Black Holes and Quantum Heat Engines

Warning! Slides have omitted citations to background papers. Apologies to all concerned. See arXiv:1905.00539 for background refs.

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Geometrizing Quantum Problems

AdS/CFT

&

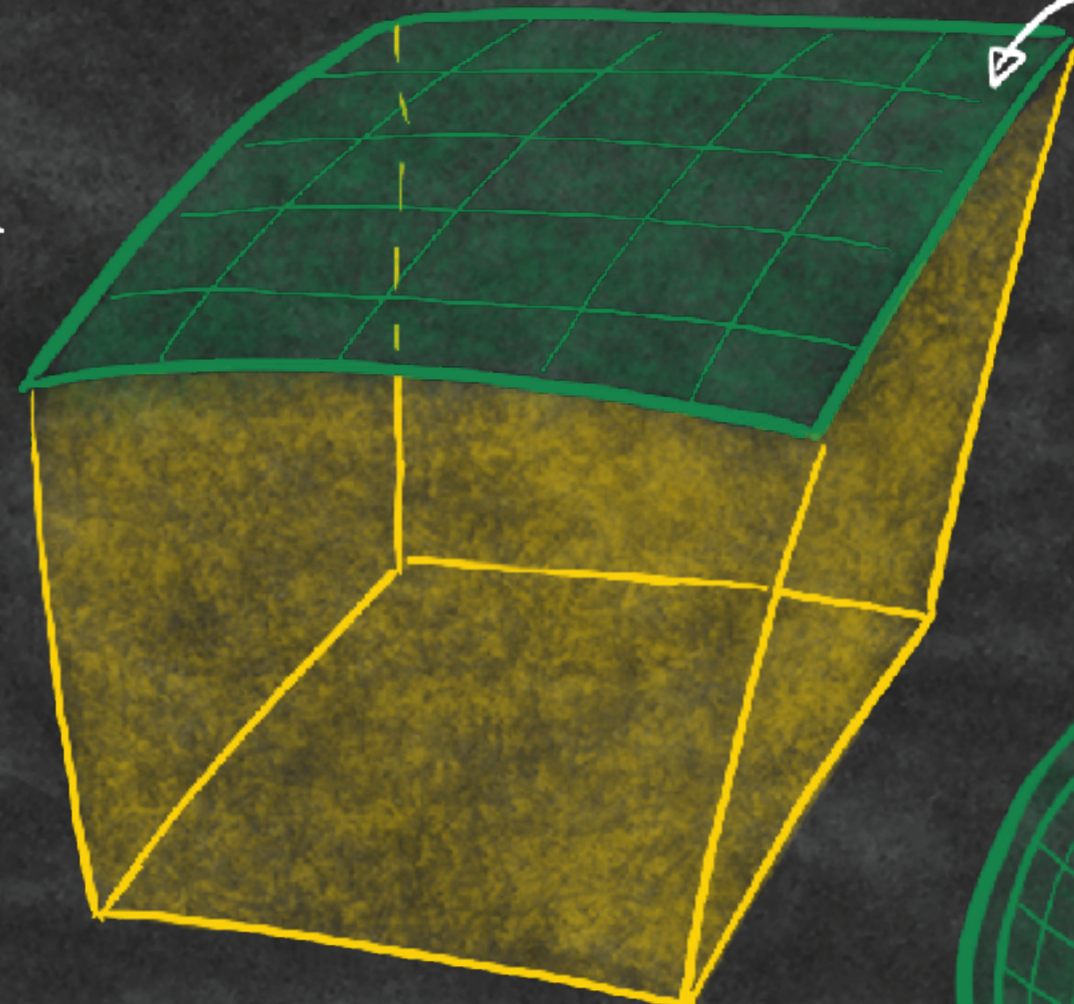
generalizations

d-dimensional

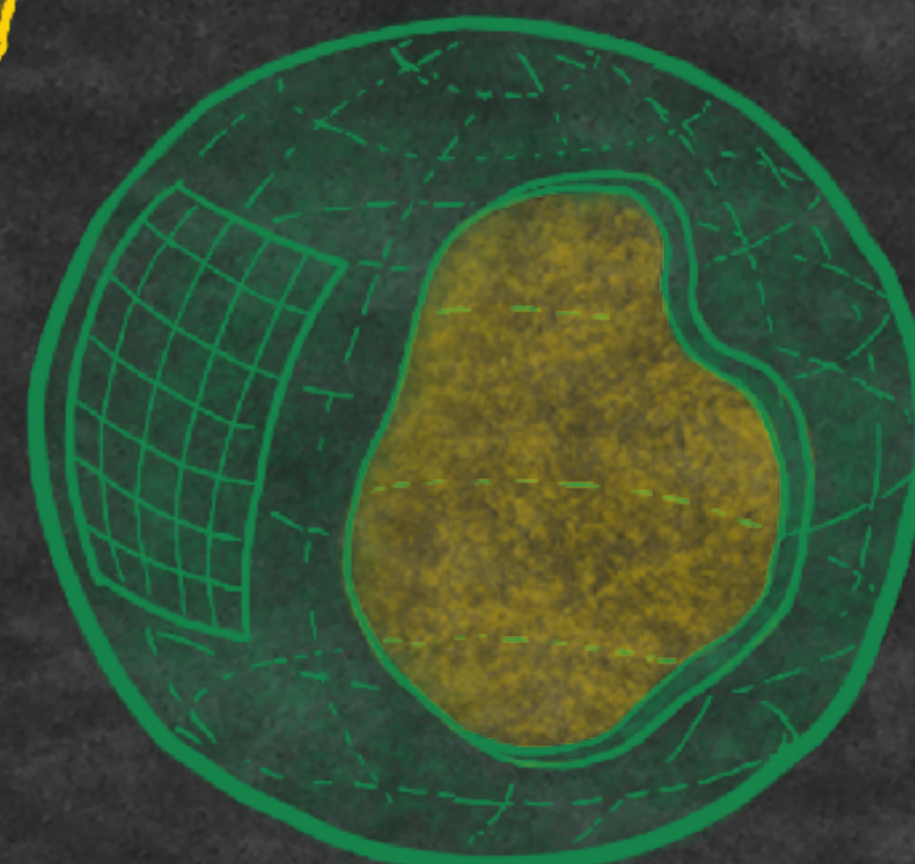
QFT

eg $SU(N)$ gauge theory

g^2, N



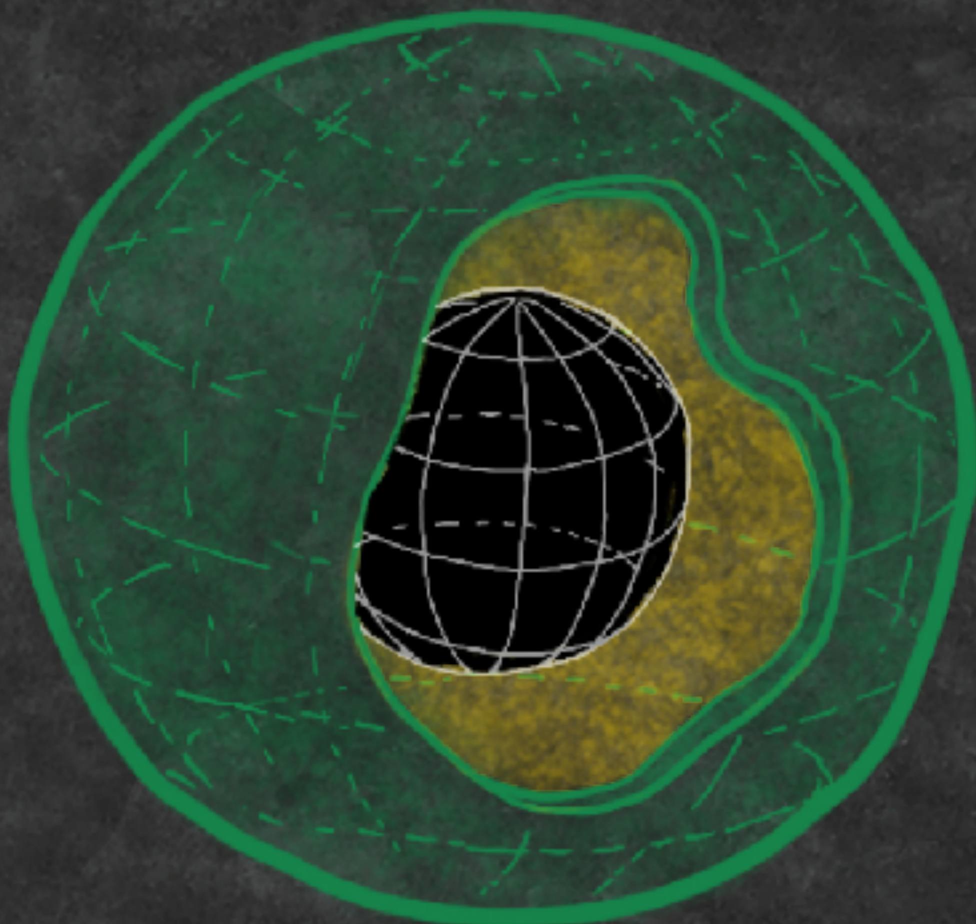
extra dimension



d+1 dimensional gravity

with c.c. $\Lambda \sim -1/\ell^2$

Thermal Physics



black hole, radius r_h

$$M = \frac{r_h^3}{2\ell^2} \longrightarrow U$$

$$\kappa = \frac{3r_h}{4\pi\ell^2} \longrightarrow T$$

$$\frac{A}{4} = \pi r_h^2 \longrightarrow S$$

