Emergent Supersymmetry

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SUSY → Constraints → Dynamics

Critical Theories

- d = 2, 3, 4, 5, 6
- Local order Parameter
- Exotic
  - No local order parameter
  - No Gaussian fixed point
- Confinement / Fractionalization
- Dualities
  - Particle - Monopole - Dyon
  - Particle - Vortex
- Emergent
  - Global Sym, Gauge Sym
  - Gravity
- Non-Renormalization Thms
  - Exact Relations (critical exponents)
  - State Counting, ...

Why Study SUSY in CM

- Model Systems
- Physical Systems

SUSY CM - Emergent

If SUSY interactions irrelevant

↳ Flows to SUSY in IR

Critical point - RG flows Large

Focus on Emergent Superconformal Sym

Ingredients

Local order parameter ϕ

Superpartner ψ

2nd Order Phase Transition $\omega_\phi = v\, k_\phi$

Space-Time SUSY $\omega_\psi = v\, k_\psi$

$\{Q^\alpha, Q^\beta\} = 2p^{\alpha\beta}$

X Thermal Fluct. $n_\phi \neq n_\psi$ Thermal S^1 b.c. ~~SUSY~~

Quantum Fluct. $T=0$ - QCP

Most General Set of Interactions
(Possibly Restricted by Global Sym)

$d=2$ SUSY Critical points : Tricritical Ising, ...

$d=3$?

N=1 SUSY Wilson-Fisher d=3

ϕ $\mathbb{R}$ $\qquad Z_2: \phi \to -\phi$

ψ $\mathbb{R}$ 2 Comp Weyl Spinor

$$Z\,\partial\phi\,\partial\phi + \frac{1}{2} m_\phi^2 \phi^2 + \frac{\lambda^2}{2}\phi^4 + \cdots$$

$$Z\, i\psi \not{\partial} \psi + \frac{1}{2} m_\psi \psi\psi + h\phi\psi\psi + \cdots$$

Two Relevant op. m_ϕ, m_ψ $\quad$ μ-critical

But $m_\psi = 0$ might be obtained everywhere in a phase

Mean Field · 2nd order Phase Transition

- $\lambda = h \qquad m_\phi = m_\psi \qquad$ SUSY

$$\delta\phi = \epsilon\psi$$

$$\delta\psi = i\sigma^\mu \epsilon \partial_\mu \phi + h\phi^2$$

Superspace (x^μ, θ) $\qquad \Phi = \phi + \theta\psi + \theta^2 F$

$$\int d^2\theta \, \frac{h}{3} \Phi^3$$

Regulate $d = 4 - \epsilon$

Keep explicit spinors after all contractions (external) in $d=3$, p^μ in $d = 4-\epsilon$ $\quad$ - preserves b=f DoF

(Motivated by Supergraphs - Bosonic)

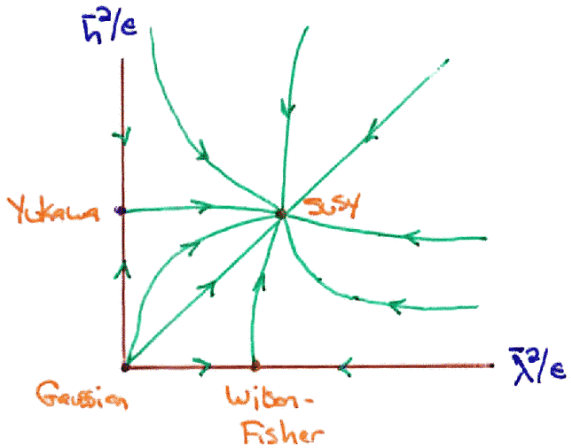
One-Loop

$$\beta_{\bar{h}^2} = -\epsilon \bar{h}^2 + 28 \bar{h}^4 \qquad \bar{h}^2 = h^2/16\pi^2$$

$$\beta_{\bar{\lambda}^2} = -\epsilon \bar{\lambda}^2 + 36 \bar{\lambda}^2 + 8 \bar{h}^2 \bar{\lambda}^2 - 16 h^4 \qquad \bar{\lambda}^2 = \lambda^2/16\pi^2$$

