# Quantum Criticality in the Bose-Fermi Kondo Model

Matthew Glossop and Kevin Ingersent

Bose-Fermi Kondo model (BFKM):

- Couples an impurity spin to both Fermi and Bose baths.
- Enters extended dynamical mean-field theory of the Kondo lattice.
- Previous impurity solutions have produced conflicting results.

New numerical renormalization-group treatment:

- Novel bath discretization and iterative solution procedure.
- Initial application to the Ising-symmetry BFKM—has interacting QCP in same universality class as the spin-boson model.
- Opens the way for decisive EDMFT studies of the Kondo lattice.

Supported by DMR-0312939

## Impurity Quantum Phase Transitions

[Recent review: M. Vojta, cond-mat/0412208]

- Impurities can undergo continuous (boundary) quantum phase transitions when coupled to noninteracting baths.
- Fermionic baths:

  suppression of the Kondo effect due to …

  - a pseudogap in the density of states;
  - competition between multiple conduction bands;
  - magnetic correlation between multiple impurities.
- Bosonic baths:

  (de)localization transition in a two-level system.
- Fermionic and bosonic baths:

  competition between Kondo screening and bosonic localization.

## Bose-Fermi Kondo Model

- Describes a local spin-half **S** coupled both to a conduction band and to three dissipative baths.
- Isotropic model has the Hamiltonian

$$H = \underbrace{J\mathbf{S}\cdot\mathbf{s} + H_{\text{band}}}_{H_{\text{Kondo}}} + \underbrace{g\mathbf{S}\cdot\mathbf{u} + H_{\text{bath}}}_{H_{\text{spin-boson}}}$$

where (for $\alpha = x, y, z$)

$$s_\alpha = \tfrac{1}{2}\sum_{\sigma,\sigma'} c_{0\sigma}^\dagger \sigma_{\sigma\sigma'}^\alpha c_{0\sigma'} \qquad u_\alpha = a_{0\alpha} + a_{0\alpha}^\dagger$$

$$H_{\text{band}} = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \qquad H_{\text{bath}} = \sum_{\mathbf{q},\alpha} \omega_{\mathbf{q}} a_{\mathbf{q}\alpha}^\dagger a_{\mathbf{q}\alpha}$$

- Anisotropic versions distinguish between

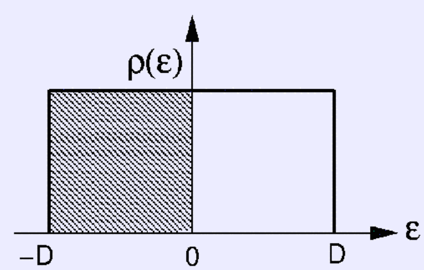
$J_z$ and $J_x = J_y = J_\perp$ $\qquad$ $g_z$ and $g_x = g_y = g_\perp$

## Bose-Fermi Kondo Model: Bath Spectra

$$H = J\mathbf{S}\cdot\mathbf{s} + H_{\text{band}} + g\mathbf{S}\cdot\mathbf{u} + H_{\text{bath}}$$

- Take a flat conduction band density of states:

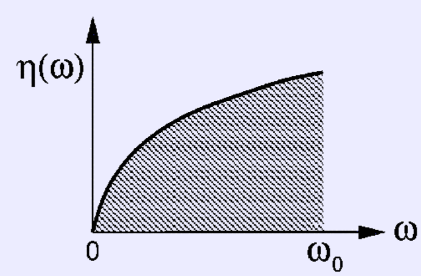
$$\rho(\varepsilon) = \rho_0 \quad \text{for} \quad |\varepsilon| < D$$




- Assume a power-law bosonic spectrum:

$$\eta(\omega) = K_0^2\,\omega_0(\omega/\omega_c)^s$$

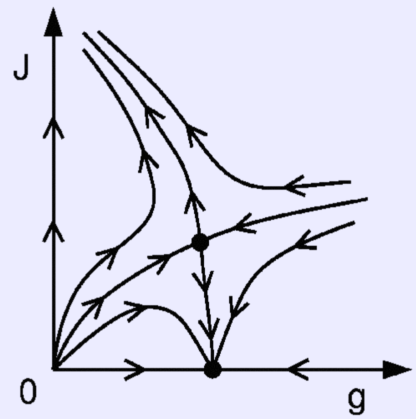
$$\text{for} \quad 0 \le \omega < \omega_0$$




- Dimensionless parameters: $\rho_0 J$ and $K_0 g$.

