Many-body generalization of the Z2 topological invariant for the quantum spin Hall effect

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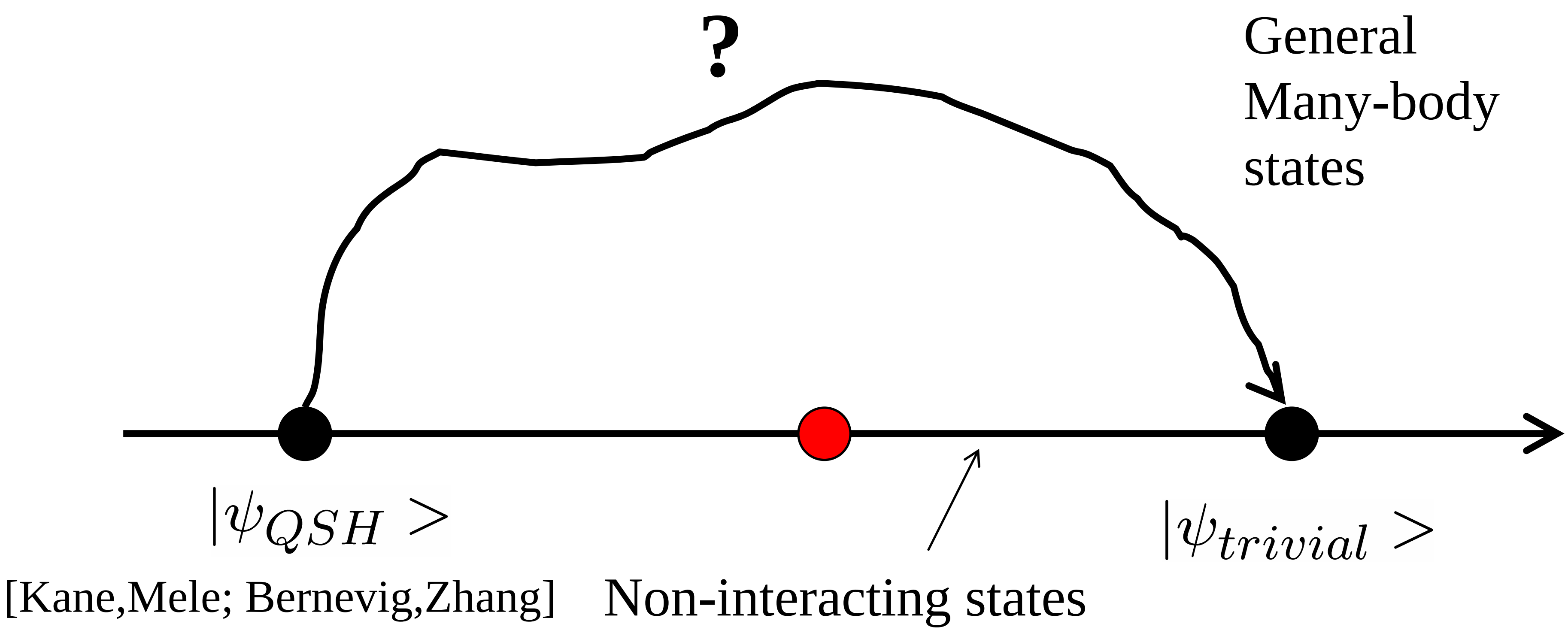
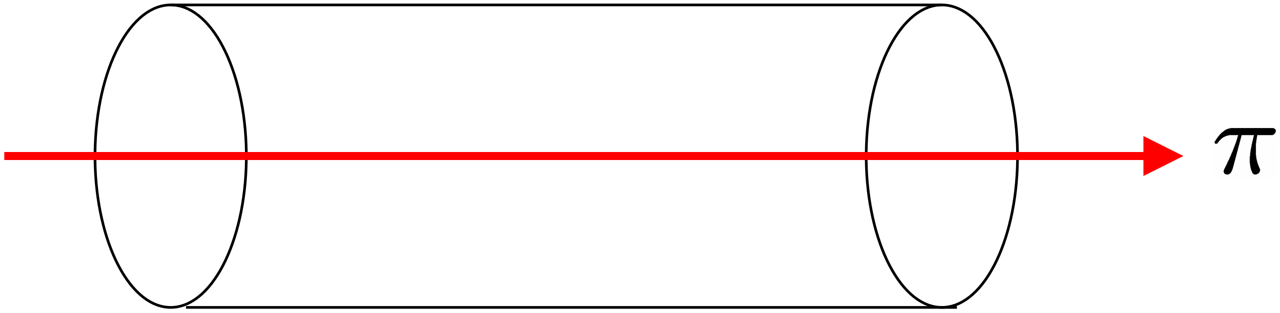
Is the QSH state really distinct from trivial insulators ?

Is there no adiabatic path that connects a QSH state and a trivial state while preserving N, T?



