

Quantum Optical Tools for Generation, Manipulation, and Diagnostic of Driven-Dissipative Many-Body States of Light

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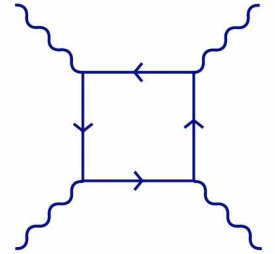
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- A. Amo, J. Bloch (C2N-CNRS)
- C. Ciuti (Paris 7)
- A. Biella, D. Rossini (SNS), R. Fazio (ICTP)
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- N. Goldman (UL Brussels)
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- P. Comaron, N. Proukakis (Newcastle)
- M. Wimmer, U. Peschel (Jena)

Why not hydrodynamics of light ?

Light field/beam composed by a huge number of photons

- in vacuo photons travel along straight line at c
- (practically) do not interact with each other
- in cavity, collisional thermalization slower than with walls and losses

\Rightarrow optics typically dominated by single-particle physics

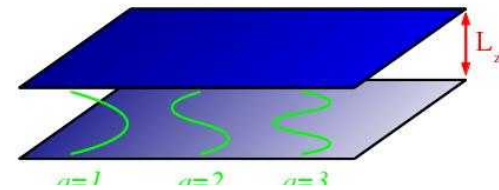


$$\sigma \sim \alpha^4 \frac{\hbar^2}{m^2 c^2} \left(\frac{\hbar \omega}{mc^2} \right)^6$$

In photonic structure:

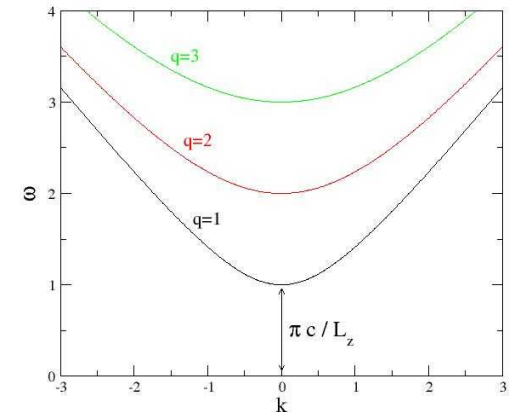
$\chi^{(3)}$ nonlinearity \rightarrow photon-photon interactions

Spatial confinement \rightarrow effective photon mass



\Rightarrow collective behaviour of a quantum fluid

Many experiments and many intriguing effects

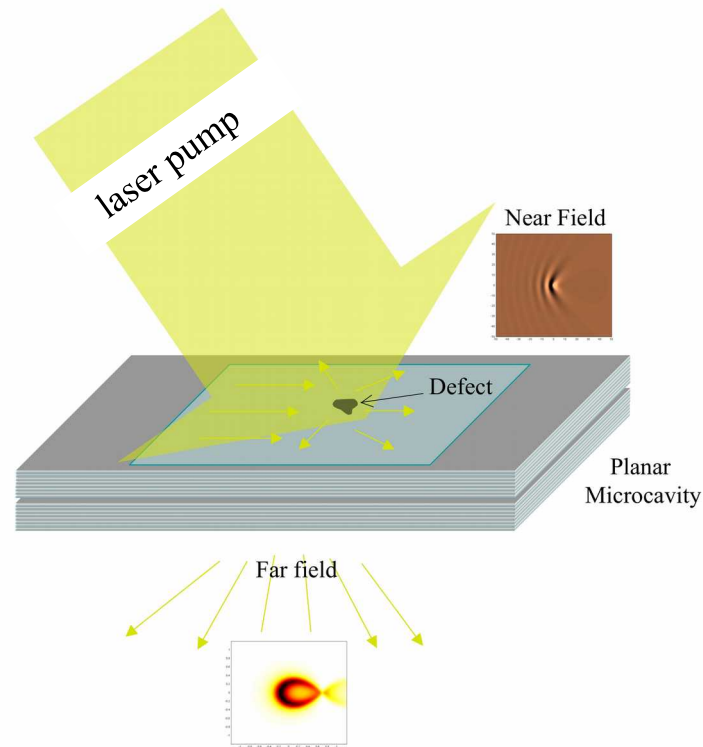


In this talk:

A few selected topics related to open quantum systems

How to exploit driven-dissipative condition to generate/manipulate/diagnostic quantum many-body states?

How to create and detect the photon gas?



Photon fluid consists of excitations on top of ground state:

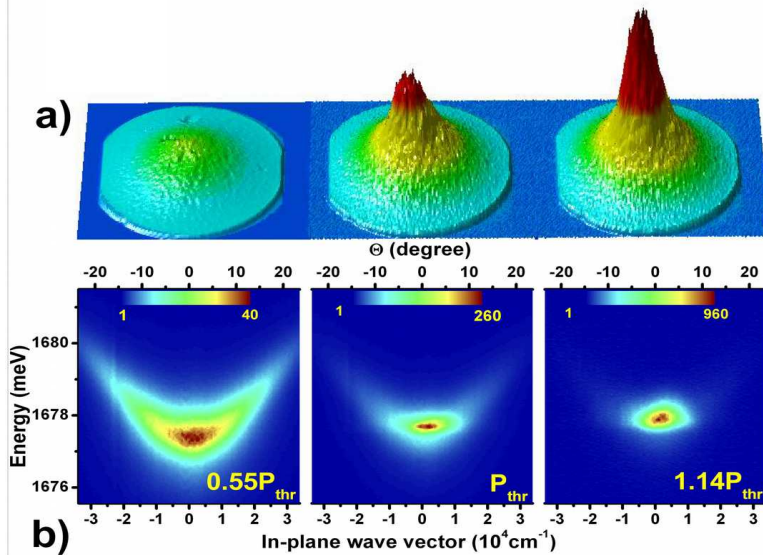
- Radiative and non-radiative losses intrinsic to optical systems
- Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium
 - Coherent laser pump: directly injects photon BEC in cavity, may lock BEC phase
 - Incoherent (optical or electric) pump: BEC transition similar to laser threshold
spontaneous breaking of U(1) symmetry
- Classical and quantum correlations of in-plane field directly transfer to emitted radiation

Part 1:

Open system features in weakly-interacting photon fluids

*non-equilibrium Bose-Einstein condensation
& non-equilibrium superfluidity*

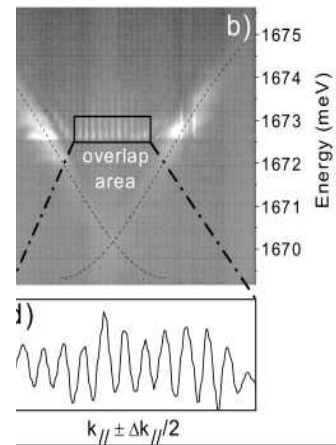
2006 - Photon/polariton Bose-Einstein condensation



Momentum distribution

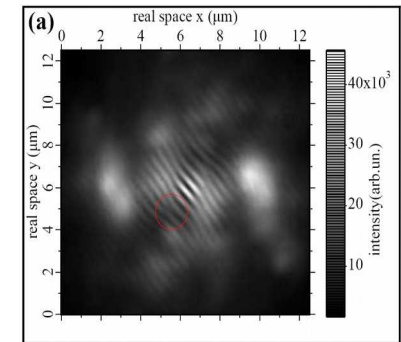
Kasprzak et al., Nature **443**, 409 (2006)

Many features very similar to atomic BEC



Interference

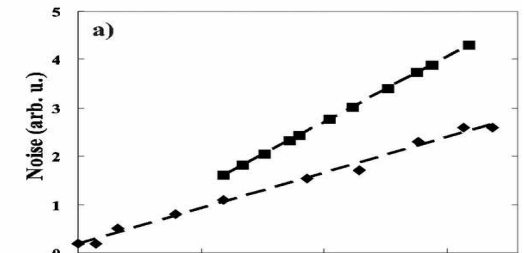
Richard et al., PRL **94**, 187401 (2005)



Quantized vortices

K. Lagoudakis *et al.*

Nature Physics **4**, 706 (2008).



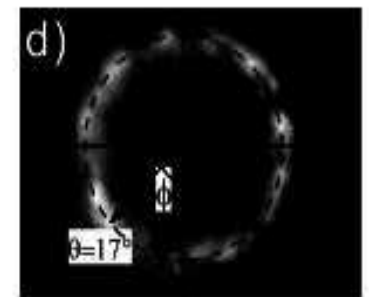
Suppressed fluctuations

A. Baas et al., PRL **96**, 176401 (2006)

But also differences due to non-equilibrium:

- BEC @ $k \neq 0$ → volcano effect
- T-reversal broken → $n(k) \neq n(-k)$
- interesting questions about thermalization

Photon/polariton BEC closely related to laser operation in VCSELs



BEC on k-space ring

M. Richard et al.,

PRL **94**, 187401 (2005)

2008 - Superfluid light (under coherent pump)

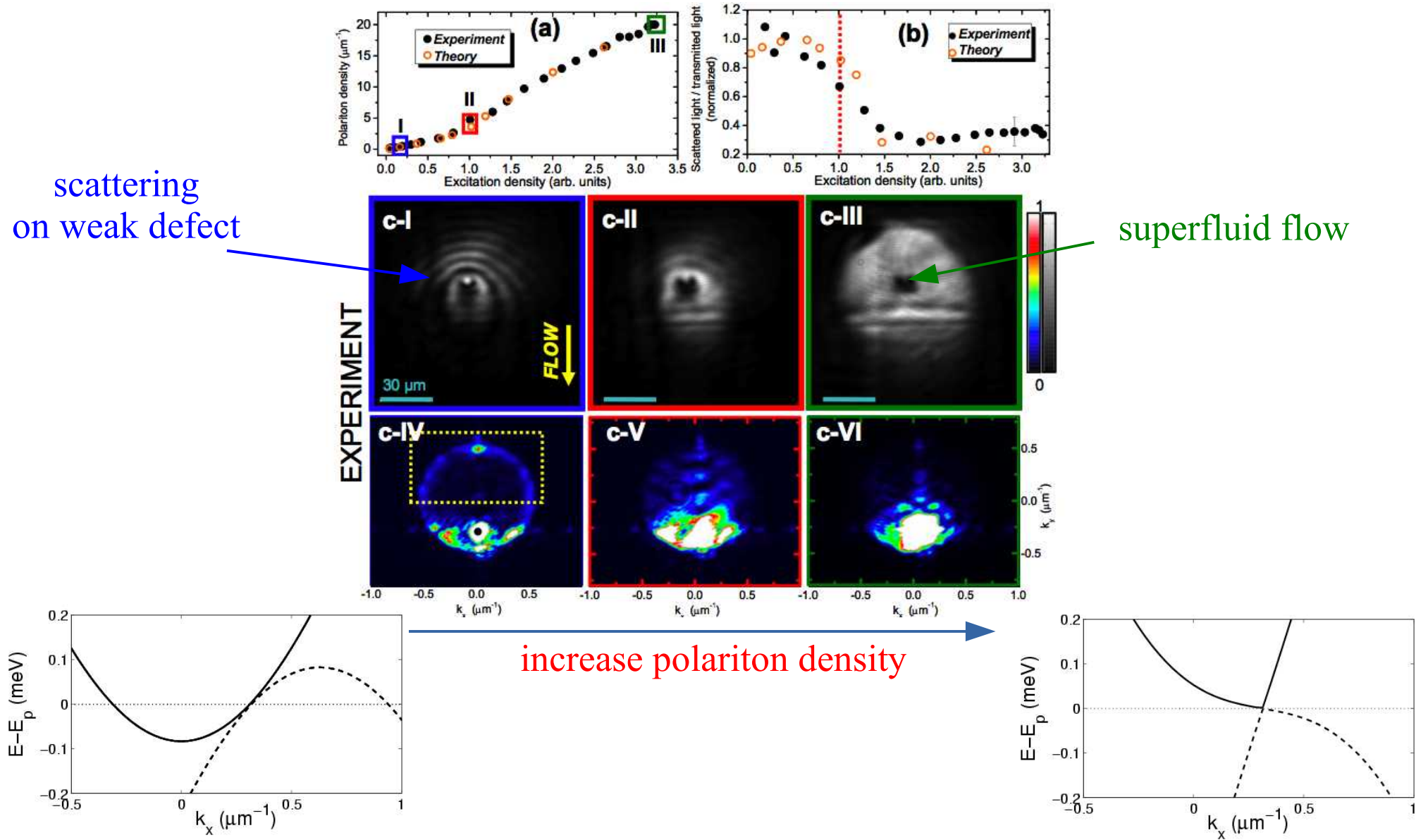


Figure from LKB-P6 group: A.Amo, J.Lefrère, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)

Theory: IC and C. Ciuti, PRL **93**, 166401 (2004)

Are non-equilibrium BECs superfluid?

Polariton superfluidity expts under **coherent pump** → **explicitly broken U(1)**
 (non-equilibrium) BEC phase transition → spontaneous breaking of U(1),
 e.g. under **incoherent pump** (or polariton lasing)

Driven-dissipative GPE

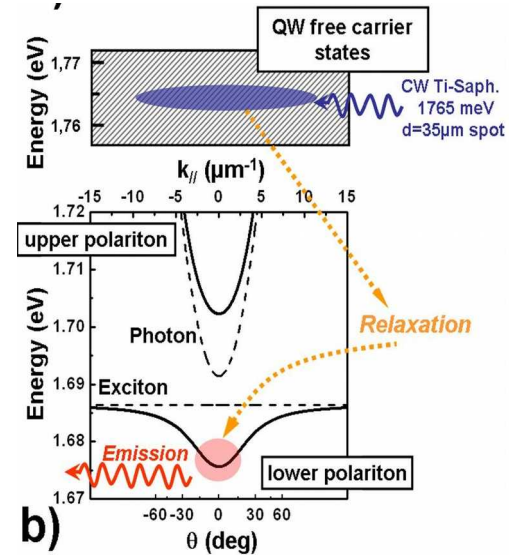
$$\begin{cases} i\frac{\partial\psi}{\partial t} = \left\{ -\frac{\hbar^2\nabla^2}{2m_{LP}} + \frac{i}{2}[R(n_R) - \gamma] + g|\psi|^2 + 2\tilde{g}n_R \right\} \psi. \\ \frac{\partial n_R}{\partial t} = P - \gamma_R n_R - R(n_R)|\psi(\mathbf{r})|^2 + D\nabla^2 n_R. \end{cases}$$

Linearize GPE around **steady state**

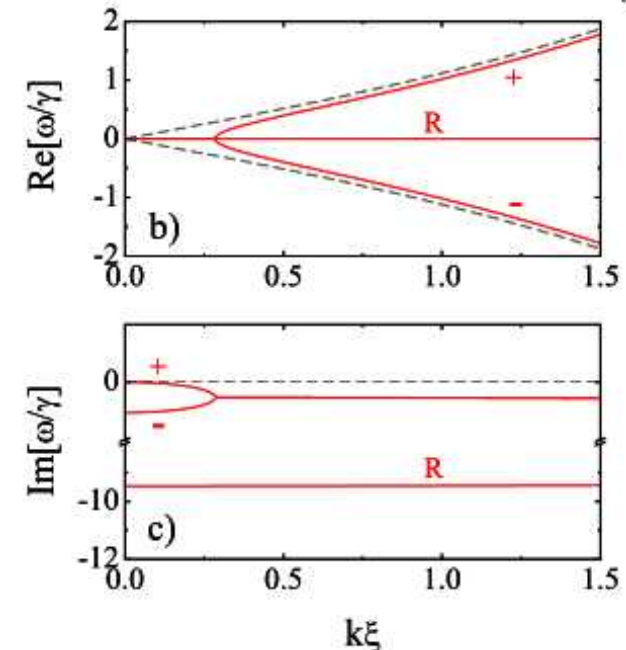
→ Reservoir R mode at $-i\gamma_R +$ **BEC density** and **phase** modes at:

$$\omega_{\pm}(k) = -\frac{i\Gamma}{2} \pm \sqrt{[\omega_{Bog}(k)]^2 - \frac{\Gamma^2}{4}}$$

- density (-) and phase (+) oscillations **decoupled** around $k=0$
- Goldstone phase (+) mode is **diffusive**



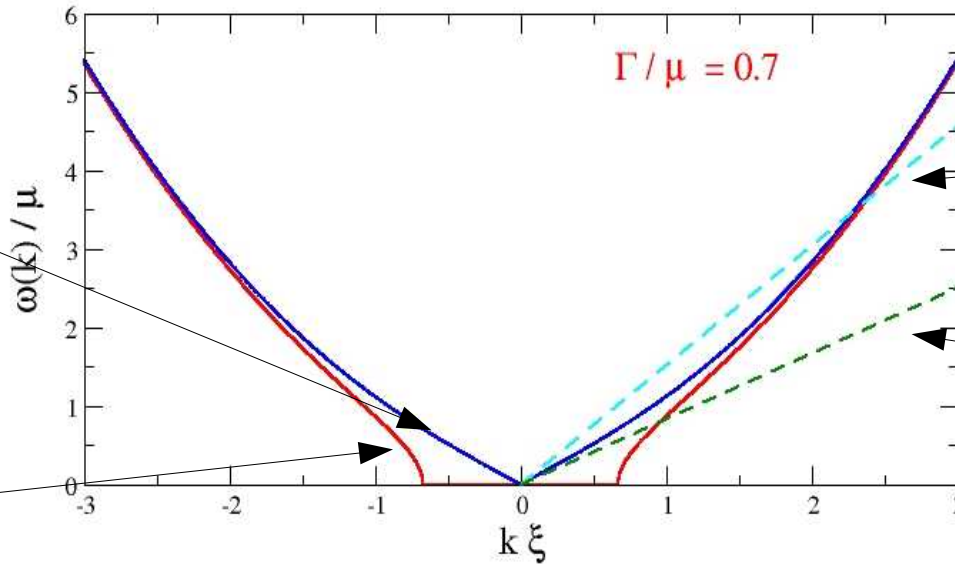
b) Figure from Kasprzak et al., Nature 2006



Consequences on superfluidity

Equilibrium
Bogoliubov
dispersion

Non-equilibrium
Bogoliubov
dispersion



Long-range coherence \rightarrow metastability of supercurrents (mode stability of ring lasers)

Interaction with defect: naïf Landau argument

- Landau critical velocity $v_L = \min_k[\omega(k)/k] = 0$ at non-equilibrium BEC
- Any moving defect expected to emit phonons

But nature is always richer than expected...

