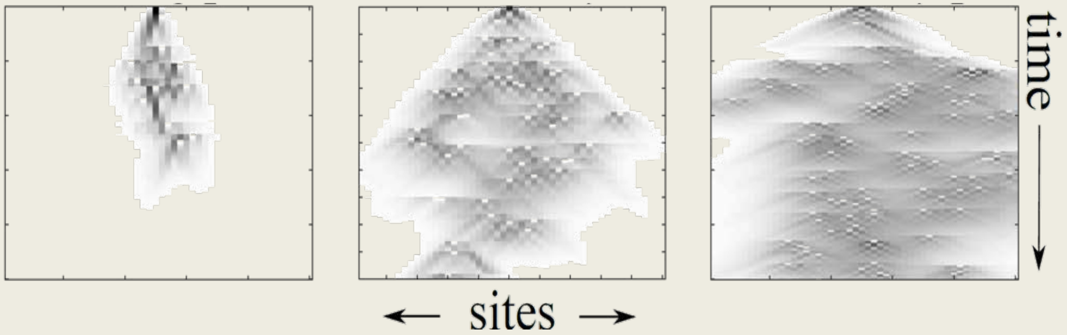
Constrained dynamics

Disorder and localisation

Non-equilibrium phase transitions

… on Rydberg Quantum Simulators



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Igor Lesanovsky

KITP – Santa Barbara
01/05/2019

Marcuzzi et al., PRL **118**, 063606 (2017)
Ostmann et al., Quant. Sci. Tech. **4**, 02LT01 (2019)
Ostmann et al., arXiv:1811.01667
F. Carollo et al., arXiv:1902.04515

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positions available

5th Granada Summer School on

Quantum Matter Out of Equilibrium

1st-5th September 2019, Granada, Spain

This School will feature tutorial style lectures introducing themes of broad interest in the areas of **dynamics of open and closed quantum systems**, **quantum state preparation**, **numerical methods and quantum control**, **measurement and feedback**, among others **from a theory and experimental perspective**.

The aim is to provide a basis for new members of the community, deepening the knowledge of more experienced ones and giving a flavour of current trends in the field. All lectures will be given by leading scientists from around the world, but participants are strongly encouraged to present and discuss their own research, especially during a dedicated poster session and contributed talks. The level of the lectures will be aimed at graduate students and young post-docs.

Registration open: ***11th February - 31st May***
Further details at the website: *www.granada-summer-school.com*

Invited speakers

Alberto Amo (Laboratoire PhLAM, University of Lille)
Mari-Carmen Bañuls (Max Planck Institute of Quantum Optics, Garching)
Hans Peter Büchler (University of Stuttgart)
Andrew Daley (University of Strathclyde)
Juan P. Garrahan (University of Nottingham)
Zoran Hadzibabic (Cavendish Laboratory, Cambridge)
Carlos Pérez-Espigares (University of Granada)
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Organising committee:

Rosario González-Férez (University of Granada)
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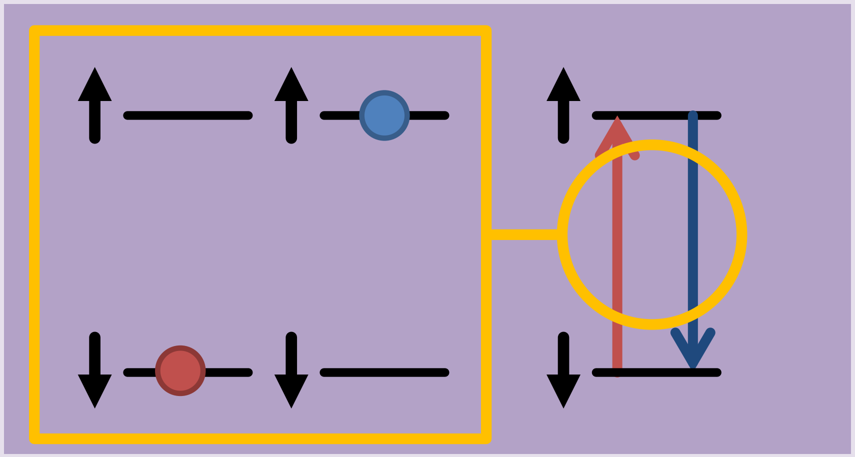
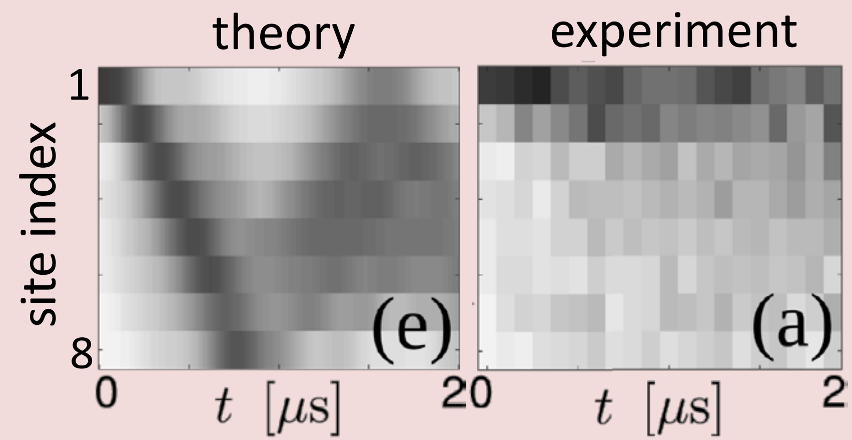
Beatriz Olmos (University of Nottingham)

