Locality Bound for Dissipative Quantum Transport

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Open Quantum Systems @ KITP
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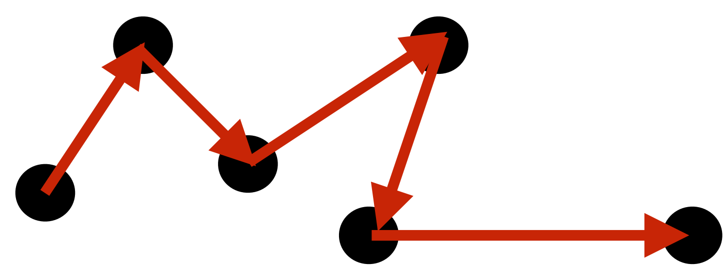
Context

- Dynamics of quantum systems simplifies if there are emergent, well-defined quasiparticles.

- Collisions of these quasiparticles leads to e.g. diffusion of energy.



$$D \sim v_{\mathrm{qp}}^2 \tau_{\mathrm{qp}} \sim v_{\mathrm{qp}} \ell_{\mathrm{qp}}$$

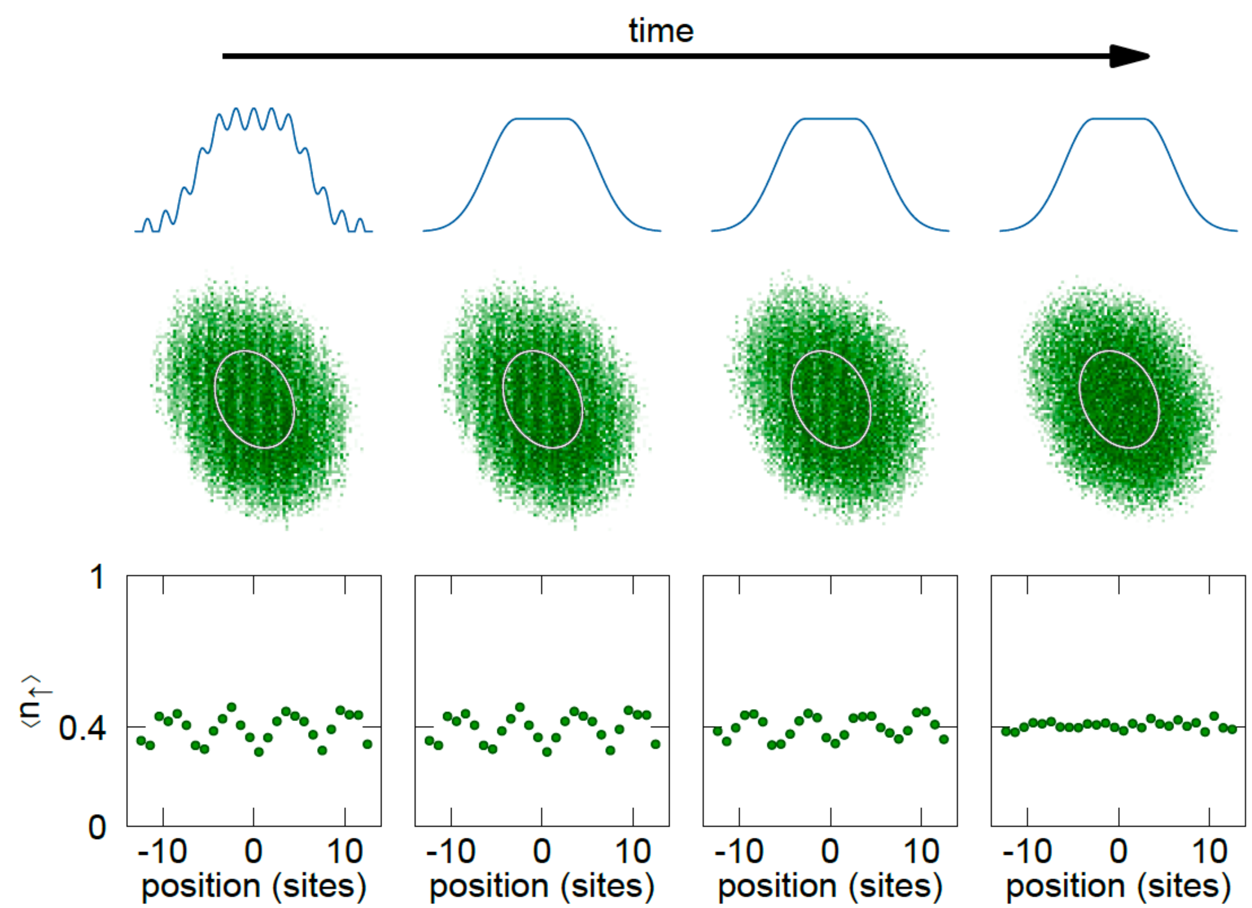
[Maxwell, Boltzmann, Einstein …]