Kramers pair with π-flux



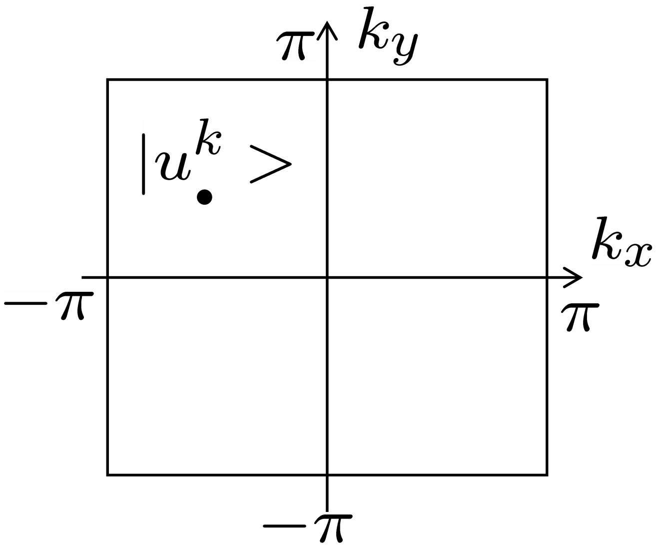
[Fu and Kane(06), Ran, Vishwanath and Lee(08), Qi and Zhang(08)]

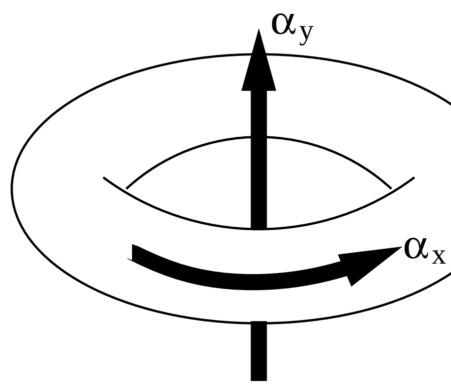
Property of excitation

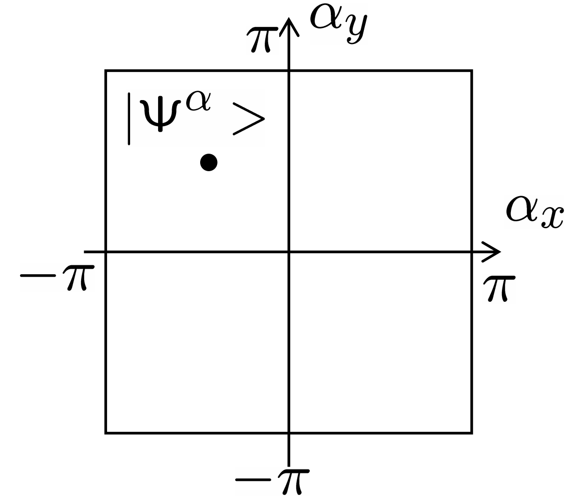
Ground state property ?

Magnetoelectric polarization in 3d [Essin, Moore and Vanderbilt(08)]

Topological order in IQHE







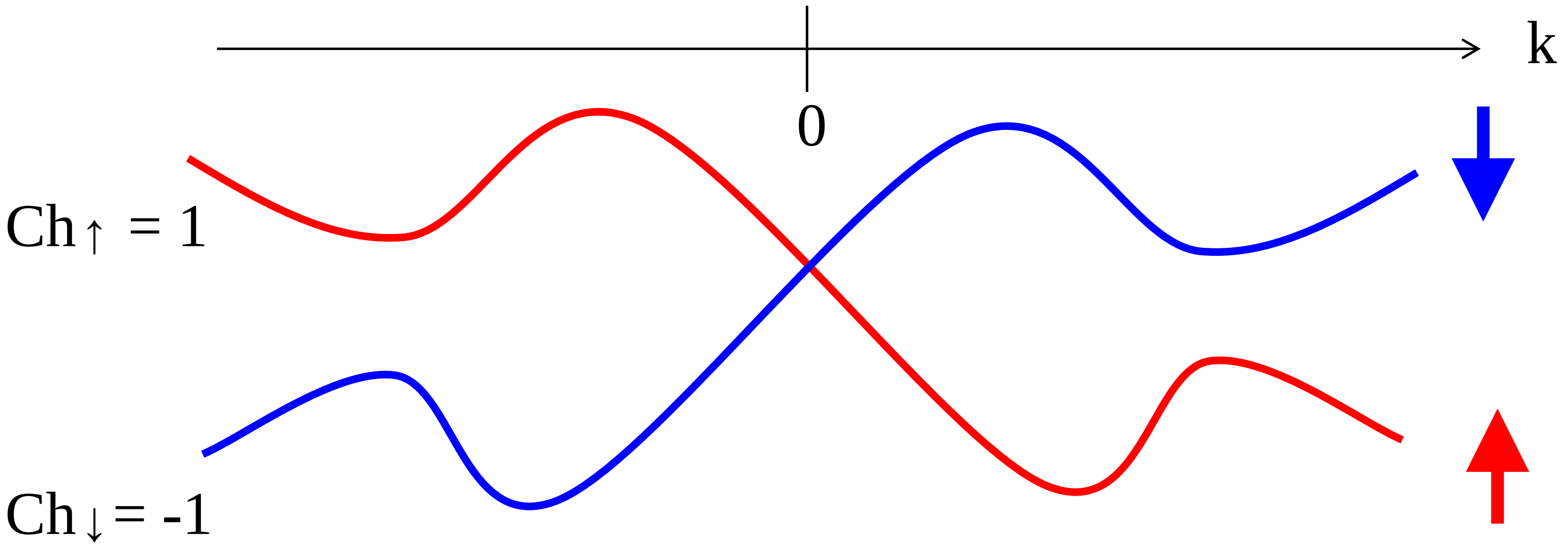
$$i\int d^2k < \nabla u^k| \times |\nabla u^k >$$

