Quantum systems and dynamical classical noise: from environmental engineering to quantum simulators

Claudia Benedetti

AQM & QTLab

“Aldo Pontremoli” Physics Department - University of Milan (Italy)

Open Quantum System Dynamics: Quantum Simulators and Simulations Far From Equilibrium
UC Santa Barbara, Kavli Institute for Theoretical Physics - April, 3rd 2019

My interests

Noisy Continuous-time quantum walks
Transport & diffusion
Non-Markovianity
Spatial search algorithm

Quantum probing
Classical noise
Quantum baths

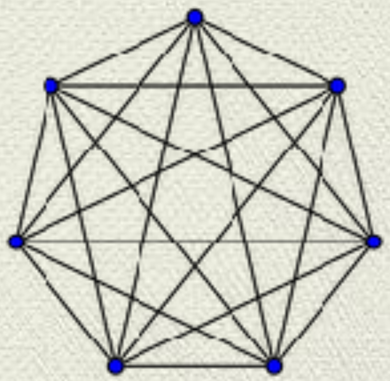
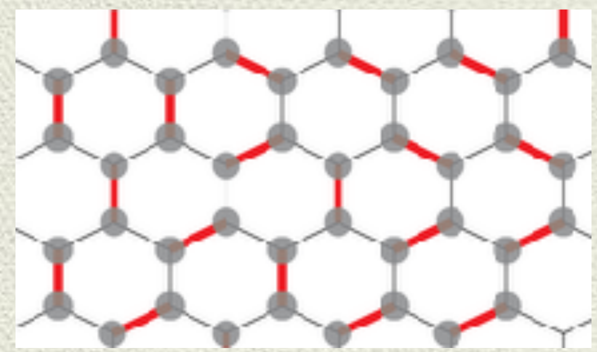
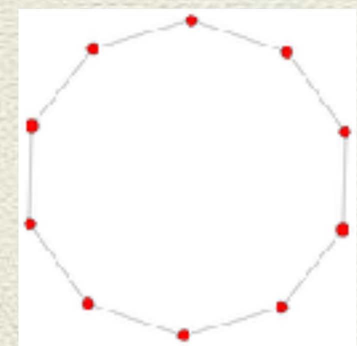
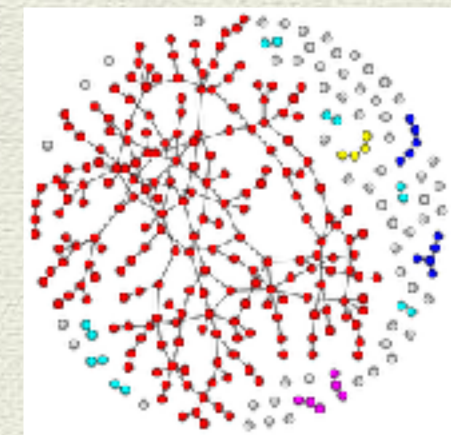
Classical noise

Decoherence in qubit systems
Non-Markovianity
QI protocols
Quantum correlations

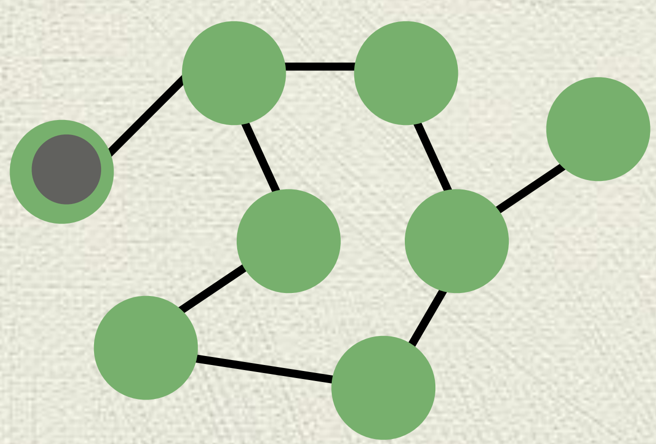
Optical quantum simulators
Single quit: dynamics & NM
Two qubits: Transition local/common noise

Continuous-time quantum walks on graphs

Graph G(V,E) → Networks

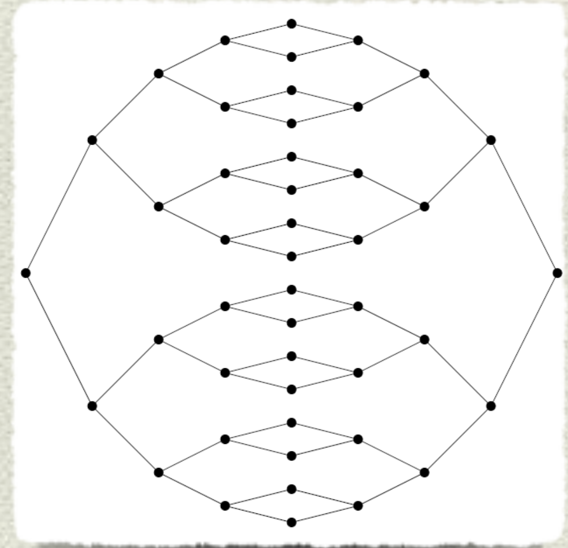
Continuous-time random walks → Continuous-time quantum walks



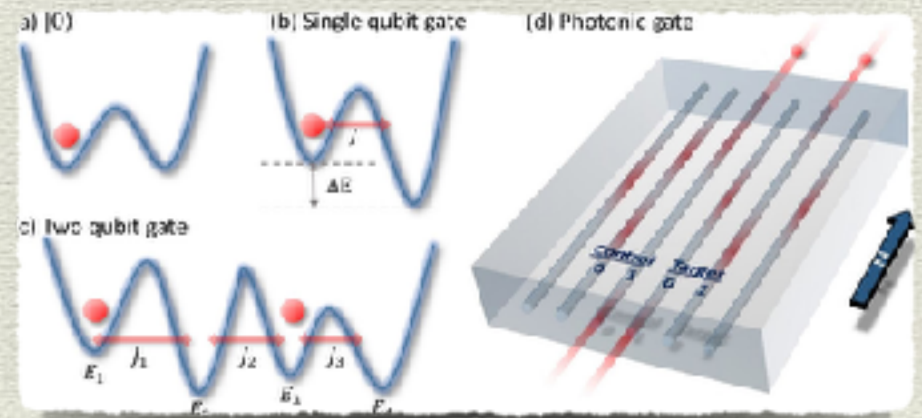
Transition probability

- Superposition of states
- Interference effects
- Transition amplitudes
- The **edges** of the graph correspond to the **tunneling** energies (or transition amplitudes)