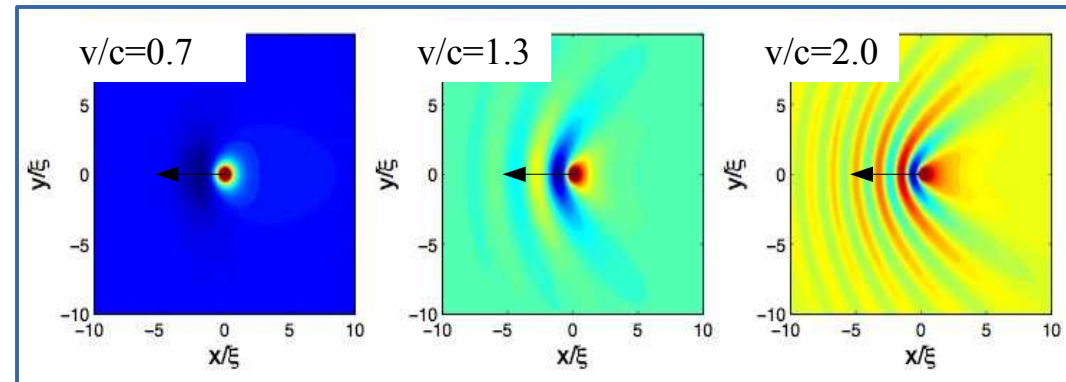
Steady-state \rightarrow well defined ω

Defect \rightarrow k not a good quantum number

(Complex) k vs. (real) ω dispersion

Low v :

- emitted k_{\parallel} purely imaginary
- no real propagating phonons
- perturbation localized around defect

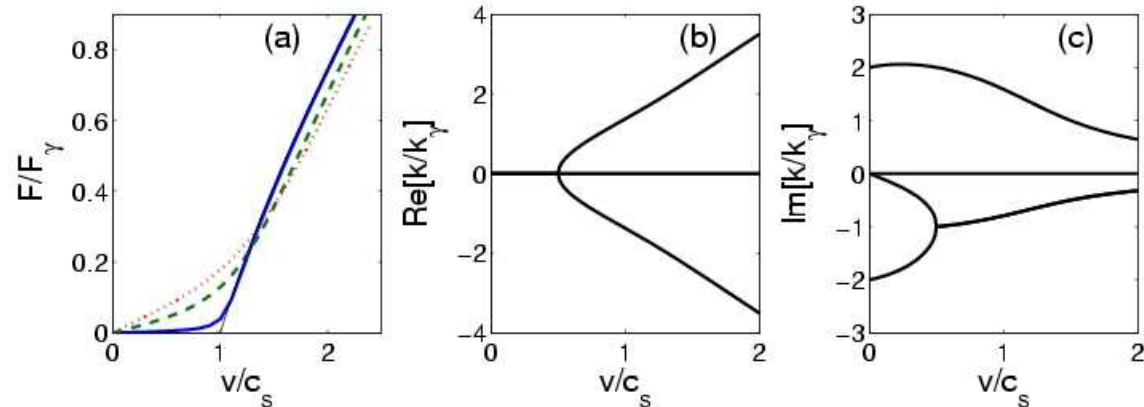


Critical velocity $v_c < c$:

- corresponds to bifurcation point
- decreases with Γ / μ

High v :

- emitted propagating phonons:
 - \rightarrow Cerenkov cone
 - \rightarrow parabolic precursors
- spatial damping of Cerenkov cone



Part 2:

Strongly interacting photons

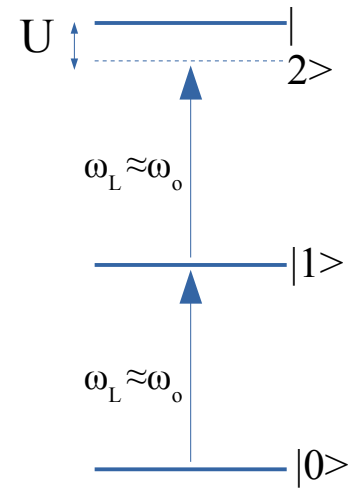
*from Tonks-Girardeau gases
to Mott insulator states*

Photon blockade

Driven-dissipative Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_0 \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i F_i(t) \hat{b}_i + h.c.$$

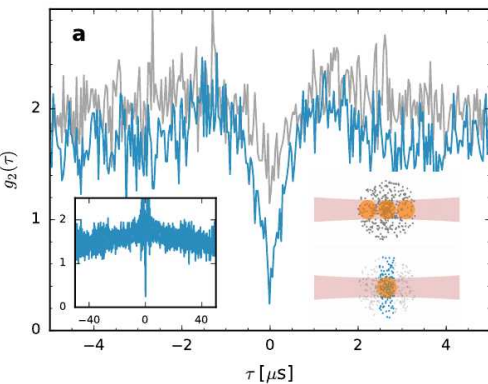
- Array of single-mode cavities at ω_0 , tunneling coupling J
- Losses $\gamma \rightarrow$ Lindblad terms in master eq.
- Polariton interactions: on-site interaction U due to optical nonlinearity
- If $U \gg \gamma, J$, coherent pump resonant with $0 \rightarrow 1$ transition, but not with $1 \rightarrow 2$ transition.



Photon blockade \rightarrow Effectively impenetrable photons

- Single-cavity blockade observed in many platforms since the 2000s, present challenge \rightarrow scale up to many-cavity geometry

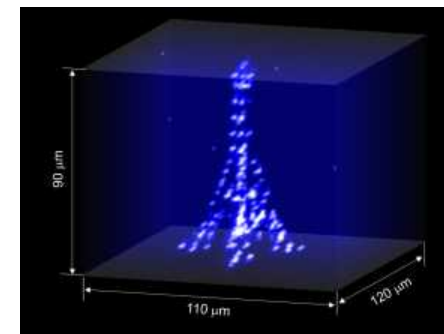
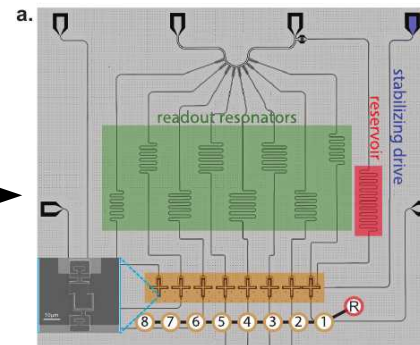
Fluid of spin excitations in lattice of Rydberg atoms.
More in Antoine's talk
Similar expt by Lukin's group



Polariton blockade via Rydberg-EIT

Circuit QED device

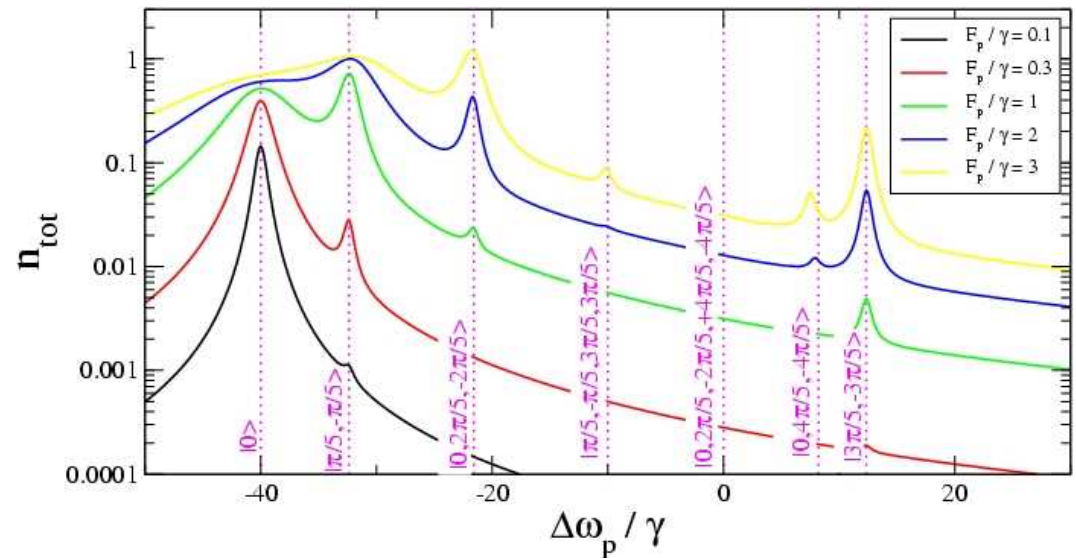
More in next talk by Jon (and in Pedram's one)



Impenetrable “fermionized” photons in 1D necklaces

Many-body eigenstates of
Tonks-Girardeau gas
of impenetrable photons

Coherent pump
selectively addresses
specific many-body states



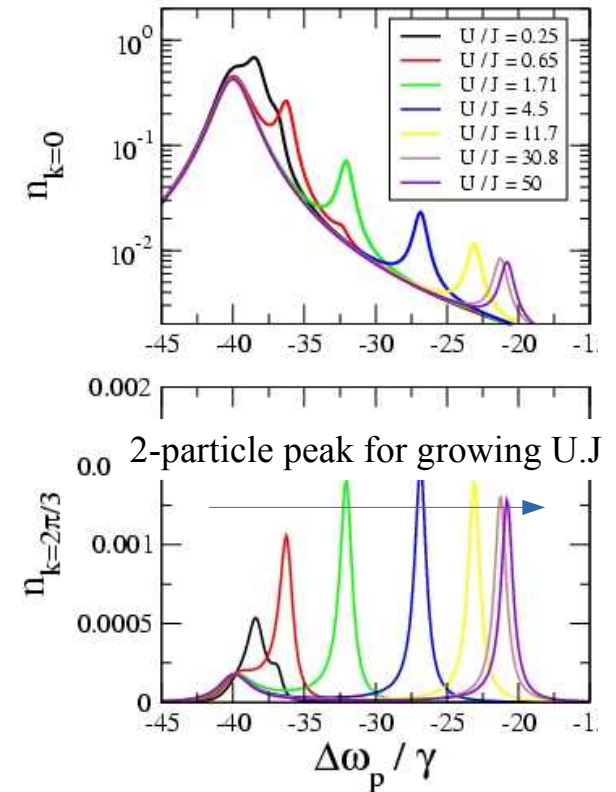
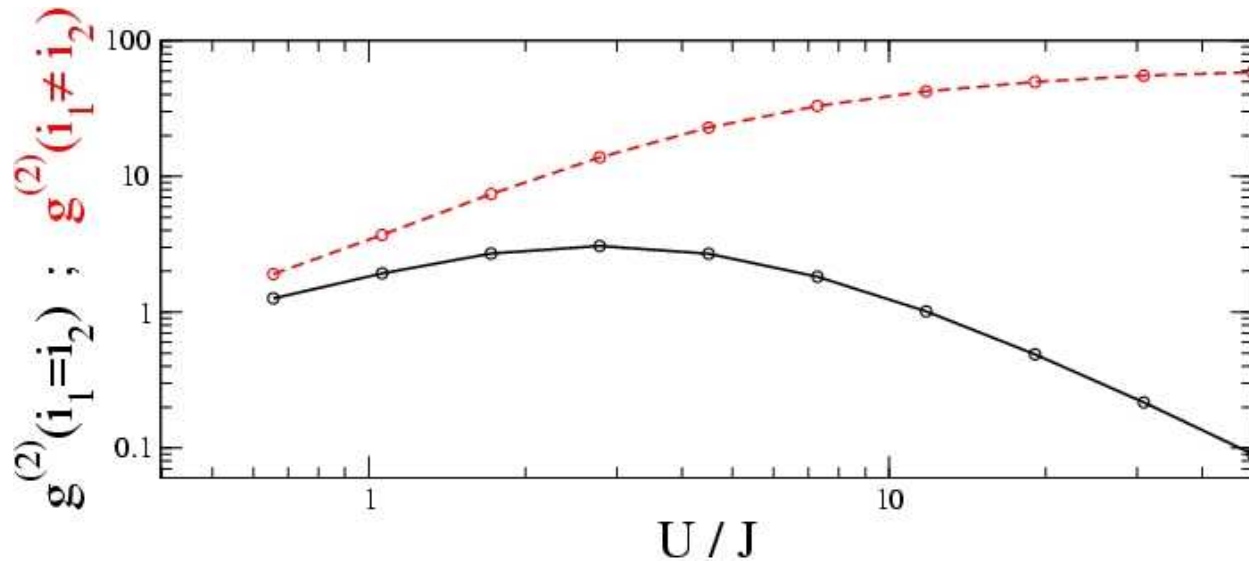
Transmission spectrum as a function pump frequency for fixed pump intensity:

- each peak corresponds to a Tonks-Girardeau many-body state $|q_1, q_2, q_3, \dots\rangle$
- q_i quantized according to PBC/anti-PBC depending on $N=\text{odd}/\text{even}$
- $U/J \gg 1$: efficient photon blockade, impenetrable photons.