Outline

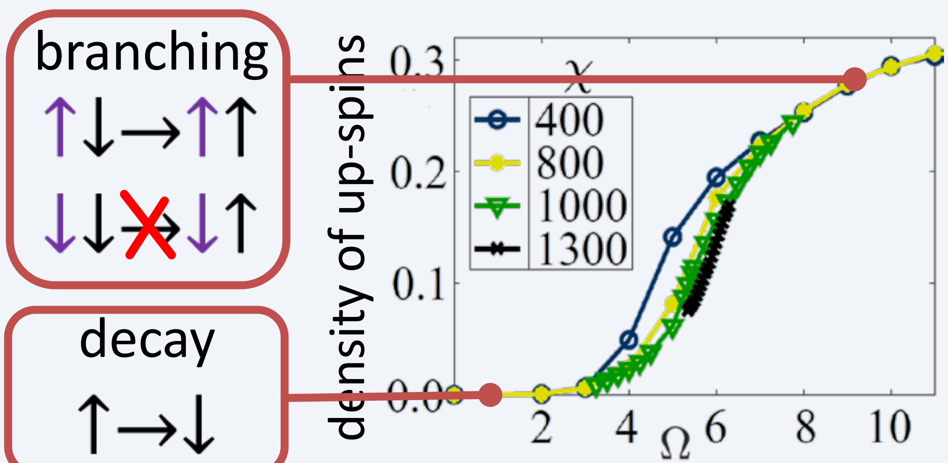
1) Constrained dynamics

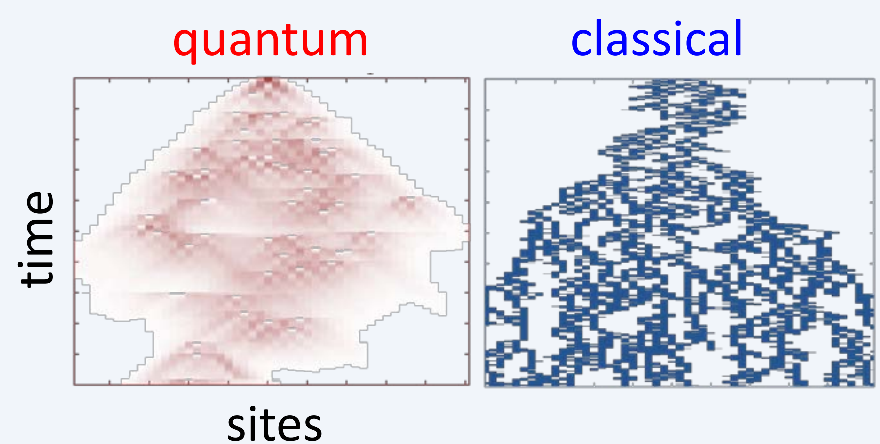


2) Disorder



3) Non-equilibrium absorbing state phase transitions

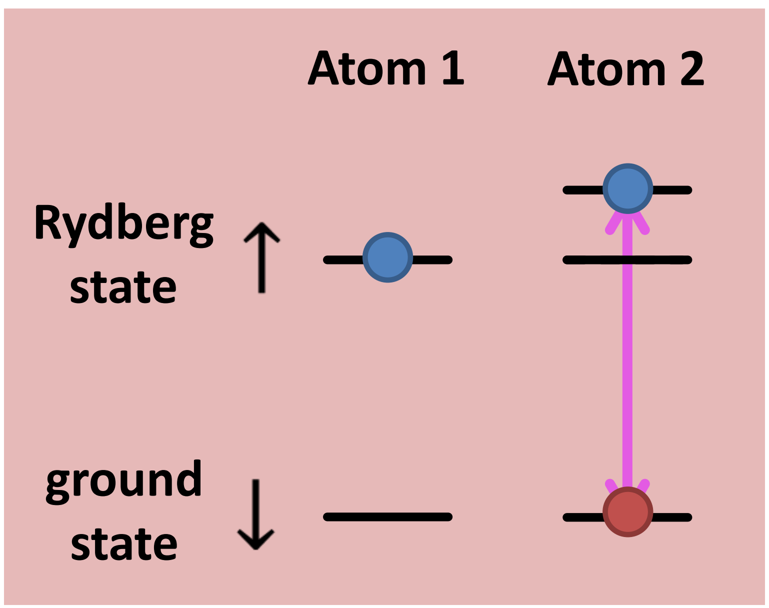




Facilitation constraint

Facilitated excitation/anti-blockade

- exploit conditional excitation to shifted energy level by applying off-resonant laser



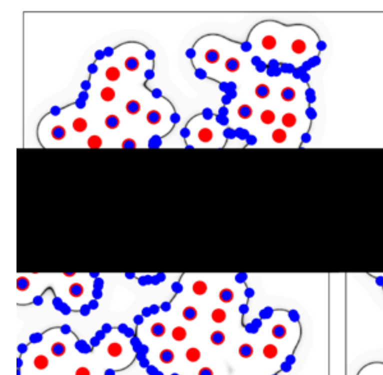
Applications

1) Dissipative state engineering, e.g. PRL **111**, 033607 (2013)

2) Quantum gates, e.g. PRA **88**, 043410 (2013)

3) "Quantum" glasses, e.g. NJP **17**, 113039 (2015)

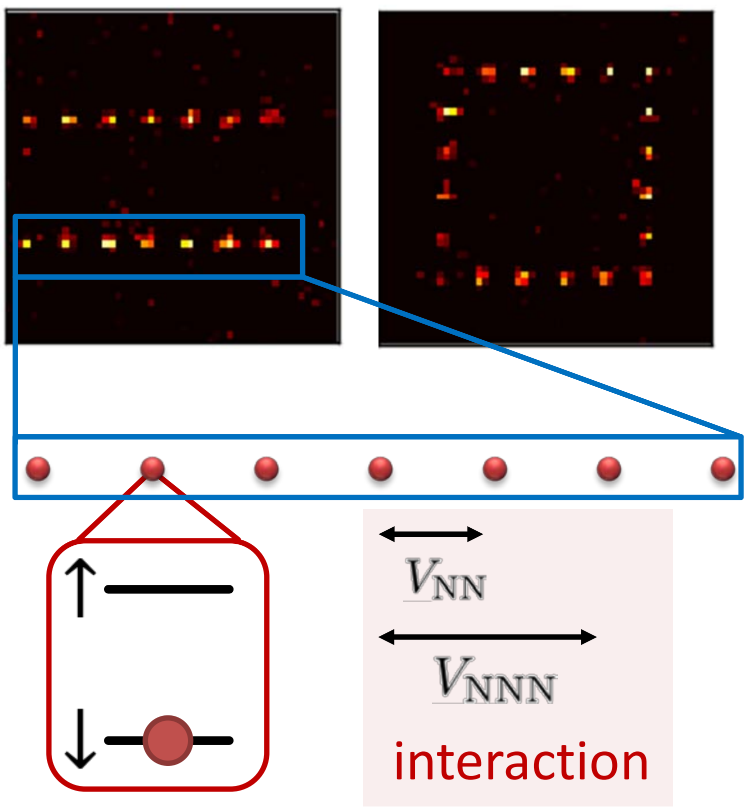
4) Growth dynamics, e.g. PRA **93**, 040701 (2016)



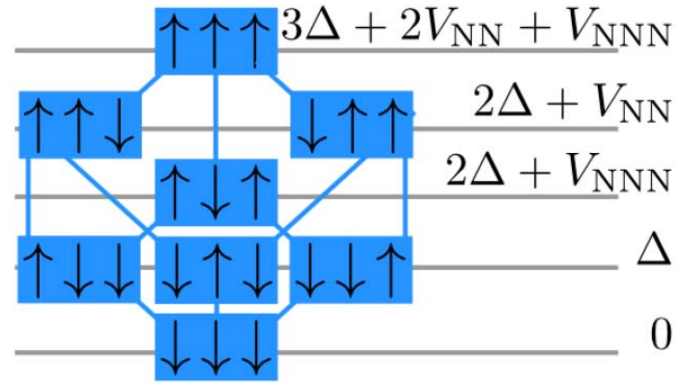
Facilitation constraint

Palaiseau experiment
[Science **354**, 1021 (2016)]

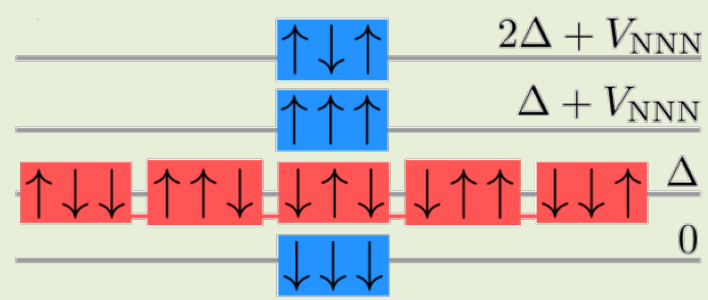
- Rydberg dynamics in large spacing optical lattices (site distance ~ 5-15 um)



Off-resonant excitation

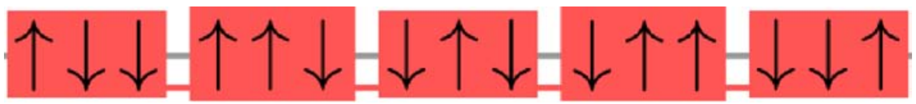


Facilitation condition (Δ+V_{NN}=0)



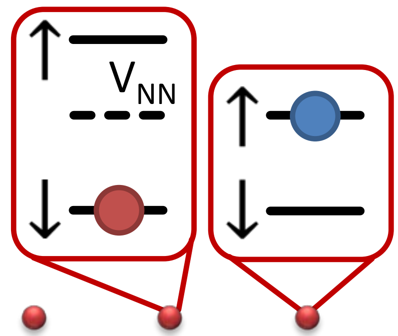
Resonant dynamics

- limit of strong interactions, i.e. V_{NN} and V_{NNN} large
- reduction to version of **tight binding model** (in Fock space)