$$i\int d^2\alpha < \nabla \psi^\alpha| \times |\nabla \psi^\alpha >$$

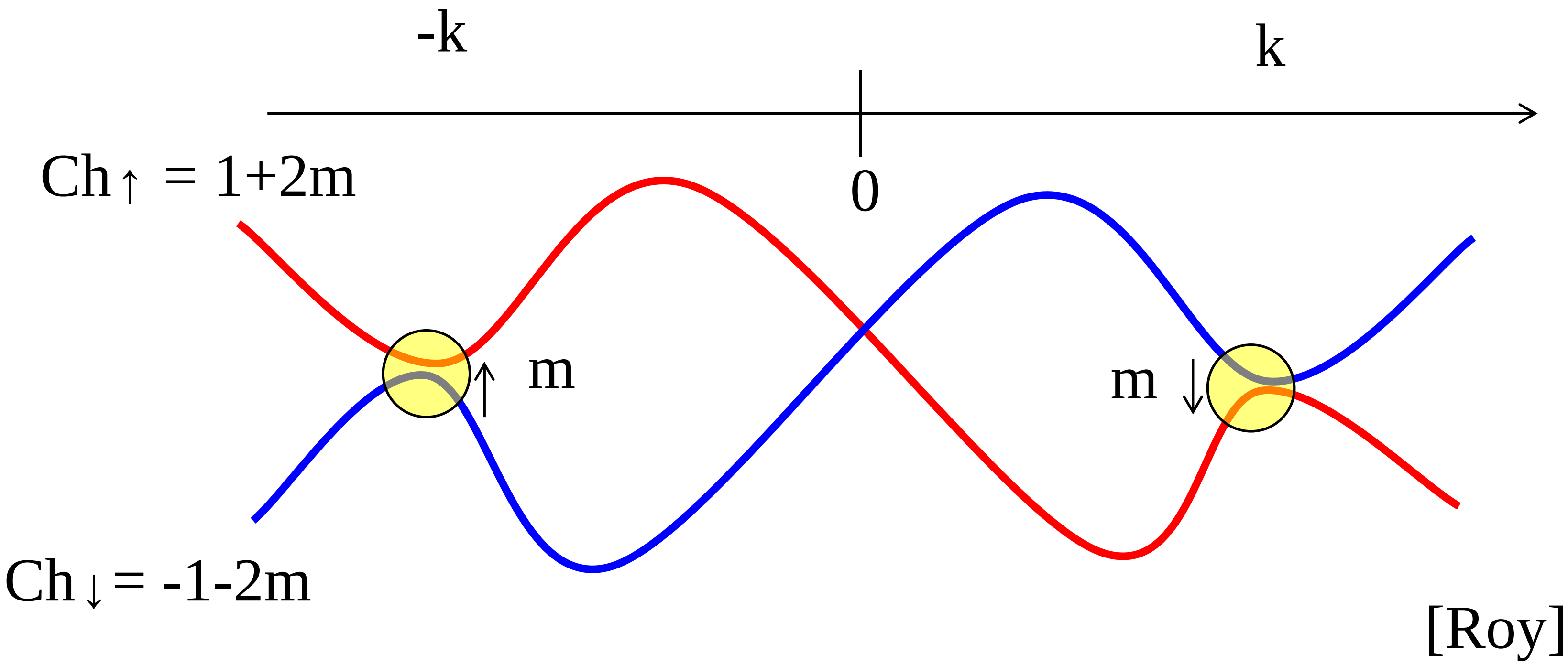
[Thouless,Kohmoto,Nightingale,M.den Nijs (82)]

[Niu, Thouless, Wu (85)]