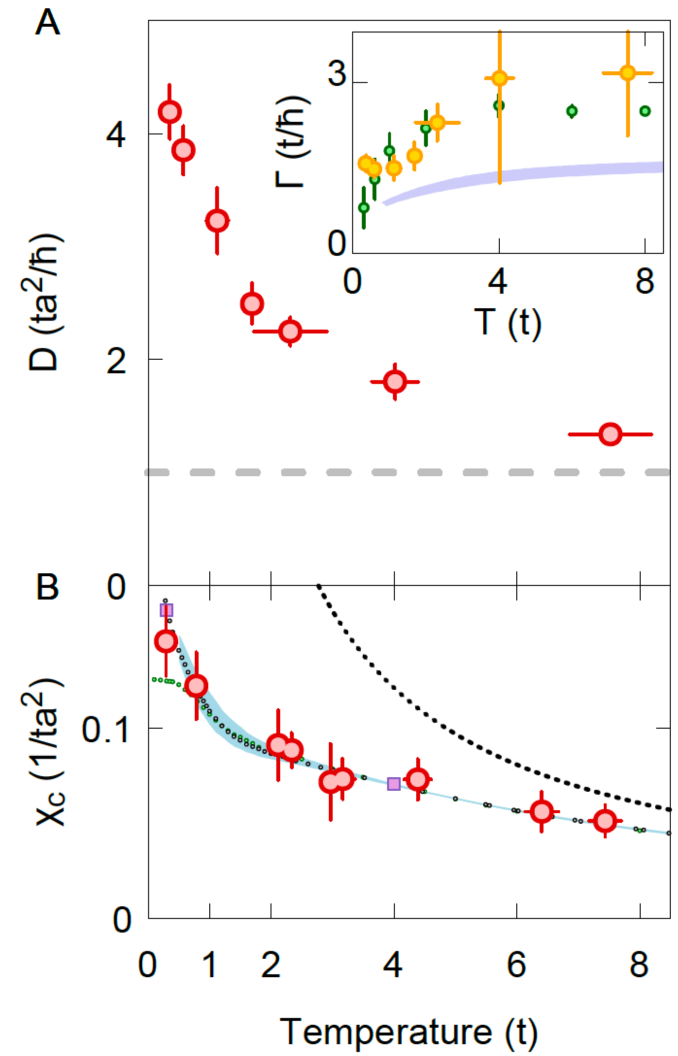
- Theoretical framework: Boltzmann equation.

