

Non-Fermi liquid behavior near magnetic quantum critical points in U-based systems

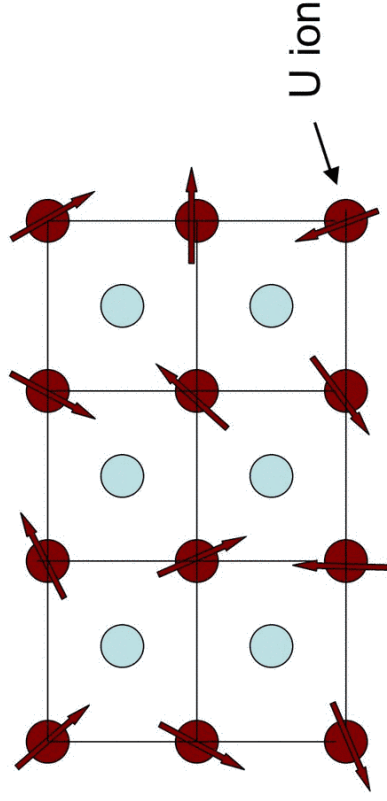
*M. Brian Maple
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- Introduction to the physics of strongly correlated f-electron materials
- Recent experiments on $M_{1-x}U_xPd_3$ ($M = Y, Sc$) systems
- $Y_{1-x}U_xPd_3$: first f-electron system to exhibit NFL behavior
- Both systems: NFL behavior near spin glass QCP
- Interplay between Kondo effect, disorder, antiferromagnetic interactions
- $Sc_{1-x}U_xPd_3$: ω/T scaling of $\chi''(\omega, T)$
- Recent experiments on $URu_{2-x}Re_xSi_2$
- URu_2Si_2 : “Hidden order?”
- $URu_{2-x}Re_xSi_2$: NFL behavior near ferromagnetic QCP

Supported by US DOE and NSF

U-based systems

- Intermetallic compounds containing sublattice of U ions



- U ion carries magnetic moment μ_{eff}
- Associated with partially-filled 5f electron shell
- Derived from Hund's rules: $S, L, J = |L - S|$ (<half-filled shell)
- Degeneracy = $2J+1$
- $\mu_{\text{eff}} = g_J[J(J+1)]^{1/2}\mu_B$
- $g_J = 1 + \{[J(J+1)+S(S+1)-L(L+1)]/2J(J+1)\}$

Los Alamos National Laboratory Chemistry Division

Periodic Table of the Elements

1A	2A	3B	4B	5B	6B	7B	8B	11B	12B	3A	4A	5A	6A	7A	8A																																																																																																						
1 H 1.008 gas	2 He 4.003 gas	3 Li 6.941 solid	4 Be 9.012 solid	5 B 10.81 solid	6 C 12.01 solid	7 N 14.01 gas	8 O 16.00 gas	9 F 19.00 gas	10 Ne 20.18 gas	11 Na 22.99 solid	12 Mg 24.31 solid	13 Al 26.98 solid	14 Si 28.09 solid	15 P 30.97 solid	16 S 32.07 solid	17 Cl 35.45 gas	18 Ar 39.95 gas	19 K 39.10 solid	20 Ca 40.08 solid	21 Sc 44.96 solid	22 Ti 47.88 solid	23 V 50.94 solid	24 Cr 52.00 solid	25 Mn 54.94 solid	26 Fe 55.85 solid	27 Co 58.93 solid	28 Ni 58.71 solid	29 Cu 63.55 solid	30 Zn 65.38 solid	31 Ga 69.72 solid	32 Ge 72.64 solid	33 As 74.92 solid	34 Se 78.96 solid	35 Br 79.90 liquid	36 Kr 83.80 gas	37 Rb 85.47 solid	38 Sr 87.62 solid	39 Y 88.91 solid	40 Zr 91.22 solid	41 Nb 92.91 solid	42 Mo 95.94 solid	43 Tc 98.91 solid	44 Ru 101.1 solid	45 Rh 102.9 solid	46 Pd 106.4 solid	47 Ag 107.9 solid	48 Cd 112.4 solid	49 In 114.8 solid	50 Sn 118.7 solid	51 Sb 121.8 solid	52 Te 127.6 solid	53 I 126.9 solid	54 Xe 131.3 gas	55 Cs 132.9 solid	56 Ba 137.3 solid	57 La* 138.9 solid	58 Ce 140.1 solid	59 Pr 140.9 solid	60 Nd 144.2 solid	61 Pm 144.9 solid	62 Sm 150.4 solid	63 Eu 152.0 solid	64 Gd 157.3 solid	65 Tb 158.9 solid	66 Dy 162.5 solid	67 Ho 164.9 solid	68 Er 167.3 solid	69 Tm 168.9 solid	70 Yb 173.1 solid	71 Lu 175.1 solid	72 Hf 178.5 solid	73 Ta 180.9 solid	74 W 183.8 solid	75 Re 186.2 solid	76 Os 190.2 solid	77 Ir 192.2 solid	78 Pt 195.1 solid	79 Au 197.0 solid	80 Hg 200.6 solid	81 Tl 204.4 solid	82 Pb 207.2 solid	83 Bi 208.9 solid	84 Po 209 solid	85 At 210 solid	86 Rn 222 gas	87 Fr 223 solid	88 Ra 226 solid	89 Ac~ 227 solid	90 Th 232.0 solid	91 Pa 231 solid	92 U 238.0 solid	93 Np 237 solid	94 Pu 244 solid	95 Am 243 solid	96 Cm 247 solid	97 Bk 247 solid	98 Cf 251 solid	99 Es 252 solid	100 Fm 257 solid	101 Md 258 solid	102 No 259 solid	103 Lr 260 solid	104 Rf 261 solid	105 Db 262 solid	106 Sg 266 solid	107 Bh 264 solid	108 Hs 277 solid	109 Mt 268 solid	110 Ds 271 solid	111 Uu 288 solid	112 Uub 285 solid	113 Nh 286 solid	114 Uuq 289 solid	115 Fl 289 solid	116 Uuh 289 solid	117 Ts 289 solid	118 Uuo 289 solid

Lanthanide Series*

Actinide Series-



element names in blue are liquids at room temperature
 element names in red are gases at room temperature
 element names in black are solids at room temperature

U-based systems

U: $[Rn]5f^36d^17s^2$ (atom)

- Trivalent (U^{3+}): $5f^3$

$S = 3/2$, $L = 6$, $J = 9/2$, degeneracy = 10

$\Rightarrow \mu_{\text{eff}} = 3.58\mu_B$

- Tetravalent (U^{4+}): $5f^2$

$S = 1$, $L = 5$, $J = 4$, degeneracy = 9

$\Rightarrow \mu_{\text{eff}} = 3.62\mu_B$

- CEF can lift degeneracy of Hund's rule multiplet

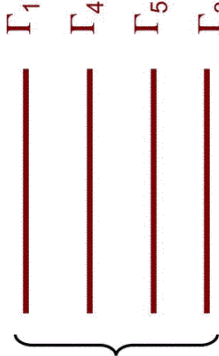
e.g., U^{4+} in cubic CEF

Γ_1 singlet, Γ_3 nonmagnetic doublet, Γ_4 & Γ_5 triplets

Example of energy level scheme

appropriate for quadrupolar

Kondo effect



U-based systems

Hybridization of localized 5f and conduction electron states

⇒ interesting correlated electron physics!