Idealised situation

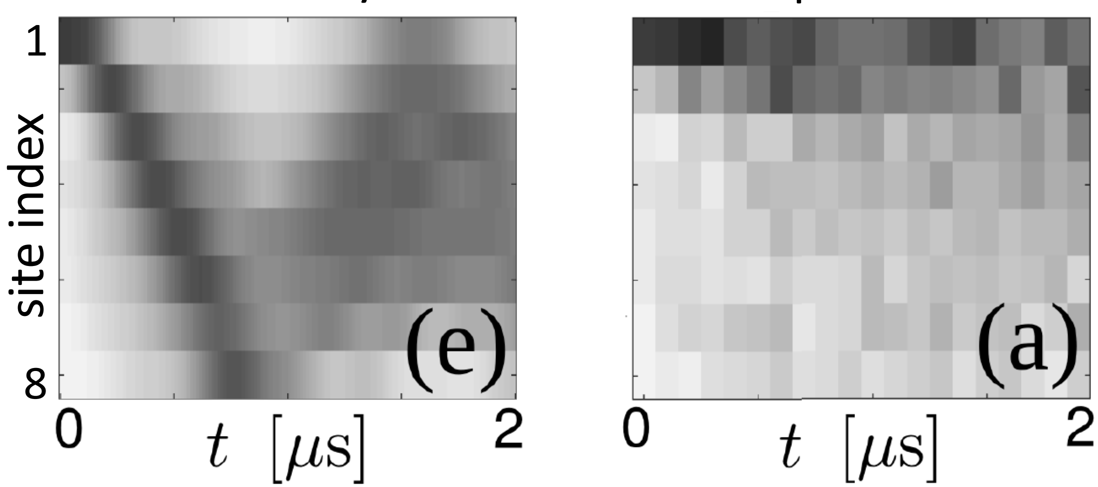
- atoms considered as points



Real-space dynamics

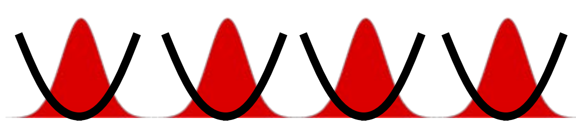
theory

experiment



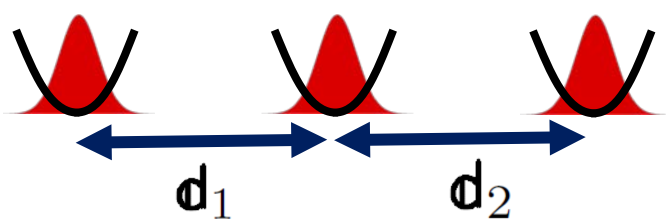
Actual situation

- finite temperature (50 uK)
- finite stiffness of trap



Disorder

Characterisation of disorder



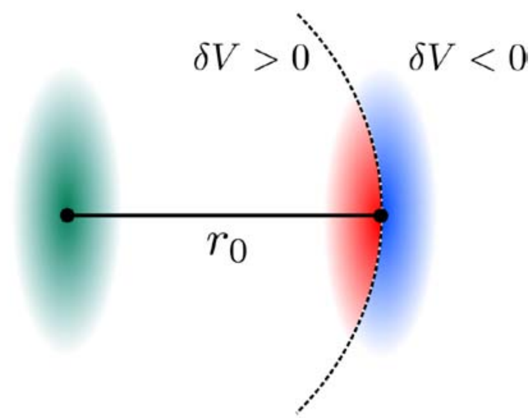
Gaussian distribution of atomic positions

$$p^{(k)}_{\mathrm{pos}}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}\,\sigma_1\sigma_2\sigma_3}\, \mathrm{e}^{-\frac{r_1^2}{2\sigma_1^2}-\frac{r_2^2}{2\sigma_2^2}-\frac{(r_3-kr_0)^2}{2\sigma_3^2}}$$

Distribution of atomic distances

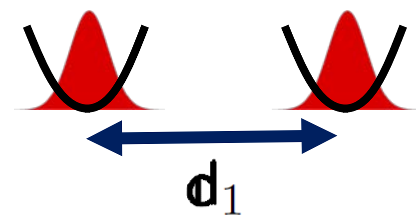
$$p_{\mathrm{diff}}(\mathbf{d}_1,\ldots,\mathbf{d}_{L-1}) = \int \left[\prod_{k=1}^{L} \mathrm{d}^3 r_k\, p^{(k)}_{\mathrm{pos}}(\mathbf{r}_k)\right]\left[\prod_{k'=1}^{L-1} \delta^{(3)}\left(\mathbf{d}_{k'} - (\mathbf{r}_{k'+1}-\mathbf{r}_{k'})\right)\right]$$

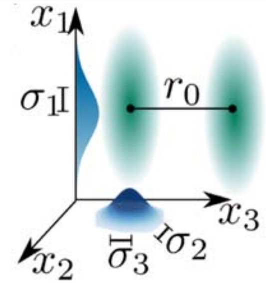
- distances are **identically** but **not independently** distributed
- disorder is correlated
- disorder enters through interactions: $V_{NN}=V_0+\delta V$ (skewed to smaller interactions)



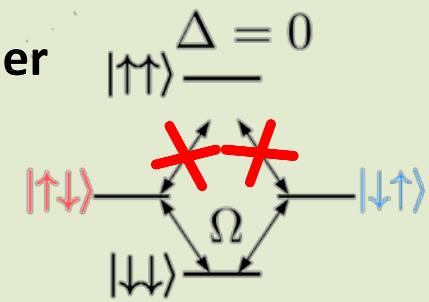
Two atoms

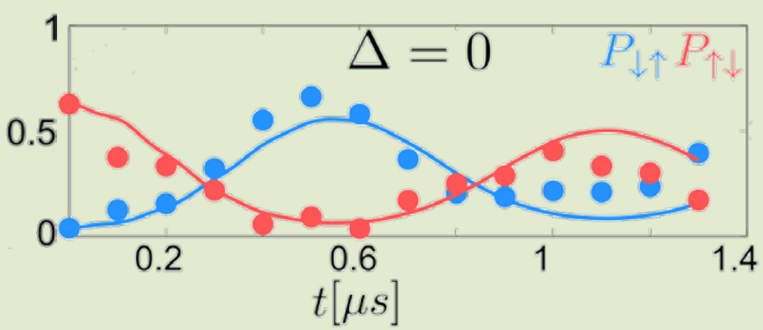
- only $|\uparrow\uparrow\rangle$–state is affected by disorder
- for two atoms disorder can be (approximately) switched on and off by controlling the detuning





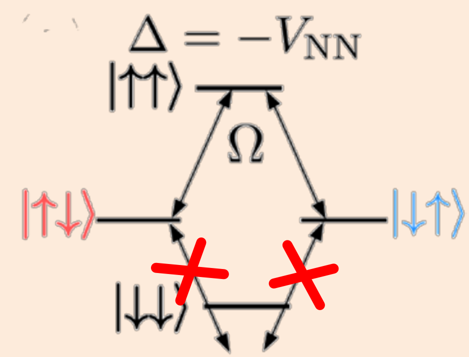
"without" disorder

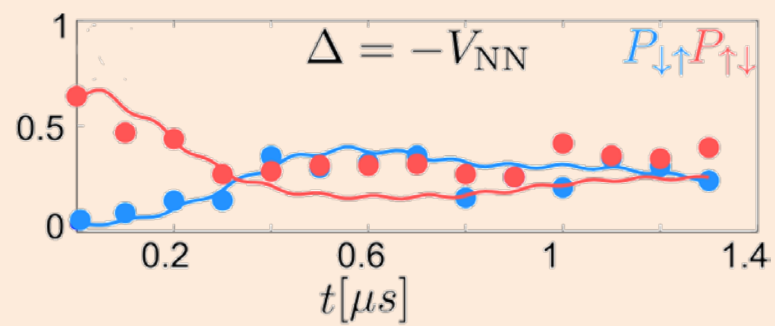




- coherent oscillations with (moderately) large amplitude
- note, imperfect initial state

with disorder



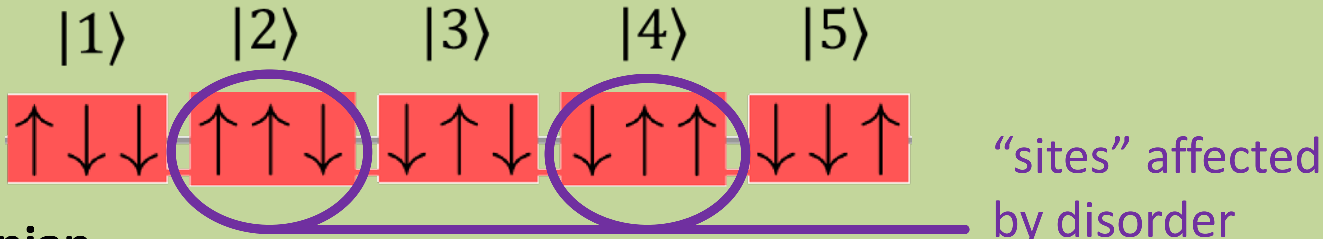


- rapid loss of contrast
- no perfect agreement between theory and experiment, but right systematics

„Many-body“ dynamics

Generalised Anderson model (“Anderson-Fock”)