On $m_\phi = m_\psi = 0$ Emergent Superconformal Sym

Conformal Sym $\Rightarrow$ Accidental Superconformal Sym



Critical Exponents

$\eta = 2\gamma_\phi$ — Order Parameter $G(r)$ Scaling Regime

$\nu^{-1} = 2 - \gamma_{m^2}$ — Correlation Length

$\omega = \frac{\partial \beta}{\partial \bar{h}^2}$ — Approach to Scaling

Most Relevant irrelevant op along $\lambda = h$

$m_\psi = 0$

N=2 SUSY Wilson Fisher d=3

ϕ $\mathbb{C}$
ψ^α $\mathbb{C}$ 2 Comp Weyl

U(1) - Allow Yukawa

$$Z\,\partial\phi\,\partial\phi + m_\phi^2\phi^*\phi + \lambda^2(\phi^*\phi)^2 + \cdots$$

$$Z\, i\bar\psi\not\partial\psi + h\phi\psi\psi + h\phi^*\psi^*\psi^* + \cdots$$

Only One Relevant Op.

$Z_2 \times Z_3$ or Z_N $N \geq 5$ or $m_\psi = 0$ in phase

$\lambda = h$ $m_\phi = 0$ SUSY ----

Regulate $d = 4 - \epsilon$

All Spinors in $d=4$, $p^\mu = 4-\epsilon$ DRED
(Supergraphs Bosonic)

One-Loop

$$\beta_{\bar h^2} = -\epsilon\bar h^2 + 12\bar h^4$$

$$\beta_{\bar\lambda^2} = -\epsilon\bar\lambda^2 + 20\bar\lambda^4 + 8\bar h^2\bar\lambda^2 - 16\bar h^4$$

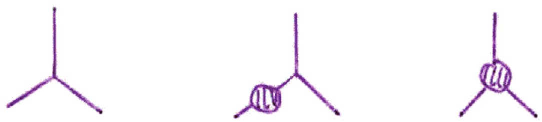
On $m_\phi = 0$ Emergent Superconformal Sym $U(1) = U(1)_R$

Conformal Sym $\Rightarrow$ Accidental SuperConformal Sym

General $\int d^dx\, g_i \mathcal{O}_i$ $\qquad \mathcal{O}_i = \phi\phi\cdots\psi\psi\cdots$

$$\beta_{g_i} = (d - D^c_{\mathcal{O}_i})\, g_i + \sum_a \gamma_{\phi_a} g_i + \gamma_{\mathcal{O}_i}$$

Classical Scaling $\qquad$ Wave Function $\qquad$ Vertex



Any Dim Reg Scheme $\gamma_\phi, \gamma_\mathcal{O}$ independent of ϵ

$\beta = 0 \qquad \epsilon = f(g_i^*) \qquad$ non-linear ; invert

$$g^* = \sum_n g^*_{n,\epsilon}\, \epsilon^n$$

Then

$$\gamma(g^*) = \sum_n \gamma_n \epsilon^n$$

Only explicit dep is ϵ'

N=2 SUSY d=3

Non-Renormalization Thm

SuperPotential Recieves no Vertex Corrections

(Exact - All orders and non-perturbatively)

↳ Holomorphy

↳ Hard to see in Components

$$Z F^*F + hF\phi\phi + h F^*\phi^*\phi^* = -\frac{h^2}{Z}(\phi^*\phi)^2$$

Exact → $\beta_{\bar h^2} = -\epsilon \bar h^2 + 6\gamma_\phi \bar h^2$ ← No Vertex Corrections!!