Non-interacting QSH state
Sz conserved case :
Chern number of each band



Non-interacting QSH state
Sz non-conserved case :
Z2 invariant

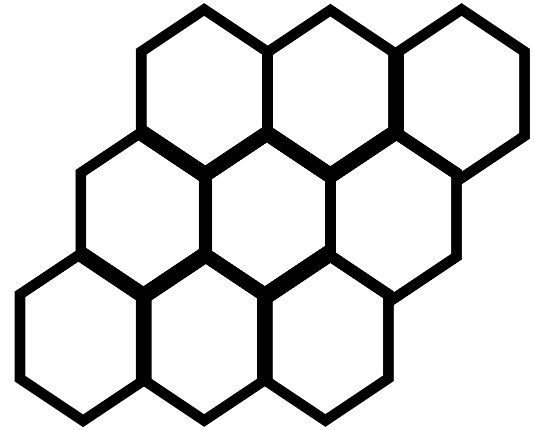


Chern number in each band changes by even numbers :
even/odd parity of the Chern number is conserved

[Kane and Mele, Balents and Moore, Fu and Kane]

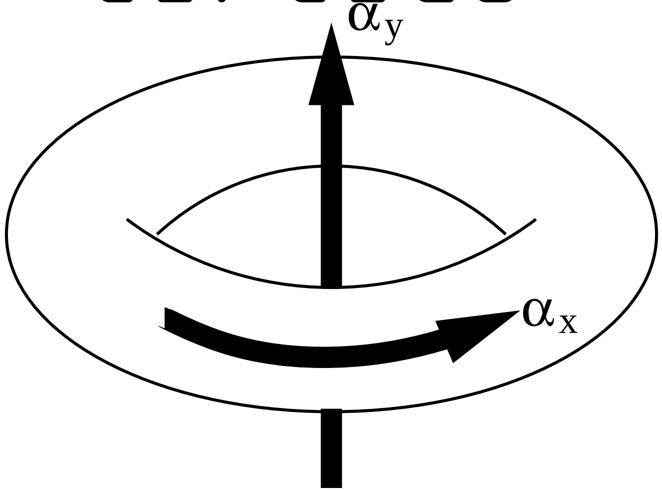
Generalization of the Z2 invariant to interacting systems

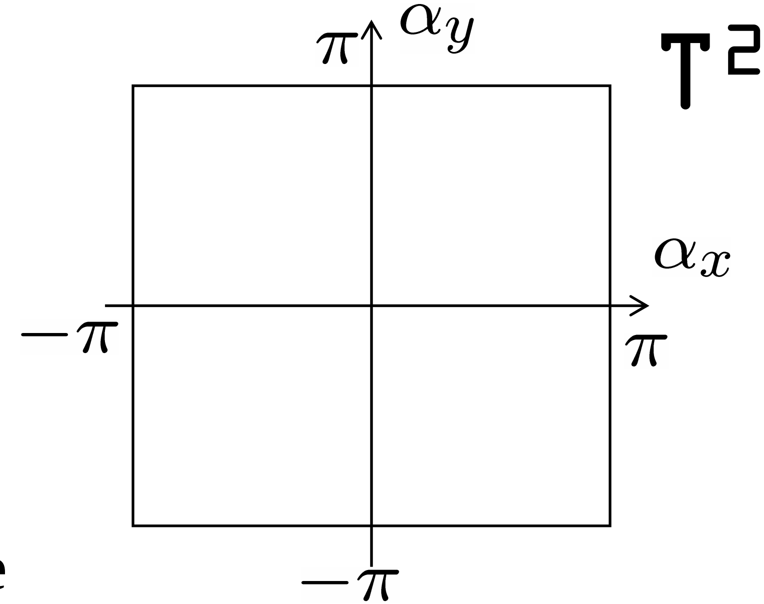
in N=(odd)x(odd) periodic lattice (torus)



[Kane and Mele (05)]