Hybridization strength:

Weak ⇒ ionic behavior (5f occupation number $\langle n \rangle$ integral)

Moderate ⇒ Kondo behavior ($\langle n \rangle \sim$ integral)

Appreciable ⇒ Valence fluctuations ($\langle n \rangle$ nonintegral)

Strong ⇒ f-bands

Heavy fermion f-electron materials

Underlying physics — significant hybridization between localized 5f- & conduction-electron states

- AFM exchange interaction
- $\mathcal{H}_{\text{ex}} = -2J\mathbf{S}\cdot\mathbf{s}$ where $J \sim -\langle V_{\text{kf}}^2 \rangle / (E_{\text{F}} - F_{\text{f}}) < 0$
- Kondo effect (lattice of U ions)
- $T_{\text{K}} \sim T_{\text{F}} \exp(-1/N(E_{\text{F}})|J|) \sim T^*$ (effective T_{F})

- $T \gg T^*$: local moment behavior

$$\chi(T) \sim N\mu_{\text{eff}}^2/3k_{\text{B}}(T+T^*)$$

$$\rho(T) \sim -\ln T$$

- $T \ll T^*$: many body singlet

Nonmagnetic heavy Fermi liquid (FL)

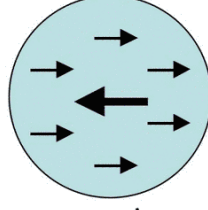
$$\chi(T) \rightarrow \chi_0 \propto m^* \propto 1/T^*$$

$$\gamma(T) = C_{\text{e}}(T)/T \rightarrow \gamma_0 \propto m^* \propto 1/T^*$$

$$R = (\chi_0/\mu_{\text{eff}}^2)/(\gamma_0/\pi^2 k_{\text{B}}^2) \approx 1 \text{ (Wilson-Sommerfeld ratio)}$$

$$\rho(T) \propto \rho_{\text{e-e}}(T) \sim AT^2 \text{ with } A \sim \gamma_0^2$$

- Heavy FL unstable to SC & magnetic order (RKKY)



Heavy fermion f-electron materials

Superconductivity

- Pairing of electrons with $L > 0$
 - L=1 (p-wave), S=1 (triplet)
 - L=2 (d-wave), S=0 (singlet)
- Pairing mechanism — spin fluctuations
- Anisotropic energy gap $\Delta(\mathbf{k}) \neq \text{const.}$
 - $\Delta(\mathbf{k})$ vanishes at points or lines on Fermi surface
- Superconducting properties $\sim T^n$ for $T \ll T_c$
 - e.g., $C_e(T) \sim T^n$ ($n=2$, line nodes; $n=3$, point nodes) [exp(- Δ/T) with const. Δ for BCS superconductor]
- Multiple superconducting states (complex T-x-P phase diagrams)
 - e.g., UPt_3 , $\text{U}_{1-x}\text{Th}_x\text{Be}_{13}$, $\text{PrOs}_4\text{Sb}_{12}$
- Sometimes occurs near x or P where $T_M \rightarrow 0$ K (quantum critical point — QCP)

Non-Fermi liquid behavior

- Materials: U, Ce, Yb intermetallic compounds
 - Chemically substituted: $\text{Y}_{1-x}\text{U}_x\text{Pd}_3$, $\text{UCu}_{5-x}\text{Pd}_x$, $\text{CeCu}_{6-x}\text{Au}_x, \dots$
 - Stoichiometric: (P=0) — UBe_{13} , CeCoIn_5 , $\text{YbRh}_2\text{Si}_2, \dots$;
 - (P>0) — CeIn_3 , $\text{CePd}_2\text{Si}_2, \dots$
- Physical properties — weak power law, logarithmic divergences in T at low $T \ll T_o$
 - $\rho(T) \approx \rho(0)[1 \pm (T/T_o)^n]$ ($1 \leq n \leq 1.5$)
 - $C(T)/T \approx - (1/T_o)\ln(T/T_o)$, $(T/T_o)^{-1+\lambda}$
 - $\chi(T) \approx \chi(0)[1 - (T/T_o)^n]$ ($n \sim 0.5$), $- (1/T_o)\ln(T/T_o)$, $(T/T_o)^{-1+\lambda}$, $C/(T^\alpha + \theta)$
 - $\chi''(\omega, T)$: ω/T scaling
- Appreciable T-dependence below $T_o \Rightarrow$ lower energy scale than Fermi liquid (FL)

Non-Fermi liquid behavior

Experiments \Rightarrow two routes to NFL behavior:

(1) Single ion & (2) inter-ionic interactions

(1) **Single ion models**

- Multichannel Kondo effect
- Single channel Kondo effect with disorder — $P(T_K)$

(2) **Interacting ion models**

- Fluctuations of order parameter above 2nd order phase transition at 0 K
- Griffiths' phase — interplay between disorder & competing Kondo and RKKY interactions

Superconductivity near magnetic QCP mediated by spin fluctuations

Recent experiments on $M_{1-x}U_xPd_3$ ($M = Y, Sc$) systems

$Y_{1-x}U_xPd_3$: First f-electron system in which NFL behavior observed

C. L. Seaman, M. B. Maple, B. W. Lee, S. Ghamaty, M. S. Torikachvili,

*K. N. Yang, L. Z. Liu, J. W. Allen, D. L. Cox, PRL **67**, 2882 '91*

$\rho(T,H)$, $C(T)$, $M(T,H)$ for $0 < x \leq 0.2$

Interpreted NFL behavior in terms of quadrupolar Kondo effect

*B. Andraka, A. M. Tsvetlik, PRL **67**, 2886 '91*

$\rho(T,H)$, $C(T,H)$, $M(T,H)$ for $x = 0.2$

Interpreted NFL behavior in terms of OP fluctuations

associated with magnetic transition suppressed to 0 K

Extensive investigations \Rightarrow situation much more complex

INS studies: U^{4+} energy level scheme in cubic CEF not yet established

Fluctuations in U concentration x on $\sim 10 \mu m$ scale (*S. Süllow et al. '91*)

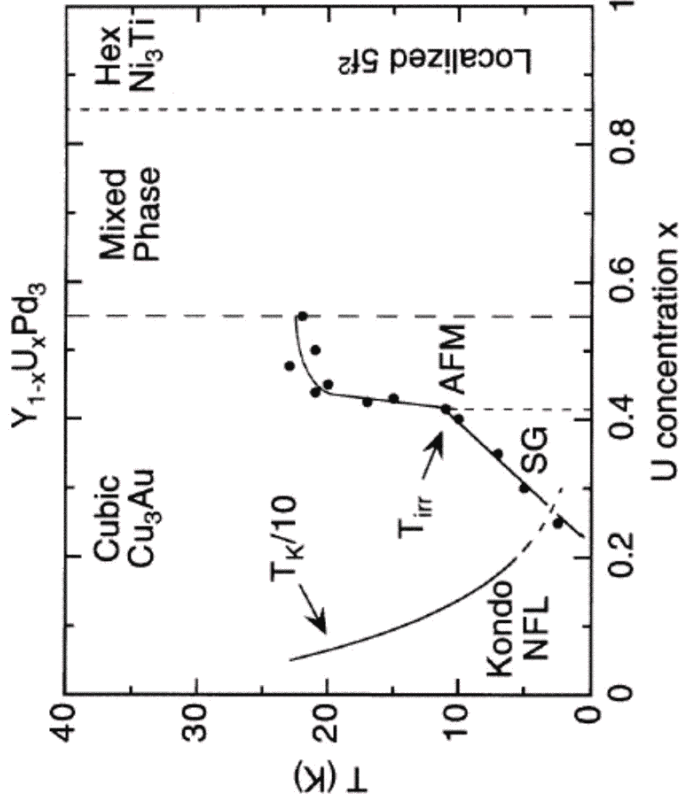
$Sc_{1-x}U_xPd_3$: NFL behavior similar to that observed in $Y_{1-x}U_xPd_3$ system

More homogeneous distribution of U

INS studies: ω/T scaling of $\chi''(\omega,T)$

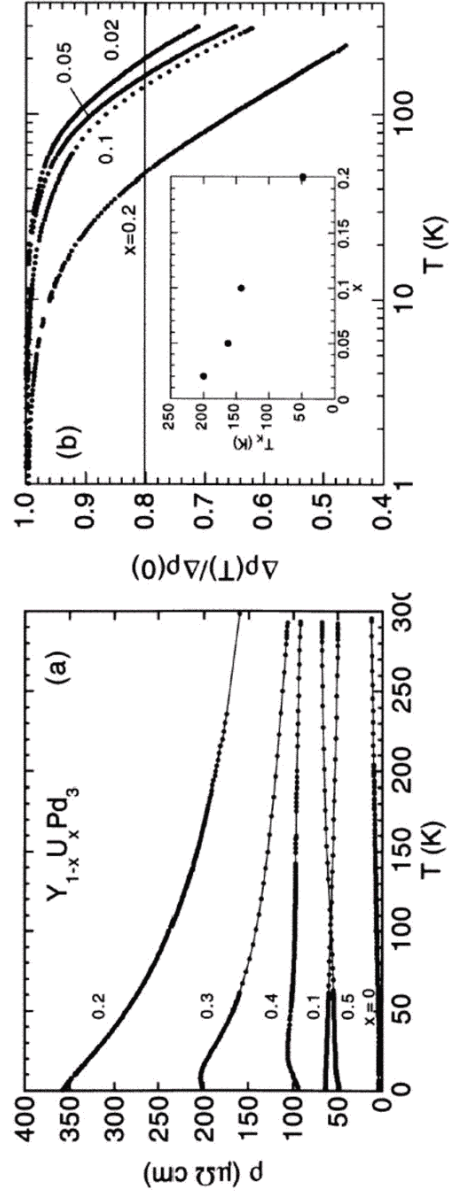
Interplay between Kondo effect, disorder, AFM interactions