N -particle state excited by N photon transition:

- Plane wave pump with $k_p=0$: selects states of total momentum $P=0$
- Monochromatic pump at ω_p : resonantly excites states of many-body energy E such that $\omega_p = E / N$

State tomography from emission statistics



Finite U/J , pump laser tuned on two-photon resonance

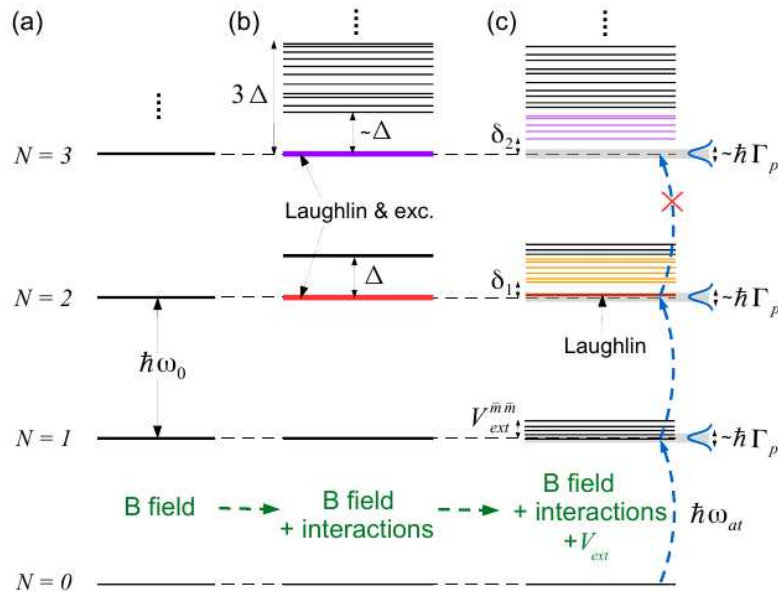
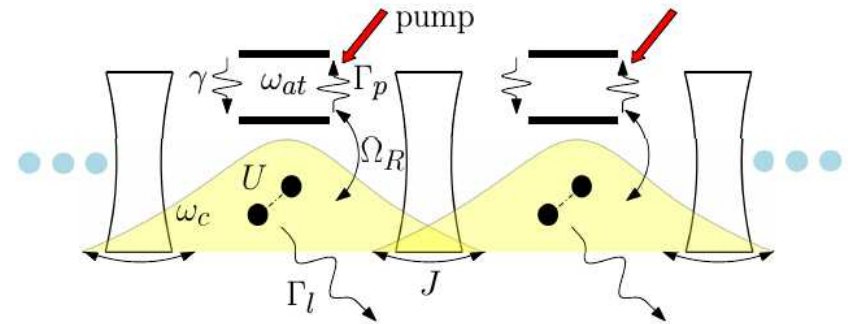
- intensity correlation between the emission from cavities i_1, i_2
- at large U/γ , larger probability of having $N=0$ or 2 photons than $N=1$
 - low $U \ll J$: bunched emission for all pairs of i_1, i_2
 - large $U \gg J$: antibunched emission from a single site
positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

How to access larger particle numbers

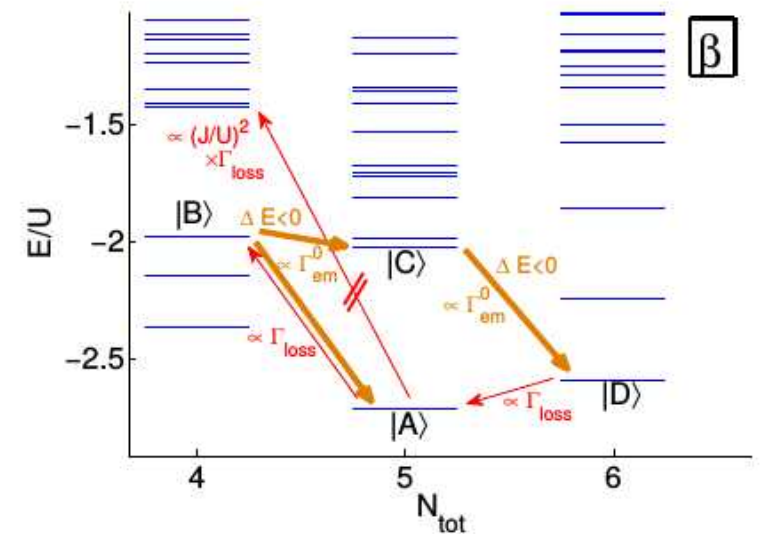
Coherent pump only able to selectively excite few-photon states

→ Frequency-dependent incoherent pumping, e.g. collection of inverted emitters

- Lorentzian emission line around ω_{at}
sophisticated schemes → other spectral shapes
- Emission only active if many-body transition is near resonance
- Injects photons until band is full (MI) or many-body gap develops (FQH)
- Many-body gap blocks excitation of higher bands



Umucalilar-IC, PRA 2017



Lebreuilly, Biella et al., PRA 2017

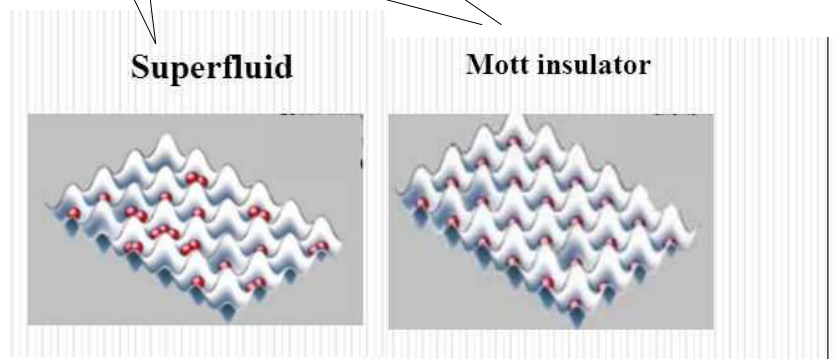
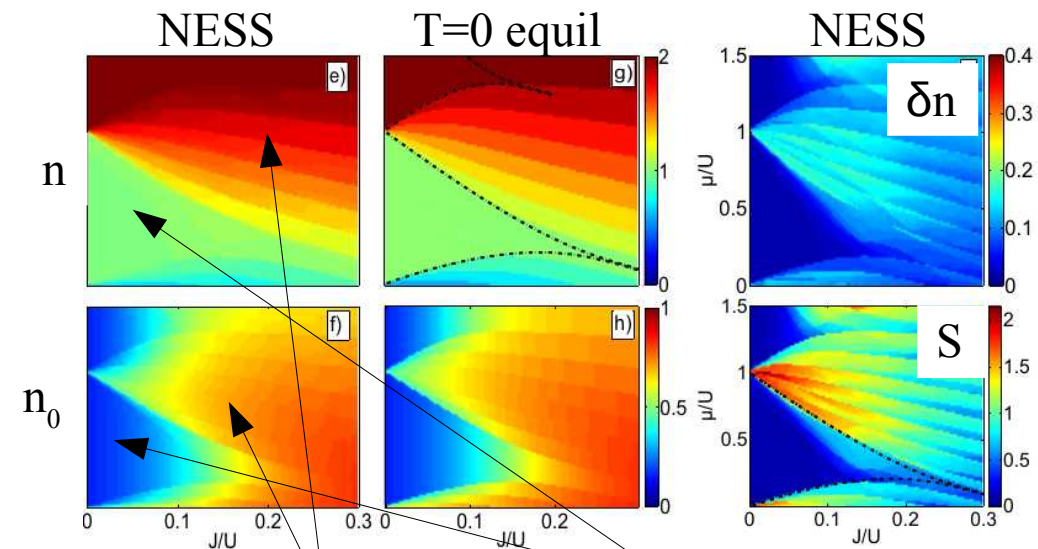
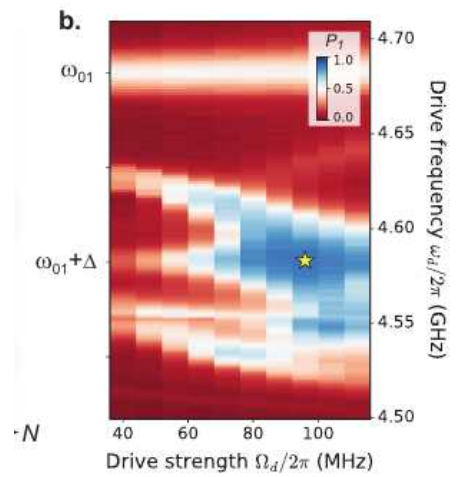
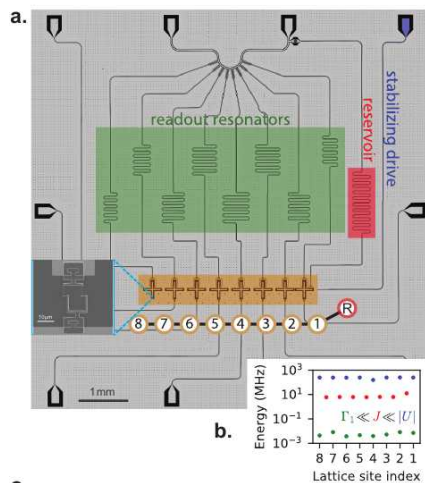
General idea: Lebreuilly et al. CRAS (2016) and Kapit, Hafezi, Simon, PRX 2014

Numerical validation for MI & FQH

- non-Markovian master equation:
frequency-dependent emission
→ rescaled jump operators
- driven-dissipative steady state stabilizes
strongly correlated many-body states
e.g. Mott-insulator, FQH...
- resembles low-T equilibrium
(but interesting deviations in some cases)
- (in principle) no restriction to small N_{ph}

$$\bar{\mathcal{L}}_{\text{em}}(\rho_{\text{ph}}) = \frac{\Gamma_{\text{em}}}{2} \sum_{i=1}^k \left[2\bar{a}_i^\dagger \rho_{\text{ph}} \bar{a}_i - \bar{a}_i \bar{a}_i^\dagger \rho_{\text{ph}} - \rho_{\text{ph}} \bar{a}_i \bar{a}_i^\dagger \right]$$

$$\langle f' | \bar{a}_i^\dagger | f \rangle = \frac{\Gamma_{\text{pump}}/2}{\sqrt{(\omega_{\text{at}} - \omega_{f',f})^2 + (\Gamma_{\text{pump}}/2)^2}} \langle f' | a_i^\dagger | f \rangle$$



Lebreuilly, Biella et al., 1704.01106 & 1704.08978
(published on PRA, 2017)

First expt: Ma *et al.* Nature 2019
All details in next talk by Jon!

Related work in Kapit, Hafezi, Simon, PRX 2014

Part 3:

Topological photonics

*from Chern insulators
towards Fractional Quantum Hall liquids*

2009 - the dawn of Topological Photonics: Photonic (Chern) topological insulator

MIT '09, Soljacic group

Original proposal Haldane-Raghu, PRL 2008

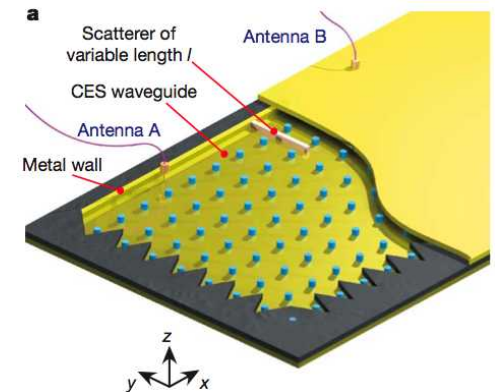
Magneto-optical photonic crystals for μ -waves

T-reversal broken by magnetic elements

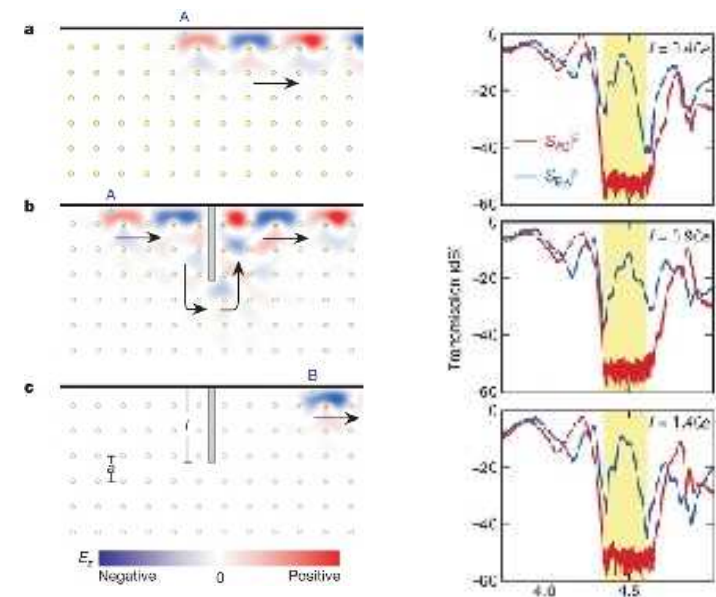
Band with non-trivial Chern number:
→ chiral edge states within gaps

Experiment:

- measure transmission from antenna to receiver
- only in one direction: unidirectional propagation
- immune to back-scattering by defects topologically protected



Wang et al., Nature 461, 772 (2009)



Wang et al., Nature 461, 772 (2009)

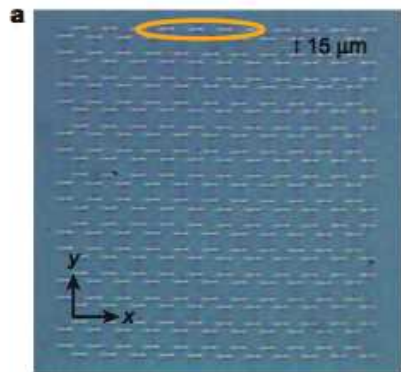
2013 - Harper-Hofstadter & Haldane models for visible photons

2D lattice of coupled cavities with tunneling phase

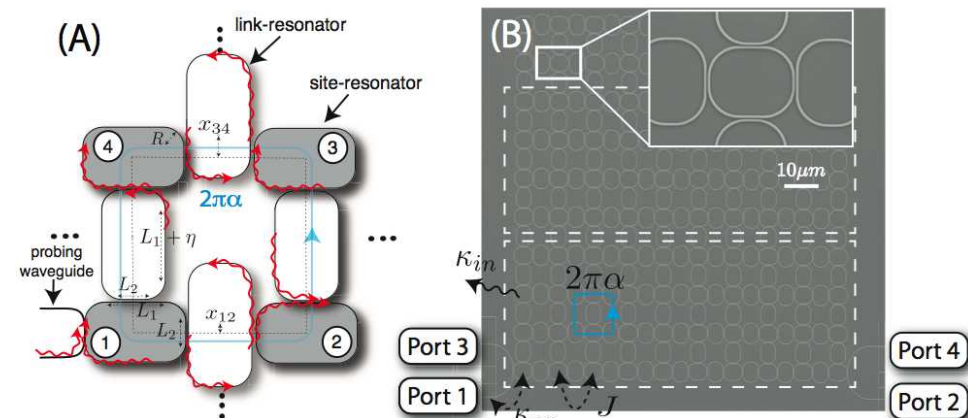
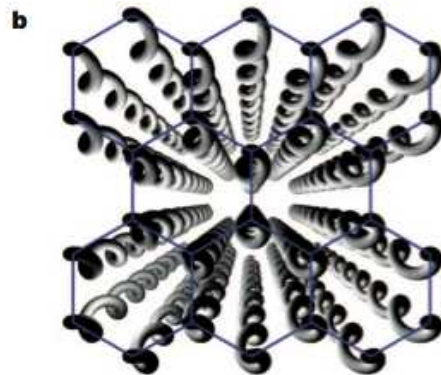
$$H = \sum_i \hbar\omega_0 \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} + \sum_i \left[\hbar F_i(t) \hat{a}_i^\dagger + \text{h.c.} \right]$$

Experiments along these lines:

- Floquet bands in helically deformed **honeycomb waveguide lattices** → Rechtsman/Segev
- **silicon ring cavities** → Hafezi/Taylor (JQI)
- **electronic circuits** with lumped elements → J. Simon (Chicago)
- **honeycomb lattice** for polaritons → A. Amo/J.Bloch (C2N)



Rechtsman, Plotnik, et al., Nature 496, 196 (2013)

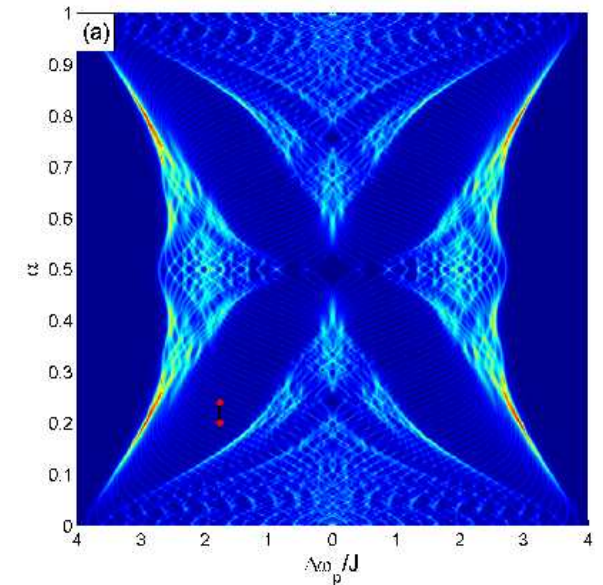


Hafezi et al., Nat. Phot. 7, 1001 (2013)