- consider Fock space configurations as lattice sites $|b\rangle$

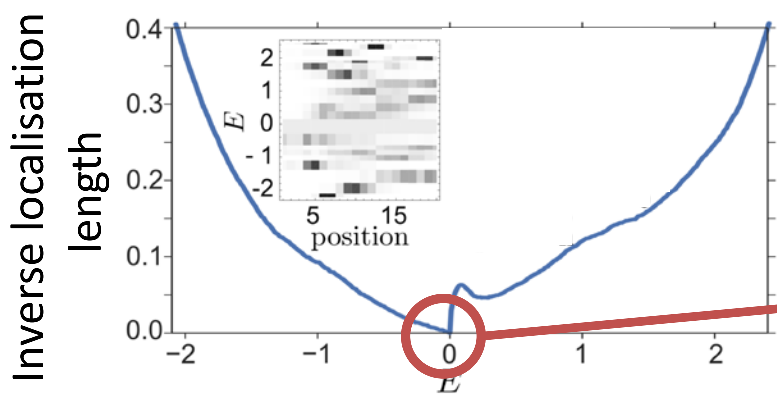


Hamiltonian

$$H_{\mathrm{A}} = \frac{\Omega}{2} \sum_{b=1}^{2L-2} \Big[|b\rangle \langle b+1| + |b+1\rangle \langle b| + h_b \, |b\rangle \langle b| \Big] \quad \text{with} \quad h_b = \frac{\delta V_{b/2}}{\Omega} \left[1 + (-1)^b\right]$$

site dependent disorder

Localisation



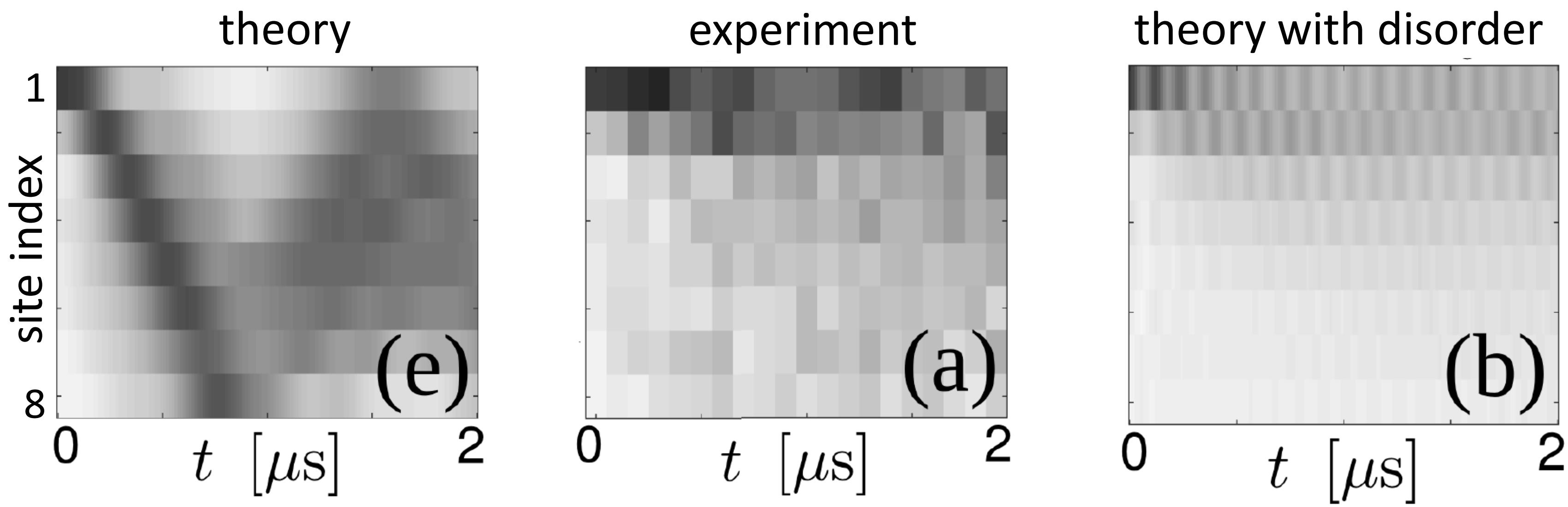
- all states are localised, but one

$$|\psi_0\rangle = \frac{1}{\sqrt{L}} \sum_b \sin\left(\frac{\pi b}{2}\right) |b\rangle$$

Localisation

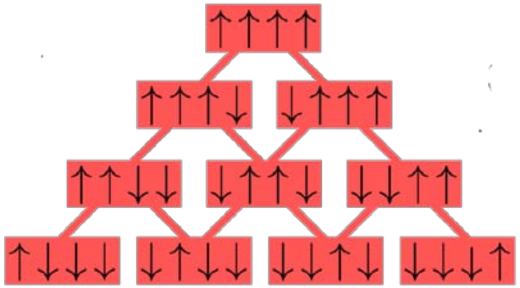
- localised eigenstates prevent spreading of initial state

Real-space dynamics



… note, this was the simplified story

- V_{NNN} not strong enough in experiment to warrant 1d approximation: more states are being accessed, cf. NJP **17**, 113039 (2015)



- for analyses of more complex Fock space structures see PRL **118**, 063606 (2017) Quant. Sci. Tech. **4**, 02LT01 (2019)

Beyond few-excitation physics

- so far: restriction to few excitation sector and focus on Fock space "lattice"

Many-body Hamiltonian

$$H_{\text{eff}} = \Omega \sum_j \boldsymbol{P_j} \sigma_j^x$$

Facilitation constraint

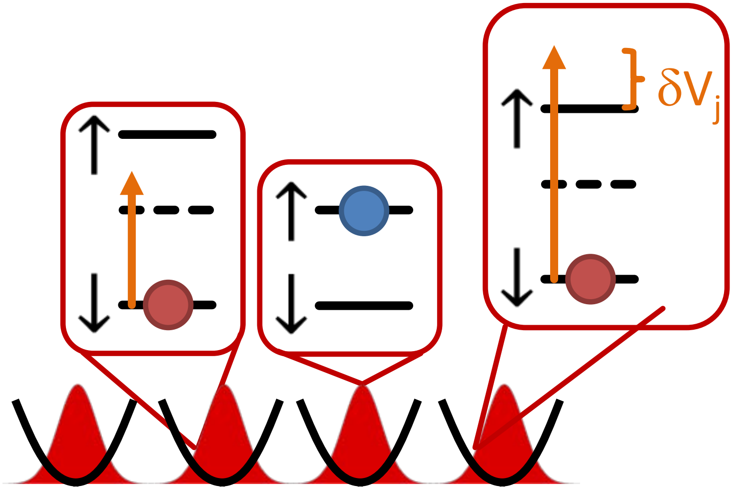
$$\boldsymbol{P_j} = \frac{1}{2}\left[1 - \sigma_{j-1}^z \sigma_{j+1}^z\right]$$

- constrained spin flips

Disorder "potential"

$$V_{\text{dis}} = \sum_j \delta V_j n_j \, n_{j+1}$$

- residual interaction (deviation from facilitation condition $\Delta = -V_{\text{NN}}$)



- Can this be brought into a more familiar shape?

Domain wall representation

Kramers-Wannier transformation

$$H_{\mathrm{eff}} = \Omega \sum_j \boldsymbol{P}_j \sigma_j^x$$

$$\left.\begin{aligned} \sigma_j^x &= \mu_j^x \mu_{j+1}^x \\ \sigma_j^y &= (-1)^{j+1} \Pi_{l=1}^{j-1} \mu_j^z \mu_{j+1}^y \mu_{j+1}^x \\ \sigma_j^z &= (-1)^{j+1} \Pi_{l=1}^{j} \mu_l^z \end{aligned}\right\} \quad H_{\mathrm{eff}} = \frac{\Omega}{2} \sum_k \left(\mu_j^x \mu_{j+1}^x + \mu_j^y \mu_{j+1}^y \right)$$

- constrained spin flips become hopping Hamiltonian (free fermions)

- disordered interactions become non-local disorder potential/interaction

$$V_{\mathrm{dis}} = \frac{1}{4} \sum_j \delta \mathrm{V}_j \left[\left((-1)^{j+1} \Pi_{l=1}^{j} \mu_l^z \right) + 1 \right] \left[\left((-1)^{j+1} \Pi_{l=1}^{j+1} \mu_l^z \right) + 1 \right]$$

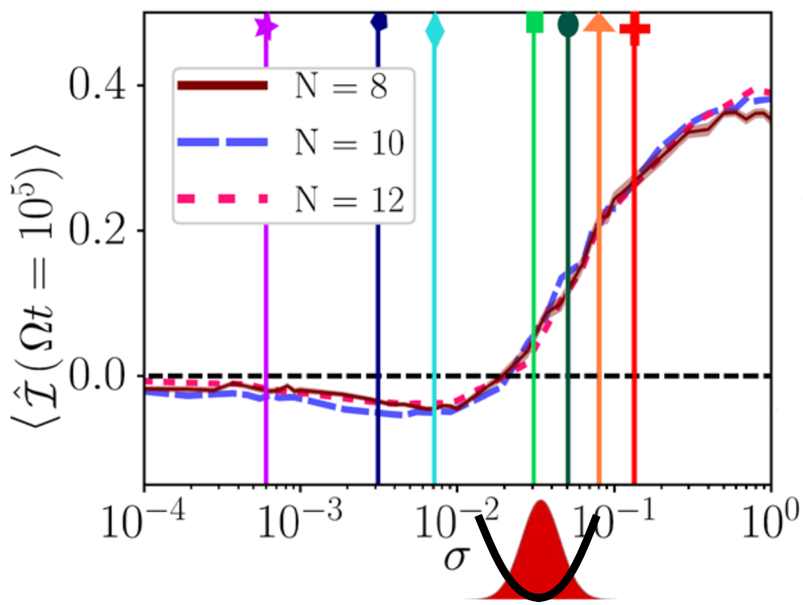
- Does this system display signatures of localisation?