$$\boxed{u \sim N^{3/2} T^3}$$

density

$$\boxed{dU = T\,dS}$$

first law

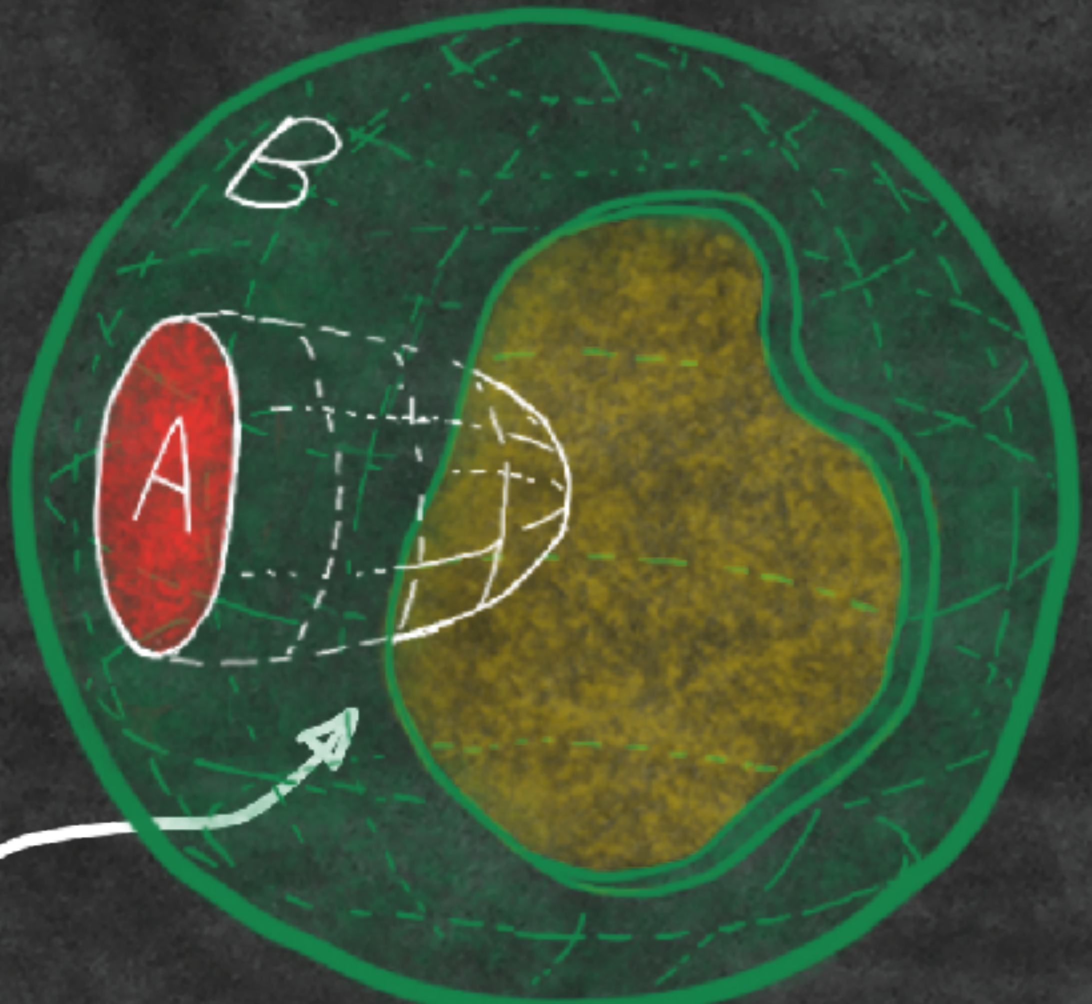
Quantum Entanglement

$$\rho = |0\rangle\langle 0|$$

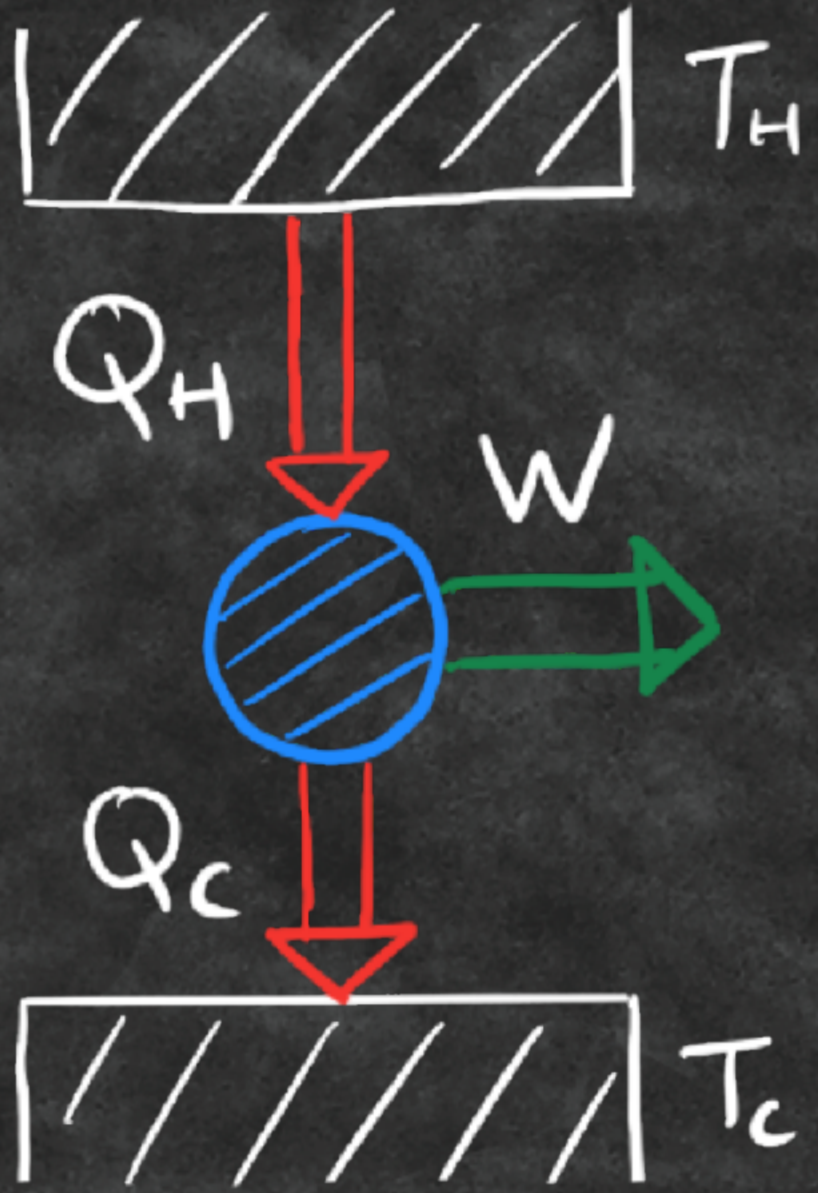
$$\rho_A = \mathrm{Tr}_B\, \rho$$

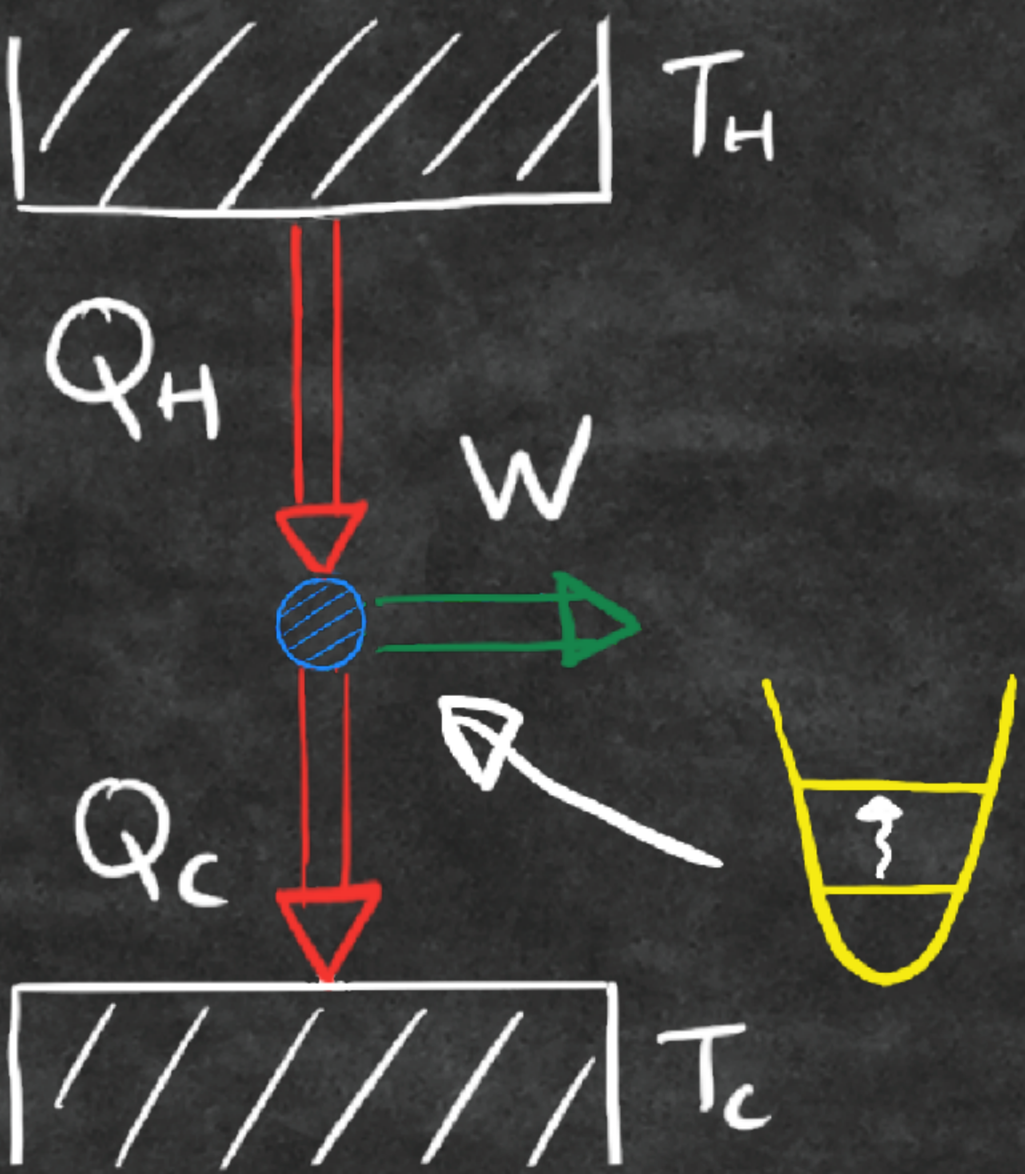
$$S_{EE} = -\mathrm{Tr}\, \rho_A \ln \rho_A$$

$$= \frac{\text{Area}}{4}$$



minimal
surface
area A





Today: Quantum Heat Engines

key: Working substance has small number of d.o.f.

Can gravity + black holes help?

- No p or V for mechanical work
- WAY too big a system:

$$\text{d.o.f} \sim N^{\#}$$

N large for good geometry description

Solution: We're in the wrong thermo...

B.H. Thermodynamics ("extended")