2013 - Imaging chiral edge states

2D square lattice of coupled cavities
at large magnetic flux

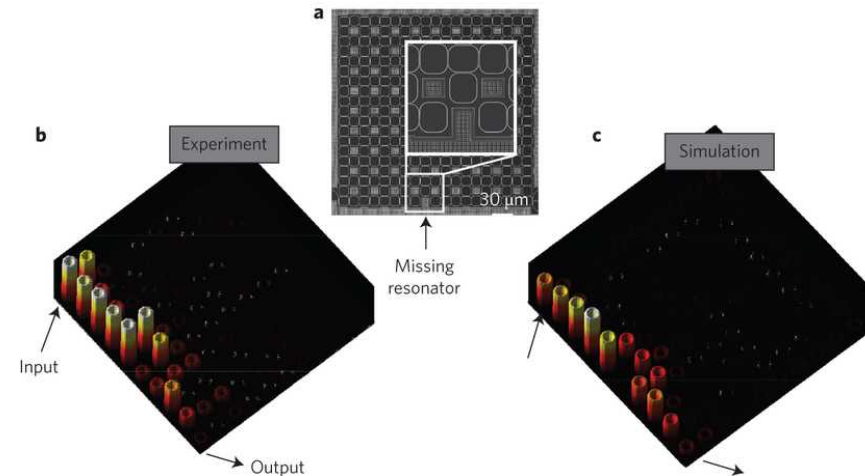
- eigenstates organize in **bulk Hofstadter bands**
- **Berry connection in k-space:** $A_{n,k} = i \langle u_{n,k} | \nabla_k u_{n,k} \rangle$



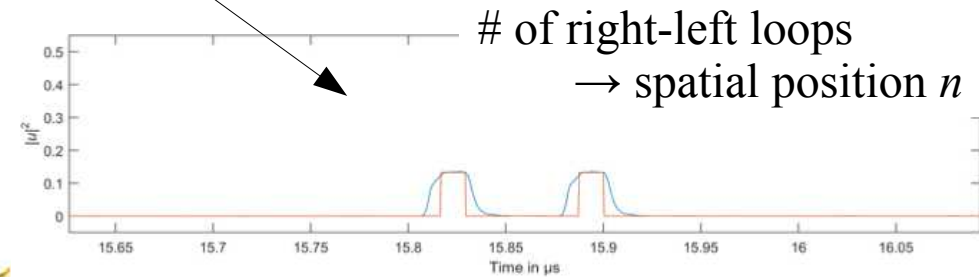
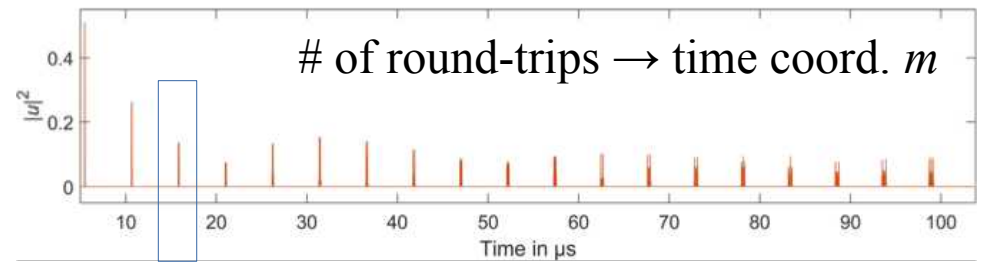
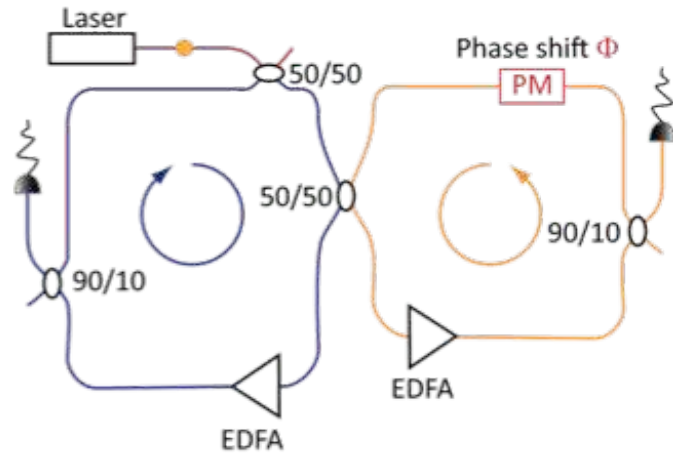
Bulk-edge correspondance:

$A_{n,k}$ has non-trivial **Chern number**
→ **chiral edge states** within gaps

- unidirectional propagation
- (almost) immune to scattering by defects

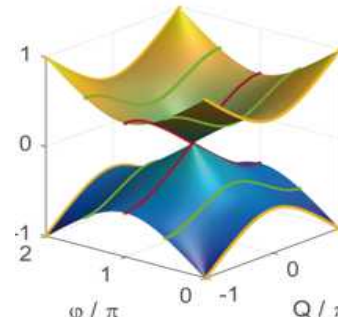


2016 - Experimental mapping of Berry curvature



Periodic temporal modulation of $\Phi(m) = \pm\varphi$:

- 1D Floquet band structure $\theta(Q, \varphi)$, φ considered as 2nd dim



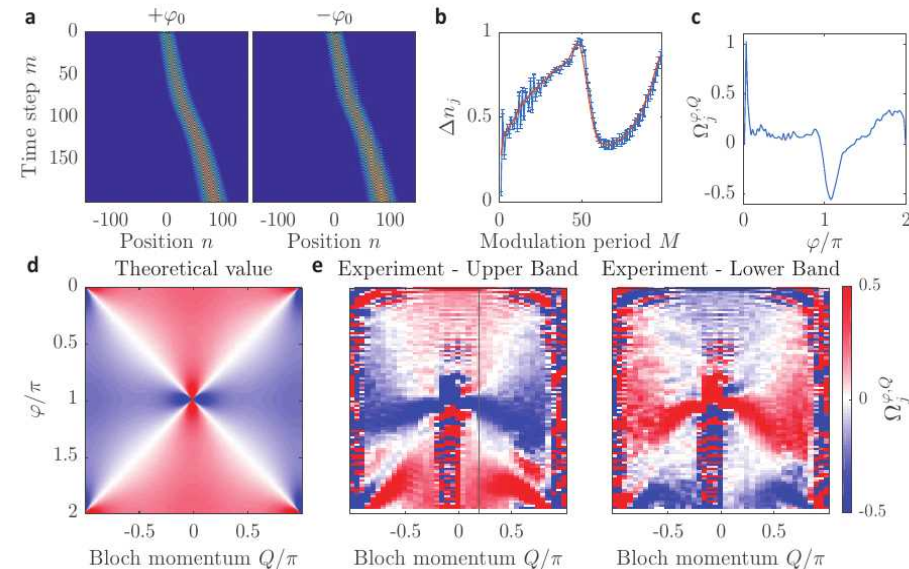
- Berry curvature

$$\Omega_j^{\varphi, Q} = \frac{\partial}{\partial \varphi} \langle \psi_j | i \frac{\partial}{\partial Q} | \psi_j \rangle - \frac{\partial}{\partial Q} \langle \psi_j | i \frac{\partial}{\partial \varphi} | \psi_j \rangle$$

- Geometrical charge pumping if φ adiabatically varied

- Look at lateral displacement along n at all times m
→ reconstruct Berry curvature $\Omega_j^{\varphi, Q}$ in whole FBZ

Cold atoms → Berry phase reconstructed via state tomography (Fläschner et al., Science '16)



Photon blockade + synthetic gauge field = FQHE for light

Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_0 \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

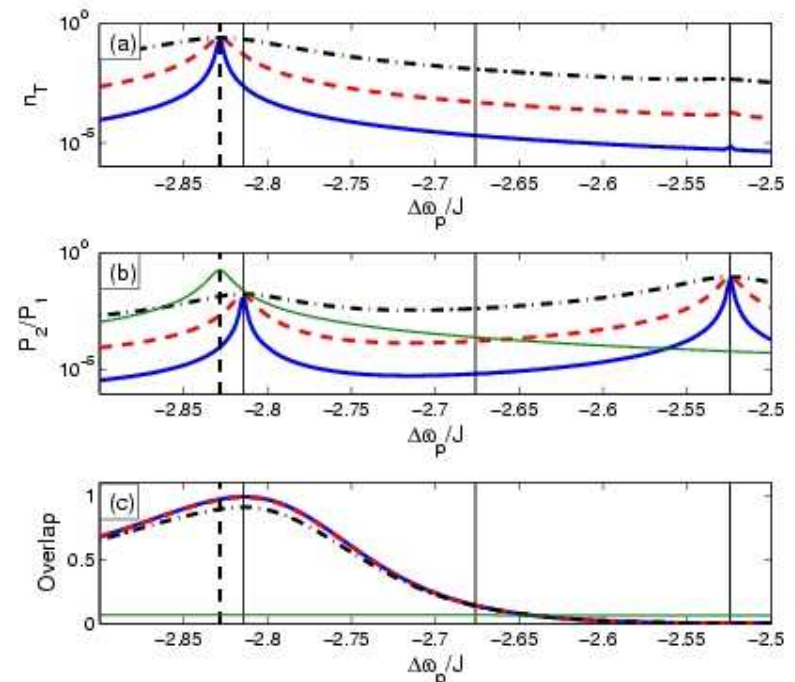
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1, \dots, z_N) = \mathcal{N}_L F_{\text{CM}}^{(l)}(Z) e^{-\pi\alpha \sum_i y_i^2} \times \prod_{i<j}^N \left(\vartheta \left[\begin{matrix} \frac{1}{2} \\ \frac{1}{2} \end{matrix} \right] \left(\frac{z_i - z_j}{L} \middle| i \right) \right)^2$$

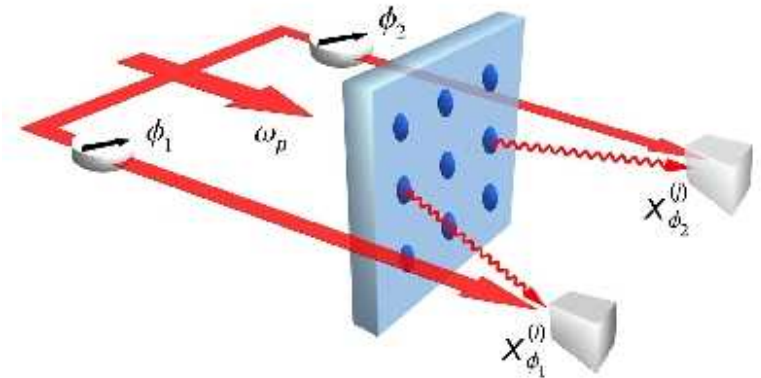
- no need for adiabatic following, etc....



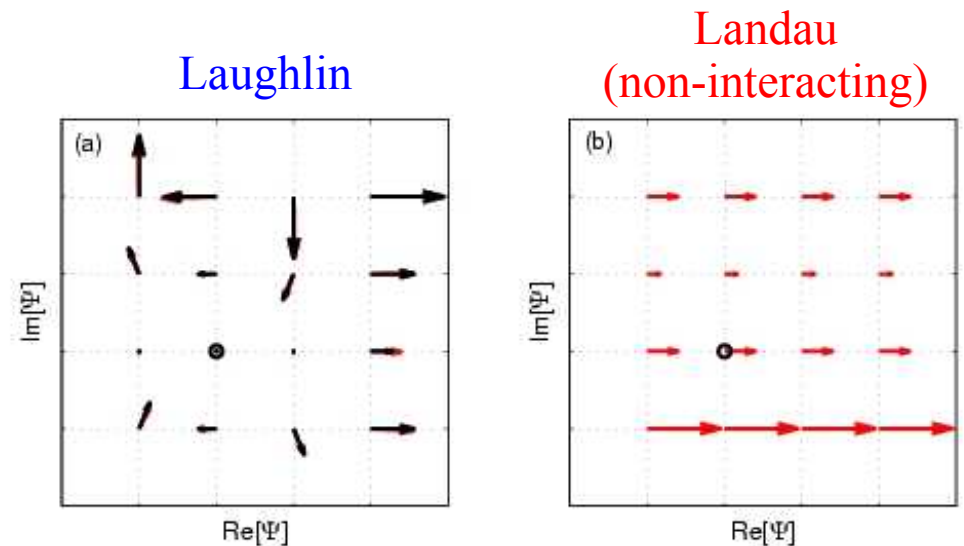
Tomography of FQH states

Homodyne detection of secondary emission

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$



Non-trivial structure of Laughlin state compared to non-interacting photons



Circuit-QED experiment

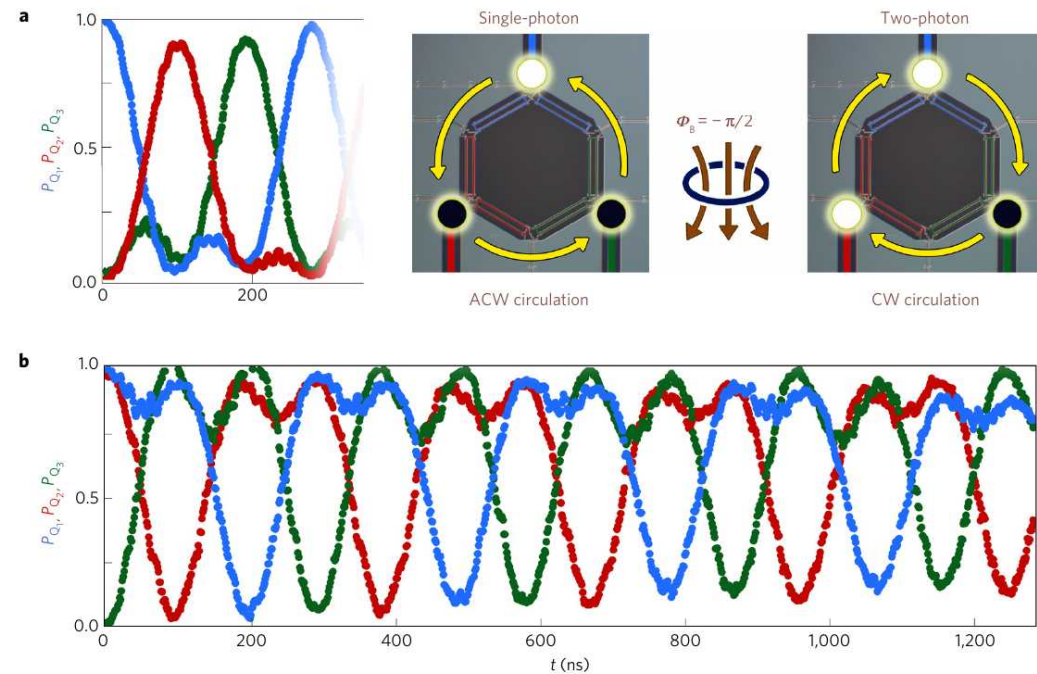
Ring-shaped array of qubits

- Transmon qubit: two-level system
→ Impenetrable photons
- Time-modulation of couplings
→ synthetic gauge field

Independently initialize sites

Follow unitary evolution until bosons lost

Monitor site occupation in time



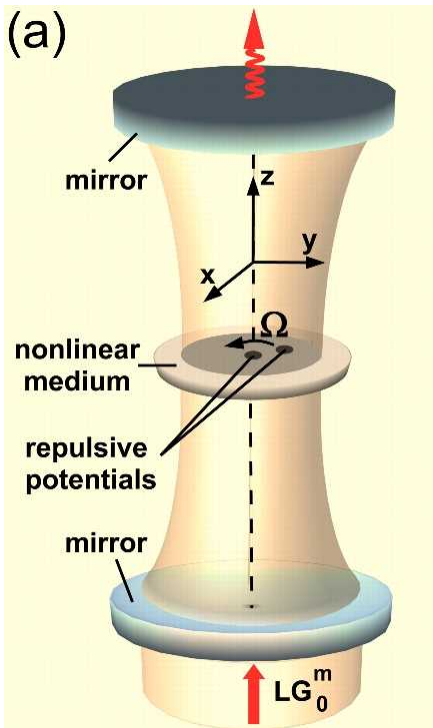
Roushan et al., Nat. Phys. 2016
More in Pedram's talk on Wed.