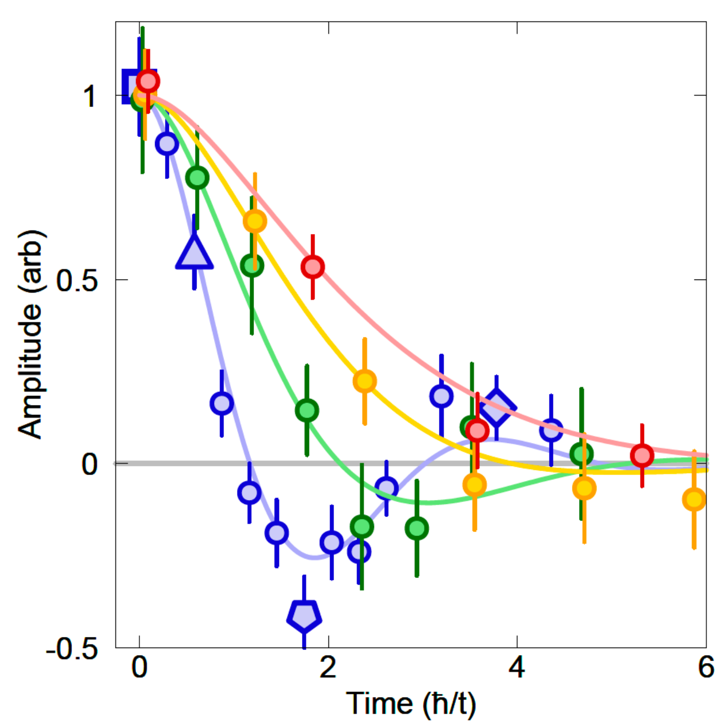
Context

- Emergence of quasiparticles from a quantum many body system is nontrivial (eg. in Fermi Liquid theory) and may not occur.
- Strongly correlated many-body quantum dynamics in space and time — what theoretical framework generalizes the Boltzmann equation?
- Ingredients: conserved charges, spatial locality, thermalization.
- What can we say about the resulting diffusion?

Measuring diffusion







QUANTUM SIMULATION

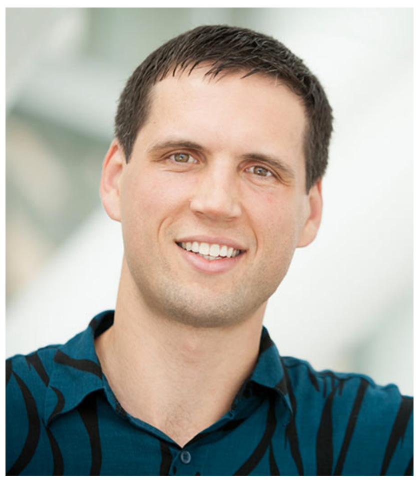
Science **363**, 379–382 (2019)

Bad metallic transport in a cold atom Fermi-Hubbard system

Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1]†

Bounding diffusion

PRL **119,** 141601 (2017) PHYSICAL REVIEW LETTERS week ending 6 OCTOBER 2017



Upper Bound on Diffusivity

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PHYSICAL REVIEW LETTERS **121,** 170601 (2018)

Locality Bound for Dissipative Quantum Transport

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A bound can identify fundamental constraints
(cf. bound on efficiency of heat engines)

Key ingredients: Conservation law + locality

Diffusion and thermalization

- Diffusion is a late time phenomenon.
- Almost all non-conserved quantities decay on a local thermalization timescale $\tau \sim \tau_{th}$.
- After τ_{th} there is a locally well-defined temperature, chemical potential, magnetization etc.
- Long wavelength inhomogeneities of these local thermodynamic variables then reach global thermal equilibrium via diffusion: $\Gamma_k = 1/\tau_k = Dk^2$.

