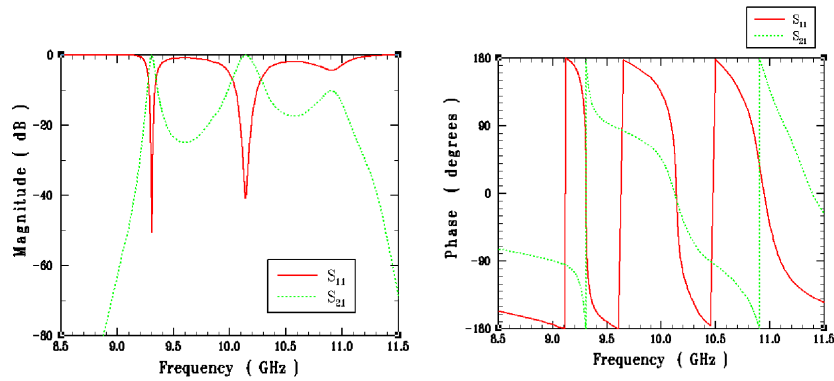
This is the measured data with the Thru data divided out.



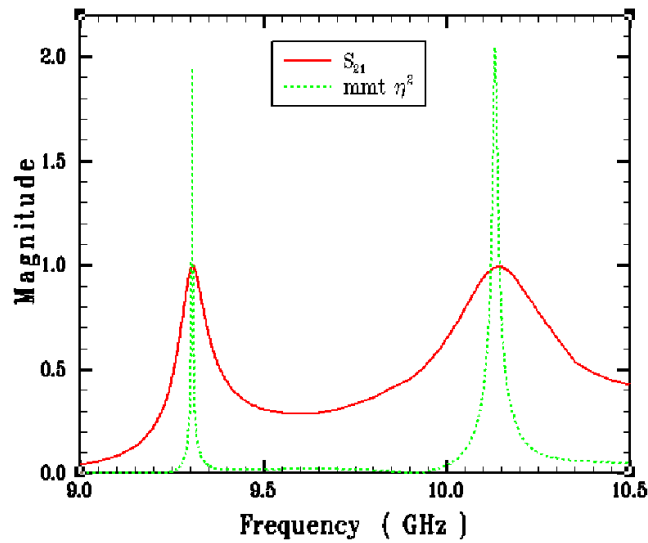




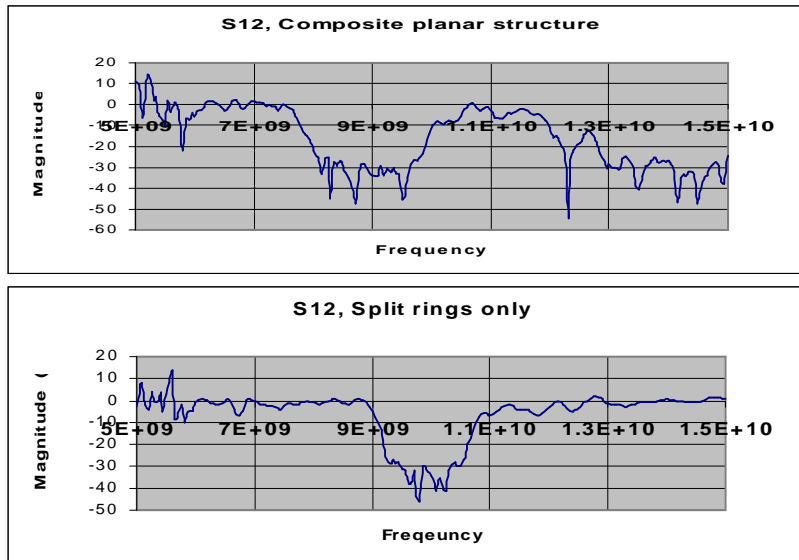
The effective permittivity and permeability were extracted from the HFSS S-parameter calculations



The composite planar MTM exhibits matching to free space in two frequency regions (and nearly three)



The S_{21} experimental data shows the predicted broad transmission band for the matched DNG medium



Effective permittivity and permeability parameters are commonly extracted from S-parameter (calculated/measured) values using the Nicolson, Ross, and Weir approach

$$\begin{aligned}
 V_1 &= S_{21} + S_{11} & k_0 &= \frac{\omega}{c} \\
 V_2 &= S_{21} - S_{11} \\
 X &= \frac{1 + V_1 V_2}{V_1 + V_2} & \sqrt{\epsilon_r \mu_r} &= \frac{\ln(Z)}{jk_0 d} \\
 Y &= \frac{1 - V_1 V_2}{V_1 - V_2} & \sqrt{\frac{\mu_r}{\epsilon_r}} &= \frac{1 + \Gamma}{1 - \Gamma} \\
 Z &= \exp(ikd) = X \pm \sqrt{X^2 - 1} & \mu_r &= \frac{1 + \Gamma \ln(Z)}{1 - \Gamma \ln(Z)} \\
 \Gamma &= Y \pm \sqrt{Y^2 - 1} & \epsilon_r &= \frac{1 - \Gamma \ln(Z)}{1 + \Gamma \ln(Z)}
 \end{aligned}$$

The effective permittivity and permeability parameters were extracted from the S-parameter values using a modified version of the Nicolson, Ross, and Weir approach

$$\exp(ikd) = \frac{V_1 - \Gamma}{1 - \Gamma V_1}$$

$$\Gamma = \frac{\exp(ikd) - V_2}{1 - V_2 \exp(ikd)}$$

$$1 - \exp(ikd) = \frac{(1 - V_1)(1 + \Gamma)}{1 - \Gamma V_1}$$

$$\sqrt{\frac{\mu_r}{\epsilon_r}} = \frac{1 - \exp(ikd) \frac{1 - V_2}{1 + V_2}}{1 + \exp(ikd) \frac{1 - V_2}{1 + V_2}}$$

$$\sqrt{\epsilon_r \mu_r} \approx \frac{1}{jk_0 d} \frac{(1 - V_1)(1 + \Gamma)}{1 - \Gamma V_1}$$

$$\mu_r \approx \frac{2}{jk_0 d} \frac{1 - V_2}{1 + V_2}$$

$$\epsilon_r \approx \left(\frac{k}{k_0}\right)^2 \frac{1}{\mu_r}$$

$$n = \sqrt{\epsilon_r} \sqrt{\mu_r}$$

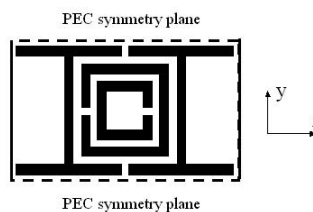
$$\epsilon_r \approx \mu_r - \frac{2}{jk_0 d} S_{11}$$