mass M $\Longrightarrow$ $H \equiv U + pV = M$ enthalpy

area A $\Longrightarrow$ $S = A/4G$ entropy

surface gravity κ $\Longrightarrow$ $T = \frac{\kappa}{2\pi}$ temp.

cosmo. const. Λ $\Longrightarrow$ $p = -\frac{\Lambda}{8\pi G}$ pressure

1st Law $H(p, S)$

$$dH = TdS + Vdp \qquad T = \left.\frac{\partial H}{\partial S}\right|_p \qquad V = \left.\frac{\partial H}{\partial p}\right|_S$$

Example: Schwarzschild in AdS

$$\Lambda = -\frac{3}{\ell^2} \quad \therefore p = \frac{3}{8\pi G \ell^2}$$

$$ds^2 = -F(r)\,dt^2 + F(r)^{-1}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$F(r) = \left(1 - \frac{2M}{r} + \frac{r^2}{\ell^2}\right)$$

horizon at largest root of $F(r) = 0$

i.e. $r_h^3 + r_h \ell^2 - 2M\ell^2 = 0$

$$S = \frac{A}{4} = \pi r_h^2$$

solve: $M = \dfrac{r_h}{2}\left(1 + \dfrac{r_h^2}{\ell^2}\right)$

Also, $\beta F' = 4\pi$ gives $T = \dfrac{1}{4\pi}\left(\dfrac{2M}{r_h^2} + \dfrac{2r_h}{\ell^2}\right)$

Now write $M = H(S,p) = \frac{1}{2}\left(\frac{S}{\pi}\right)^{1/2}\left(1+\frac{8Sp}{3}\right)$

check: $T = \left.\frac{\partial H}{\partial S}\right|_p$

$$= \frac{1}{2}\cdot\frac{1}{2}\left(\frac{1}{\pi}\right)^{1/2} S^{-1/2}\left(1+\frac{8Sp}{3}\right)$$

$$+\frac{1}{2}\left(\frac{1}{\pi}\right)^{1/2} S^{1/2}\left(0+\frac{8}{3}p\right)$$

$$= \frac{1}{4}\,\frac{1}{\pi^{1/2}}\,\frac{1}{S^{1/2}}\left(1+8Sp\right)$$

$$= \frac{1}{4\pi r_h}\left(1+3r_h^2/\ell^2\right) = \frac{\left(2M + 2r_h^3/\ell^2\right)}{4\pi r_h^2}$$