Lieb-Robinson velocity

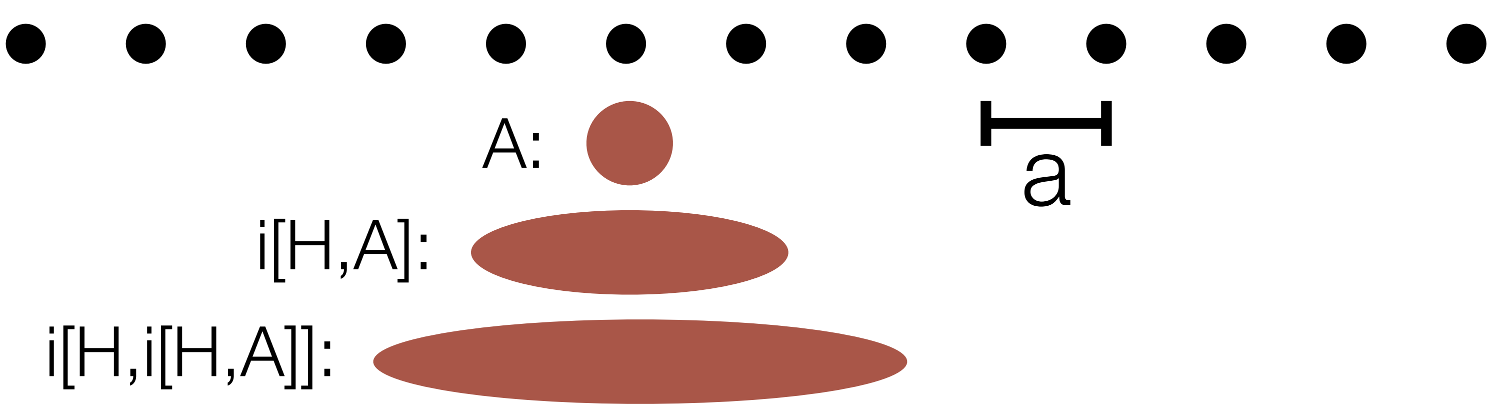
- Even non-relativistic systems have a ‘lightcone’: bounded propagation of signals from locality.

$$||[A(t,x),B(0,0)]|| \lesssim ||A||\,||B||e^{-\mu(|x|-vt)}$$

[Lieb-Robinson 72]

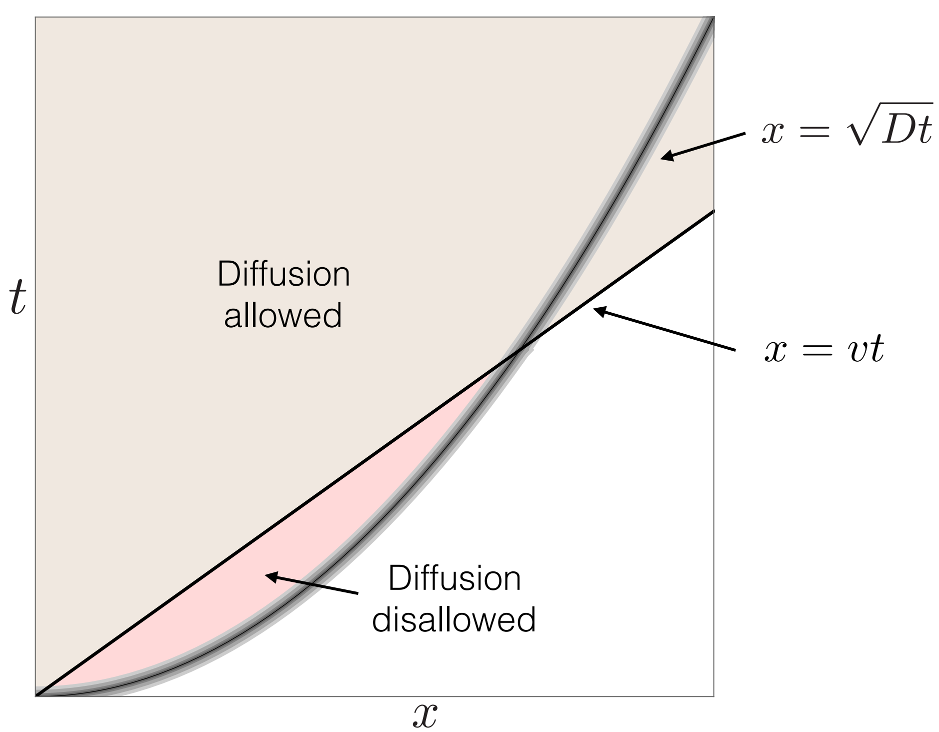
- The “Lieb-Robinson” velocity: $v \sim \frac{J\,a}{\hbar}$

- This is a microscopic, state-independent velocity. It describes the growth of operators under time evolution.