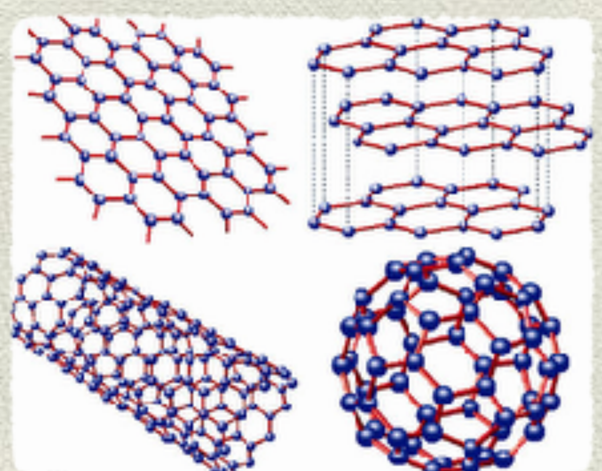
Motivations



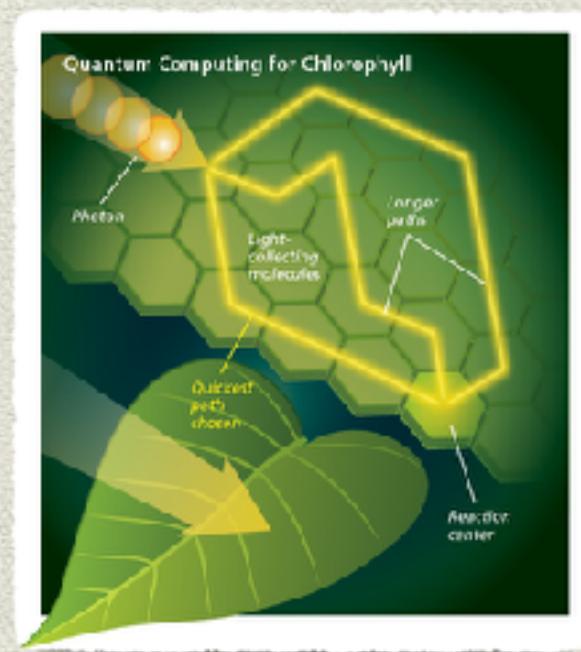
Building blocks for quantum algorithms



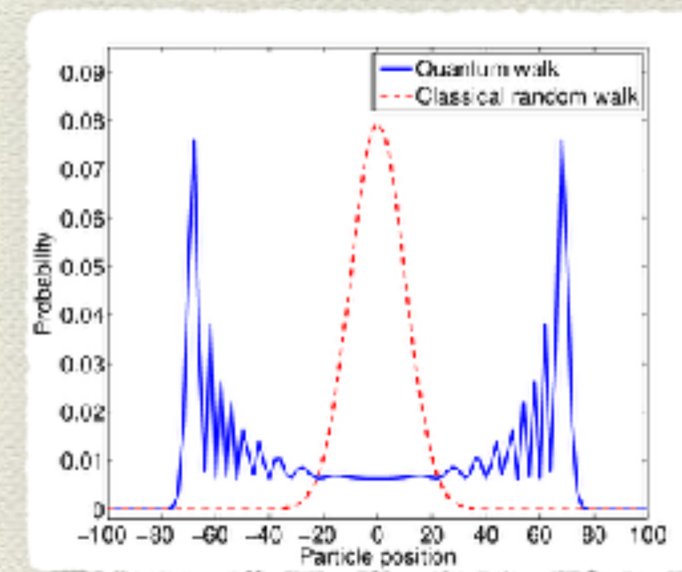
Model for universal quantum computation



Transport properties



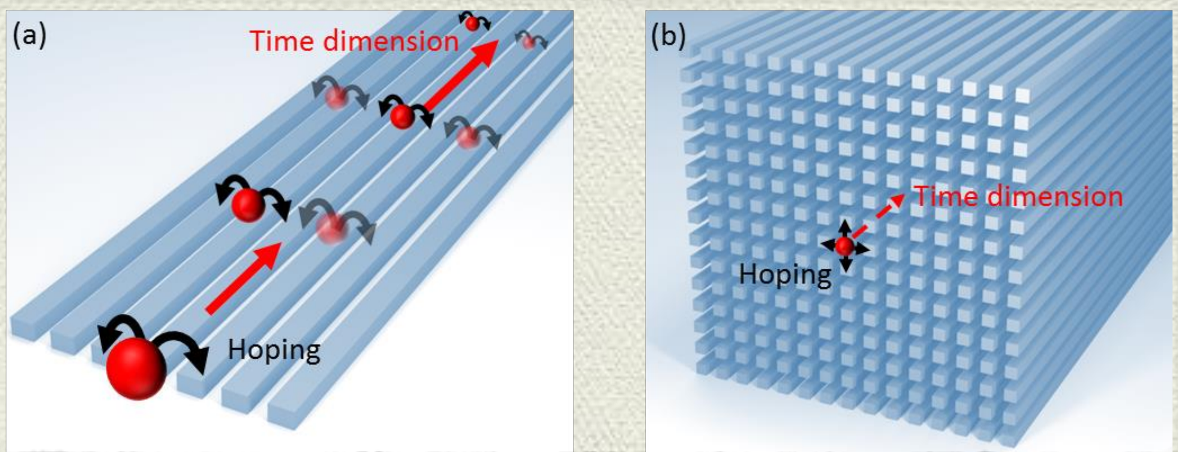
Quantum biology



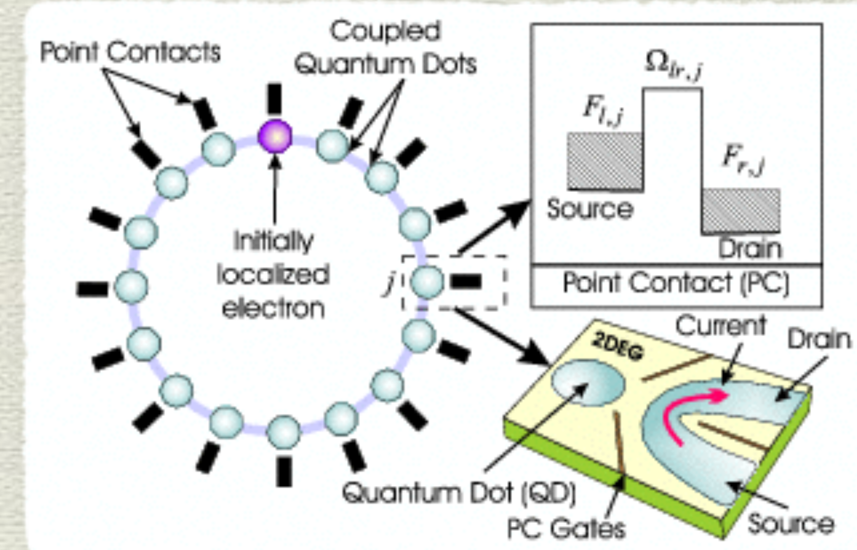
Decoherence

Quantum-to-classical transition

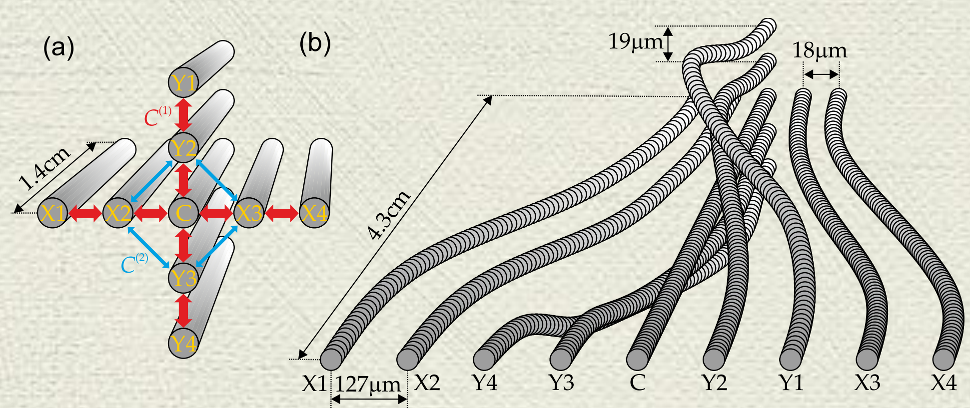
Motivations



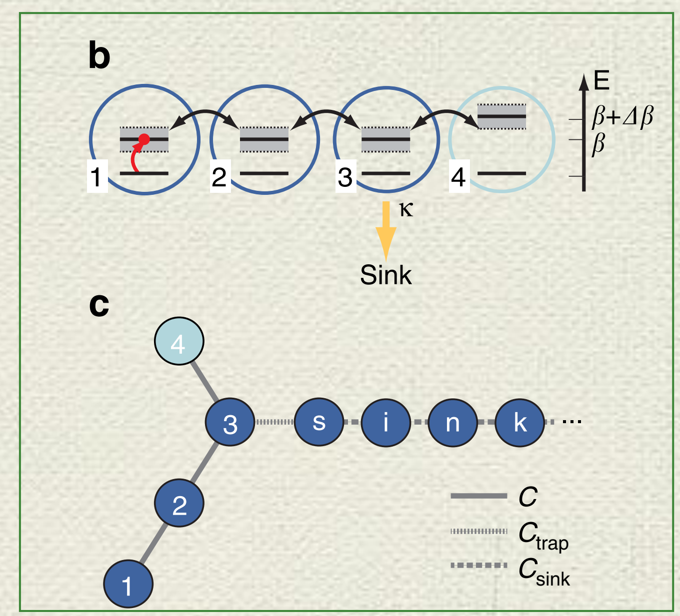
Coupled waveguides - Sci Adv 4, eaat317 (2018)



Quantum dots - PRA 73, 012313 (2006)



Coupled waveguides - PRL 112 143604 (2014)

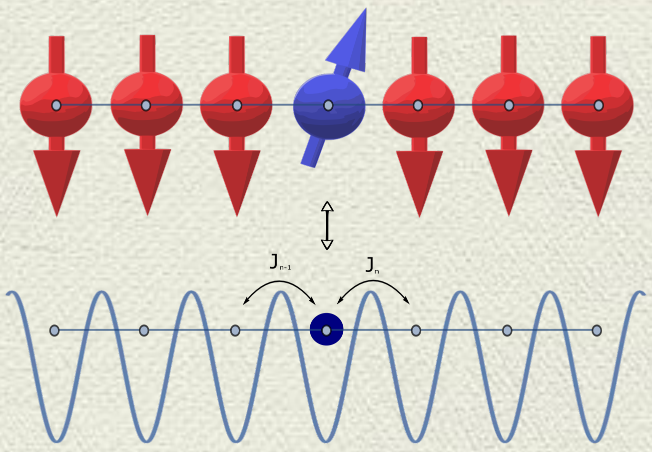


Nat Comm 7, 11282 (2016)

Experimental implementations of Qws are realized on different platforms

Imperfections (defects) and noise may affect the implementation of a lattice

CTQW on the line



$$H = \sum_k \epsilon_k |k\rangle\langle k| + \sum_k J_k \Big(|k\rangle\langle k+1| + |k+1\rangle\langle k| \Big)$$

Diagonal terms
On-site energy

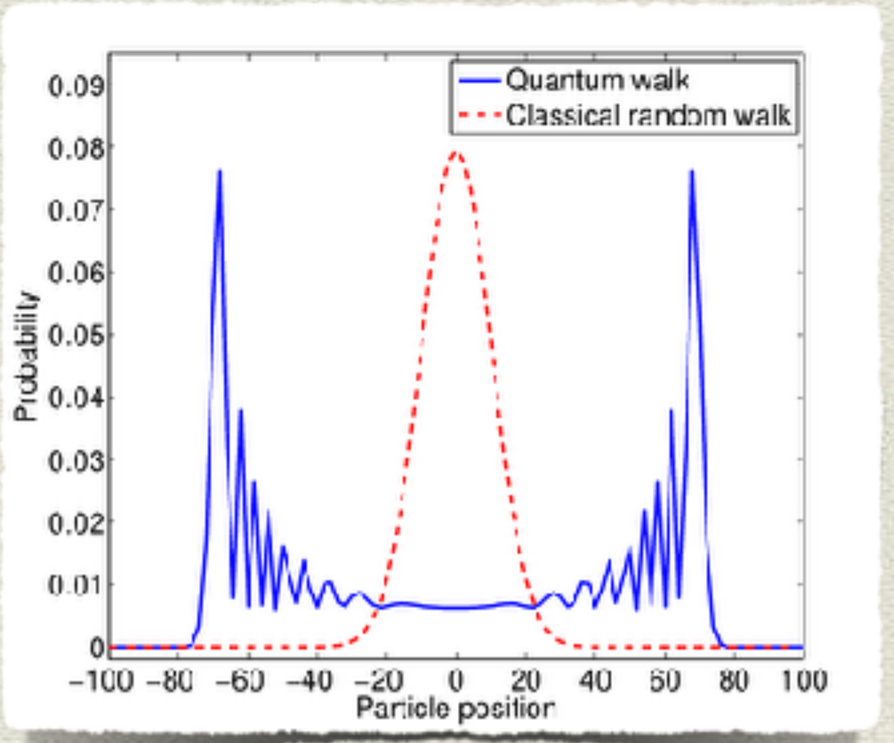
Off-diagonal terms
Tunneling energy

Eigenvectors & eigenvalues of H:

$$|\Phi_\theta\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-i\theta j} |j\rangle$$

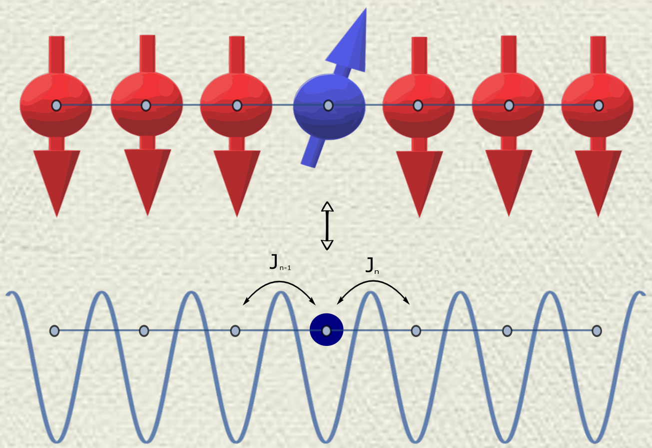
$$E_\theta = 2 - 2\cos\theta$$

$$\theta = \frac{2n\pi}{N}$$



EPL, **124** (2018) 60001

Noisy CTQW on the line



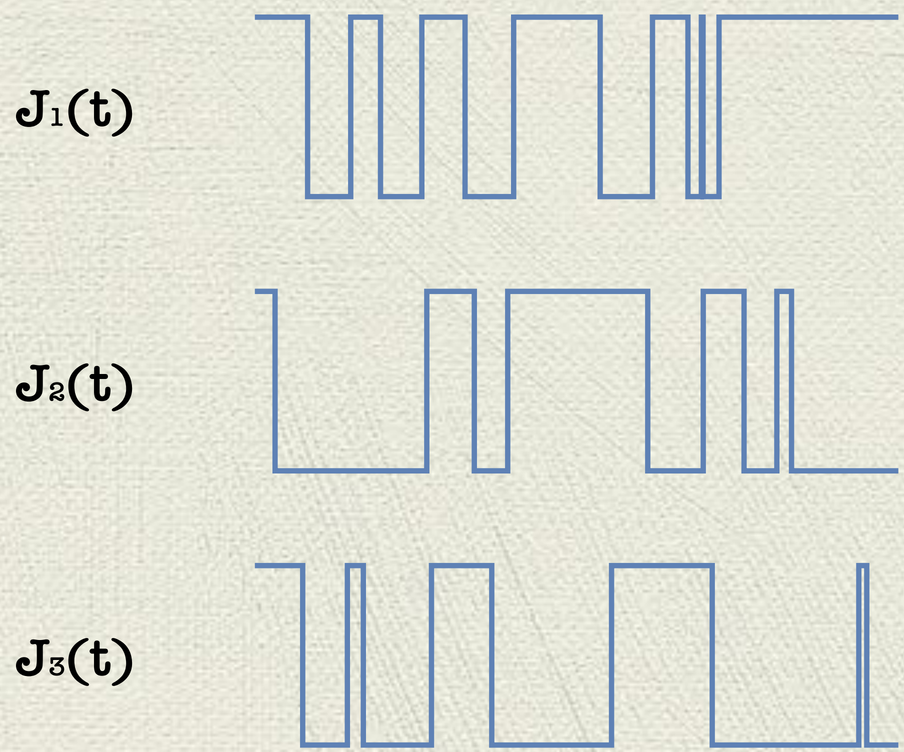
$$H = \sum_k \epsilon |k\rangle\langle k| + \sum_k J_k(t) |k\rangle\langle k+1| + \text{hc}$$

Constant on-site energies

Stochastic tunneling energies

Dynamical percolation noise

Every tunneling energy J_k fluctuates randomly in time → Random telegraph noise



$$J_k(t) = J_0 + \nu X_k(t)$$

$$\nu \in [0, J_0] \qquad X_k(t) = \pm 1$$

γ Switching rate

$$\langle X_k(0) X_j(t) \rangle = \delta_{kj}\, e^{-2\gamma t}$$

Noisy CTQW on the line: generalized percolation noise

$$H = \sum_k J_k(t)\Big(|k\rangle\langle k+1| + |k+1\rangle\langle k|\Big)$$

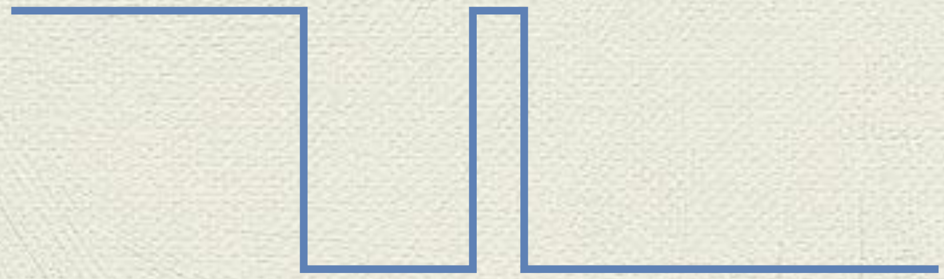
$$J_k(t) = J_0 + \nu X_k(t)$$

$\nu \in [0, J_0]$ $\quad X_k(t) = \pm 1$

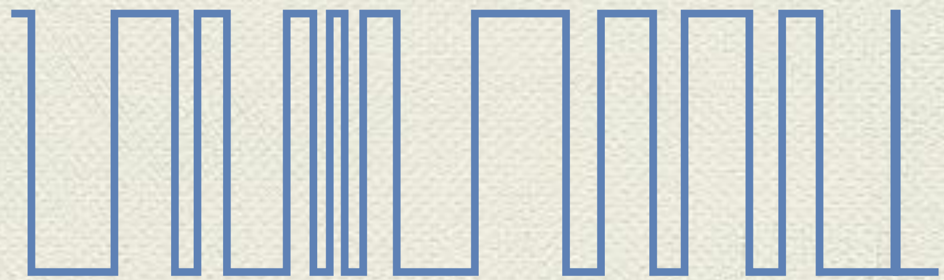
γ **Switching rate**

$$\langle X_k(0) X_j(t)\rangle = \delta_{kj}\, e^{-2\gamma t}$$

Slow noise



Fast noise

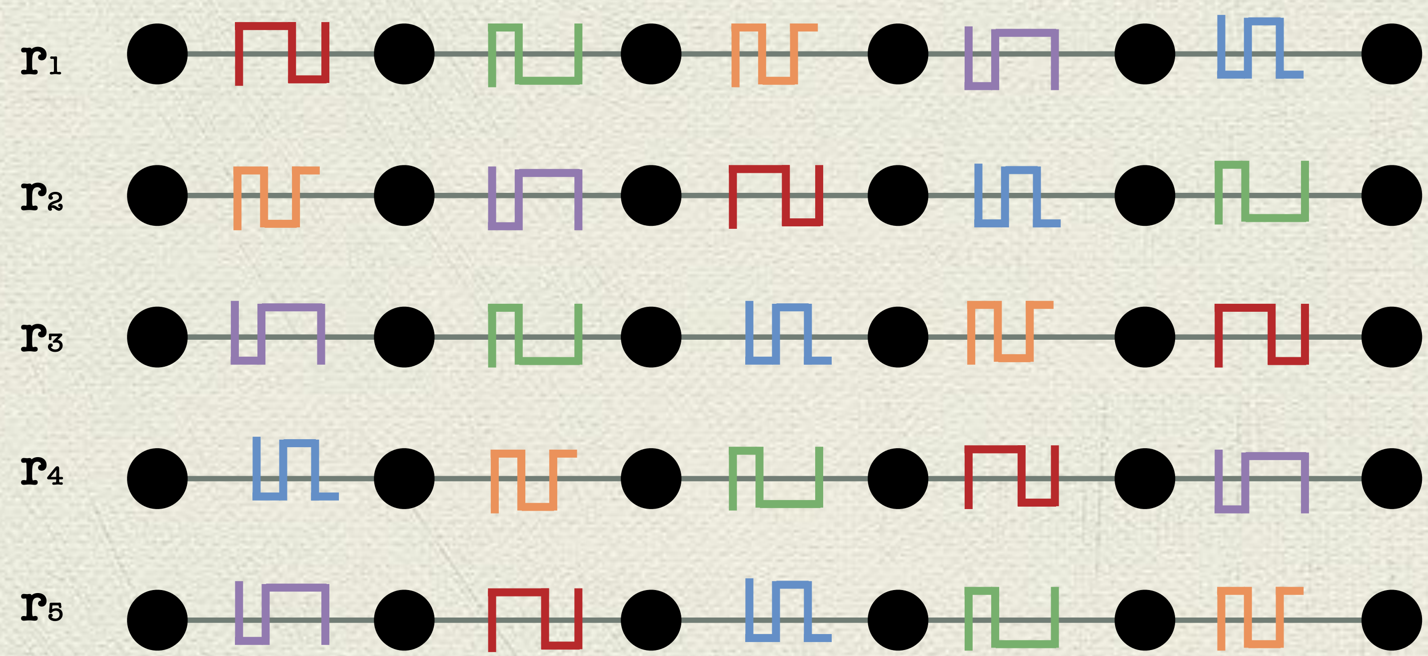


The dynamics: ensemble average

$$H_r(t) = \sum_k J_k^r(t)\Big(|k\rangle\langle k+1| + |k+1\rangle\langle k|\Big)$$ Single realization Hamiltonian