Enhanced result when $S_{11} \sim 0$

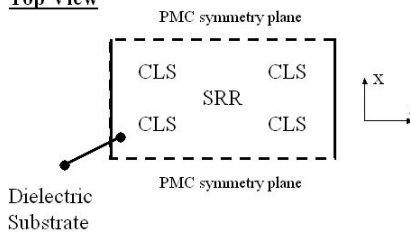


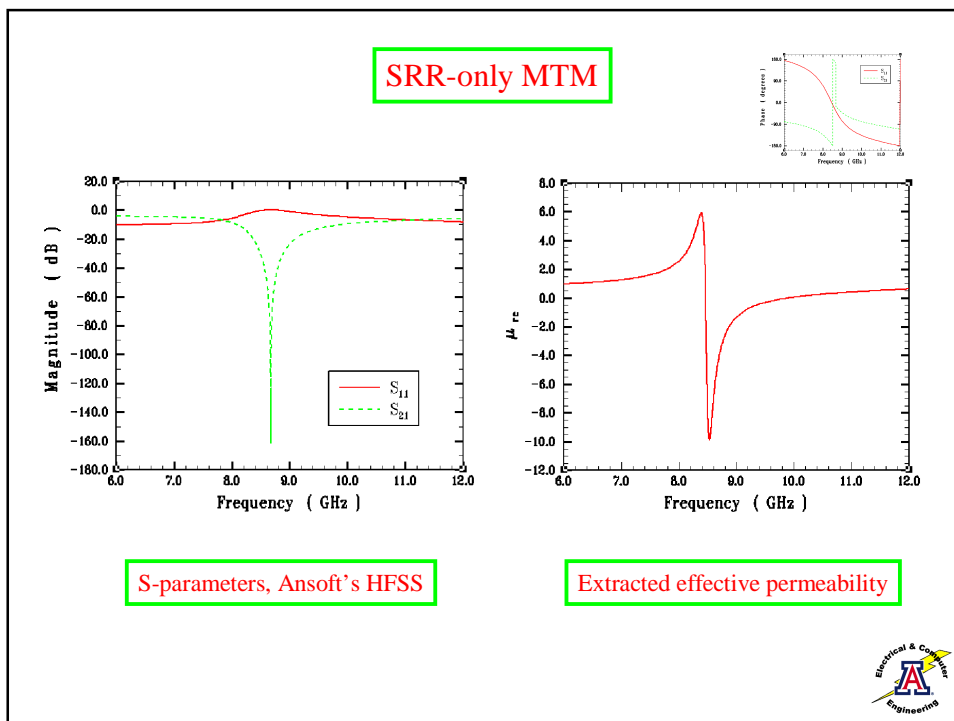
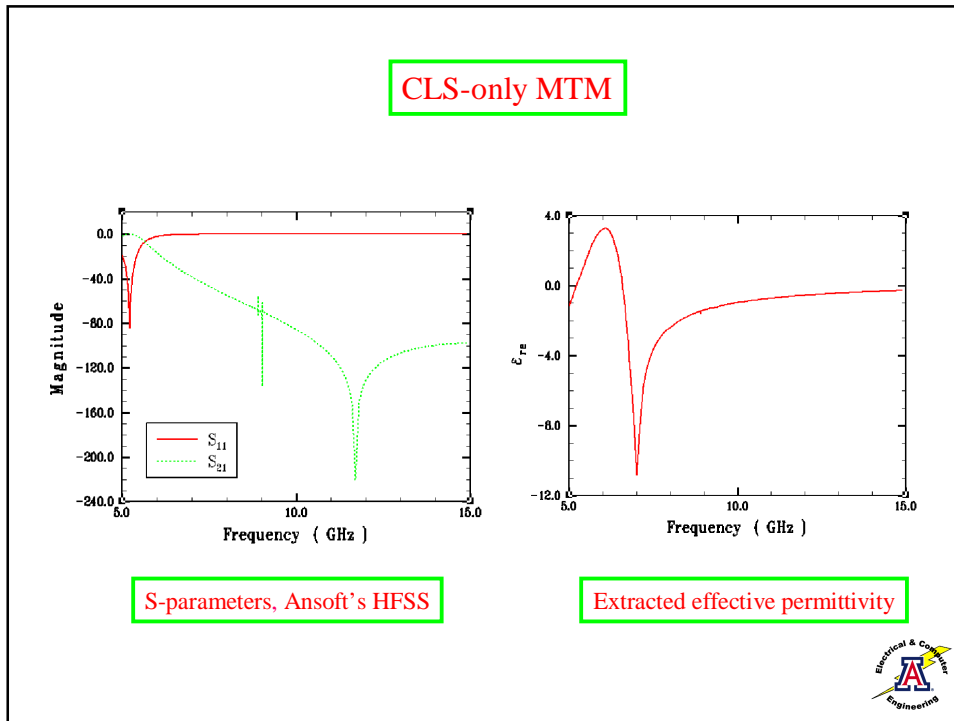
A simplified MTM was designed to isolate the electric and magnetic elements and the corresponding effects

Side View



Top View





The effective material responses raise interesting issues

$$\chi_e = \frac{\omega_p^2 \chi_L}{-\omega^2 + j\Gamma\omega + \omega_0^2} \Rightarrow \epsilon_r \approx 1 - \frac{\omega_p^2 \chi_L}{\omega^2 - \omega_0^2}$$

Lorentz Model

$$\epsilon_r \approx 1 - \frac{\omega_p^2}{\omega^2}$$

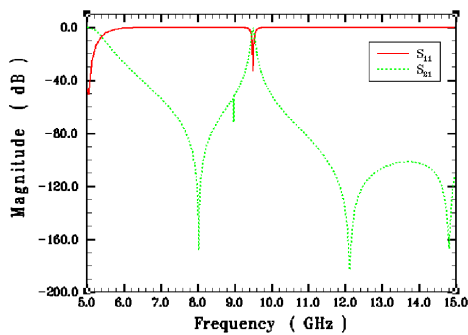
Drude Model

$$\chi_m = \frac{-\chi_\gamma \omega^2 + j\chi_\beta \omega_p \omega + \chi_\alpha \omega_p^2}{-\omega^2 + j\Gamma\omega + \omega_0^2} \Rightarrow \mu_r \approx 1 - \frac{|\chi_L| \omega^2}{\omega^2 - \omega_0^2}$$