“Many”-body effect:

two-photon state → opposite rotation compared to one-photon state
(similar to cold-atom experiment in Greiner's lab: Tai et al., Nature 2017)

Continuous space FQH physics

Single cylindrical cavity. No need for cavity array



same form \rightarrow Coriolis $F_c = -2m\Omega \times v$
 \rightarrow Lorentz $F_L = e v \times B$

Photon gas injected by Laguerre-Gauss pump
 with finite orbital angular momentum

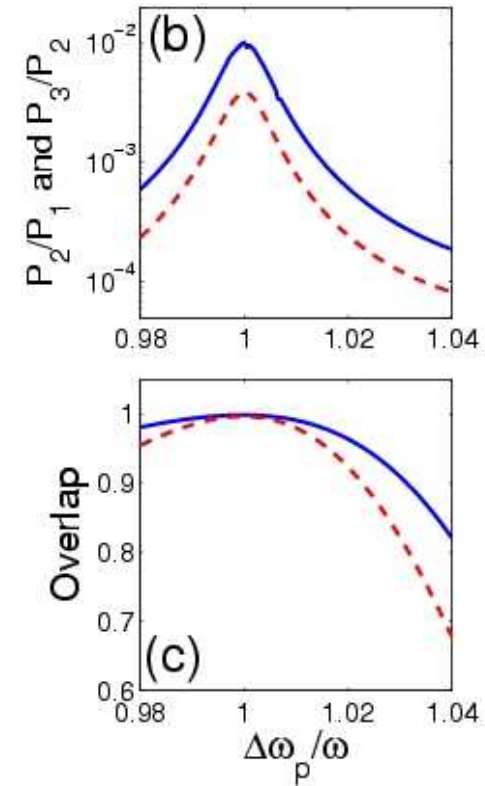
Strong repuls. interact., e.g. layer of Rydberg atoms

Resonant peak in transmission due to Laughlin state:

$$\psi(z_1, \dots, z_N) = e^{-\sum_i |z_i|^2 / 2} \prod_{i < j} (z_i - z_j)^2$$

Overlap measured from quadrature noise of transmitted light

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$

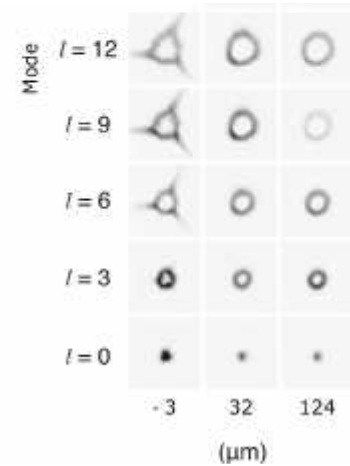
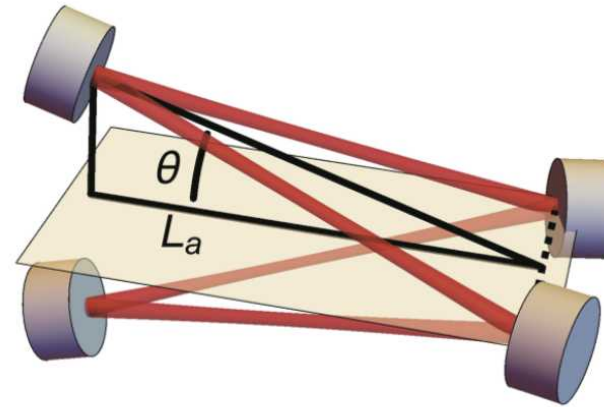


Experiment @ Chicago

A far smarter design

Non-planar ring cavity:

- Parallel transport \rightarrow synthetic B
- Landau levels for photons observed

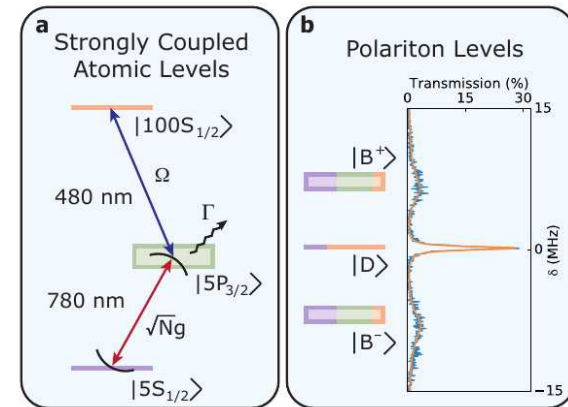


Crucial advantages:

- Narrow frequency range relevant
- Integrated with Rydberg-EIT reinforced nonlinearities

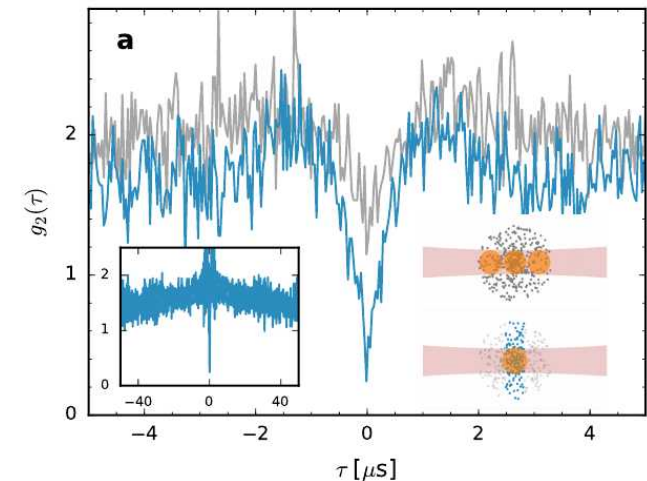
Polariton blockade on lowest (0,0) mode

- Equivalent to $\Delta_{\text{Laughlin}} > \gamma$: [Laughlin physics coming soon!](#)



Easiest strategy for Laughlin

- Coherent pumping \rightarrow multi-photon peaks to few-body states
- Laughlin state \rightarrow quantum correlations between orbital modes
(Umucalilar-Wouters-IC, PRA 2014)



How to access larger fluids?

Coherent pump:

- Able to selectively generate few-body states
- Limited by (exponentially) decreasing matrix element for larger systems

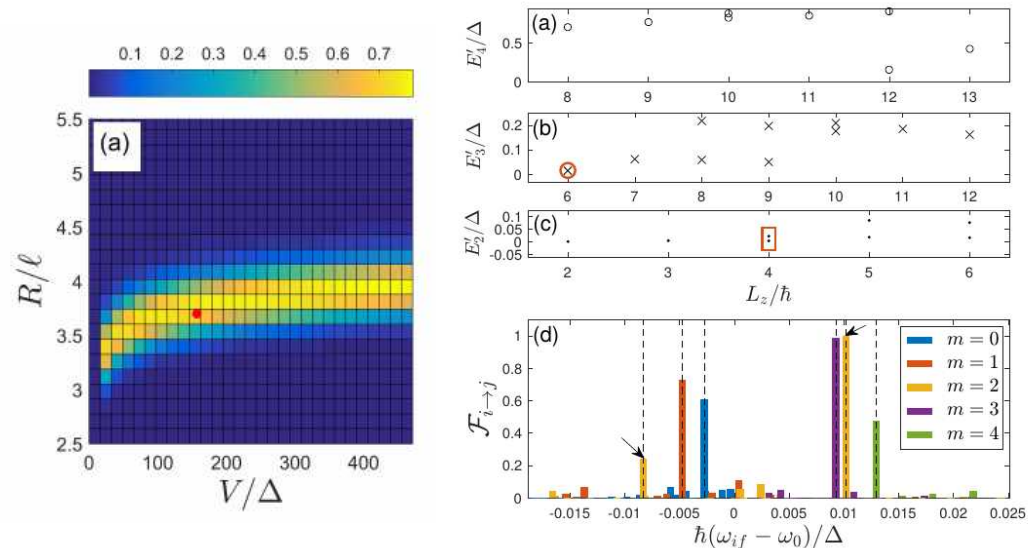
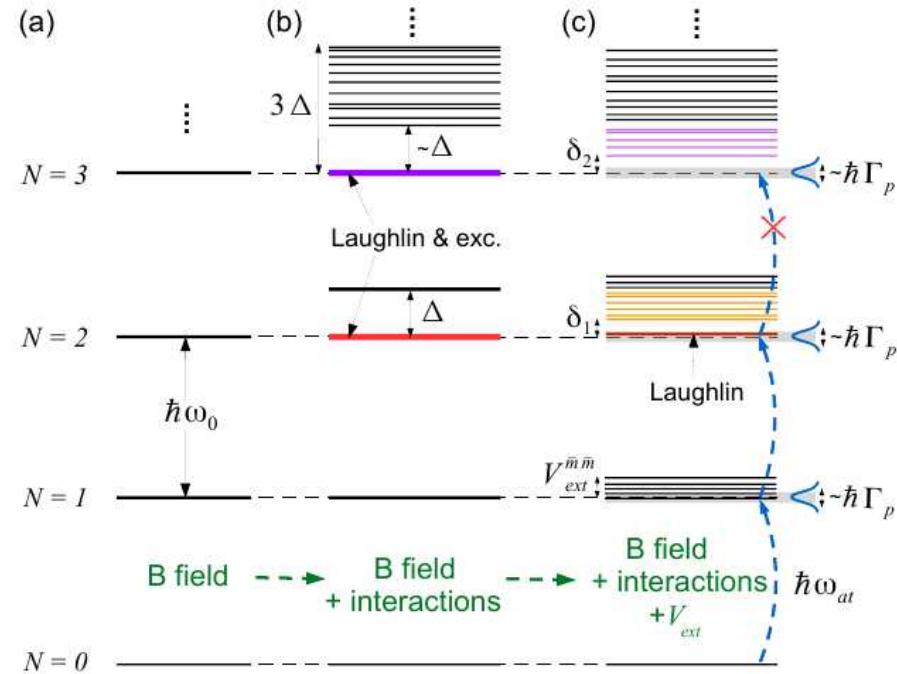
Frequency-dependent incoherent pump:

- Interactions \rightarrow many-body gap Δ
- Edge excitations not gapped. Hard-wall confinement gives small δ
- Non-Markovianity blocks excitation to higher states

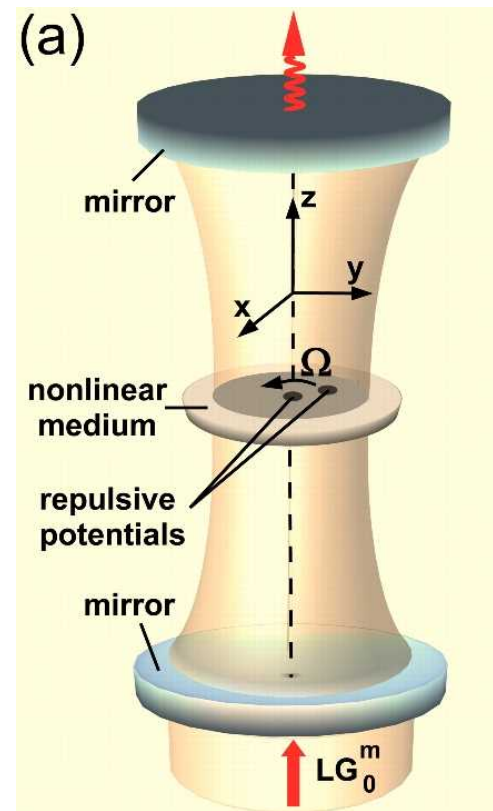
Calculations only possible for small systems:

- Overlap with Laughlin still large
- Spectroscopy of emitted light gives info on many-body states & excitations
- Excitations localized mostly on edge

Open question: what are ultimate limitations of this pumping method?

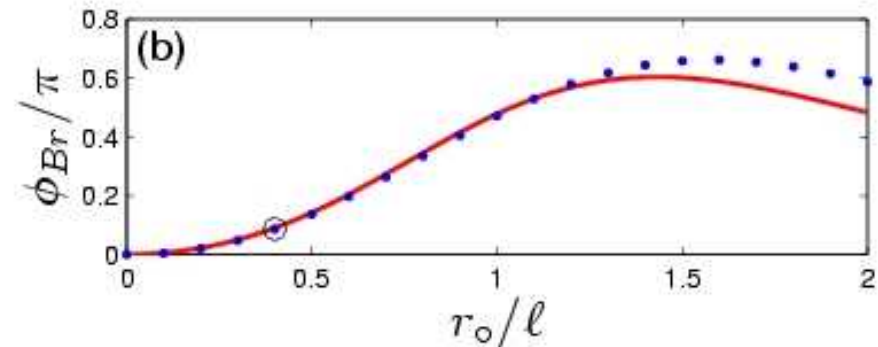
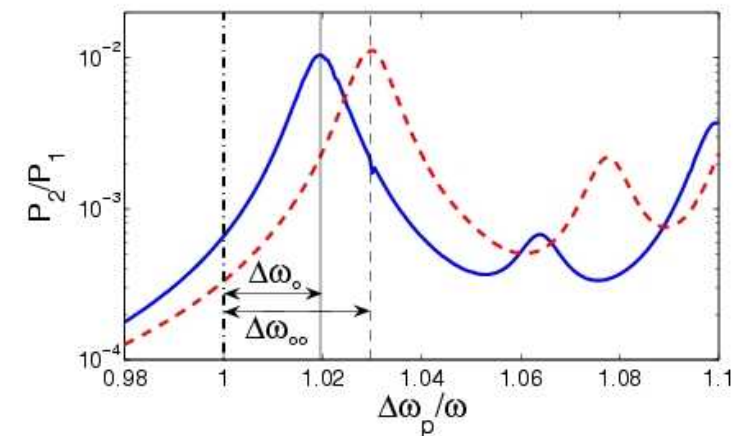


Theorists' speculations: anyonic braiding phase

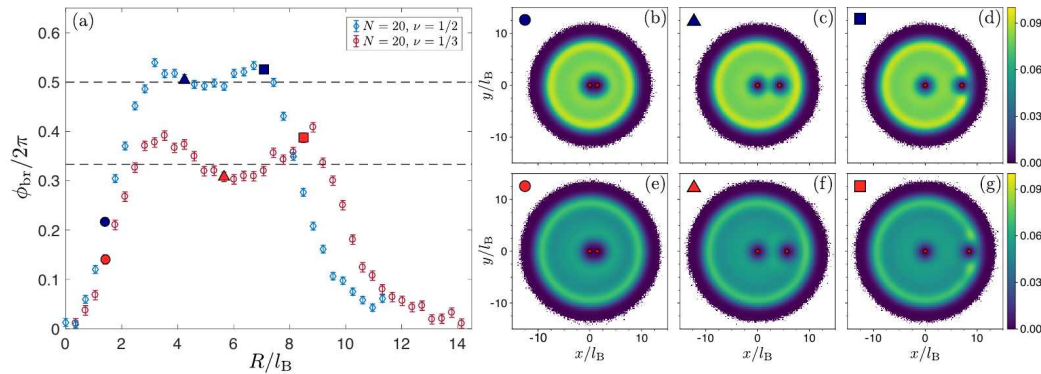


- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
 - create quasi-hole excitation in quantum Hall liquid
 - position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase ϕ_{Br} when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period T_{rot} along circle

$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_o) T_{rot} [2\pi]$$



Observing anyonic statistics via time-of-flight measurements



Braiding phase \rightarrow Berry phase when two quasi-holes are moved around each other

$$\varphi_B(R) = i \oint_R \langle \Psi(\theta) | \partial_\theta | \Psi(\theta) \rangle d\theta.$$

Braiding operation can be generated by rotations, so braiding phase related to L_z

$$\varphi_B(R) = \frac{1}{\hbar} \oint_R \langle \Psi(\theta) | L_z | \Psi(\theta) \rangle d\theta = \frac{2\pi}{\hbar} \langle L_z \rangle$$

Self-similar expansion of lowest-Landau-levels $\rightarrow L_z$ can be measured in time-of-flight via size of the expanding cloud

$$\langle r^2 \rangle_{\text{tof}} = \frac{1}{N} \left(\frac{\hbar t}{\sqrt{2} M l_B} \right)^2 \left(\frac{\langle L_z \rangle}{\hbar} + N \right) = \left(\frac{\hbar t}{2 M l_B^2} \right)^2 \langle r^2 \rangle$$

Can be applied to both cold atoms or to fluids of light looking at far-field emission pattern