T-x phase diagram of $Y_{1-x}U_xPd_3$ system



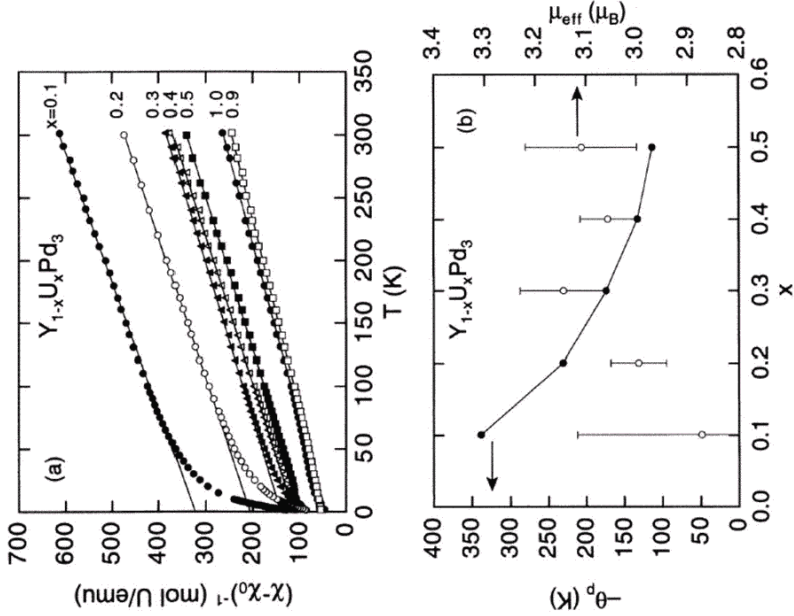
C. L. Seaman et al., *PRL* **67**, 2882 '91
 D. A. Gajewski, N. R. Dilley, R. Chau, M. B. Maple, *J. Phys.: Condens. Matter* **8**, 9793 '96

Electrical resistivity ρ vs T for the $Y_{1-x}U_xPd_3$ system (high T)



M. B. Maple, R. P. Dickey, J. Herrmann, M. C. de Andrade, E. J. Freeman,
 D. A. Gajewski, R. Chau, *J. Phys.: Condens. Matter* **8**, 9773 '96

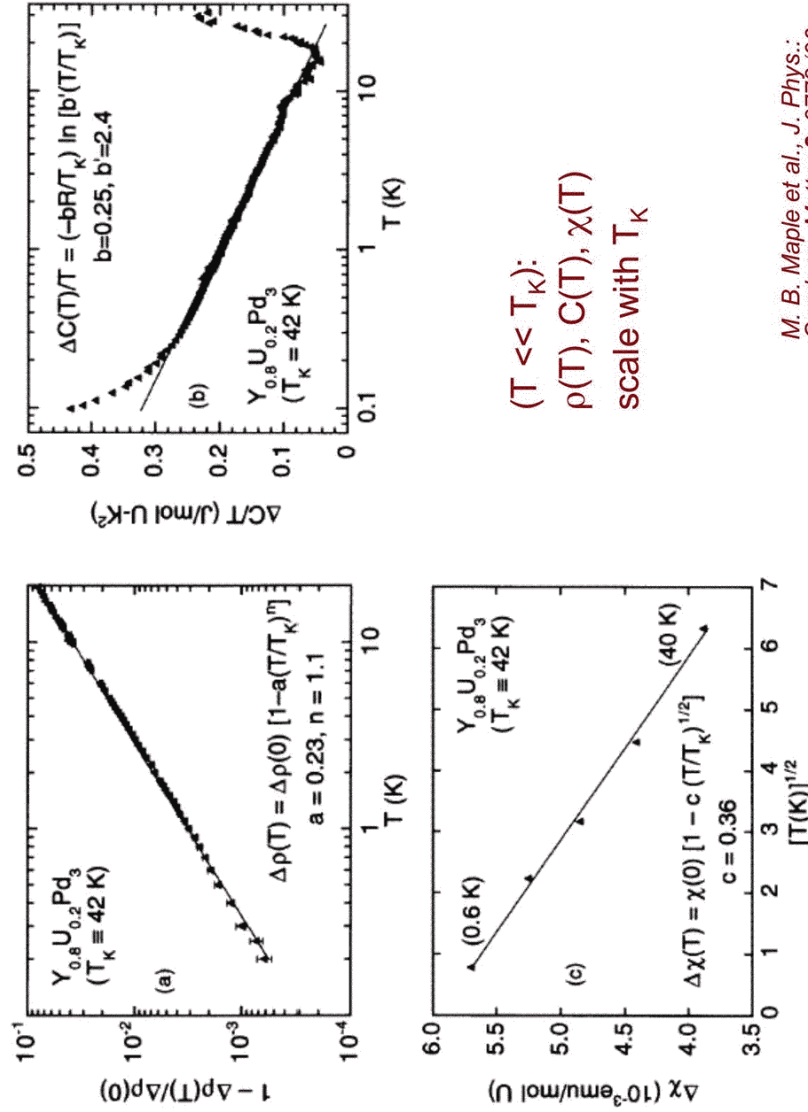
Magnetic susceptibility χ vs T for the $Y_{1-x}U_xPd_3$ system (high T)



For Kondo systems:
 $\chi(T) = N\mu_{eff}^2/3k_B(T - \theta_p)$
 $\theta_p \approx -3T_K$

M. B. Maple et al., *J. Phys.: Condens. Matter* **8**, 9773 '96

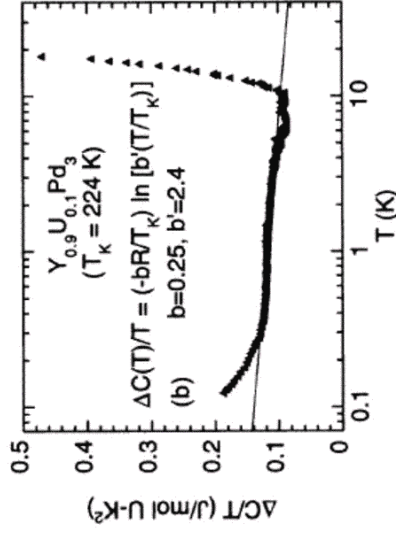
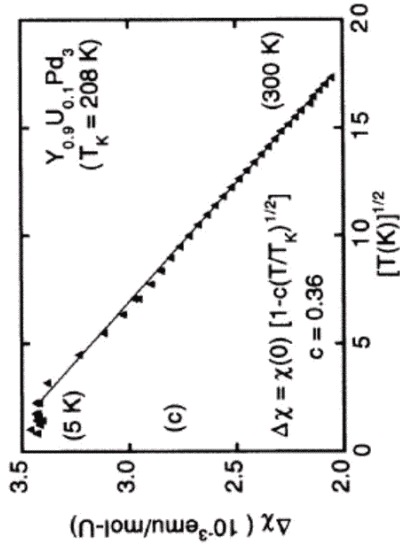
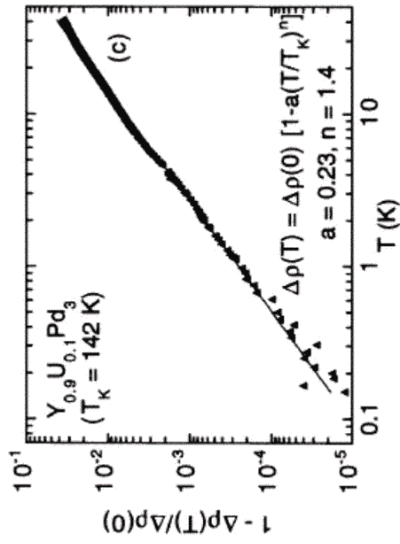
Low-T NFL behavior in $\rho(T)$, $C(T)$, $\chi(T)$ for $Y_{1-x}U_xPd_3$



($T \ll T_K$):
 $\rho(T)$, $C(T)$, $\chi(T)$
 scale with T_K

M. B. Maple et al., *J. Phys.: Condens. Matter* **8**, 9773 '96

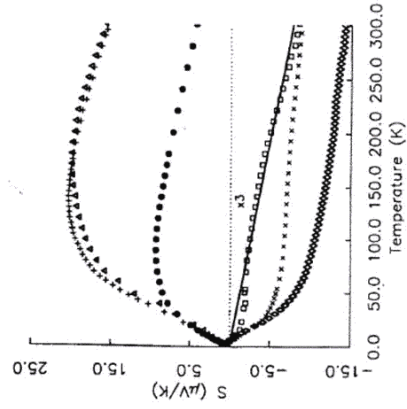
Low-T NFL behavior in $\rho(T)$, $C(T)$, $\chi(T)$ for $Y_{1-x}U_xPd_3$