Bounding diffusion

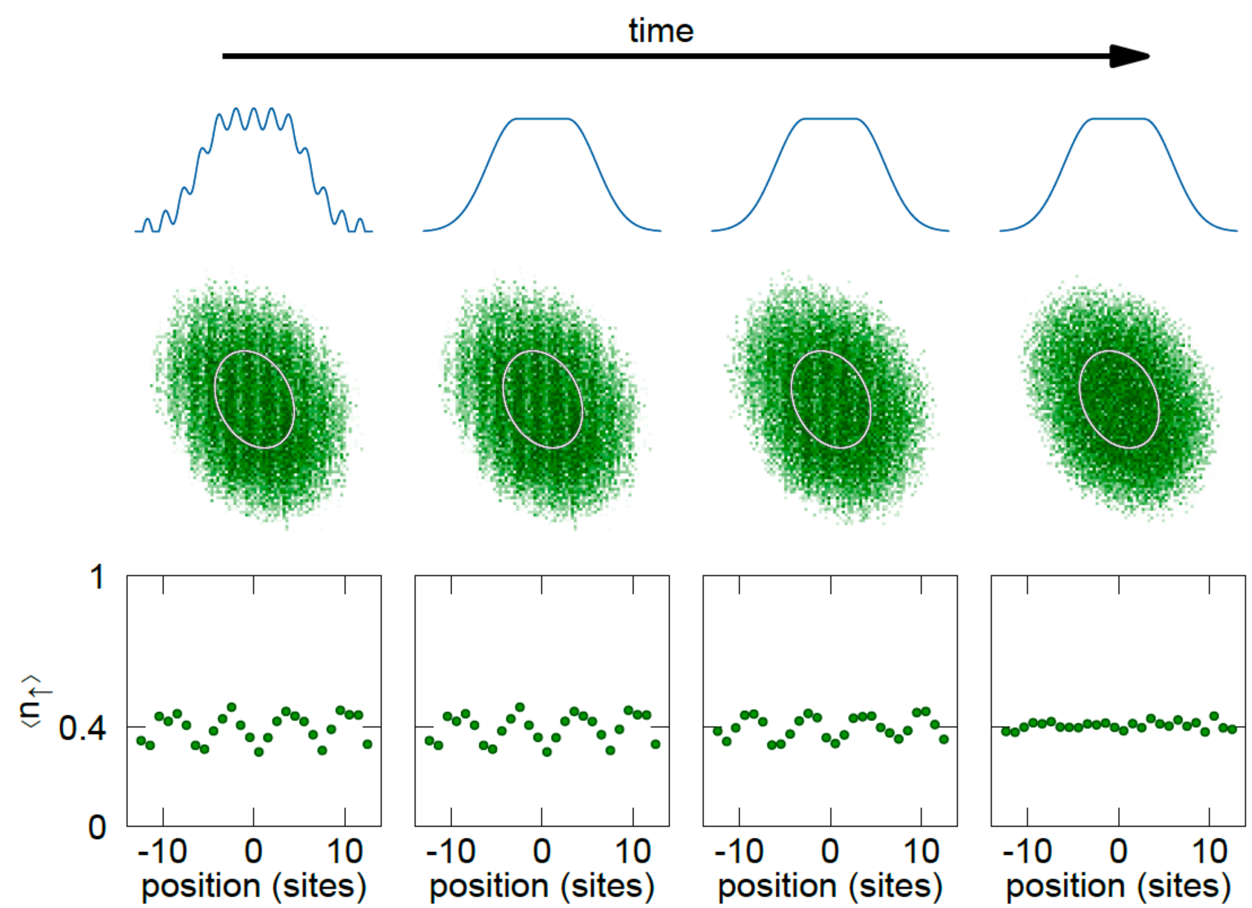
- The LR velocity clearly bounds ballistic transport (e.g. in ordered phases: $v_{\text{spin wave}} < v_{\text{LR}}$).
- It also bounds diffusivity: [Inspired by [Blake PRL 16]]