Imbalance

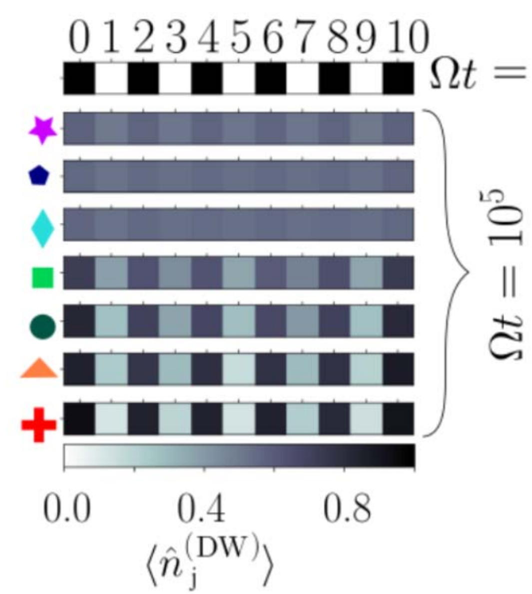
Ostmann et al., arXiv:1811.01667

- initial state: $|\Psi(t=0)\rangle_{\text{spin}} = |\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow\downarrow\cdots\rangle$ many excitations, unlike before
  (staggered domain-wall configuration)
- "loss of memory" of initial state can be quantified by **imbalance** (population difference between odd and even sublattice)

$$\hat{\mathcal{I}} = \frac{1}{N-1}\sum_{j=1}^{N-1}(-1)^j\left[\hat{\mathrm{n}}_j\left(\mathbb{1}-\hat{\mathrm{n}}_{j+1}\right)+\left(\mathbb{1}-\hat{\mathrm{n}}_j\right)\hat{\mathrm{n}}_{j+1}\right]$$



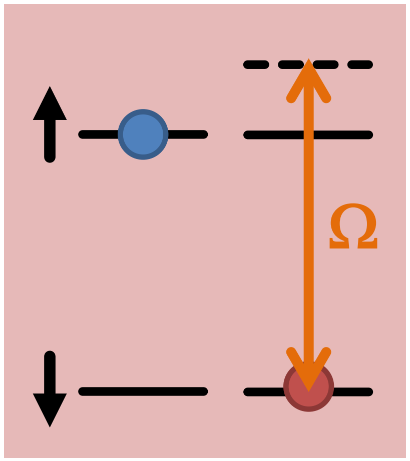
- small disorder: thermalisation
- strong disorder: non-ergodic



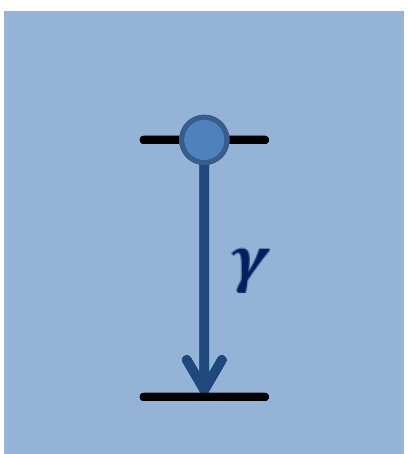
- Localisation also shown in entanglement entropy and energy level spacing statistics [Ostmann et al., arXiv:1811.01667]

Non-equilibrium phase transition

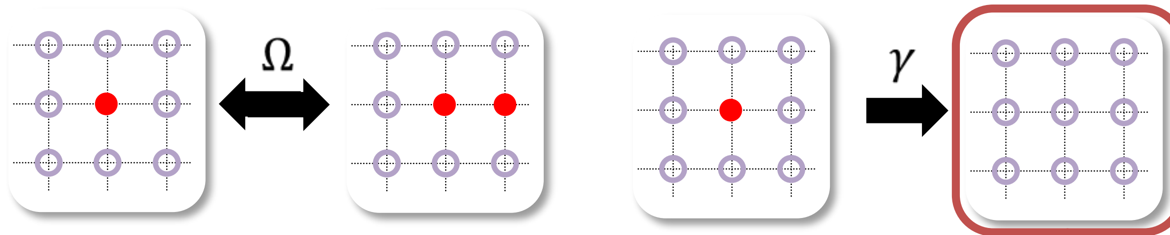
Facilitated excitation



Decay

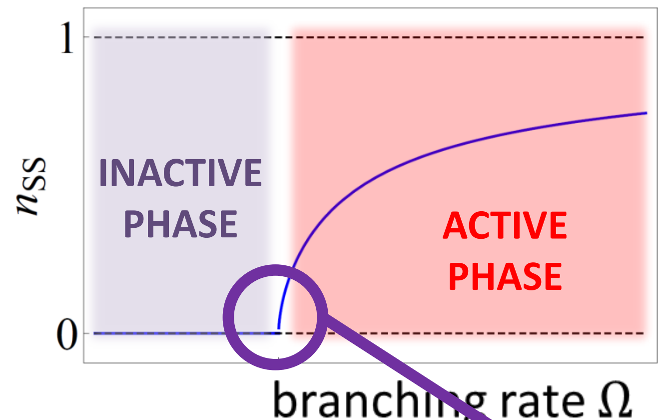


Contact process – "Game of life"



Unique absorbing state

Phase diagram



*observed (to some extent) with Rydbergs [PRA **96**, 041602(R) (2017)]*

Classical process

- continuous transition (**directed percolation** universality)

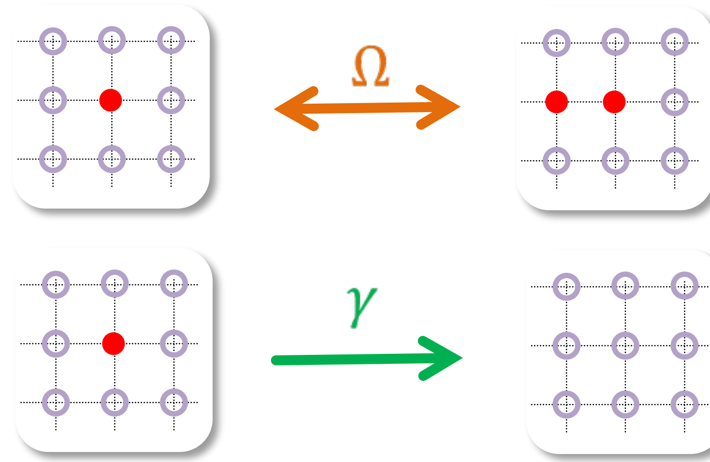
Questions:
What happens in when branching is "quantum"?
What when quantum and classical branching compete?