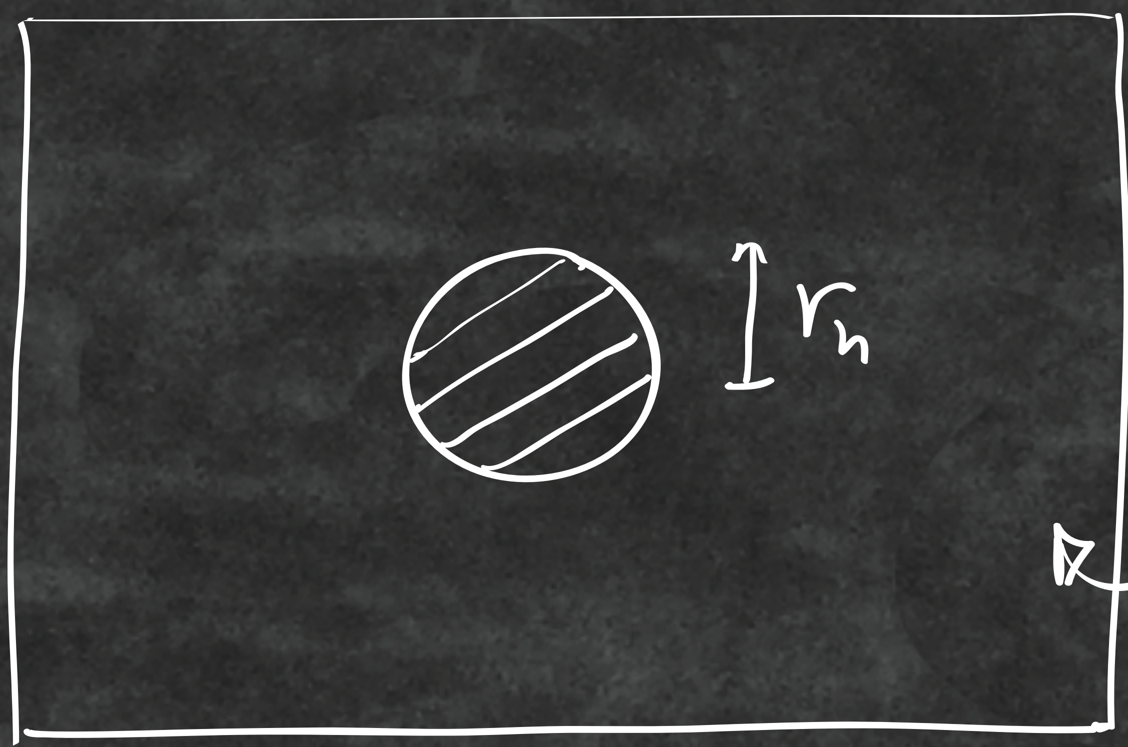
Now for something new...

$$H = \frac{1}{2}\left(\frac{S}{\pi}\right)^{1/2}\left(1 + \frac{8Sp}{3}\right)$$

$$V = \left.\frac{\partial H}{\partial p}\right|_S = \frac{1}{2} S^{1/2} \frac{1}{\pi^{1/2}} \frac{8}{3} S$$

$$= \frac{4}{3}\frac{S^{3/2}}{\pi^{1/2}} = \frac{4}{3}\pi r_h^3 \quad !!$$

The 'naive' volume!



energy density

$$\rho = \frac{\Lambda}{8\pi G}$$

so cost is ρV

What about the degrees of freedom?

Let's check: use specific heat

$$C(T) = T\frac{\partial S}{\partial T}$$

$$dU = TdS - pdV$$

- $\boxed{C_V(T)}$ tells us about available d.o.f.

  $C_p(T)$ also includes energy used for work.

For black holes, usual $C \to C_p$ in extended.

So what is C_V? (largely overlooked)

In fact, for our favourite black holes:

$$C_V = 0 \; !$$

follows from V and S both being geometrical

$$V = \frac{4\pi}{3} r_h^3 \qquad S = \pi r_h^2$$

So, $\boxed{\text{No degrees of freedom!!}}$

(Well, no "traditional" ones.)

So, what to do?

Use black holes w $C_V \neq 0$

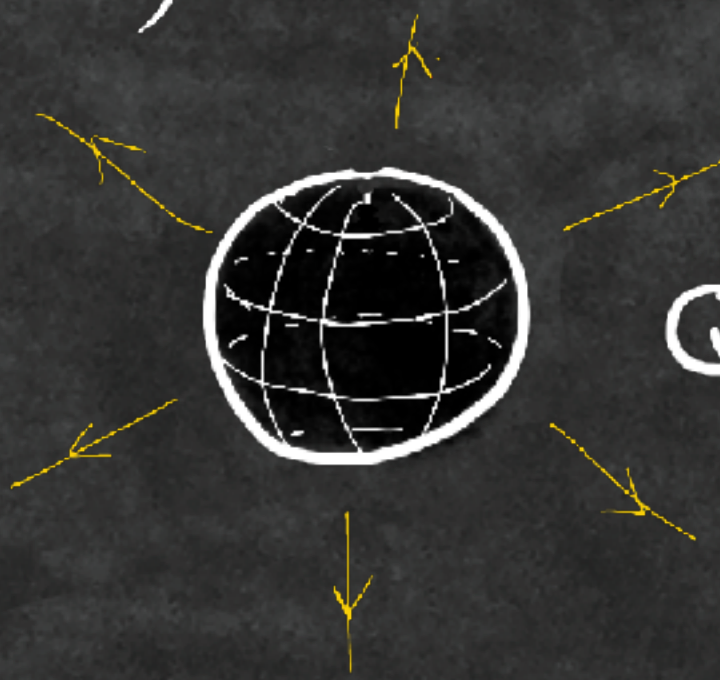
e.g. Kerr

$$V = \frac{2}{3\pi H}\left(S\left(S + \frac{8p}{3}S^2\right) + 2\pi^2 J^2\right) \qquad \text{(Dolan)}$$

$$H = \frac{1}{2}\sqrt{\frac{\left(S + \frac{8p}{3}S^2\right)^2 + 4\pi\left(1 + \frac{8pS}{S}\right)J^2}{\pi S}}$$

So, what to do?