$$U_r(t) = \mathcal{T}\int_0^t e^{-iH_r(s)\,ds}$$ Evolution operator

$$\rho(t) = \langle U_r(t)\rho_0 U_r^\dagger(t)\rangle$$



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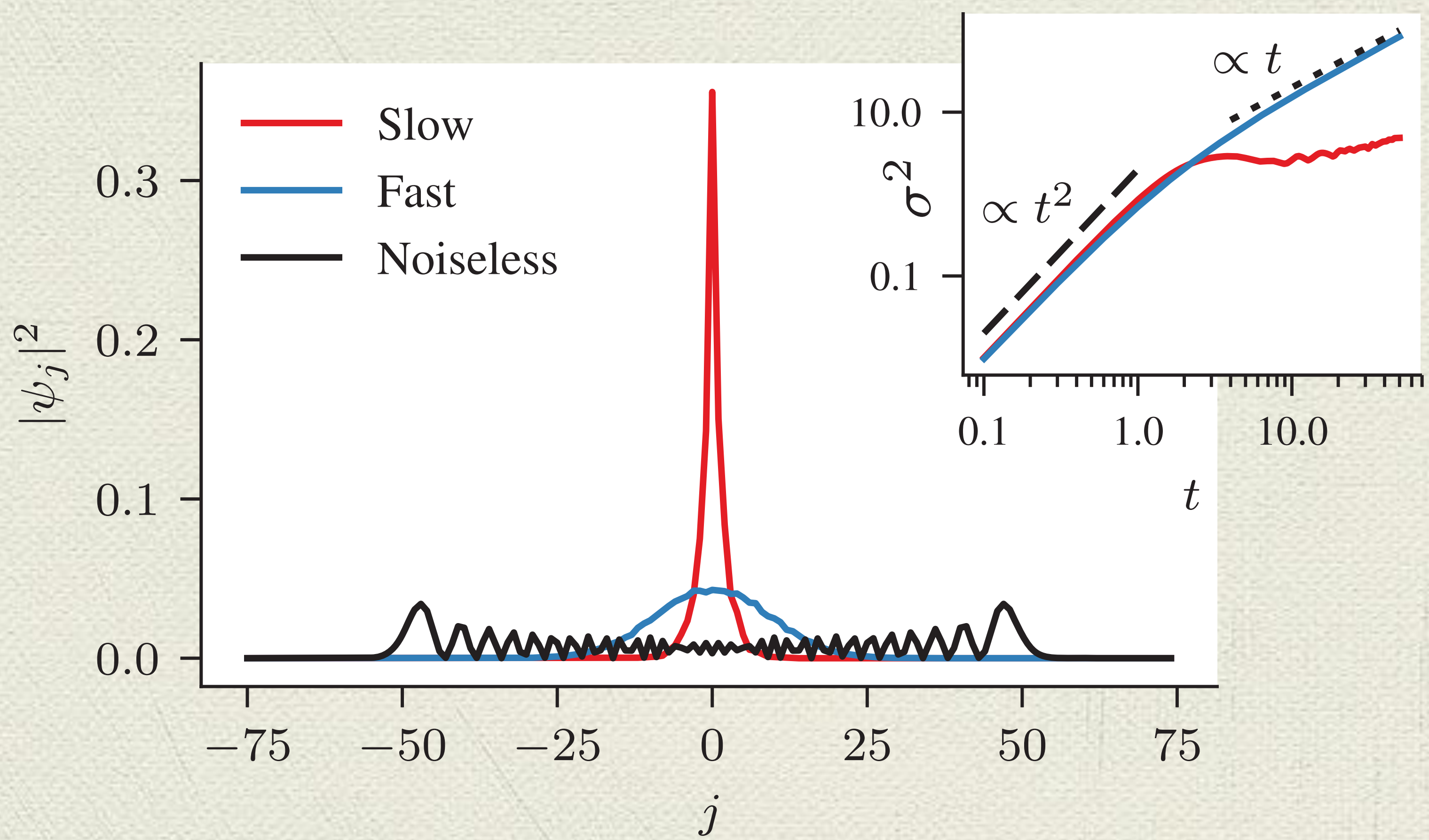


Fig. 2: (Color online) Probability distribution of the walker at $t = 50$ for slow ($\gamma = 0.01$, red) and fast ($\gamma = 1$, blue) noise. The noiseless walker is shown in black for comparison. Inset: the variance σ^2 as a function of time. The black lines are visual guides for different propagation regimes: ballistic (dashed) and diffusive (dotted). With fast noise we can see a transition from the ballistic to the diffusive propagation, while slow noise causes temporary localization of the walker.

PRA 77, 022302 (2008)
PRL 106, 180403 (2011)
PRA 93, 042313 (2016)

Spatially correlated noise

With: M. Rossi, M. Borrelli, S. Maniscalco, M. Paris

PHYSICAL REVIEW A **96**, 040301(R) (2017)
EPL, **124** (2018) 60001

The tunneling amplitudes are grouped into spatial domains
Synchronized domains

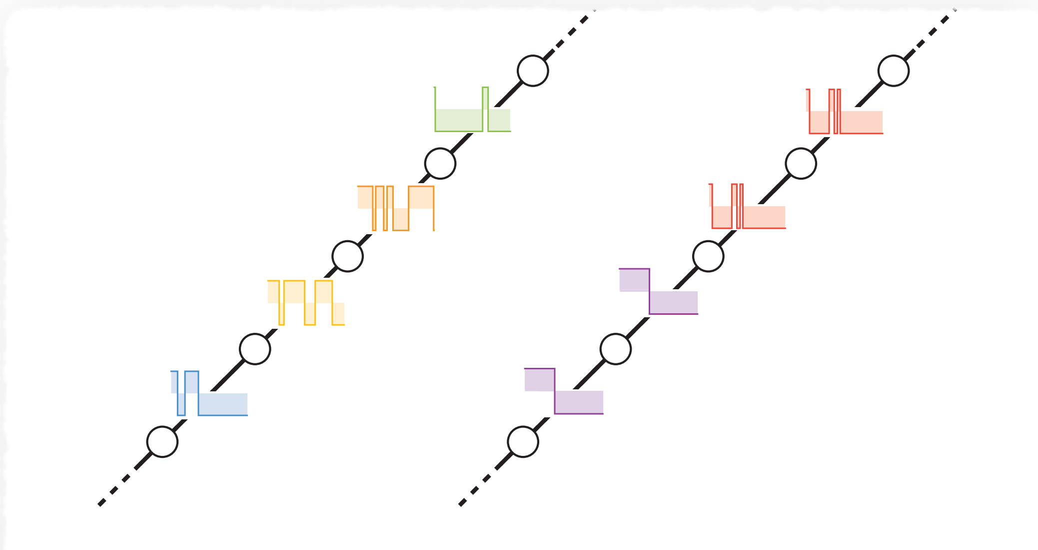
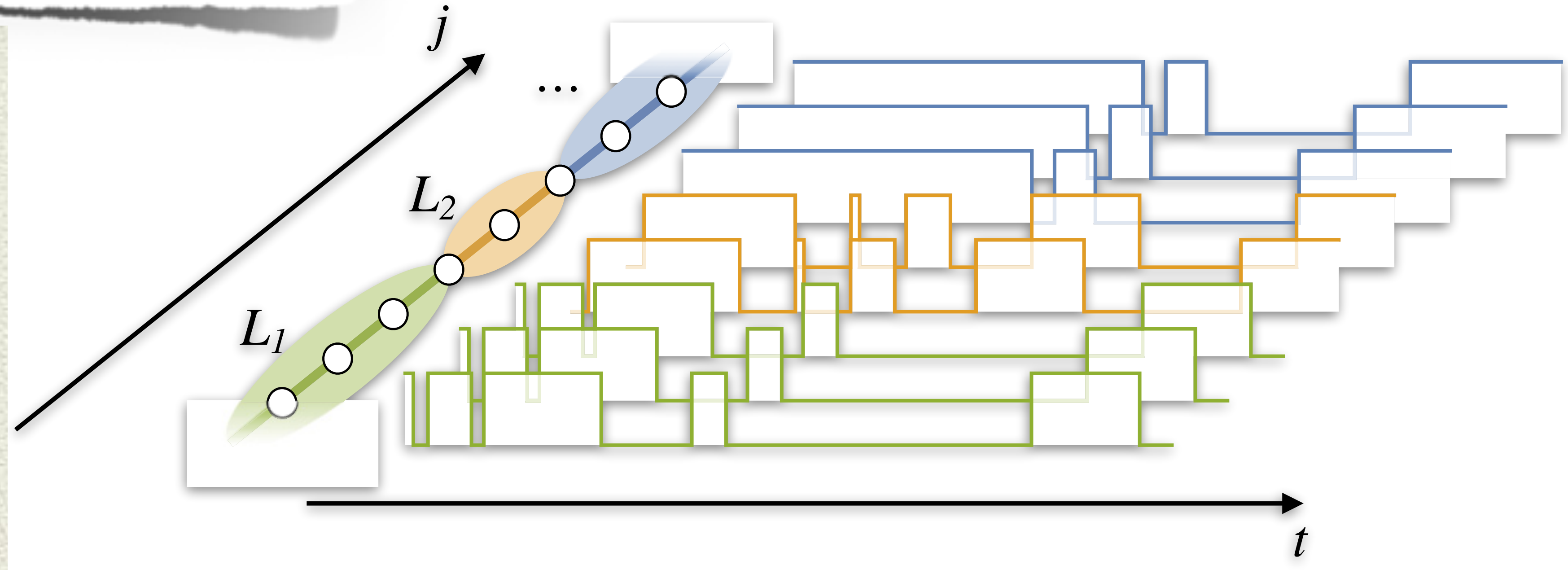
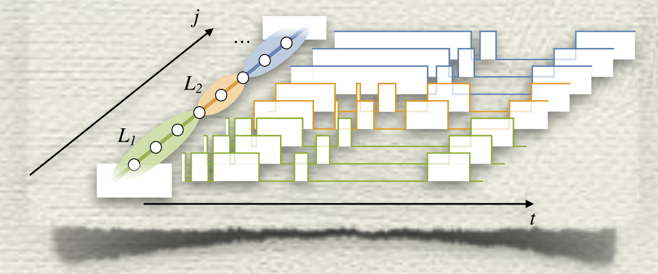


Fig. 1: (Color online) Pictorial representation of the lattice described in eq. (9), with uncorrelated noise sources (left) and spatially correlated noise (right).

FIG. 1. Schematic representation of the random spatial domains $\{L_1, L_2, \ldots, L_M\}$ for a single realization of the noise, generated according to Eq. (3) and of average length $\bar{L}_p$. Tunneling amplitudes within the same domain fluctuate synchronously in time and according to the same stochastic process. Different domains evolve independently from each other.