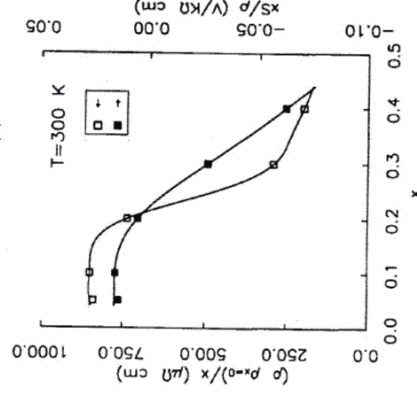
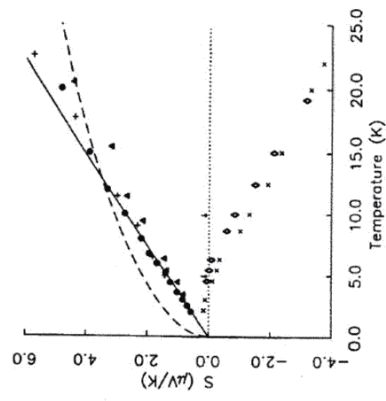
($T \ll T_K$):
 $\rho(T)$, $C(T)$, $\chi(T)$
scale with T_K

M. B. Maple et al., *J. Phys.: Condens. Matter* **8**, 9773 '96

Thermoelectric power S vs T for the $Y_{1-x}U_xPd_3$ system

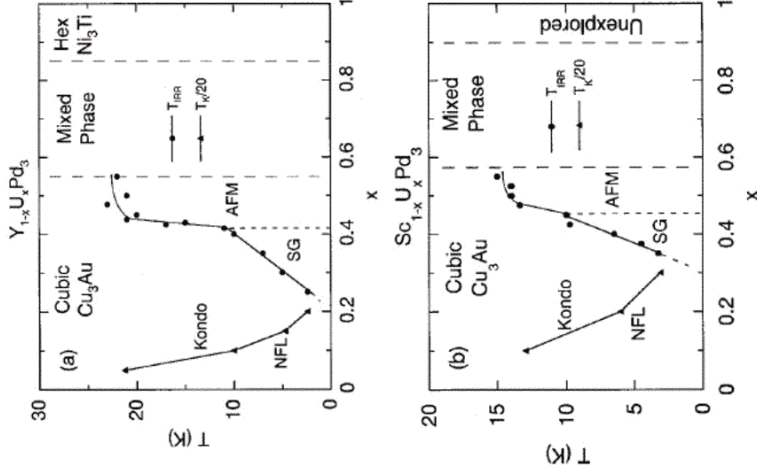


- Symbols:
 X=0 (star)
 X=0.03 (square)
 X=0.1 (plus)
 X=0.2 (triangle)
 X=0.3 (x)
 X=0.4 (diamond)

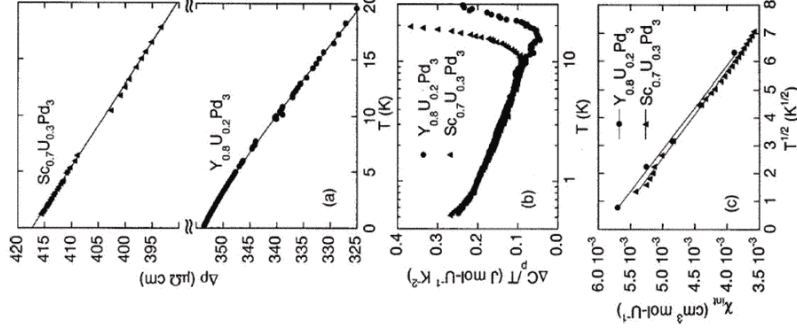


J. McCarten, S. E. Brown, C. L. Seaman,
M. B. Maple, *PRB* **49**, 6400 '94

T-x phase diagrams & low T properties of $M_{1-x}U_xPd_3$ ($M = Y, Sc$)

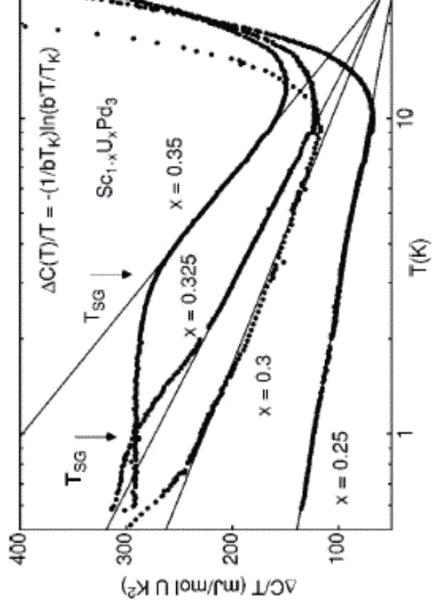
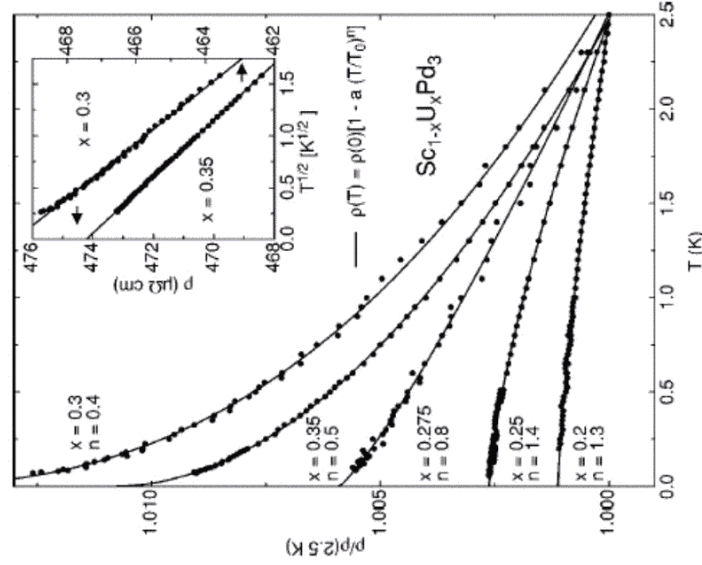


- Similar T-x phase diagrams & low T NFL characteristics in $\rho(T)$, $C(T)/T$, $\chi(T)$
- T_K decreases with x
- AFM & SG phases
- SG QCP
M=Y: $x_c \approx 0.2$
M=Sc: $x_c \approx 0.3$
- NFL behavior associated with SG QCP



D. A. Gajewski, R. Chau, M. B. Maple, PRB 62, 5496 '00

$\rho(T)$ and $C(T)/T$ of the $Sc_{1-x}U_xPd_3$ system near SG QCP

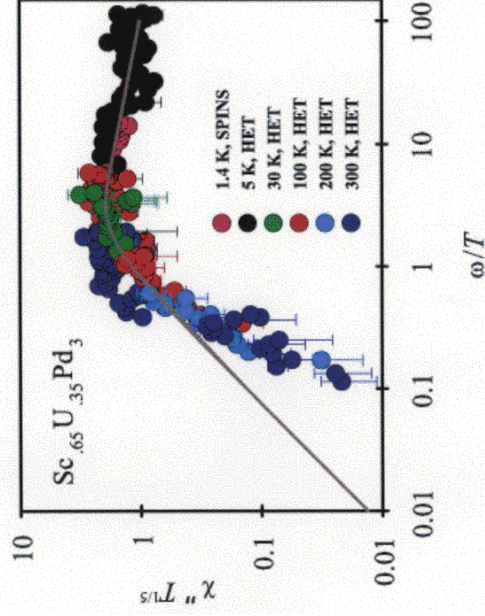


Near SG QCP ($x_c \approx 0.3$)

$$\rho(T) \approx \rho(0)[1 - (T/T_0)^{1/2}]$$

$$C(T)/T \approx -\ln(T/T_0)$$

Inelastic neutron scattering on $\text{Sc}_{1.65}\text{U}_{0.35}\text{Pd}_3$: ω/T scaling



$$\chi''(q, \omega, T) = 1/[AT^\alpha F(\omega/T)]$$

with $\alpha = 1/5$

No q-dependence \Rightarrow no evidence for U-U correlations

Single impurity critical scaling associated with spin glass transition suppressed to 0 K