also, STU

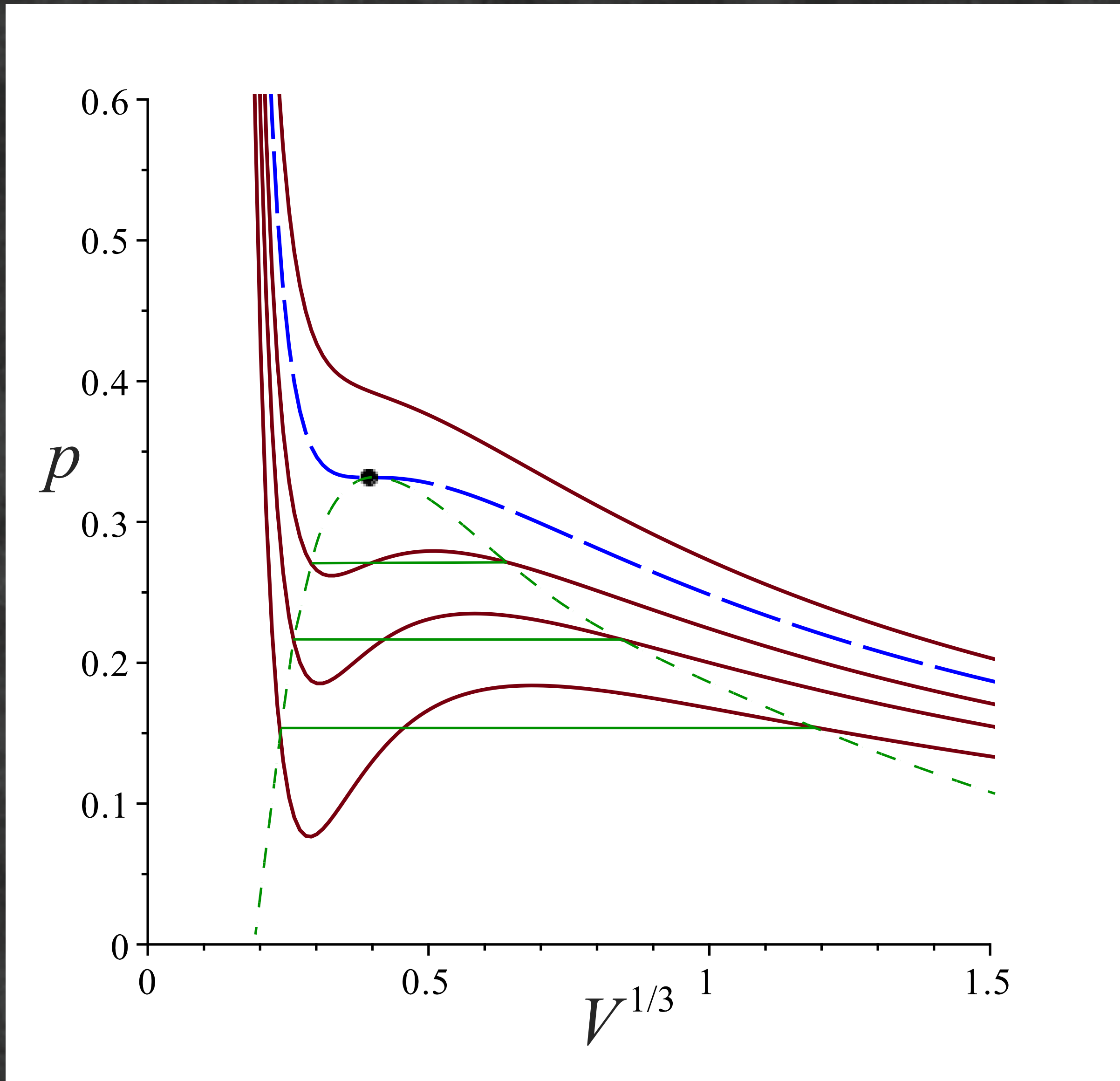


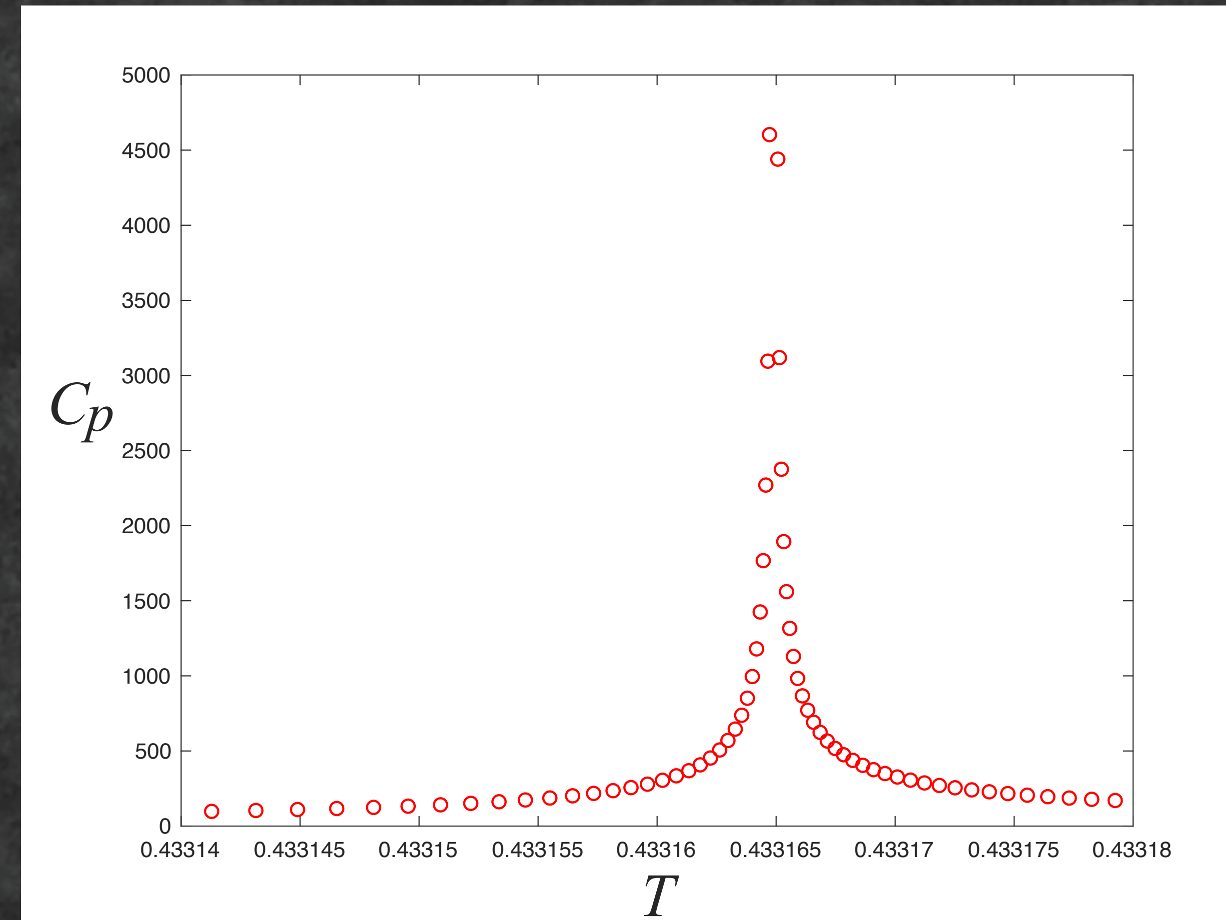
Q

b.h. charged non-diag.
under

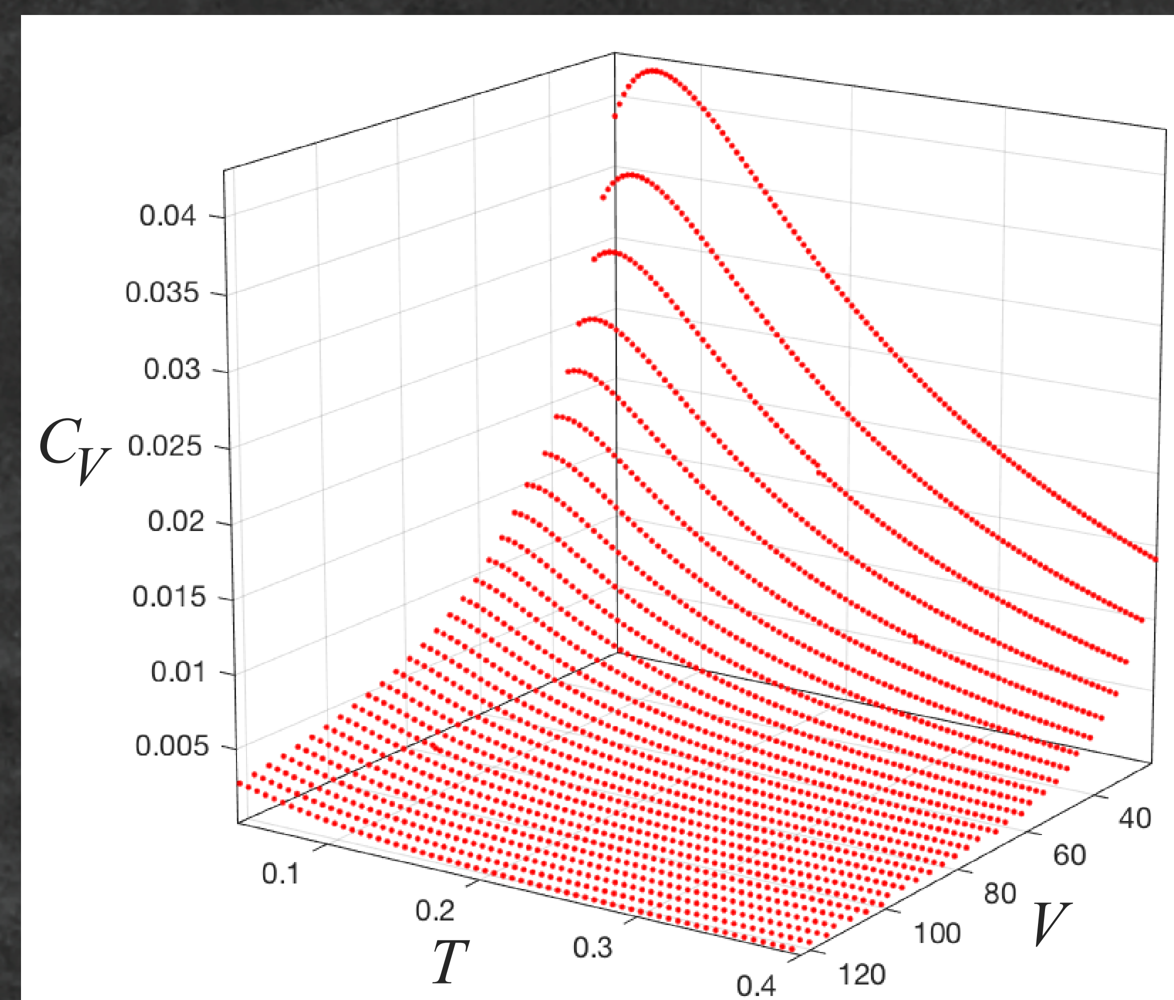