Spatially correlated noise



The tunneling amplitudes are grouped into spatial domains
Synchronized domains

$$C(t) = \langle X_k(0)X_j(t)\rangle = \begin{cases} e^{-2\gamma t} & \text{if } j,k \text{ belong to the same domain} \\ 0 & \text{otherwise} \end{cases}$$

The domains are created randomly: too adjacent links are correlated with probability p

For each noise realization the spatial correlations will form **M domains** of lengths $\{L_1, L_2, \ldots, L_M\}$ corresponding to M independent noise evolutions.
The probability P_M of having M domains in a particular realization is

$$P_M = \begin{pmatrix} N-1 \\ M-1 \end{pmatrix} (1-p)^{M-1} p^{N-M}$$

which corresponds to the **average domain length**

$$\bar{L}_p = \frac{p^N - 1}{p - 1}$$

$$\bar{L}_1 \equiv \lim_{p\to 1} \bar{L}_p = N$$

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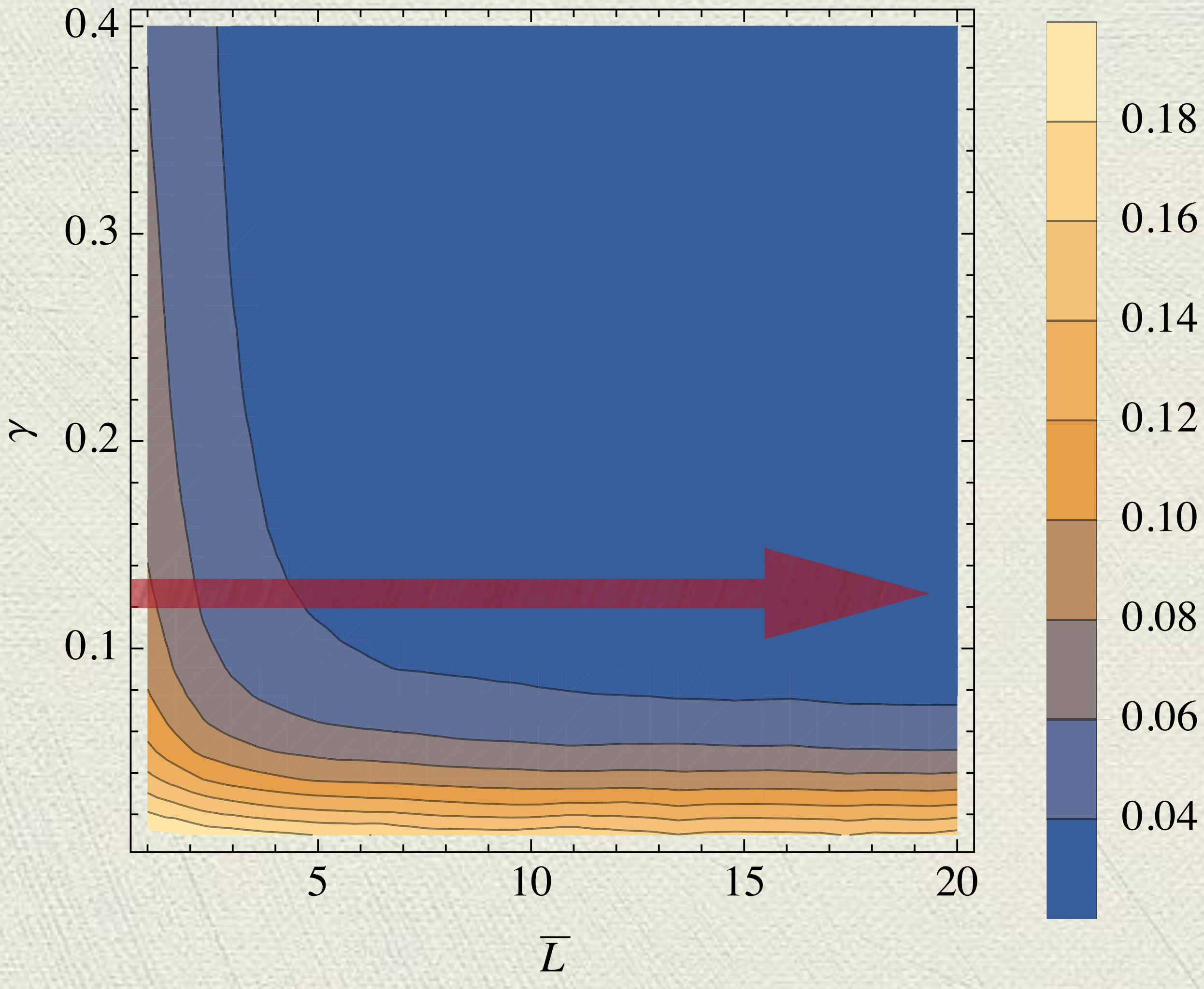
PHYSICAL REVIEW A **96**, 040301(R) (2017)

Spatially correlated noise Diffusion vs Localization

Inverse partecipato ratio

$$\mathcal{I}(t) = \sum_{j=1}^{N} \langle j|\bar{\rho}(t)|j\rangle^2$$

Delocalization $\frac{1}{N} \leq \mathcal{I}(t) \leq 1$ Localization



Initial localized state

Spatial correlations tend to break localization

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PHYSICAL REVIEW A **96**, 040301(R) (2017)

Diffusion vs Localization: Gaussian wave packet

$$|\psi_0\rangle = \frac{1}{2\pi\Delta}\sum_j e^{-\frac{j-x_0}{2\Delta^2}} e^{-ip_0 j}|j\rangle$$

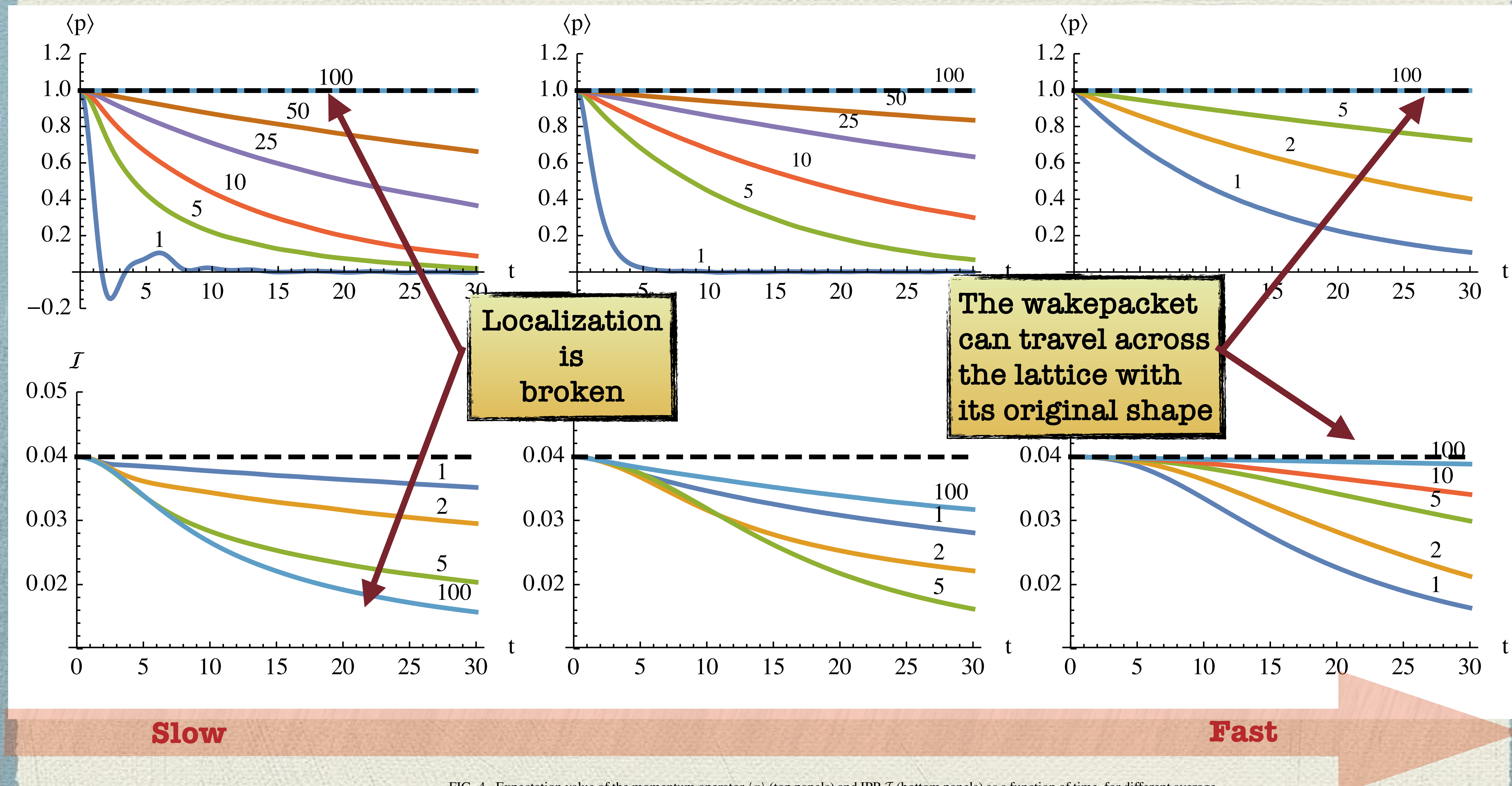


FIG. 4. Expectation value of the momentum operator $\langle p\rangle$ (top panels) and IPR $\mathcal{I}$ (bottom panels) as a function of time, for different average domain lengths $\bar{L}$, for $\gamma = 0.1$ (left), 1 (center), and 10 (right), with lattice size $N = 100$. The black dashed line indicates the noiseless case. The initial state is (8), with $k_0 = \pi/2$, $\Delta = 10$.

2-particle QW

PHYSICAL REVIEW A **95**, 022106 (2017)

Noisy quantum walks of two indistinguishable interacting particles

Ilaria Siloi,[1] Claudia Benedetti,[2] Enrico Piccinini,[3] Jyrki Piilo,[4] Sabrina Maniscalco,[4]
Matteo G. A. Paris,[2,5,6] and Paolo Bordone[1,6]

$$H_2 = H_0 + H_{\text{int}},$$

$$H_0 = H_1 \otimes \mathbb{I} + \mathbb{I} \otimes H_1,$$

$$H_{\text{int}} = U(|j-k|) \sum_{j,k=1}^{N} |j,k\rangle\langle j,k|,$$

$$U(|j-k|) = \begin{cases} U & \text{if } j = k, \\ U/3 & \text{if } j = k+1. \end{cases}$$

$$|\Psi_0^{\pm}\rangle = \frac{1}{\sqrt{2}}(|j,k\rangle \pm |k,j\rangle) \quad \text{with } j \neq k.$$

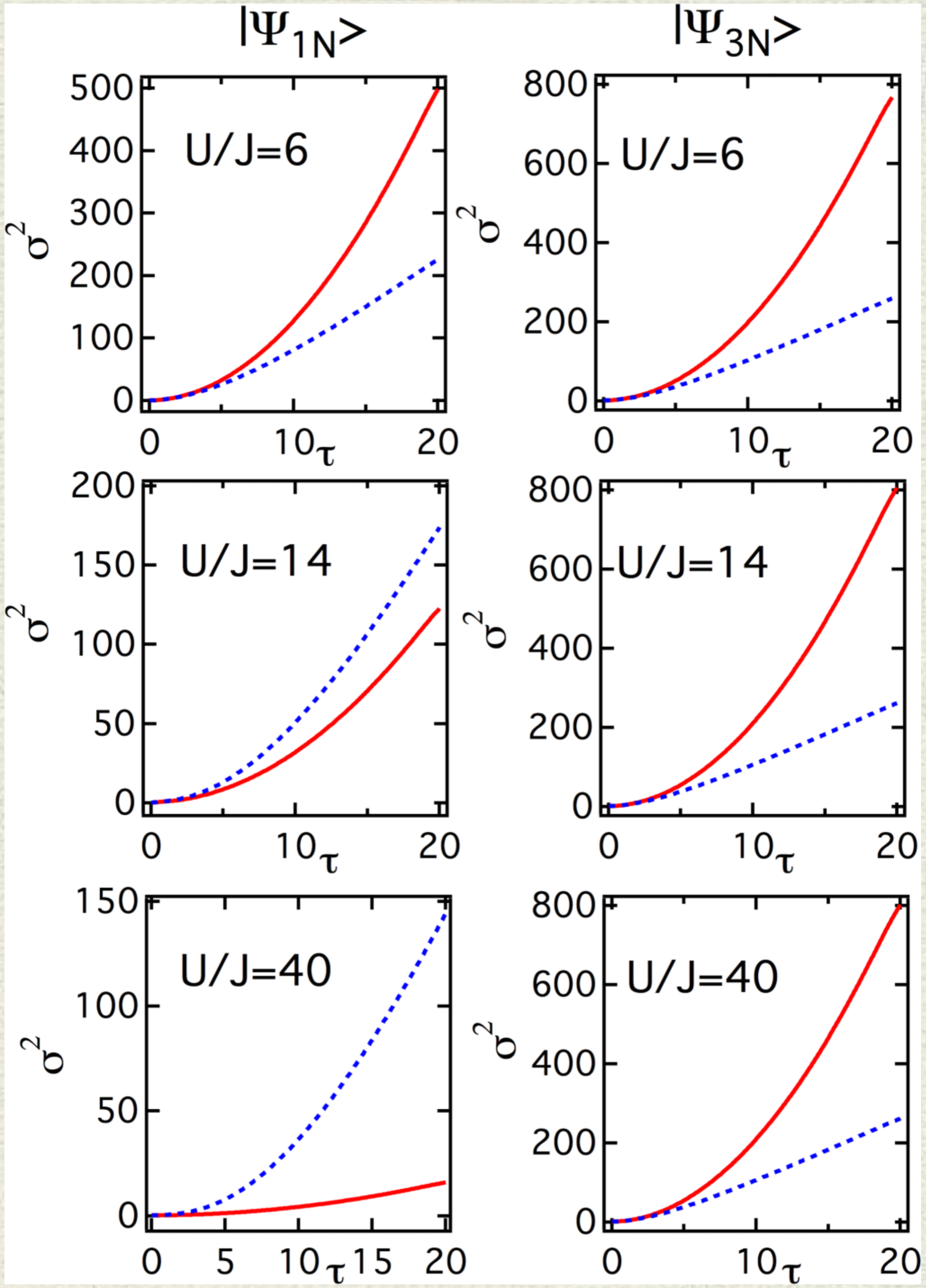
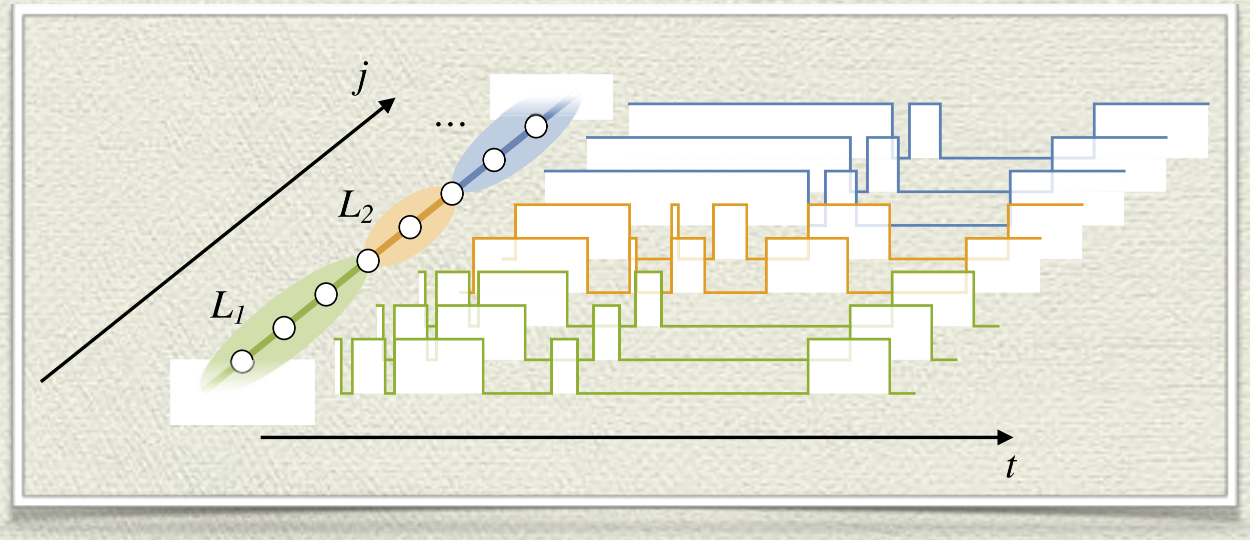