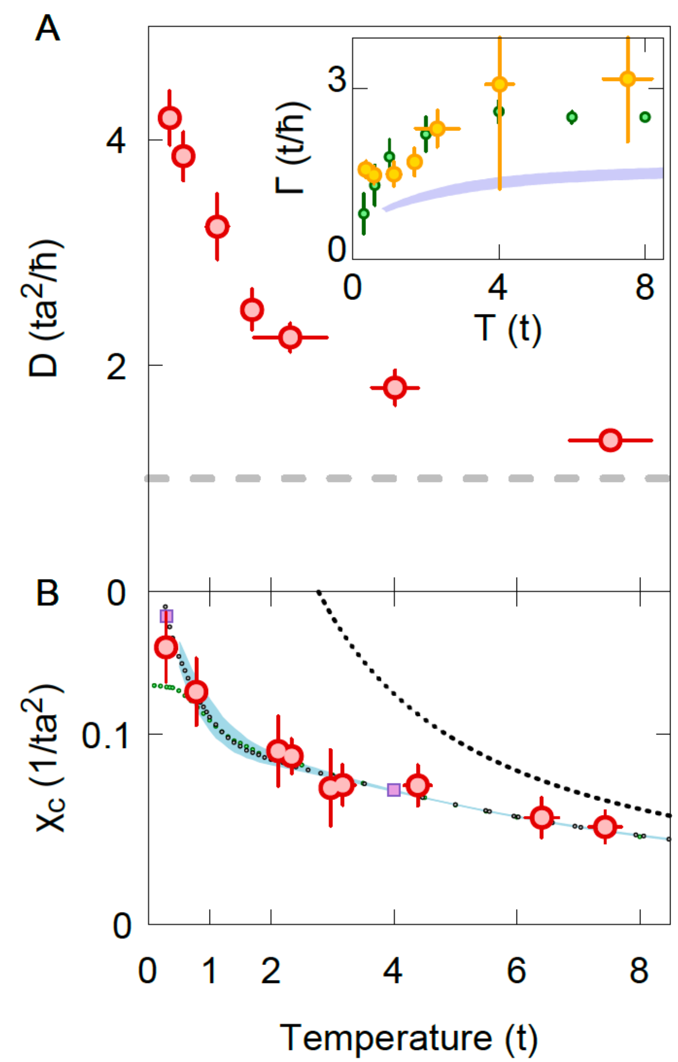


LR causality implies disallowed region must not be diffusive — i.e. must occur before local thermalization, so that:

$$D \lesssim v_{\text{LR}}^2 \tau_{\text{th}}$$

Measuring diffusion

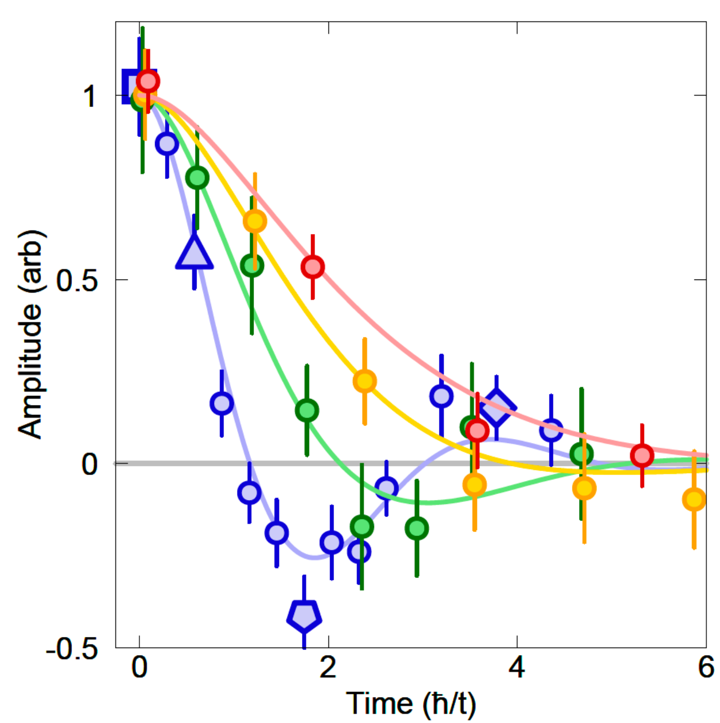




Obeys bound:

$$D \lesssim \frac{(ta)^2}{\hbar^2}\frac{1}{\Gamma}$$

v_{LR}^2 τ_{th}



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Beyond quasiparticles

- In systems with a finite on-site Hilbert space (spins, fermions), diffusion is bounded by:
  — The Lieb-Robinson velocity
  — The local thermalization time

- These concepts do not make reference to quasiparticles.