Quasi-Hole structure vs. anyon statistics

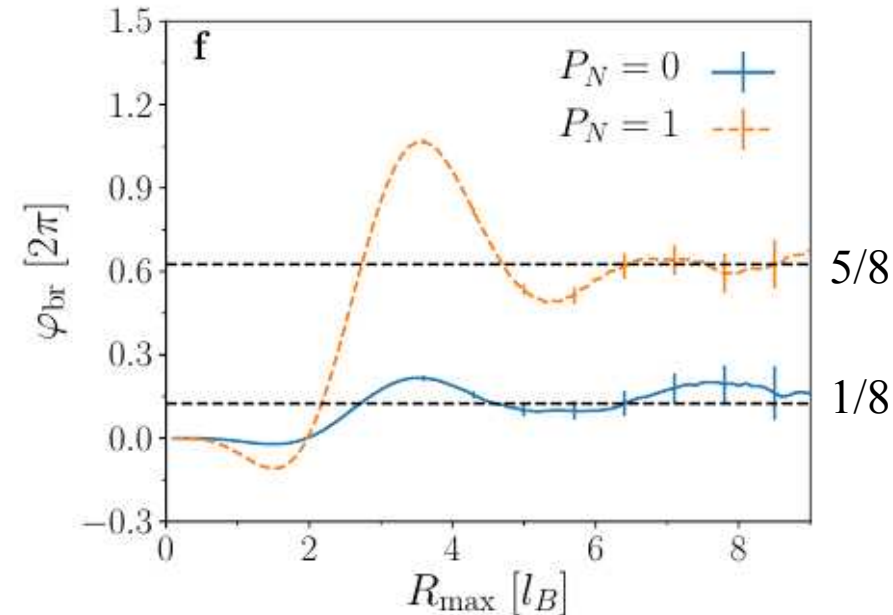
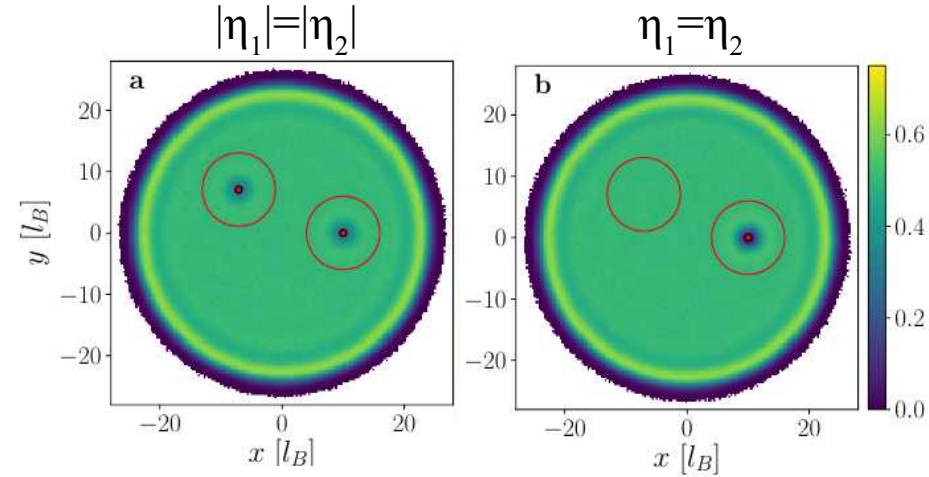
- Compare (two) single quasi-holes and overlapping pairs of quasi-holes:

$$\frac{\varphi_{\text{br}}}{2\pi} = \frac{1}{\hbar} \left[\langle \hat{L}_z \rangle_{|\eta_1|=|\eta_2|} - \langle \hat{L}_z \rangle_{\eta_1=\eta_2} \right].$$

- Relates to difference of density profiles:

$$\frac{\varphi_{\text{br}}}{2\pi} = \frac{N}{2l_B^2} \left[\langle r^2 \rangle_{|\eta_1|=|\eta_2|} - \langle r^2 \rangle_{\eta_1=\eta_2} \right],$$

- Incompressibility \rightarrow external region unaffected
- Reduces to local density difference around QH core, i.e. variance of density depletion
- Insensitive to spurious excitation of (ungapped) edge states
- Numerical calculation for Moore-Read states: allows to distinguish fusion channels of even/odd total particle number



Conclusions and perspectives

Superfluid hydrodynamics in dilute fluid of light:

- Superfluid flow past an obstacle, nucleation of topological defects (2009-2011)
- Complex flows showing analog black hole horizons, on-going experimental quest for signatures of analog Hawking emission in two-point intensity correlations (2015-)
- Mechanical effects of superfluid light (2017-)

1-body magnetic and topological effects for photons in synthetic gauge field:

- Unidirectional and topologically protected edge states (2009-)
- Geometrical properties of bulk & anomalous current (2016-)
- Landau levels in cylindrical trap; smart tricks to generate conical geometry (2016-)

First steps in strongly correlated many-body physics:

- Photon blockade in many platforms: CQED with atoms and solids, circuit-QED, Rydberg atoms,...
- Mott-insulator → recent experimental observation @ Chicago – listen at next talk!
- Chain of strongly interacting bosons in synthetic gauge field
→ recent experimental observation @ GoogleLabs!
- Few-body Laughlin states → don't get asleep!

Some open questions

Topological states of matter:

- Meso- to macro-scopic number of photons typically required
 - Fabricate a large enough cavity or cavity array with low disorder: circuit-QED or Rydberg polaritons in optical cavities or Rydberg atoms
 - Create and stabilize desired many-photon state: what is best pumping scheme (coherent vs. frequency-dependent incoherent vs ...) ?
 - How to theoretically guide experiment? Lindblad ME not enough, non-Markovianity needed, not clear how to combine with DMRG-like methods
- Probe topological nature of many-photon state
 - Edge spectroscopy: optical signatures of fractional charge / statistics? Anyon interference?
 - Non-Abelian anyons, schemes to probe topological degeneracy
- How stable is topological degeneracy against pumping/losses?
 - Losing one photon → multiple QHs created, still exchange needed to affect topological state.
 - Fine if immediately refilled. But what if some escape? What kinetics of creation and refilling?

New material platforms and spectral domains:

- Available platforms challenging to work with. Promising alternatives
 - Integrated solid-state devices in the visible/near-IR/telecom, to be included in photonic circuits
 - Mid- and Far-IR domains: huge dipoles of intersubband transitions, ultra-strong light-matter coupling, modified quantum vacuum state, integrable in quantum cascade structures

If you wish to know more...

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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(published 21 February 2013)

I. Carusotto and C. Ciuti, *Reviews of Modern Physics* **85**, 299 (2013)

CIRCUMNAVIGATING AN OCEAN OF INCOMPRESSIBLE LIGHT

A JOURNEY ACROSS THE EXCITING PERSPECTIVES OF QUANTUM FLUIDS OF LIGHT

IACOPO CARUSOTTO

INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, Povo, Italy

I. Carusotto, *Il Nuovo Saggiatore – SIF magazine* (2013)

Topological Photonics

Review article arXiv:1802.04173 by Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, [Simon](#), Zilberberg, [IC](#), *RMP* **91**, 015006 (2019)

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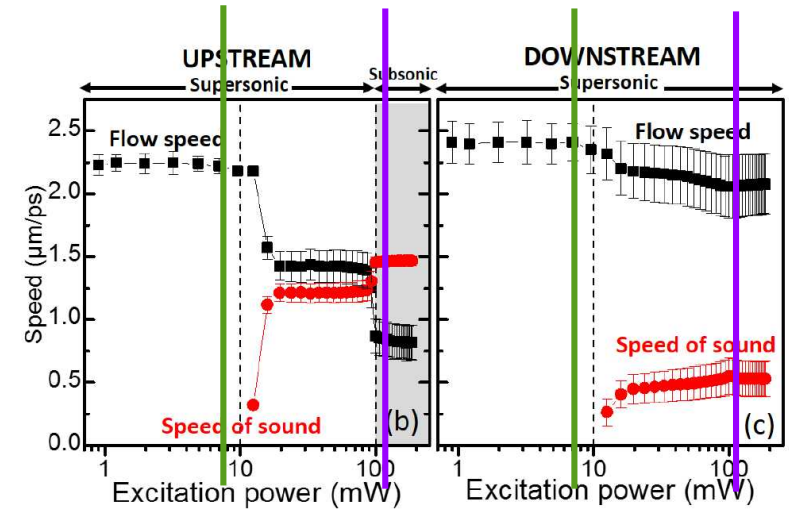
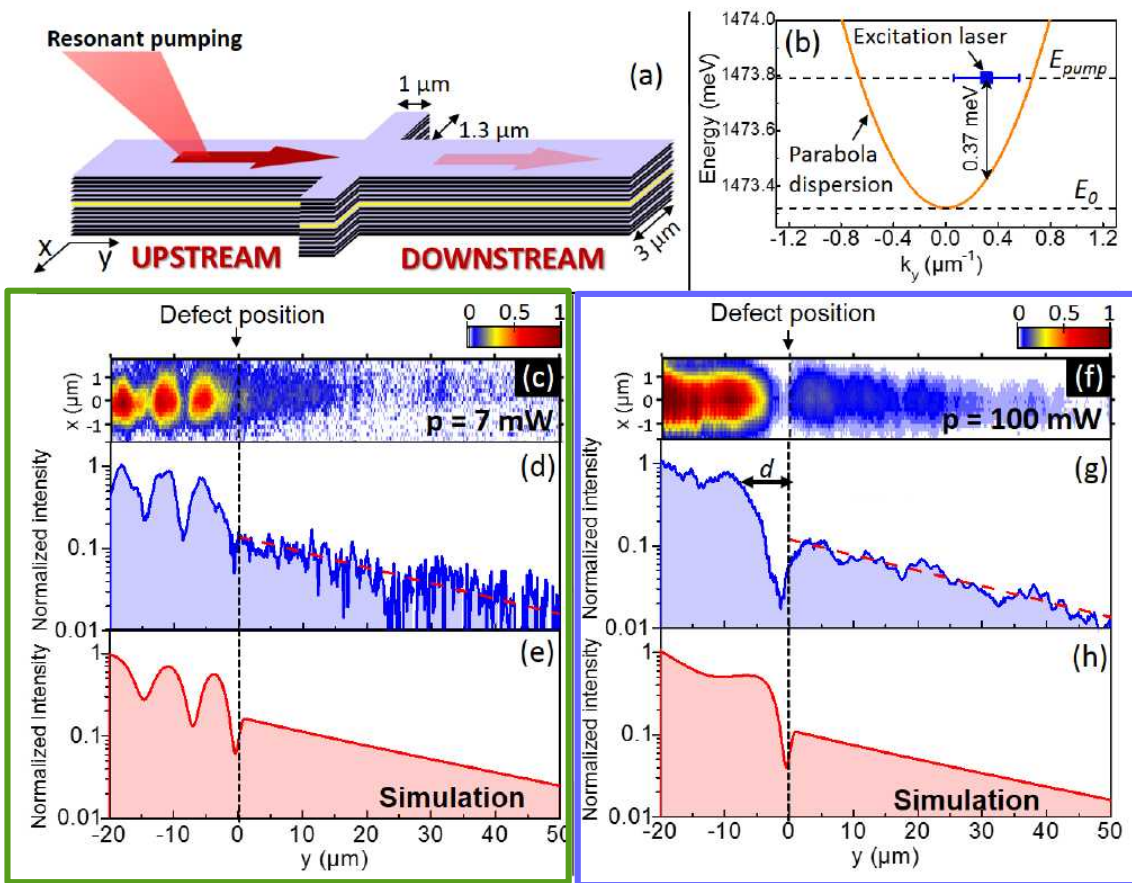
European
Commission

Horizon 2020
European Union funding
for Research & Innovation

PhouS
Photons for Quantum Simulation



2015 - Acoustic Black Hole in flowing fluid of light



BH created!

The hunt for
Hawking radiation
is now open!!

H.-S. Nguyen, Gerace, IC, *et al.*, PRL **114**, 036402 (2015)

Analogous (more advanced!) experiments with atoms: J. Steinhauer, Nat. Phys 2014 & 2016.

2017-8 - Topological lasing

What happens if one adds gain to a topological model ?

First experiments on topological lasing:

- St.Jean et al., Nat. Phot 2017 (1D-SSH model)
- A bit later: Khajavikhan's group, 2017 (1D-SSH); Bahari et al., Science 2017 (2D magnetic photonic crystal)

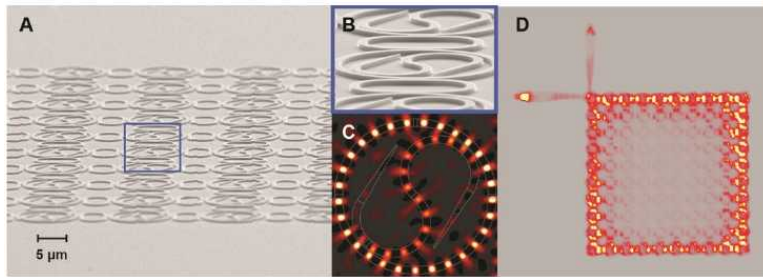
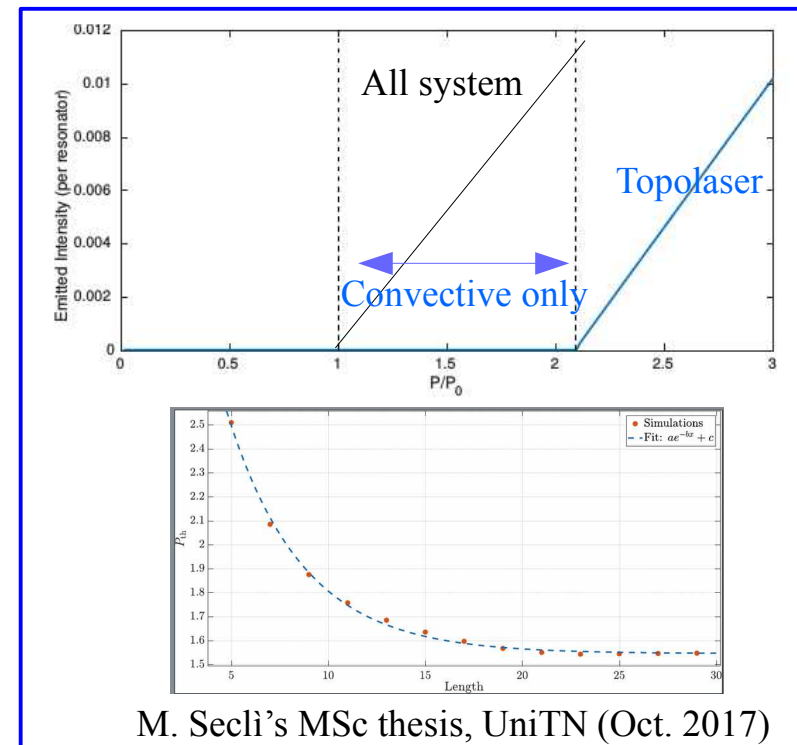


Figure from Bandres et al., Science 2018
Theory in Harari et al., Science 2018

System: array of Si-based ring resonators with optically pumped III-V amplifier layer.
Tai-Ji shape to break inversion symmetry

Pump position selects **edge mode** → unidirectional emission;
robust to disorder; high slope efficiency, but:

- **Convective** vs. **absolute instability**:
 - different threshold of edge-mode lasing
 - unusual statistical properties in between
- **Topological effects** visible in **lasing threshold** for high number of pumped sites
- **Topological robustness against mode jumps**:
 - Random choice of lasing mode
 - Extra-slow relaxation of fluctuations
 - What consequences for quantum noise?