FIG. 6. Single-particle variance $\sigma^2(t)$ as a function of time for two fermions starting from next-neighbor sites $|\Psi_{1N}\rangle$ and third-neighbor sites $|\Psi_{3N}\rangle$. Each panel considers a different interaction strength U/J, and compares the noiseless evolution (solid red line) with the one in fast noise regime (dotted blue line), whose amplitude and switching time are, respectively, $g_0 = 0.9$ and $\gamma = 10.0$.

EPL, **124** (2018) 60001
PHYSICAL REVIEW A **96**, 040301(R) (2017)

Still work to do!



TO DO:

- Test the robustness of **perfect state transfer** against dynamical noise
- **2** particles - **2D** lattices with spatial domain

- Noise on the **on-site energies** in spatial domains
- **Other** kinds of noise, e.g. Gaussian
- Being able to characterize the defects in a network using a QW as a **probe**

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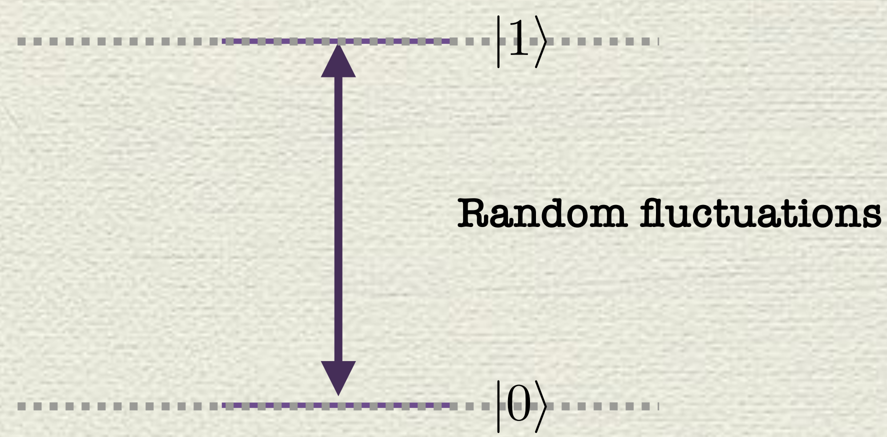
PHYSICAL REVIEW A **96**, 040301(R) (2017)

All optical simulator: 1 quit

2 nodes → 1 qubit



$$H = \omega_0 \sigma_z + X(t)\sigma_z$$

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$$

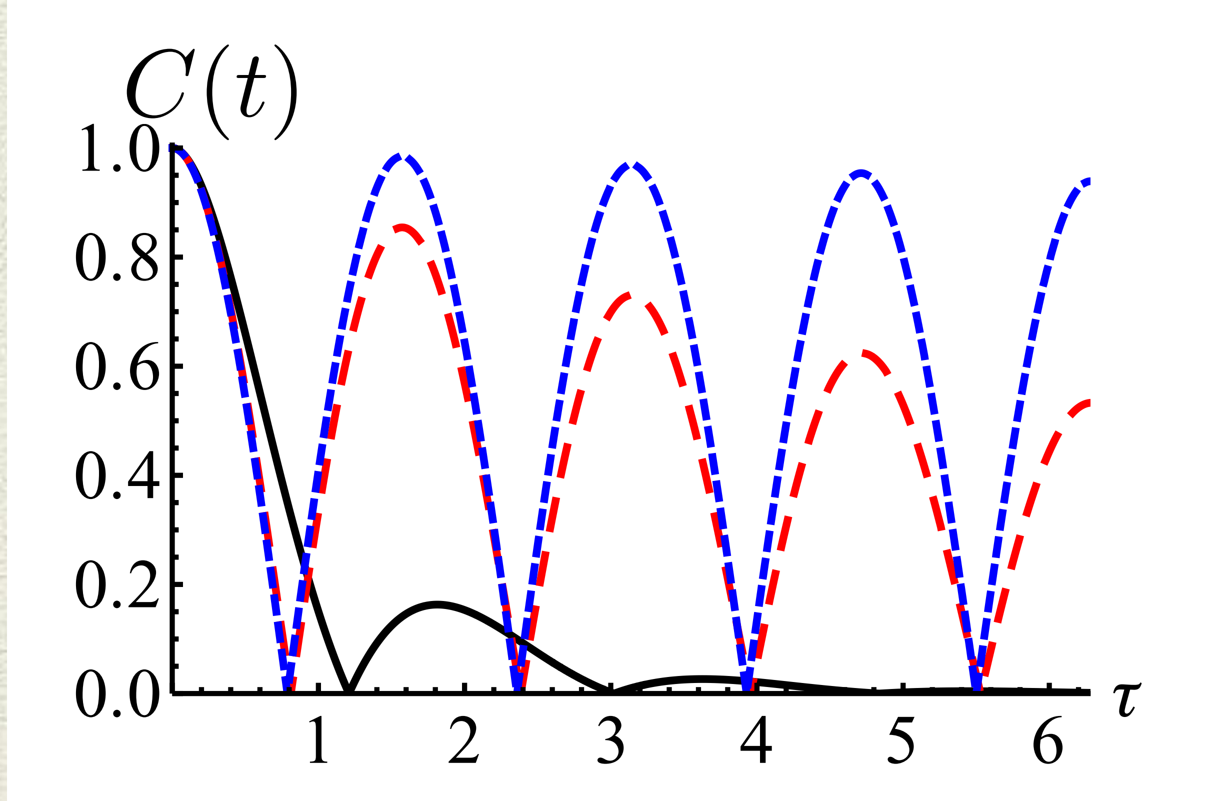
$$\bar{\rho}(t) = \frac{1}{2}\begin{pmatrix} 1 & \langle e^{-2i\phi(t)}\rangle \\ \langle e^{2i\phi(t)}\rangle & 1 \end{pmatrix}$$

$$\phi(t) = \int_0^t X(s)ds$$

APPLIED PHYSICS LETTERS 110, 081107 (2017)

Coherences (BLP Non-Markovianity)

$$C(t) = \langle e^{-2i\phi(t)} \rangle$$



$\gamma = 1$

$\gamma = 0.1$

$\gamma = 0.01$

The coherence factor decays monotonically in time for $\gamma > 2$

Random telegraph noise

$$C(t) = e^{-\gamma t} \left[\cos(\delta t) + \frac{\gamma}{\delta} \sinh(\delta t) \right]$$

$$\delta = \sqrt{\gamma^2 - (2\nu)^2}$$

Phys. Rev. B 74, 024509 (2006)

New J. Phys. 11, 025002 (2009)

PHYSICAL REVIEW A **89**, 012114 (2014)

APPLIED PHYSICS LETTERS 110, 081107 (2017)

All optical quantum simulator of qubit noisy channels

With: S. Cialdi, M. Rossi, B. Vacchini, D. Tamascelli, S. Olivares, and M. Paris

APPLIED PHYSICS LETTERS 110, 081107 (2017)

Aim: Simulate a single qubit noisy channel originating from the interaction with a fluctuating field

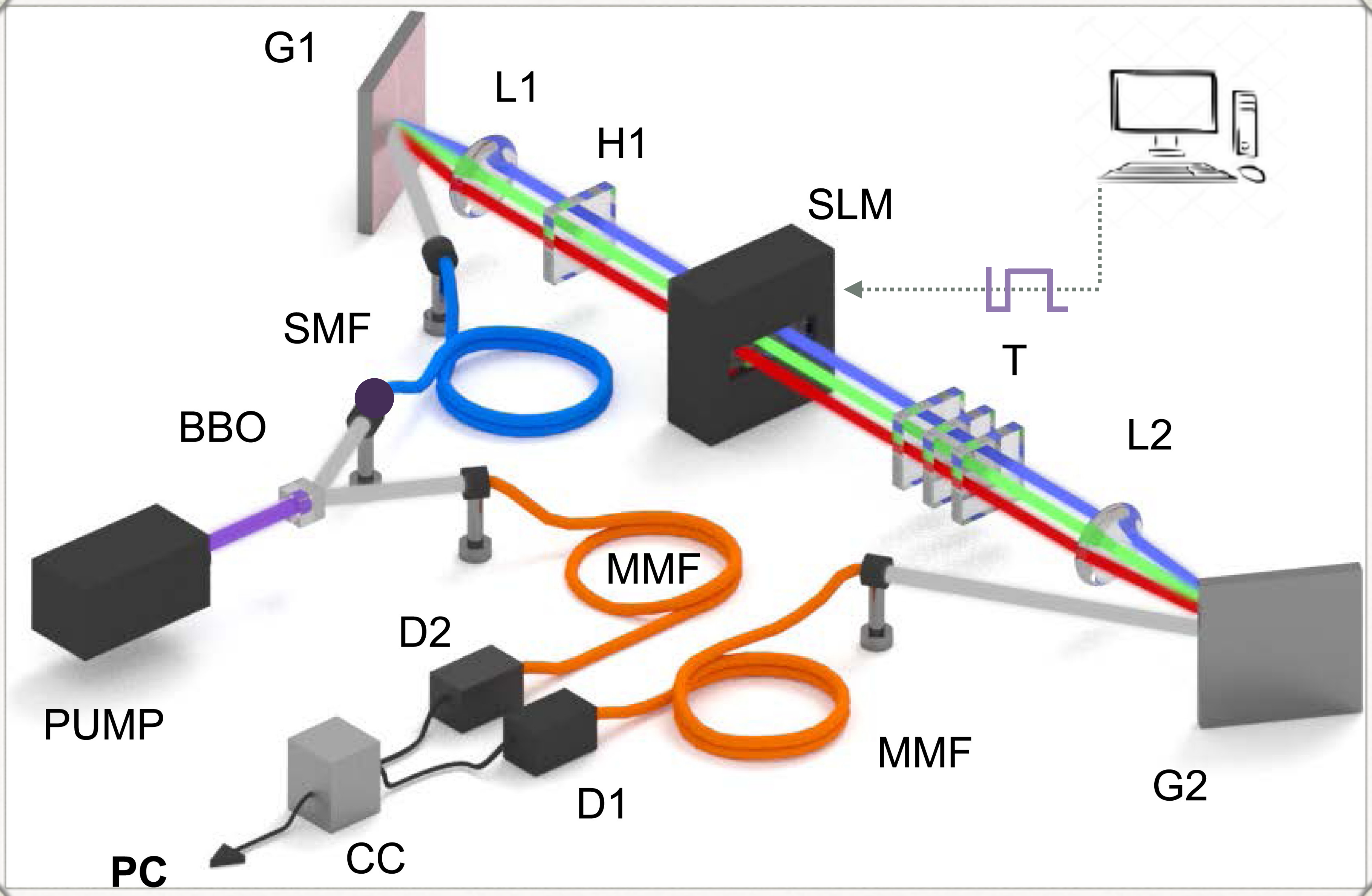
How: All-optical setup

Implementation: Employ the polarization degrees of freedom of a **single photon** and exploit its **spectral components** to average over the realizations of the stochastic dynamics.