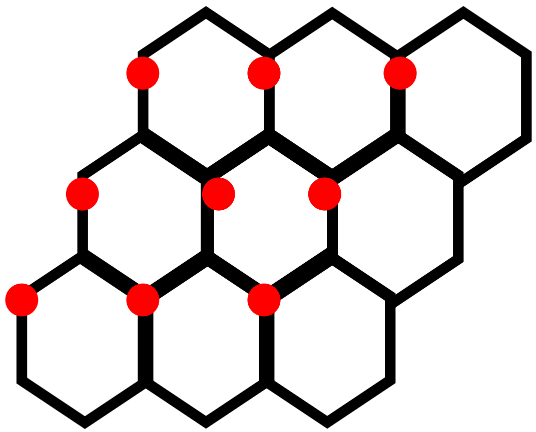
Thread fluxes along two circles





$|\Psi^{\alpha}>$ 2N-particle ground state

Many-body Kramers doublet

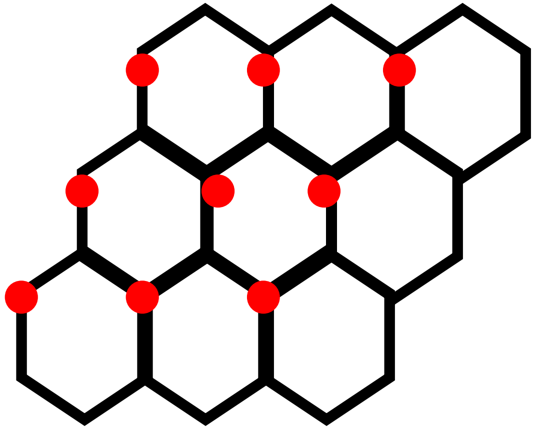


$|\Psi^{\alpha}>$ 2N-particle ground state

N (odd)-particle states

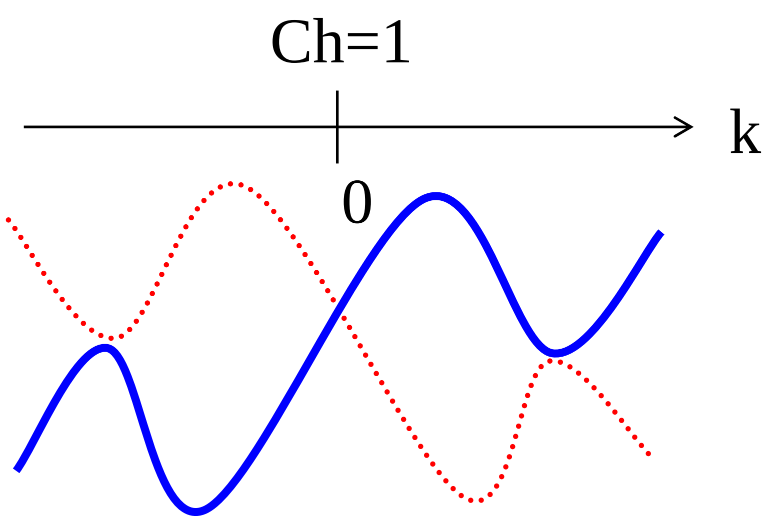
$$|\uparrow(\alpha)> \; = \; \Pi_{i=1}^{N} c_{r_i\uparrow}|\Psi^{\alpha}>$$
$$|\downarrow(\alpha)> \; = \; \Pi_{i=1}^{N} c_{r_i\downarrow}|\Psi^{\alpha}>$$

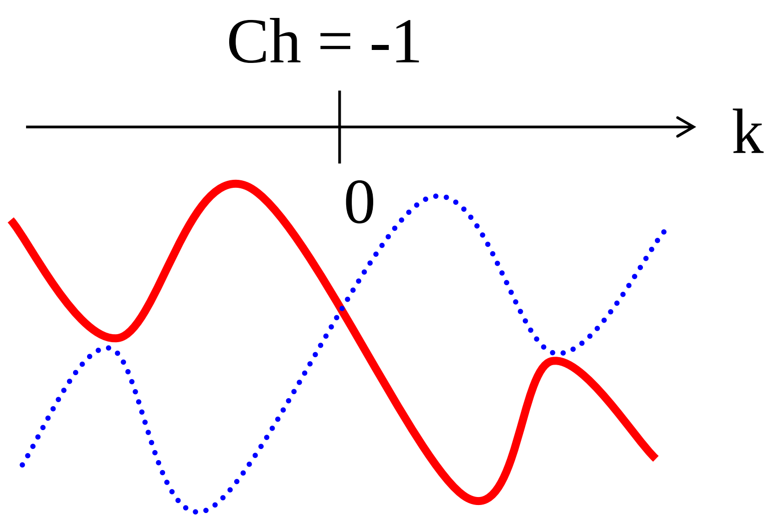
Many-body Kramers' doublet



$$|\uparrow(\alpha)> \;=\; \Pi_{i=1}^{N} c_{r_i\uparrow}|\Psi^\alpha>$$
$$|\downarrow(\alpha)> \;=\; \Pi_{i=1}^{N} c_{r_i\downarrow}|\Psi^\alpha>$$

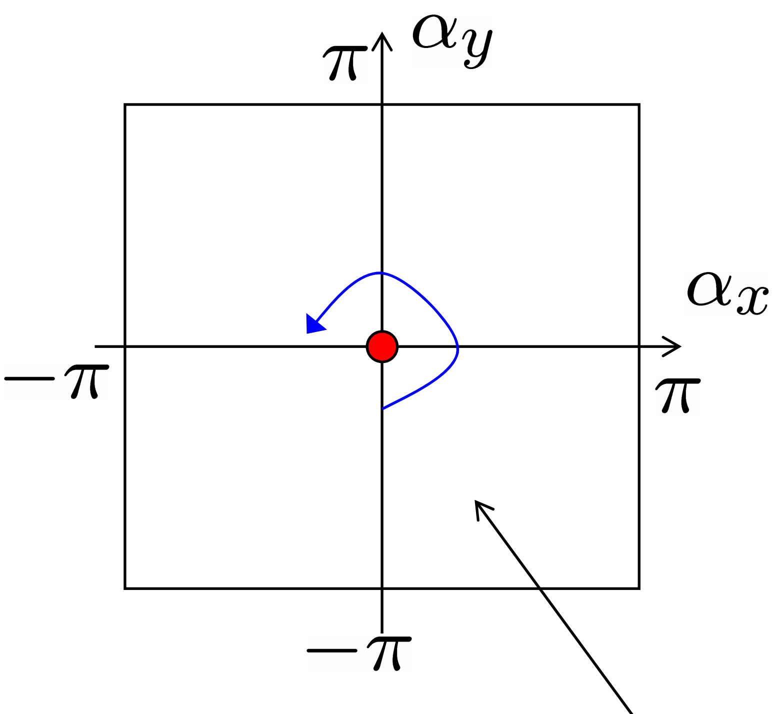
In the non-interacting and Sz conserved limit :