Homogenous in $\bar h^2$

$$\gamma^*_\phi = \frac{\epsilon}{6} \qquad \eta = \frac{\epsilon}{3}$$

ϵ-expansion for η exact at $O(\epsilon)$

Superconformal Algebra

J^μ_D, J^μ_R Same super μ-plet

$$D^*_\phi = \frac{d-1}{2} R_\phi \qquad R_{\int d^2\theta} = -2 \qquad R_\phi = 2/3$$

$$D^*_\phi = \frac{1}{3}(d-1) = 1 - \frac{\epsilon}{3} \qquad D^c_\phi = 1 - \frac{\epsilon}{2}$$

$\therefore \gamma^*_\phi = \frac{\epsilon}{6} \qquad \eta = \frac{\epsilon}{3}$ by superconformal Sym (emergent)

$\nu^{-1} = 2 - \gamma_{m^2}$: $m^2 \phi^* \phi$ $\cancel{\text{susy}}$

- Extend β-fcn in Superspace
  Exact Relation ν, ω

$h^2 = \frac{h_0^+ h_0}{Z^3} \rightarrow H^2$ Background Superfield

with $Z = Z(1 - m^2 \theta^2 \bar{\theta}^2)$, $\int d^4\theta Z \bar{\Phi}^+ \Phi \subset m^2 \phi^* \phi$

$\beta_{H^2} = -\epsilon H^2 + 6\gamma_\phi H^2$ $\gamma_\phi = \gamma_\phi(H^2)$

$\gamma_{H^2} = 6\gamma_\phi$

$\gamma_{H^2}\big|_{\theta^2\bar{\theta}^2} = 3m\gamma_m^2$

$\therefore$

$\gamma_m^2 = 6\bar{h}^2 \frac{\partial \gamma}{\partial \bar{h}^2}$ Exact on $\lambda = h$
Away from Critical pt.

But

$\omega = \frac{\partial \beta^*}{\partial \bar{h}^2} = 6\bar{h} \frac{\partial \gamma^*}{\partial \bar{h}^2}$

$\therefore$ $\omega = \gamma_{m^2}$ $\omega = 2 - \nu^{-1}$ Exact!

SUSY
$N=2$

$\frac{\epsilon}{3}$ (Exact)

$\frac{1}{2} + \frac{\epsilon}{4} + O(\epsilon^2)$

$2 - \nu^{-1}$ (Exact)

	Wilson Fisher $N=0$	SUSY $N=1$
η	$0 + \mathcal{O}(\epsilon^2)$	$\frac{\epsilon}{7} + \mathcal{O}(\epsilon^2)$
ν	$\frac{1}{2} + \frac{\epsilon}{12} + \mathcal{O}(\epsilon^2)$	$\frac{1}{2} + \frac{\epsilon}{7} + \mathcal{O}(\epsilon^2)$
ω	$\epsilon + \mathcal{O}(\epsilon)$	$\epsilon + \mathcal{O}(\epsilon^2)$

Mean Field: 2nd order Phase Transition

Q - Fluctuations of other Fields coupled to order Parameter might ruin → 1st order

i) $\beta_{m^2} = -2m^2 + 2\gamma_\phi m^2$

— Continuing β_{m^2} into Superspace

Inherits Homogeneity in m^2

Can't develop Tachyon

ii) $\bar{\lambda}^* > 0$

Stable

∴ Phase Transition Remains 2nd order

Quantum Super Critical Point Stable

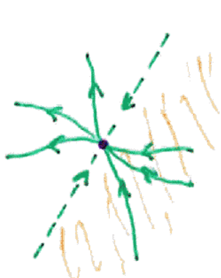
- Generalizations

$U(1)_R$ c.f. $O(2)$ Sym

Lorentz Viol. $v_\phi \neq v_\psi$

μ- Component

More SUSY (Requires Gauge Fields)



Entire Region
Flows to SUSY in IR

Simple Models - Retain 2nd Order
General Model - Fluctuation induced 1st order
by tachyon -
QSCP unstable
└ Effects t=finite

Summary

Emergent Superconformal Sym
Possible in Simple d=3 Systems

Single Relevant Direction Possible

Stable

Provide new Examples Quantum Super Critical Points

With N=2 d=3 Exact Relations Among η, ν, ω

Lessons

Exact Results - SUSY sufficient

Non-susy Exact Results