Solid line: $F(\omega/T) = \exp[\alpha\Psi(1/2 - i\omega/2\pi T)]$

Proposed for AFM QCP — Q. Si *et al.*, *Nature* **413**, 804 '01

S. D. Wilson, P. Dai, D. T. Adroja, S.-H. Lee, J.-H. Chung, J. W. Lynn, N. P. Butch, M. B. Maple, *PRL* '05 (in press)

Inelastic neutron scattering on $\text{Sc}_{1.65}\text{U}_{0.35}\text{Pd}_3$: ω/T scaling

Similar behavior: $\chi''(q, \omega, T) \propto 1/AT^\alpha F(\omega/T)$

$\text{UCu}_{5-x}\text{Pd}_x$ ($x = 1, 1.5$); $\alpha = 1/3$

Near AFM QCP

M. Aronson *et al.*, *PRL* **75**, 725 '95; *PRL* **87**, 197205 '01

$\text{CeCu}_{5.9}\text{Au}_{0.1}$; $\alpha = 0.75$

A. Schröder *et al.*, *PRL* **80**, 5623 '98

Near AFM QCP

Absence of energy scale, other than T itself \Rightarrow microscopic origin of the NFL behavior involves individual ions near 0 K spin glass phase transition

Although impurities may play role, they do not appear to be primary cause

The URu_{2-x}Re_xSi₂ system

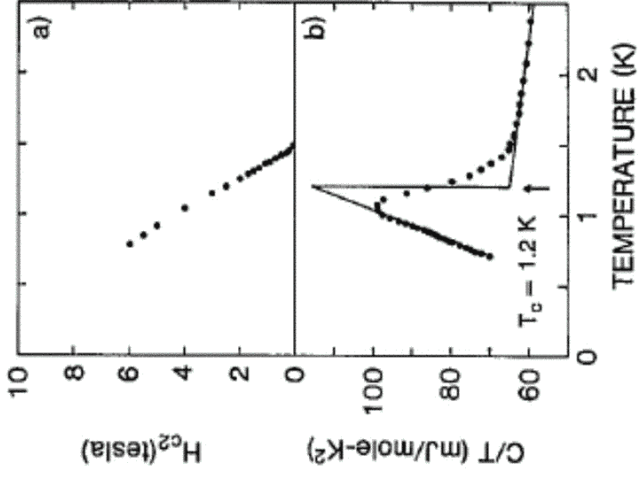
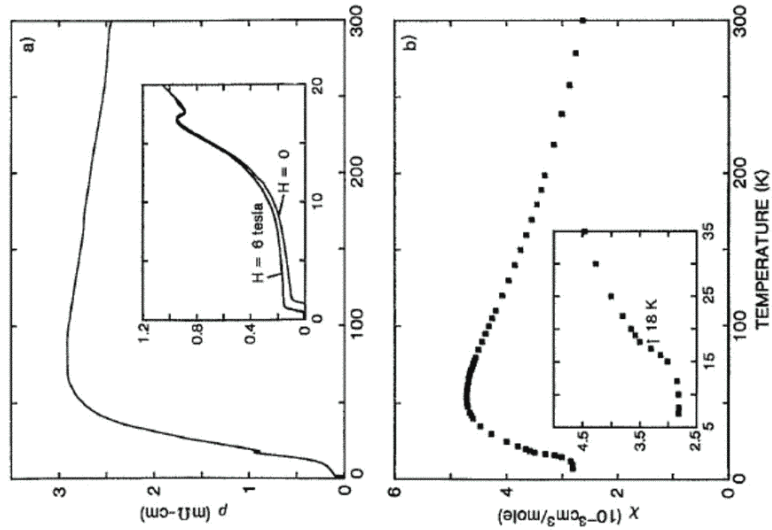
NFL behavior in the vicinity of FM QCP

- Substitution of other elements for Ru (i.e., URu_{2-x}M_xSi₂) depresses T_N and T_c. *Dalichaouch et al. '90*
 - FM instability observed in URu_{2-x}M_xSi₂ for M=Tc, Re at higher x *Dalichaouch, Torikachvili, Maple, Giorgi '89*
 - Similar behavior for T_c, Re — FM extends over range of x values, maxima in θ_c , μ_s , γ at nearly same x
 - No anomalies in C(T) & $\rho(T)$ due to FM near θ_c
 - FM determined from M(H,T) measurements & analyzed by means of modified Arrott plots
 - $M^{1/\beta}$ vs $(H/M)^{1/\gamma}$
 - Standard Arrott plots: $\beta = 1/2$, $\gamma = 1$ (MFT)
 - Critical exponents β , γ , δ defined by
 - $M(t, H=0) \propto t^\beta$; $M(t=0, H) \propto H^{1/\gamma}$; $\gamma = \beta(\delta - 1)$
 - Analysis based on Arrott-Noakes equation
 - $(H/M)^{1/\gamma} = (T - \theta_c)/T_1 + (M/M_1)^{1/\beta}$
- where T₁ & M₁ are normalization factors

Parent compound URu₂Si₂

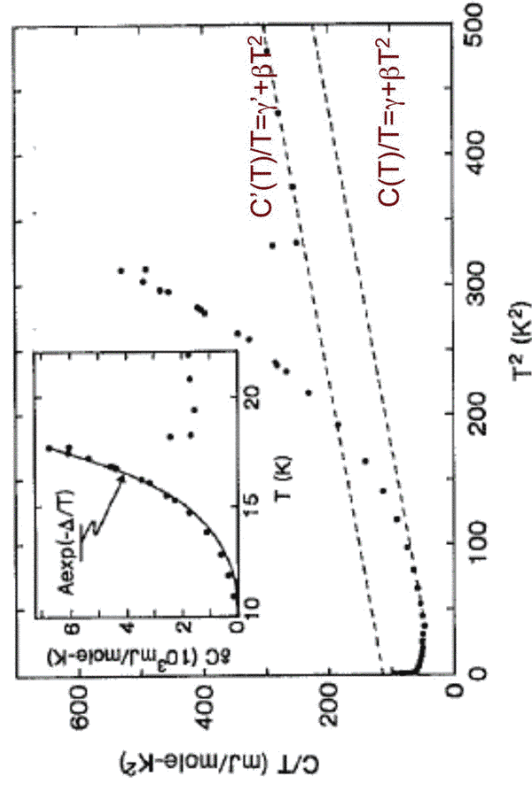
- Moderately heavy Fermi liquid: $m^* \approx 25 m_e$
- Superconductivity: T_c = 1.5 K, $\Delta_s \approx 0.1$ meV
- Hidden order (HO) phase: T_o = 17.5 K, $\Delta \approx 11$ meV
- AFM (U 5f): $\mu \approx 0.02-0.04 \mu_B/U$, ($\parallel c$ -axis), (100) modulation
- Transport, thermal, magnetic properties:
- Single crystal specimens*
 - Palstra, Menovsky, van den Berg, Dirkmaat, Kes, Nieuwenhuys, Mydosh '85*
- Polycrystalline specimens*
 - Schlabbitz, Baumann, Pollit, Rauchschwalbe, Mayer, Alheim, Bredl ZP '86*
 - Maple, Dalichaouch, Kohara, Rossel, Torikachvili, McElfresh, Thompson PRL '86*
- Neutron scattering**
- Single crystal specimens*
 - Broholm, Kjems, Buyers, Matthews, Palstra, Menovsky, Mydosh '86*
- Polycrystals (INS)**
 - Walter, Loong, Loewenhaupt, Schlabbitz '86*