## Perturbative Solutions of the BFKM

- Model has been solved via expansion in $\epsilon = 1 - s$ [Si & Smith '99, Sengupta '00, Zhu & Si '02, Zárand & Demler '02].
- A quantum critical point separates Kondo and bosonic regimes.
- Critical point couplings $\rho_0 J_c$ and $K_0 g_c$ are of order $\epsilon$.
  Exception: Ising symmetry ($g_\perp = 0$), for which $\rho_0 J, K_0 g_c \sim O(1)$.
- At the QCP, $\chi_{\text{loc}}$ shows power laws in $\omega$ and $T$ with $\epsilon$-dependent exponents.

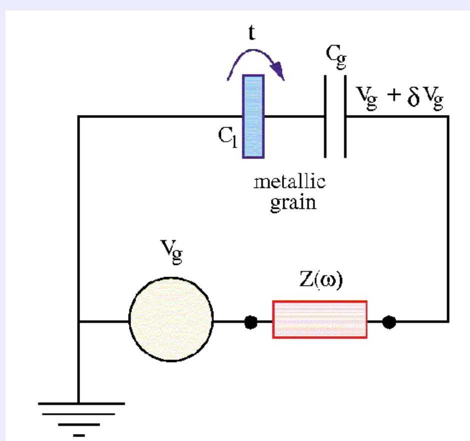



- Large-$N$ multichannel BFKM has also been studied [Zhu et al. '04].

## Application I: Coulomb Blockade in a Noisy Dot

[K. Le Hur, PRL 92, 196804 (2004)]

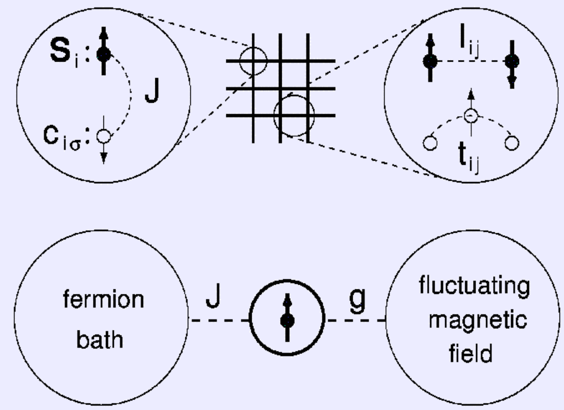
- Consider a metallic box …
  … in a strong magnetic field,
  … grounded via a point contact,
  … subject to a noisy gate voltage.
- Can map charge fluctuations onto an anistropic BFKM with …
  … an Ohmic bath ($s = 1$);
  … $J_\perp \propto t$;
  … $g_z \propto R = Z(\omega = 0)$.




- Predicts a Kosterlitz-Thouless transition at $R = R_c$.

## Application II: EDMFT Treatment of the Kondo Lattice

- Extended dynamical mean-field theory includes some spatial fluctuations [Si and Smith '96, Kajueter & Kotliar].
- Fermionic band accounts for local dynamical correlations.
- Dissipative baths represent a fluctuating magnetic field due to other local moments.
- Band and bath densities of states must be found self-consistently.




## What is the Nature of the QPT in EDMFT?

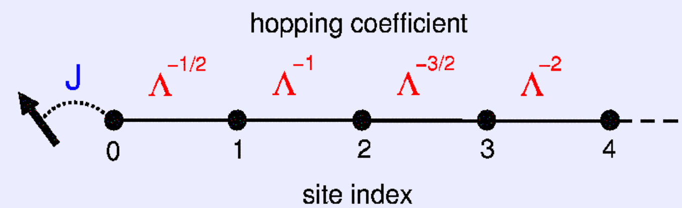
- EDMFT equations have been solved using various impurity solvers.
- $\epsilon$-expansion [Si et al. '01, '03] finds two types of QCP:
  - conventional spin-density-wave type;
  - locally critical QCP—reproduces some features of $CeCu_{6-x}Au_x$ and $YbRh_2Si_2$, but corresponds to $\epsilon = 1^-$.
- Quantum Monte Carlo yields conflicting results:
  - Anderson lattice has no locally critical QPT; transition is 1st order [Sun & Kotliar '03].
  - Kondo lattice has 1st order transition at $T > 0$, but a locally critical QCP at $T = 0$ [Grempel & Si, '03, Zhu et al. '04].
- To resolve this discrepancy, need nonperturbative $T = 0$ solutions.