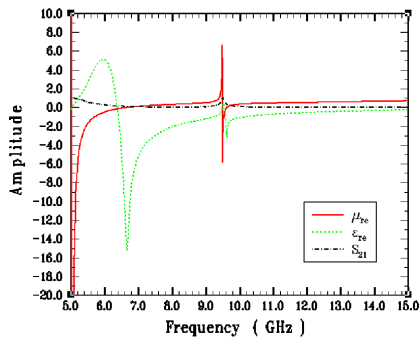
Two Time Derivative Lorentz Model



Composite MTM structure



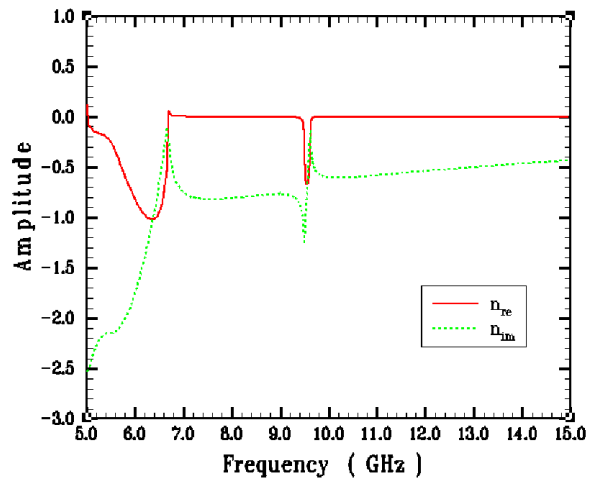
S-parameters, Ansoft's HFSS



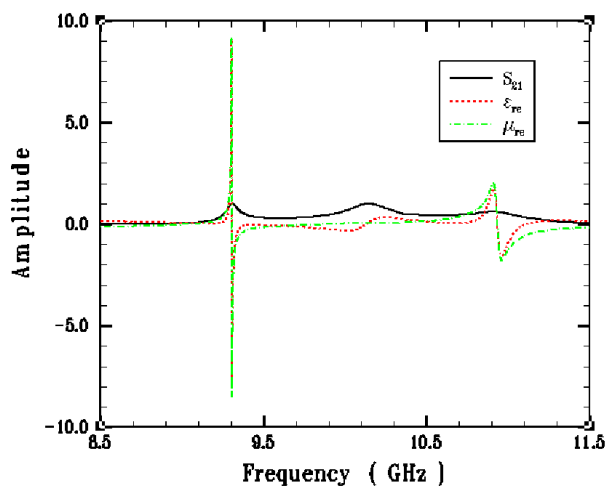
Extracted effective permittivity and permeability

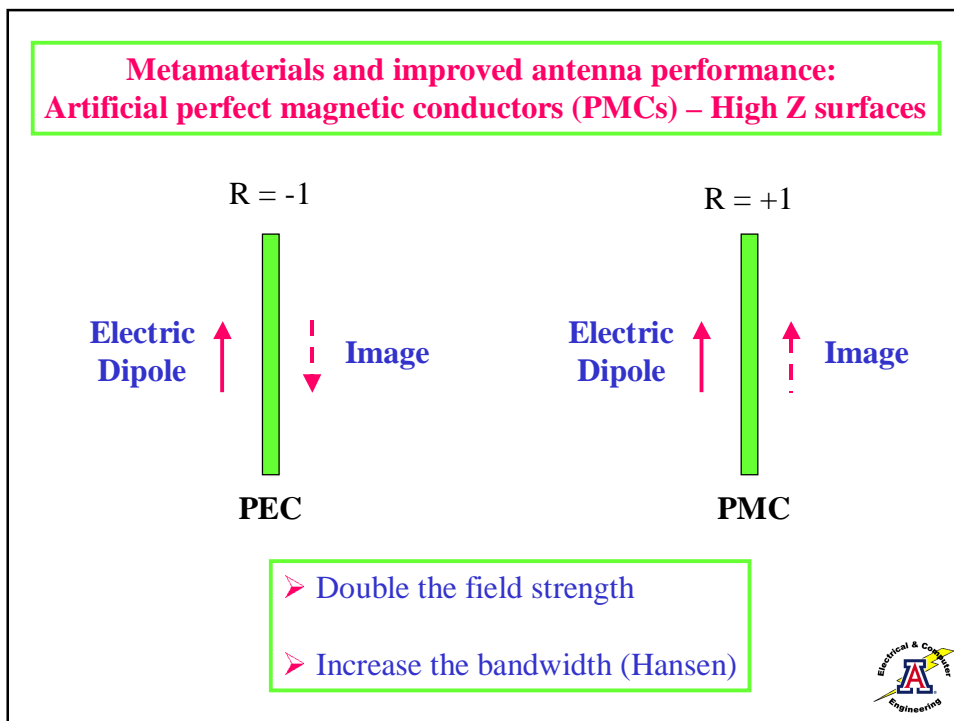
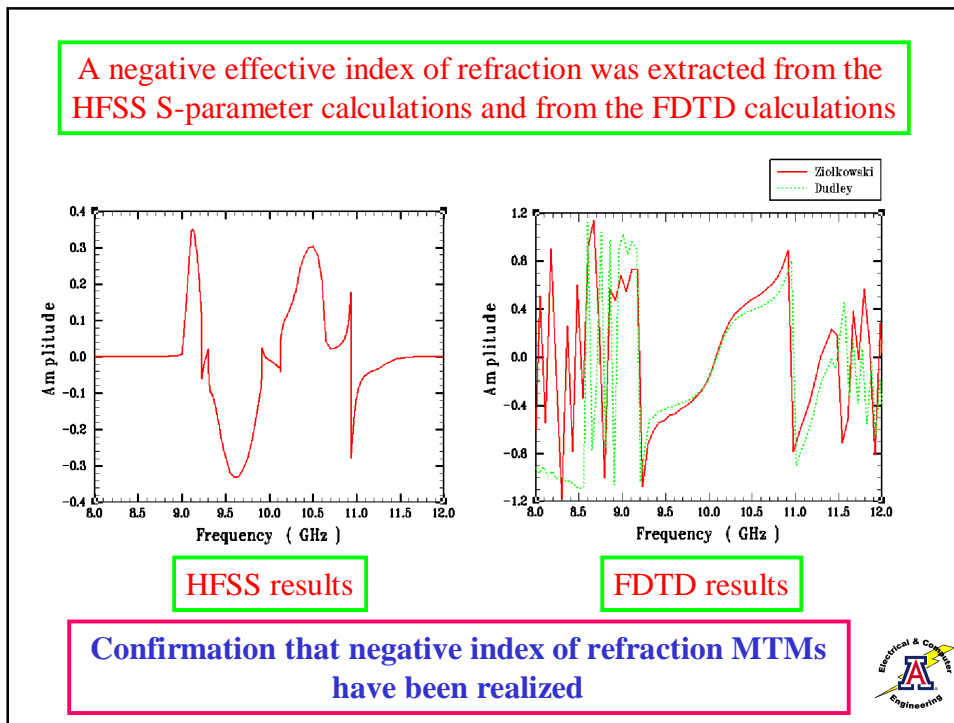