Normal and superconducting state properties of URu₂Si₂



Maple, Dalichaouch, Kohara, Rossel, Torikachvili, McElfresh, Thompson, PRL '86

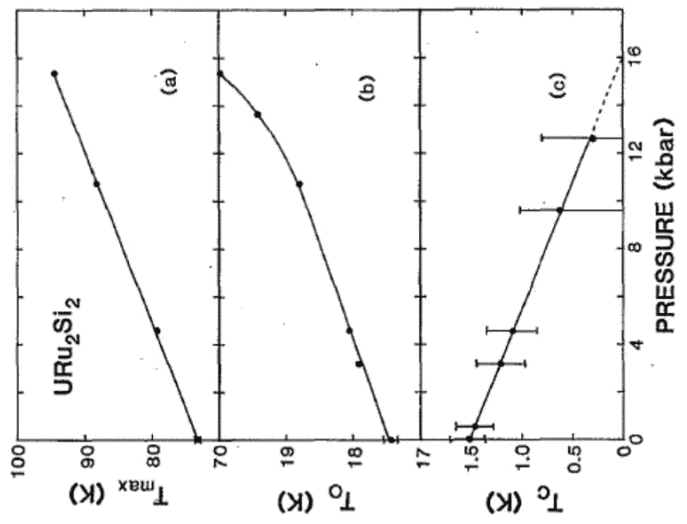
Specific heat of URu₂Si₂



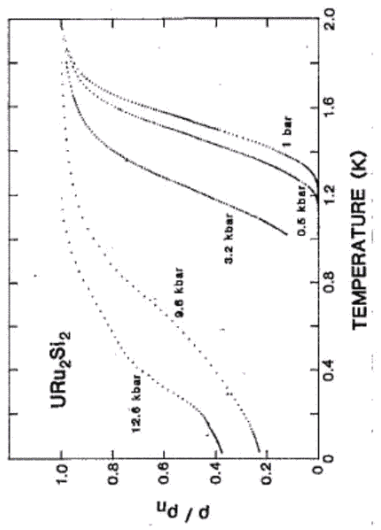
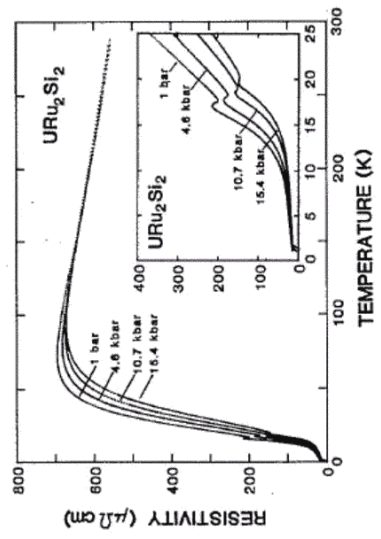
BCS-type mean field transition at $T_0 = 17.5$ K
 $\delta C \approx A \exp(-\Delta/T)$; $\Delta \sim 10^2$ K ~ 10 meV
 AFM ($\mu \approx 0.02 \mu_B/U$) coexists with SC
 $\gamma(0)/\gamma' \approx 0.6 \Rightarrow \sim 40\%$ Fermi surface removed by SDW or CDW
 SC & SDW or CDW compete for Fermi surface!

Maple, Dalichaouch, Kohara, Rossel, Torikachvili, McElfresh, Thompson, PRL '86

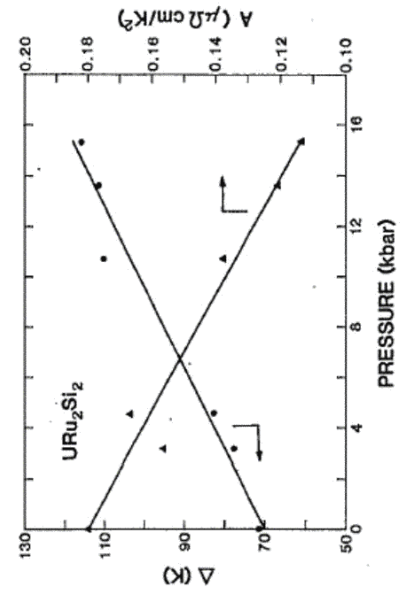
Effect of pressure on competing electronic states in URu₂Si₂



McElfresh, Thompson, Willis, Maple, Kohara, Torikachvili '87



Electrical resistivity ρ(T) of URu₂Si₂ under pressure



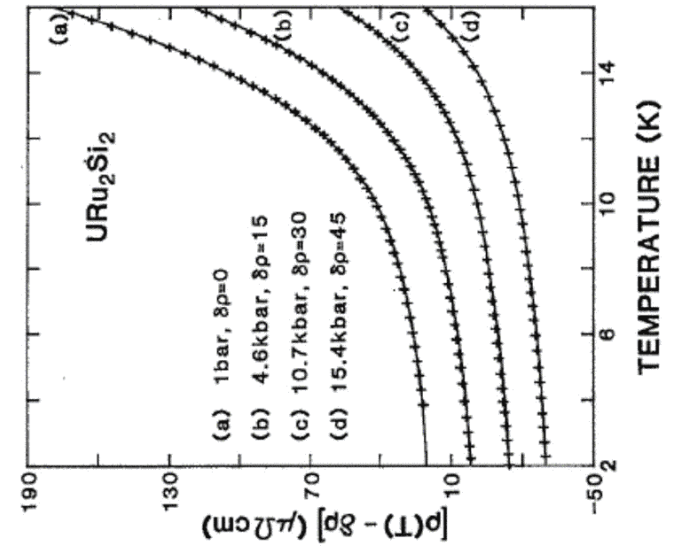
$$\rho(T) = \rho_0 + AT^2 - B(T/\Delta)(1 + 2T/\Delta)\exp(-\Delta/T)$$

Hessel Anderson '80

Palstra, Menovsky, Mydosh '86

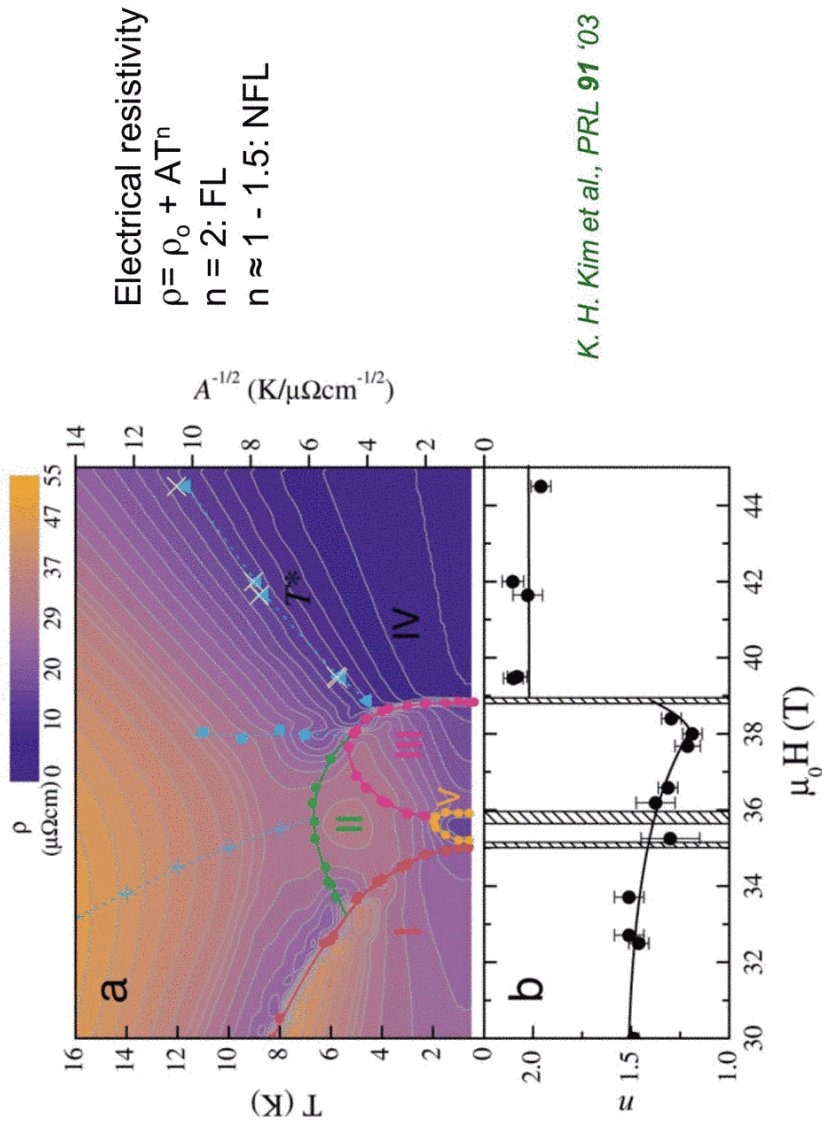
$$\rho(T): \text{URu}_2\text{Si}_2 (P=0)$$

$$A \propto T_K^{-2} \propto T_{\text{max}}^{-2}$$



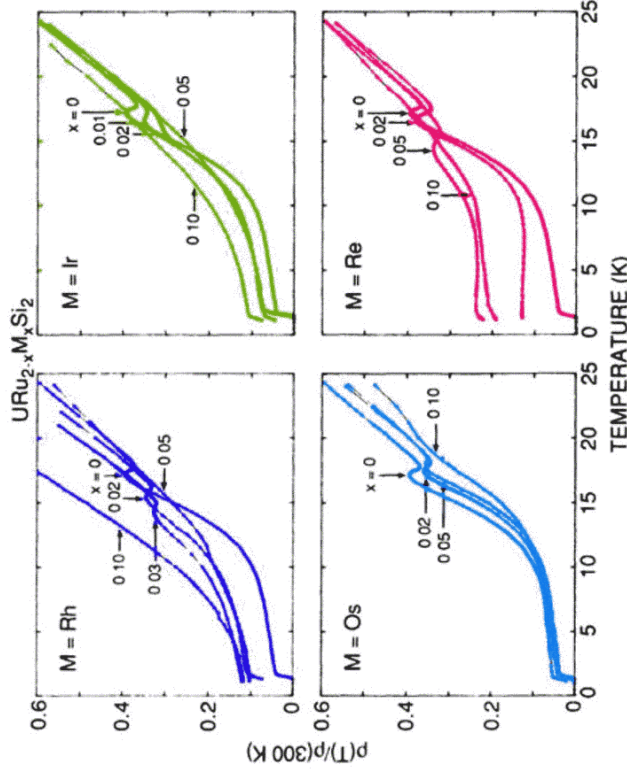
McElfresh, Thompson, Willis, Maple, Kohara, Torikachvili '87