## Numerical Renormalization-Group Method [Wilson '74]

- NRG replaces a continuum of fermionic states by a discrete set having energies $\varepsilon = \pm D, \pm D\Lambda^{-1}, \pm D\Lambda^{-2}, \ldots \quad (\Lambda > 1)$.
- Then the kinetic energy is converted to a tight-binding form:

$$H_{\text{band}} = \sum_{\sigma} \sum_{n=0}^{\infty} \Lambda^{-n/2} \left( c_{n\sigma}^{\dagger} c_{n-1,\sigma} + \text{h.c.} \right),$$

where only $c_{0\sigma}$ couples to the impurity.

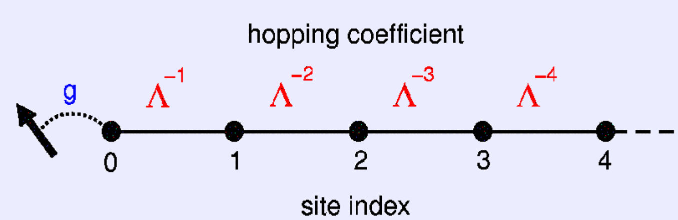



- The exponential decay of the hopping permits iterative solution via diagonalization of progressively longer chains.

## Discretizing a Bosonic Bath

- Can use the same energy discretization as for fermions.
- No negative-ω states ⇒ hopping decays faster:

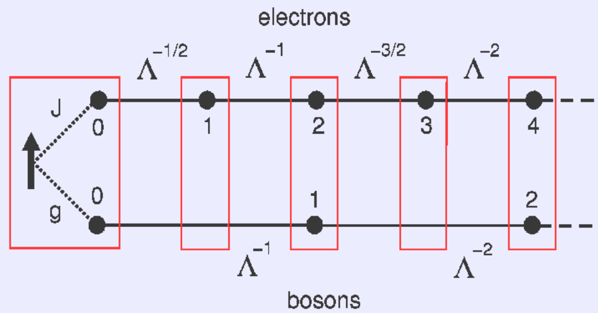
$$H_{\text{bath}} = \sum_{\alpha} \sum_{n=0}^{\infty} \Lambda^{-n} \left[ t \left( a_{n\alpha}^{\dagger} a_{n-1,\alpha} + \text{h.c.} \right) + e\, a_{n\alpha}^{\dagger} a_{n\alpha} \right]$$




- This discretization has been used to study the spin-boson model [Bulla et al., '03, '04].

## Combining Fermionic and Bosonic Baths

- Seek an iterative procedure that treats simultaneously fermionic and bosonic degrees of freedom of the same energy.
- One method—add a bosonic site every other iteration:



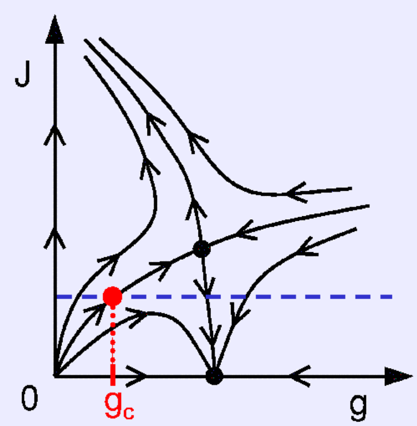

- Iterate until reach a scale-invariant fixed point describing the ground state.

## NRG Results: Bose-Fermi Kondo and Anderson Models

- Have first studied the Ising-symmetry BFKM ($g_\perp = 0$):

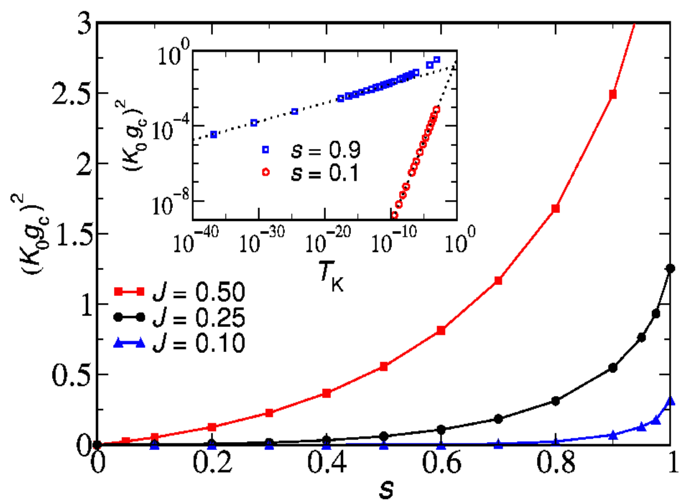
$$H_{\mathrm{imp}} = J\mathbf{S}\cdot\mathbf{s} + g S_z u_z .$$

  - Only one bosonic bath
    ⇒ computationally most tractable.
  - Probably most relevant to $CeCu_{6-x}Au_x$.