What:
Pump+BBO crystal
Spatial light modulators: apply a computer-imposed random phase to H component for every pixel
Lens
Gratings
Half/Quarter wave plates
Polarizers
Detectors

APPLIED PHYSICS LETTERS 110, 081107 (2017)

All optical quantum simulator of qubit noisy channels



diode **pump** laser @405.5nm using a **BBO** crystal (1mm thick); **SMF**: single-spatial-mode and polarization preserving fiber; **MMF**: multimode fiber; **G1-G2**: gratings (1714 lines/mm); **L1-L2**: lens(f=500mm); **H1**, half-wave-plate; **SLM**: spatial light modulator (640 pixels, 100 μm/pixel); **T**, tomographic apparatus; **Q**:quarter-wave plate; **P**, polarizer; **C**, optical coupler; **D1-D2**: single photon detectors; **CC**: coincidences counter. The acquisition time is of 10s for each measure of coincidence counts. The inset shows the measured PDC spectrum.

$$\rho_{SE} = |H\rangle\langle H| \otimes \int d\omega\, |f(\omega)|^2\, |\omega\rangle\langle\omega|$$

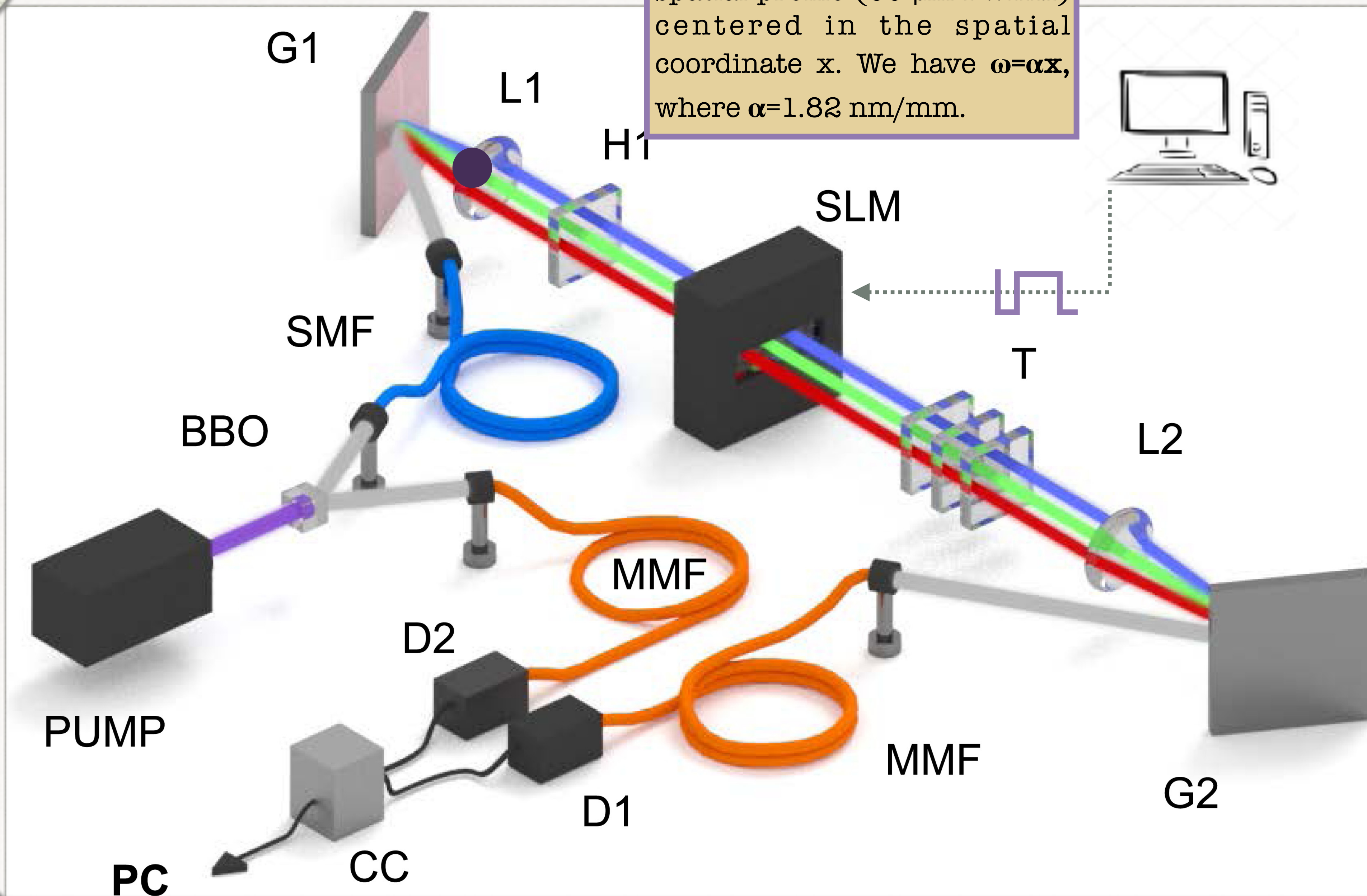
Polarization:qubit

Spectral degrees of freedom: environment

APPLIED PHYSICS LETTERS 110, 081107 (2017)

All optical quantum simulato… …annels

Each **spectral** component **ω** is characterized by a **Gaussian** spatial profile (60 μm FWHM) centered in the spatial coordinate x. We have **ω=αx,** where **α**=1.82 nm/mm.



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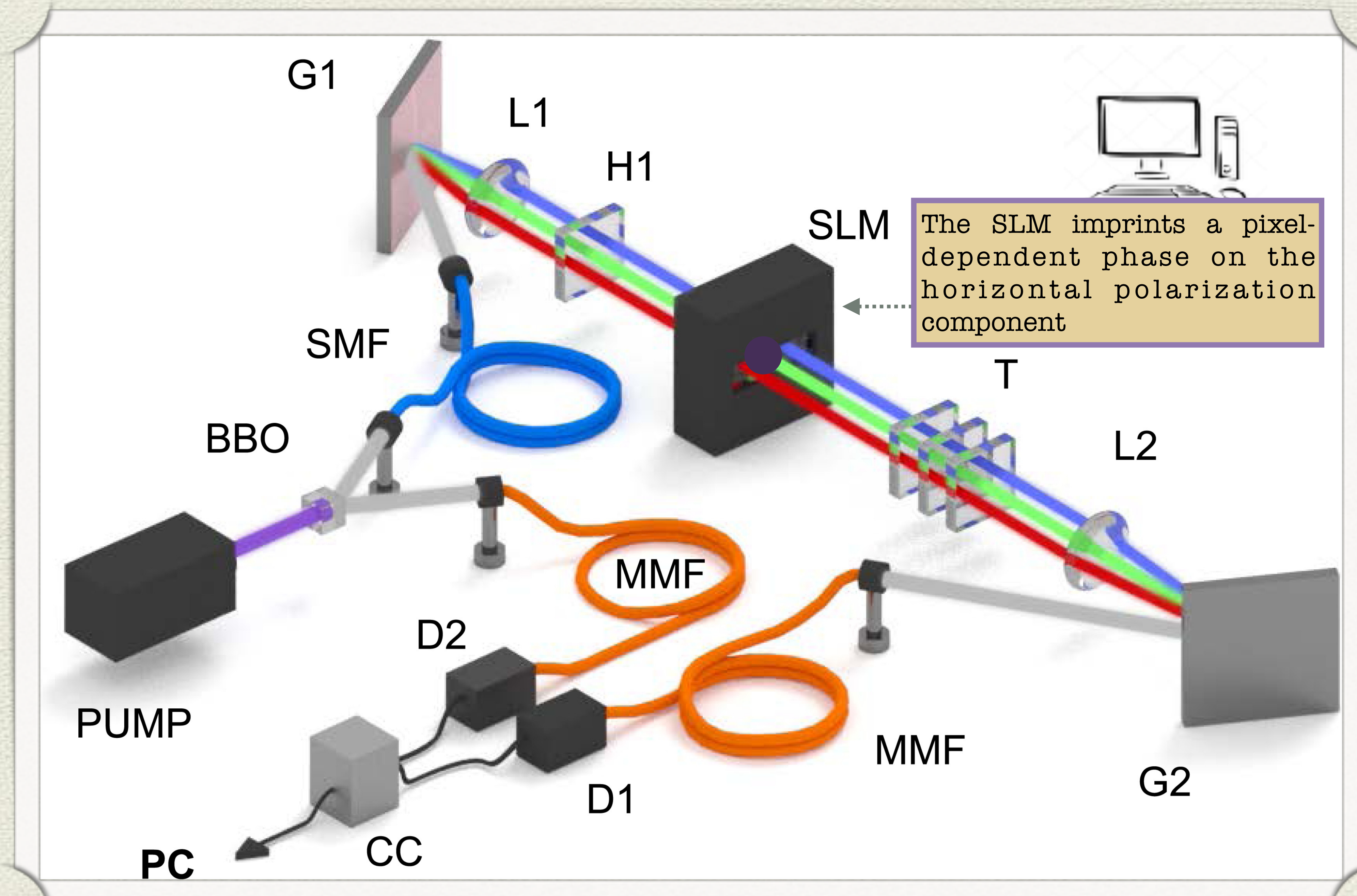
APPLIED PHYSICS LETTERS 110, 081107 (2017)

All optical quantum simulator of qubit noisy channels



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Polarization:qubit

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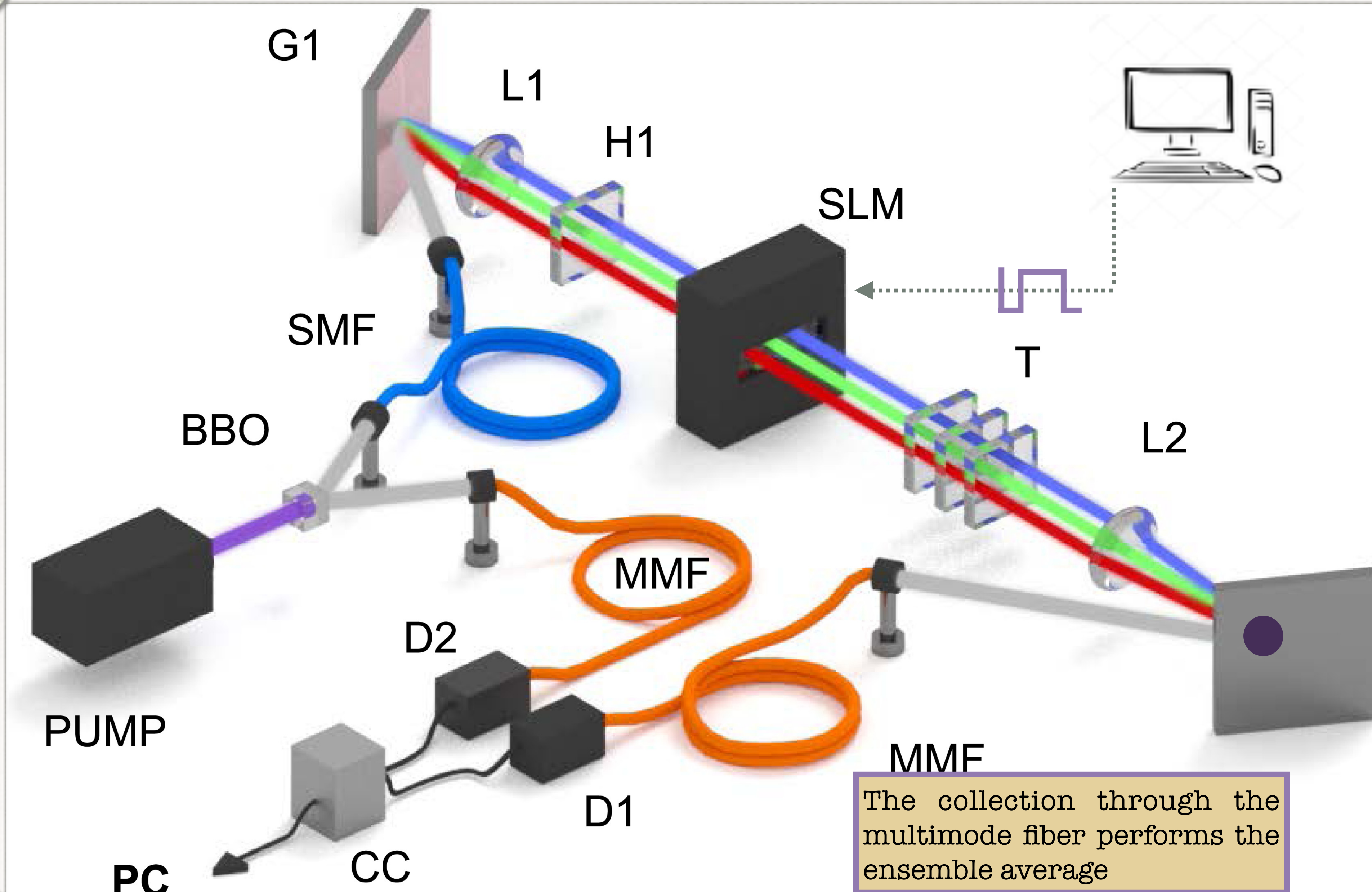


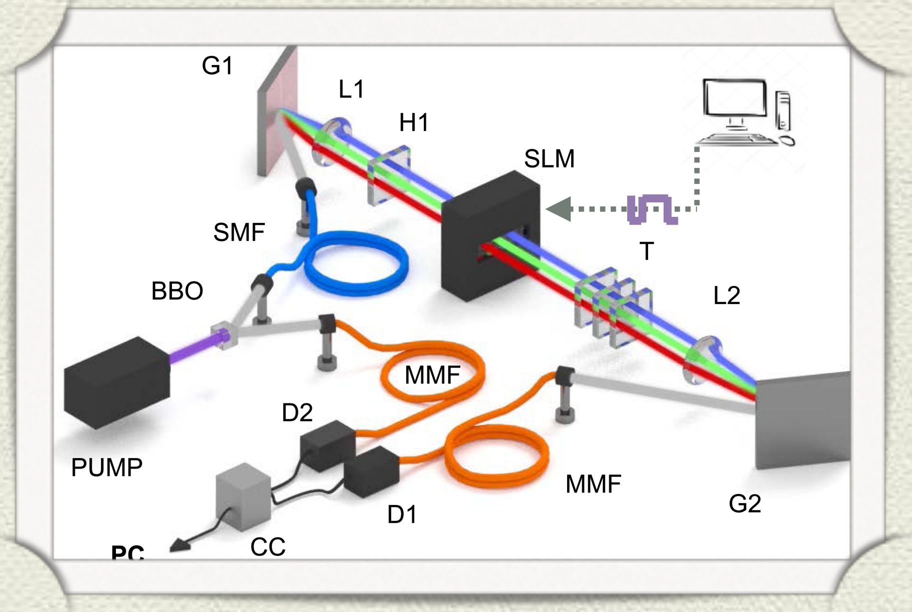
The collection through the multimode fiber performs the ensemble average

diode **pump** laser @405.5nm using a **BBO** crystal (1mm thick); **SMF**: single-spatial-mode and polarization preserving fiber; **MMF**: multimode fiber; **G1-G2**: gratings (1714 lines/mm); **L1-L2**: lens(f=500mm); **H1**, half-wave-plate; **SLM**: spatial light modulator (640 pixels, 100 μm/pixel); **T**, tomographic apparatus; **Q**:quarter-wave plate; **P**, polarizer; **C**, optical coupler; **D1-D2**: single photon detectors; **CC**: coincidences counter. The acquisition time is of 10s for each measure of coincidence counts. The inset shows the measured PDC spectrum.