- Next: establish rigorous version of this bound in context of open quantum dynamics.

Lindbladian dynamics

- Rigorous argument: 'Lindbladian' quantum evolution:

$$\dot{O}(t) = i\sum_{\mathbf{x}}[H_{\mathbf{x}}, O(t)] + c\sum_{\mathbf{x},\alpha}\left(2L_{\mathbf{x}}^{\alpha\dagger}O(t)L_{\mathbf{x}}^{\alpha} - L_{\mathbf{x}}^{\alpha\dagger}L_{\mathbf{x}}^{\alpha}O(t) - O(t)L_{\mathbf{x}}^{\alpha\dagger}L_{\mathbf{x}}^{\alpha}\right)$$

- Thermalizing degrees of freedom are integrated out but evolution is local in time (Markovian).

- Technical simplification: in thermal case diffusion only occurs in a particular state, need to work with thermal expectation values. In dissipative context, operators themselves will diffuse.

Lindbladian diffusion

- Translation invariant — label operators by momentum k.

- Strategy: Conserved charge operator C implies small k eigenoperator (note: operators decay):

$$\dot{C}_k = -Dk^2 C_k + \mathcal{O}(k^3)$$

With

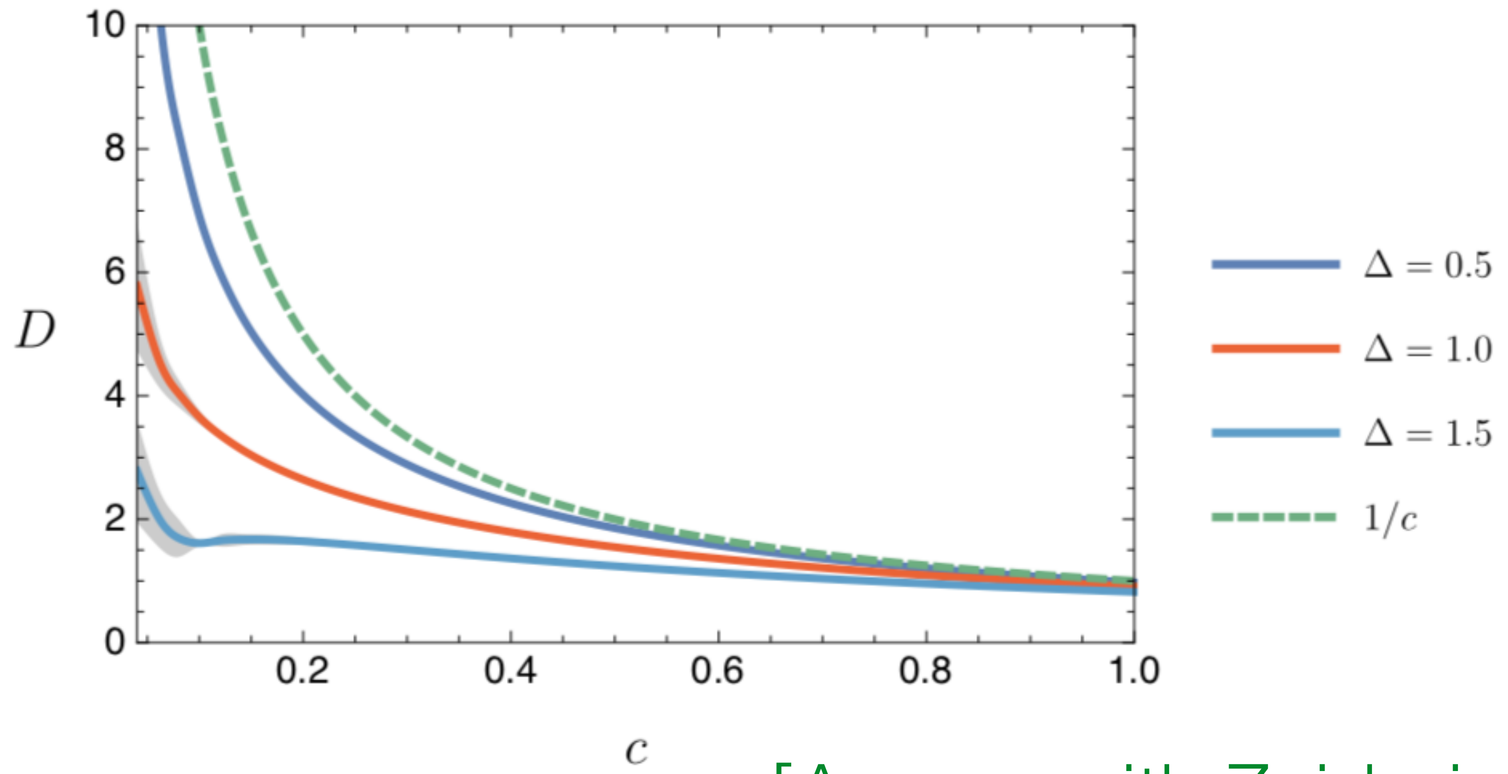
$$\partial_t|_{\mathbf{k}} = \sum_{i \geq 0} k^i \mathcal{L}^i$$