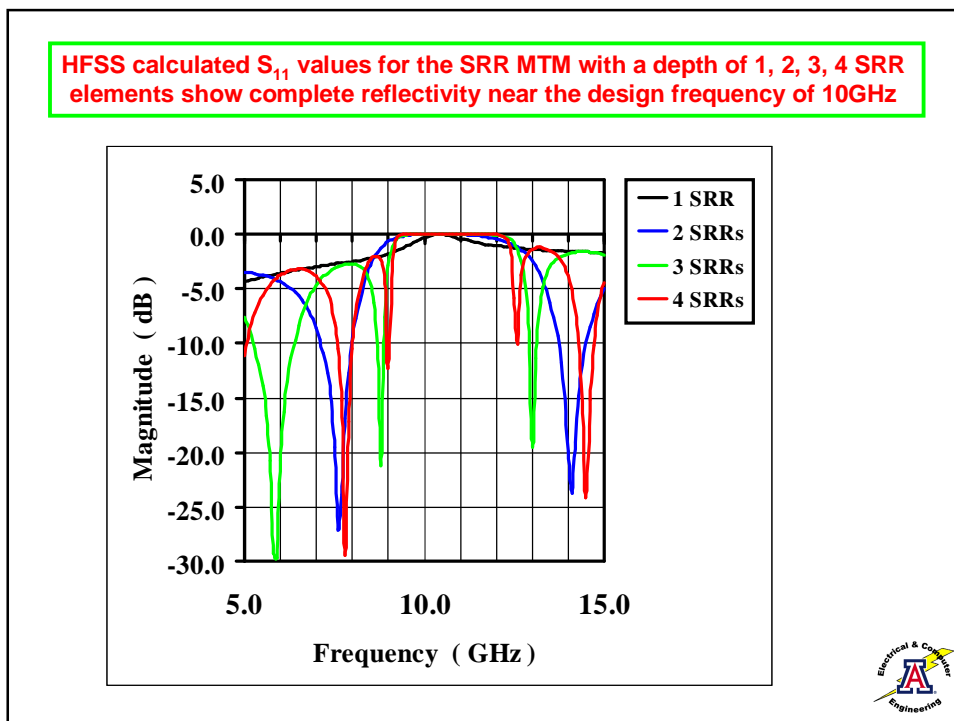
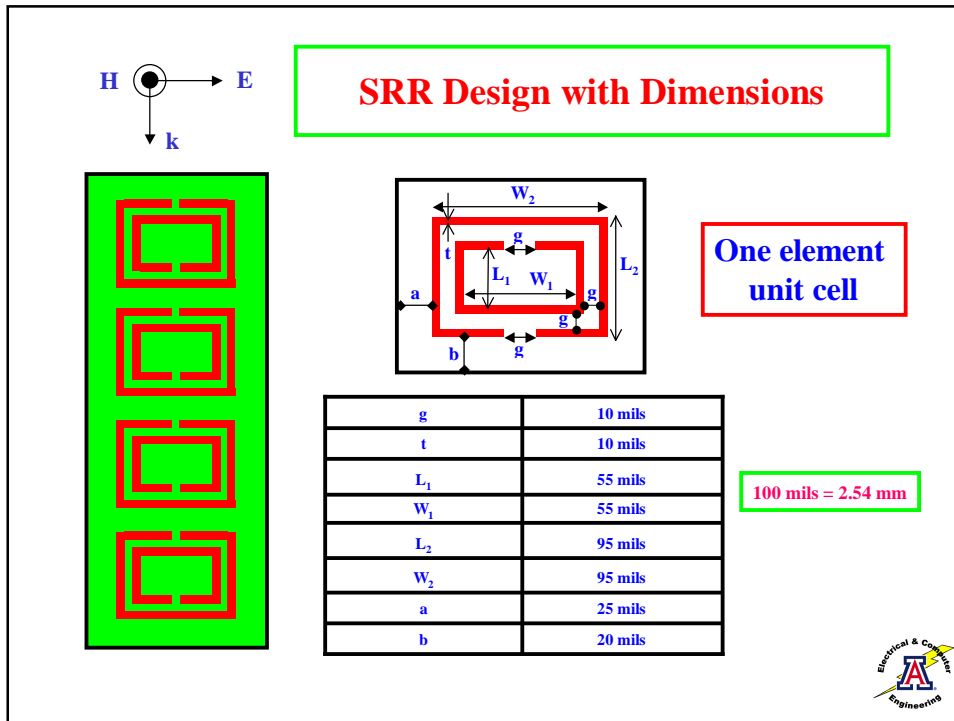
The effective index of refraction of the simplified MTM is negative in the frequency region where matching occurs



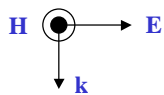
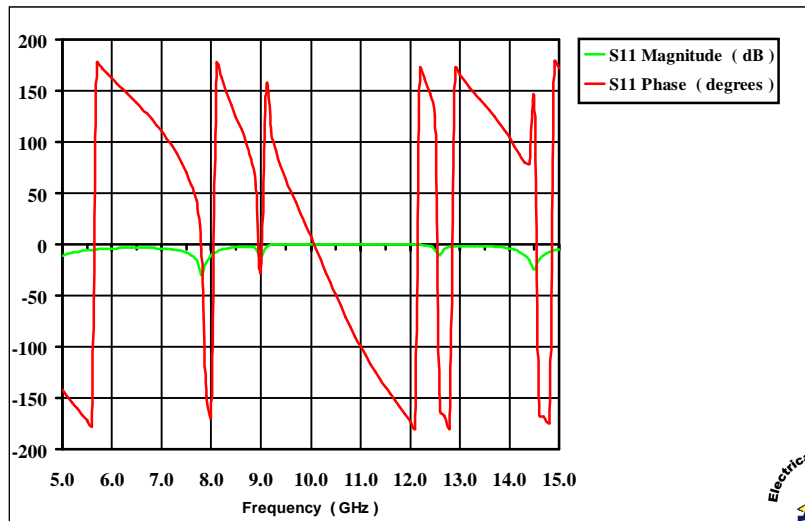
The effective permittivity and permeability of the composite planar MTM were extracted from the HFSS S-parameter calculations



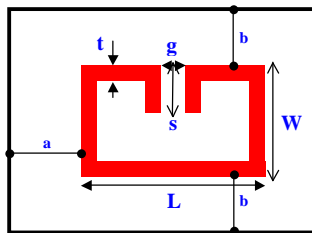
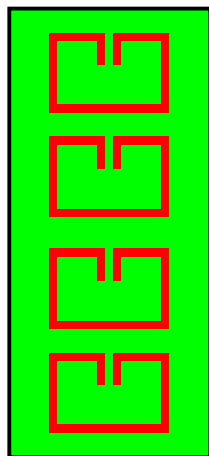




HFSS calculated S_{11} values for the SRR MTM with a depth of 4 SRR elements show that it acts as a PMC near the design frequency of 10GHz