Quantum contact process

Competing processes

Quantum facilitation
[Quantum branching]

decay

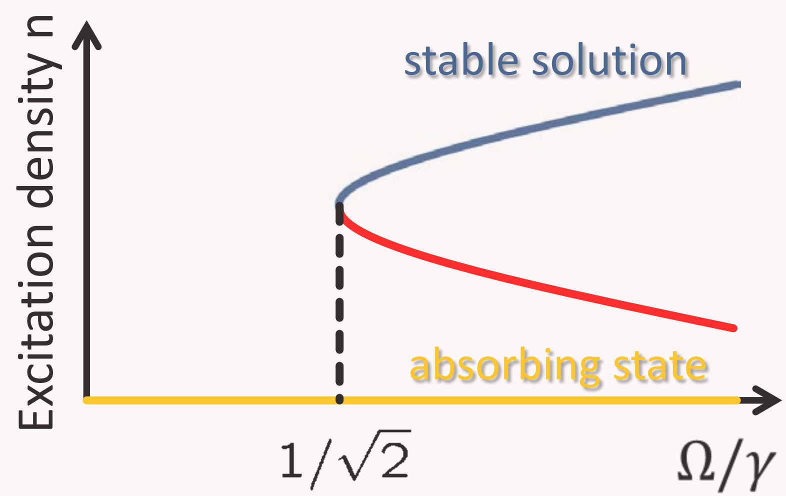


F. Carollo et al. arXiv:1902.04515 (2019)

Mean field ‚equation of state'

$$n\left[n^2 - \frac{1}{2}n + \frac{\gamma^2}{32\,\Omega^2}\right] = 0$$

- Suggests that transition in quantum process is of „1st order"
- recent results challenge that this is the case in a 1d-system
  [Roscher et al., PRA **98**, 062117 (2018)]
  [Odor et al., J. Stat. Mech. P08024 (2009)]

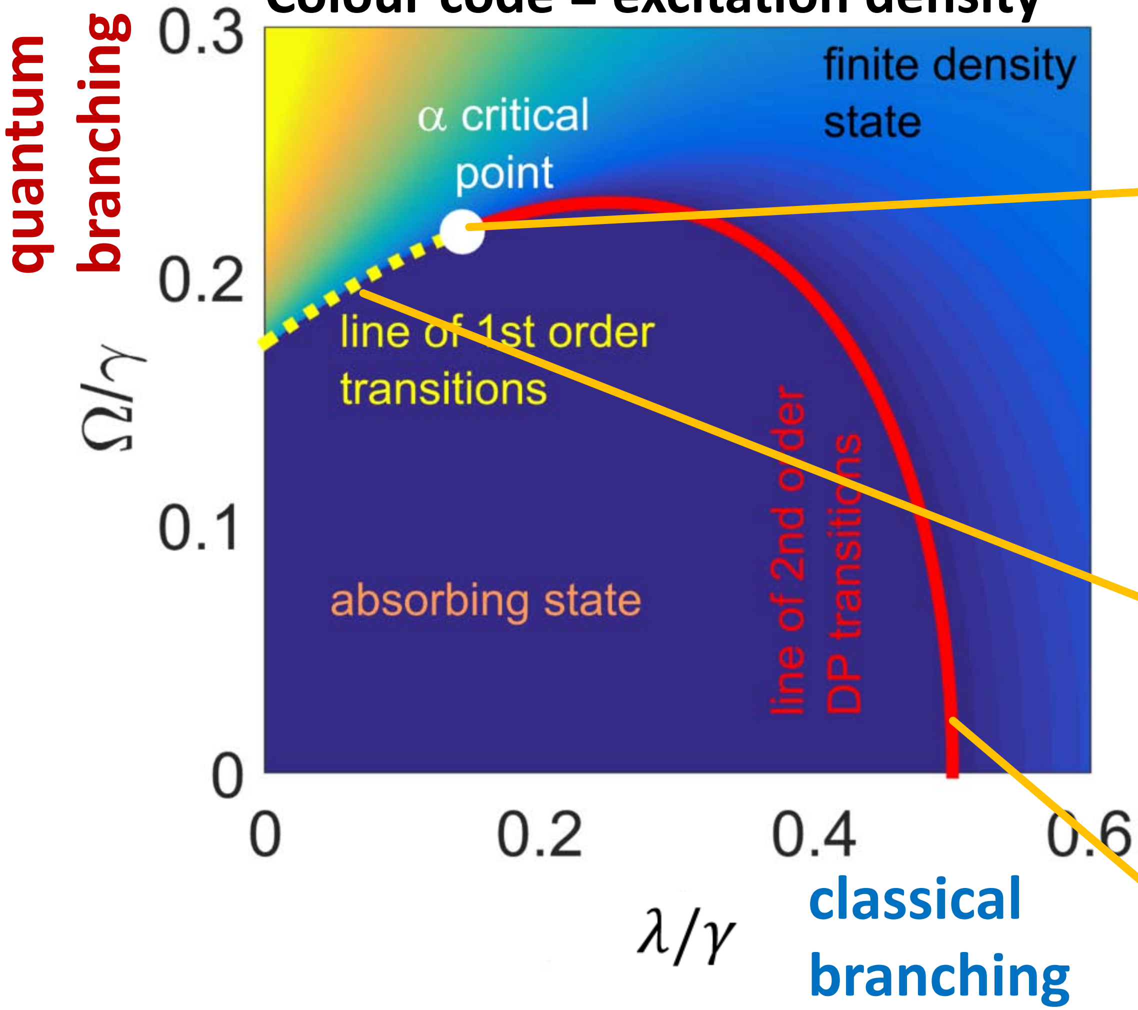


Phys. Rev. Lett. **116**, 245701 (2016)
Phys. Rev. B **95**, 014308 (2017)

Classical vs quantum (mean field)

Phase diagram

Colour code = excitation density



Tri-critical directed percolation

First order transition in limit where quantum processes dominate

Continuous transition (directed percolation) in classical limit

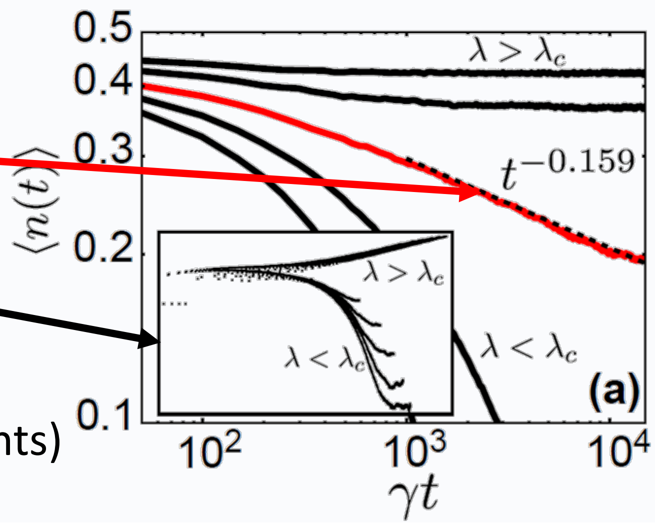
Phys. Rev. Lett. **116**, 245701 (2016); Phys. Rev. B **95**, 014308 (2017)

Classical vs quantum dynamics in 1d

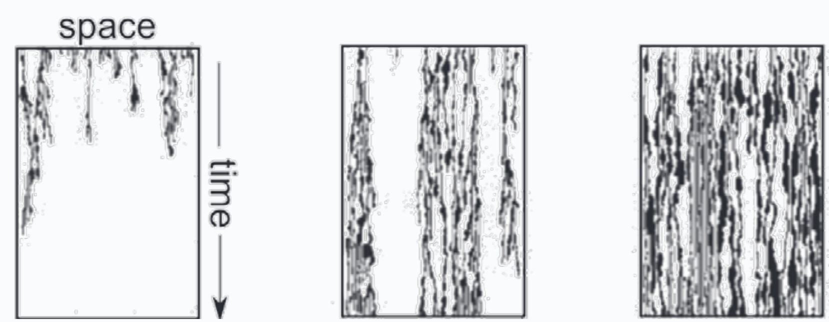
Classical

critical curve

collapsed data (rescaling with critical exponents)

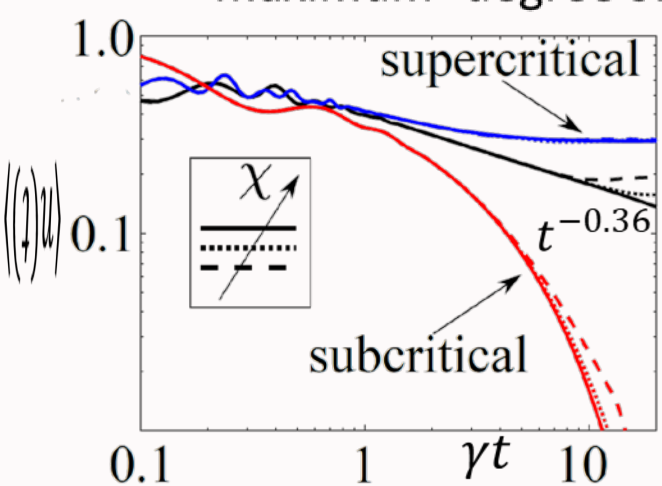


subcritical **critical** **supercritical**

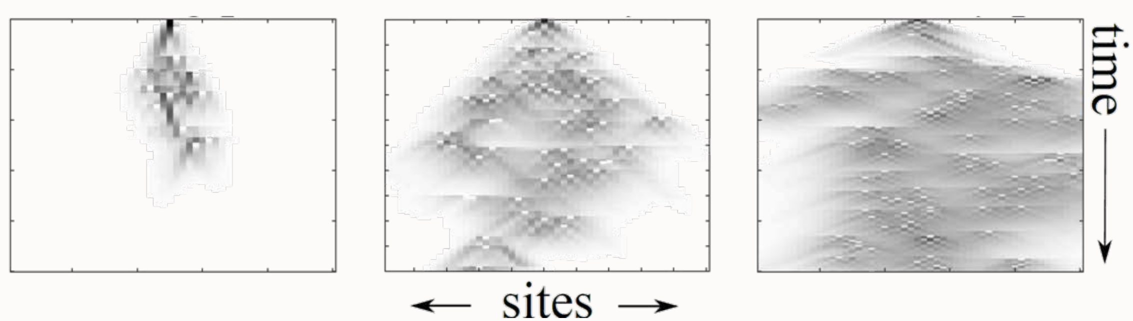


Quantum

- numerics is hard (iTEBD algorithm)
- maximum "degree of entanglement" is bond dimension χ