$$D = -\sum_{E_a^0 \neq 0} (C|\mathcal{L}^1|E_a^0) \frac{1}{E_a^0} (E_a^0|\mathcal{L}^1|C)$$

‘Solvable’ example

- XXZ model with on-site dephasing. Truncate to operators below a certain length.

$$H_x = X_x X_{x+1} + Y_x Y_{x+1} + \Delta Z_x Z_{x+1}$$
$$L_x = Z_x$$



[Agrees with Znidaric, Prosen, …]

Bounding the diffusivity

$$
\begin{aligned}
D &= |\int_0^\infty dt (C|\mathcal{L}^1 e^{\mathcal{L}^0 t} \mathcal{L}^1 |C)| \\
&= |\int_0^\infty dt (C|\mathcal{L}^1 e^{\mathcal{L}^0 t} |J)| \\
&\leq \int_0^\infty dt |(C|\partial_k|\dot{J}(t))| \\
&\leq \int_0^\infty dt || \partial_k|\dot{J}(t))|| / || \, |C)||
\end{aligned}
$$

Bounding the diffusivity

- Lieb-Robinson bound

$$||\mathcal{P}_l[O(t)]|| \leq A'||O||e^{(v_{\mathrm{LR}}t-l)/\xi}$$

- And the moment

$$i\partial_k O = \int_0^\infty dl \mathcal{P}_l[O] - \int_{-\infty}^0 dl(1-\mathcal{P}_l[O])$$

- ‘Single-mode ansatz’: All local operators other than C’s decay on timescale τ (property of Lindbladian).

$$||\dot{J}(t)|| \leq \frac{A}{\tau}||J||e^{-t/\tau}$$

Bounding the diffusivity

$$D \leq \alpha v_{\mathrm{C}} v_{\mathrm{LR}} \tau + \beta v_{\mathrm{C}} \xi$$

with $v_{\mathrm{C}} = ||J||/||C||$

New in rigorous version:

Extra $\beta v_C \xi$ term. If thermalization becomes very fast, diffusivity can still saturate at scale set by microscopic interaction range. Picture for resistivity saturation?

Questions

- The Lieb-Robinson velocity is microscopic, and will control high-temperature transport. Is there a non-quasiparticle velocity at finite temperatures? Is it the butterfly velocity (quantum chaos)?

- Dissipative Markovian dynamics simplifies theory (and is interesting in its own right!). Again tied to infinite temperatures. Can a finite temperature be introduced while preserving tractability?

Questions

- The local thermalization time also controls transport. At infinite temperature it can itself be bounded in terms of local couplings in the Hamiltonian. What are constraints on thermalization time in finite temperature systems? When does the 'Planckian' time $\tau = \hbar/kT$ emerge?

- Is our formula for the diffusivity of open quantum systems useful in experiments?