2-d Hilbert space in T^2

$$P(\alpha) = \frac{|\uparrow(\alpha)\rangle\langle\uparrow(\alpha)|}{\langle\uparrow(\alpha)|\uparrow(\alpha)\rangle} + \frac{|\downarrow(\alpha)\rangle\langle\downarrow(\alpha)|}{\langle\downarrow(\alpha)|\downarrow(\alpha)\rangle}$$



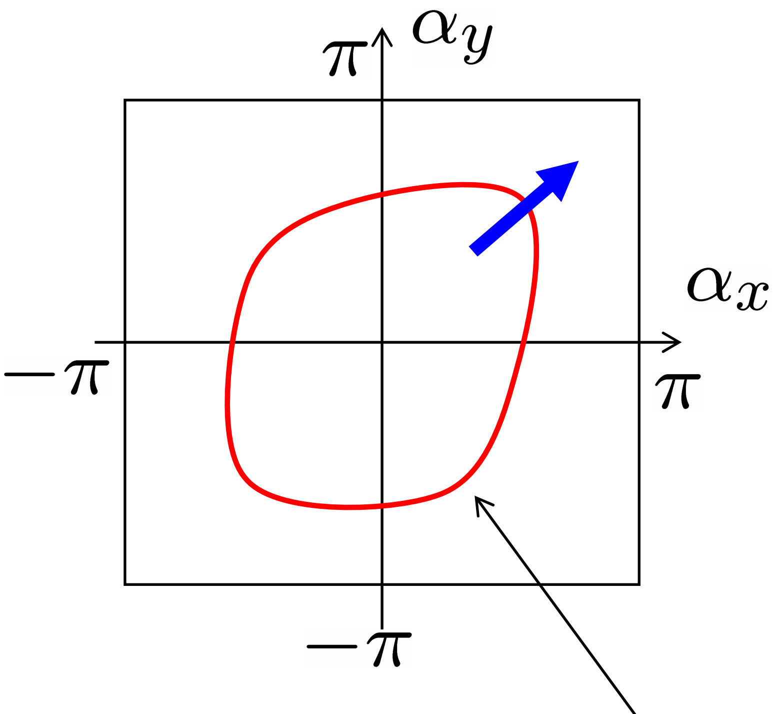
$$|\uparrow(\alpha)> = (\alpha_x + i\alpha_y)|\uparrow>$$

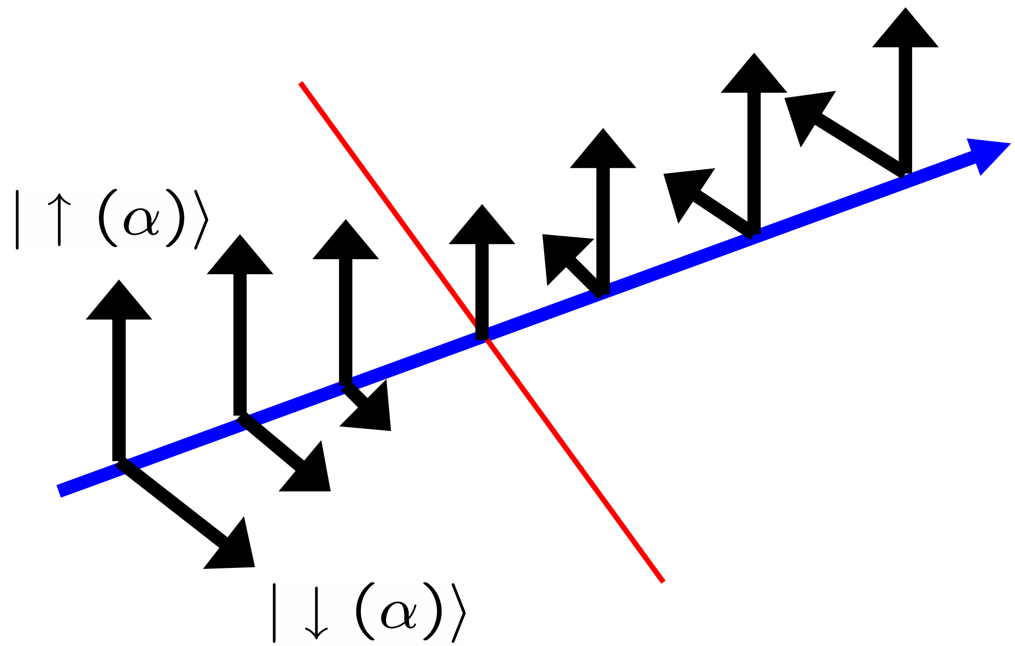
$$|\downarrow(\alpha)> = (\alpha_x - i\alpha_y)|\downarrow>$$