Field-induced phases in URu₂Si₂



Transition metal substitutions for Ru in URu₂Si₂

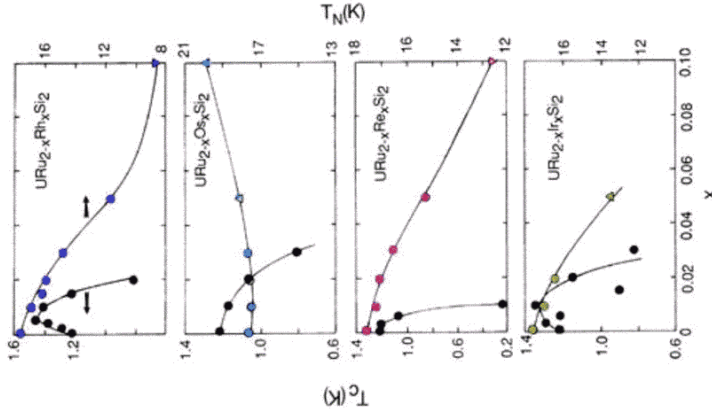
URu_{2-x}M_xSi₂ (M = Rh, Os, Re, Ir)



Dalichaouch, Maple, Chen, Kohara, Rossel, Torikachvili, Giorgi '90

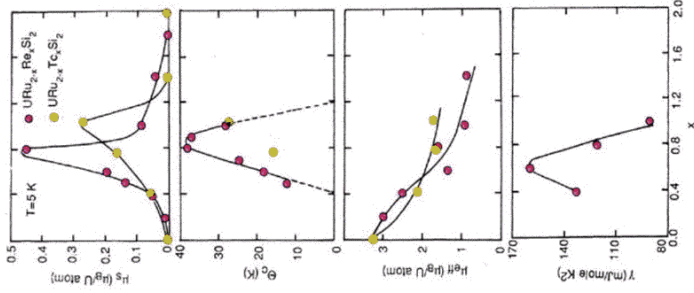
Transition metal substitutions for Ru in URu₂Si₂

URu_{2-x}M_xSi₂ (M = Rh, Os, Re, Ir)



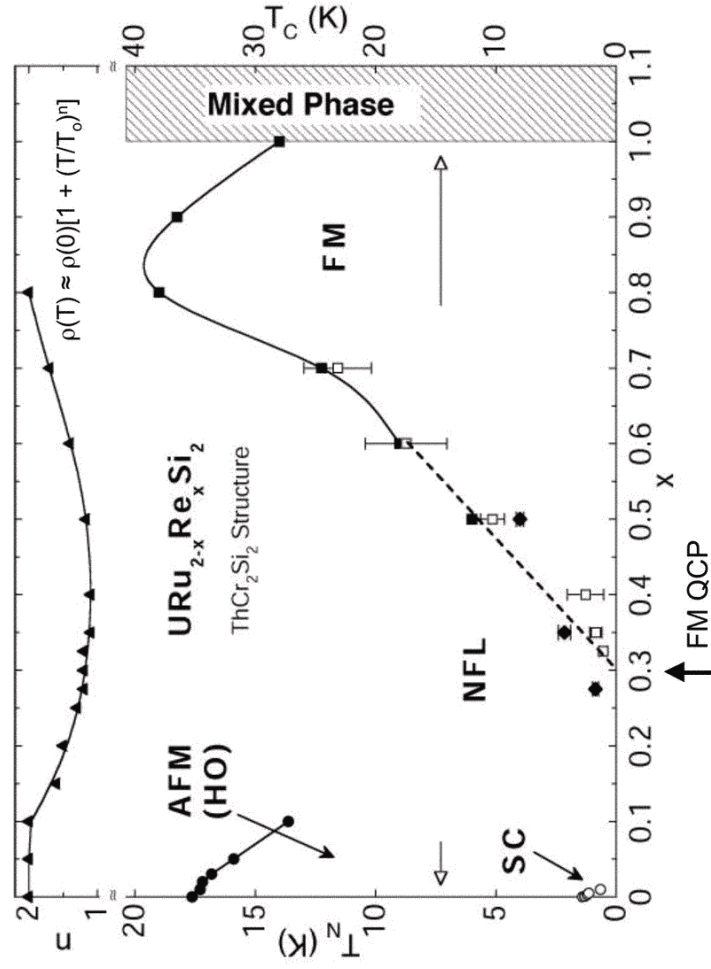
Dalichaouch, Maple, Chen, Kohara, Rossel, Torikachvili, Giorgi '90

URu_{2-x}M_xSi₂ (M=Re, Tc) ⇒ FM!



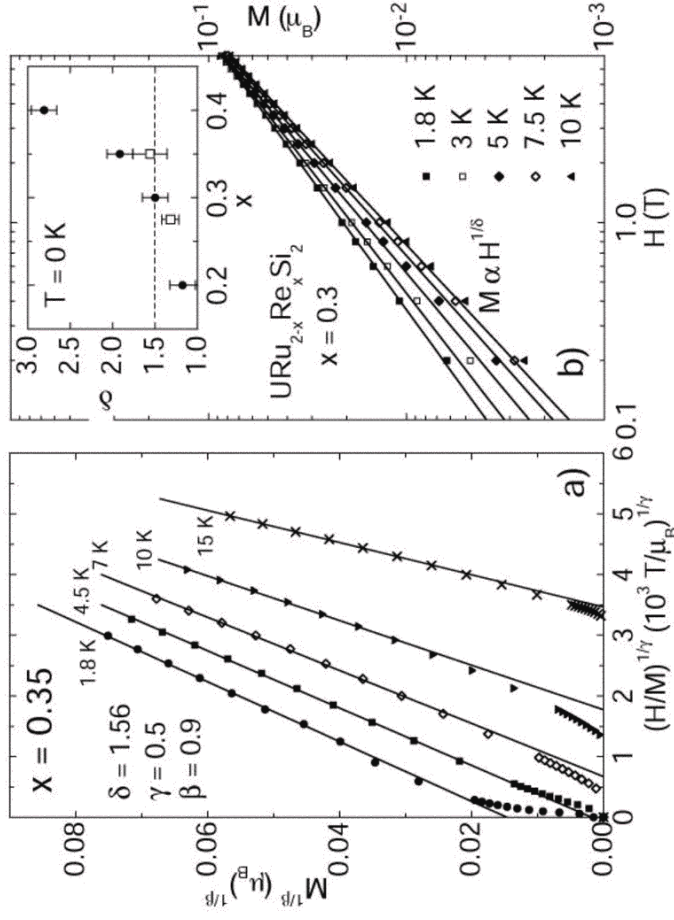
Dalichaouch, Torikachvili, Maple, Giorgi '89