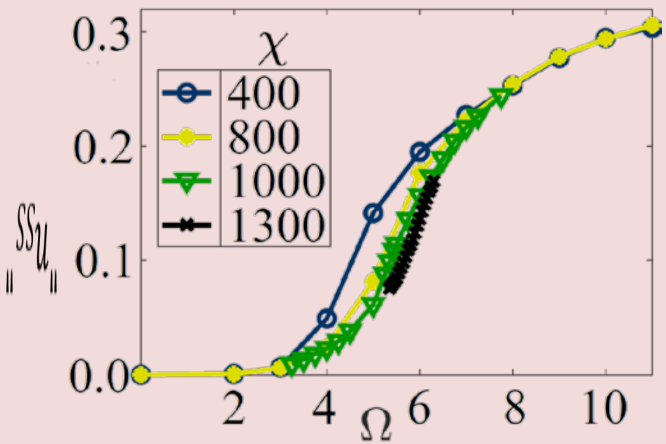


subcritical **critical** **supercritical**

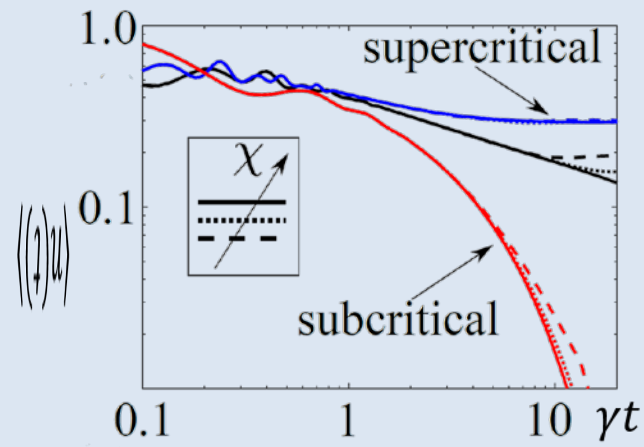


Critical exponents

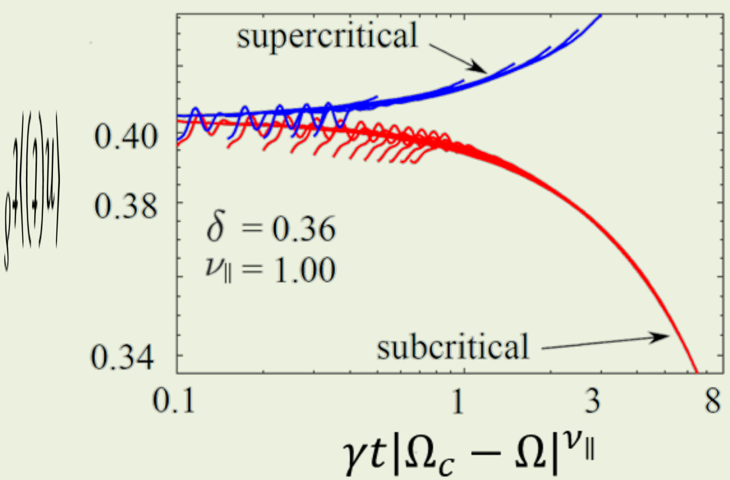
Density dependence near criticality: $n_{SS} \propto |\lambda - \lambda_c|^{\beta}$



Time dependence at criticality $(\Omega = \Omega_c)$: $n(t) \propto t^{-\frac{\beta}{\nu_{\parallel}}} = t^{-\delta}$



Collapse onto master curves



δ	0.36
β	0.36
$\nu_{\parallel}$	1.00
$\nu_{\perp}$	0.5

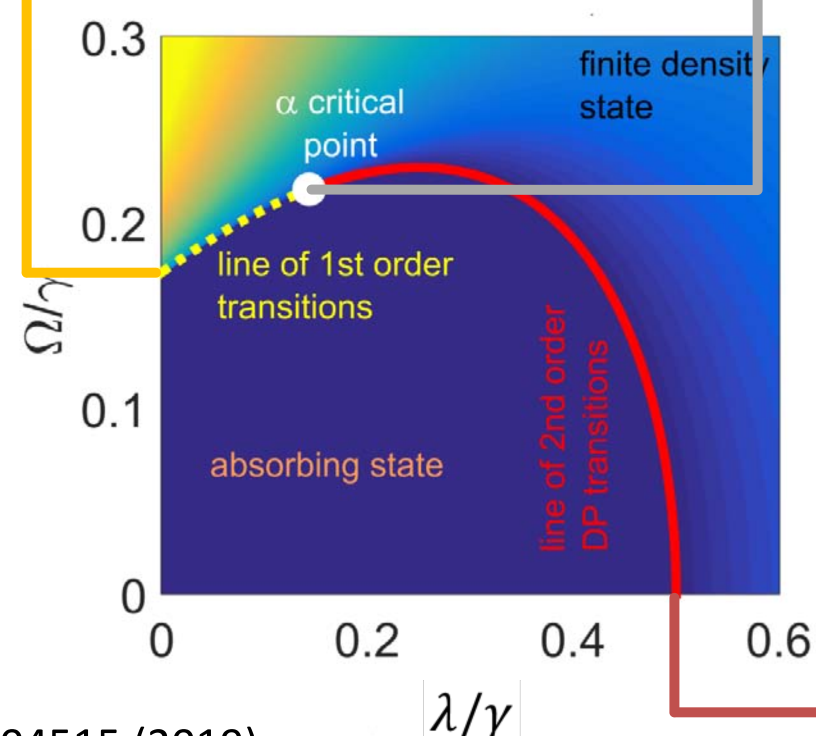
Which universality class is that?

F. Carollo et al. arXiv:1902.04515 (2019)

Critical exponents

	QCP (1D)	DP (1D)	DP (2D)	α CP (2D)
δ	0.36	0.16	0.45	0.35
β	0.36	0.28	0.58	0.35
$\nu_{\|}$	1.00	1.73	1.30	1.03
$\nu_{\perp}$	0.5	1.10	0.73	0.52

Roscher et al., PRA 98, 062117 (2018)



F. Carollo et al. arXiv:1902.04515 (2019)

Summary and other research

- Rydberg atoms permit natural realisation of kinetic constraints

Disorder

<u>Few-particle limit</u>

- Anderson localisation in Fock-space

<u>Many-body limit</u>

- map to fermions with non-local disorder potential
- Many-body localisation

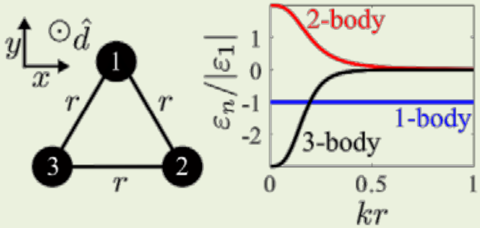
Facilitation competing with decay

- contact process
- absorbing state phase transition
- quantum vs. classical

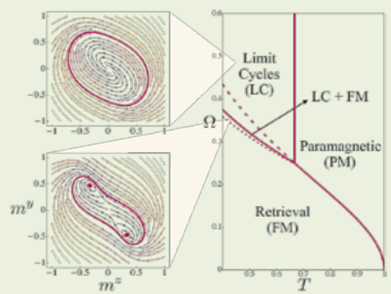
Dissipative Time Crystals [PRL **122**, 015701 (2019)]