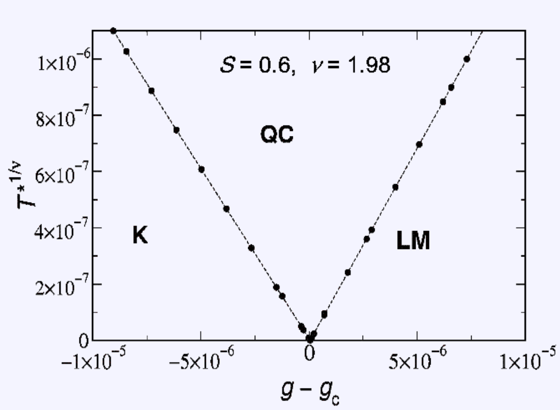

- For convenience, take $\omega_0 = D = 1$.
- Results will be presented at fixed $J$ ⇒ QPT is located at $g = g_c$.

## Critical Coupling $g_c$




$$(K_0\, g_c)^2\, T_K^s \sim T_K \approx \exp(-1/\rho_0 J)$$

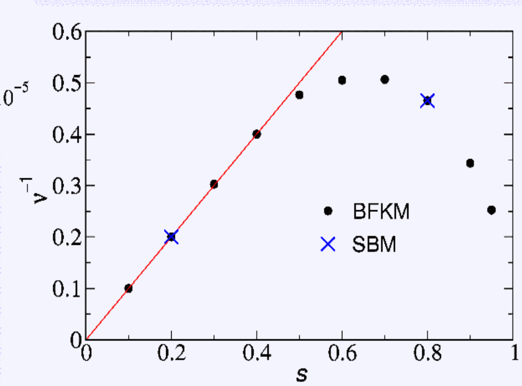
## Crossover Temperature Scale $T^*$




Identify a crossover scale

$$T^* \sim |g - g_c|^\nu$$

from many-body spectrum or physical properties.



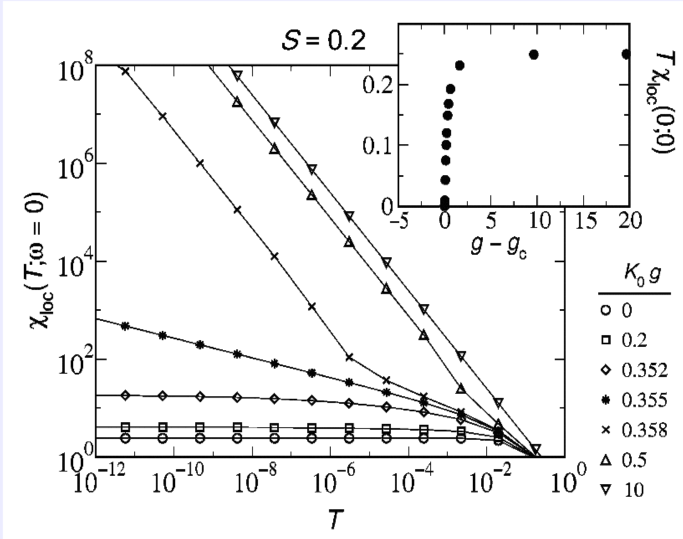

Exponent ν diverges as

$s \to 0^+$ and $s \to 1^-$.

Have interacting QCP only

for $0 < s < 1$.

## Response $\chi_{\mathrm{loc}}$ to a Local Magnetic Field




$\chi_{\mathrm{loc}}(\omega=0, g=g_c) \sim T^{-x}$ with $x = s$ (agrees with ϵ-expansion)

## Response to a Local Magnetic Field (continued)

- All critical exponents agree with those for the spin-boson model.

  ⇒ Ising BFKM and SBM belong to same universality class.