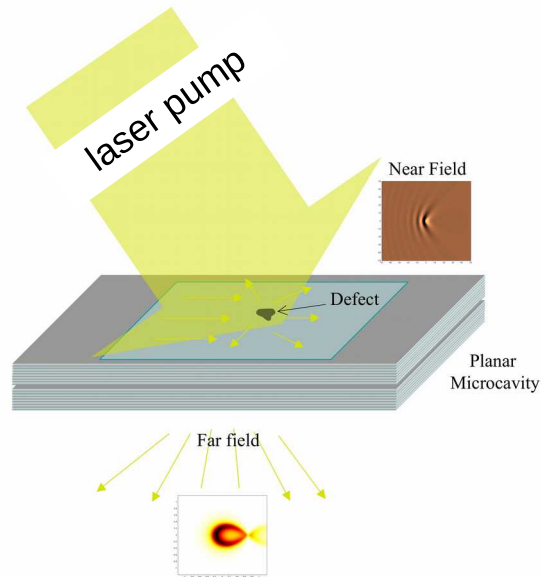


Part 2:

Quantum fluids of light
with a *unitary* dynamics

Field equation of motion

Planar microcavities & cavity arrays



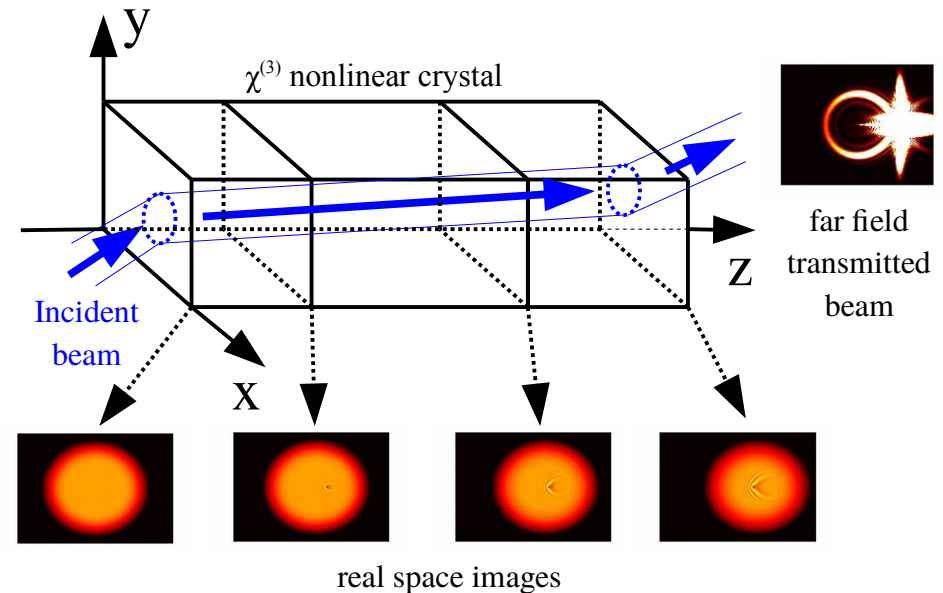
Pump needed to compensate losses:
 driven-dissipative dynamics in real time
 stationary state \neq thermodyn. equilibrium

Driven-dissipative CGLE evolution

$$i \frac{dE}{dt} = \left[\omega_o - \frac{\hbar \nabla^2}{2m} + V_{ext} + g |E|^2 + \frac{i}{2} \left(\frac{P_0}{1 + \alpha |E|^2} - \gamma \right) \right] E + F_{ext}$$

Quantum correl. sensitive to dissipation

Propagating geometry



Monochromatic beam

Incident beam sets initial condition @ $z=0$

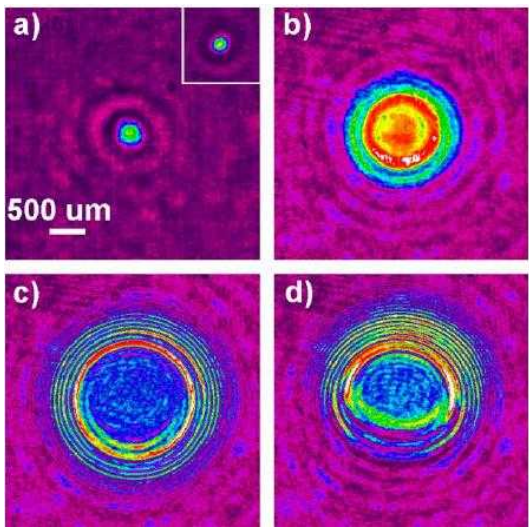
MF \rightarrow Conserv. paraxial prop. \rightarrow GPE

$$i \frac{dE}{dz} = \left[-\frac{\hbar \nabla_{xy}^2}{2\beta} + V_{ext} + g |E|^2 E \right] E$$

- V_{ext} , g proportional to $-(\epsilon(r)-1)$ and $\chi^{(3)}$
- Mass \rightarrow diffraction (xy)

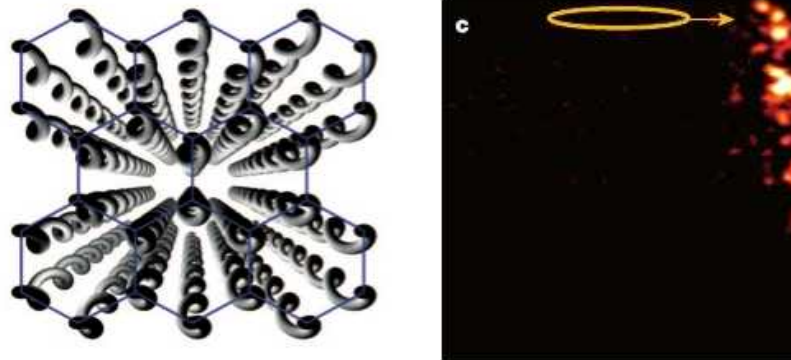
First expts with (almost) conservative QFL's

Dispersive superfluid-like shock waves



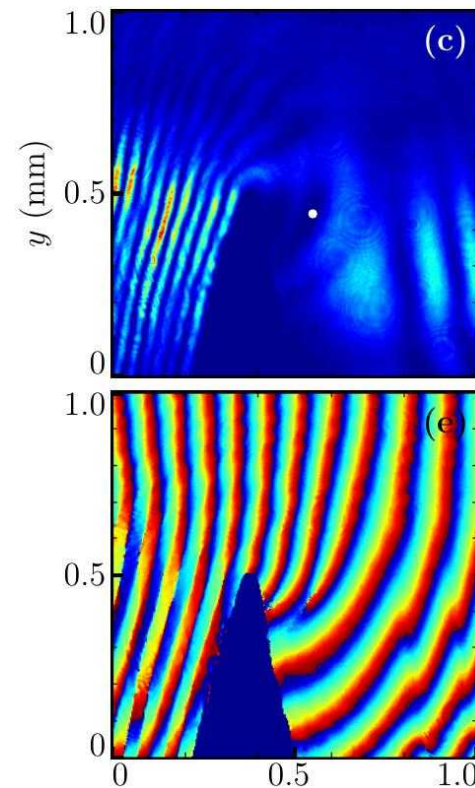
Wan et al., Nat. Phys. 3, 46 (2007)

Chiral edge states in (photonic) Floquet topological insulator



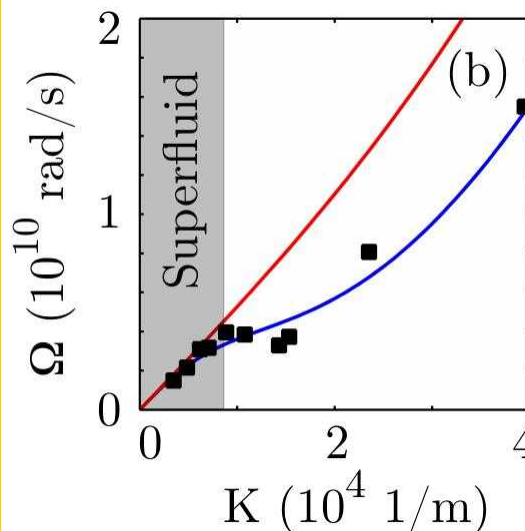
Rechtsman, et al., Nature 496, 196 (2013)

Hydrodynamic nucleation of quantized vortices

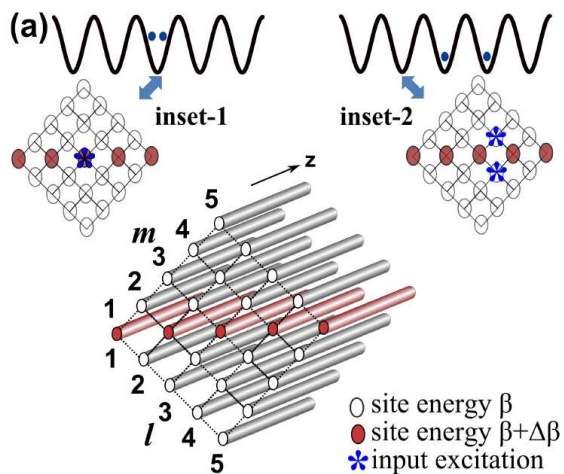


D. Vocke et al., arXiv:1511.06634

Bogoliubov dispersion of collective excitations



D. Vocke et al. Optica (2015)



Quantum simul. of 2-body physics
Mukherjee et al., arXiv:1604.00689

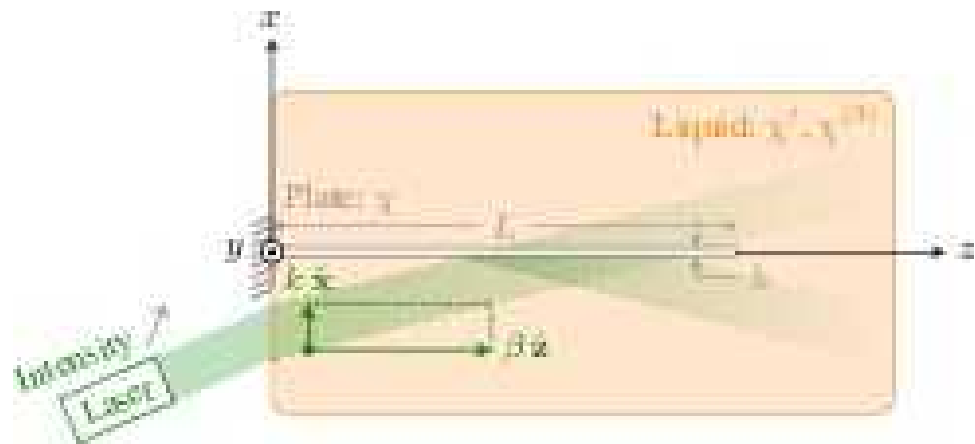
Frictionless flow of superfluid light (I)

All superfluid light experiments so far:

- Planar microcavity device with stationary obstacle in flowing light
- Measure response on the **fluid density/momentum pattern**
- Obstacle typically is defect **embedded in semiconductor material**
- **Impossible to measure mechanical friction force exerted onto obstacle**

Propagating geometry more flexible:

- Obstacle can be solid dielectric slab with different refractive index
- Immersed in **liquid nonlinear medium**, so can move and deform
- **Mechanical force measurable from magnitude of slab deformation**



Frictionless flow of superfluid light (II)

Numerics for **propagation GPE** of **monochromatic laser**:

$$i \partial_z E = -\frac{1}{2\beta} (\partial_{xx} + \partial_{yy}) E + V(r) E + g |E|^2 E$$

with $V(r) = -\beta \Delta \varepsilon(r) / (2\varepsilon)$ with rectangular cross section and $g = -\beta \chi^{(3)} / (2\varepsilon)$

For growing light power, **superfluidity** visible:

- Intensity modulation disappears
- Suppression of opto-mechanical force

An intermediate powers:

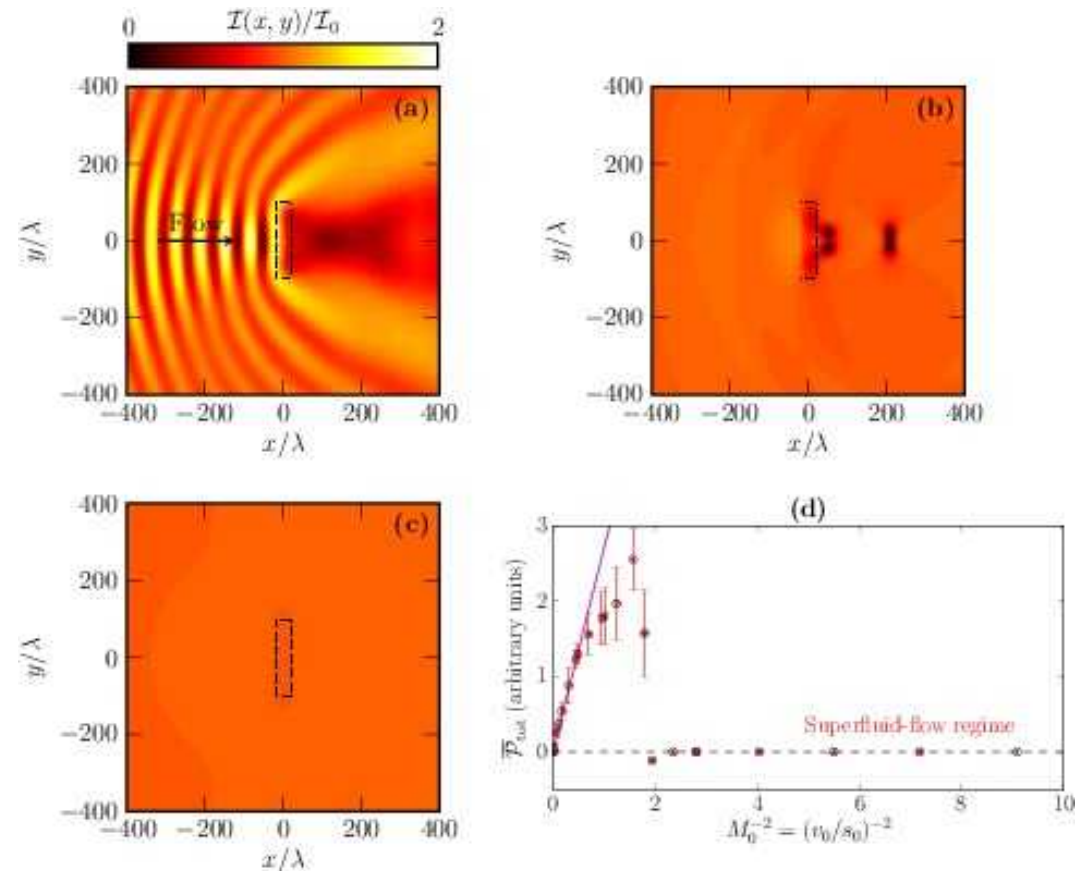
- Periodic nucleation of vortices
- **Turbulent** behaviours

Fused silica slab as obstacle

→ deformation almost in the μm range

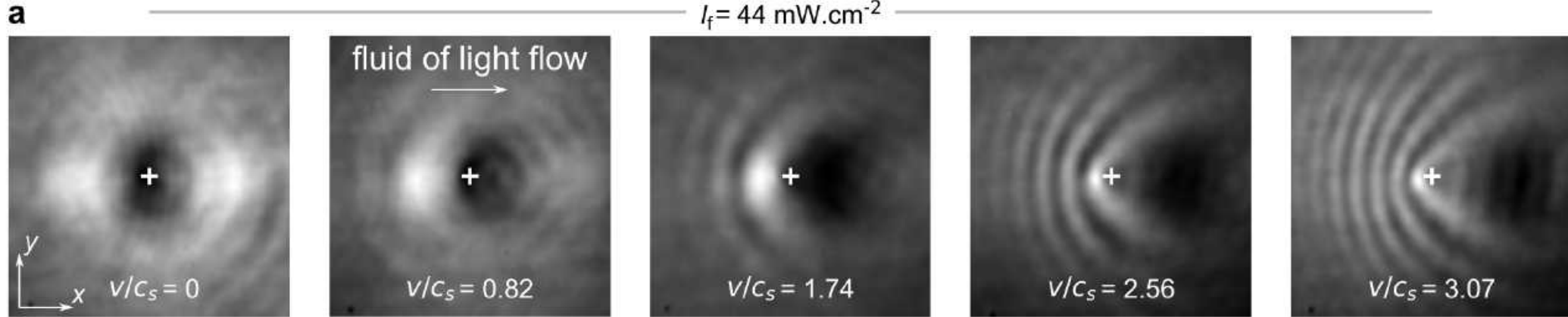
Experiment in progress

→ surrounding medium in fluid state
but local nonlinearity (e.g. atomic gas)



Nice experiments: Michel et al., Nat. Comm. 2018

Increasing incidence angle, i.e. speed

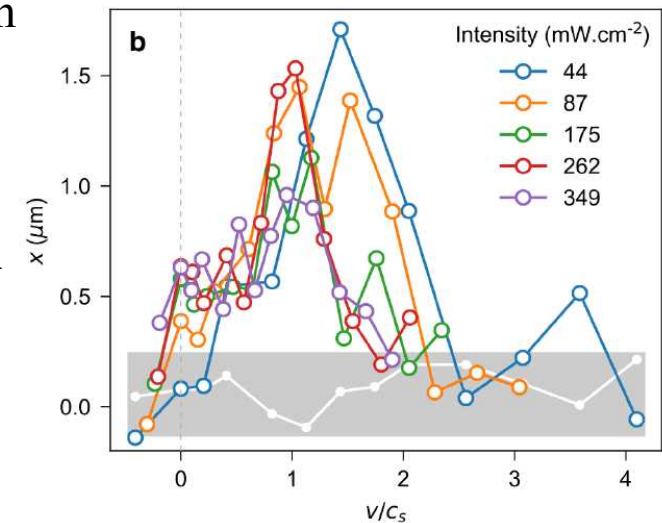
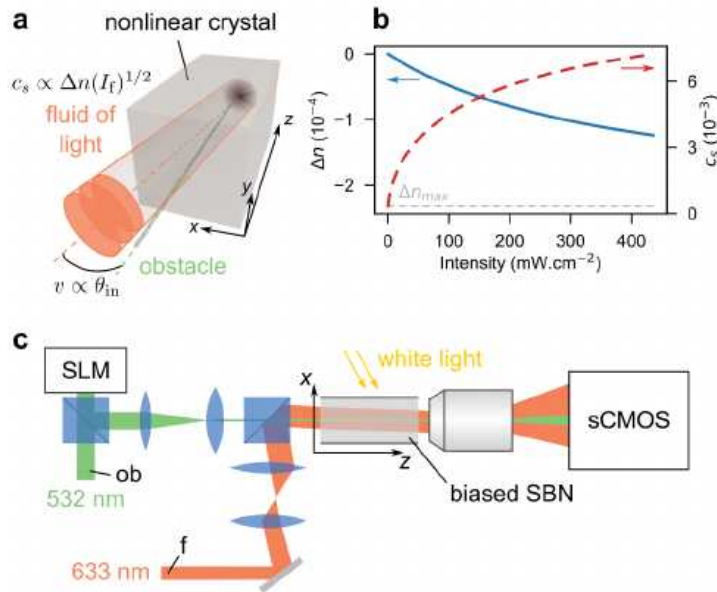


Superfluid

Cherenkov cone shrinks with v/c_s

First attempt to measure absence of mechanical effect of superfluid light.