CLL Design with Dimensions

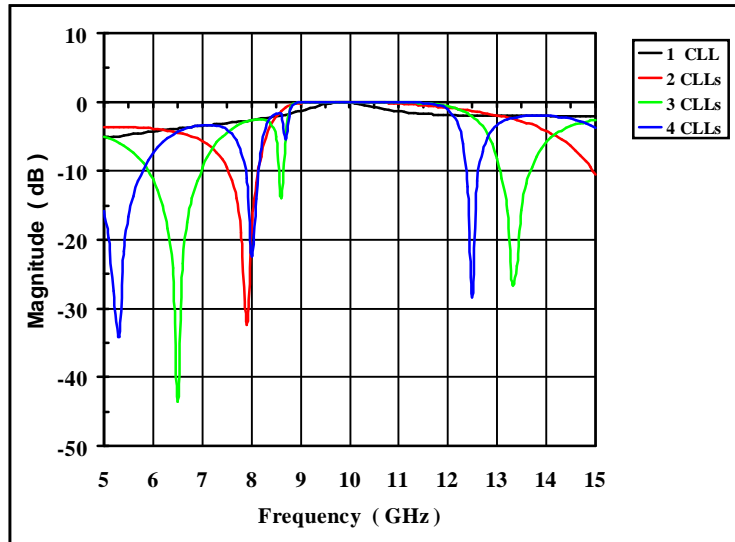


One element unit cell

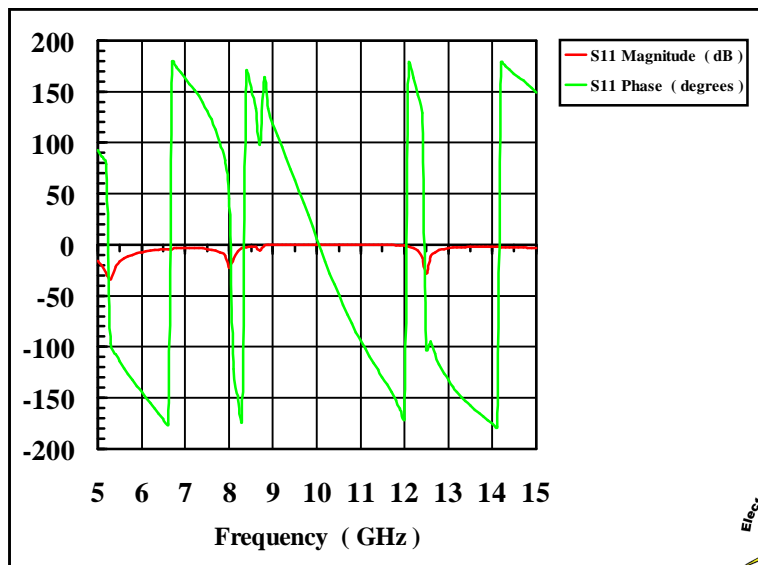
t	10 mils
$G2$	10 mils
s	17.2 mils
W	80 mils
L	120 mils
a	40 mils
b	40 mils

100 mils = 2.54 mm

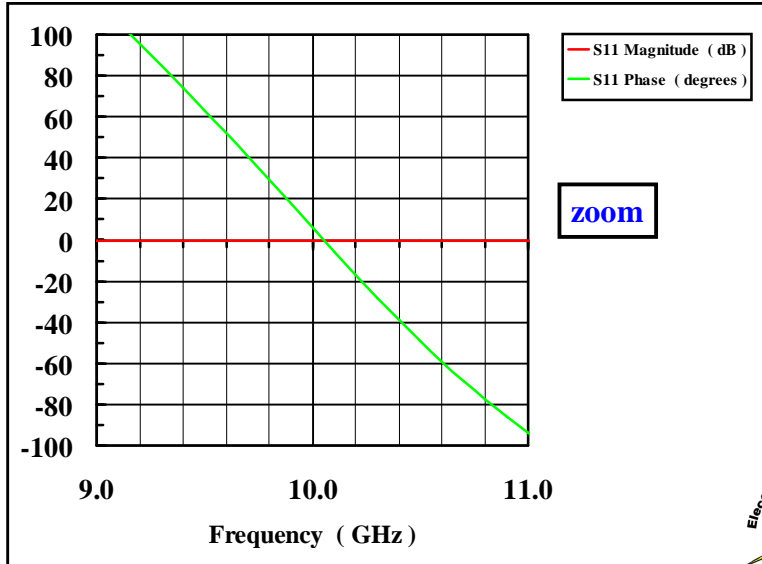
HFSS calculated S_{11} values for the SRR MTM with a depth of 1, 2, 3, 4 CLL elements show complete reflectivity near the design frequency of 10GHz



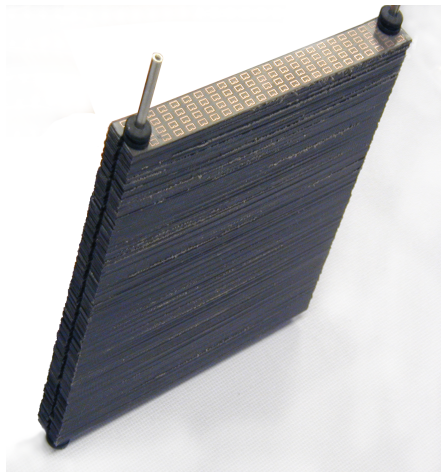
HFSS calculated S_{11} values for the CLL MTM with a depth of 4 CLL elements show that it acts as a PMC near the design frequency of 10GHz



HFSS calculated S_{11} values for the CLL MTM with a depth of 4 CLL elements show that it acts as a PMC near the design frequency of 10GHz



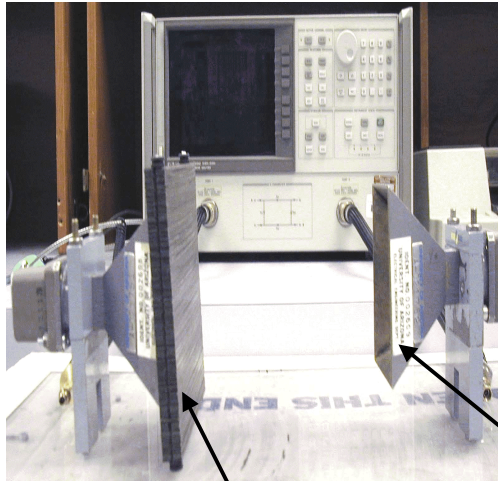
“Final” CLL Metamaterial Structure



➤ “Precisely cut, aligned” and assembled of 161 strips



The CLL MTM was measured with a free space measurement system



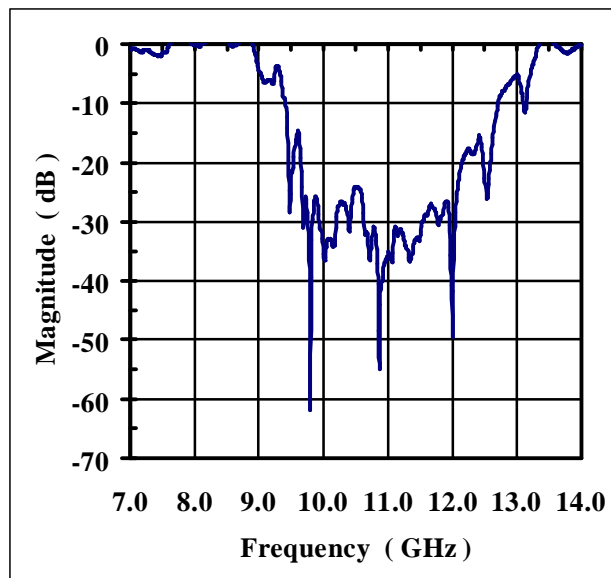
- Measure the CLL MTM with its designed orientation
- Measure the CLL MTM with a 90° rotation
- HP 8720C network analyzer to measure the S-parameters