$$d(\alpha) = det \begin{vmatrix} <\uparrow(\alpha)|\uparrow(\alpha)> & <\uparrow(\alpha)|\downarrow(\alpha)> \\ <\downarrow(\alpha)|\uparrow(\alpha)> & <\downarrow(\alpha)|\downarrow(\alpha)> \end{vmatrix} = 0$$

2-d Hilbert space in T^2

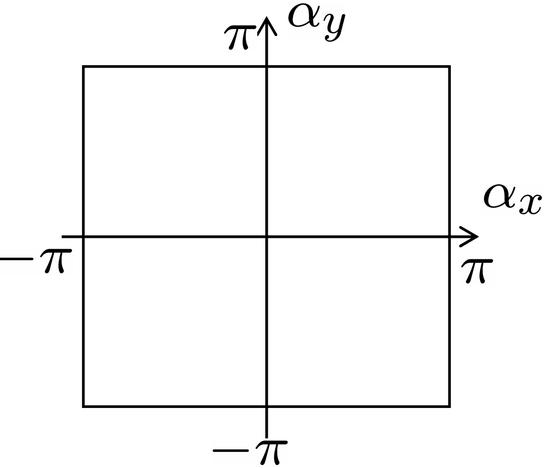
$$P(\alpha) = \frac{|\uparrow(\alpha)\rangle\langle\uparrow(\alpha)|}{\langle\uparrow(\alpha)|\uparrow(\alpha)\rangle} + \frac{|\downarrow(\alpha)\rangle\langle\downarrow(\alpha)|}{\langle\downarrow(\alpha)|\downarrow(\alpha)\rangle}$$





$$d(\alpha) = det \begin{vmatrix} <\uparrow(\alpha)|\uparrow(\alpha)> & <\uparrow(\alpha)|\downarrow(\alpha)> \\ <\downarrow(\alpha)|\uparrow(\alpha)> & <\downarrow(\alpha)|\downarrow(\alpha)> \end{vmatrix} = 0$$

SU(2) Berry gauge field

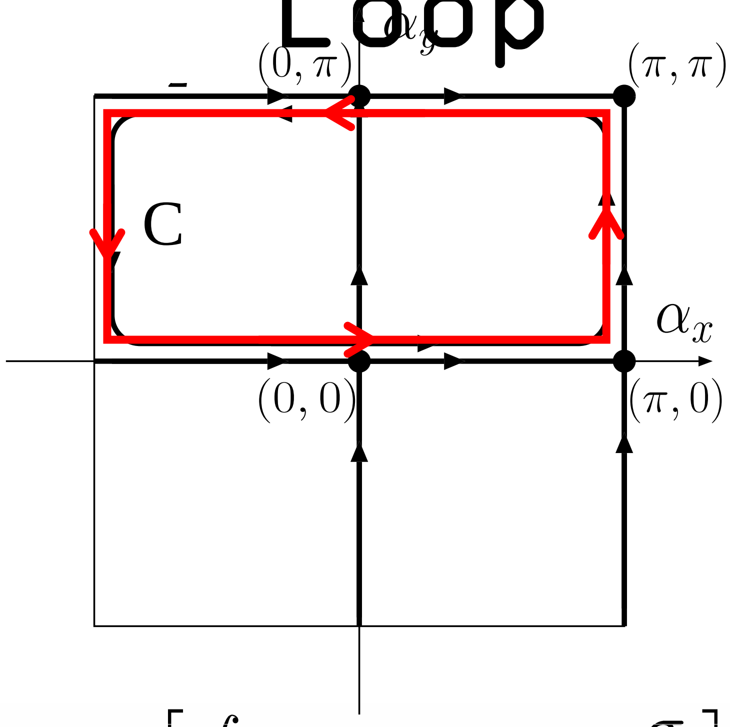


$$A_\mu^{nm}(\alpha) \;=\; \langle n(\alpha)|\frac{\partial}{\partial\alpha^\mu}|m(\alpha)\rangle.$$

$$v(-\alpha)w(\alpha) = \Theta v(\alpha)$$

$$A_\mu(-\alpha) \;=\; w(\alpha)A_\mu^T(\alpha)w^\dagger(\alpha) - w(\alpha)\partial_\mu w^\dagger(\alpha)$$