Dynamical phase transitions in chiral atom chains [arXiv:1902.08525 (2019)]

Dressed dense atomic gases [arXiv:1902.02989 (2019)]



Quantum generalisations of neural networks [JPA **51**, 115301 (2018), PRA **99**, 032126 (2019)]



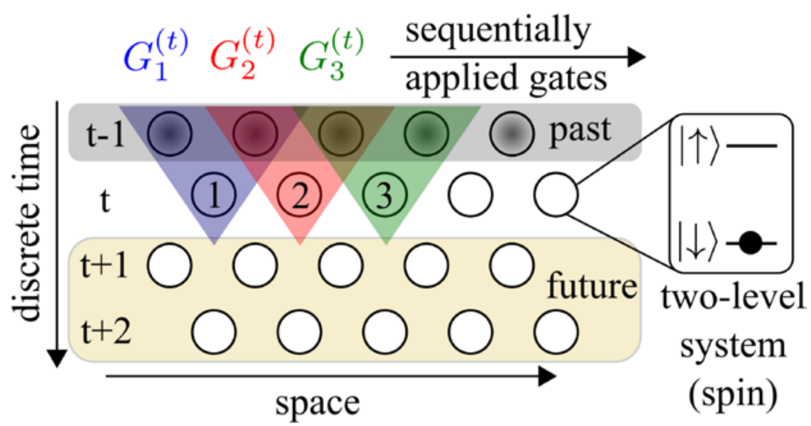
Questions/Problems

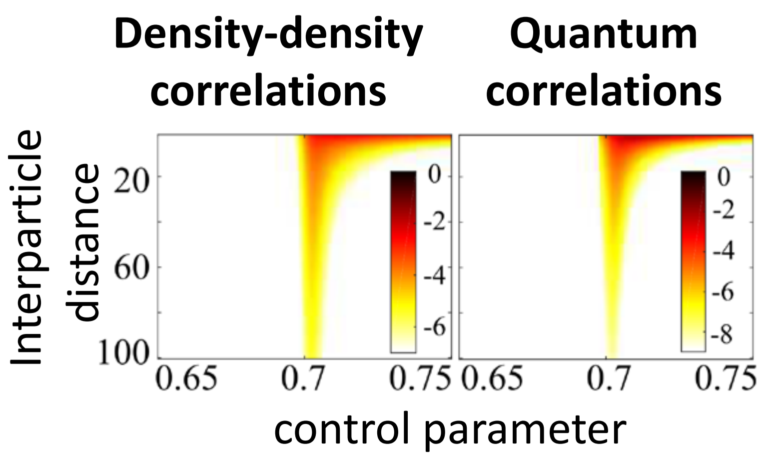
Problems:

- long time dynamics difficult to access numerically
  [need numerics that evolves many-body state (open and closed) in real time]
- methods that target stationary state of absorbing state phase transitions are drawn into absorbing state

Questions:

- What makes the simulation of open quantum systems hard? (entanglement vs. operator space entanglement)
- Is there "quantumness" at the phase transition (entanglement, discord)?
- Is it beneficial to use discrete instead of continuous dynamics?





Quantum Science and Technology **4**, 02LT02 (2019)

Open questions

Antoine

Rydberg: role of exp. imperfections and dephasing ("bad" dissipation)

Limits preparation of ground state

Limits duration of interaction driven dynamics

+ role of complex atomic structure

$\Rightarrow$ Model of dissipation?? Use dissipation to prepare MB states??

Optical dipoles ("good" dissipation)**:**

Strength of interaction-induced non-linearity?

Mapping atomic correlations onto light correlations?

Exp.: structure at sub-λ scale = hard

$\Rightarrow$ use low-lying Rydberg states??

Igor

Open quantum systems with absorbing states

Problems:

- long time dynamics of open quantum systems with absorbing states is difficult to access numerically
  [need numerics that evolves many-body state (open and closed) in real time]
- methods that target stationary state of absorbing state phase transitions are drawn into absorbing state

Questions:

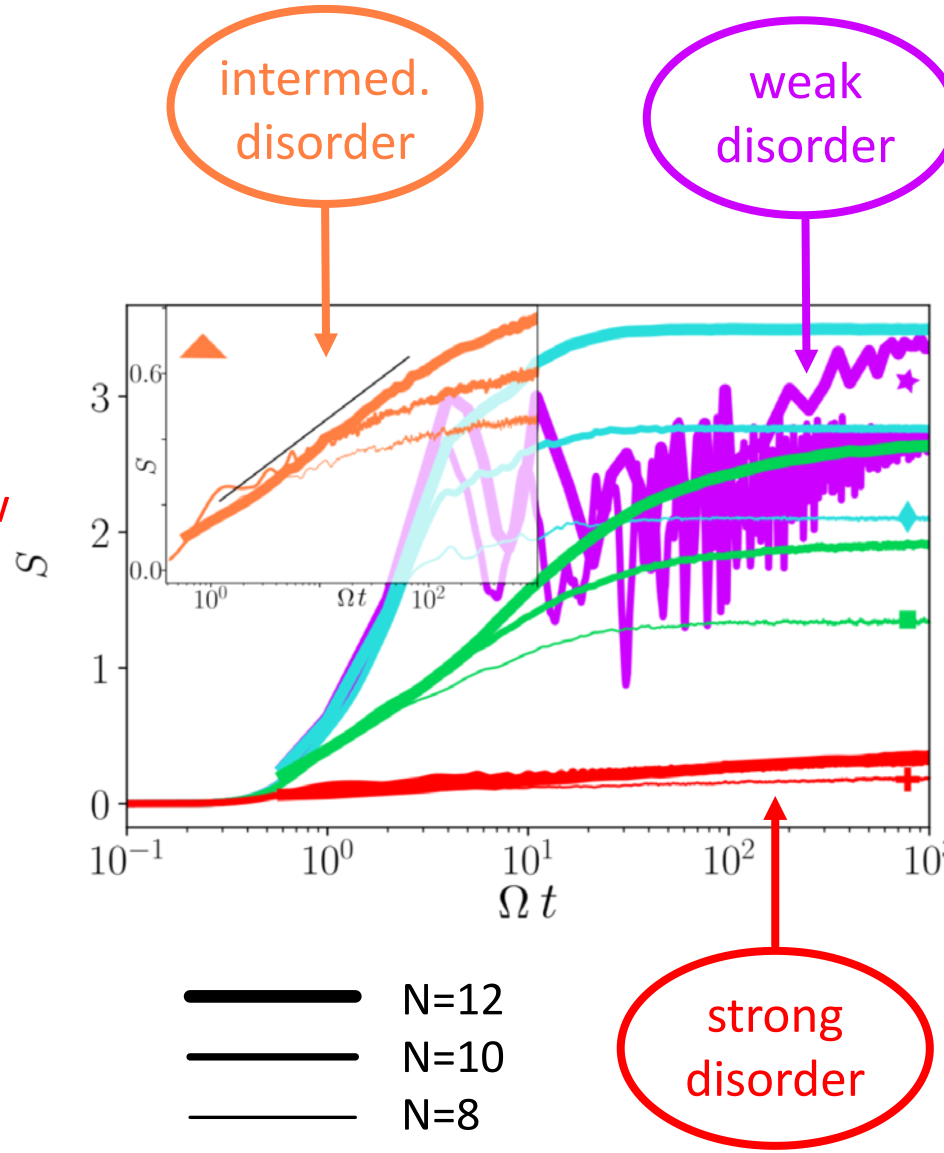
- What makes the simulation of <u>open</u> quantum systems hard? (entanglement vs. operator space entanglement)
- Is there "quantumness" at the diss. phase transitions (entanglement, discord)?

Supplementary slides

Half-chain entanglement

Entanglement entropy of reduced state of a half-chain

- <u>weak disorder:</u> strong oscillations and entanglement growth
- <u>strong disorder:</u> non-ergodic, slow entanglement growth as system decays into non-interacting components
- <u>intermediate disorder:</u> seemingly logarithmic growth of entanglement: many-body localisation?
[Serbyn et al., PRL **110**, 260601 (2013)]



Level spacing statistics

Level statistics ratio

$$r_n = \frac{\min\{\Delta_n, \Delta_{n+1}\}}{\max\{\Delta_n, \Delta_{n+1}\}}$$ With energy gaps $\Delta_n = |E_n - E_{n+1}|$

- allows to quantify how integrable/ergodic a quantum system is