T-x phase diagram of the URu_{2-x}Re_xSi₂ system



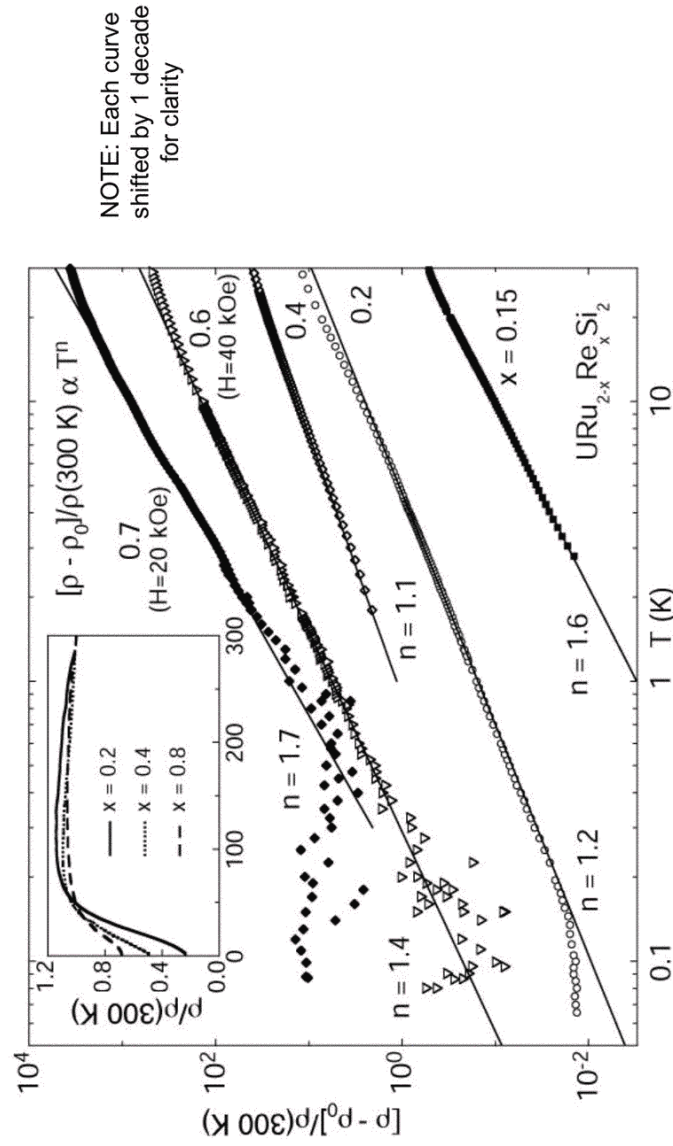
E. D. Bauer, V. S. Zapf, P.-C. Ho, E. J. Freeman, C. Sirvent, M. B. Maple, PRL '05 (in press)

URu_{2-x}Re_xSi₂: scaled Arrott plots



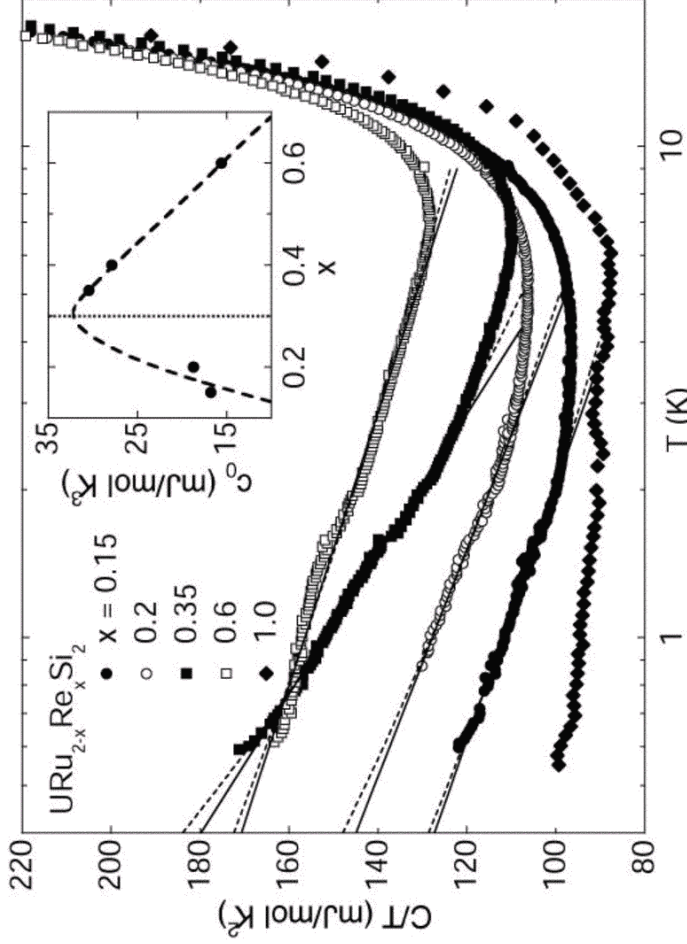
E. D. Bauer, V. S. Zapf, P.-C. Ho, E. J. Freeman, C. Sirvent, M. B. Maple, PRL '05 (in press)

URu_{2-x}Re_xSi₂: ρ(T) at low T



E. D. Bauer, V. S. Zapf, P.-C. Ho, E. J. Freeman, C. Sirvent, M. B. Maple, PRL '05 (in press)

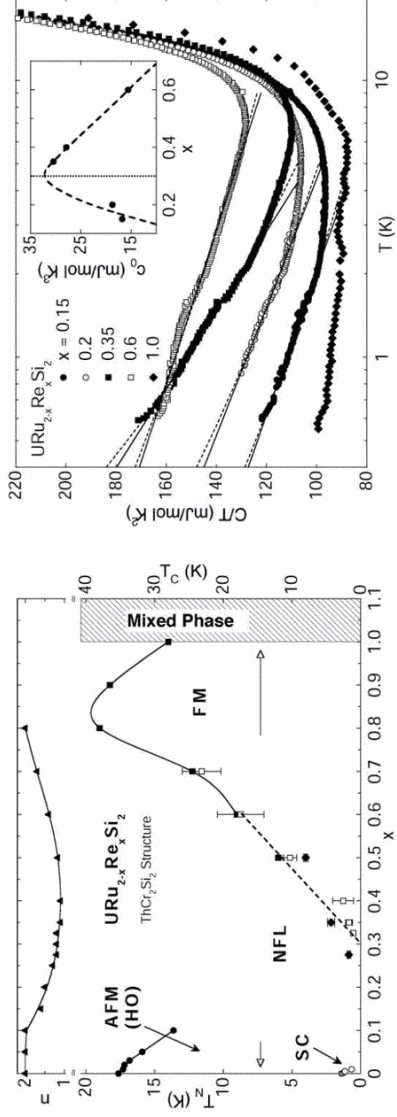
URu_{2-x}Re_xSi₂: C(T) at low T



$C(T)/T = \gamma_0 - c_0 \ln T$
(or T^{-n} where $n \approx 0.1 - 0.2$)

E. D. Bauer, V. S. Zapf, P.-C. Ho, E. J. Freeman, C. Sirvent, M. B. Maple, PRL '05 (in press)

T-x phase diagram and C(T) of the URu_{2-x}Re_xSi₂ system



NFL behavior:

$\sim 0.15 < x < \sim 0.6$ (extends into FM phase $\sim 0.3 < x < \sim 1.0$)

$C(T)/T = \gamma_0 - c_0 \ln T$ (or T^{-n} where $n \approx 0.1 - 0.2$)

$\rho(T) \propto T^n$ ($n \approx 1 - 1.5$)

Deviation from FL behavior greatest at $x_c \approx 0.3$ (FM QCP) where

c_0 maximum & n minimum ($n \approx 1$)

$\delta(0)$ varies with x ; $\delta(0) = 1.5$ at $x_c \approx 0.3$

$\chi(T) \propto T^{-n}$ in PM region ($\sim 0.15 < x < \sim 0.3$)

($n \approx 0.2$ for $x = 0.1$, $n \approx 0.4$ for $x = 0.3$)

Possible scenarios for NFL behavior in $URu_{2-x}Re_xSi_2$

- Griffiths-McCoy phase
Competition between Kondo effect & RKKY interaction + disorder \Rightarrow magnetic clusters embedded in a nonmagnetic FL \Rightarrow NFL characteristics

Castro Neto, Castilla, Jones '98

$$C(T)/T \propto \chi(T) \propto T^{-n} \quad (n \approx 0.2 - 0.3)$$

- Could account for NFL behavior in PM & FM phases in vicinity of FM QCP
Spin fluctuation models

Millis '93; Moriya, Takamoto '95

2D:	$\rho(T) \propto T^{5/3}$ $C(T)/T \propto -\ln T$ $\chi(T) \propto T^{-4/3}$ $\theta_c \propto \delta - \delta_c ^{3/4}$	3D:	$\rho(T) \propto T^{4/3}$ $C(T)/T \propto T^{-1/3}$ $\chi(T) \propto T^{-1}$ or $1/T \ln T$ $\theta_c \propto \delta - \delta_c $
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- Quantum critical behavior of itinerant FMs incorporating effects of nonmagnetic disorder

Belitz, Kirkpatrick '01

Near x_c $C(T)/T \propto -\ln T$
 $\beta = 2, \gamma = 1, \delta = 1.5$