$$|\psi(t)\rangle = \frac{1}{2}\left(e^{-2i\phi_r(t)}|H\rangle + |V\rangle\right)$$

APPLIED PHYSICS LETTERS 110, 081107 (2017)

All optical quantum simulator of qubit noisy channels



$$|x\rangle = |\omega(x)\rangle$$ the spectral components are spatially dispersed

$$|x\rangle = \sum_r \eta_r(x)\,|\eta_r\rangle \qquad \sum_r |\eta_r\rangle\langle\eta_r| = I$$

r^{th} pixel

$$|\eta_r(x)|^2$$ Probability that the component x passes through the r-th pixel

$$U(t) = \exp\left[-2i\,|H\rangle\langle H| \otimes \sum_r \phi_r(t)\,|\eta_r\rangle\langle\eta_r|\right]$$

$$U(t)\,|H\rangle \otimes |\eta_r\rangle = e^{-2i\phi_r(t)}\,|H\rangle \otimes |\eta_r\rangle$$

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All optical quantum simulator of qubit noisy channels

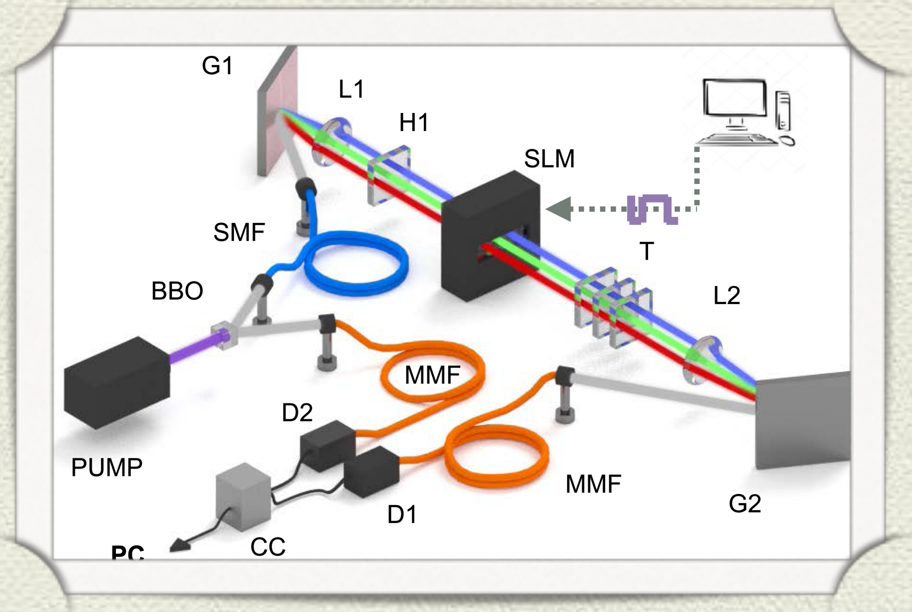

Fig. 4.1 Schematic diagram of our experimental setup. Pump, 405.5 nm laser diode; BBO, Beta barium borate nonlinear crystal; SMF, single-spatial-mode and polarization preserving fiber; MMF, multimode fiber; G1–G2, gratings; L1–L2, lens; H1, half-wave-plate; SLM, spatial light modulator; T, tomographic apparatus; D1–D2, single photon detectors; CC, coincidences counter

$$\rho_s(t) = \frac{1}{2}\sum_r A_{rr}\begin{pmatrix} 1 & e^{-2i\phi_r(t)} \\ e^{2i\phi_r(t)} & 1 \end{pmatrix}$$

$$A_{rr} = \int dx\, |f(x)|^2\, |\eta_r(x)|^2 \simeq \frac{1}{n}$$

$$C(t) = \frac{1}{n}\sum_r e^{-2i\phi_r(t)}$$

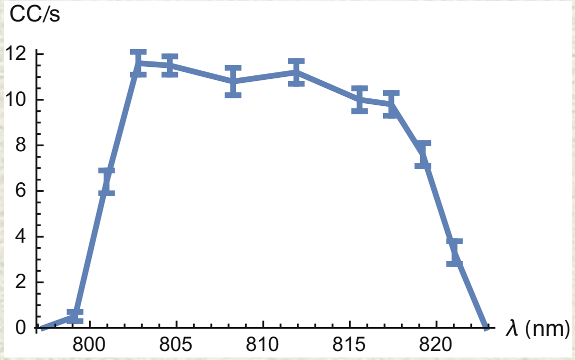

Fig. 4.2 The measured spectrum of the PDC. We can see that it is almost flat in the region 802–817 nm

Due to imperfections in the apparatus, the state becomes

$$\rho_s^{exp}(t) = p\,\rho_s(t) + (1-p)\rho_{mix}$$

$$\rho_{mix} = \frac{1}{2}\big(|H\rangle\langle H| + |V\rangle\langle V|\big)$$

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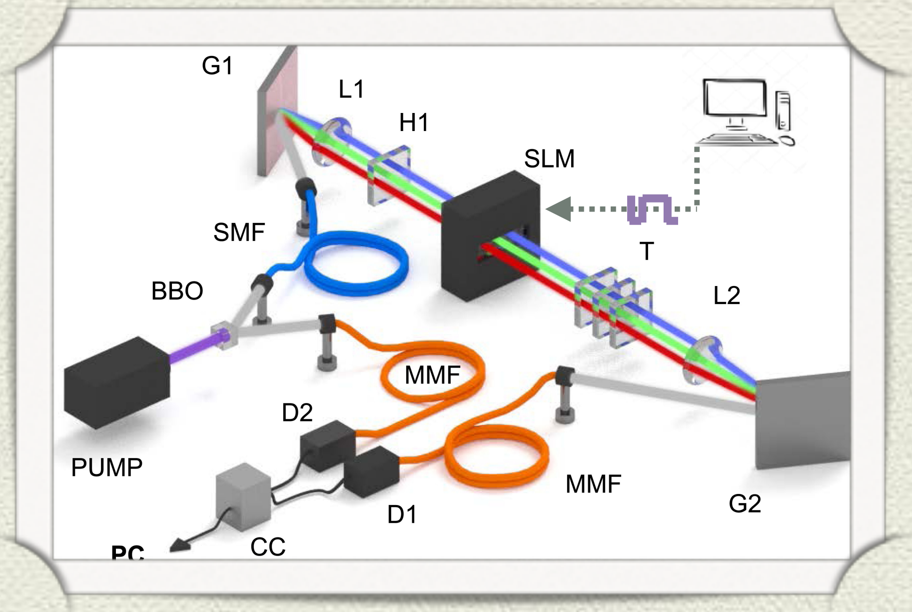

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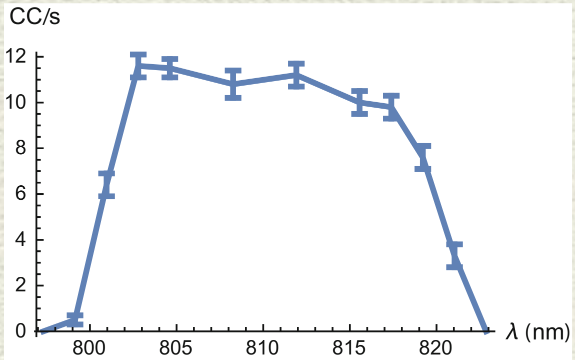

Fig. 4.2 The measured spectrum of the PDC. We can see that it is almost flat in the region 802–817 nm

$$C(t) = \frac{1}{n} \sum_r e^{-2i\phi_r(t)}$$

The relevant quantity to be measured is

$$\langle H | \rho_S^{exp}(t) | V \rangle = \frac{1}{2} p \langle e^{-2i\phi_r(t)} \rangle_n$$

No need for full tomography. Just one projective measurements

$$\langle + | \rho_S^{exp} | + \rangle = \frac{1}{2} \left(1 + p\, \Re \langle e^{-2i\phi_r(t)} \rangle_n \right)$$

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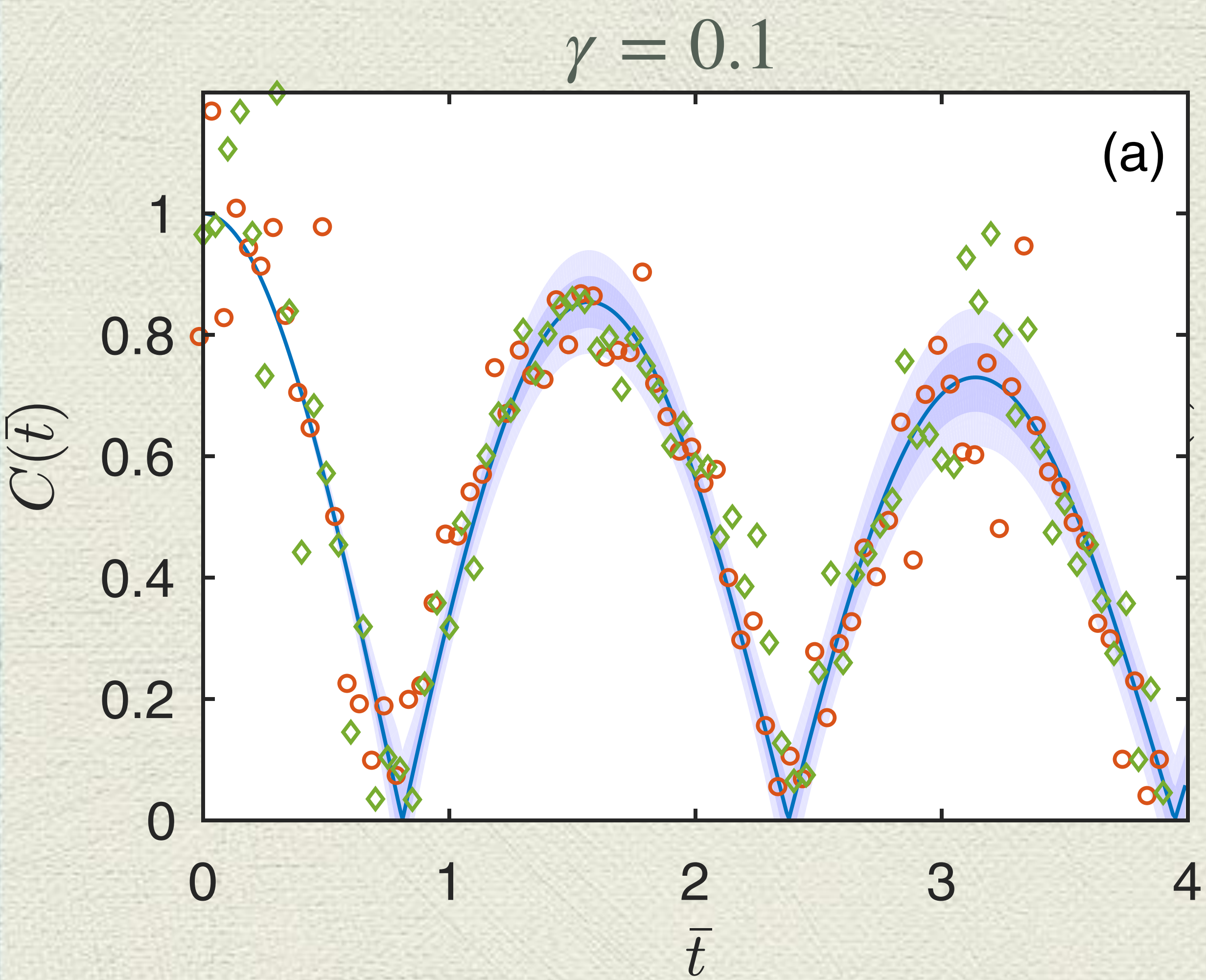
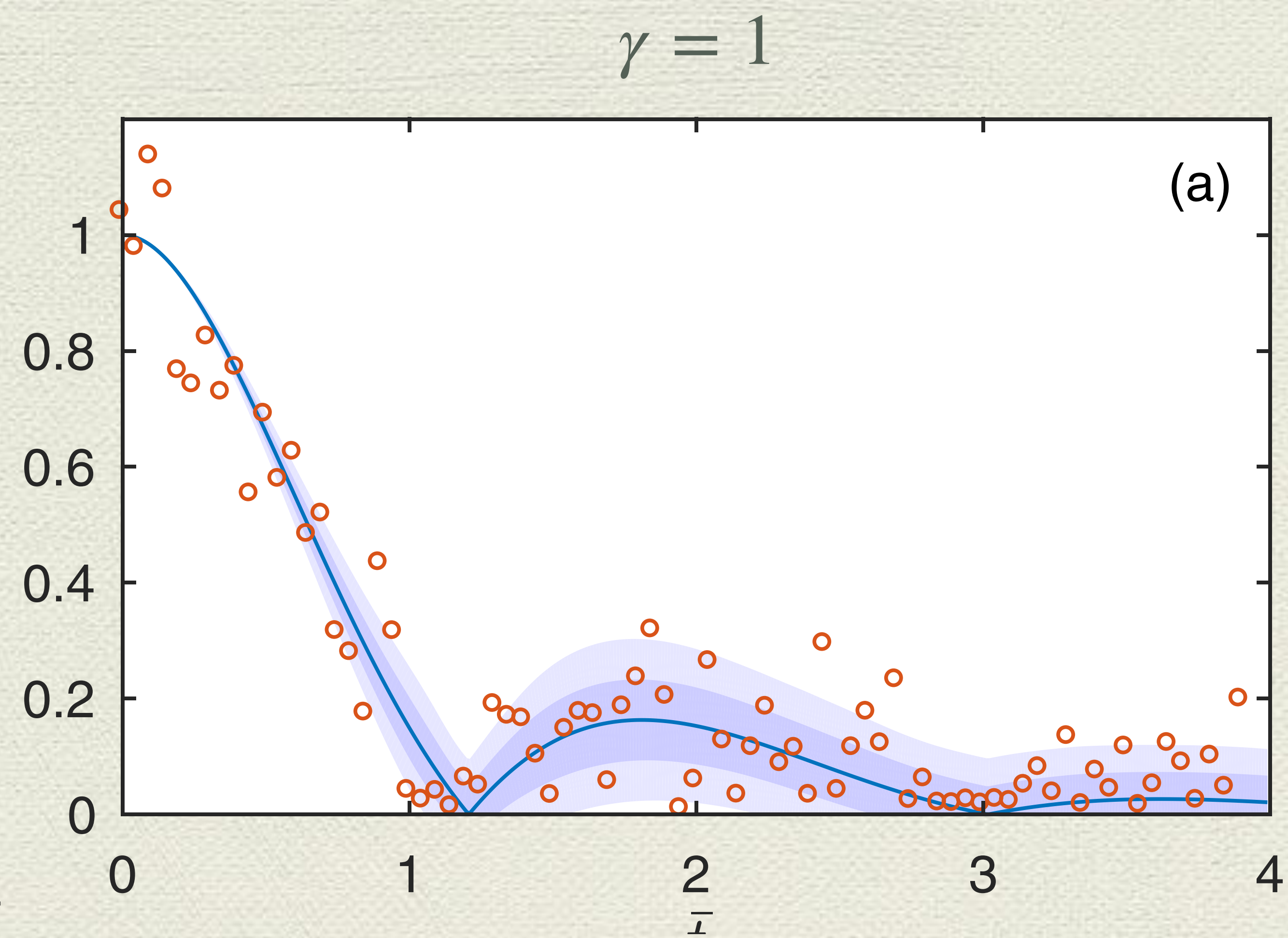
All optical quantum simulator of qubit noisy channels