All-optical analog:
lateral shift of obstacle beam



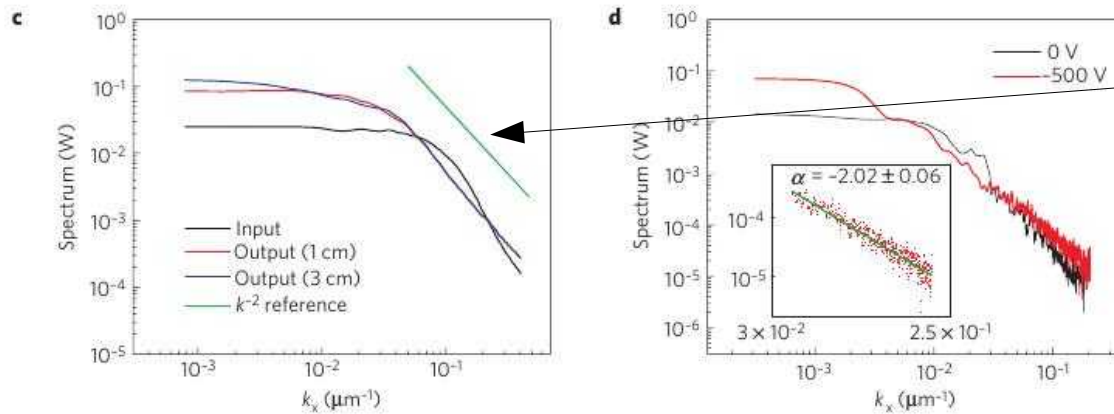
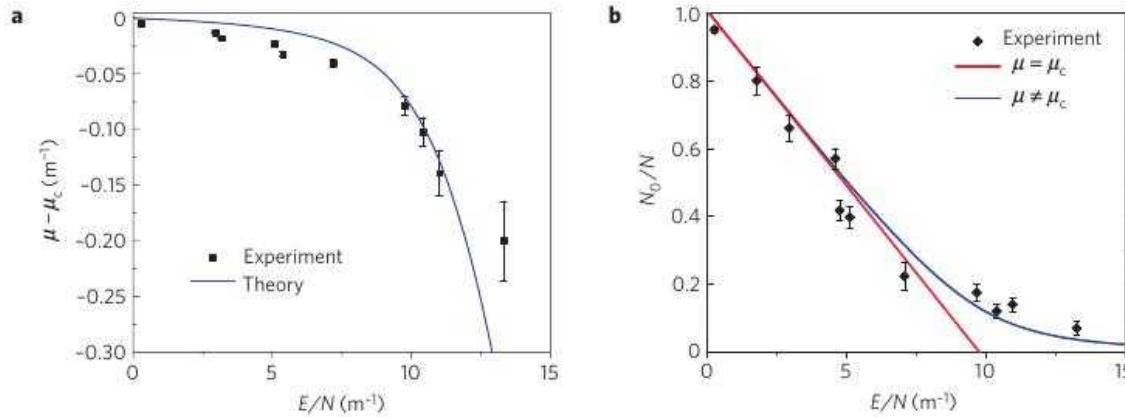
Condensation of classical waves

Monochromatic beam
but spatially noisy profile

Slow nonlinearity
→ remains monochromatic
Evolution during propagation
→ classical GPE

Thermalizes to condensate
plus thermal cloud with
Rayleigh-Jeans $1/k^2$ high- k tail

- What about quantum effects?
- How to recover Planckian?



Fourier space (k)

E/N (m^{-1}) = 13.3

10.4

7.1

4.6

1.8

How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field with spatially local $\chi^{(3)}$

$$i \frac{\partial E}{\partial z} = -\frac{1}{2\beta_0} \left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \right) - \frac{1}{2D_0} \frac{\partial^2 E}{\partial t^2} + V(r)E + g|E|^2 E$$

Propagation coordinate $z \rightarrow$ time

Physical time \rightarrow extra spatial variable, dispersion $D_0 \rightarrow$ temporal mass

(similar to Michael's description of light propagation in EIT medium)

Upon quantization \rightarrow conservative many-body evolution in z : $i \frac{d}{dz} |\psi\rangle = H |\psi\rangle$

with
$$H = N \iiint dx dy dt \left[\frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + V \hat{E}^\dagger \hat{E} + \hat{E}^\dagger \hat{E}^\dagger \hat{E} \hat{E} \right]$$

Same z commutator $[\hat{E}(x, y, t, z), \hat{E}^\dagger(x', y', t', z)] = \frac{c \hbar \omega_0 v_0}{\epsilon} \delta(x-x') \delta(y-y') \delta(t-t')$

Difficulty for Rydberg-EIT: interactions non-local in x, y , and $z \rightarrow$ approximated as non-local in t

P.-E. Larré, IC, *Propagation of a quantum fluid of light in a cavityless nonlinear optical medium:*

General theory and response to quantum quenches, PRA **92**, 043802 (2015)

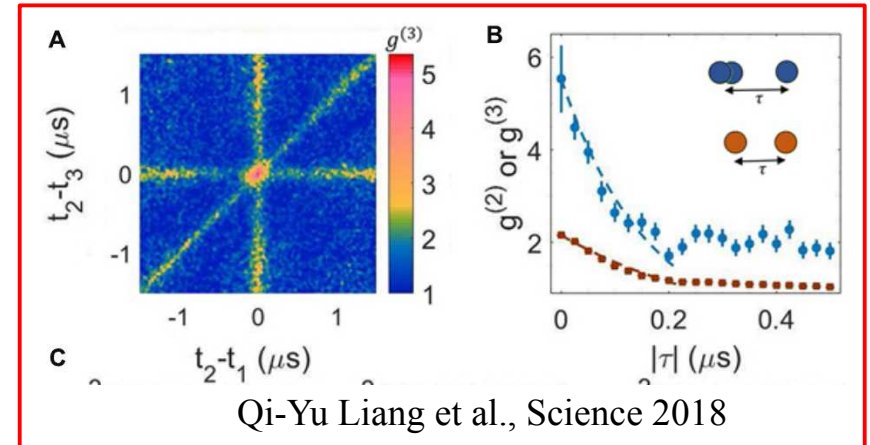
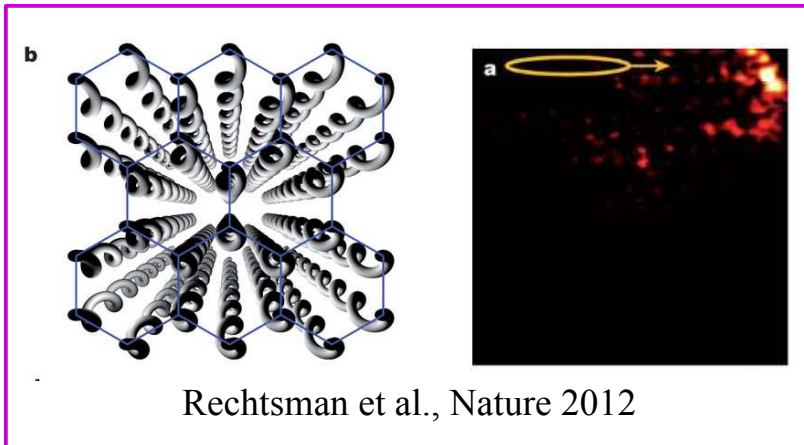
See also old work by Lai and Haus, PRA 1989

A quite generic quantum simulator

Quantum many-body evolution in z :

$$i \frac{d}{dz} |\psi\rangle = H |\psi\rangle \quad \text{with:} \quad H = N \iiint dx dy dt \left[\frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + V \hat{E}^\dagger \hat{E} + \hat{E}^\dagger \hat{E}^\dagger \hat{E} \hat{E} \right]$$

- Physical time t plays role of extra spatial coordinate
- Same z commutator: $[\hat{E}(x, y, t, z), \hat{E}^\dagger(x', y', t', z)] = \frac{c \hbar \omega_0 v_0}{\epsilon} \delta(x-x') \delta(y-y') \delta(t-t')$



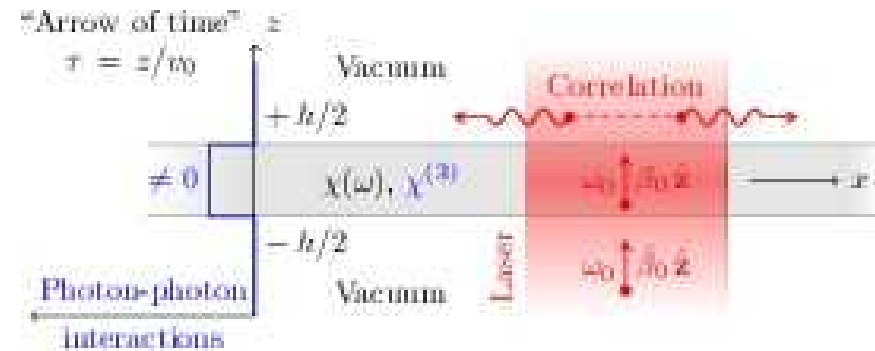
Can realize (or quantum simulate) wide variety of physical systems:

- Arbitrary splitting/recombination of waveguides \rightarrow quench of tunneling
- Modulation along z \rightarrow Floquet topological insulators
- In addition to photonic circuit \rightarrow many-body due to photon-photon interactions
- Pioneering experiments of few-body physics, e.g. two- and three-photon bound states

Dynamical Casimir emission at quantum quench (I)

Monochromatic wave @ normal incidence
into slab of weakly nonlinear medium

→ **Weakly interacting Bose gas**



Air / nonlinear medium interface

→ **sudden jump** in interaction constant when moving along z

Mismatch of Bogoliubov ground state in air and in nonlinear medium

→ emission of phonon pairs at opposite k on top of fluid of light
(sort of Dynamical Casimir Effect for phonons)

Propagation along z

→ **conservative quantum dynamics**

Important question: what is quantum evolution at late times? Thermalization?

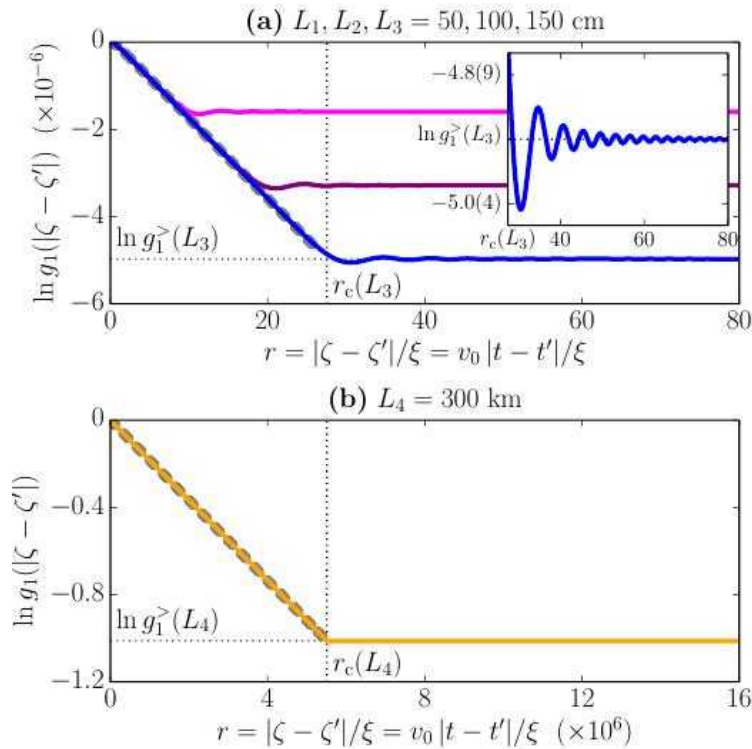
“Pre-thermalized” 1D photon gas

Perfectly coherent light injected into 1D optical fiber:

- quantum quench of interactions $\sim \chi^{(3)}$
- pairs of Bogoliubov excitations generated

Resulting **phase decoherence** in $g^{(1)}(t-t')$:

- **Exponential decay** at short $|t-t'| < 2z / c_s$
(c_s = speed of Bogol. sound)
- Plateau at long $|t-t'| > 2z / c_s$
- Low-k modes eventually tends to **thermal** $T_{\text{eff}} = \mu / 2$
- Hohenberg-Mermin-Wagner theorem prevents long-range order in 1D quasi-condensates at finite T

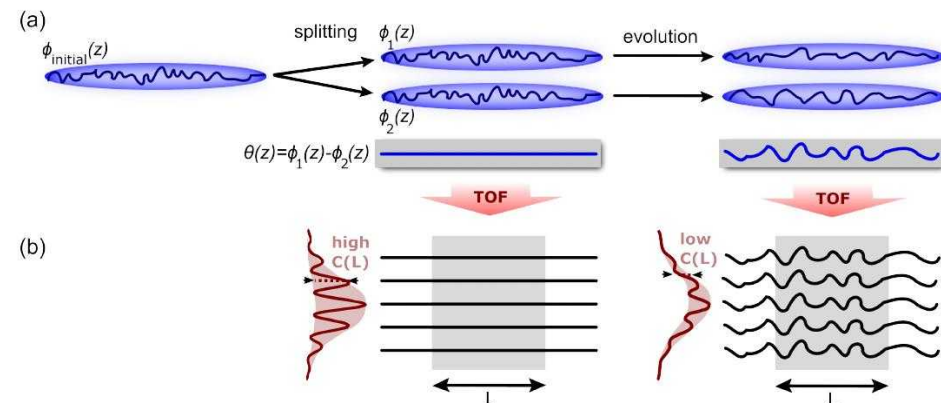


Effect small for typical Si fibers, still potentially harmful on long distances

Decoherence slower if tapering used to “adiabatically” inject light into fiber

Related cold atom expts by J. Schmiedmayer when 1D quasi-BEC suddenly split in two

Nature Physics 9, 640–643 (2013)



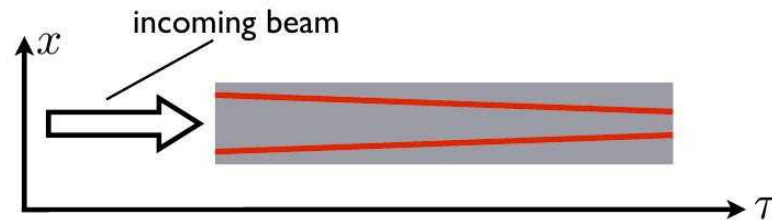
Evaporative cooling of light

Quantum Hamiltonian under space-z / time-t mapping:

$$H = N \iiint dx dy dt \left[\frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + g \hat{E}^\dagger \hat{E}^\dagger \hat{E} \hat{E} \right]$$

In 3D bulk crystal after **long propagation distances**:

- equilibration in transverse k and frequency ω leads to **Bose-Einstein distribution**
- temperature and chemical potential fixed by **incident distribution** $I(k, \omega)$



Harmonic trap in xy plane + selective absorption of most energetic particles:

- Energy **redistributed by collisions**; photon gas **evaporatively cooled**
- Incident incoherent (in both space and time) field eventually gets to **BEC state**
- NOTE: fast and coherent optical nonlinearity $\chi^{(3)}$ essential !!

Novel source of coherent light

A. Chiochetta, P.-É. Larré, IC, EPL (2016).

Intriguing (and not yet fully understood) experiment, Krupa et al., Nat. Phot. 2017