MUT

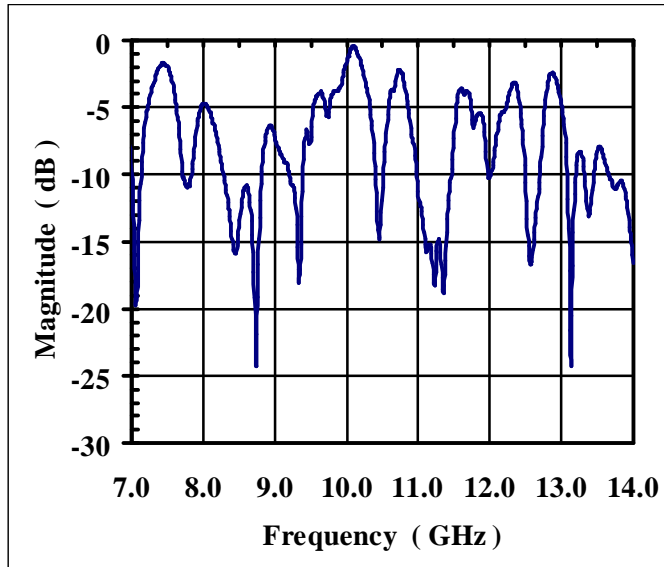
X-band rectangular horn



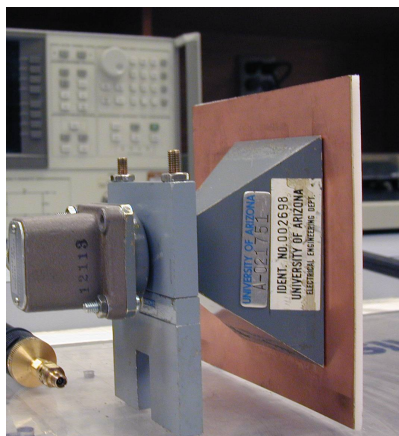
Measured magnitude of S_{21} for the CLL MTM slab



Measured magnitude of S_{11} for the CLL MTM slab

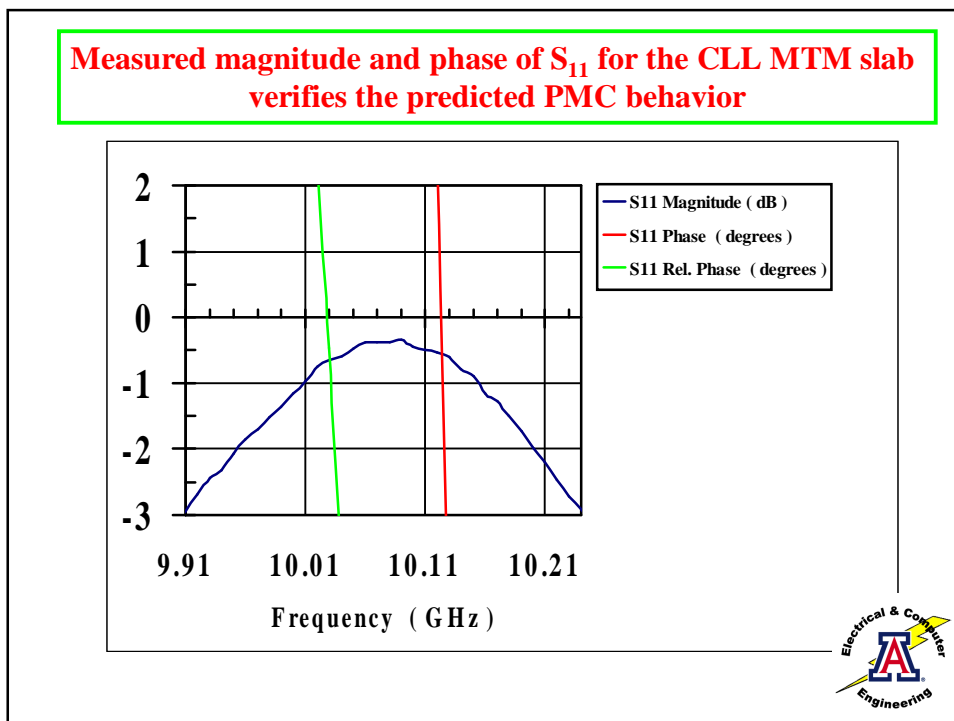
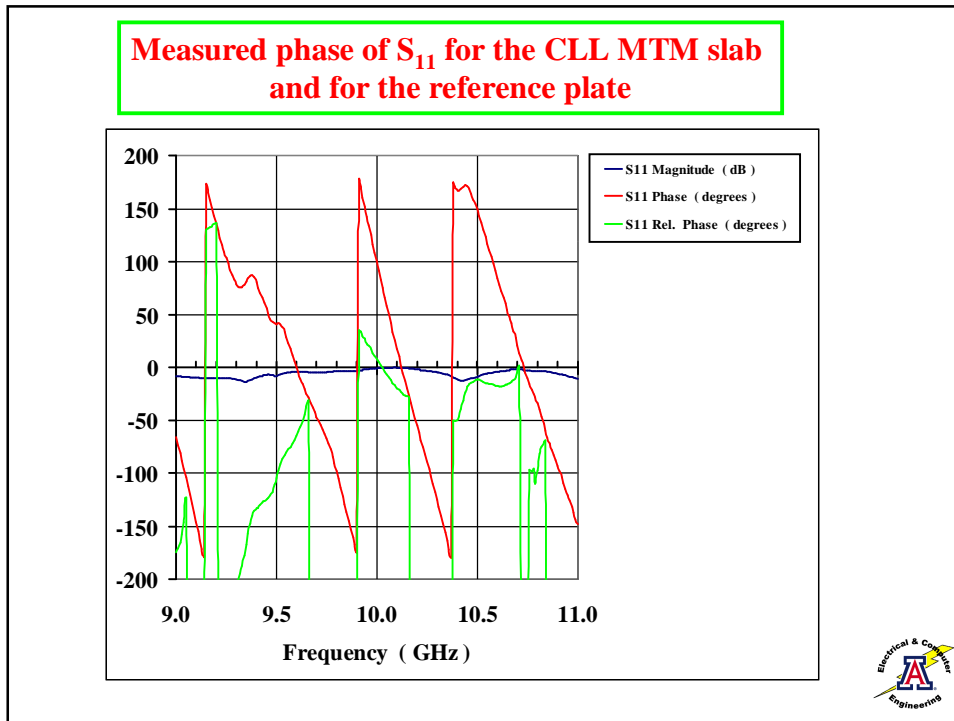