- Static exponents are consistent with a critical free energy

  $$F_{\mathrm{imp}} = T\, f\left(\frac{g-g_c}{T^{1/\nu}}, \frac{h}{T^b}\right) \qquad h = \text{local magnetic field}$$

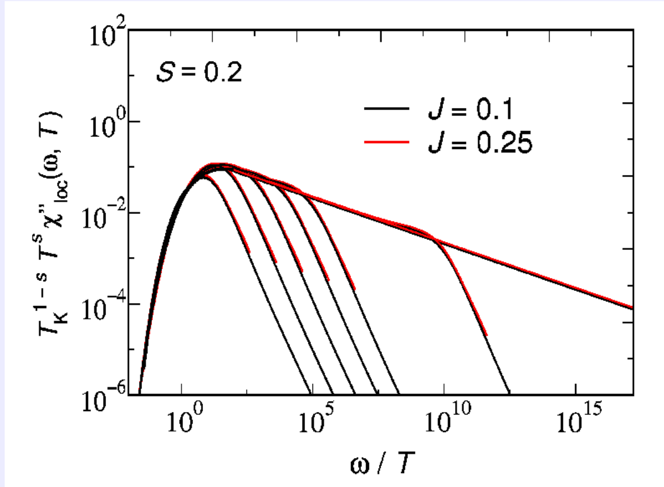
  i.e., exponents obey hyperscaling ⇒ QCP is interacting.

- Local spin dynamics obey

  $$\chi''_{\mathrm{loc}}(\omega, T=0, g=g_c) \sim |\omega|^{-y}\,\mathrm{sgn}\,\omega \quad \text{with} \quad y = x = s$$

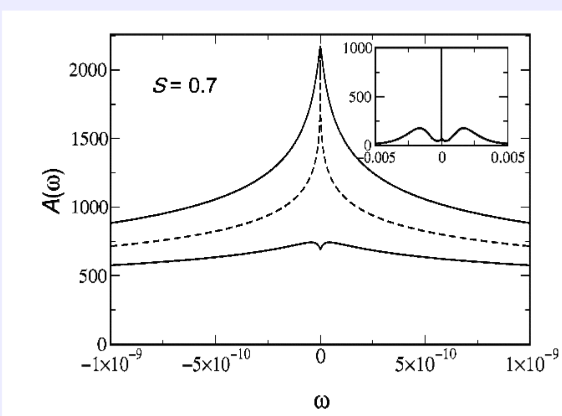
  consistent with $\omega/T$ scaling.

## $\omega/T$ Scaling



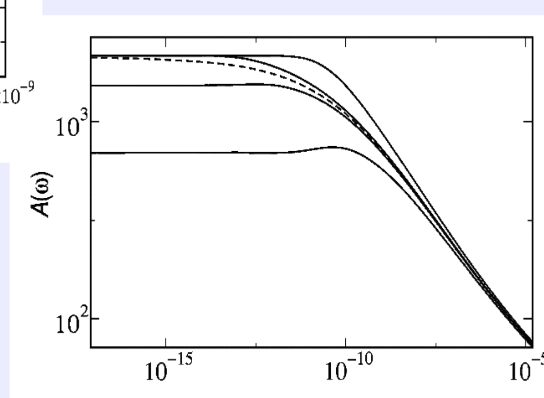

For $|\omega| \ll T_K$ , $T_K\, \chi''_{\rm loc}(\omega, T, g=g_c) = \left(\frac{T}{T_K}\right)^{-x} \Phi_x\left(\frac{\omega}{T}\right)$

## Bose-Fermi Anderson Model: Impurity Spectral Function




Throughout Kondo regime, resonance width $\approx T_K$, and

$$A(\omega=0) = 1/\pi\Gamma_0 .$$




Throughout bosonic regime, have maximum at $\omega = \pm\omega^*$,

and

$$A(\omega=0) < 1/\pi\Gamma_0 .$$

## Summary

- We have developed a new nonperturbative numerical method for Bose-Fermi quantum impurity models at $T = 0$ and $T > 0$.
- Initial application to the Ising-symmetry BFKM:
  - Continuous QPT exists for power-law bosonic baths with exponents $0 < s < 1$.
  - Critical exponents coincide with those of the spin-boson model.
  - QCP exhibits hyperscaling and $\omega/T$ scaling.
  - Destruction of the Kondo resonance at the QCP is directly seen in the impurity spectral function.
- Opens the way to resolve controversy surrounding the EDMFT treatment of the Kondo lattice.