Quantized SU(2) Wilson Loop



$w(\alpha) = \pm i\sigma_2$

@ TRS points

$$W[C] = \frac{1}{2}\mathrm{Tr}\, P \exp\left[\oint_C d\alpha^\mu \mathbf{a}_\mu(\alpha)\cdot\frac{\sigma}{2i}\right]$$
$$= \mathrm{Pf}\left[\tilde{w}(0,0)\right]\mathrm{Pf}\left[\tilde{w}(\pi,0)\right]\mathrm{Pf}\left[\tilde{w}(\pi,\pi)\right]\mathrm{Pf}\left[\tilde{w}(0,\pi)\right]$$

= 1 for trivial insulator

= -1 for topological insualtor

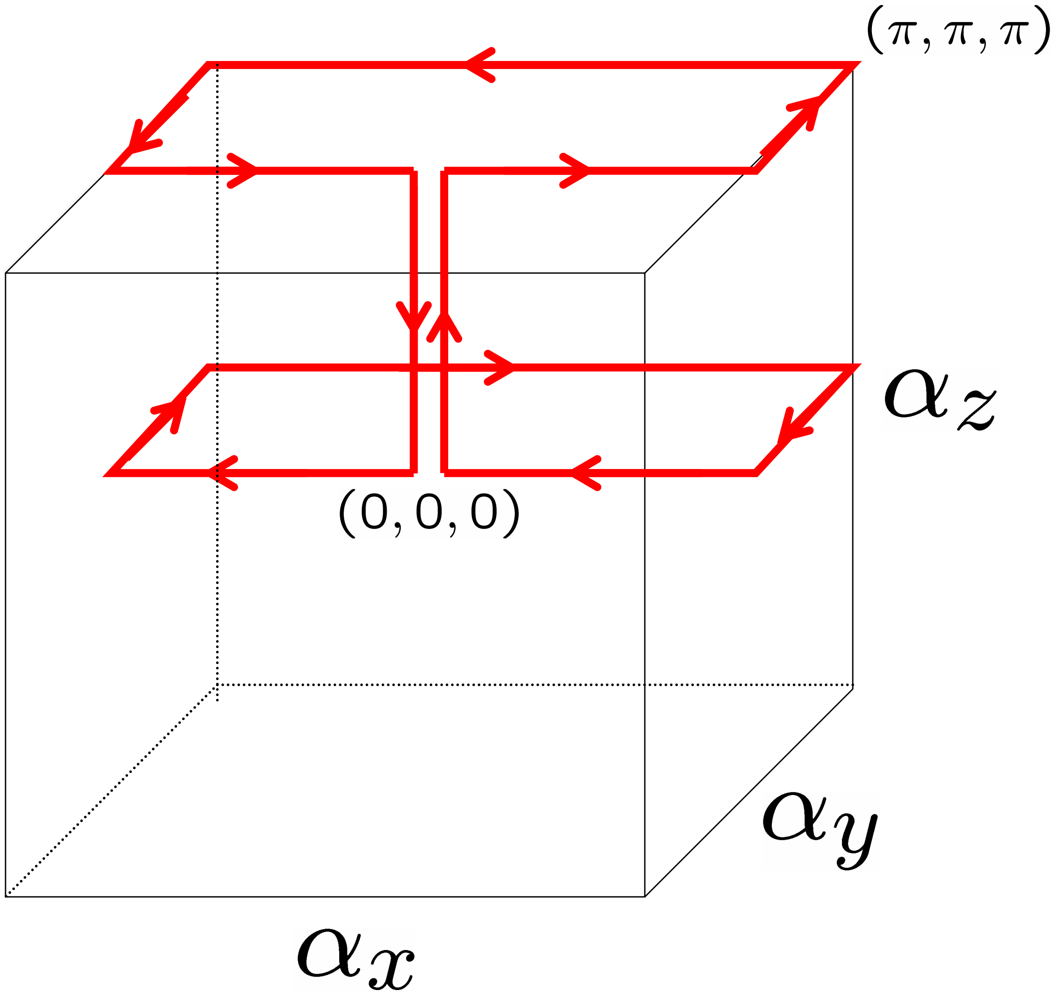
Z2 invariant is well defined except for

- N or T is broken
- Collapse of bulk gap (phase transition)

Any two states which can be adiabatically connected in the N,T preserving Hilbert space should have the same Z2 number

Any two states which have different Z2 number can not be adiabatically connected in the N,T preserving Hilbert space

3d case



$$W[C] \;=\; \frac{1}{2}\mathrm{Tr}\,P\exp\left[\oint_C d\alpha^\mu \mathbf{a}_\mu(\alpha)\cdot\frac{\sigma}{2i}\right]$$

Summary

- Z2 invariant can be generalized to the cases with many-body interactions

- \`Topological insulator’ can not be adiabatically connected to trivial insulators without going through a phase transition

Ref : SL and S. Ryu, PRL **100** 186807 (‘08)

Discussion

- Dependence of Z2 invariant on the way the Kramers doublet is constructed; more than one Z2 invariants ?

- Is there a many-body topological insulator which is distinct from non-interacting topological insulators ?