Phase Reference Plane measurement was achieved with a reference copper plate

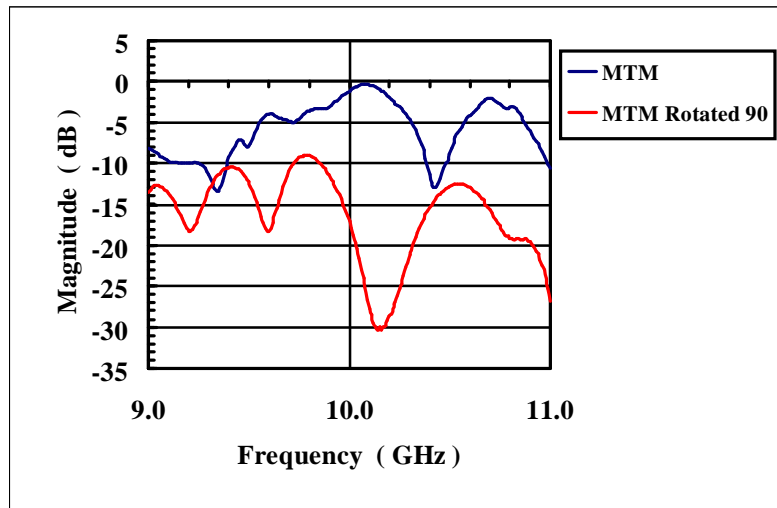


➤ A copper plate is placed over the mouth of the flange of the transmit antenna





Measured magnitude of S_{11} for the CLL MTM slab in 90° fixed rotation demonstrates the expected anisotropy



The relationships representing the power radiated by and the antenna Q of an electrically small antennas are well-known

Complex power for a small dipole in free space:

$$P = \int_0^{2\pi} \int_0^\pi \frac{1}{2} E_\theta H_\phi^* r^2 \sin \theta d\theta d\phi = \eta \frac{\pi}{3} \left| \frac{I_0 l}{\lambda} \right|^2 \left[1 - \frac{j}{(kr)^3} \right]$$

Antenna Q:

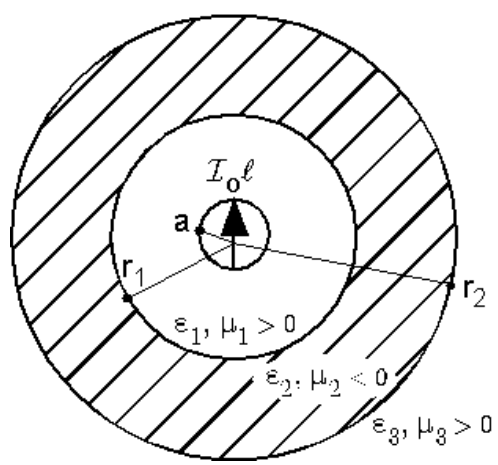
Changes sign for DNG medium

$$Q = \frac{1 + 2(ka)^2}{(ka)^3 [1 + (ka)^2]} \approx \frac{1}{(ka)^3} \quad \text{for } ka \ll 1$$



Can a DNG spherical shell be used to improve the power radiated by an electrically small antenna??

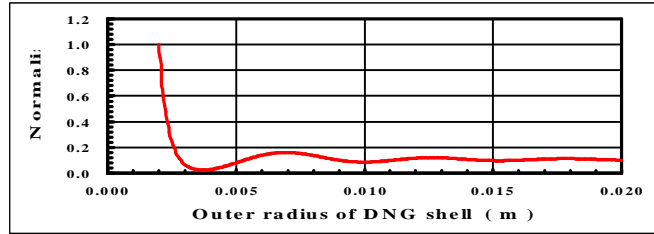
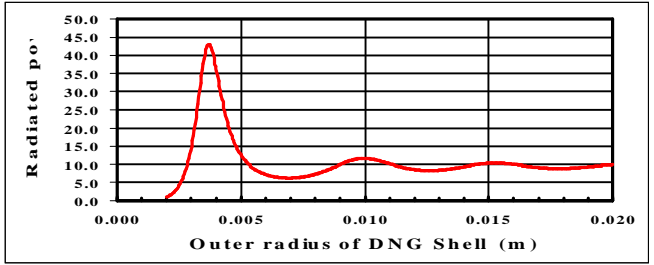
We have solved analytically the problem of an electrically small linear dipole antenna surrounded by a spherical shell of DNG material



Allison Kipple, PhD candidate

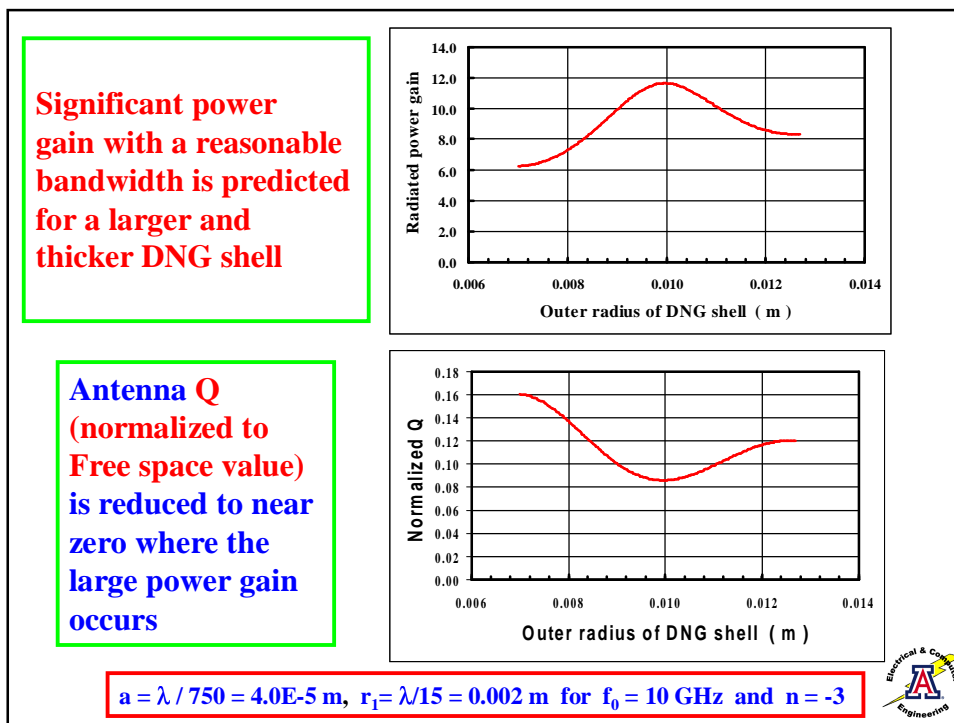
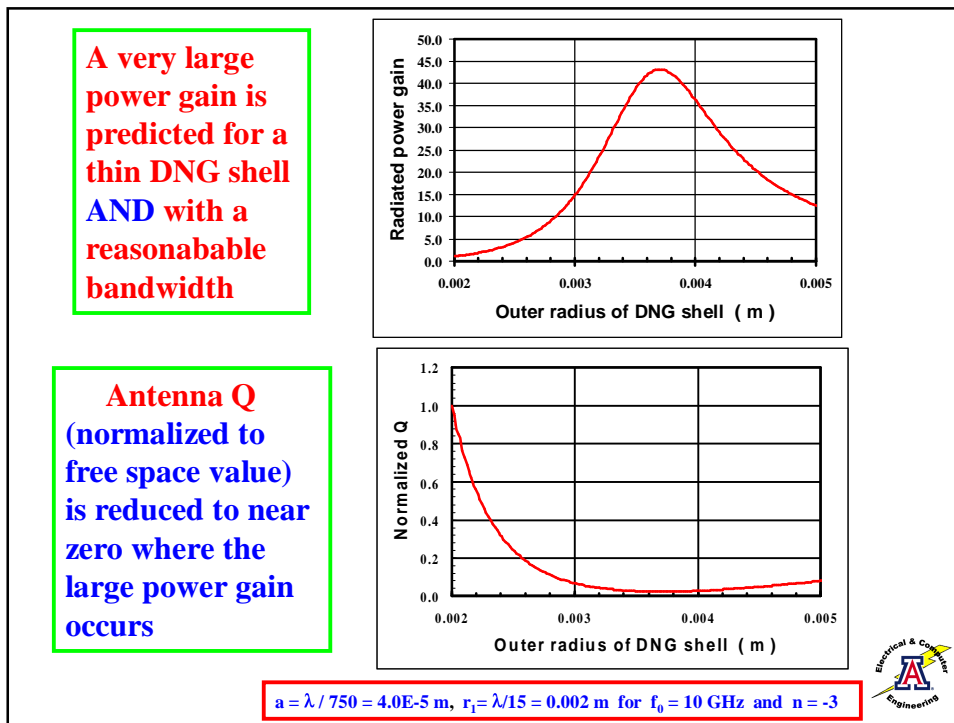