$$U(1) \times U(1) \times U(1) \times U(1)$$
$$Q_1 \quad Q_2 \quad Q_2 \quad Q_3$$

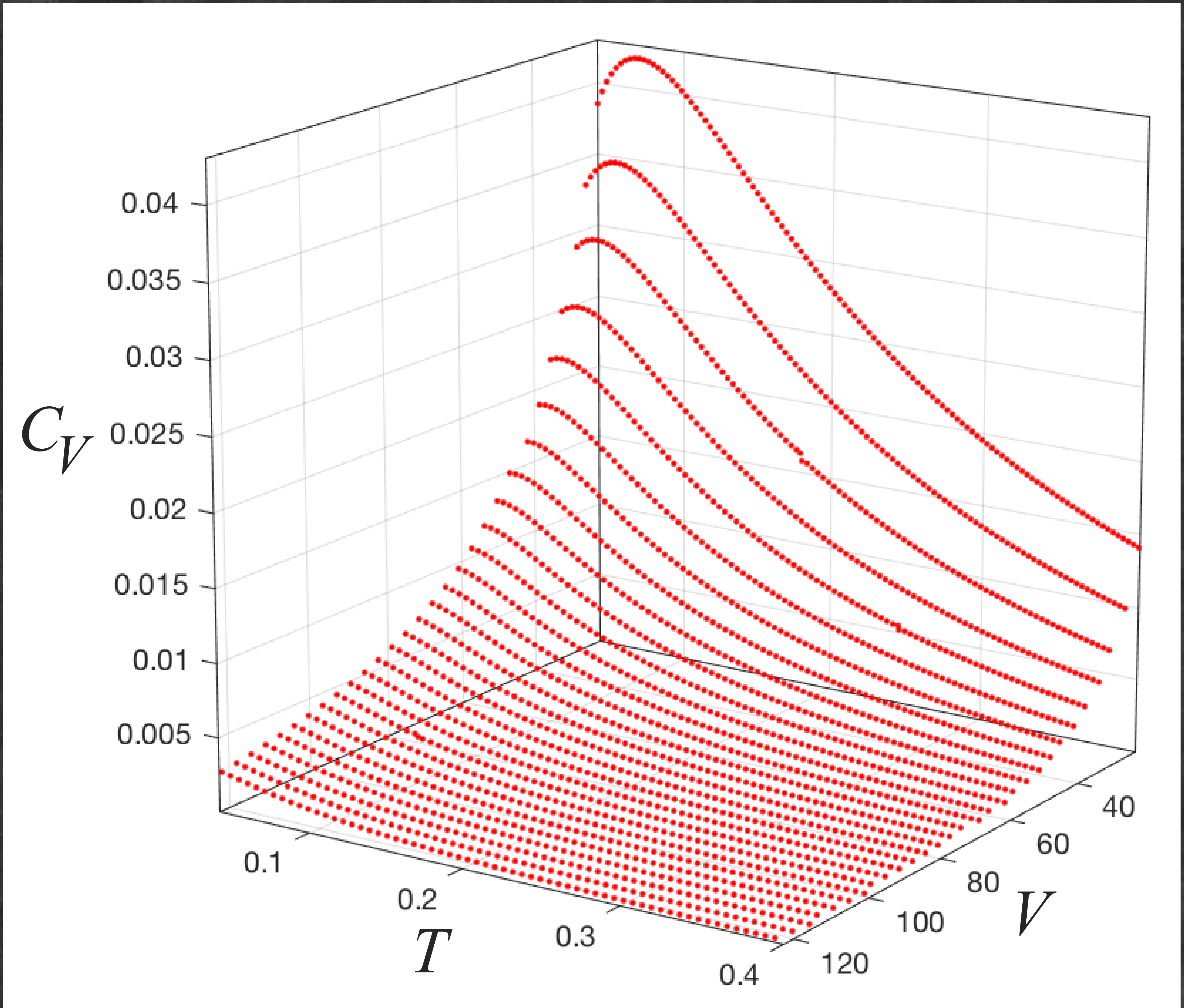
(RN is
all Q's equal)