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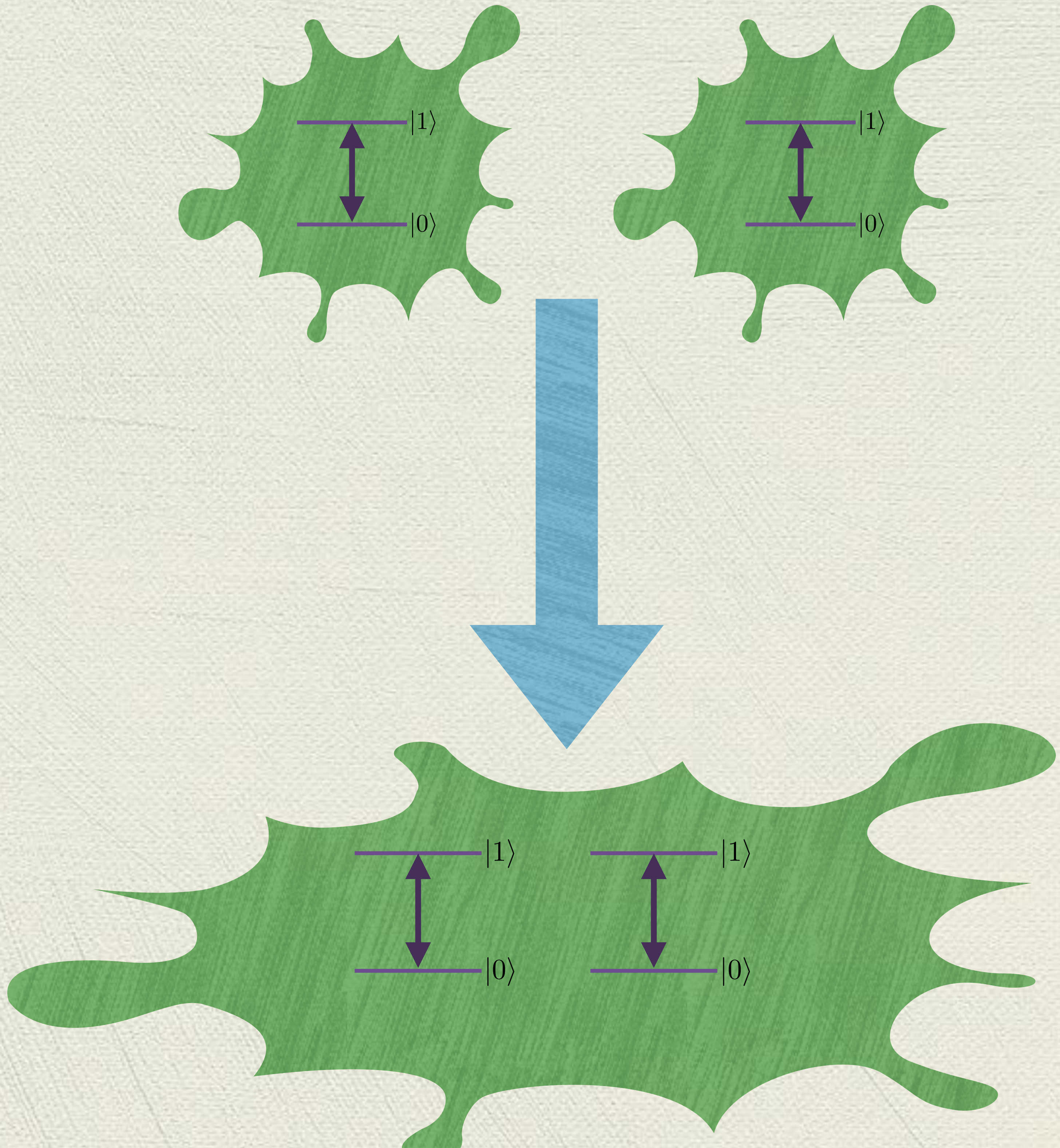
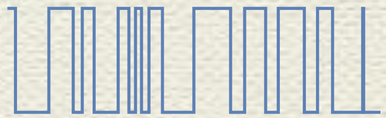
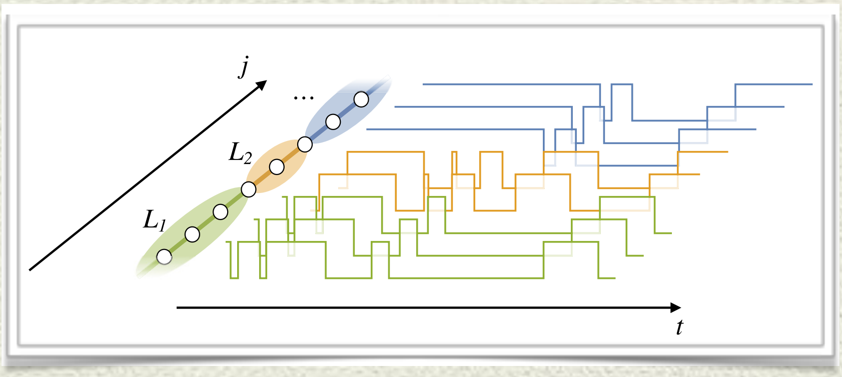
Work in progress: 2 qubits simulator

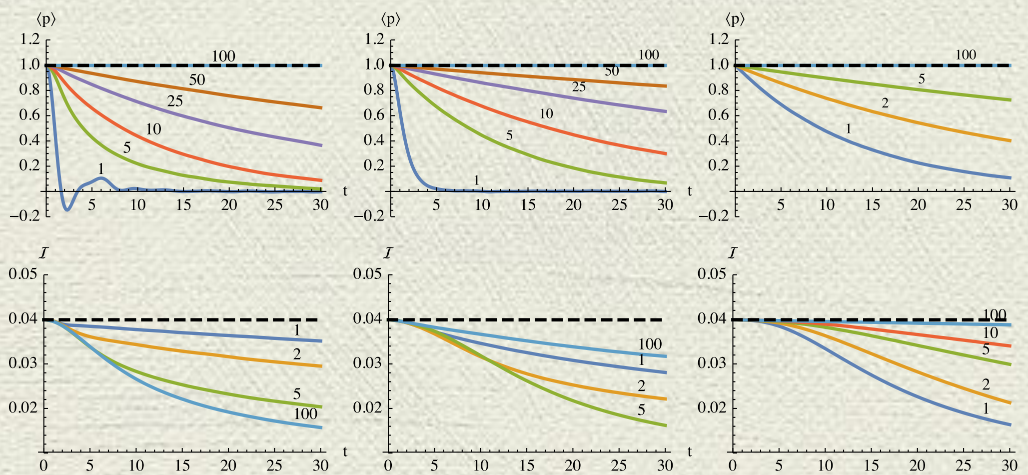
Study the transition local/global environment

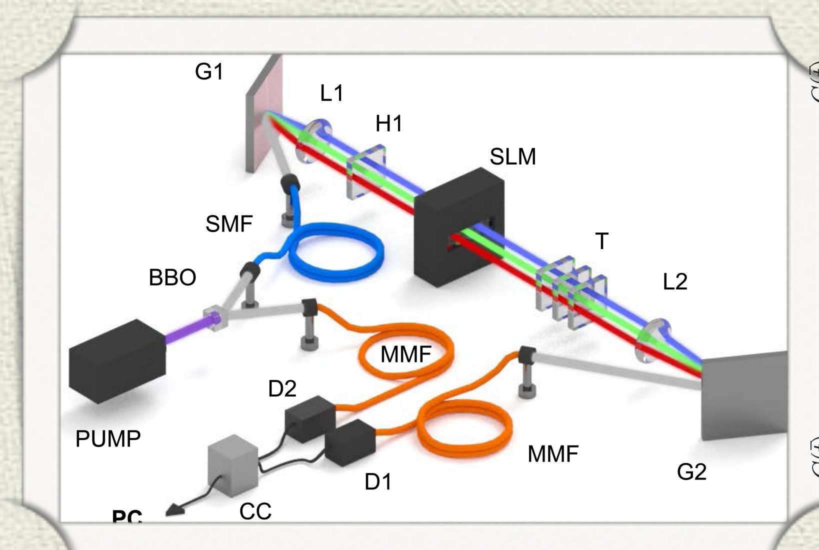


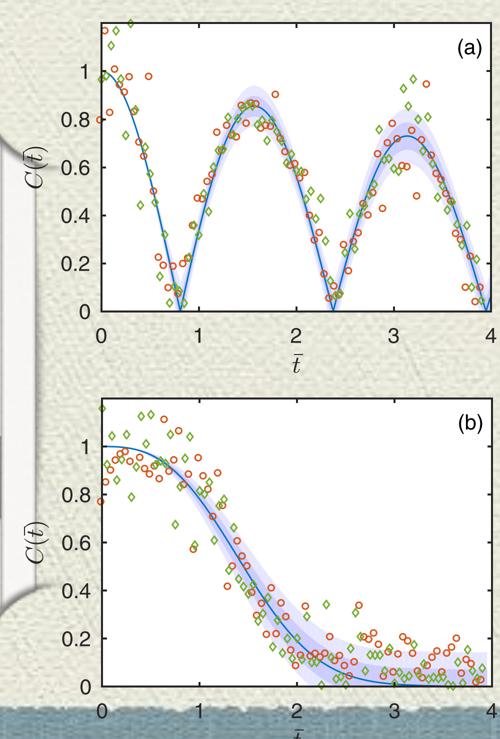
Conclusions



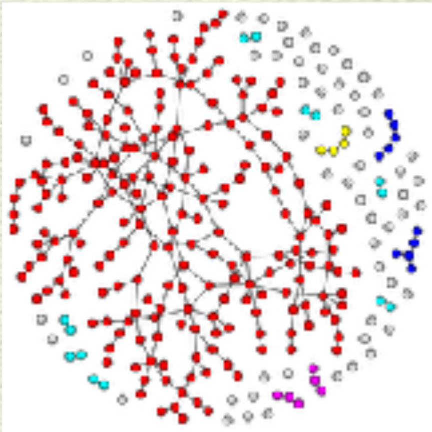


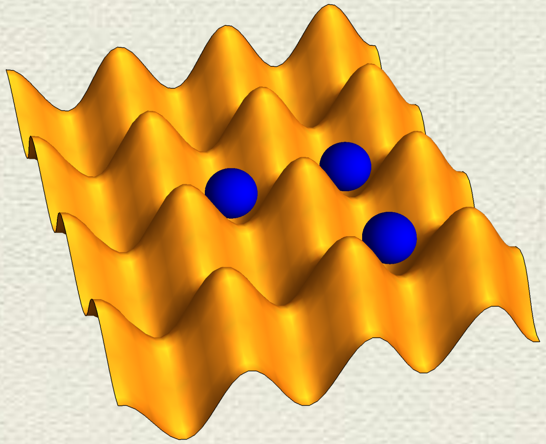






What's next?





Thank you!

Unrelated question: anybody expert in Spatial search algorithms by continuous-time quantum walks?