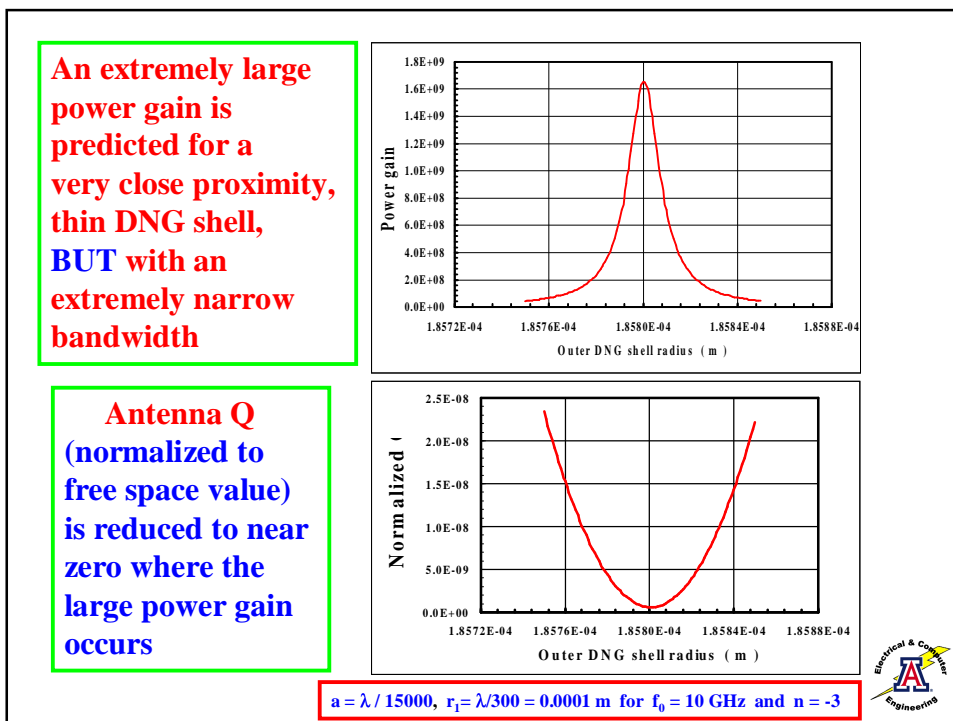
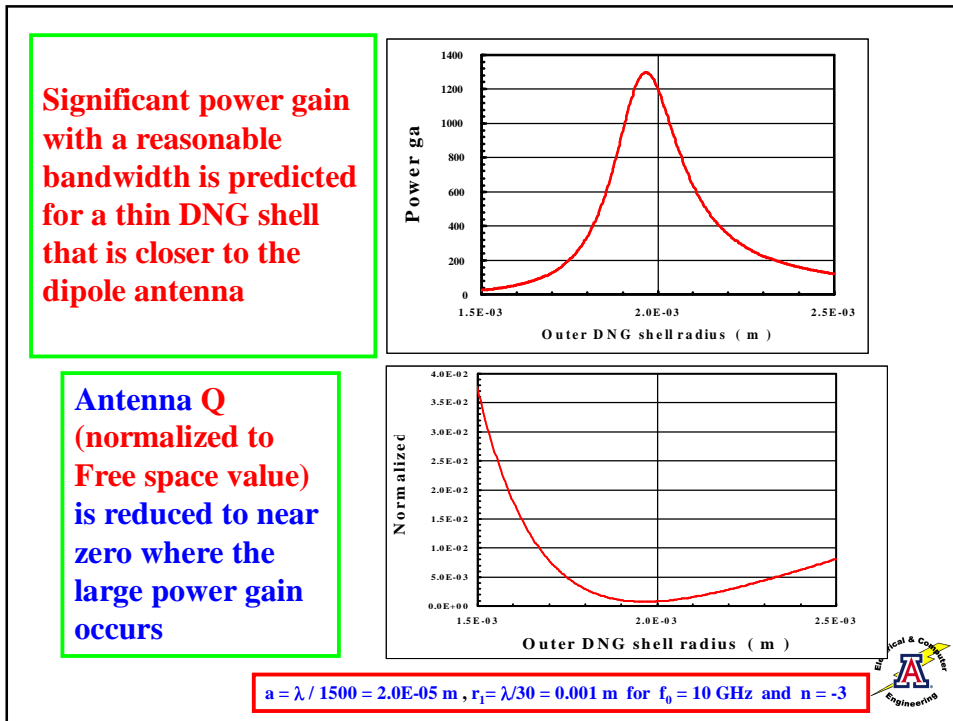
Calculating the radiated power normalized to the small dipole value as a function of the radius r_2 , we find that there several regions of where the radiated power is enhanced significantly and, correspondingly, where Q is reduced



$a = \lambda / 750 = 4.0E-5 \text{ m}$, $r_1 = \lambda / 15 = 0.002 \text{ m}$ for $f_0 = 10 \text{ GHz}$ and $n = -3$







Compact metamaterials having negative index of refraction have been designed, fabricated and tested experimentally

- **HFSS and FDTD simulators have been used to design several DNG ($\epsilon < 0$ and $\mu < 0$) metamaterials (MTMs)**
- **Extraction formula have been derived to determine the MTM's effective permittivity and permeability**
- **Experimental results confirm the realization of DNG MTMs that are matched to free space and have a negative index of refraction**
- **Several potential applications have been studied: Efficient Electrically Small Antennas (EESAs)**



Thank you for listening 😊

Special Issue of IEEE Antennas and Propagation on Metamaterials

R. W. Ziolkowski and N. Engheta, Guest Editors

Contributions due October 1, 2002