New exciting results!
See: arXiv:1905.00539





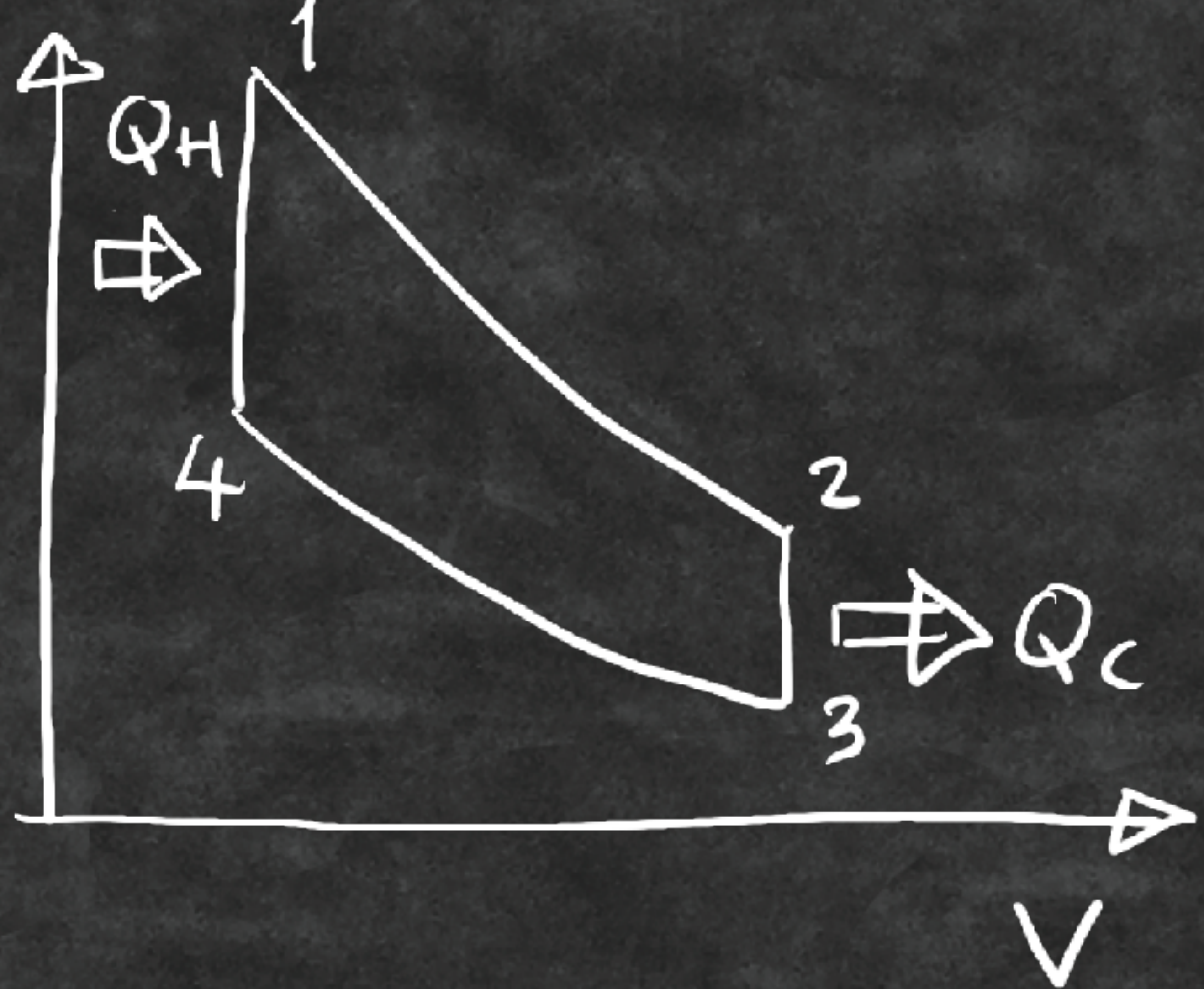
C_V
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0.1
0.2
0.3
0.4
T
40
60
80
100
120
V

Schottky-like Peaks!

- A finite window of energies.
- Height, position set by Q & J.

This gives us the "small" system we need!

Holographic heat engines as Quantum heat engines



Otto cycle

In each case a cycle is a tour through a family of models (SHO, or QFT)

Summary + Discussion

- Are these solvable models useful?
- What issues in experiment + theory can they help with?

(Power vs. effeciency?
Running near criticality?)

- Can we model friction /noise?
- Models of heat baths?
- Do QHE do other cycles?

Thank you!